Developing a Growth Model of Household Heterogeneity, Human Capital Investment, and Impacts of Disaster Events

Muneta Yokomatsu[a], Thomas Schinko[a], Junko Mochizuki[b], Armon Rezai[c,a]

[a]Population and Just Societies Program, International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria;
[b]Biodiversity and Natural Resources Program, International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria;
[c]Institute for Political Economy of Public Policy, Vienna University of Economics and Business (WU), Vienna, Austria.

ABSTRACT

This study develops a simple monetary growth model of the sluggish factor-price adjustment to examine GDP growth and distributional implications of climate-related disasters with a special focus on human capital investment in developing countries. The model demonstrates an endogenous business cycle through integration of nonlinear factors associated with the money-demand function and the human-capital-investment function. The results of the numerical analysis suggest that there is the possibility that a disaster occurring in an economy experiencing unemployment increases GDP in the short run but hampers growth in the long run. This is due to the interruption of human-capital investment, implying that the widespread view that a disaster causes short-run adverse GDP impacts may not always hold true and negative indirect impacts may manifest in the long-term. On such a path, development in disaster mitigation infrastructure could reduce human-capital gaps in the long run by supporting continued post-disaster human-capital investment opportunities for the poor. The study further points out the methodological potential of the nonlinear dynamic model for analyzing indirect risks. The nonlinear feature of our model derives long-term non-monotonic impacts on economic dynamics that are sensitive to small changes in initial values and direct disaster damages. This allows for the estimation of various qualitative and quantitative market responses associated with a macroeconomic situation such as a boom and recession, and the possibilities of lagged influences.

KEYWORDS

Natural hazards; Economic growth; Household heterogeneity; Human capital investment; Developing countries

1. Introduction

While the existing body of literature generally agrees that a catastrophic disaster leads to the destruction of production factors, thereby resulting in adverse impacts on the short-run macroeconomy, the impacts on long-term growth and inequality remain contested. Although it may be a socially accepted idea that disasters widen the gap between the rich and the poor, factors that crucially make the poor more economically vulnerable to disasters have not been clarified in the growth model framework; for a counter-example, it could be a part of disaster-related phenomena that wealthier

E-mail: yoko@iiasa.ac.at

households have more assets exposed to a disaster, which may reduce the disparity of asset ownership, and that unemployed workers are hired at disaster reconstruction sites and improve their economic situation. At the same time, if wealthier households prefer to form their wealth by holding money or investing in other means of protection such as insurance, a disaster will lead to the reduction of their vulnerability, while such behaviors may somehow influence the real side of economy such as production. Overall, the growth and distributional impacts of a disaster consist of multiple factors that are intertwined, posing significant challenges to sound economic policy analysis.

This study formulates a minimum-size macroeconomic dynamic model to simulate the impacts of a disaster on the dynamics of inequality. We apply a Keynesian framework of sluggish prices in labor and capital markets, associated with quantity adjustment in the factor markets. In addition to involuntary unemployment, we focus on the supply side of the labor market with a focus on how poor households keep enough time for learning with the motivation of becoming high-skilled workers in the future. On such a topic, we easily come up with a guess that both demand and supply would be increased immediately after a disaster because of increased demand for goods in the reconstruction process. However, this is not always the case; multiple conditions including the technological (e.g., substitutability between physical capital and labor) and financial (e.g., asset preference and price) interact with one another, resulting in complicated non-linear dynamics of the system.

For a simple computation work, we formulate the household problem by a sequence of the two-period optimization without the recursive relation of dynamic programming. The second-period utility function is defined on a set of the state variables of each household, which is intended to work like a pseudo-value function. Based on such a setting, we can derive the closed-form solutions, which include the interior- and corner-point ones, of household behaviors. We find that the money-demand function and the human-capital-investment function are associated with the non-linear factors which take the main role of causing an endogenous business cycle. The numerical simulation demonstrates a business cycle where boom and recession are repeated in turn on many parameter sets even in the absence of a disaster although ex-ante risk is always present. In this study, we only focus on the qualitative aspects of the model's behavior; however, in order to facilitate interpretation of the simulation results we assume that one interval corresponds to five years or more. Moreover, in order to deal with the dynamics of household heterogeneity, we focus on the change in the density function that is defined on the state-variable space where the values of the state vector characterize the individual state of each household.

In recent years, there are an increasing number of dynamic models for analyzing impacts of natural hazards and disaster risk reduction. While challenges are made with the purpose of developing an integrated framework (e.g., Akao & Sakamoto, 2018), many models are developed based on specific concerns in terms of subjects as variables and market structure. With respect to the former, recent work includes a focus on relationships between different capitals (e.g., Hallegatte, Jooste, & Mcisaac, 2022) and co-benefit of disaster-risk-reduction measures (e.g., Yokomatsu, Mochizuki, Joseph, Burek, & Kahil, 2022) while the latter include applications of DSGE models (e.g., Isoré & Szczerbowicz, 2017; Keen & Pakko, 2007), disequilibrium models (e.g., Hallegatte, Hourcade, & Dumas, 2007), Keynesian models with a focus on distribution (e.g., Rezai, Taylor, & Foley, 2018), and agent-based models (e.g., Choquette-Levy, Wildemeersch, Oppenheimer, & Levin, 2021; Hochrainer-Stigler & Poledna, 2016; Naqvi & Rehm, 2014). The interests and elements that make up this study such as the Keynesian framework, household heterogeneity, and income distribution are each shared with these studies.

Moreover, our model also comprises structural components that were developed in the areas of economics unrelated to climate/disaster issues. We apply the one type of Keynesian-model framework associated with sluggish price adjustment (e.g., Ono, 2001). The analysis of business cycles with exogenous impulses began with Frisch (1933) and has accumulated theoretical developments and applications, along with endogenous cycles and deterministic chaos (e.g., Benhabib, 1992; Bischi, Chiarella, & Gardini, 2010; Goodwin, 1990). Moreover, the approach to dealing with household heterogeneity by focusing on the density function of individual state variables is taken by prior studies including Lasry and Lions (2007) and Achdou, Han, Lasry, Lions, and Moll (2022) who worked on the system known as the backward-forward mean-field-game system. However, because we formulate the discrete-time and non-recursive optimization problem, we do not apply such a system but simply work with the Markov chain of distribution of the household states.

Our modeling motivation is supported by the findings of empirical studies and discussions. As briefly mentioned at the beginning, there is some consensus that natural disasters have a negative impact on macroeconomy in the short run due to human damages, destruction of structures, and so on, which results in slowdowns in production (e.g., AG van Bergeijk & Lazzaroni, 2015; Noy, 2009; Raddatz, 2007). On the other hand, the discussion on the long-run effects of natural disasters is inconclusive (e.g., Shabnam, 2014); some studies describe the expansionary disaster effects caused by "creative destruction" (e.g., Skidmore & Toya, 2002), while others make contrasting conclusions that natural disasters have a negative long-term impact (e.g., Noy, 2009; Raddatz, 2007). Other results include various assertions including important effects of disasters on growth (Albala-Bertrand, 1993), the lack of partial correlation between natural disaster risk and economic growth (Cuaresma, 2022), different effects across disasters and economic sectors (Loayza, Olaberria, Rigolini, & Christiaensen, 2012), impacts being dependent on political situation (Cavallo, Galiani, Noy, & Pantano, 2013), greater magnitude of long-term disaster damage in developing societies (UNISDR, 2009), and the relationship between natural disasters and the poverty trap wherein the poorest households struggle most with shocks (Carter, Little, Mogues, & Negatu, 2007).

In recent years, the impact of disasters on income inequality has been better measured and analyzed in some countries due to the development of surveys and data (e.g., Bista, 2020; Pleninger, 2022). As for our special focus on human capital formation in developing countries, many articles have reported detrimental effects of disaster events on education by damaging complementary infrastructure such as school buildings and access roads (e.g., J. Baez, De la Fuente, & Santos, 2010; Petal et al., 2015); increasing child work participation rates, which results in the removal of children from schools (e.g., J. E. Baez & Santos, 2007; De Janvry, Finan, Sadoulet, & Vakis, 2006; Jacoby & Skoufias, 1997); and causing nutritional deficiencies that prevent continuous learning (e.g., Alderman, Hoddinott, & Kinsey, 2006). Cuaresma (2010) applies cross-country and panel regressions to figure out a robust negative partial correlation between secondary school enrollment and natural disaster risk.

This study makes a unique contribution to development of a dynamic model for examining impacts of disaster events and a macroeconomic situation on changes in multi-dimensional household heterogeneity. Each household is characterized by four-dimensional state variables: a financial asset, human capital, a physical household asset, and physical production capital, and they have different responses either to a disaster or production with one another. For example, we assume the step-function

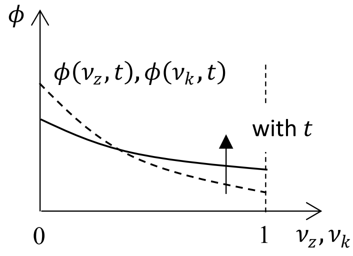


Figure 1.: Density of damage rate

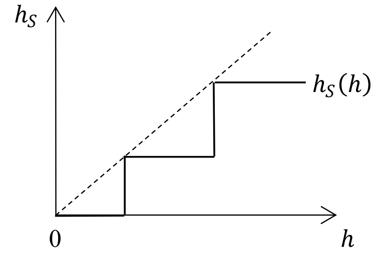


Figure 2.: Classes of human capital

property in the productivity of human capital, which makes its formation process different from that of physical production capital. Human capital investment is influenced by a business cycle, and it also causes the cycle. Unlike many dynamic models, including canonical DSGE models, which are analyzed by attributing them to a (log-) linearized framework, we maintain the nonlinear structure of the system, so that the model can illustrate a disaster impact that appears in the future, which is a part of the indirect risks of a disaster event. To the best of our knowledge, it is the first attempt to formulate a model of the four-dimensional household heterogeneity to examine direct and indirect disaster impacts.

The rest of this paper is organized as follows: Section 2 formulates the model; Section 3 presents the numerical simulation results; Section 4 discusses implications and future issues; and Section 5 concludes the study.

2. Model

2.1. Disaster

A one-sector closed-economy model is formulated. The model is dynamic with a discrete-time horizon. In each period of time t $(= 1, 2, \cdots)$, a disaster arrives with probability λ and destroys a part of physical household assets and production capital, which are damaged by the rates ν_z and ν_k, respectively. While the arrival rate λ is assumed to be constant throughout, the distributions of the damage rates ν_z and ν_k over the interval $[0, 1]$ change over time with climate change. Both arrival and damage rates are independent of previous occurrences. The density functions of the damage rates are given as follows;

$$\phi(\nu_z, t) := \phi_0 \exp(-\phi_1 \nu_z), \tag{1a}$$
$$\phi(\nu_k, t) := \phi_0 \exp(-\phi_1 \nu_k), \tag{1b}$$
$$\text{where} \quad \phi_0 := \phi_1[1 - \exp(-\phi_1)]^{-1}, \tag{1c}$$
$$\phi_1 := \phi_{10} - \phi_{11} t. \tag{1d}$$

ϕ_{10} (> 0) and ϕ_{11} (≥ 0) are constant parameters while ϕ_0 and ϕ_1 change with t so that they meet $\int_0^1 \phi(\nu_z, t) d\nu_z = 1$ and $\int_0^1 \phi(\nu_k, t) d\nu_k = 1$ as illustrated in Fig.1. For simplicity, we assume that ϕ_{10} and ϕ_{11} of the two density functions have the same value. Moreover, all (z, k) owned by heterogeneous households are exposed to the same density functions above in the ex-ante sense, but given different ex-post values by a disaster. The expected damage rates, which are thus equal to the ex-post average damage rates, are given by

$$\nu_{zE} := \mathrm{E}[\nu_z] = \int_0^1 \nu_z \phi(\nu_z, t)\ d\nu_z$$
$$= [\phi_1\{1 - \exp(-\phi_1)\}]^{-1} \{1 - (1 + \phi_1) \exp(-\phi_1)\}, \tag{2a}$$
$$\nu_{kE} := \mathrm{E}[\nu_k] = \nu_{zE}. \tag{2b}$$

We assume that the disaster damage rates are reduced by the stock of disaster mitigation infrastructure D_M by the following factors;

$$\chi_z = \exp\{-\chi_{z1}(D_M - D_{M0})\}, \tag{3a}$$
$$\chi_k = \exp\{-\chi_{k1}(D_M - D_{M0})\}, \tag{3b}$$

where D_{M0} is the initial value of D_M, and χ_{z1} and χ_{k1} are positive parameters. Note that we use the term "mitigation" to indicate "disaster mitigation" which means damage reduction at the time of a disaster. Our usage is different from the one in the climate area where "mitigation" means a reduction of greenhouse gas emissions into the atmosphere or increasing emission sequestration in carbon sinks such as forests.

2.2. Household

Households are heterogeneous with respect to four state variables $(a(t), h(t), z(t), k(t))$ where $a(t)$ represents a financial asset, $h(t)$, human capital, $z(t)$, a physical household asset, $k(t)$, physical production capital, and t represents a period of time. Distribution of households in the four-dimensional space $(a(t), h(t), z(t), k(t))$ is represented by the density function $g(a(t), h(t), z(t), k(t))$ that meets

$$\int_a \int_h \int_z \int_k g(a(t), h(t), z(t), k(t))\ dk\ dz\ dh\ da \quad \equiv 1. \tag{4}$$

Hereafter, we omit the notation "(t)" for brevity as long as we do not need clarification on it. Moreover, we denote the quadruple integral with respect to (a, h, z, k) by the single integral with respect to $\boldsymbol{s} := (a, h, z, k)$, with which expression of the above Eq.(4) is reduced to be $\int g(\boldsymbol{s})\ d\boldsymbol{s} \equiv 1$, for example. The total population is assumed to be constant throughout and standardized to be unity.

A financial asset a is composed of bond b and money m; namely $a \equiv b + m$. Human capital h is defined by knowledge and skill and is formed by investing time in learning. We define human capital $h(t)$ by a continuous variable, while actual contribution to the productivity of the firm, which we call the class of human capital h_S, is given by a step function: $h_S := h_S(h)$ as illustrated in Fig.2. A reason behind this step-function formulation is supported by several facts that are more often observed in developing countries: (i) the classes that are identified by the graduation of each stage of schooling, for example, are often an observable index based on which jobs or positions are assigned, and (ii) unexpected interruption of learning in the middle of a school stage caused by a large-scale disaster prevents young people from acquiring an organized skill and knowledge at the applicable level. Without such formulation, human capital would become theoretically indifferent to physical production capital, and a model would lose an essential aspect associated with an issue of education disruption caused by a disaster. On the other hand, because we do not consider health and injury in the model although they are one of the factors that compose working capacity in the real world, we assume that human capital is not directly damaged by a disaster. We further assume that human capital investment is conducted by allocating a portion of the time to learning, and is thus associated with a decrease in labor income as an opportunity cost.

A physical household asset z includes dwellings, furniture, and other durable goods that directly bring utility to households who use them. Firms are owned by households by means of physical production capital k. The formation processes of the four state variables are represented as follows;

$$
\begin{aligned}
a' &= (1+r)a + \{wh_S \cdot (1-\eta_h)l_D + r_K k_D + \xi\}(1-\phi_\tau) \\
&\quad -\upsilon_\tau - c - Rm - \eta_z z - \eta_k k, && \text{(5a)} \\
h' &= h \cdot (1+\iota\eta_h)(1-\delta_h), && \text{(5b)} \\
z' &= z \cdot (1+\eta_z)(1-\delta_z)(1-\varepsilon_\lambda \nu_z \chi_z), && \text{(5c)} \\
k' &= k \cdot (1+\eta_k)(1-\delta_k)(1-\varepsilon_\lambda \nu_k \chi_k), && \text{(5d)}
\end{aligned}
$$

where (a', h', z', k') is the state in the next period. r is the real interest rate, w, the real wage rate, η_h, the human capital investment rate, l_D, the employed labor, r_K, the real rate of return to physical production capital, k_D, the employed physical capital, ξ, the firm's profit, ϕ_τ, the income-tax rate, υ_τ, the lumpsum tax, c, consumption, R, the nominal interest rate, η_z, the investment rate of a physical household asset, and η_k, the investment rate of physical production capital. $R = r + \pi_E$ holds by Fisher's equation where π_E is the expected inflation rate of the commodity price, implying that the opportunity cost of holding money is composed of a gain of interest and a decrease in the value of money (e.g., Fisher, 1930). Moreover, ι is the coefficient of forming of human capital, and $\delta_h, \delta_z, \delta_k$ are the depreciation rates of h, z, k, respectively. ε_λ is the indicator of disaster occurrence; namely, $\varepsilon_\lambda = 0$ if a disaster does not occur in a concerned period, and $\varepsilon_\lambda = 1$ if a disaster occurs.

In each period t, each household focuses on its utility in the current period t and the next period $t+1$ and maximizes the following two-period utility function:

$$
\begin{aligned}
&U(t) := u_1(\cdot) + \beta u_2(\cdot) && \text{(6a)} \\
&\text{where} \\
&u_1(\cdot) := \gamma_c \frac{(c-\underline{c})^{1-\theta}}{1-\theta} + \gamma_m \frac{m^{1-\theta}}{1-\theta} + \gamma_z \frac{\{z(1+\eta_z)\}^{1-\theta}}{1-\theta} && \text{(6b)} \\
&u_2(\cdot) := \gamma_a \frac{(a'-\underline{a})^{1-\theta}}{1-\theta} + \gamma_h \left\{1 + \gamma_{hh} \cdot \left(1 - \frac{h_{S+1}-h}{h_{S+1}-h_S}\right)\right\} \cdot \frac{h'^{\,1-\theta}}{1-\theta} \\
&\qquad + \gamma_{zz} \frac{\mathrm{E}[z']^{1-\theta}}{1-\theta} + \gamma_k \frac{\mathrm{E}[k']^{1-\theta}}{1-\theta} && \text{(6c)}
\end{aligned}
$$

$u_1(\cdot)$ is the sub-utility function of the variables in the current period, and $u_2(\cdot)$ is one of the variables in the next period. β $(0 \leq \beta \leq 1)$ is a discount factor, and θ is a degree of relative risk aversion. $\gamma_c, \gamma_m, \gamma_z, \gamma_a, \gamma_h, \gamma_{hh}, \gamma_{zz}, \gamma_k$ are positive parameters that determine weights of the terms. In the current-period utility function defined by Eq. (6b), $\underline{c}$ (≥ 0) is the subsistence consumption (e.g., Kraay & Raddatz, 2007). The money-in-utility form (Sidrauski, 1967) is applied to easily derive the demand function for money. We assume that households can enjoy the level $z(1+\eta_z)$ of a household asset in the current period before it gets depreciated. The next-period utility function defined by Eq.(6c) is composed of the state variables in the next period. $\underline{a}$ (< 0) is the lowest level of the financial asset that is introduced for the technical reason of making $(a' - \underline{a})$ always positive, considering that a' itself could be negative when a household takes a negative position of a bond. The second term related to the utility of h' includes the motivation for the human capital investment where the closer h is to the next class h_{S+1}, the more strongly a household is motivated to continue learning. $\mathrm{E}[z']$ and $\mathrm{E}[k']$ are the expected levels of a household asset and physical capital, respectively, that are given by

$$\mathrm{E}[z'] = z \cdot (1 + \eta_z)(1 - \delta_z)(1 - \lambda \cdot \nu_{zE} \cdot \chi_z), \tag{7a}$$

$$\mathrm{E}[k'] = k \cdot (1 + \eta_k)(1 - \delta_k)(1 - \lambda \cdot \nu_{kE} \cdot \chi_k). \tag{7b}$$

We assume that households are faced with the borrowing constraint:

$$b(t) \geq b_{\mathrm{Lim}} \quad \text{for any } t \tag{8}$$

where b_{Lim} is the borrowing limit that meets $-\infty < b_{\mathrm{Lim}} < 0$ (e.g., Aiyagari, 1994). From the identity $a \equiv b + m$, demand for money in Period t is constrained by the following area:

$$0 \leq m(t) \leq a(t) - b_{\mathrm{Lim}}. \tag{9}$$

The household problem is represented as follows:

$$\max_{c, m, \eta_h, \eta_z, \eta_k, a'} U(\cdot) \tag{10a}$$

$$\text{subject to} \quad 0 \leq \eta_h \leq 1, \ \eta_z \geq -1, \ \eta_k \geq -1, \tag{10b}$$

$$\text{Eqs. (5a)-(5d), (7a), (7b), (9).}$$

The inequality constraints in Eq.(10b) imply that the total available time in each period is standardized to be one, and η_h is equivalent to the time for learning in that period. Moreover, a household can also sell a part of its physical household asset and physical production capital by choosing η_z and η_k in the area $-1 \leq \eta_z, \eta_k \leq 0$, respectively. Due to the inequality constraints that may lead to corner point solutions, there are multiple patterns of optimal solutions. Among them, the typical case of interior point solutions is shown in Appendix A.

2.3. Firm

Firms are homogeneous and have constant returns-to-scale technology with respect to labor and capital:

$$F(L_D(t), K_D(t), A(t)) := A(t)\{\alpha_L L_D(t)^\rho + \alpha_K K_D(t)^\rho\}^{\frac{1}{\rho}} \tag{11}$$

where $L_D(t)$ and $K_D(t)$ represent labor and capital demands, respectively. $A(t)$ is the total factor productivity that increases by the exogenous rate g_A. The notation of $A(t)$ in the parentheses of $F(\cdot)$ is omitted hereafter. α_L, α_K, and ρ are parameters that are constant throughout. The labor is measured in terms of the effective labor unit that is defined by the product of human capital and working time.

Labor and physical capital supplies are given respectively by the following:

$$\bar{L}(t) := \sum_{S=1}^{S_M} h_S \int_S^{S+1} \{1 - \eta_h\} g(\boldsymbol{s}) d\boldsymbol{s} \tag{12a}$$

$$\bar{K}(t) := \int_0^{\infty} k \; g(\boldsymbol{s}) d\boldsymbol{s} \tag{12b}$$

where the low-case variables k and η_h represent the levels of one household of the state $\boldsymbol{s}$. It is assumed that $h_1 = 0$ and $h_{S_M+1} = \infty$. $\{1 - \eta_h\}$ is a time for working, whose value is determined by each household.

The factor-price markets are assumed to be sluggish. We assume that the increase rates of the nominal wage rate and the nominal return rate of physical capital are given by

$$\frac{W(t+1) - W(t)}{W(t)} \equiv \pi_W(t) := \mu + \kappa_W \cdot \left\{ \frac{L_D(t)}{\bar{L}(t)} - 1 \right\}, \tag{13a}$$

$$\frac{R_K(t+1) - R_K(t)}{R_K(t)} \equiv \pi_{RK}(t) := \mu + \kappa_{RK} \cdot \left\{ \frac{K_D(t)}{\bar{K}(t)} - 1 \right\}, \tag{13b}$$

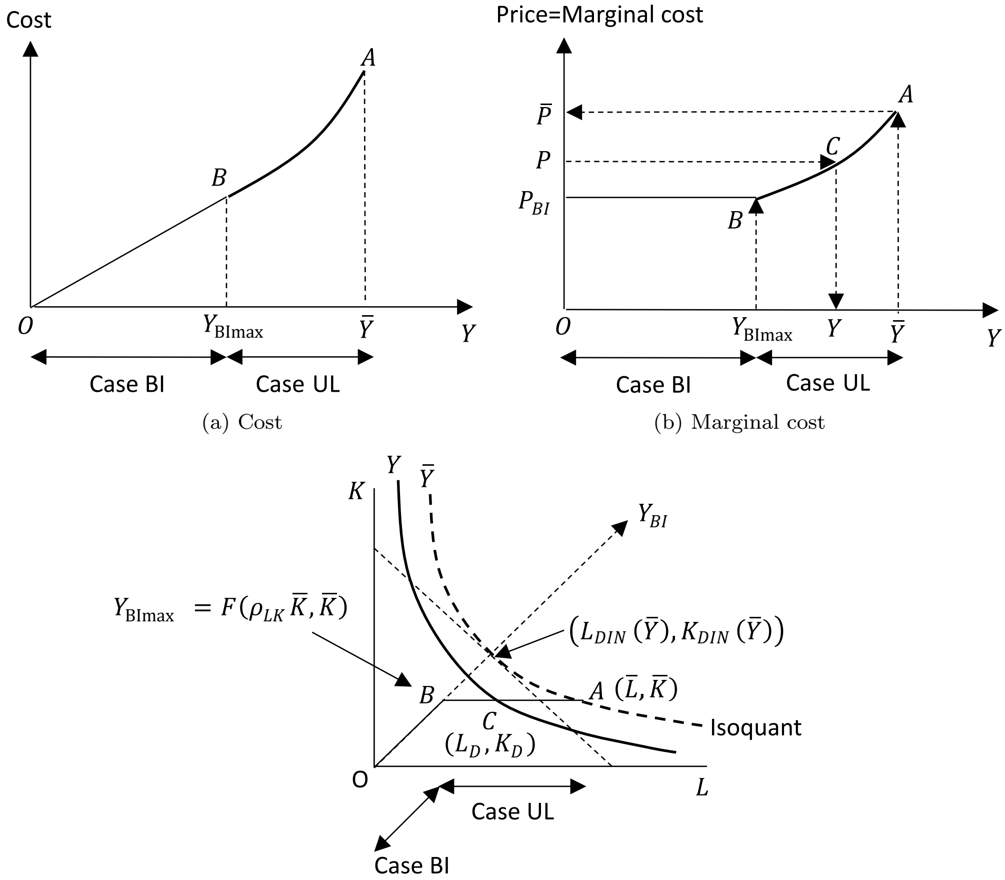
where μ is the increase rate of the money supply. κ_W and κ_{RK} are parameters of the non-negative values that reflect the speed of the price adjustment. Eq.(13a) ((13b)) implies that if the supplied labor (physical capital) is fully employed at Period t, the nominal wage rate (rate of return to physical capital) in Period $t+1$ is increased by the rate of an increase in the money supply, while, if there is unemployment, the increase rate is smaller than that.

Because the Period-t factor prices are given as of the beginning of Period t, the factor markets are closed by the quantity adjustment and associated with unemployment although the production technology is represented by the homogeneous function of degree one with respect to labor and physical capital. Figure 3 illustrates a case of unemployment of labor. Suppose $\bar{Y} := F(\bar{L}, \bar{K})$ is the full-employment production level. Because the representative firm determines the level of production Y so that its marginal cost is equalized with commodity price P, it can happen that $Y < \bar{Y}$ that is associated with $L_D < \bar{L}$. Moreover, depending on the provided (W, R_K) and Y, the input bundle (L, K) is not necessarily the interior point solution of the cost-minimizing problem; Case BI (balanced inputs) in Fig.3 indicates a case where the factor demands (L_D, K_D) are given by the interior point solution represented by $(L_{DIN}(Y), K_{DIN}(Y))$ derived in the problem:

$$\min_{L_D, K_D} WL_D + R_K K_D \tag{14a}$$

$$\text{subject to} \;\; F(L_D, K_D) = Y, \tag{14b}$$

while Case UL (unemployment of labor) applies if $K_{DIN}(Y)$ exceeds the stock $\bar{K}(t)$ (equivalently, $Y > Y_{\text{BImax}}$): the demand for labor is determined at Point C in the interval AB in Fig. 3c, namely in the area $\rho_{LK}\bar{K} := L_{DIN}(Y_{\text{BImax}}) < L_D < \bar{L}$, where $\bar{L} - L_D$ is not employed and the marginal cost of production is increasing (Figs. 3a, 3b). Case UK (unemployment of capital) can occur in the same manner. The firm's



(a) Cost

(b) Marginal cost

(c) Inputs

Figure 3.: Input and output for production

profit is derived as

$$\xi := PY - (WL_D + R_K K_D). \tag{15}$$

2.4. Government

Money is supplied based on the increase rate of money, which meets

$$\mu \equiv \frac{M_S(t+1) - M_S(t)}{M_S(t)}, \tag{16}$$

where $M_S(t)$ represents the nominal money supply. μ is assumed to be constant. The government invests in infrastructure for disaster mitigation D_M that develops by

$$D_M(t+1) = (1 - \delta_D)D_M(t) + \zeta(t), \tag{17}$$

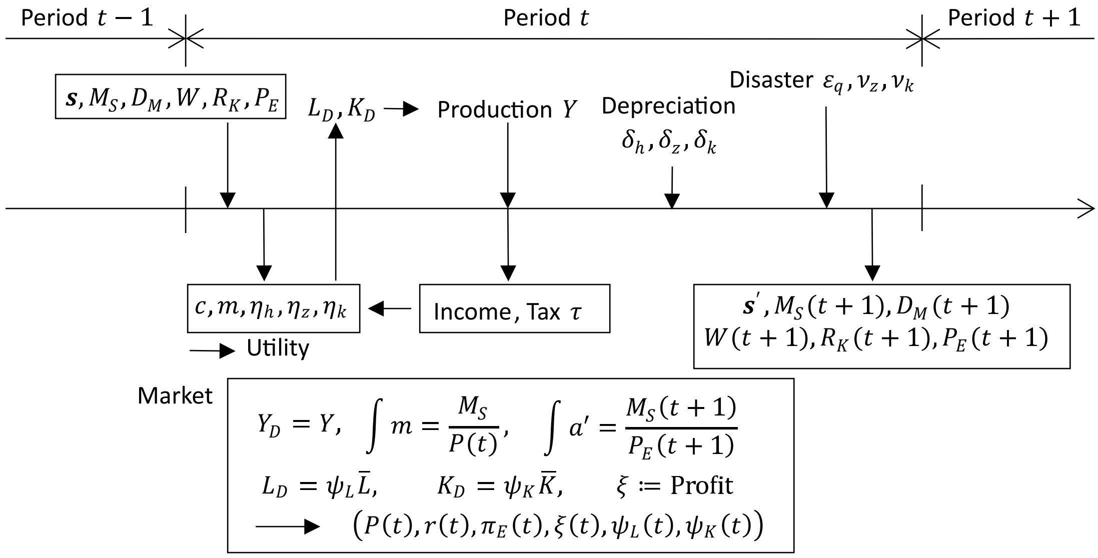


Figure 4.: Event sequence

where $\zeta(t)$ represents the investment, which is financed by seignorage and tax, namely

$$\zeta(t) = \mu \frac{M_S(t)}{P(t)} + \int \tau \; g(\boldsymbol{s}) d\boldsymbol{s} \tag{18a}$$

$$\text{where} \quad \tau := \phi_\tau \{ w h_S \cdot (1 - \eta_h) l_D + r_K k_D + \xi \} + \upsilon_\tau \tag{18b}$$

and ϕ_τ and υ_τ are the income-tax rates and the lumpsum tax, respectively, and are assumed to be constant. We assume that there is no other government's consumption and investment.

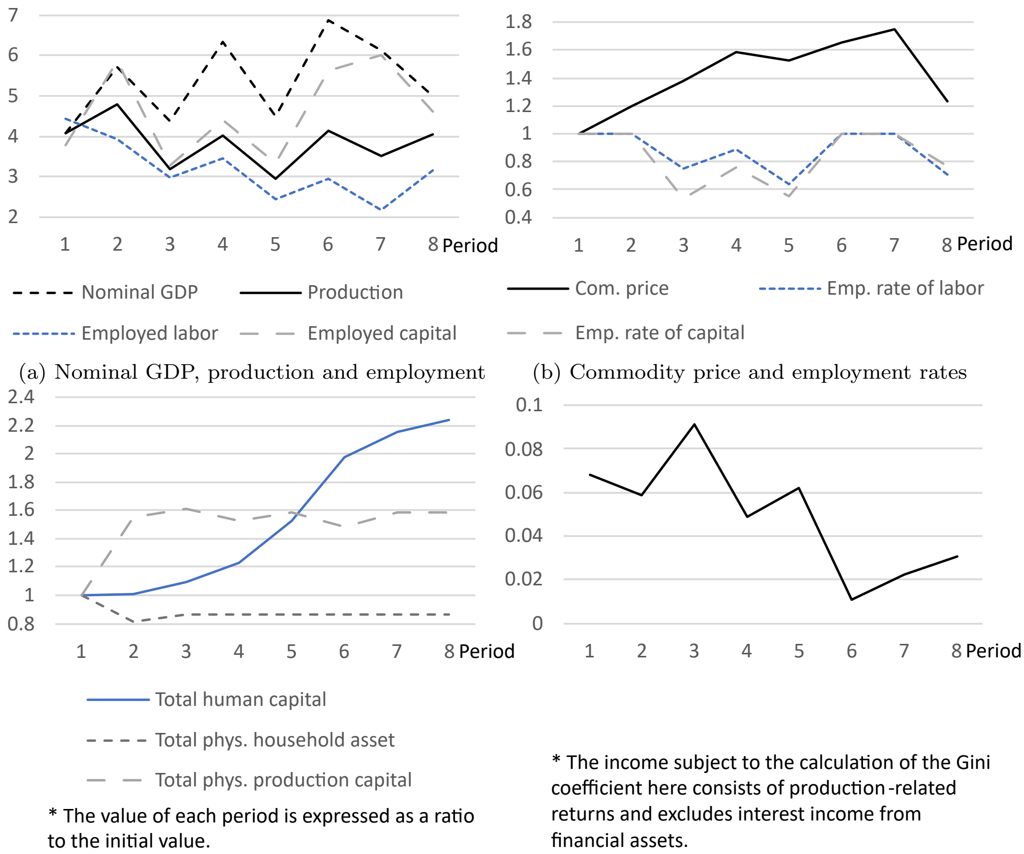
2.5. Market

Figure 4 illustrates the sequence that variables are determined. We assume that disaster randomly arrives at the end of each period, therefore, direct impacts of the period-t disaster appear in the decrease in z and k in Period $t+1$. We further assume that, due to the timing of a disaster, a realized value of the commodity price $P(t)$ and the expected inflation rates are related in the following manner:

$$P_E(t) = \{1 + \pi_E(t-1)\} \cdot P(t-1), \tag{19a}$$

$$P(t) = \{1 + \varepsilon_P(t)\} \cdot P_E(t). \tag{19b}$$

$P_E(t)$ represents the expected price that is obtained based on the expected inflation rate in Period $t-1$. Realized price $P(t)$ generally differs from $P_E(t)$ after the realization of stochastic factors related to a disaster. The market closure is given by a set of the



(a) Nominal GDP, production and employment (b) Commodity price and employment rates

(c) Total stocks (d) Gini coefficient of income

Figure 5.: Process of macroeconomic variables of No-disaster case (Case 0)

following equations:

$$Y_D := \int \{c + \eta_z z + \eta_k k\}\ g(\boldsymbol{s})d\boldsymbol{s} + \zeta = Y, \tag{20a}$$

$$\int m\ g(\boldsymbol{s})d\boldsymbol{s} = \frac{M_S}{P}, \tag{20b}$$

$$\int a'\ g(\boldsymbol{s})d\boldsymbol{s} = \frac{M_S(t+1)}{P_E(t+1)}, \tag{20c}$$

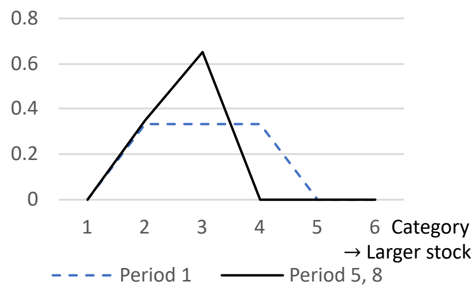
$$L_D = \psi_L \bar{L}, \tag{20d}$$

$$K_D = \psi_K \bar{K}, \tag{20e}$$

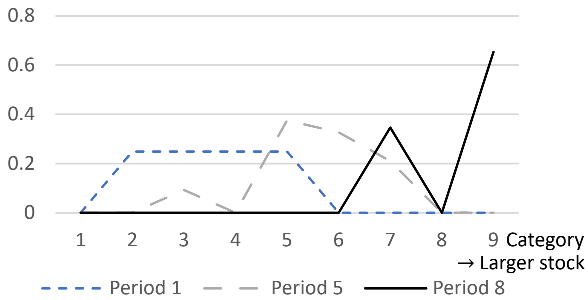
and Eq.(15) that defines the profit ξ. From the six conditions, $(P, r, \pi_E, \xi, \psi_L, \psi_K)$ are determined. The bond market is not independent and automatically closed. Eq.(20c) is derived from $\int b' = 0$, $b' = a' - m'$, and $\int m' = M_S(t+1)/P_E(t+1)$.

3. Numerical example

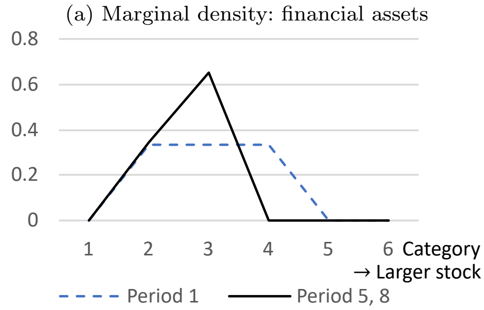
We examine the characteristics of the model by numerical simulation. Since all the parameters and the initial values of the state variables are given hypothetical values,



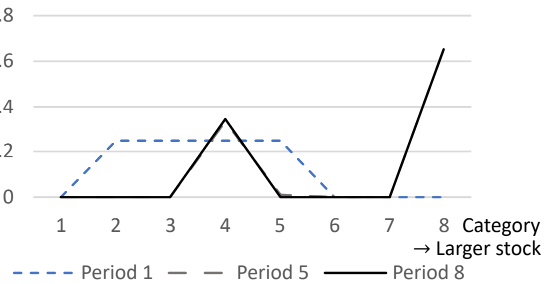
(a) Marginal density: financial assets



(b) Marginal density: human capital



(c) Marginal density: household assets

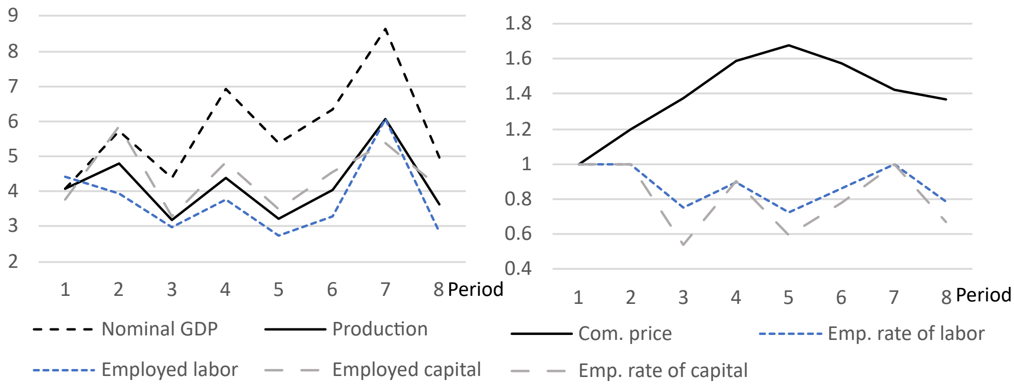


(d) Marginal density: production capital

Figure 6.: Change in distribution of household heterogeneity (Case 0)

qualitative aspects of the model behaviors are concerned here. While a quantitative assessment of a disaster and policy impacts in a specific country is of the next interest, it is an indispensable step to figure out varieties of behaviors that can be illustrated. The set of parameter values used in this example is listed in Appendix B.

The non-linear factors, which are included in the money-demand function and the human-capital-investment function, result in an endogenous business cycle. The non-linear dynamics are so sensitive to a small change in parameters, initial conditions, and disaster shocks that the resulting paths of the model economy are not easily predictable. Figure 5 shows changes in the endogenous variables in the specific case that we call "Case 0" where a disaster did not arrive as a result despite the existence of the risk in the ex-ante sense. Although the economy did not take an exogenous disaster shock, the production level, which is equivalent to real GDP in this model, oscillates with the employed labor and capital (Fig.5a). This oscillation is caused by the sluggish price adjustment as follows; in each period, the nominal factor prices, i.e., the wage rate and the rate of return to capital, are given; if they are small, demand for labor and capital increases, and production increases in that period; then, in the next period, the factor prices become higher due to Eqs.(13a)(13b), so employment decreases and production declines; in the following period, the factor prices fall, so employment increases. This repetitive element is the dominant factor in the production process in this case. Involuntary unemployment, which is indexed by the (un)employment rate where the learning time η_h is excluded from the denominator, also occurs in Periods 3-5, and 8 (Fig.5b). The economy is under the trend of inflation in the first four periods but is faced with some fluctuation of the price later. The Gini coefficient of income, although the absolute value is small due to the settings of parameter values and initial distribution of the four state variables, moves in the adverse direction against the process of the production level in most of the periods, implying that the economic recession expands a disparity between the high- and low-income households (Fig.5d). In Fig.6, changes in the distribution of the households' state variables $g(a, h, z, k)$ are



(a) Nominal GDP, production and employment (b) Commodity price and employment rates

Figure 7.: Process of macroeconomic variables of a sample disaster case (Case 1)

represented by the marginal distribution in terms of the values of the marginal density function. The horizontal axes represent categorized levels of the stocks and indicate that the stocks are larger in higher categories. While the distributions of a and z turn out to be converged as the single-peaked ones, those of h and k have double peaks in the final period, i.e., Period 8, implying that there is a group of households that are left behind.

We now pay attention to one sample process of disaster occurrences; "Case 1" is defined as a case in which a disaster arrives at the end of Periods 3 and 6, and therefore, Periods 4 and 7 are the periods immediately following the disaster. Figure 7 shows the process of the selected macroeconomic variables, and Fig.8 illustrates the disaster impacts by the differences of several variables from those of Case 0 by the black dashed lines in Figs.8a-8c. Although a part of the physical production capital is destroyed, productions are larger than those of Case 0 in Periods 4 and 7 due to the reconstruction demands for physical capital k and a physical household asset z (Fig.8a). In particular, according to the production cycle in Case 0, Period 7 was supposed to be a period of recession in terms of production (Fig.5a). However, in Case 1, production is further increased from that of Period 6 (Fig.7a). These increases in production are achieved through increases in labor supply, and in Period 4, also through increases in the employment rate of labor and capital (Figs.5b,7b). At the same time, human capital investment is decreased (Fig.8b), resulting in a decrease in the total stock (Fig.8c). It is also figured out in Fig.8d that the slowdown of development of the total human capital stock is caused by households who were in the second (i.e., the lower-human-capital) group in the final period of Case 0 because the distribution has a longer tail for the second group of Case 1. It should also be noted that production largely drops in Period 8 in Case 1, compared to other recession periods and Period 7 in Case 0 (Figs.5a,7a), resulting in the production level in Period 8 that is smaller in Case 1 (Fig.8a). This implies that a disaster could increase the real GDP in the short run but hamper the growth in the medium or long run due to the interruption of human-capital investment. Such a pattern may not be consistent with the widespread understanding that a disaster results in adverse impacts on the short-run macroeconomy while the impacts on long-term growth remain contested.

We examine the impact of a policy of consecutive investment in disaster mitigation infrastructure through income taxation, and call it "Case 2". As in Case 1, a disaster occurs at the end of Periods 3 and 6, and Periods 4 and 7 are the immediate

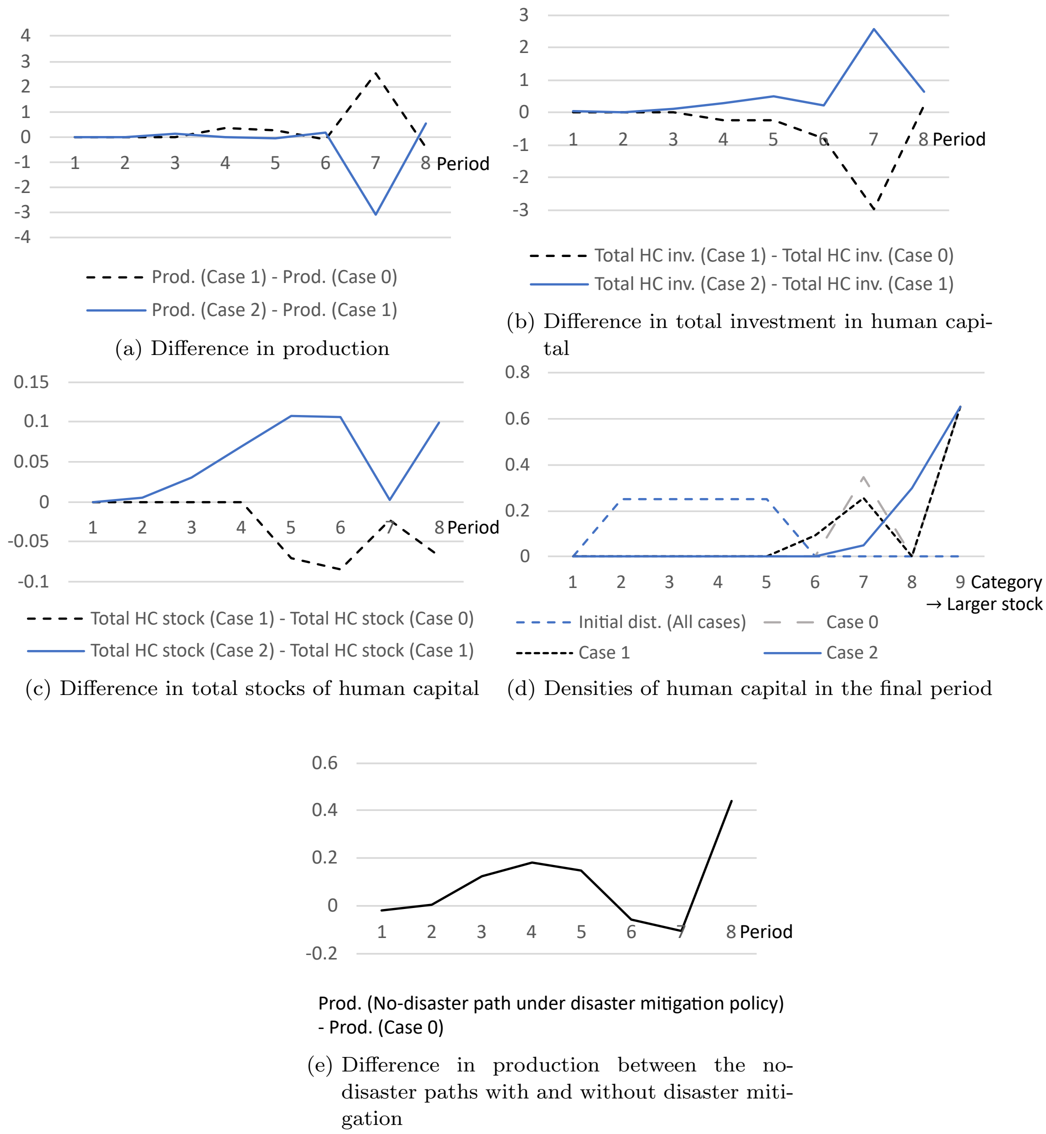


Figure 8.: Impacts of disasters and disaster mitigation (Cases 0-2)

aftermath. Figure 8 includes the comparisons with Case 1 illustrated by the blue solid lines (Figs.8a-8c). In this case, the investment in disaster mitigation is found to have a crowding-out effect on investment in physical production capital, but the increased human capital stock results in no significant difference in the production levels except in Period 7. The disaster mitigation policy supports the continued post-disaster human-capital investment opportunities, and as a result, a part of households who were in the low-human-capital group in Case 1 move to the high-human-capital group. Now, the distribution of human capital has only one peak in the marginal density function (Fig.8d). The policy contributes to reducing the human capital gaps. Figure 8e shows that production with the disaster mitigation policy is greater than production without it for most periods when they are compared in the no-disaster case. In this case, the expansion of production is not achieved through the deepening of the physical production capital, which is often pointed out as one of the effects of disaster-risk-reduction investment (e.g., Ishiwata & Yokomatsu, 2018), but mainly through the expansion of labor and capital employment.

4. Discussion

In addition to the results presented in Section 3, numerical experiments were conducted with various values of exogenous variables. As a result, a variety of qualitatively different behaviors were observed. For example, the case with a stricter borrowing constraint was examined and the results are described in Appendix C. Although the scope of the effects of a severe borrowing constraint that can be revealed in this case is limited, we find that the severe constraint deepens the total physical production capital and increases real GDP in the long run, while increasing the human capital gap among households.

Another example implies that, unlike Cases 0-2 where we observed that the level of production followed a zigzag path, when the decline in purchasing power due to lower factor prices and income becomes dominant, demand may decline and production and the commodity price may continue to fall. As a result, employment also declines, leading to a deflationary spiral. In some cases, even in the immediate aftermath of a disaster, employment of labor declines. However, if human capital is accumulated during such periods, it will exert its strength in the direction of bringing back production.

Moreover, the configuration of a dynamic path of the economy is affected by not only the initial factor prices, but also the balance between physical production capital stock and human capital stock. Where this balance is irregular, in some a gradual adjustment may occur only to be disrupted again when a disaster strikes, effectively reintroducing irregular dynamics.

In addition, human-capital-investment behavior is mediated by pecuniary externalities among households. When labor supply, measured in the efficient labor units, is increased by households of the high-human-capital group, this lowers the wage rate (per efficient labor unit). This has two effects on households of the low-human-capital group. One is the effect of encouraging human capital investment by reducing the opportunity cost of learning (i.e., human capital investment). On the other hand, if the decline in income makes the subsistence-consumption constraint binding, their time for learning will be bound at zero.

The impact of investment in disaster mitigation on human capital investment is composed of several effects that include (i) the income effect, (ii) the substitution effect, (iii) the choice-opportunity-provision effect, and (iv) the externality-reinforcement effect, which are described in the following sentences in the text and supplemented in the last paragraph of Appendix A. In terms of the income effect, the reduction in damage to a physical asset and production capital due to disaster mitigation changes income, which in turn changes all investment and consumption, including human capital investment. Although the reduction in damage to production capital increases the production capacity, the actual levels of production and income may be decreased depending on employment and other factors. It should further be noted that taxation of such disaster mitigation policies itself has a negative impact on the level of disposable income. In the substitution effect, a decrease in the expected damage rates of physical household assets and physical production capital lowers the relative effective prices of those investments (i.e., increases investment efficiency in them), and thereby, in turn, raises the relative effective price of human capital investment, which has no disaster-damage risk originally. Disaster mitigation thereby works on reducing human capital investment.

The choice-opportunity-provision effect can be considered part of the income effect, but it is unique in that it is related to the inequality constraint regarding the subsistence consumption. That is, after a disaster, households with income below a certain level will have zero human capital investment, which is obtained as the corner point solution of the optimization problem where those households try to ensure the subsistence consumption. The human capital gap between high- and low-income households widens. However, if disaster mitigation allows the latter's income to exceed the subsistence consumption condition during a disaster, positive human capital investment will be chosen as the interior point solution. This effect differs from the income effect in that here disaster mitigation selectively affects only low-income households, thus promoting a narrowing of the human capital gap.

The externality-reinforcement effect is the effect of increasing the above-mentioned pecuniary externalities between income groups regarding human capital investment. That is, disaster mitigation first affects the behaviors of the high-income group, which is able to choose the interior point solution, which in turn changes the human-capital-investment behavior of the low-income group through changes in the market wage rate and the rate of return to capital. This differs from the choice-opportunity-provision effect in that it is via the behaviors of the high-income group while the low-income group is affected by a time lag.

As discussed above, the impact of disaster mitigation on human capital investment is determined by the superposition of multiple effects. As for the example in Section 3, from the fact that human capital investment is larger in Case 2 than that of Case 1 in the immediate aftermath (Fig.8b), it can be estimated that the positive income effect and the choice-opportunity-provision effect had a dominant impact on human capital investment. In another example, we observed the substitution effects dominating, with disaster mitigation reducing human investment. In some cases, the sign of the total effect changed from period to period, even on a single sample path. As a result, changes in the Gini coefficient of income are also complex. Understanding which effects and paths have a high probability of occurring is an important issue for the future.

One of the limitations of this model is that we apply the two-period optimization problem, two major issues of which may be described as follows. First, the role of loans is limited; in the current model, the impact of increased borrowing comes only through the constrained demand for money in the next period. The reason is that such a short-term optimization problem cannot lead to a long-term repayment plan for households. In the future, a framework in which borrowing limits are endogenously determined by the possibility of repayment should be considered. Second, the levels of human capital and physical production capital are direct inputs of the second-period utility function. In particular, it is necessary to clarify the implications of the treatment of human capital, as it is a major concern of this study. This model implies that the motivation for human capital accumulation is not for the increase in future income, but for the utility of human capital itself. This would be a drawback from a viewpoint of traditional optimization problems. On the other hand, the value of education is also pointed out, such as the value of knowledge gained to improve the quality of life and human security independently of income growth (e.g., Sen et al., 2002). From this perspective, a framework in which human capital itself is incorporated into the utility function has some meaning.

The model in this study describes the impacts of disasters that appear with a time lag. As shown in the numerical example in Section 3, the impact can be considered indirect in that lower production levels than that in the immediate aftermath of disaster appear in subsequent periods. In this context, this study emphasizes, first and foremost, the problem that interruptions and withdrawals of children and youth from the learning process affect macroeconomic indicators, such as GDP, with a time lag. That time lag can extend over a decade or more in the real world. However, in one sense, these problems are relatively easy to predict to some extent if they can be captured as characteristics of the relevant cohort through observations of school attendance and labor market conditions immediately after a disaster.

On the other hand, even after the physical damage caused by a disaster has been measured, there are nonlinear dynamics that make it extremely difficult to predict the long-term effects of a disaster. In some cases, chaos occurs, and the unpredictability faced even within a deterministic framework becomes problematic, which is also seen as an indirect risk or uncertainty. Such nonlinear dynamics are derived as a result of the superposition of individual effects, and as a practical matter, they become a quantitative problem. In the future, it will be necessary to develop a framework for quantitatively evaluating indirect disaster risks with a time lag. In such a framework, distributional characteristics of probabilities such as conditional expectation, variance, and value-at-risk for each future period for a variable of interest are derived through Monte Carlo simulations which are associated with error distribution at the time of evaluation. The pattern of variation should also be examined, for example, by deriving the autocorrelation coefficient. In addition, it may be necessary to examine chaos-related indicators such as Lyapunov exponents based on the output data (e.g., Wolf, Swift, Swinney, & Vastano, 1985) to obtain information to examine the potential for disaster shocks to cause chaos in the subsequent dynamics. And if chaos does occur, the process of considering whether control of that chaos (e.g., Ott, Grebogi, & Yorke, 1990) is feasible and appropriate for the context of the problem under consideration and the structure of the model would also be included in the risk assessment. The development of the evaluation method described above will enable us to introduce a new perspective to the discussion on the short- and long-term impact of disasters.

5. Conclusion

This study formulated a model of monetary growth under disaster risk that takes into account household heterogeneity. Then, a qualitative analysis of economic dynamics was conducted through numerical simulations. It was pointed out that the impacts of disaster risk reduction policies on human capital investment behavior include four effects: the income effect, the substitution effect, the choice-opportunity-provision effect, and the externality-reinforcement effect. Furthermore, the existence of indirect impacts with time lags due to human capital investment behavior and business cycles was clarified. The study further pointed out the methodological potential of the nonlinear dynamic model for analyzing indirect risks and discussed directions for the development of an evaluation framework.

In addition to the important issues discussed in the previous section, this study leaves much to be done in the future. For example, although this study did not address policies other than investment in disaster mitigation, it is necessary to consider the creation of effective demand including ones during normal times, which could also change the impacts of a disaster. The effects of distributional policies such as progressive taxation should also be analyzed. The next important step is to conduct a case study using real data. Methods of calibration and introduction of error distributions should be investigated for this purpose.

Disclosure statement

The authors declare no conflict of interest.

Acknowledgement

This work was supported by the project "Distributional Implications of Climate-Related Disasters -A Macroeconomic Assessment (DIoD)" [Jubiläumsfondsprojekt Nr. 18654] funded by Oesterreichische Nationalbank (OeNB).

References

Achdou, Y., Han, J., Lasry, J.-M., Lions, P.-L., & Moll, B. (2022). Income and wealth distribution in macroeconomics: A continuous-time approach. *The review of economic studies*, *89*(1), 45–86.

AG van Bergeijk, P., & Lazzaroni, S. (2015). Macroeconomics of natural disasters: Strengths and weaknesses of meta-analysis versus review of literature. *Risk analysis*, *35*(6), 1050–1072.

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, *109*(3), 659–684.

Akao, K.-I., & Sakamoto, H. (2018). A theory of disasters and long-run growth. *Journal of Economic Dynamics and Control*, *95*, 89–109.

Albala-Bertrand, J.-M. (1993). Natural disaster situations and growth: A macroeconomic model for sudden disaster impacts. *World Development*, *21*(9), 1417–1434.

Alderman, H., Hoddinott, J., & Kinsey, B. (2006). Long term consequences of early childhood malnutrition. *Oxford economic papers*, *58*(3), 450–474.

Baez, J., De la Fuente, A., & Santos, I. V. (2010). Do natural disasters affect human capital? an assessment based on existing empirical evidence.

Baez, J. E., & Santos, I. V. (2007). Childrens vulnerability to weather shocks: A natural disaster as a natural experiment. *Social science research network, New York*.

Benhabib, J. (1992). *Cycles and chaos in economic equilibrium*. Princeton University Press.

Bischi, G. I., Chiarella, C., & Gardini, L. (2010). *Nonlinear dynamics in economics, finance and social sciences* (Vol. 123). Springer.

Bista, R. B. (2020). Does disaster change income and wealth distribution toward extremity of inequality and poverty? analysis of flood and landslides in the vulnerable locations of nepal. In *Forum for social economics* (pp. 1–15).

Carter, M. R., Little, P. D., Mogues, T., & Negatu, W. (2007). Poverty traps and natural disasters in ethiopia and honduras. *World Development*, *35*(5), 835-856. Retrieved from `https://www.sciencedirect.com/science/article/pii/S0305750X07000149`

Cavallo, E., Galiani, S., Noy, I., & Pantano, J. (2013). Catastrophic natural disasters and economic growth. *Review of Economics and Statistics*, *95*(5), 1549–1561.

Choquette-Levy, N., Wildemeersch, M., Oppenheimer, M., & Levin, S. A. (2021). Risk transfer policies and climate-induced immobility among smallholder farmers. *Nature Climate Change*, *11*(12), 1046–1054.

Cuaresma, J. C. (2010). Natural disasters and human capital accumulation. *The World Bank Economic Review*, *24*(2), 280–302.

Cuaresma, J. C. (2022). Natural disasters and economic growth: revisiting the evidence. In *Handbook on the economics of disasters* (pp. 134–149). Edward Elgar Publishing.

De Janvry, A., Finan, F., Sadoulet, E., & Vakis, R. (2006). Can conditional cash transfer programs serve as safety nets in keeping children at school and from working when exposed to shocks? *Journal of development economics*, *79*(2), 349–373.

Fisher, I. (1930). *The theory of interest (new york, mcmillan).* Porcupine Press Reprinted, Philadelphia.

Frisch, R. (1933). Propagation problems and impulse problems in dynamic economics. *Essays in Honor of Gustav Cassel*.

Goodwin, R. M. (1990). Chaotic economic dynamics..

Hallegatte, S., Hourcade, J.-C., & Dumas, P. (2007). Why economic dynamics matter in assessing climate change damages: illustration on extreme events. *Ecological economics*, *62*(2), 330–340.

Hallegatte, S., Jooste, C., & Mcisaac, F. J. (2022). Macroeconomic consequences of natural disasters.

Hochrainer-Stigler, S., & Poledna, S. (2016). Modelling dependent risk with copulas: An application on flooding using agent-based modelling. *Geoinformatics Research Papers*, *4*(BS4002).

Ishiwata, H., & Yokomatsu, M. (2018). Dynamic stochastic macroeconomic model of disaster risk reduction investment in developing countries. *Risk Analysis*, *38*(11), 2424–2440.

Isoré, M., & Szczerbowicz, U. (2017). Disaster risk and preference shifts in a new keynesian model. *Journal of Economic Dynamics and Control*, *79*, 97–125.

Jacoby, H. G., & Skoufias, E. (1997). Risk, financial markets, and human capital in a developing country. *The Review of Economic Studies*, *64*(3), 311–335.

Keen, B. D., & Pakko, M. R. (2007). Monetary policy and natural disasters in a dsge model. *Federal Reserve Bank of St. Louis Working Paper Series*(2007-025).

Kraay, A., & Raddatz, C. (2007). Poverty traps, aid, and growth. *Journal of Development Economics*, *82*(2), 315–347.

Lasry, J.-M., & Lions, P.-L. (2007). Mean field games. *Japanese journal of mathematics*, *2*(1), 229–260.

Loayza, N. V., Olaberria, E., Rigolini, J., & Christiaensen, L. (2012). Natural disasters and growth: Going beyond the averages. *World Development*, *40*(7), 1317–1336.

Naqvi, A. A., & Rehm, M. (2014). A multi-agent model of a low income economy: simulating the distributional effects of natural disasters. *Journal of Economic Interaction and Coordination*, *9*(2), 275–309.

Noy, I. (2009). The macroeconomic consequences of disasters. *Journal of Development economics*, *88*(2), 221–231.

Ono, Y. (2001). A reinterpretation of chapter 17 of keynes's general theory: Effective demand shortage under dynamic optimization. *International Economic Review*, *42*(1), 207–236.

Ott, E., Grebogi, C., & Yorke, J. A. (1990). Controlling chaos. *Physical review letters*, *64*(11), 1196.

Petal, M., Wisner, B., Kelman, I., Alexander, D., Cardona, O., Benouar, D., ... others (2015). School seismic safety and risk mitigation. *Encyclopedia of Earthquake Engineering*, 2450–2468.

Pleninger, R. (2022). Impact of natural disasters on the income distribution. *World Development*, *157*, 105936.

Raddatz, C. (2007). Are external shocks responsible for the instability of output in low-income countries? *Journal of Development Economics*, *84*(1), 155–187.

Rezai, A., Taylor, L., & Foley, D. (2018). Economic growth, income distribution, and climate change. *Ecological Economics*, *146*, 164–172.

Sen, A., et al. (2002). Basic education and human security. *Extraído de: http://www. humansecuritychs. org/activities/outreach/Kolkata. pdf*.

Shabnam, N. (2014). Natural disasters and economic growth: A review. *International Journal of Disaster Risk Science*, *5*(2), 157–163.

Sidrauski, M. (1967). Rational choice and patterns of growth in a monetary economy. *The American Economic Review*, *57*(2), 534–544.

Skidmore, M., & Toya, H. (2002). Do natural disasters promote long-run growth? *Economic inquiry*, *40*(4), 664–687.

UNISDR, G. (2009). Global assessment report on disaster risk reduction. *United Nations*

International Strategy for Disaster Reduction Secretariat Geneva, Switzerland.
Wolf, A., Swift, J. B., Swinney, H. L., & Vastano, J. A. (1985). Determining lyapunov exponents from a time series. *Physica D: nonlinear phenomena*, *16*(3), 285–317.
Yokomatsu, M., Mochizuki, J., Joseph, J., Burek, P., & Kahil, T. (2022). Macroeconomic co-benefits of drr investment: assessment using the dynamic model of multi-hazard mitigation cobenefits (dynammics) model. *Disaster Prevention and Management: An International Journal*(ahead-of-print).

Appendix A. Interior optimum of household problem

The household problem represented by Eqs.(10a)(10b) in 2.1 contains multiple inequality constraints. Including the case splitting due to the influence of market variables, there are about 20 different case splits for obtaining the optimal solution by numerical analysis. Among them, one representative case is shown below, where the market nominal interest rate is positive, households have access to borrowing, the available budget is above the subsistence income level, and money demand and human capital investment are determined by the interior point solution.

When focusing on the interior point solution, the following substitutions of the control variables may provide a better outlook for understanding the structure of the problem.

$$\tilde{c} := c - \underline{c} \ > 0, \quad \tilde{a}' := a' - \underline{a} \ > 0, \tag{A1a}$$

$$\tilde{\eta}_z := 1 + \eta_z \ > 0, \quad \tilde{\eta}_k := 1 + \eta_k \ > 0, \tag{A1b}$$

Since the inequality constraint (10b) is not binding in this case, we can deal with the above substituted variables that are guaranteed to take positive values. Equation (5a) is transformed into the following equation:

$$\tilde{c} + Rm + P_h \eta_h + z\tilde{\eta}_z + k\tilde{\eta}_k + \tilde{a}' = y_{B0}, \tag{A2a}$$

where

$$P_h := wh_S l_D(1 - \phi_\tau), \tag{A2b}$$

$$y_{B0} := (1 + r)a + (wh_S l_D + r_K k_D + \xi)(1 - \phi_\tau) - \upsilon_\tau + z + k - \underline{c} - \underline{a}. \tag{A2c}$$

Equation (A2a) implies that R and P_h are the effective prices of money and human capital investment, respectively, which are also interpreted as opportunity costs. y_{B0} means the amount of available financial resources, net of subsistence consumption $\underline{c}$ and the lowest financial asset $\underline{a}$. Equation (A2c) implies that it is also possible to sell a physical household asset z and capital k. The interior point solutions of the optimization problem, which are attached asterisks, meet the following relations.

$$\tilde{c}^* = c^* - \underline{c} = \frac{\gamma_c{}^{\frac{1}{\theta}}}{\Gamma_{B1}} y_{B1}, \quad m^* = \frac{\left(\gamma_m R^{-1}\right)^{\frac{1}{\theta}}}{\Gamma_{B1}} y_{B1}, \tag{A3a}$$

$$h \cdot (1 + \iota\eta_h^*) = \frac{\left(\bar{\gamma}_h P_h^{-1} h\right)^{\frac{1}{\theta}}}{\Gamma_{B1}} y_{B1}, \quad z\tilde{\eta}_z^* = z \cdot (1 + \eta_z^*) = \frac{\bar{\gamma}_z^{\frac{1}{\theta}}}{\Gamma_{B1}} y_{B1}, \tag{A3b}$$

$$k\tilde{\eta}_k^* = k \cdot (1 + \eta_k^*) = \frac{\bar{\gamma}_k^{\frac{1}{\theta}}}{\Gamma_{B1}} y_{B1}, \quad \tilde{a}'^* = a'^* - \underline{a} = \frac{(\beta\gamma_a)^{\frac{1}{\theta}}}{\Gamma_{B1}} y_{B1}, \tag{A3c}$$

where

$$\bar{\gamma}_h := \iota\beta\gamma_h\bar{\gamma}_{hh}(1 - \delta_h)^{1-\theta}, \quad \bar{\gamma}_{hh} := 1 + \gamma_{hh} \cdot \left(1 - \frac{h_{S+1} - h}{h_{S+1} - h_S}\right), \tag{A3d}$$

$$\bar{\gamma}_z := \gamma_z + \beta\gamma_{zz}(1 - \delta_z)^{1-\theta}(1 - \lambda \cdot \nu_{zE} \cdot \chi_z)^{1-\theta}, \tag{A3e}$$

$$\bar{\gamma}_k := \beta\gamma_k(1 - \delta_k)^{1-\theta}(1 - \lambda \cdot \nu_{kE} \cdot \chi_k)^{1-\theta}, \tag{A3f}$$

$$\Gamma_{B1} := \gamma_c{}^{\frac{1}{\theta}} + R^{1-\frac{1}{\theta}}\gamma_m{}^{\frac{1}{\theta}} + \iota^{-1} P_h{}^{1-\frac{1}{\theta}} \bar{\gamma}_h^{\frac{1}{\theta}} h^{\frac{1}{\theta}-1} + \bar{\gamma}_z^{\frac{1}{\theta}} + \bar{\gamma}_k^{\frac{1}{\theta}} + (\beta\gamma_a)^{\frac{1}{\theta}}, \tag{A3g}$$

$$y_{B1} := y_{B0} + \iota^{-1} P_h. \tag{A3h}$$

y_{B1} means the effective disposable budget. The fraction on the right-hand-side of each of the six equations of Eqs.(A3a)-(A3c), which is a multiplier of y_{B1}, represents the effective budget share assigned to the term on the left-hand-sides. $\bar{\gamma}_z$ and $\bar{\gamma}_k$, which compose the effective budget share, include not only the weights of the utility function but the disaster arrival rate, the expected damage rates, and the disaster mitigation effects. A portion of the market prices, R and w, is also included in the composition of the share. Hence a set of the effective budget shares changes every period.

The four effects of disaster mitigation on human capital investment, which were discussed in Section 4, may be explained by focusing on equations of the optimal choices introduced above. (i) The income effect is related to the change in y_{B1}. y_{B0}, which is a part of y_{B1}, includes the distribution of real GDP, which is naturally expected to increase after disaster damage is mitigated. However, a sign of the impacts on real GDP is not certain when also considering the possibility that larger reconstruction demand stimulates production. At the same time, a reduction in damage to z and k that are included in y_{B0} in Eq.(A2c) would increase household purchasing power. (ii) The substitution effect of disaster mitigation is derived from the change in the effective budget shares. A decrease in the expected damage rates increases $\bar{\gamma}_z$ and $\bar{\gamma}_k$ if θ is smaller than one (Eqs.(A3e)(A3f)), resulting in an increase in Γ_{B1} (Eq.(A3g)) and finally a negative impact on human capital investment (Eq.(A3b)). (iii) The choice-opportunity-provision effect is explained in the way that households that would have been forced to choose the corner point solution $\eta_h = 0$ are given the opportunity to choose $\eta_h > 0$ by making their available resources larger than one under the constraint through disaster mitigation. (iv) The externality-reinforcement effect is brought by changes in market variables such as w, r_K, and so on. One of the prominent cases may be the process by which disaster mitigation policies encourage investment in human capital by high-income households, resulting in a decrease in w, followed by a decrease in P_h and y_{B0} (Eqs.(A2b)(A2c)), and finally a change in human capital investment by low-income households (Eq.(A3b)).

Table B1.: Values of main exogenous variables for the numerical example

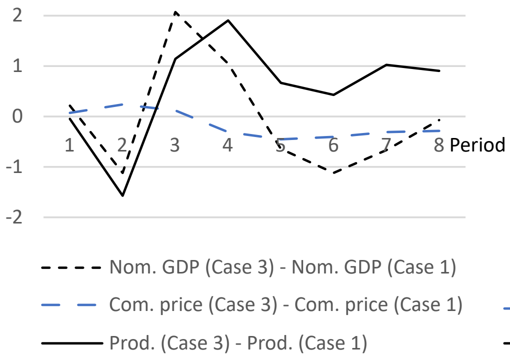
Parameter	θ	β	γ_c	γ_m	γ_z	γ_a	γ_h	γ_{hh}	γ_{zz}	γ_k	$\underline{c}$
Value	0.8	0.9	1.6	0.3	1.2	5.4	1.1	0.0	0.5	4.05	1.2
Parameter	g_A	ρ	α_L	α_K	ι	κ_W	κ_{RK}	$\delta_h, \delta_z, \delta_k, \delta_D$			
Value	0.01	-0.67	0.5	0.5	0.6	0.8	0.8	0.03			
Parameter	λ	ϕ_{10}	ϕ_{11}	χ_{z1}, χ_{k1}		b_{Lim}	h-levels to enter a higher h_S				
Value	0.4	0.9	0.5	1.5		-7.0	(2.0, 3.4, 7.8, 10.6)				
Stock/price		M_S	D_M	W	R_K	P_E					
Initial value		2.8	1.0	0.31	0.26	1.0					
Individual state				a		h		z		k	
Interval of initial distribution				[-1.6,7.2]		[3.4,7.8]		[1.7,3.8]		[2.7,4.9]	
Policy parameter		μ	ϕ_τ (Cases 0,1,3)			ϕ_τ (Case 2)		υ_τ			
Value		0.0	0.0			0.03		0.0			

Appendix B. Values of parameters and initial states for the numerical example

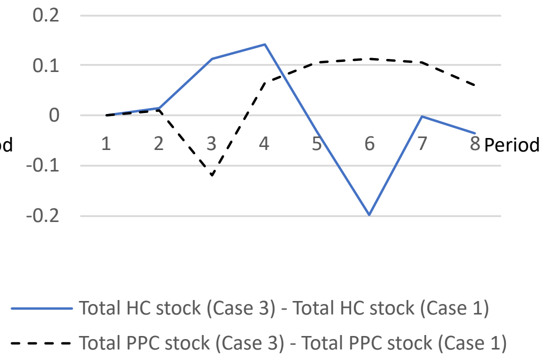
Table B1 shows the values of the main parameters, initial states, and prices that are used in the numerical example in Section 3. Those values are not based on real data but are determined by assumption. The initial distribution of households' states is assumed to be uniformly distributed over the intervals noted in the table. The results of the small absolute values of the Gini coefficient shown in Fig.5d for instance are partially due to the assumption that the initial distributions of the four state variables are independent of one another. The simulation results were found to be highly sensitive to the values of some parameters and initial states. The configuration of the grid in (a, h, z, k) space is also influential in relation to the rounding process. A simulation strategy should be developed that takes into account the computational cost and acceptable level of rounding error.

Appendix C. Case of the severe borrowing constraint

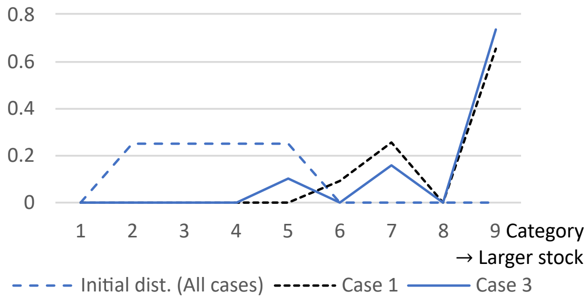
The case of the severe borrowing constraint is designated as "Case 3", where b_{Lim} is given the value -0.5. As in Case 1, a disaster occurs at the end of Periods 3 and 6. Disaster mitigation policy is not considered. The results are shown in Fig.C1, comparing several variables with Case 1. In this model, because each household's decision is made by maximizing its utility only for the two periods that follow and there is no long-term budget constraint, the direct impact of the borrowing constraint is limited; it mainly prohibits households from going into large debt for the purpose of holding money as represented by Eq.(9). As a result, the demand for money is suppressed, and the commodity price rises at the beginning, although after the first hit of a disaster at the end of Period 3, the price moves below that of Case 1 (Fig.C1a). Human capital investment in the aftermath of the disaster is discouraged (Fig.C1b). On the other hand, the borrowing constraint results in the deepening of physical production capital (Fig.C1b) and an increase in real GDP in the long run (Fig.C1a). The total stock of human capital in the final period is slightly smaller than in Case 1 (Fig.C1b), but the stock level of each household is more widely distributed (Fig.C1c). There is now the third group that is left behind the second one. Based on the results, we may make



(a) Difference in Nominal GDP, production, and price



(b) Difference in Total stocks of human capital and physical production capital



(c) Densities of human capital in the final period

Figure C1.: Comparison between Case 1 and Case 3

inferences that severe borrowing constraints remove children of low-income households from schools and more access to loans could increase the chances of continuous learning.