Simulating interacting multiple natural-hazard events for lifecycle consequence analysis

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ABSTRACT: Among different types of natural-hazard interactions (simply multi-hazard interactions hereinafter), some occur through the nature of the hazards themselves, regardless of the presence of any physical assets: they are often called "Level I" (or occurrence) interactions. In such cases, one hazard event triggers or modifies the occurrence of another (e.g., severe wind and flooding; liquefaction and landslides triggered by an earthquake), thus creating a dependency between the parameters characterising such hazard events. They differ from "Level II" (or consequence) interactions, which instead occur through impacts/consequences on physical assets/components and systems (e.g., accumulation of physical damage or social impact due to earthquake sequences, landslides due to the earthquake-induced collapse of a retaining structure). Multi-hazard Life Cycle Analysis (LCA) aims to quantify the consequences (e.g., repair costs, downtime, and casualty rates) expected throughout a system's service life, accounting for both Level I and Level II interactions. Nevertheless, the available literature generally considers these interactions mainly defining relevant taxonomies, often qualitatively, without providing a computational framework to simulate a sequence of hazard events in terms of their occurrence times and features and resulting consequences. This paper aims to partly fill this gap by identifying modelling approaches associated with different Level I interactions. It describes a simulation-based approach for generating multi-hazard scenarios (i.e., a sequence of hazard events and associated features through the system's life cycle) based on the theory of competing Poisson processes. The proposed approach incorporates the different types of interactions in a sequential Monte Carlo sampling method. The method outputs potential sequences of events throughout a system's life cycle, which can be integrated into LCA frameworks to quantify interacting hazard consequences. A simple application is presented to illustrate the potential of the proposed method.

1. INTRODUCTION

Risk modelling and quantification for multiple natural hazards (or multi-hazard risk) cannot generally be regarded as the superposition of approaches for individual hazard events. Instead, multi-hazard risk modelling and quantification approaches must account for the interactions among different hazard events and corresponding impacts/consequences. Significant effort has been put into establishing a common nomenclature for multi-hazard risk analysis (e.g., Han et al. 2007, Kappes et al. 2012) to improve communication among various end users. Most past studies recognise that a realistic assessment of multi-hazard impacts should separate occurrence interactions (which ignore the physical

assets/components affected by the hazard events) from consequence interactions (which happen through the physical assets/components). Zaghi et al. (2016) denoted the former as Level I interactions and the latter as Level II interactions. Level I interactions occur because multiple hazard types are dependent in terms of frequency/characteristics or because the occurrence of one hazard type triggers or intensifies another one. An extensive review of Level I interactions has been conducted by Gills & Malamud (2014), which classified interactions based on the physical relationships between their occurrences and investigated several hazard types to determine which ones could trigger or intensify others. The works mentioned above generally classify interactions from a qualitative perspective without providing the computational tools to integrate them into a simulation-based framework for risk modelling and quantification. As such, the problem of simulating sequences of events that account for the identified interactions remains mostly unexplored. There have been a few studies attempting to address such a task. However, they are either limited in scope (e.g., site-specific and scenario-based studies such as Adachi & Ellingwood 2008 and Marzocchi et al. 2012) or consider all types of interactions in the same way, disregarding their specific characteristics (Mignan et al. 2014).

In this paper, we propose a simulation-based procedure to generate sequences of hazard events (in terms of their time of occurrence and features) throughout the life cycle of an engineering system. The proposed approach considers all possible types of Level I interactions identified in the literature, each modelled with appropriate methods. We separate concurrent from successive interactions and further distinguish interactions where a primary hazard event immediately triggers a secondary one from interactions where a primary hazard event changes the rate of occurrence of a secondary hazard type. The proposed simulation method assumes that hazard events act as competing homogeneous Poisson point processes (simply Poisson processes

hereinafter). While limiting, this assumption can be easily circumvented (i.e., by transforming non-homogeneous Poisson processes into equivalent homogeneous Poisson processes; e.g., Westcott 1977) and allows for efficient simulations of hazard scenarios. Those can then be incorporated into frameworks for Level II interactions to quantify consequences for Life Cycle Analysis (LCA) purposes.

2. METHODOLOGY

Each considered hazard type is associated with certain *event characteristics* (e.g., rupture characteristics and magnitude in the case of earthquakes) that do not depend on a specific location (or site) of interest. We denote the curves relating the event characteristics to their corresponding occurrence/exceedance rates as *event curves* (e.g., magnitude-frequency distributions in seismic hazard analysis or intensity-duration-frequency in flood hazard modelling). The event characteristics are typically translated into relevant (local) *intensity measures* that quantify the location-/site-specific effects and the corresponding physical impacts caused by the event. Appropriate methods for local-intensity calculation can be found in the literature and typically depend on the hazard type considered. For example, Ground Motion Models (GMMs) (Douglas and Edwards 2016) can be used to translate earthquake characteristics into earthquake-induced ground-motion intensity measures such as peak ground parameters and spectral accelerations. These methods can be integrated within a probabilistic framework to obtain curves that relate each intensity measure to its associated frequency of exceedance. Such curves are denoted as *hazard curves* (e.g., the curves for wind speed and surge depth in Apivatanagul et al. 2011). Because both event and hazard curves are used interchangeably in the proposed formulation for the same purpose on a case-by-case basis, we arbitrarily introduce the term *rate curves* to refer collectively to both cases and the term *severity measure* to refer to both event characteristics and intensity measures.

The severity measure associated with the occurrence of the i-th hazard type h_i is denoted as m_i, and the corresponding rate curve is denoted as $\lambda(m_i)$. The rate of occurrence of hazard type h_i can be obtained from the rate curve as

$$\lambda_i = \lambda(m_{i,min}) \qquad (1)$$

where $m_{i,min}$ is the minimum value of interest of the severity measure (e.g., for earthquakes, it could be the minimum magnitude of engineering interest). In other words, the rate of occurrence of h_i is the rate of exceedance of its minimum severity measure. If a hazard type is associated with multiple severity measures, rate surfaces define their joint rate. For example, intensity-duration-frequency surfaces are a standard tool to quantify the mean return period of given rainfall heights and durations (Fadhel et al. 2017). They define the mean return period (reciprocal to the rate) as a function of both severity measures. The rates obtained from these curves/surfaces are then used in event simulation, assuming that the event occurrences follow a homogeneous Poisson process (for example, both the Bartlett-Lewis and the Neyman-Scott models for storm generation rely on this assumption; Ritschel et al. 2017). In homogeneous Poisson processes, the interarrival times between event occurrences of hazard type h_i follow an exponential distribution with parameter λ_i. A critical assumption of such processes is that events occur independently, a somewhat restrictive assumption for specific hazard types that change the underlying conditions of the environment affecting the rate of subsequent events (e.g., rate of aftershocks after a mainshock of a given magnitude and location). To account for such phenomena, the homogeneous Poisson assumption is typically retained after appropriate transformations of non-homogeneous rates into homogeneous ones (e.g., Iervolino et al. 2014, 2022).

2.1. Types of Level I interactions and corresponding input

The proposed methodology accounts for the types of interactions identified in the literature, namely concurrent and successive interactions (e.g.,

Zaghi et al. 2016). *Concurrent interactions* between two or more hazard types can be identified whenever the hazard types/events tend to occur simultaneously and/or to overlap for a period of time (e.g., storm surge, waves, and strong wind that co-occur during a hurricane). In the case of *successive interactions*, instead, a causal relationship exists between a primary hazard type/event and one or more secondary hazard types/events. According to these causal relationships, two broad categories can be identified within successive interactions. We denote as *Type A* the interactions where the secondary hazard type/event (or multiple secondary hazard types/events) is triggered immediately after the occurrence of the primary hazard type (e.g., liquefaction immediately following an earthquake). In contrast, *Type B* interactions are those where the rate of occurrence of the secondary hazard type (or multiple secondary hazard types) increases (or, more in general, changes) following the occurrence of the primary hazard type (e.g., aftershocks following a mainshock). The resulting classification of interactions is a combination of the qualitative classifications proposed by Zaghi et al. (2016) (concurrent vs successive) and Gill and Malamud (2014) (interactions where a hazard event is triggered vs interactions where the probability of a hazard event is increased). Figure 1 shows a portion of the interaction matrix from Zaghi et al. (2016), which includes flood (F), heavy rain (HR), earthquake (E), and landslide (L). The successive interactions have been further separated as Type A (L→F, F→L, HR→L, E→L, L → L) and Type B (E → E). This further classification is not present in Zaghi et al. (2016), but it is necessary to differentiate Type A and B interactions in the modelling framework proposed here.
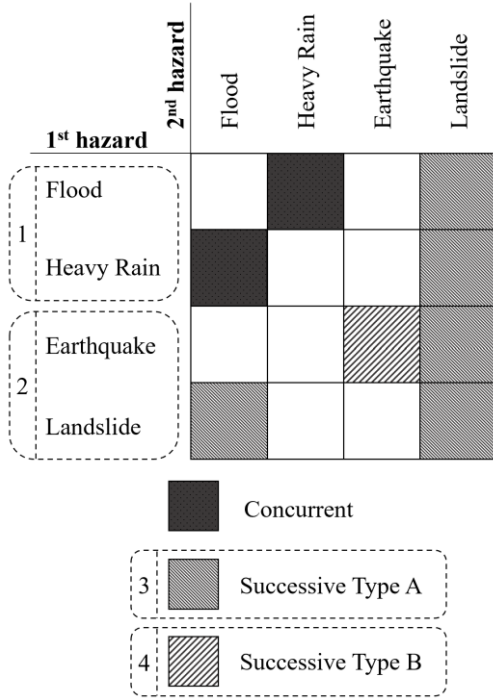
Figure 1: Example of interaction taxonomy

Each type of interaction requires different information to be modelled, provided in the list below. For successive interactions, we provide descriptions for the specific case when h_2 is a secondary hazard type following the occurrence of a primary hazard type h_1. Depending on the identified interactions, any hazard type could be the primary or the secondary hazard. Also, the discussion can be extended to the case with multiple secondary hazard types.

- *Concurrent* interactions are defined by the joint rate of exceedance of the severity measures of all hazard events involved (e.g., the joint exceedance of a given snow depth and a given wind speed, as shown in Wang and Rosowski, 2013). This results in rate surfaces which can be interpreted in the same way as the ones for single hazard types with multiple severity measures.
- *Successive Type A* interactions are defined by the probability of the occurrence of h_2 conditioned on the severity of h_1, i.e. $P(h_2|m_1)$. In cases where the severity measure of h_2 (m_2) is of interest, a conditional probability distribution of such quantity ($f(m_2|m_1)$) is also provided

(conditioned on the severity measure of h_1). An example of this type of interaction can be found in Neri et al. (2008), where the probability of several secondary hazard events, such as floods and landslides, is conditioned on the occurrence of the volcanic eruption of Mount Vesuvius. Neri et al. (2008) also provide the variability in the severity measures associated with secondary hazard events.

- *Successive Type B* interactions are defined by the change in the rate curves of h_2 following the occurrence of h_1. Mainshocks and aftershocks give a classic example of successive type B interactions. Following a mainshock, the rate of aftershocks is typically modified in terms of the characteristics of the mainshock using the modified Omori law (Utsu 1970) (or more advanced models; e.g., Iacoletti et al., 2023). Rates for this type of interaction typically decay with time, resulting in non-homogenous Poisson processes. The non-homogeneous processes can be translated into an equivalent homogeneous process with a given memory mem_i. By taking the inverse of such memory, we can also define a "memory loss rate" $\zeta_i = 1/mem_i$. This rate determines how often the system loses its memory of h_1 and the occurrence rates of h_2 go back to the original level. We call rate curves not affected by Type B interactions *original rate curves* and rate curves affected by Type B interactions *conditional rate curves*.

We incorporate the above information into a sequential Monte Carlo (MC) simulation method based on a Poisson process with state-dependent rates. The procedure outputs life cycle hazard scenarios, i.e. sequences of hazard events (i.e., time of occurrence and associated severity measure) throughout the life cycle of the system of interest. Figure 2 summarises the input required to apply the proposed formulation.
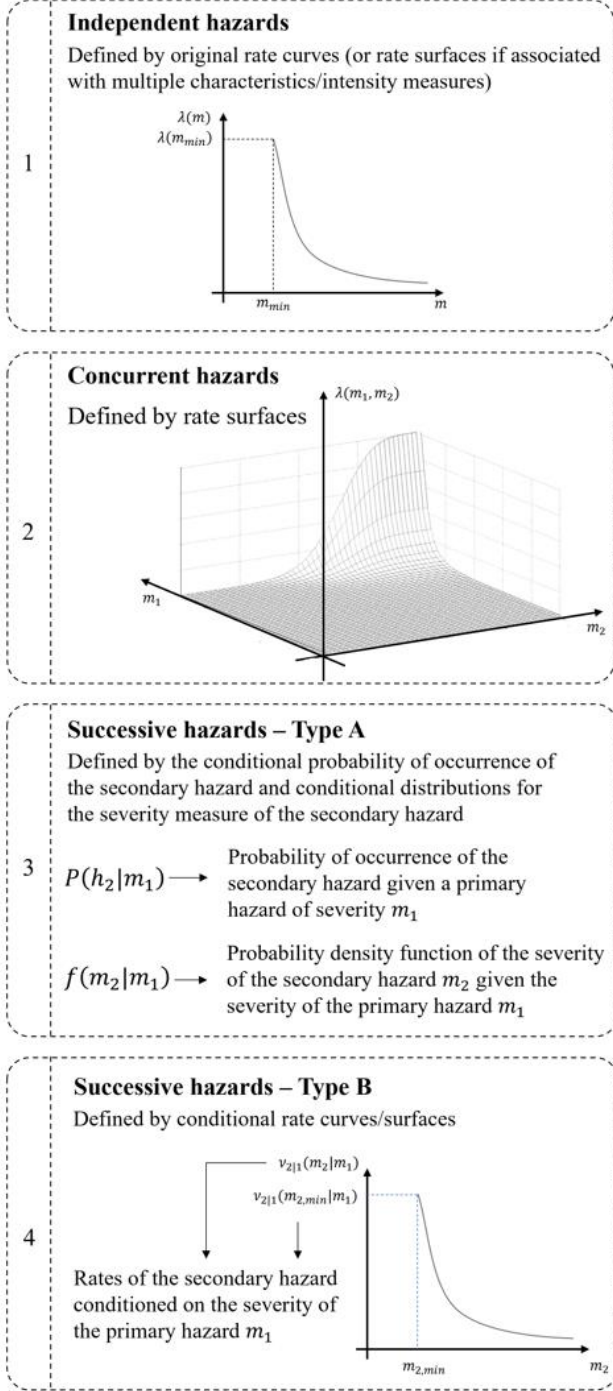
Independent hazards

Defined by original rate curves (or rate surfaces if associated with multiple characteristics/intensity measures)

1

Concurrent hazards

Defined by rate surfaces

2

Successive hazards – Type A

Defined by the conditional probability of occurrence of the secondary hazard and conditional distributions for the severity measure of the secondary hazard

3

$P(h_2|m_1) \longrightarrow$ Probability of occurrence of the secondary hazard given a primary hazard of severity m_1

$f(m_2|m_1) \longrightarrow$ Probability density function of the severity of the secondary hazard m_2 given the severity of the primary hazard m_1

Successive hazards – Type B

Defined by conditional rate curves/surfaces

4

Rates of the secondary hazard conditioned on the severity of the primary hazard m_1

Figure 2: Input required for the proposed formulation

Because of the simplifying assumptions, the simulation of such scenarios is computationally efficient. It can be repeated multiple times to obtain relevant statistical quantities such as (i) the probability of having a given number of hazard types in a given time span; (ii) the probability of a

combination of hazard events; and (iii) the distribution of the severity measures of the hazard events and joint distributions for dependent hazard events. These quantities, as well as the simulated scenarios, can be incorporated into formulations for Level II interactions (e.g., Otárola et al. 2023) to obtain the expected consequences of the hazard events throughout the life cycle of a system.

2.2. Scenario simulation
The simulation of the scenario starts in a neutral state where the rates for each hazard type h_i ($i = 1, ..., N$) are defined based on the corresponding original rate curves. We define J as the number of primary hazard events that, at any given time, have affected the rate of any secondary hazard type. Because the system has no memory of any previous hazard events at this point, we set $J = 0$. Assuming that the occurrence of each of the hazard events is Poissonian, the theory of competing Poisson processes determines that the rate of occurrence of the first hazard event is equal to the sum of the rates of the individual hazard types and that the probability that the i-th hazard type h_i is the first to occur is

$$P(H = h_i) = \frac{\lambda_i}{\sum_{n=1}^{N} \lambda_n} \qquad (2)$$

After the simulation of a hazard event, three phases follow: Phase 1 is the assessment of the hazard type and severity; Phase 2 is the simulation of Type A interactions; and Phase 3 is the reassessment of the rates based on Type B interactions.

In Phase 1, the hazard type is simulated consistently with Eq. (2), and its associated severity measure m_i can be obtained from the rate curve of the i-th hazard type $\lambda_i(m_i)$ by formulating its Cumulative Distribution Function (CDF) as

$$F_M(m_i) = 1 - \frac{\lambda_i(m_i)}{\lambda_i(m_{min})} \qquad (3)$$

In the case of hazard types associated with multiple severity measures and/or concurrent

hazards, all severity measures are obtained from the rate surfaces rather than rate curves. A similar relationship to Eq. (3) can be obtained for the multi-dimensional case.

In Phase 2, the Type A interactions are simulated. Each secondary hazard event is simulated based on the conditional probabilities and distributions described in Section 2.1.

In Phase 3, the rates of each hazard type are re-assessed to account for the Type B interactions. In particular, for each Type B interaction: (i) we substitute the original rate curves for each secondary hazard type with the corresponding conditional rate curve; (ii) we introduce an additional "memory loss" Poisson event with rate $\zeta_i = 1/mem_i$ to the pool of possible events; (iii) we set $J = J + 1$.

We can then simulate the following event, which can be either the occurrence of a new hazard event (with rate λ_i) or a memory loss event (with rate ζ_i). The theory of competing Poisson processes determines that the rate of the next event occurrence is equal to the sum of the rates of the individual events (which now include both hazard events and memory loss events). The probability that the next event is the occurrence of the i-th a hazard type ($\{H = h_i\}$) is

$$P(H = h_i) = \frac{\lambda_i}{\sum_{n=1}^{N} \lambda_n + \sum_{j=1}^{J} \zeta_j} \qquad (4)$$

and the probability that the next event to occur is a loss of memory of the i-th hazard type ($\{H = h_i^*\}$) is

$$P(H = h_i^*) = \frac{\zeta_i}{\sum_{n=1}^{N} \lambda_n + \sum_{j=1}^{J} \zeta_j} \qquad (5)$$

If the next simulated event is a hazard event, Phases 1-3 are repeated. If the simulated event is the memory loss of the i-th hazard type, we set $J = J - 1$ and remove the Poisson event with rate ζ_i from the pool of possible events. The flowchart in Figure 3 details the described sequential MC approach. Every type of interaction is included in the proposed procedure and incorporated based on its specific characteristics.
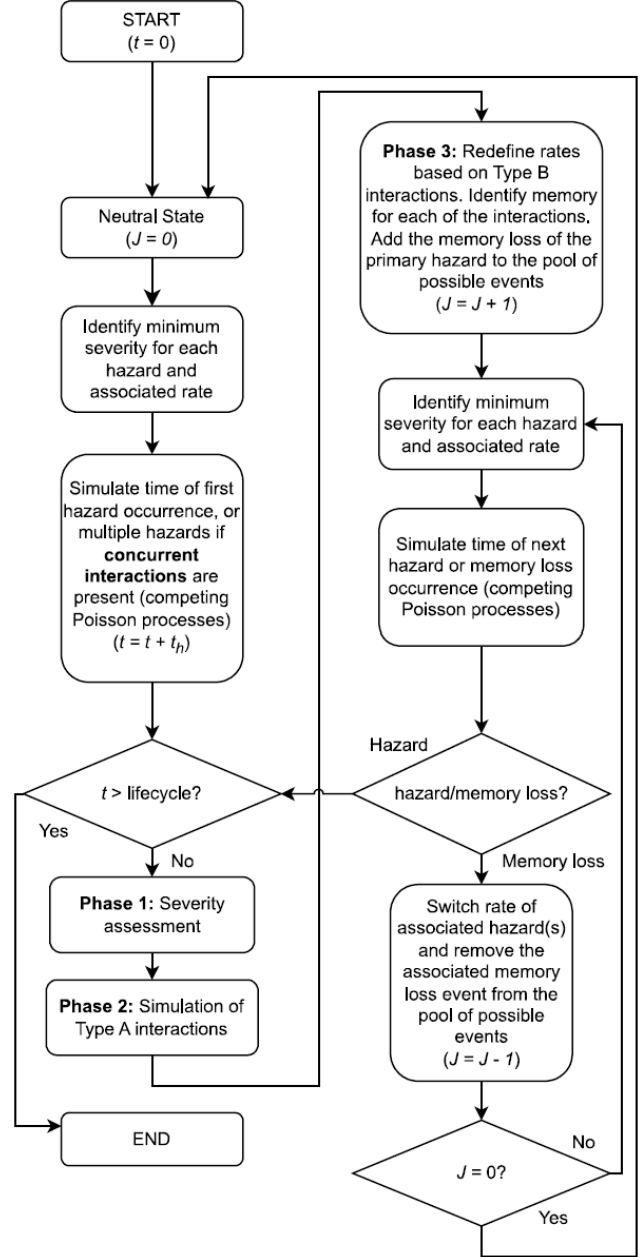

Figure 3: Proposed simulation method

3. NUMERICAL EXAMPLE

We hereby showcase the simulation of scenarios (i.e., sequences of events throughout the system's life cycle) using the sequential MC method detailed in the previous sections. The hazard types considered are the ones in the interaction matrix shown in Figure 3 (i.e., a portion of the hazard interaction matrix provided in Zaghi et al., 2016).
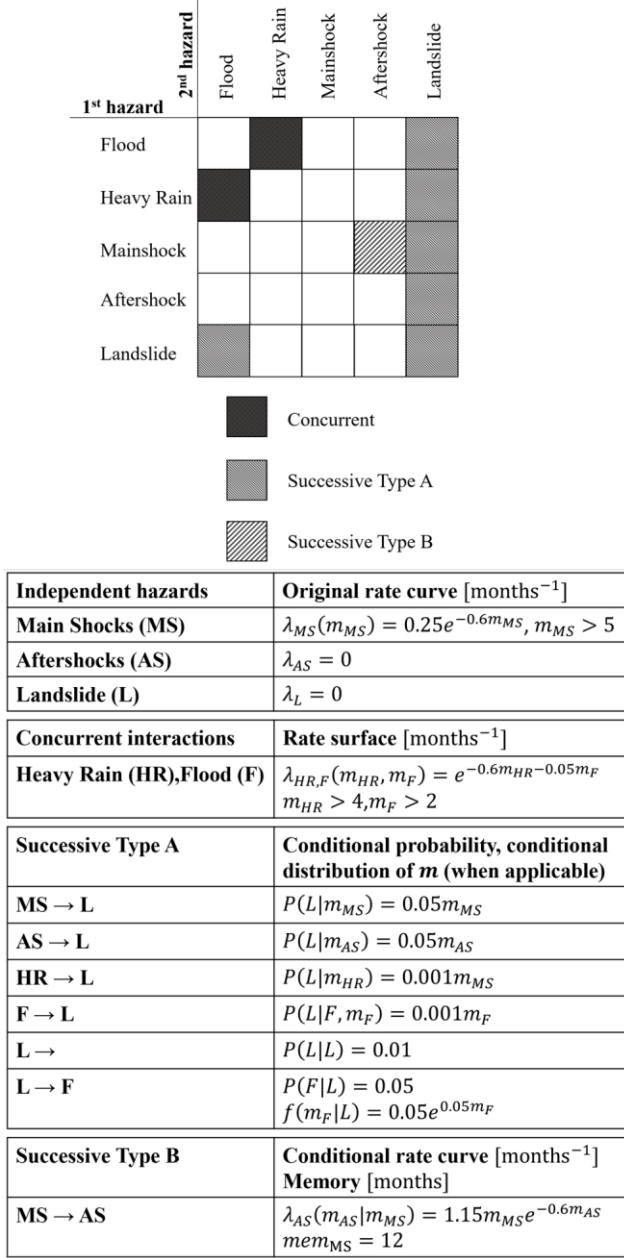
1st hazard \ 2nd hazard	Flood	Heavy Rain	Mainshock	Aftershock	Landslide
Flood		Concurrent			Successive Type A
Heavy Rain	Concurrent				Successive Type A
Mainshock				Successive Type B	Successive Type A
Aftershock					Successive Type A
Landslide	Successive Type A				Successive Type A

- ■ Concurrent
- ▨ Successive Type A
- ▨ Successive Type B

Independent hazards	Original rate curve [months^{-1}]
Main Shocks (MS)	$\lambda_{MS}(m_{MS}) = 0.25e^{-0.6m_{MS}}, m_{MS} > 5$
Aftershocks (AS)	$\lambda_{AS} = 0$
Landslide (L)	$\lambda_L = 0$

Concurrent interactions	Rate surface [months^{-1}]
Heavy Rain (HR),Flood (F)	$\lambda_{HR,F}(m_{HR}, m_F) = e^{-0.6m_{HR}-0.05m_F}$ $m_{HR} > 4, m_F > 2$

Successive Type A	Conditional probability, conditional distribution of m (when applicable)
MS → L	$P(L\|m_{MS}) = 0.05m_{MS}$
AS → L	$P(L\|m_{AS}) = 0.05m_{AS}$
HR → L	$P(L\|m_{HR}) = 0.001m_{MS}$
F → L	$P(L\|F, m_F) = 0.001m_F$
L →	$P(L\|L) = 0.01$
L → F	$P(F\|L) = 0.05$ $f(m_F\|L) = 0.05e^{0.05m_F}$

Successive Type B	Conditional rate curve [months^{-1}] Memory [months]
MS → AS	$\lambda_{AS}(m_{AS}\|m_{MS}) = 1.15m_{MS}e^{-0.6m_{AS}}$ $mem_{MS} = 12$

Figure 4: Input for the numerical example

Compared to Zaghi et al. (2016), earthquakes have been separated into mainshocks and aftershocks. This distinction allows us to separate the rate curves for the two hazard types, with aftershocks having rate = 0 before the occurrence of a mainshock and a rate defined by a conditional rate curve after the occurrence of the mainshock. The distinction also allows the system to retain a memory of the mainshock after the occurrence of the first aftershock (without this distinction, the simulated occurrence of the first aftershock would

redefine the rates of subsequent aftershocks and the effects of the main shock would be forgotten). The interaction matrix and the parameters for the case study are shown in Figure 4. While the interactions are meant to represent a realistic scenario, the selected numerical values for the severity measures and the associated rates are ideal and are only used for demonstration purposes. As such, units for the severity measures are also disregarded.

Figure 5 shows an example scenario generated using the proposed sequential MC method. The severity measure of the events is proportional to the diameter of the circles used to represent their occurrence (for simplicity, landslide events do not have an associated severity in this case study). Because of the underlying assumptions, aftershocks only occur in the aftermath of the mainshock, and their severity measures depend on the severity measure of the mainshock that triggered them. Heavy rain and flood (concurrent events) co-occur with rates and severity measures generated based on their joint rate surface. A sequence of landslide events can also be found around years 20-30 in the system's life cycle, triggered by a mainshock-aftershock sequence that occurred during those years.
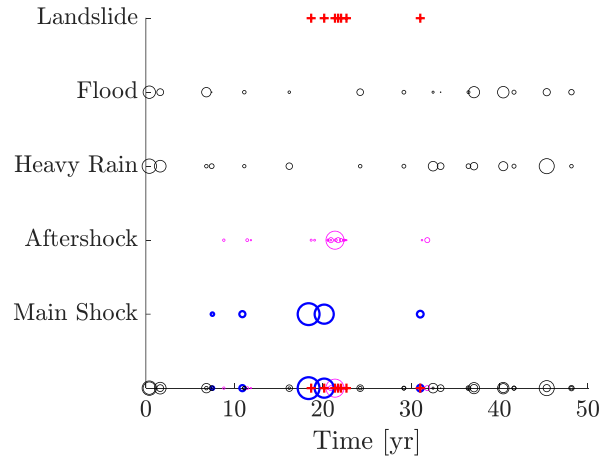

Figure 5: Simulated scenario

The simulated sequence reflects the dependencies highlighted in the literature among the different hazard types, and due to the selected modelling assumptions, it is computationally efficient to obtain. Consequently, it is possible to

generate multiple sequences such as the one shown in Figure 4. Those can then be used to obtain the relevant statistical quantities listed in Section 2.1.

4. CONCLUSIONS

The paper proposed a simple simulation-based approach to account for different types of hazard interactions in generating a multi-hazard scenario, i.e., a sequence of events (and their associated characteristics/severities) throughout the system's life cycle. We accounted for concurrent interactions, successive interactions where the secondary hazard event is immediately triggered by the primary (Type A), and successive interactions where the primary hazard event affects the secondary (Type B) occurrence rate. Each interaction is incorporated in the simulation differently; concurrent hazards are modelled based on the rate surface that defines the joint rate of the associated severity measures. Successive Type A interactions are incorporated through the conditional probability of occurrence of the secondary hazard type(s) and the conditional distribution of the associated severity measure; successive Type B interactions are modelled through the modification of the rate curve of the secondary hazard type(s). The different hazard events are then assumed to be a set of competing Poisson processes. The simulation of one scenario is computationally efficient and can be repeated to obtain relevant statistics of hazard occurrences. Such statistics can be used in analytical methods for Life Cycle Analysis (LCA) to obtain the expected value of impact/consequence metrics of interest to end users throughout the system's service life. The simulated scenarios can also be integrated into simulation-based frameworks for Level II interactions, i.e., the interactions between the effects of the hazard events.

5. ACKNOWLEDGEMENTS

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