

SYNTHESIS OF OPTIMIZED COMPLIANT MECHANISMS FOR ULTRA-PRECISION APPLICATIONS

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ABSTRACT

Compliant mechanisms for ultra-precision applications are often required to achieve highest accuracy over largest possible ranges of motion along multiple axes. The typical synthesis approach for such high demands is based on the substitution of the revolute joints of a suitable rigid-body model with optimized flexure hinges. However, during the transition from rigid-body model to compliant mechanism, the effects of multiple input parameters are still widely unknown. Among them are the degrees of freedom of the rigid-body model, the integration of the drive elements, as well as the coupling of mechanisms to achieve multiple motion axes. The following contribution expands the fundamentals of the synthesis of compliant mechanisms based on rigid-body models for their application in ultra-precision technologies. Based on the investigation of the aforementioned parameters as well as the knowledge gained from previous research work, a novel synthesis method has been developed.

Index Terms – compliant mechanisms, synthesis, ultra-precision, embodiment design

1. INTRODUCTION

Further development of ultra-precision technologies demands increasingly accurate motions in ever-expanding working spaces. Examples include micro- and nanopositioning systems [1] and mass measuring instruments [2]. Interest lies currently in the use of compliant mechanisms due to their high motion repeatability and applicability in vacuum and clean room environments. Their synthesis often compromises between achievable accuracy and range of motion as well as footprint. The elastic limit of the material represents the main constraint. For a given design space, largest motion ranges are mainly achieved using structures with distributed compliances, whereas highest motion accuracies require concentrated compliances, i.e., flexure hinges.

In the specialized literature, numerous synthesis methods have been proposed. The Freedom and Constraint Topology (FACT) method presented in [3] provides an extensive library of geometric shapes that represent regions where compliant elements should be placed to achieve motion along a specific number of axes. Compliant mechanisms can also be developed using one or more existing modules that perform certain functions according to the Building-Block method [4]. In the Rigid-Body Replacement method, idealized links and joints of a suitable rigid-body model are replaced with compliant elements [5]. Topology Optimization is also frequently used to synthesize free-form compliant structures with optimal properties for a defined set of boundary conditions [6]. Among these, the Rigid-Body Replacement method



offers the most potential for the straightforward synthesis of ultra-precision compliant mechanisms due to its intuitiveness and predictable behavior.

The focus of the previous methods lies mainly on the layout of the compliant mechanism. However, for ultra-precision applications, other often-neglected design parameters directly affect the achievable performance. Among them are the effects of the kinematic structure as well as the integration of the drive elements. Currently, there is no systematic synthesis approach in the literature that takes these parameters into account. The following contribution looks to investigate these effects and integrate them into the method presented in [5] for the synthesis of compliant mechanisms for ultra-precision applications.

2. KINEMATIC STRUCTURE

In high-precision technologies, most operations performed by compliant mechanisms are an extension of positioning [7]. Complex motions are mainly achieved by combining translational and rotational motions along one or more axes. Mechanisms for purely linear and rotational motions represent, thus, the cornerstone of high-precision compliant systems. In this section, kinematic structures for producing rectilinear guided motions as well as their coupling into higher-order systems are investigated and compared.

2.1 Guiding mechanisms

Compliant elements are ideally subjected to pure bending due to their higher compliance in comparison to axial, shear or torsional directions. As such, they are commonly used as idealized revolute joints, i.e., flexure hinges. Rotations around a fixed axis can also be realized using multiple linkage configurations. However, a single flexure hinge has been proved to produce the lowest deviation of the ideal rotational axis [8]. Rectilinear motions can only be achieved by either subjecting compliant elements to complex loading or through linkage configurations with flexure hinges. Linkages for approximate linear motions are well known in the literature [9-11]. Figure 1 shows some common rigid-body models. Linear motion of the output stage can be achieved by coupling multiple parallel-crank linkages (Figure 1a) or two identical rectilinear point-guiding mechanisms, e.g., Figures 1b and 1c, in a parallel arrangement. In a parallel-crank linkage, the crank works as the guiding mechanism of point/joint A.

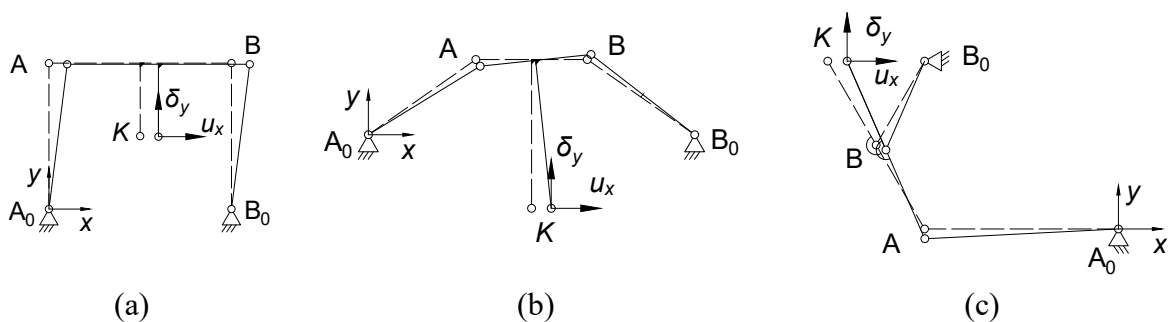


Figure 1: Rigid-body models: (a) parallel-crank; (b) Roberts linkage; (c) Evans linkage

The synthesis of compliant mechanisms based on rigid-body models presented in [5] requires the revolute joints to be replaced by flexure hinges with an optimized power-function contour [12]. By setting the geometric parameters and material properties, the exponent of the power function n can be determined with the rotation angle of each joint according to the rigid-body model. Computer-aided design tools, design equations and graphs can be used for this purpose

[13-15]. Figure 2 shows the compliant mechanisms designed using the rigid body models of Figure 1 for a deflection $u_x = +10$ mm. The planar mechanisms are to be cut out of a plate of width $w = 6$ mm. The flexure hinges have a minimum height of $h = 0.3$ mm, maximum height of $H = 10$ mm and length of $L = 10$ mm. The material is a high-strength aluminum with a permissible elastic strain of $\varepsilon = 0.5\%$.

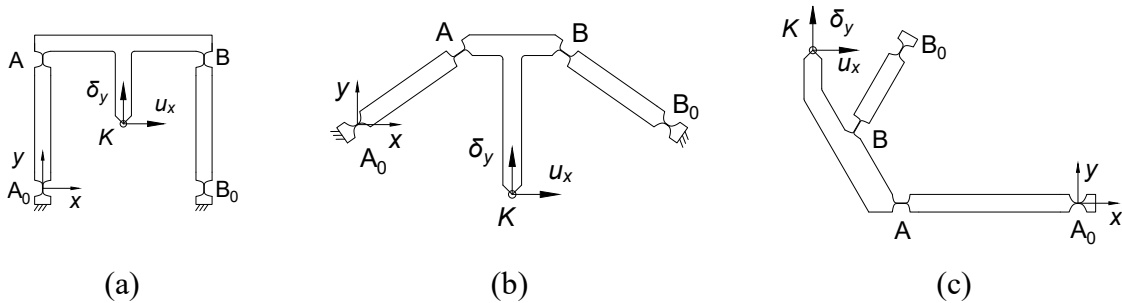


Figure 2: Compliant mechanisms: (a) parallel-crank ($n_{A0} = n_A = n_{B0} = n_B = 6$); (b) Roberts linkage ($n_{A0} = n_{B0} = 3$, $n_A = 10$, $n_B = 8$); (c) Evans linkage ($n_{A0} = 3$, $n_A = 9$, $n_B = 16$, $n_{B0} = 5$)

The motion behavior of the compliant mechanisms (CM) is investigated using finite element simulations and compared to the rigid-body models (RBM). In the finite element model, the flexure hinges are modeled using 20-node 3D elements connected with 1D beam elements [16]. A fine mesh is used on the flexure hinges to ensure high result accuracy. Figure 3a shows the guiding deviation of all models. Both the Roberts and Evans mechanisms achieve guiding deviations below 1%. The path deviation to their rigid-body models is higher than the parallel-crank due to the lower rotational precision of contours with higher exponents. Still, the Evans and Roberts linkages can be used to synthesize very precise plane-guiding mechanisms.

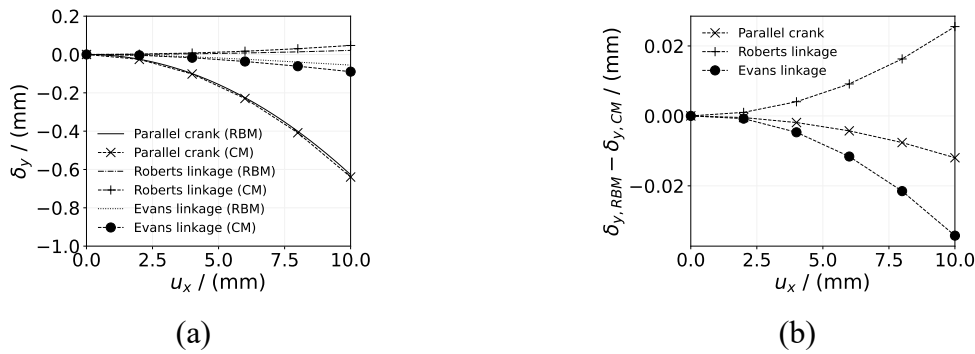


Figure 3: Motion behavior of guiding mechanisms: (a) guiding; (b) path deviations

2.2 Kinematic constraints

In rigid-body models, certain degrees of freedom (DoF), which depend on the kinematic constraints of the system, are assumed to be ideal. In contrast, compliant systems can deform in any direction under specific loading and boundary conditions. Thus, compliant mechanisms based on kinematically over- and under-constrained rigid-body structures can still realize a desired output motion. Typical examples are the serial and parallel double parallel-crank mechanisms which compensate for the guiding deviation of the single parallel crank. This is also valid for other plane-guiding mechanisms. Figure 4 shows the influence of the degrees of freedom of the rigid-body model on the elasto-kinematic properties of the derived compliant mechanism. Kinematic structures based on the parallel cranks used for realizing an almost rectilinear motion are investigated due to their good accordance to the rigid-body model. For

the sake of comparison, the deflection angle of the joints of the parallel cranks, and thus, the design of the flexure hinges ($n = 4$) is kept equal.

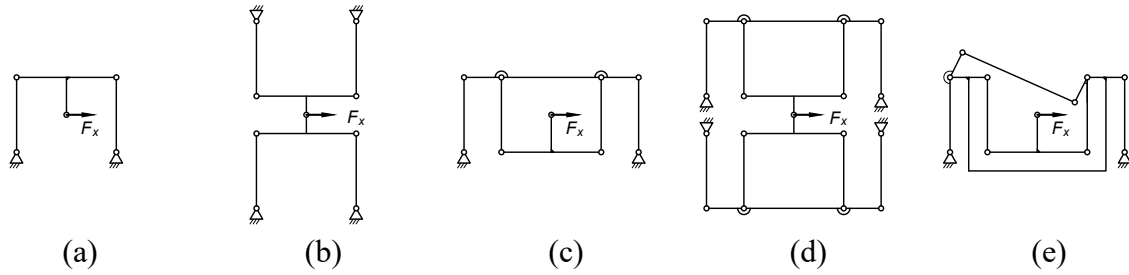


Figure 4: Rigid-body models for parallel-crank based guiding mechanism:

(a) DoF=1; (b) DoF = -1; (c) DoF = 2; (d) DoF = 1; (e) DoF = 1

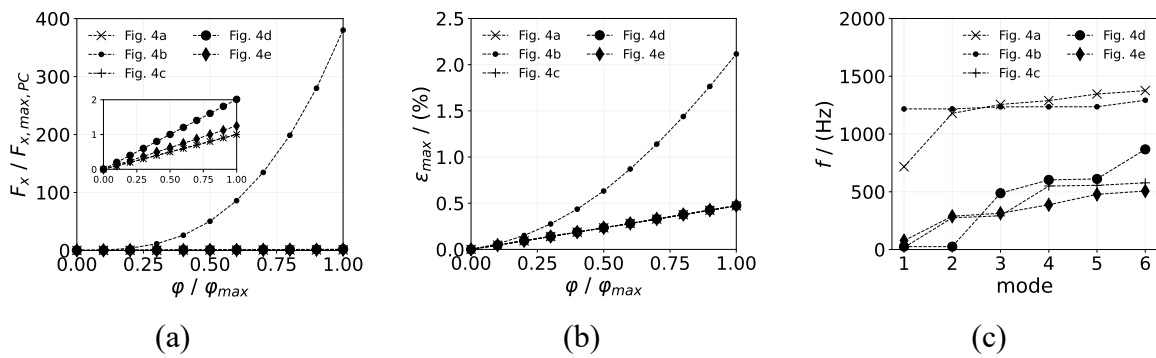


Figure 5. Influence of degrees of freedom of the rigid-body model on:

(a) restoring force; (b) elastic strain; (c) natural frequencies

The results in Figures 5a and 5b show that the elastic strain and stiffness of mechanisms based on an over-constrained rigid-body model (Figure 4b) increases significantly with the deflection angle. This is due to the hinges being subjected to axial loads caused by the additional constraints, resulting in higher actuation forces and reduced motion ranges. On the other hand, mechanisms based on an under-constrained rigid-body model (Figure 3c) possesses lower natural frequencies (under a constrained coupling point). This signifies lower lateral stiffnesses, which may lead to stability problems. In the case of kinematically defined models, the vibration modes depend on the substructures composing the mechanism. For example, the intermediate stages of the models in Figures 3d and 3e can still move independent of the output stage and frame as a parallel mechanism with two redundant cranks. In the case of the simple parallel-crank, the first mode of vibration is bending on the out-of-plane direction, for which the stiffness is very high.

2.3 Structure coupling

To produce positioning systems with degrees of freedom higher than 1 according to the rigid-body model, multiple mechanisms with DoF = 1 can be coupled either in a serial or parallel arrangement. In addition, the drive elements can also be coupled in series (carried) or in parallel (fixed). Table 1 shows the possible coupling combinations for producing mechanisms with two degrees of freedom based on parallel-crank linkages. In the case of parallel structure coupling, 2 redundant parallel cranks are needed to decouple the drive element from the guide. The result is a special case of mechanism with two degrees of freedom (DoF = 1 according to the Grübler equation [17]).

Table 1. Coupling of parallel-crank linkages into mechanism with DoF = 2

		drive	
		serial (carried)	parallel (fixed)
structure	serial		
	parallel		

The motion behavior of the mechanisms in Table 1 are compared using finite element models. Figures 6-9 show the results of the positioning deviations in x and y as well as the yaw error θ of the different structures. In the case of serial structures with a carried drive, the yaw error of each stage adds up, but the positioning deviations remain independent from one another. When both drives are fixed, there is a relative lateral motion between the carried stage and its drive. This leads to a change on the force application point, friction and a deflection of the carried stage at $u_y = 0$. While the positioning deviation in y -direction depends only on the yaw error, the overall reproducibility is affected by the friction due to the relative motion. In the case of parallel structures with fixed drives, the axes are not independent from one another. The input motion of one axis can affect the position of the other axis depending on the support stiffness of the guiding mechanism. A carried (and stiff) actuator instead limits the deviation in that axis caused by the deformation of the intermediate stage. For quasistatic applications, serial structures with carried drives are to be preferred due to their predictable behavior and small footprint. Serial structures also facilitate the integration of rotational degrees of freedom with independent drives.

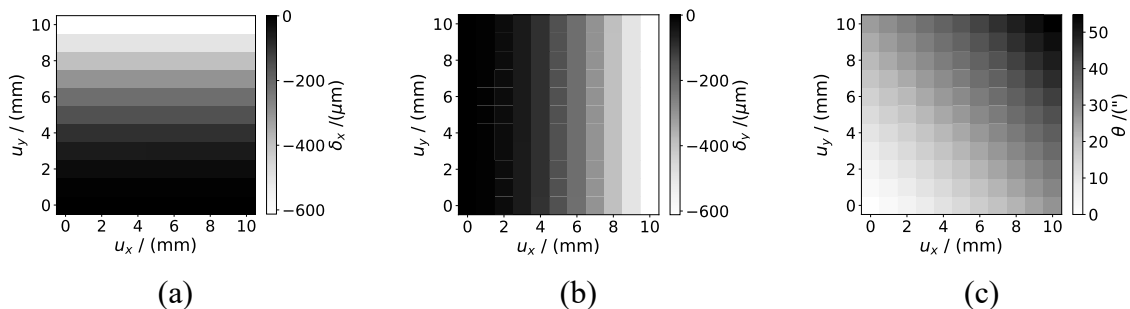


Figure 6. Positioning deviations of serial structure with carried drive: (a) δ_x , (b) δ_y , (c) θ

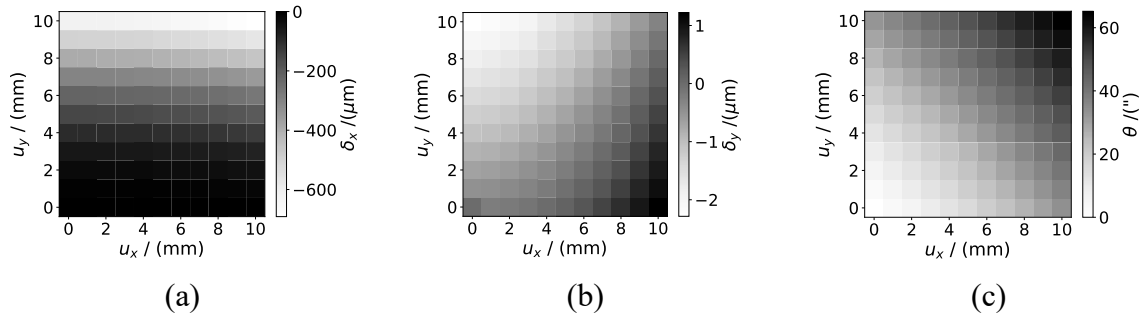


Figure 7. Positioning deviations of serial structure with fixed drives: (a) δ_x , (b) δ_y , (c) θ

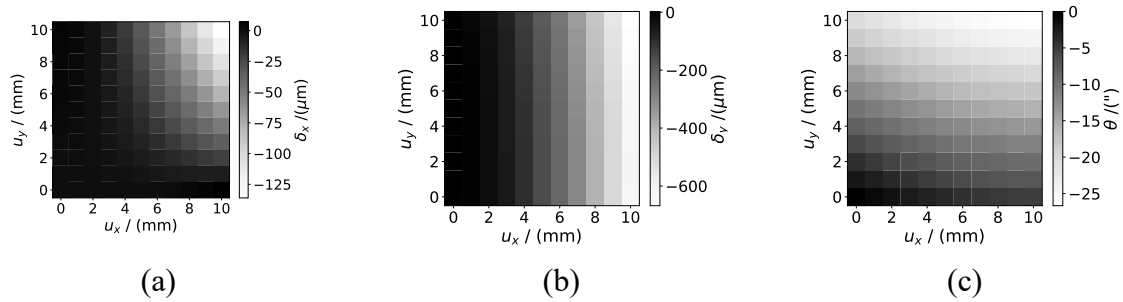


Figure 8. Positioning deviations of parallel structure with carried drive: (a) δ_x , (b) δ_y , (c) θ

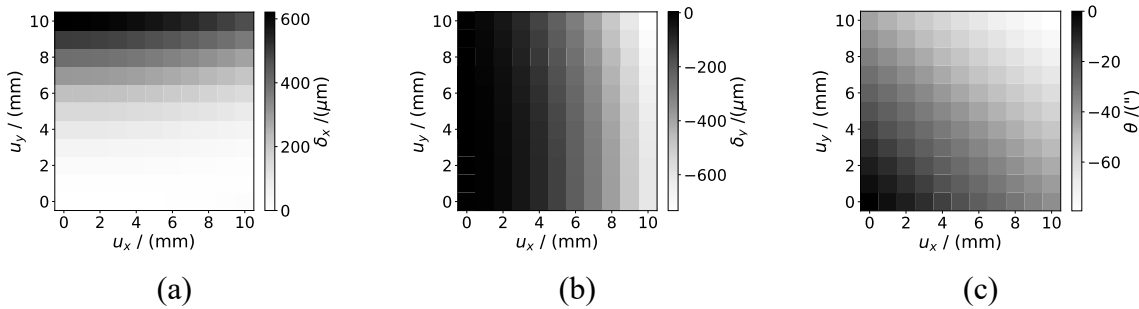


Figure 9. Positioning deviations of parallel structure with fixed drives: (a) δ_x , (b) δ_y , (c) θ

3. DRIVE ELEMENTS

3.1 Load application point

The influence of the load application point on the motion behavior is investigated on the parallel-crank linkage. Figure 10 shows the variation of the positioning deviations when the load is not on the couple point K (see Figure 2a). When the force is applied on the coupler AB, the effect on the positioning deviations is neglectable. However, when applied on another link, the positioning deviation increases due to the asymmetric loading on the hinges. Of particular interest is the motion deviation in the guided direction, which highly increases in this case. In addition, load application to links other than the output link would result in a change in output resolution or range of motion, as well as a high relative angle between the input and the link. This further increases the motion deviation, as shown in Section 3.2. As such, drive elements are to be integrated on the output stage.

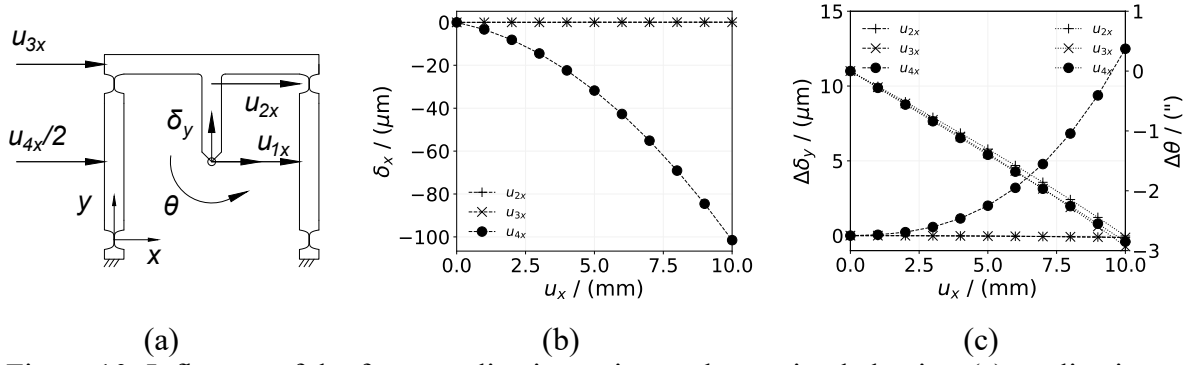


Figure 10: Influence of the force application point on the motion behavior: (a) application points; (b) lost motion δ_x ; (c) guiding deviations δ_y and θ with respect to u_{1x}

3.2 Coupling interface

For the constraint-free motion transfer between drive and compliant guiding mechanism, a coupling interface with DoF = 5 is required. This can be done with a ball-plane contact pair. Due to the guiding deviation and yaw error, there is a relative motion between elements of the contact pair. Figure 11 shows the influence on the maximum positioning deviation and yaw error relative to an ideal punctual force in motion direction ($\Delta\delta_y = \Delta\delta_{y,interface} - \Delta\delta_{y,ideal}$). If the ball is in the drive side, the yaw error produces a parasitic force component outside of the guiding direction. If the ball is on the mechanism side, the guiding deviation produces a change of the force application point. Under ideal conditions of no friction nor misalignment, both types of contact pair have a negligible influence on the motion behavior. However, the ball-plane contact (Figure 11a) is less sensitive to small misalignments.

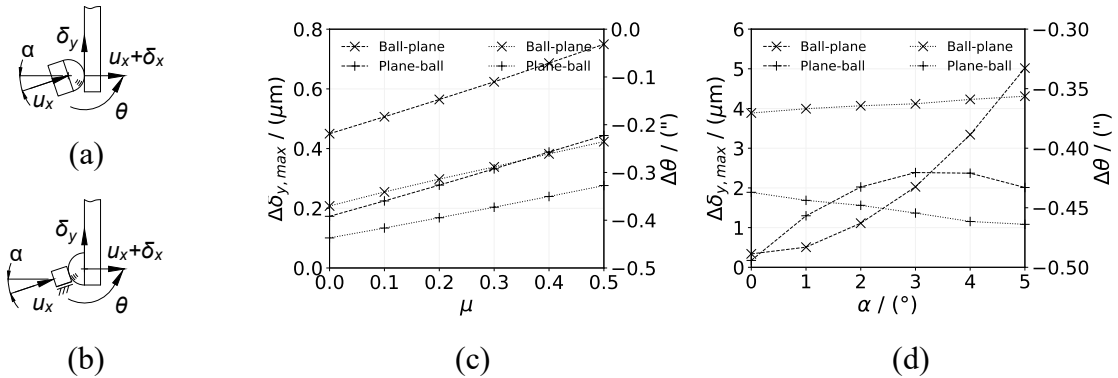


Figure 11: Influence of the drive coupling interface on the motion behavior: (a) ball-plane interface; (b) plane-ball interface; (c) effect of friction; (d) effect of misalignment

4. SYNTHESIS OF ULTRA-PRECISION COMPLIANT MECHANISMS

In [5], the synthesis method presented comprised of the selection of a suitable rigid-body model and the replacement of the revolute joints with flexure hinges with an optimized power-function contour. Based on the results of this and previous contributions [5,12-15,18,19], the method can be expanded for the use in positioning stages with multiple degrees of freedom:

- I. Synthesis of suitable rigid-body models with DoF = 1 for each DoF of the positioning stage;
- II. Serial coupling of the different sub-mechanisms with DoF = 1 into the multi-DoF positioning stage;

- III. Replacement of the revolute joints with flexure hinges with optimized power-function contours using design equations, graphs or tools based on the maximum deflection angles,
- IV. Computer-aided optimization of the hinge contours to minimize the guiding deviation of each stage;
- V. Integration of carried drives with the output stage of each sub-mechanism using a coupler which allows transmission only in the desired direction, e.g., a ball element on the drive side and a plane element on the mechanism side.

5. CONCLUSION

The following contribution investigates neglected parameters in the synthesis of compliant mechanisms such as the degrees of freedom of the rigid-body model, the structural coupling of sub-mechanisms to produce higher degrees of freedom as well as the integration of the drive elements. Based on the results, the synthesis method presented in [5] is expanded for the design of multi-degree-of-freedom positioning systems for ultra-precision quasistatic applications. Ongoing work focuses on the integration of further design parameters such as the orientation of the flexure hinges to further reduce guiding deviation.

REFERENCES

- [1] R. Hebenstreit, R. Theska, K. Wedrich, and S. Strehle, “Conceptual design of a positioning device with subatomic resolution and reproducibility”, in Proceedings of the 20th International Conference of the European Society for Precision Engineering and Nanotechnology. June 8th-12th June 2020, virtual conference, euspen, Bedford, UK, pp. 305–306, 2020.
- [2] L. Keck, F. Seifert, D. Newell, S. Schlamming, R. Theska, and D. Haddad, “Design of an enhanced mechanism for a new Kibble balance directly traceable to the quantum SI”, EPJ Techn Instrum 9 (7), 2022.
- [3] J. B. Hopkins and M. L. Culpepper. “Synthesis of multi-degree of freedom, parallel flexure system concepts via Freedom and Constraint Topology (FACT) – Part I: Principles”, Precision Engineering 34 (2), pp. 259–270, 2009.
- [4] L. L. Howell, S. P. Magleby, and B. M. Olsen, Handbook of compliant mechanisms. Wiley, Chichester, 2013.
- [5] S. Linß, P. Gräser, S. Henning, F. Harfensteller, R. Theska, and L. Zentner, “Synthesis Method for Compliant Mechanisms of High-Precision and Large-Stroke by Use of Individually Shaped Power Function Flexure Hinges”, in Advances in Mechanism and Machine Science, Springer International Publishing, Cham, pp. 1569–1578, 2019.
- [6] M. I. Frecker, G. K. Ananthasuresh, S. Nishiwaki, N. Kikuchi, and S. Kota. “Topological Synthesis of Compliant Mechanisms Using Multi-Criteria Optimization”, Journal of Mechanical Design 119 (2), pp. 238–245, 1997.
- [7] Z. Wu and Q. Xu, “Survey on Recent Designs of Compliant Micro-/Nano-Positioning Stages”, Actuators 7(1), pp. 5, 2018.
- [8] P. Gräser, S. Linß, L. Zentner, and R. Theska. “Investigations of different compliant manipulator concepts for a high-precise rotational motion”, in Proceedings of the 19th International Conference of the European Society for Precision Engineering and Nanotechnology. June 3rd-7th June 2019, Bilbao, ES. euspen, Bedford, pp. 64–67, 2019.
- [9] S. Wan and Q. Xu, “Design and analysis of a new compliant XY micropositioning stage based on Roberts mechanism”, Mechanism and Machine Theory 95, pp. 125–139, 2015.

- [10] P. Gräser, S. Linß, L. Zentner, and R. Theska, “Design and Experimental Characterization of a Flexure Hinge-Based Parallel Four-Bar Mechanism for Precision Guides”, In *Microactuators and Micromechanisms*, Springer International Publishing, Cham, pp. 139–152, 2017.
- [11] N. T. Pavlovic, N. D. Pavlovic and M. Milosevic, “Design of a compliant slider-crank mechanism”, in 56. IWK – Internationales Wissenschaftliches Kolloquium, TU Ilmenau, Ilmenau, 2011.
- [12] S. Linß, T. Erbe, and L. Zentner, "On polynomial flexure hinges for increased deflection and an approach for simplified manufacturing", in 13th World Congress in Mechanism and Machine Science 2011. Guanajuato, Mexico, 19 - 23 June 2011. Curran, Red Hook, NY, A11_512, 2013.
- [13] S. Linß, P. Schorr, and L. Zentner, “General design equations for the rotational stiffness, maximal angular deflection and rotational precision of various notch flexure hinges”, *Mech. Sci.* 8(1), pp. 29–49, 2017.
- [14] S. Henning, S. Linß, and L. Zentner, “detasFLEX – A computational design tool for the analysis of various notch flexure hinges based on non-linear modeling”, *Mech. Sci.* 9(2), pp. 389–404, 2018.
- [15] S. Linß, „Ein Beitrag zur geometrischen Gestaltung und Optimierung prismatischer Festkörpergelenke in nachgiebigen Koppelmechanismen”, Univ.-Verl. Ilmenau, Ilmenau, 2015.
- [16] M. Roesner and R. Lammering, “Kinematic modeling of flexure hinges and compliant mechanisms”, *Proc. Appl. Math. Mech.* 14 (1), pp. 189–190, 2014.
- [17] M. Grübler, *Getriebelehre - Eine Theorie des Zwanglaufes und der ebenen Mechanismen* (Repr. [der Ausg. Berlin, 1917]), VDM Verlag Dr. Müller, Saarbrücken, 2007.
- [18] S. Henning, S. Linß, P. Gräser, J. D. Schneider, R. Theska, and L. Zentner, “Optimization of Compliant Path-Generating Mechanisms Based on Non-linear Analytical Modeling”, in *Microactuators, Microsensors and Micromechanisms. MAMM 2020. Mechanisms and Machine Science*. Springer, Cham, pp. 25–35, 2021.
- [19] P. Gräser, S. Linß, F. Harfensteller, M. Torres, L. Zentner, and R. Theska, „High-precision and large-stroke XY micropositioning stage based on serially arranged compliant mechanisms with flexure hinges”, *Precision Engineering* 72, pp. 469–479, 2021.

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