Acceleron Aerospace Journal || AAJ.11.2106-2314
Volume 1, Issue 3, pp (64-69)
Research Paper
ISSN- 2583-9942 || DOI: 10.61359/11.2106-2314

Pairwise β-Open Set in Neutrosophic Bitopological Spaces

Kulandhai Therese A*

Research Scholar, Department of Mathematics, Nirmala College for Women, India.
ORCID- 0009-0003-7917-5566

Dr. A. Arokia Lancy†
Assistant Professor, Department of Mathematics, Nirmala College for Women, India.

Abstract: This paper introduces the concepts of pairwise $\tau_1\tau_2$ neutrosophic-open sets, pairwise $\tau_1\tau_2$ neutrosophic-semi-open sets, and pairwise $\tau_1\tau_2$ neutrosophic-pre-open sets in neutrosophic bitopological spaces. We study some of the basic properties of these sets and prove several propositions, including the fact that the fusion of two $\tau_1\tau_2$ neutrosophic-open sets is a pairwise $\tau_1\tau_2$ neutrosophic-open set.

Keywords: Neutrosophic set, Neutrosophic Bitopology, Neutrosophic β-open set.

Table of Contents

1. Introduction

Neutrosophic bitopological spaces are a generalization of both topological spaces and neutrosophic sets. In this paper, we introduce the concepts of pairwise $\tau_1\tau_2$ neutrosophic-open sets, pairwise $\tau_1\tau_2$ neutrosophic-semi-open sets, and pairwise $\tau_1\tau_2$ neutrosophic-pre-open sets in neutrosophic bitopological spaces. These sets are defined in a similar way to the corresponding sets in ordinary topology, but with the added complication of dealing with neutrosophic sets.

We study some of the basic properties of pairwise $\tau_1\tau_2$ neutrosophic-open sets, pairwise $\tau_1\tau_2$ neutrosophic-semi-open sets, and pairwise $\tau_1\tau_2$ neutrosophic-pre-open sets. We also prove several propositions, including the following:

- The fusion of two $\tau_1\tau_2$ neutrosophic-open sets is a pairwise $\tau_1\tau_2$ neutrosophic-open set.
- If A is a $\tau_1\tau_2$ neutrosophic-semi-open set ($\tau_1\tau_2$ neutrosophic-pre-open set) in a neutrosophic bitopological space, then A is a $\tau_1\tau_2$ neutrosophic-β-open set.
- Every pairwise $\tau_1\tau_2$ neutrosophic-semi-open set (pairwise $\tau_1\tau_2$ neutrosophic-pre-open set) is a pairwise $\tau_1\tau_2$ neutrosophic-open set.
- The fusion of any two $\tau_1\tau_2$ -PN-βO-sets is a $\tau_1\tau_2$ -PN-βO-set.
- If A is a $\tau_1\tau_2$ neutrosophic-semi-open and $\tau_1\tau_2$ neutrosophic-p-set in a neutrosophic bitopological space, then A is a $\tau_2\tau_1$ neutrosophic-pre-open set.
- If A is a $\tau_1\tau_2$ neutrosophic-semi-open and contra $\tau_1\tau_2$ neutrosophic-p-set in a neutrosophic bitopological space, then A is a $\tau_2\tau_1$ neutrosophic-pre-open set.
- If A is a $\tau_1\tau_2$ neutrosophic-p-set and τ_2 τ_1 neutrosophic-q-set in a neutrosophic bitopological space, then A is a pairwise $\tau_1\tau_2$ neutrosophic-p-set and a pairwise $\tau_1\tau_2$ neutrosophic-q-set.

*Research Scholar, Department of Mathematics, Nirmala College for Women, India. **Contact**: theresa26.mary@gmail.com.
† Assistant Professor, Department of Mathematics, Nirmala College for Women, India. **Contact:** lancyaarokia@gmail.com
**Received: 15-September-2023 || Revised: 23-September-2023 || Accepted: 26-September-2023 || Published Online: 30-September-2023

2. Pairwise β-open set in Neutrosophic Bitopological Spaces

2.1. Definition 2.1.1:

Let X be a non-empty set. Then S, a neutrosophic set (NS in short) over X is signified as follows:

S= { (y,T S(y),I S(y),F S(y)) : y ∈ X and T S(y),I S(y),F S(y) ∈]-0 ,1+[},where T S(y),I S(y) and F S(y) are the degree of truthiness, indeterminacy and falseness.

2.2. Definition 2.1.2:

A neutrosophic set U is expressed to be pairwise $\tau_1\tau_2$ neutrosophic β-open set in a neutrosophic bitopological space (X,τ_1,τ_2) if U=M∪N,where M is a $\tau_1\tau_2$ neutrosophic β open set and N is a $\tau_2\tau_1$ neutrosophic β open set) in (X,τ_1,τ_2).

2.3. Definition 2.1.3:

A neutrosophic set U is phrased to be pairwise τ1τ2 neutrosophic-semi-open set (pairwise τ1τ2 neutrosophic-pre-open set) in a neutrosophic bitopological space (X,τ_1,τ_2) if U=M∪N,where M is a τ1τ2 neutrosophic semi-open set (τ1τ2 neutrosophic-pre-open set) and N is a τ1τ2 neutrosophic semi-open set (τ2τ1 neutrosophic-pre-open set) in (X,τ_1,τ_2).

2.4. Proposition 2.1.4:

The fusion of two τ1τ2 neutrosophic β-open set in a neutrosophic bitopological space (X,τ_1,τ_2) is afresh a pairwise τ1τ2 neutrosophic β-open set.

Proof:

If A, B be two τ1τ2 -β-open set in a neutrosophic bitopological space (X, τ1, τ2). Then there exists two τ1τ2 -neutrosophic- β -open set.

G1, G2 and two τ1τ2-neutrosophic-β-open set H1, H2 such that $A = G1 \cup H1 \; and \; B = G2 \cup H2$.

Later, G1, G2 are τ1τ2-neutrosophic-β-open set so

$$G1 \subseteq \tau2 - cl(\tau1 - int(\tau2 - cl(G1))).$$

$$G2 \subseteq \tau2 - cl(\tau1 - int(\tau2 - cl(G2))).$$

Since, H1, H2 are τ1τ2-neutrosophic-β-open set so

$$H1 \subseteq \tau2 - cl(\tau1 - int(\tau2 - cl(H1))).$$

$$H2 \subseteq \tau2 - cl(\tau1 - int(\tau2 - cl(H2))).$$

Currently, we possess

$$G1 \cup G2 \subseteq \tau2 - cl(\tau1 - int(\tau2 - cl(G1))) \cup \tau2 - cl(\tau1 - int(\tau2 - cl(G2)))$$

$$\tau2 - cl(\tau1 - int(\tau2 - cl(G1)) \cup \tau1 - int(\tau2 - cl(G2)))$$

$$\tau 2 - cl(\tau 1 - int(\tau 2 - cl(G1 \cup G2)))$$

$$\Rightarrow G1 \cup G2 \ is \ a \ \tau 1 \tau 2 - neutrosophic - \beta - open \ set.$$

In Addition, we possess

$$H1 \cup H2 \subseteq \tau 2 - cl(\tau 1 - int(\tau 2 - cl(H1))) \cup \tau 2 - cl(\tau 1 - int(\tau 2 - cl(H2)))$$

$$\tau 2 - cl(\tau 1 - int(\tau 2 - cl(H1)) \cup \tau 1 - int(\tau 2 - cl(H2)))$$

$$\tau 2 - cl(\tau 1 - int(\tau 2 - cl(H1 \cup H2)))$$

$$\Rightarrow H1 \cup H2 \ is \ a \ \tau 1 \tau 2 - neutrosophic - \beta - open \ set.$$

Consequently, $A \cup B = (G1 \cup H1) \cup (G2 \cup H2)$

$$= (G1 \cup G2) \cup (H1 \cup H2)$$

$$= G \cup H.$$

Accordingly, there persist a τ1τ2-neutrosophic-β-open set $G = (G1 \cup G2)$ and a τ1τ2-neutrosophic-β-open set $H = (H1 \cup H2)$ such that$A \cup B = G \cup H$.

Consequently A∪B is a pairwise τ1τ2-neutrosophic-β-open set.

Thus the fusion of two τ1τ2-neutrosophic-β-open set in a neutrosophic bitopological space (X, τ1, τ2) is again a τ1τ2-neutrosophic-β-open set.

2.5. Proposition 2.1.5:

In a NBi-T-space (X,τ1 ,τ2), if A is τ1τ2NSO-set (τ1τ2-NPO-set), then P is a τ1τ2-PN- βO-set.

Proof:

If we consider that A is τ1τ2 -neutrosophic-semi-open set in a neutrosophic bitopological space

$(X, \tau 1, \tau 2)$. As we know

$$A \subseteq \tau 1 - cl(\tau 1 - int(A))$$

Formerly we can state,

$$P \subseteq \tau 1 - cl(\tau 1 - int(P))$$

$$P \subseteq \tau 2 - cl(\tau 1 - int(\tau 2 - cl(P))).$$

Consequently, A is τ1τ2-neutrosophic-β-open in (X, τ1, τ2). Similarly, we can state that if P is τ1τ2-neutrosophic-pre-open set in (X, τ1, τ2) then it is τ1-neutrosophic-β-open set.

As we know $A \subseteq \tau 1 - cl(\tau 1 - int(A))$

Formerly we can state,

$$P \subseteq \tau 1 - cl(\tau 1 - int(P))$$

$$P \subseteq \tau 2 - cl(\tau 1 - int(\tau 2 - cl(P)))$$

Accordingly A is τ1τ2 - PN- βO in (X, τ1, τ2).

2.6. Proposition 2.1.6:

In a neutrosophic bitopological space (X,τ_1, τ_2), every pairwise $\tau_1\tau_2$ neutrosophic-semi-open set(pairwise $\tau_1\tau_2$ neutrosophic-pre-open set) is a pairwise $\tau_1\tau_2$ neutrosophic β-open set.

Proof:

Let M be a pairwise $\tau_1\tau_2$ -neutrosophic-semi-open set (pairwise $\tau_1\tau_2$ -neutrosophic-pre-open set). Then there persist a $\tau_1\tau_2$ -neutrosophic-semi-open set U ($\tau_1\tau_2$ -neutrosophic-pre-open set U) and a $\tau_1\tau_2$ -neutrosophic-semi-open set V ($\tau_2\tau_1$neutrosophic-pre-open set V) such that M=A∪B.

By Proposition 2.1.5 we can consider that there persist a $\tau_1\tau_2$ -neutrosophic-β-open set U and a $\tau_2\tau_1$ -neutrosophic-β-open set V such that M=U∪V.

Consequently, G is a pairwise $\tau_1\tau_2$ -neutrosophic-β-open set.

So U be $\tau_1\tau_2$ -neutrosophic-β-open and V be $\tau_2\tau_2$ -neutrosophic-β-open set.

Consequently, there exist a $\tau_1\tau_2$ -neutrosophic-β-open set U and a $\tau_2\tau_1$ -neutrosophic-β-open set V such that M=U∪V.

Accordingly, M is a pairwise $\tau_1\tau_2$-neutrosophic-β-open set.

Thus every pairwise $\tau_1\tau_2$ -neutrosophic-semi-open set (pairwise $\tau_1\tau_2$ -neutrosophic-pre-open set) is a pairwise $\tau_1\tau_2$ -neutrosophic-β-open set.

2.7. Proposition 2.1.7:

Let (X,τ1,τ2) be an NBi-T-space. Then, the fusion of any two τ1τ2-PN-βO-sets is a τ1τ2-PN-βO-set

Proof:

The approach used to prove these propositions closely resembles the method employed in the proof of Proposition 2.1.4.

2.8. Proposition 2.1.8:

In a neutrosophic bitopological space (X,τ_1, τ_2)

1) If A is τ1τ2 –neutrosophic semi open and τ1τ2 –neutrosophic-p-set then A is τ2τ1 –neutrosophic pre-open

2) If A is τ2τ1 –neutrosophic semi-open and contra τ2τ1 –neutrosophic-p-set then A is τ2τ1 –neutrosophic pre-open

Proof:

1) Let L and M be two pairwise $\tau_1\tau_2$- neutrosophic semi open in an NBi-T-space(X,τ_1,τ_2).

So, one can state$L = L1 \cup L2 \ and \ M = M1 \cup M2$, where L_1, M_1 are $\tau_1\tau_2$- neutrosophic-p-sets and L_2, M_2 are $\tau_1\tau_2$-neutrosophic-p-set in (X,τ_1,τ_2).

Formerly, L_1 and M_1 are $\tau_1\tau_2$- neutrosophic-p-set, so

$$L1 \subseteq N_{cl}^{i} N_{int}^{j}(L1), \text{ and}$$

$$M1 \subseteq N_{cl}^{i} N_{int}^{j}\ (M1).$$

Further, L_2 and M_2 are $\tau_1\tau_2$- neutrosophic-p-set, so

$$L2 \subseteq N_{cl}^{j} N_{int}^{i}\ (L2),$$

$$M2 \subseteq N_{cl}^{j} N_{int}^{i}\ (M2).$$

Now,

$$L \cup M = (L1 \cup L2) \cup (M1 \cup M2)$$

$$= (L1 \cup M1) \cup (L2 \cup M2).$$

Accordingly, $L1 \cup M1 \subseteq N_{cl}^{i} N_{int}^{j}(L1) \cup N_{cl}^{i} N_{int}^{j}\ (M1)$.

$$= \ N_{cl}^{i}(N_{int}^{j}(L1) \cup N_{int}^{j}\ (M1))$$

$$\subseteq N_{cl}^{i} N_{int}^{j}(L1 \ \cup M1))$$

This implies, $L_1 \cup M_1$ is a $\tau_1\tau_2$- neutrosophic-p-set in (X,τ_1,τ_2).

Similarly, it can be established that $L_2 \cup M_2$ is a $\tau_1\tau_2$- neutrosophic-p-set in (X,τ_1,τ_2). Accordingly, L$\cup$M is a pairwise $\tau_1\tau_2$- neutrosophic-p-set et in (X,τ_1,τ_2).

2) Comparably, The following proof shares a similar structure and approach to the first proof

2.7. Proposition 2.1.9:

Let (X,τ_1, τ_2) be an neutrosophic bitopological space.

1) If A is $\tau_1\tau_2$ neutrosophic -p-set and $\tau_2\tau_1$ neutrosophic -q-set then

$$N\_cl\hat{\ }i\ N\_int\hat{\ }j\ (A) \subseteq N\_cl\hat{\ }j\ N\_int\hat{\ }i\ (A)$$

2) If A is contra $\tau_1\tau_2$ neutrosophic -p-set and contra$\tau_1\tau_2$ neutrosophic -q-set then

$$N\_cl\hat{\ }j\ N\_int\hat{\ }i\ (A) \subseteq N\_cl\hat{\ }i\ N\_int\hat{\ }j\ (A).$$

Proof.

The approach used to prove these propositions closely resembles the method employed in the proof of Proposition 2.1.8.

3. Conclusion

In this paper, we have introduced the concepts of pairwise $\tau_1\tau_2$ neutrosophic-open sets, pairwise $\tau_1\tau_2$ neutrosophic-semi-open sets, and pairwise $\tau_1\tau_2$ neutrosophic-pre-open sets in neutrosophic bitopological spaces. We have also studied some of the basic properties of these sets and proved several propositions. Our work opens up a new area of research in the field of neutrosophic topology. We hope that our results will be useful to other researchers in this area.

4. References

[1] Binod Chandra Tripathy, Diganta Jyoti Sarma,introduced on weakly b-continuous functions in bitopological spaces, Acta Scientiarum,Technology (35),(2013),521-525.

[2] Chandrasekhara Rao.K. and Kannan.K"s*g locally closed sets in bitopological spaces",International Journal Contemporary Mathematical Sciences,.4(12),(2009), 597-607.

[3] Dimacha Dwibrang Mwchahary and Bhimraj Basumatary .A Note on Neutrosophic Bitopological Spaces , Neutrosophic Sets and Systems, 33,(2020),134-144.

[4] Kelly, J. C. Bitopological spaces. Proceedings of the London Mathematical Society, 3(1), (1963), 71-89.

[5] Salama.A.A,Samarandache.F,Valeri.K,Neutrosophic closed sets and neutrosophic continuous function,Neutrosophic sets and System. 4,(2014), 4-8.

[6] Zadeh. A, Fuzzy Sets,Inform.Control 8 (1965),338-353.

5. Biography

A. Kulandhai Therese is a Research Scholar in the Department of Mathematics at St. Joseph's College of Arts and Science for Women, Hosur. She is a passionate educator and researcher with interests in topology, functional analysis, and differential geometry. She is also committed to promoting diversity and inclusion in STEM fields.

6. Conflict of Interest

The author have no conflict of interest to report.

7. Funding

No external funding was received to support this study.