UNIVERSITY OF CAPE TOWN

DEPARTMENT OF ACCOUNTING

AN EMPIRICAL EVALUATION OF THE CAPITAL ASSET PRICING MODEL IN SOUTH AFRICA

by

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ABSTRACT

This thesis presents an empirical evaluation of the validity of the Capital Asset Pricing Model (CAPM) in South Africa. More specifically, the behaviour of share prices on the Johannesburg Stock Exchange during the eight years from 1973 to 1980 is evaluated. The study is the first direct test of the CAPM in South Africa.

The methodology employed is a cross-sectional regression technique which has been used successfully in testing overseas security markets. An extension to the usual methodology is made by comparing the results obtained using a published market-index with those obtained using an internally generated index.

The historical development and the derivation of the CAPM is discussed in the thesis, as is the relationship between the CAPM and the Efficient Markets Hypothesis.

The results indicate a strong possibility that the CAPM is a valid model in a South African context. Refinements to the research methodology strengthen this conclusion.

A potential problem in the interpretation of the results of tests of this sort is also discussed, as is a recent extension to the theory.

The overall conclusion is that the CAPM is a valid model, however further research is required to establish this with greater certainty.

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CHAPTER 1

INTRODUCTION AND THESIS ORGANISATION

1.1 INTRODUCTION

The past 25 years have seen the origin of modern finance theory, its expansion into a widely accepted branch of microeconomics and a vast amount of empirical research testing the concepts and predictions of the various branches of finance. The bulk of this research has taken place on the New York Stock Exchange, while another major area of research has been the London Stock Exchange. Empirical research in South Africa has been very limited.

One of the fundamental concepts underlying finance theory is the relationship between the risk of an investment, and the return that the investment will yield. The Capital Asset Pricing Model (CAPM) is a model developed in the 1960's by Sharpe and Lintner, and refined by various theorists since then, which attempts to describe the relationship between the expected return of an investment and the commensurate risk. In addition, the model describes the equilibrium pricing relationship between various risky assets.

Empirical research in the USA has shown that the model does appear to be a valid description of the market place, notwithstanding certain anomalies that have been reported.

In view of the key position that the CAPM has in finance theory, it is important to ascertain empirically whether the actual market behaviour corresponds with the predicted behaviour. No direct study of the CAPM has been carried out in South Africa. This thesis tests the validity of the CAPM by examining the pricing relationship between securities quoted on the Johannesburg Stock Exchange.

The major aim of this thesis is thus to aid in the understanding of the functioning of the Johannesburg Stock Exchange, by ascertaining whether or not the CAPM suitably describes the pricing of risky securities on the market. Secondary aims of the study are : to present a cohesive analysis of the historical derivation of the CAPM; to present a review of tests of capital market behaviour that have been carried out in South Africa, together with a review of the major empirical tests of the CAPM that have been conducted overseas; and finally, but by no means least important, to stimulate further research into the South African capital market.

1.2 ORGANISATION OF THE THESIS

The thesis can, for descriptive purposes, be conveniently broken down into three parts.

The first part, covering Chapters 2 to 6, presents a theoretical background to the development of the CAPM, together with a review of empirical work. The development of the CAPM is presented in a manner which is not usually followed. Chapter 2 introduces the concept of utility together with the underlying 'rules' of utility theory. Markowitz Portfolio analysis is presented in the same chapter. This details the formation of an efficient portfolio. Chapter 3 presents the actual Capital Asset Pricing Model, as developed by Sharpe (1964) and Lintner (1965). The model is derived using Sharpe's original derivation. The final section of Chapter 3 presents a discussion on the general significance of the CAPM, its place in finance theory and the prevailing financial paradigm. The relationship between the Efficient Markets Hypothesis and the CAPM is discussed in Chapter 4. Chapter 5 presents a review of empirical tests that have been carried out on the South African capital markets. Finally, Chapter 6 reviews the major empirical tests of the CAPM that have been performed on other markets, notably the New York Stock Exchange.

The second part of the thesis consists of the research work carried out. The model used in the present study is presented in a general context in Chapter 7, without analysing the reasons for adopting the specific procedures. Chapters 8 through to 11 describe the actual methodology and results. Initial research is described in Chapter 8, with the results presented in the following chapter. Chapter 10 analyses possible shortcomings in the initial approach, and presents a refined methodology. The results of the second test are presented in Chapter 11. These results are compared to the initial results in this chapter also.

The third part of the thesis is a reversion to theory. Certain criticisms of tests of the CAPM, as raised by Roll (1977; 1978) are presented and discussed in Chapter 12. Finally, the thesis concludes with Chapter 13, wherein overall conclusions are reached, and suggestions for further research are noted.

CHAPTER 2

MARKOWITZ DIVERSIFICATION: THE ORIGIN OF MODERN PORTFOLIO THEORY

2.1 INTRODUCTION

In 1952, Harry Markowitz published a seminal paper dealing with the selection criteria for combining individual securities into portfolios for investment purposes. The fundamental innovation with Markowitz's approach was the mathematical insights which he lent to what had been, until then, a non-mathematical, fairly abstract approach to investing in different securities.(1)

Markowitz's insights came in breaking down the investment decision relating to any one stock into two components, risk and return. These components were then expressed in mathematical terms, and certain 'rules' were developed which aided the placing of different shares into various portfolios.

Before discussing the Markowitz model, and the derivative thereof, the Sharpe-Lintner Model, it is considered necessary to give a brief introduction to Utility Theory. This is because, at the most basic level, the choice of securities, or, indeed any choice, can be expressed in terms defined by this theory.

Another reason for introducing utility theory is that Markowitz's work is in fact based to a large extent on the assumptions of utility theory. Although there is no specific mention of the theory in his 1952 paper, he makes direct reference to the work of von Neumann and Morgenstern (1953) in Markowitz (1959), wherein he expands his original work to cover the general case. This is covered in section 2.4.

Utility theory and the axioms which follow in the next section are thus implicit in all discussions relating to the Capital Asset Pricing Model.

The aim of this chapter is to present the base upon which all portfolio theory is built (i.e. the assumptions and 'rules' of utility theory), then to lead the reader through Markowitz's derivation of the efficient portfolio (which term will be described later in the chapter) in both a finite and a general sense, and finally to present the complete theory by means of which an investor can make decisions as to the optimal portfolio of shares he desires to hold. This portfolio is of course based on his own desires and interpretations of various factors, and this is where the concepts outlined by utility theory become important.

The following section thus gives a brief overview of the theory.

2.2 UTILITY THEORY

Utility theory attempts to provide a framework for assessing choice under conditions of uncertainty. In simple terms, it attempts to predict an individual's choices from a simple, but hopefully accurate, representation of his feelings. This can only be done on a simple level if the individual's actions conform to a few reasonable rules of behaviour.

It has been said that utility measures the

> "Magnitude of satisfaction someone derives from something, (it) is a subjective index of preference. If a person is faced with a decision, the alternative with the highest utility is the preferred choice".(2)

The fundamental assumptions of utility theory have been discussed by von Neumann and Morgenstern (1953), who highlight the following six key axioms:

1. COMPLETE AND CONSISTENT PREFERENCES

Given a choice between A and B, a person can tell whether he prefers A to B, which can be symbolically represented by:

$$U(A) \succ U(B)$$

or, he can be indifferent between A and B, in which case,

$$U(A) = U(B)$$

where U (A) and U (B) are functions representing utility.

2. TRANSITIVE NATURE OF CHOICE

Preference over A over B and B over C implies preference of A to C.

This can be represented as follows:

If $U(A) > U(B)$, and $U(B) > U(C)$,

then $U(A) > U(C)$

3. EQUAL UTILITY IMPLIES EQUAL DESIRABILITY

If an individual is indifferent in choosing between two items, their utility is equal. Following on from this,

if $U(A) = U(D)$, and $U(A) > U(B)$,

then $U(D) > U(B)$

4. UTILITY CAN BE OBTAINED BY CHOOSING PART OF ONE CHOICE AND PART OF ANOTHER

This axiom contends that: if $U(A) > U(B)$, and $U(B) > U(C)$, then, some combination of A and C posseses equal utility to B.

viz: $P(A) \cdot U(A) + P(C) \cdot U(C) = U(B)$

where P represents the proportion of the respective choices combined.

5. ADDING IRRELEVANT OBJECTS DOES NOT CHANGE THE RANKING OF CHOICES

If $U(A) > U(B)$, then $(U(A) + U(C)) > (U(B) + U(C))$, where C has no impact on either A or B.

6. THE EXPECTED UTILITY MAXIM

The utility of a risky object is equal to the expected utility of the possible outcomes. If an object has a possible outcome, its expected utility can be expressed as

$$E(U)' = \sum_{i=1}^{n} P(O_i) \cdot U(O_i)$$

where O_i is the i'th outcome and $P(O_i)$ is the probability of O_i occurring.

The above assumptions are fundamental to all utility theory, and can be used to generate a utility function, or in other words, a mathematical expression detailing an individual's preferences.

Recall that in order to represent an individual's feelings, some simple rules of behaviour are required. These simple rules of behaviour discussed above, in relation to an investor, are as follows:

- greater return is preferred to lesser return

and

- lower risk is preferred to higher risk.

Note that at this stage risk and return have not yet been formally defined. If the two characteristics, risk and return, referred to above are plotted in a two-dimensional grid, it is possible to obtain various combinations of these two characteristics which have equal utility. This is illustrated below, in Figure 2.1.

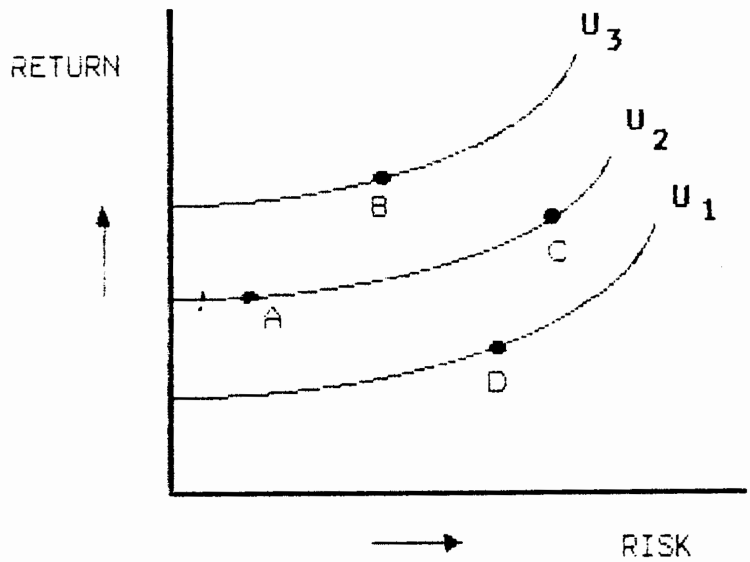


Figure 2.1: Utility functions plotted in Risk-Return space

The six basic axioms described earlier in this section ensure that the lines of preferred utility (U_1, U_2 and U_3 above) do not intersect, and also lead to the conclusion that U_3 has greater utility than U_1, in the above diagram. This is because of the assumption that the investor has a preference for greater return and dislikes greater risk.

The upward slope of the curves in Figure 2.1 is a result of the assumption made above that greater return is required in order to assume higher risk.

Referring to Figure 2.1, and assuming that positions A to D represent different investment opportunities for the investor, the following points emerge.

Clearly, B is preferred above all other investments, whilst D is the least preferred investment choice. A and C on the other hand, represent a choice of investment which the investor may have difficulty in making, as his utility is the same for either investment choice. Thus the utility functions U_2 etc are also known as indifference curves, or utility isoquants. Along these curves, the individual is indifferent to either position: i.e. his total utility is the same at any point on the curve.

The following section will show how Markowitz Portfolio Theory is linked to utility theory.

2.3 MARKOWITZ PORTFOLIO SELECTION

2.3.1 Introduction

As mentioned at the beginning of this chapter, the key factors which Markowitz identified in determining an optimal selection of securities for inclusion in a portfolio were the expected return of the portfolio, and the risk of the portfolio of securities selected.

The selection of securities in order to form some preferred portfolio introduces choice to the investor. He has to make a choice amongst various alternatives. The previous section introduced the fundamental axioms of utility theory. The reason for this was that the choice involved in selecting a portfolio can be described in terms of utility theory. In achieving the optimal portfolio, the individual has to assess his own indifference curves and make his choice based on this assessment. It is thus obvious that utility theory underlies Portfolio Theory.

The melding of utility theory with Portfolio Theory is shown in section 2.4.

Markowitz's original theory will be examined in detail in this section as this forms the basis of the entire Capital Asset Pricing Model theory.

2.3.2 Markowitz's key assumption

The essence of Markowitz's theory is that the investor considers the expected return of a portfolio as a desirable thing (i.e. he desires greater return) and the variance of that return as an undesirable thing. Markowitz called this rule the "Expected Return-Variance of Returns" rule, otherwise known as the E-V rule.

Markowitz proposed that

> "The portfolio with maximum expected return is not necessarily the one with minimum variance. There is a rate at which the investor can gain expected return by taking on variance, or reduce variance by giving up expected return."(3)

The next subsection will trace the development of the theory, and show how Markowitz proved the validity of such an assumption.[4]

2.3.3 Mathematical terms defined

The following elementary concepts and relationships are necessary in order to continue the discussion:[5]

1. Mean

Let Y be a random variable (e.g. the price of a security) and let p_1 be the probability that the Y takes on the value y_1, p_2 the probability that $Y = y_2$ and so on.
The expected value (or mean) of Y is as follows:

$$E = p_1 y_1 + p_2 y_2 + \dots + p_n y_n$$

2. Variance

The variance (V) of Y is as follows:

$$V = P_1 (y_1 - E)^2 + p_2 (y_2 - E)^2 + \dots + p_n (Y_n - E)^2$$

The variance represents the average square deviation of Y from its expected value.

The standard deviation (σ) of Y is:

$$\sigma_y = V^{\frac{1}{2}}$$

3. <u>Combination of random variables</u>

The weighted sum of a number of random variables is also a random variable. If

$$R = a_1 R_1 + a_2 R_2 + \ldots + a_n R_n$$

Where $R_1 \ldots R_n$ are random variables,

then R is a random variable.

4. <u>Expected value of a weighted sum</u>

The expected value of a weighted sum of random variables is:

$$E(R) = a_1 E(R_1) + a_2 E(R_2) + \ldots + a_n E(R_n)$$

5. <u>Variance of a weighted sum</u>

$$V(R) = \sum_{i=1}^{n} a_i^2 V(Y_i) + 2 \sum_{i=1}^{n} \sum_{i>1}^{n} a_i a_j \sigma_{ij}$$

where $\sigma_{ij} = E(R_i - E(R_i))(R_j - E(R_j))$

which is equivalent to the covariance between asset i and asset j.

However, the variance of any individual variable, R_i, is the same as the covariance,

i.e. $V(R_i) = \sigma_{ii}$,

thus

$$V(R) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j \sigma_{ij}.$$

The above properties will all be used in analysing the characteristics of a portfolio.

2.3.4 The Portfolio defined

Let R_i = return on i'th security

μ_i = expected value of R_i

σ_{ij} = covariance between R_i and R_j

X_i = the percentage of the investor's assets, which are allocated to the i'th security.

Thus from the definitions in 2.3.3 above, the yield (R) on the portfolio is:

$$R = \Sigma R_i X_i,$$

where 1) R_i and thus R are all random variables

2) $\Sigma X_i = 1$

3) $X_i > 0$, for all i.

The expected return (E) on the portfolio is:

$$E = \sum_{i=1}^{n} X_i \mu_i$$

and the variance (V) is:

$$V = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_{ij}.$$

2.3.5 Efficient portfolios introduced

Assuming that the investor has fixed ideas about the values of μ_i and σ_{ij}, there are various combinations of expected return (E) and variance (V) available, depending on the choice of portfolio as influenced by the proportion of funds invested in various securities (i.e. the X_i's). Figure 2.2 below shows all the assumed possible E-V combinations.(6)

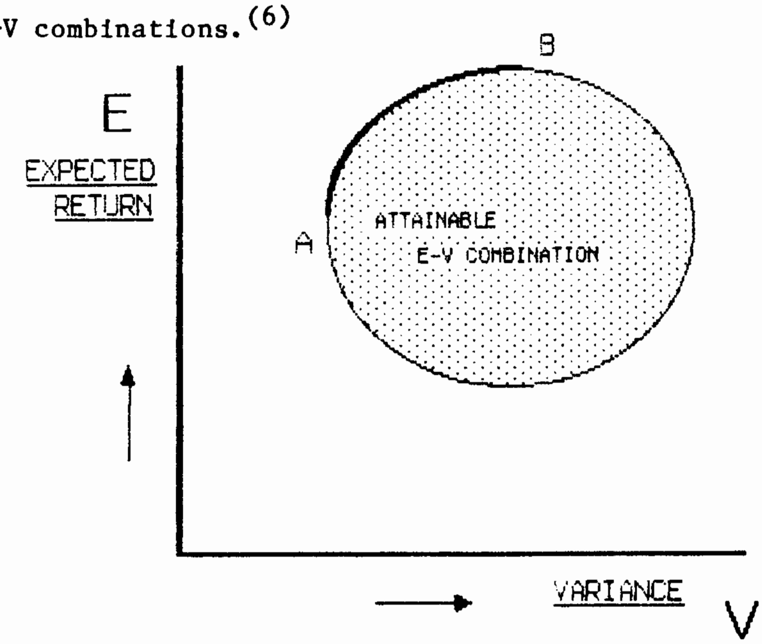


Figure 2.2: Possible E-V combinations

Source: Markowitz (1952: 82) (adapted)

It is obvious that, given the initial assumption that the investor desires increasing return together with decreasing risk, the dark line (A-B above) represents portfolios with minimum V for a given level of return, and maximum return for a given V.

Markowitz termed portfolios of this sort EFFICIENT PORTFOLIOS.

2.3.6 The three-security case

In the three-security case, Markowitz showed that the general model introduced in 2.3.4 reduces to

$$E = \sum_{i=1}^{3} X_i \mu_i \quad (2.1)$$

$$V = \sum_{i=1}^{3} \sum_{j=1}^{3} X_i X_j \sigma_{ij} \quad (2.2)$$

with

$$\sum_{i=1}^{3} X_i = 1 \quad (2.3)$$

and $X_i > 0$ for $i = 1, 2$ and 3.

From (2.3) above,

$$X_3 = 1 - X_1 - X_2 \quad (2.4)$$

which can be substituted into (2.1) and (2.2) yielding, in a general sense,

$$E = E\ (X_1,\ X_2)$$

$$V = V\ (X_1,\ X_2)$$

and: $X_1 > 0,\ X_2 > 0,\ 1 - X_1 - X_2 > 0.$

The three-security case has thus been reduced to a two-dimensional geometrical problem, the solution of which is shown in Figure 2.3.

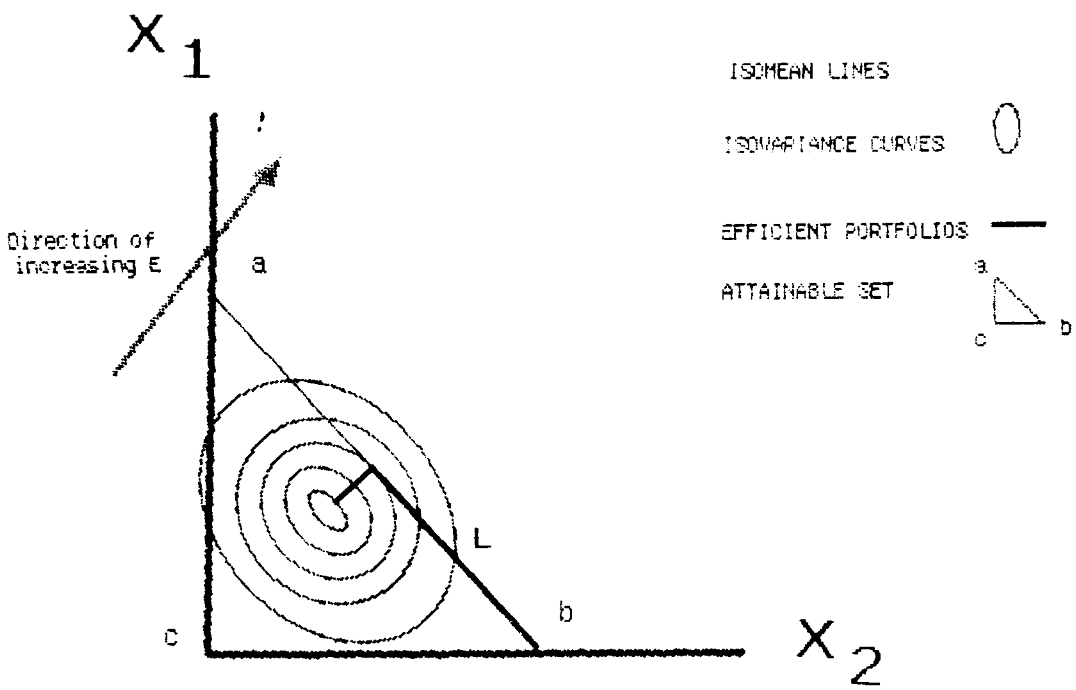


Figure 2.3: Solving for the efficient portfolio in the three-security case

Source: Markowitz (1952: 85) (adapted)

The isomean line is defined as the set of all portfolios with a given expected return. This is, in reality, a set of parallel lines as shown in Figure 2.3 above.

Similarly, the isovariance line plots combinations of the two portfolios yielding equal variances. It can be proved that these form a series of concentric ellipses as shown.

The point of an isomean line at which V takes on its lowest value is the point at which the isomean line is tangential to the isovariance ellipse. Plotting the tangential points yields the line 'L'. This is the so-called critical line which yields the set of efficient portfolios. Note that the set of efficient portfolios is forced to continue along the hypotenuse of the triangle marking the attainable set.

Using the values of the isomean and isovariance lines in Figure 2.3, each of which represents a given E and V value respectively, it is possible to plot the different E and V combinations for different combinations of X_1 and X_2. This is done in Figure 2.4, overleaf.

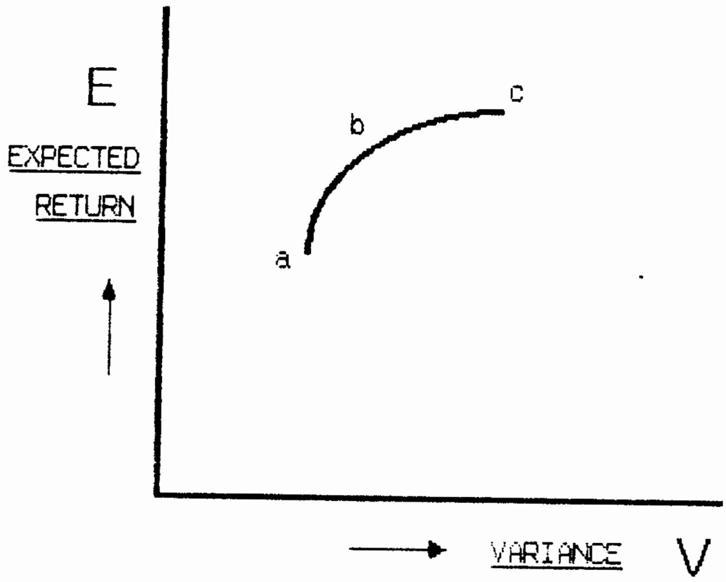


Figure 2.4: Efficient portfolios plotted in the risk-return space: the three-security case

Note that 'a', 'b' and 'c' above represent the points 'a','b' and 'c' on Figure 2.3.

2.3.7 The four-security case

The introduction of a fourth security to the analysis does not represent a major problem. Without going into the mechanics of the exercise, it is again possible to remove one-dimension, thus reducing the problem to a three-dimensional solution.

The three-dimensional solution can then be plotted as shown in Figure 2.5 overleaf.

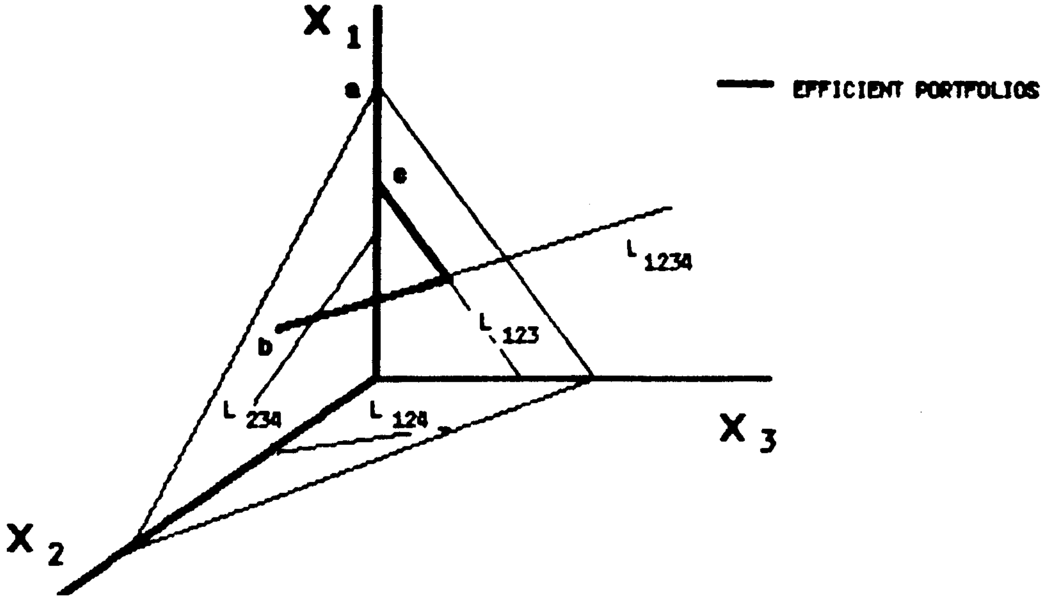


Figure 2.5: Solving for the efficient portfolio in the four-security case

The point of maximum return is represented by 'a' on Figure 2.5, while the point of minimum variance is shown by 'b'.

Again, it is possible to plot the different E and V values for various combinations of securities. This is shown in Figure 2.6.(7)

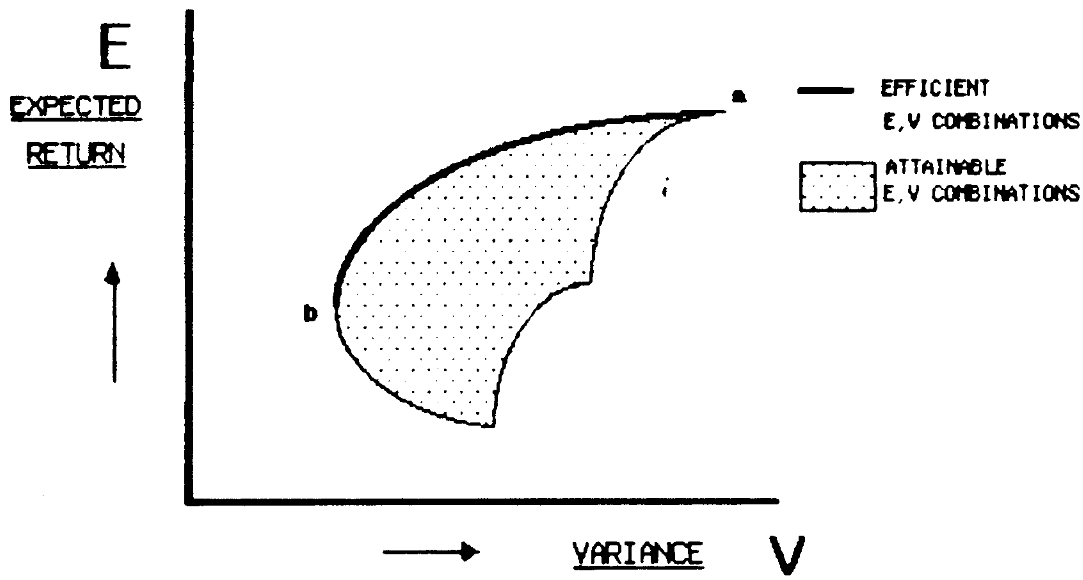


Figure 2.6: Efficient portfolios plotted in the risk-return space - the four-security case

Points 'a' and 'b' represent 'a' and 'b' on Figure 2.5. The reason for the scalloped edge of the E-V combination is that any two combinations of the individual securities lead to a reduced variance, as shown in Figure 2.4.

2.3.8 Extension of the E-V plot to n-securities

The principles shown in the previous section can be extended to n-securities. In this case, the attainable area as assumed in Figure 2.2, would reduce to that shown below (Figure 2.7).

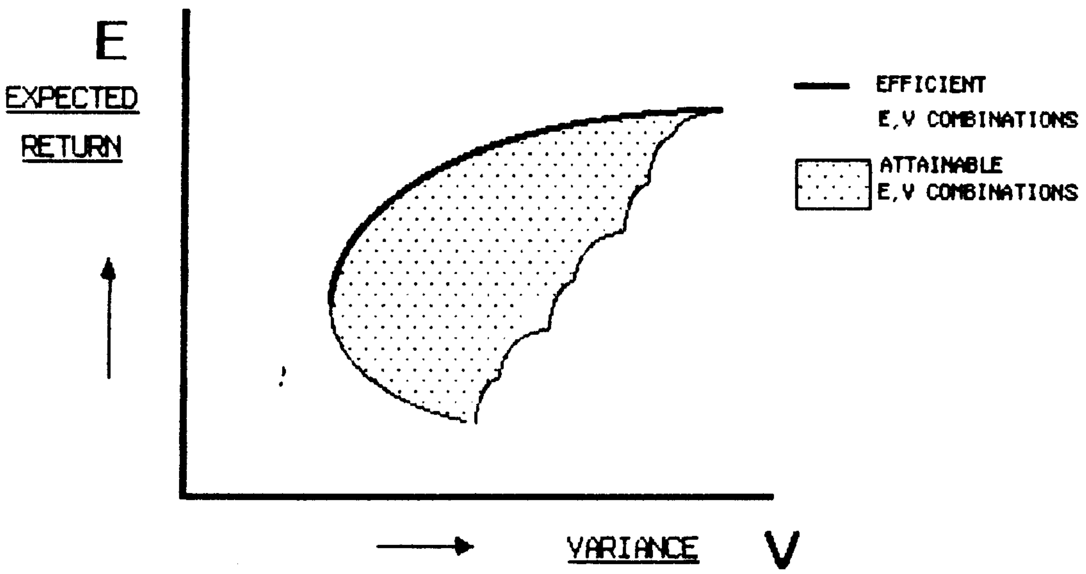


Figure 2.7: Efficient portfolios plotted in the risk-return space - n-securities

The reason for the formation of the scalloped edge is explained in 2.3.7 above.(8)

Figure 2.7 thus represents the entire set of feasible portfolios, given the investor's knowledge of μ_i and σ_{ij}.

Assuming that there is some method of calculating the means of various securities and the covariances between them, it is theoretically possible to calculate the E-V space shown above. This is discussed further in the next section.

2.4 MARKOWITZ PORTFOLIO THEORY: A SUMMARY

As mentioned in 2.3.2, Markowitz subsequently expanded his original article to the general case (Markowitz: 1959), and suggested a technique (or algorithm) for finding the efficient frontier. The following is a brief overview of the complete theory.

2.4.1 Assumptions[9]

- Investors form probability distributions about the future performance of securities.

- These distributions have finite means and variances.

- The returns relative to risk decrease beyond some point.

- An individual's preferences are a function only of the expected return and variance of a portfolio.

- For any given expected return on a portfolio, the portfolio with the smallest variance is preferred above all others; and, for any given portfolio variance, the portfolio with the maximum expected return is preferred to all others.

The last assumption, (the Mean-Variance rule, or M-V rule) was the significant innovation that reduced the problem of portfolio selection to a quadratic programming problem.

2.4.2 Stages in solving the problem

The three stages in solving the problem are:

- Measurement
- Calculation and
- Final selection.

Measurement

The expectations about the mean, variance and co-variance of all the securities are formed by the investor.

Calculation

The quadratic programming problem is solved to yield the set of efficient portfolios which together lead to the efficient frontier.

Final Selection

The utility theory as explained in section 2.2 is now introduced by bringing in the utility function and the indifference curves of the individual.

The point on the efficient frontier that maximises the individual's utility function is selected. This can be shown as follows, by combining Figure 2.1 with Figure 2.7.

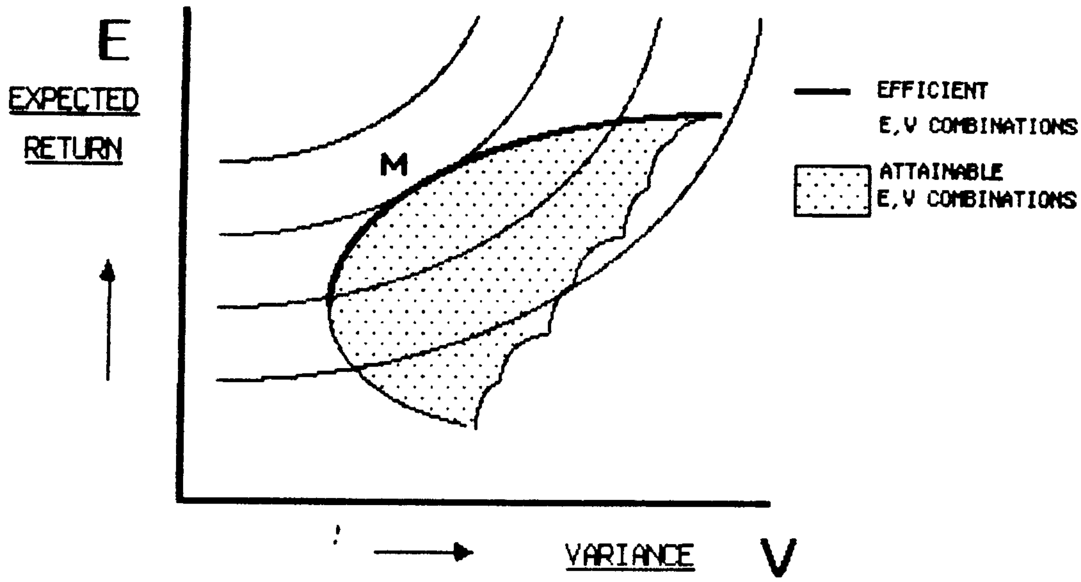


Figure 2.8: Choice of efficient portfolio

Clearly, point M maximises the individual's utility. This is thus the desired portfolio to hold.

2.4.3 Problems with Markowitz Theory

The two major drawbacks inherent in the general application of the theory as noted in 2.4.2 above, were:

a) measurement of the various means, variances and covariances
   and

b) the solution to the quadratic programming problem.

Both of these problems required a vast amount of computer time to solve them, and thus the cost involved was too high to warrant general acceptance and application of the theory.

As a result of attempts to solve these problems, and also because of the dynamic nature of the academic discussions relating to portfolio theory in the 1950's and early 1960's, significant advancements in the theory were made.

The most important of these was formulated by Sharpe, in 1963. Sharpe realised that in order to compute the various covariances between shares, as long as one assumed that individual shares were not correlated with each other, the computations became far simpler and much quicker. This innovation came to be known as the Diagonal Model.

2.5 SUMMARY

This chapter has traced the development of the efficient frontier from first principles, including utility theory because of the significance of utility theory to any situation in which choice is involved. It has also shown how a combination of risky securities can result in lower overall risk. In addition, Markowitz's general solution to the problem of portfolio selection has been presented. The Capital Asset Pricing Model will be examined in the next chapter.

CHAPTER 2 - FOOTNOTES

(1) Diversification amongst securities in investing had always been advocated. The rationale behind this was however more to do with 'not putting all one's eggs into one basket', than with a detailed, logical approach as to how to diversify one's investments.

(2) Refer Francis and Archer (1979) page 245.

(3) See Markowitz (1952) page 77.

(4) Markowitz's original article dealt with the specific case where only three and then four assets were available for investment. The concepts and proofs derived were subsequently shown to be true for the n'th case (see Markowitz (1959)). In this chapter the specific three asset and four asset cases only will be dealt with, as the general case is merely an extension of this proof. The intention of the chapter is to present Markowitz's theory as an introduction to portfolio theory, not to analyse his article and book in a rigorous method.

(5) The notation and formulae used here are exactly the same as those used in Markowitz (1952).

(6) The axes of any E-V plot are conventionally shown as in Figure 2.2. This is in order that the independent variable is shown on the horizontal axis, and the dependent variable on the vertical axis. Markowitz showed the axes the other way around. This has been changed in order to maintain uniformity throughout the thesis.

(7) This explanation is somewhat different to that given by Markowitz. It does however yield an identical result.

(8) The formation of the scalloped edge can be easily seen as follows. All possible assets exist within the feasible area (shaded area) in Figure 2.7. However, only some of the assets fall on the lower boundary of the feasible area. For exactly the same reasons as outlined in section 2.3.7, a curve such as is shown in Figure 2.4, develops between each and every pair of assets falling on this lower boundary. Each 'scallop' on the scalloped edge thus represents the frontier between two securities.

(9) Refer also to section 3.4.2.

CHAPTER 3

THE CAPITAL ASSET PRICING MODEL

3.1 INTRODUCTION

The previous chapter introduced the concept of an efficient portfolio using Markowitz's original techniques. In addition, the insights brought by Markowitz to the theory of finance were noted.

The development of the true Capital Asset Pricing Model (CAPM) will be examined in this chapter, as will the assumptions underlying it. The historical background to the CAPM development will be explored first, and then the model itself will be derived.

3.2 THE CAPITAL ASSET PRICING MODEL: AN OVERVIEW

Before deriving the model, it is considered necessary to present a brief overview of the CAPM.

Two benefits will be gained from this. First, a global view of the model will be obtained, which will help in understanding the derivation; and, secondly, the model will be presented in easy to understand, unambiguous terms.

The CAPM attempts to explain how the prices of assets will adjust in equilibrium, and in addition it attempts to relate the risk of an asset to its expected return.

The model predicts that:
"Prices of assets will adjust until in equilibrium the assets are all placed on a single straight line relative to their return and risk".

Having made the above prediction, the model goes on to say that the return of an asset is a direct function of its risk. A convenient measure of the risk is the so-called beta co-efficient, which is described as follows:

$$\beta = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2}.$$

This function describes the risk of any asset in terms of the covariance of its returns with the market, and with the risk of the market itself.(1) In addition, an empirical estimate of the beta-coefficient can be obtained by regressing the actual returns achieved on a share with the actual market returns. This 'historical' beta can then be used as a surrogate for the true ex-ante beta.(2)

3.3 DEVELOPMENT OF THE CAPM

3.3.1 Markowitz and Portfolio Theory

The work of Markowitz (1959), together with that of Tobin (1958) and Hicks (1962) developed a normative framework within which asset choice under conditions of risk was examined.

The essence of Markowitz Portfolio Theory is rationality. Markowitz assumed that investors were rational, risk averse, return-maximising individuals. As a result of this rationality assumption inherent in his model, he developed a theory which was normative and attempted to maximise investor's perceived preferences and desires, assuming rationality of action.

Markowitz's model attempted to provide a solution to the problem of portfolio selection. This solution was based on the expected utility of the individual, and can be closely linked to the work of von Neumann and Morgenstern (1953).[(3)]

Tobin (1958) expanded Markowitz's analysis and showed that under certain conditions the portfolio selection problem and the resulting investment choice can be broken down into two distinct phases: first, the choice of a unique combination of risky assets which is optimum, and second, the choice of allocation of the funds to be invested between the optimal asset combination and a single riskless asset.

The dichotomy of choice mentioned above was examined by Hicks (1962), who focused on the conditions under which the choice occurs.

All three of these works used the mean-variance approach suggested by Markowitz (1952). They did not however, extend their analysis beyond the individual to the general case in which there are numerous individuals operating in a market, each attempting to obtain the optimal combination of assets for himself.

Sharpe (1963) changed the whole tone of portfolio theory by introducing a positive model. This model was based on certain assumptions which, when built up, became a predictive model which could be used in an attempt to explain reality and actual events.

It is important to note that the concept of rationality was not discarded by Sharpe, as he still bases his model on the Portfolio Theory assumptions (amongst others).

The equilibrium position of a capital market was thus examined first by Sharpe. This is described in the next sub-section.

3.3.2 Sharpe's Diagonal Model

Sharpe (1963) in his paper "A simplified model for Portfolio Analysis", developed a relatively quick method whereby the optimal portfolio for an individual could be obtained. He called this the "Diagonal Model", due to the fact that the variance-covariances matrix between securities used in the analysis is diagonal as all values in the matrix are zero except on the top left to bottom right diagonal. The crux of this work was his assumption suggested, albeit indirectly, by Markowitz,[4] that:

> ".....the returns of various securities are related only through common relationships with some basic underlying factor."
>
> (Sharpe (1963) Section IV).

The factor suggested by Sharpe was an index of overall performance of the market. By regressing the actual returns of an individual security against the actual returns of the market in the same time-period, a characteristic line for the security would be obtained. This characteristic line could then be used to generate the inputs for the portfolio selection procedure.(5)

The formula for the characteristic line of a security is thus of the form:(6)

$$R_{jt} = a_j + b_j R_{It} + \varepsilon_{jt} \qquad (3.1)$$

where R_{jt} = return on share j in time t

R_{It} = return on some market index (I) in time t

ε_{jt} = random-error term of the regression analysis.

The graphical representation of the market model is as shown below:

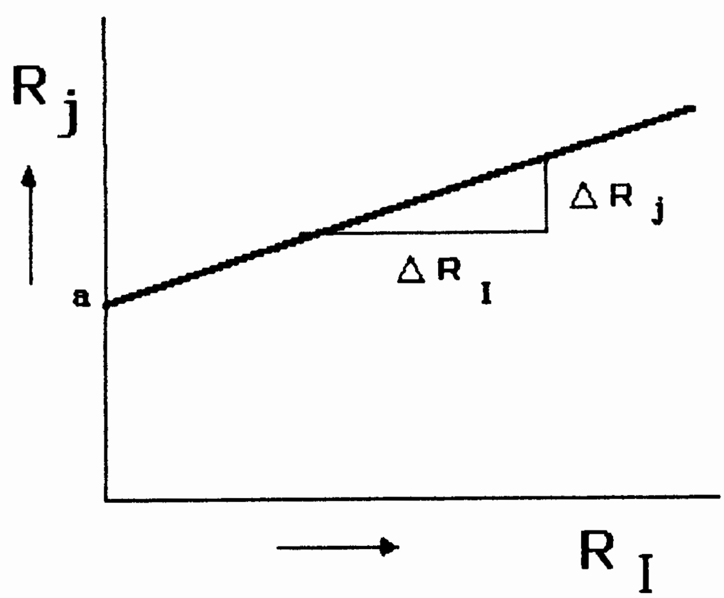


Figure 3.1: The market model

where : $b_j = \frac{\Delta \ R_j}{\Delta \ R_I}$

$= $ slope of the line, sometimes known as beta

a $=$ the regression intercept.

In the market model, the variance in R_j is caused by two factors:

$$Var\ (R_j) = Var\ (a + b_j\ R_I + \varepsilon_i)$$
$$\Rightarrow\ \doteq Var\ (b_j R_I) + Var\ (\varepsilon_i) \qquad (3.2)$$

OR: Total risk = systematic risk + unsystematic risk.

The scatter of the R_j observations around their mean is the evidence of the total risk of the asset, σ_{Rj}. Part of this scattering is however due to an underlying relationship with the return on the market index, as shown by the slope of the regression line. In other words, much of the variability of the returns in R_j is attributable to the change in R_I. This portion of risk attributable to a common cause is called 'systematic risk' by Sharpe (1964), and 'undiversifiable risk' by Treynor (1965).

The systematic or undiversifiable risk is the minimum level of risk that can be achieved by means of diversifying across a large group of assets.

The unsystematic component of total risk, the var (ε_i) in equation (3.2) above, has been called 'unsystematic risk' by Sharpe (1964), 'residual variance' by Lintner (1965) and 'diversifiable risk' by Treynor (1965).

A Markowitz efficient investor would succeed in diversifying away the unsystematic component of total risk, by investing in an efficient portfolio. However, the systematic portion would remain.

Sharpe's Diagonal Model did not lead directly to the CAPM, however, it did generate some further questions, and it certainly was an important step towards the formulation of the model.

3.3.3 The Formalisation of the CAPM

In 1964 Sharpe published his first paper on capital asset pricing. The two questions for which he attempted to find answers were:

1) What is the appropriate measure of the risk of a capital asset?

   and

2) What is the equilibrium relationship between the asset's risk and its one-period expected return?

Lintner (1965), in "Security Prices, Risk and Maximal Gains from Diversification", attempted to answer the same questions, although in a more general sense. He derived a measure of the risk of an individual security within a portfolio of assets which appeared to contradict Sharpe's results. Lintner contended that his results were different and more general than Sharpe's. Sharpe, in reply to Lintner, agreed that their results were conflicting and that Lintner's paper superseded his own.(7)

Fama (1968), resolved the apparent conflict between the two models by pointing out certain inconsistencies in the specification of Sharpe's market model. Neither Sharpe nor Lintner had highlighted these inconsistencies, hence the conflict. Fama then showed that both Sharpe and Lintner were addressing the same issues. It is perhaps for this reason that the CAPM is usually jointly attributed to Sharpe and Lintner, as shown in its title: The Sharpe-Lintner Capital Asset Pricing Model.

3.4 DERIVATION OF THE SHARPE-LINTNER CAPM

3.4.1 Introduction

The derivation of the Sharpe-Lintner CAPM can be broken down into three phases. These are:

1) the selection of an optimal investment policy for an individual,
2) the attaining of portfolio equilibrium in the capital market, and
3) the equilibrium pricing of individual securities in the market.

These phases are examined in 3.4.3 et seq.

As mentioned in 3.3.3 above, the two contradictory approaches of Sharpe and Lintner were reconciled by Fama. In this work it is not considered necessary to detail the differences between Sharpe and Lintner. Instead, the derivation used will be as proposed by Fama, wherein the two were reconciled.

However, before deriving the CAPM, it is necessary to examine the underlying assumptions.

3.4.2 Assumptions underlying the CAPM

As capital market theory is based on portfolio analysis, the assumptions of portfolio theory are also assumptions for capital market theory. There are however, additional assumptions underlying the CAPM. Both the initial and the additional assumptions are detailed below.

Assumptions of Portfolio Analysis:

- the rate of return of an investment is seen by investors as conforming to some probability distribution.
- investor's estimates of risk are proportional to the variability of return they visualise.

- investors base their decisions on only two factors: the expected return of a security, and its variance (or, alternatively, its standard deviation). Higher return and lower risk is the preferred outcome. In short, investors are rational, expected utility maximisers.
- no individual can affect the market price by buying or selling securities.

The above assumptions, if conformed to by an investor, will mean that the investor will prefer Markowitz efficient portfolios over other portfolios.

In addition to these assumptions, the following assumptions are necessary in order to derive the CAPM:

- money may be borrowed or lent at a risk-free rate of interest
- all investors have homogeneous expectations
- all investors have the same one-period investment horizon
- all investments are infinitely divisible
- there are no taxes and no transaction costs for buying and selling securities
- inflation is fully reflected in the level of interest rates
- all available information is freely available to everyone.

The above assumptions can be summarised into four main ones:

1. Investors are Markowitz-efficient diversifiers
2. Securities markets are perfect
3. Investors all share homogeneous expectations
4. Unlimited lending and borrowing exists.

If all the above conditions are present, capital market theory can be developed. The assumptions appear onerous, however it is possible to relax some of them.

3.4.3 Individual's Optimal Investment Policy

This is the first phase of the derivation of the CAPM.

Recall from Chapter 2, section 2.4.2 that the optimal portfolio choice will depend on the individual's utility function. For convenience sake, this figure (Figure 2.8) is reproduced below:

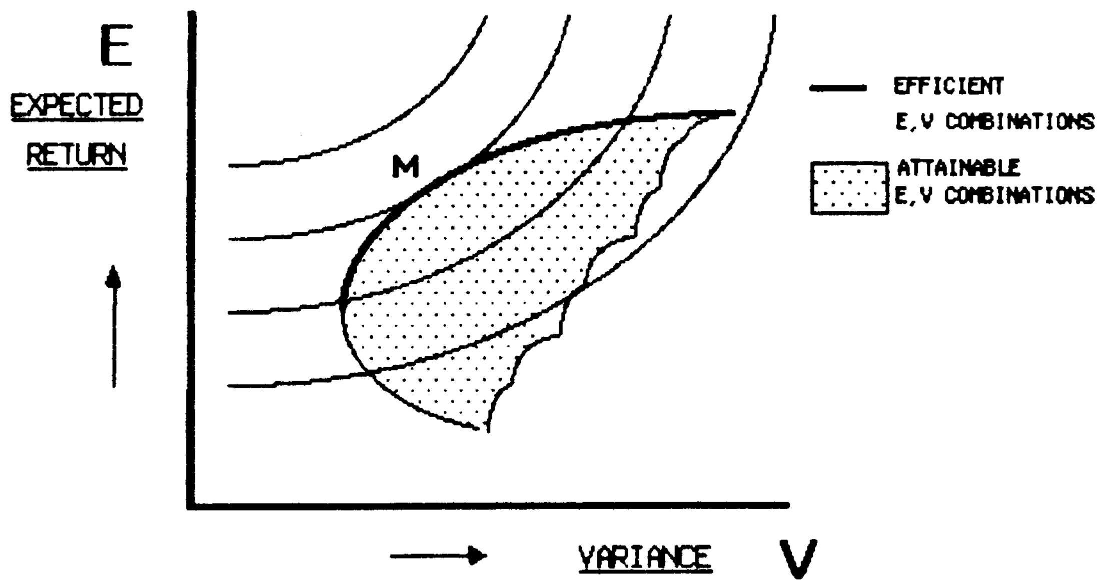


Figure 3.2: Optimal efficient portfolio choice

M represents the optimal portfolio for the individual having increasing utility functions U_1, U_2 and U_3.

Now assume the existence of a riskless asset P, the variance (and hence standard deviation) of this asset is zero, and its mean (M_p) is equal to the pure rate of interest. If an investor were to place α of his investment in P (i.e. lend to the riskless asset) and the remainder in some asset or portfolio of assets A, his expected rate of return (E(R)) would be:

$$E(R) = \alpha E(R_p) + (1 - \alpha) E(R_A)$$

and $$\sigma_R = (\alpha^2 \sigma^2_{Rp} + (1 - \alpha)^2 \sigma^2_{RA} + 2r_{pA} . \alpha . (1 - \alpha) \sigma_{Rp} \sigma_{RA})^{\frac{1}{2}}$$

where r_{pA} = correlation coefficient between P and A

but $\sigma_{Rp} = 0$,

thus $\sigma_R = (1 - \alpha) \sigma_{RA}$,

which implies that all combinations of risky assets with a riskless asset must have values of E(R) and σ_R which lie along a straight line between P and the two components' (expected return and risk) positions when plotted in the risk-return space.

Thus various possibilities of lending are possible, as shown in Figure 3.3 overleaf.

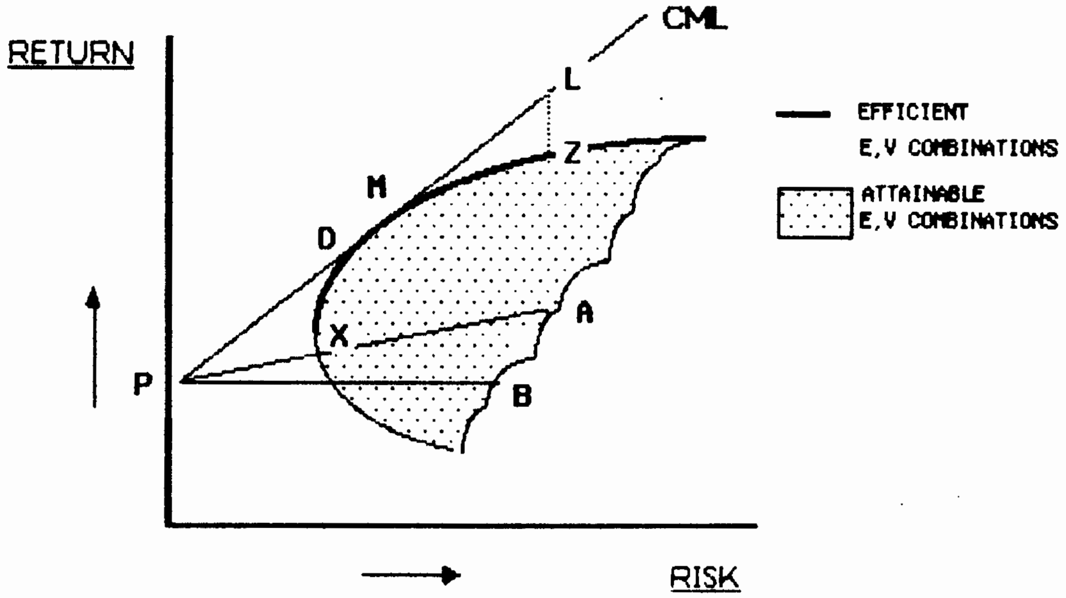


Figure 3.3: Efficient portfolio with lending introduced

Source: Sharpe (1964: 432) (adapted)

It is immediately obvious that, of the three possibilities shown in Figure 3.3 above, the preferred one for the investor will be the line PM. By investing a certain amount of his funds in the riskless asset (P), and the remainder in the optimal Portfolio (M), he can position himself along the line PM, in terms of his expected returns and the commensurate risk involved. For a given level of risk, the investor has achieved a higher return than he would have achieved had he not invested some of his capital in P. This is shown by the line PDZ in Figure 3.3.

The above case has dealt only with the situation where lending is allowed. If borrowing is now introduced into the model (at the same rate as the lending) again one investment plan dominates the others. The investments again lie along the line PM. However, it is now possible to 'gear oneself up' and achieve a higher return than was possible before (as shown by the line LZ in Figure 3.3).

At this stage of the analysis the existence of borrowing and lending, together with the effect of the introduction of the riskless asset, has been introduced. The results have shown that, from an individual's point of view, given the assumptions noted in 3.4.2, it is desirable to place oneself somewhere along the line PML as shown in Figure 3.2. The next stage of the theory requires an investigation of the conditions under which equilibrium in the capital market will arise.

3.4.4 Equilibrium in the Capital Market

As discussed in 3.3.3 above, by introducing a riskless asset into the model, a straight line emanating from the point of expected return of the riskless asset and tangential to the optimal portfolio frontier results.

As a result of the fact that the line PM in Figure 3.2 is the optimal investment choice, all investors will desire to hold some portion of portfolio M. By definition, equilibrium in a market suggests that there is no excess demand in the market. That is, all securities in the market must belong to some owner. Since all investors unanimously desire to hold M, in equilibrium M must be a huge portfolio containing all marketable assets in proportion to their value in relation to the total market value.

Again, in equilibrium, P must be the rate of interest that equates the demand for loans to the supply of money.

Point M on PML is called the Market Portfolio, whereas PML is commonly termed the Capital Market Line, or CML. In equilibrium, investors hold a proportion of their funds in M, and can adjust their return in relation to the risk they are prepared to bear, by borrowing or lending at the rate of interest P.

3.4.5 Equilibrium Pricing of Individual Securities

The second stage of Sharpe's derivation will now be examined.

The equilibrium situation as discussed above involves only efficient portfolios. The next stage in the analysis is to determine the behaviour of individual security prices in an equilibrium situation.

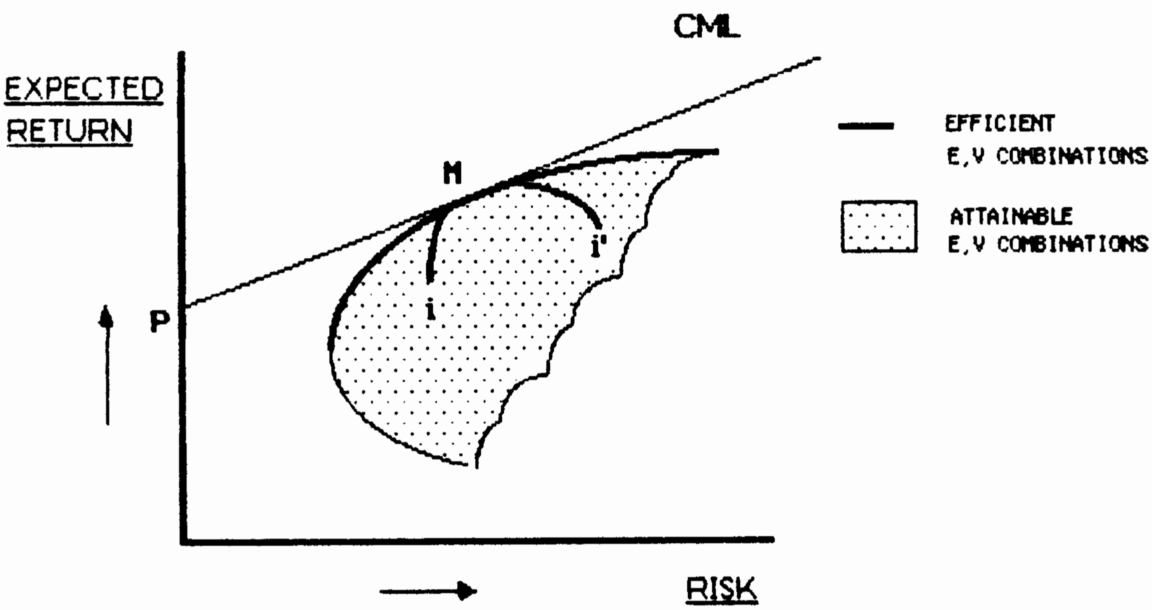


Figure 3.4 Equilibrium in the capital market

Figure 3.4 above refers to the equilibrium situation. Let i be any individual security. This security will lie within the shaded area of possible investments. It will not lie on the efficient frontier, as diversification will not have reduced the total risk of the asset to the systematic portion only.

Let α be the proportion of funds invested in i. In any portfolio consisting only of i and M, the market portfolio, the optimal portfolio will move along the curve iMi' as α varies. Let $(1 - \alpha)$ be the weight of M in the hypothetical portfolio.

In equilibrium, the price of any security such as i must adjust so that excess demand for it is zero. Note further that curve iM must be tangential to the CML in order to reflect the equality of the rate of exchange available in the market with the investor's marginal rate of transformation of risk for return. If the curve intersected the CML at any point other than at tangency, this would imply that some combination of assets and liabilities was more efficient than the CML, which is impossible, since the CML represents the efficient boundary of feasible security combinations.

Sharpe's insight came at this point, as he noted that the equilibrium conditions at tangency imply the appropriate risk measure for the individual security, <u>and</u>, they imply the equilibrium relationship between the risk and the expected return of the asset.

Returning to the portfolio mentioned above, the return on it will be:[8]

$$R_Y = \alpha R_i + (1 - \alpha) R_M. \qquad (3.3)$$

Now, assume there is an investor investing along the CML in both the riskless asset, P, and the market portfolio M. Let the proportion invested in P be α and in M be $1 - \alpha$; then,

$$R_X = \alpha R_P + (1 - \alpha) R_M \qquad (3.4)$$

In equilibrium, the curve iMi' is tangential to the CML. At this point

$$\frac{\delta \sigma (R_y)}{\delta E (R_Y)} = \frac{\delta \sigma (R_X)}{\delta E (R_p)} \qquad (3.5)$$

This is where Sharpe's insight becomes critical.[9] Using the chain rule to derive expressions for (3.5) above, and evaluating these when $\alpha = 0$, equation (3.5) becomes:

$$\frac{\text{cov} (R_i ; R_M) - \sigma^2 (R_M)}{E (R_i - E(R_M)) \; \sigma (R_M)} = \frac{\sigma (R_M)}{E(R_M) - R_P} \qquad (3.6)$$

Solving (3.6) for $E(R_i)$ yields:

$$E (R_i) = R_P + \frac{(E(R_M) - R_P) . \text{cov} (R_i ; R_M)}{\sigma^2 (R_M)} \qquad (3.7)$$

Equation (3.7) thus implies that the expected return on the i'th security equals the riskless rate of return plus the product of the slope coefficient,

$$\frac{E(R_m) - R_p}{\sigma^2 R_m}$$

, multiplied by the covariance of the returns for security i with the market. Put in simpler terms, expected return on the i'th security is a linear function of its systematic risk as measured by COV (i,M). This relationship is shown in Figure 3.5 below.

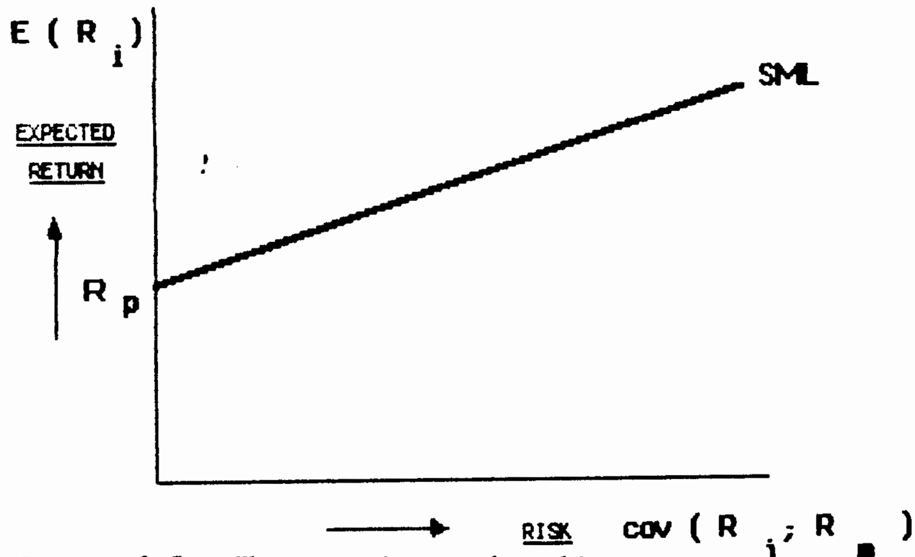


Figure 3.5: The security market line

The line described in this manner is the CAPM, or as it is sometimes known, the Security Market Line (SML).

Equation (3.7) above is thus the answer to the second question posed in 3.3.3. The equilibrium relationship between an asset's risk and its expected return is linear, with the y-intercept equal to the risk free rate of interest. Equation (3.7) can easily be rearranged to yield the risk-premium, thus answering the first question:

$$E(R_i)-R_F = \frac{E(R_M - R_F) \cdot cov(R_i;R_m)}{\sigma^2_{Rm}} \qquad (3.8)$$

The risk-premium (i.e. the amount of extra return obtained for incurring additional risk) is a function of the return of the market as a whole, the risk of the market as a whole, the risk-free rate of interest, and, finally, the covariance between the return of the asset i and the market return.

3.4.6 The CAPM expressed in terms of the Market Model

It is obvious that the line in Figure 3.5 is very similar to that produced by the market model, as shown in Figure 3.1.

Consider again the portfolio of asset i and the market portfolio, M, discussed in 3.4.5. The standard deviation of this portfolio is given by:

$$\sigma_Y = (\alpha^2\sigma^2_{Ri} + (1-\alpha)^2\sigma^2_{Rm} + 2r_{im}\alpha(1-\alpha).\sigma_{Ri}\sigma_{Rm})^{\frac{1}{2}}$$

at $\alpha = 0$ $$\frac{\delta\,\sigma_Y}{\delta\alpha} = -1/\sigma_Y\,(\sigma^2_{Rm} - r_{im}\sigma_{Ri}\sigma_{Rm})$$

but $\sigma_Y = \sigma_{Rm}$ at $\alpha = 0$, thus

$$\frac{\delta\,\sigma_Y}{\delta\alpha} = -\sigma_{Rm} - r_{im}\,\sigma_{Ri} \qquad (3.9)$$

The expected return on the combination will be:

$$E(R_Y) = \alpha\,(E_{Ri}) + (1-\alpha)\,E(R_m) \qquad (3.10)$$

Thus at all values of α ,

$$\frac{\delta E_Y}{\delta \alpha} = (E(R_M) - E(R_i))$$

and, at $\alpha = 0$

$$\frac{\delta \sigma_Y}{\delta E_Y} = \frac{\sigma_{RM} - r_{im}\sigma_{Ri}}{E(R_M) - E(R_i)} \tag{3.11}$$

Now, let the equation of the capital market line be of the form:

$\sigma_{RY} = f(E_R - P)$. Line iMi' is tangential to this line when α equals 0 and, as the plot of $E(R_i)$ and σ_{Ri}.
lies on the line, equation (3.11) can be restated as follows:

$$\frac{\sigma_{RM} - r_{im}\sigma_{Ri}}{E(R_M) - E(R_i)} = \frac{\sigma_{RM}}{E(R_M) - P} \tag{3.12}$$

The slope of a regression line is however:

$$b_i = \frac{cov(R_i; R_M)}{\sigma_m^2}$$

and, as $cov(R_i; R_M) = \frac{r_{im}\ \sigma_i\ \sigma_m}{\sigma_m^2}$

$$\text{so } b_i = \frac{r_{im}\ \sigma_i}{\sigma_m} \tag{3.13}$$

Now: multiplying equation (3.12) by σ_{Rm}, yields

$$\frac{\sigma^2_{Rm} - r_{im}\ \sigma_i\ \sigma_m}{E\ (R_m) - E\ (R_i)} = \frac{\sigma^2_{Rm}}{E(R_m) - P}$$

$$\Rightarrow \quad \sigma^2_{Rm} - r_{im}\ \sigma_i\ \sigma_m\ (E\ (R_m) - P) = \sigma^2_{Rm}\ (E(R_m) - E(R_i))$$

$$\Rightarrow \quad E\ (R_m)\ \sigma^2_{Rm} - E\ (R_m)\ r_{im}\ \sigma_i \sigma_m - P\ \sigma^2_{Rm} + P.r_{im}\ \sigma_i \sigma_m$$

$$= \quad E\ (R_m)\ \sigma^2_{Rm} - E\ (R_i)\ \ \sigma^2_{Rm}$$

thus,

$$(P - E\ (R_m))\ r_{im}\ \ \sigma_i \sigma_m = (\ - E\ (R_i) - E\ (R_m))\ \ \sigma^2_m$$

$$+ (E(R_m) + P)\sigma^2_m$$

$$\text{thus} \quad \frac{r_{im}\ \sigma_i\ \sigma_m}{\sigma^2_m} = \frac{- E\ (R_i) + P}{P - E\ (R_m)}$$

$$\text{thus} \quad - E\ (R_i) + P = (P - E\ (R_m))\ \frac{r_{i\ m}\ \sigma_i}{\sigma_m}$$

$$\text{thus} \quad E\ (R_i) = P - b_i\ (P - E\ (R_m))$$

$$\text{or:} \quad E\ (R_i) = P + b_i\ (E\ (R_m) - P) \qquad (3.14)$$

Thus, the expected return on the i'th security is equal to the riskless rate of return (P) plus the product of b_i and the risk premium on the market portfolio. Equation (3.14) is thus the CAPM, or SML, expressed in terms of the slope-coefficient of the market model, otherwise known as beta. This can be represented as follows:

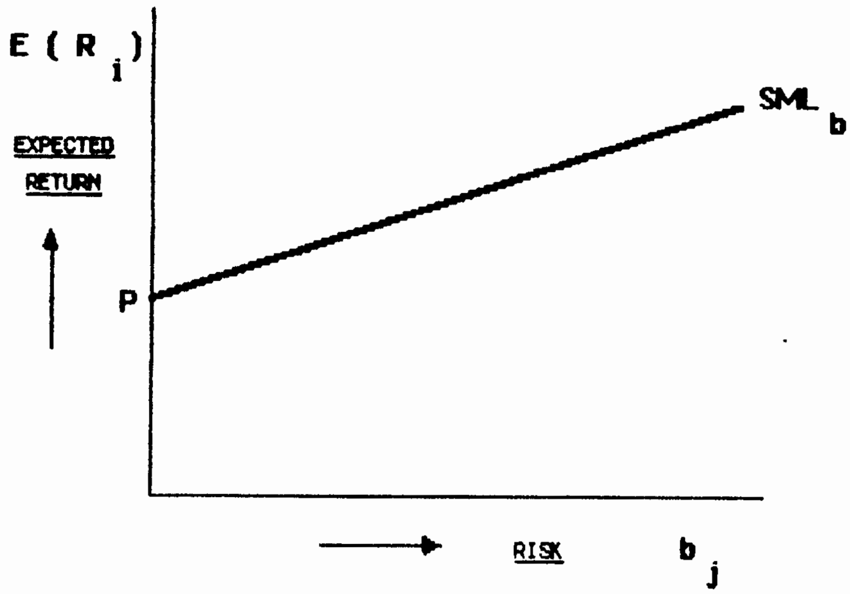


Figure 3.6: Security market line expressed in terms of beta

Comparing equation (3.14) with (3.7), it is immediately obvious that the two are identical, as, by removing the σ^2_{Rm} term from inside the bracket in (3.7) immediately yields a term identical to that in (3.14). Figure 3.6 is thus identical to (3.7) except for the horizontal scale, which differs by a factor of $1/\sigma^2_m$. (10)

3.4.7 Significance of the CAPM expressed in terms of the Market Model

The importance of expressing the CAPM in terms of the market model is that the CAPM is essentially an ex-ante model, as it deals with expectations about securities, and their related risks and returns.

The market model is however, an ex-post plotting of the actual returns on securities and markets, which can be used in an ex-ante, or predictive, sense to formulate expectations for use in evaluating the relationships between shares, as predicted by the CAPM.

3.5 BLACK'S ZERO-BETA PORTFOLIO

One of the assumptions underlying the CAPM (see section 3.4.2) was that unrestricted lending and borrowing at a risk-free rate of interest was possible. Black (1972) examined the nature of the CAPM if this assumption was removed i.e. there is no risk-free asset which has constant returns.

Black showed that portfolios exist which are uncorrelated with the true market portfolio. What this implies is that their returns have zero covariance with the market portfolio, and they have the same systematic risk. In other words, their beta is zero.

Although many such portfolios may exist, only one of them lies on the efficient frontier of the investment opportunity set. This is the minimum zero-beta portfolio, and it is unique.

Black then showed that by combining an investment in the zero-beta portfolio with an investment in the efficient portfolio (as explained in section 3.4.3) yields a Capital Market Line (CML), the equation of which is:

$$E(R_p) = E(R_z) + \frac{(E(R_m) - E(R_z))}{\sigma_m} \sigma_p \quad (3.15)$$

where $E(R_p)$ = expected return on the portfolio

$E(R_z)$ = expected return on the zero-beta portfolio

$E(R_m)$ = expected return on the market portfolio

σ_p = standard deviation of the portfolio

σ_m = standard deviation of the market portfolio.

From equation (3.15) it can be shown that the expected rate of return on any risky asset, whether or not it lies on the efficient frontier, must be a linear combination of the rate of return on the zero-beta portfolio and the market portfolio.(11)

The required rate of return on any risky asset i is thus

$$E(R_i) = (1 - \beta_i) E(R_z) + \beta_i E(R_m)$$

where $\beta_i = \dfrac{\text{cov}(R_i ; R_m)}{\sigma_m^2}$

which can be rearranged as follows:

$$E(R_i) = E(R_z) + \beta_i (E(R_m) - E(R_z)) \qquad (3.16)$$

It is patently obvious that equation (3.16) is exactly equal to the CAPM formula shown by equation (3.14). Black thus showed that the CAPM does not require the existence of a riskless asset. Beta is still the appropriate measure for the risk of an asset and the linearity of the model is still present.

3.6 GENERAL SIGNIFICANCE OF THE CAPM

3.6.1 Introduction

The CAPM as presented in this chapter appears to be a restrictive and narrow theory related only to the pricing of securities on a stock exchange. This is however not the case, as this section will show.

The CAPM is in fact one of the fundamental 'keystones' of modern finance, and as such plays a vital role in the overall financial paradigm. This section will examine the CAPM's position in the general finance theory, and will also present a brief philosophical discussion on the role of finance theory, and the possible direction in which the theory will develop..

3.6.2 The place of the CAPM in Finance Theory

Modern Finance Theory is a specialised branch of applied microeconomics. The development of the theory can be traced back to 1958, when Markowitz was developing his theory on portfolio selection, as discussed in Chapter 2. At the same time Modigliani and Miller were formulating a theory on the capital of a firm, and the valuation of different firms.

Copeland and Weston (1983) have identified the following six key facets of finance theory. Each facet is, from a theoretical point of view, internally consistent.

The facets are:

- utility theory
- state-preference theory
- mean-variance theory and the CAPM
- arbitrage pricing theory
- option pricing theory, and
- the Modigliani-Miller theorems.

The common thread running throughout these areas of finance is the attempt to determine how scarce resources are allocated by a price system based on the valuation of risky assets.

Utility theory has been discussed as the introduction to Markowitz's work, in Chapter 2. It focuses on the basis of rational decision making where risky alternatives exist. State-preference theory, CAPM theory, the arbitrage pricing theory and option theory all home in on the specific objects which are being chosen. The theory of choice combined with the object of choice lead to a determination of how risky alternatives will be valued (or priced) in an equilibrium situation. The pricing of assets in turn provides a signalling effect to the economy, aiding in the allocation of the resources. The final facet of utility theory, the Miller-Modigliani theorems, provides answers to the implications to the firm of the way in which it structures its capital, and to its dividend policy. The theorems attempt to determine whether the method of financing affects the value of the firm.

The above brief outline has given a broad overview of finance theory. The theory is positive, in that it makes hypotheses and then builts on them to yield predictions which are, if the theory is correct, accurate and meaningful predictions of the empirical situation. This explains the importance of empirical tests such as the one carried out in this study. They attempt to see whether the predictions are reflected empirically, and thus whether the theories are correct.

The preceding discussions have highlighted the major aspects of modern finance theory. The six facets of the theory are however not mutually exclusive, as explained below.

It has already been shown (in Chapter 2) that the link between utility theory and Markowitz Portfolio Theory is very close. Similarly, Chapter 3 explained the link between mean-variance portfolio theory (Markowitz Theory) and the CAPM. Thus it can be seen that the CAPM is in fact interlinked with all other aspects of finance. The next section will show how the CAPM inter-relates with the financing decision, the sixth key facet of portfolio theory.

3.6.3 The CAPM in a broader context

The previous section presented a broad overview of the central components of modern finance theory. The CAPM, although a separate component, was shown to be closely linked to other areas of finance. Notwithstanding this linkage, the CAPM still appears to be a distinct and specialised component of the overall theory. This section will show that this is not the case.

Recall from 3.6.2 above that the final area of modern portfolio theory identified concerns the optimal capital structure of firms, and the valuation of firms. Arguably, the single most important concept to do with this facet of finance is the concept of the cost of capital. The cost of capital can be defined as the minimum risk-adjusted rate of return which a project must earn in order to be acceptable to shareholders of a company.

It can be shown that the firm's mix of debt and equity financing do not affect the cost of capital. In addition, it is possible to calculate the cost of capital in the situation where risk is involved. This is not shown here as it is not considered crucial to the discussion that follows.

The derivation of the cost of capital under conditions of risk is the other important area in finance where the CAPM is extensively used. The CAPM provides an excellent theory for the pricing of risk. This can easily be combined with the cost of capital definition of the Modigliani-Miller model, to achieve a unified approach to the cost of capital.

The implication of the above discussion is that a weighted average cost of capital for the firm can be derived which can then be used in evaluating investment decisions, the risk of which is different to the risk of the firm as a whole. The CAPM is used to evaluate the required rate of return for the project. If the actual rate of return is higher than the CAPM predicted rate of return, the project should be accepted. Following on from this, the CAPM can be used to calculate the new weighted average cost of capital of the firm.

The CAPM can thus be seen to be a critical part of the evaluation process which every financial manager should follow in evaluating investment decisions. Far from being a theoretical model which can be used only for evaluating the equilibrium pricing of risky assets, it is part of the on-going business activities of any firm.

3.6.4 An alternative perspective of Modern Finance Theory

The preceding sections have presented a brief overview of modern finance theory, and have also shown specifically where the CAPM fits into this financial framework.

The finance theory has been described by Findlay and Williams (1985) as neoclassical in that it is built upon certain assumptions which are an integral part of neoclassical economic theory in general. These assumptions are:

- the methods and procedures used by natural scientists can be easily used by social scientists (e.g. mathematical procedures, statistically valid conclusions etc.)
- all future events can be expressed in terms of probability density functions
- the past is an adequate guide to the future
- simple arbitrage mechanisms can be relied on to provide continuity within and between markets.

Findlay and Williams argue that these assumptions are unnecessarily restrictive in the development of the theory, and that this has resulted in an intellectual stalemate, in which the various 'camps' of thought are unable to reject their point of view, but the same is true for the other point of view. In addition, they say that these assumptions are contradicted by observation.

The solution to this deadlock, they suggest, is an analysis of financial theory from a post-Keynesian point of view. This school of thought is based on a view of the world as being constantly changing and unpredictable through time.

The real world in which decisions are taken and economic consequences result has been described by Findlay and Williams as a world of failed expectations, in which the key problem is not risk, but fundamental uncertainty. They argue that the process of computing means and variances for distributions is a fruitless exercise, as the actual outcome may never even approximate the expected outcome.

The alternative model suggested by the post-Keynesian school is a normative one which is based on fact. The assumptions and actions of observers are assumed to be based on actual observations (e.g. individuals lie and engage in self-delusion; society cannot adjust quickly to new prices). No complete alternative theory is presented in the paper.

The foregoing discussion has been presented in order to give some insight into the 'state of the art' thinking on finance. The analysis presented is radical in its abstraction from the groundwork laid by Markowitz, Tobin, Modigliani and others, however it does give some indication of the evolutionary route along which the theory of finance may develop.

In view of the fact that no formal alternative theory is defined, it is difficult to counter the views expressed by Findlay and Williams. Their views are however certainly not mainstream finance thought. In the opinion of this researcher the positive theory of finance appears to offer a greater reliability for prediction than a normative theory. The future is uncertain, it is granted, however, this is one of the aspects implied in the notion of risk. Positive models attempt to reduce the uncertainty about the future, and this is where their strength lies.

In addition, the past may be the best indicator of the future. Certainly it is one of the few indicators that exists, and thus it is considered worthwhile to build models incorporating past knowledge. The final rejoinder to Findlay and Willaims is that the positive models of finance are not static. As empirical evidence has highlighted weaknesses in current models, so new models have been developed. The arbitrage pricing theory (see Chapter 13) is an excellent example of this. It was developed to explain anomalies in the CAPM.

In short, in this researcher's opinion, the positive neoclassical paradigm offers a more beneficial theory than the post-Keynesian paradigm suggested by Findlay and Williams.

3.6.5 Conclusion

This subsection has shown an overview of finance theory, and where the CAPM fits into the framework. The importance of the CAPM to financial managers has also been shown, as has the link between the CAPM and the financing decision. Finally, an alternative paradigm for finance theory has been examined, and refuted. The aim of all the above discussion has been to place the CAPM in its correct context in finance theory.

3.7 SUMMARY

This chapter has traced the development of the CAPM from the holding of an efficient portfolio, through to establishing the theoretical relationship between securities when their prices are in equilibrium. In addition, it has been shown that the existence of a riskless asset is not a necessity for the derivation of the model. The general significance of the CAPM, and its place in modern finance theory have also been presented. The next chapter highlights the link between the Efficient Markets Hypothesis and the CAPM.

CHAPTER 3 - FOOTNOTES

(1) The market model is described in section 3.3.1 below.

(2) The ex-ante/ex-post distinction is described in section 3.4.7.

(3) Refer section 2.2 for a review of the basic axioms of utility theory, as described by von Neumann and Morgenstern (1953).

(4) See Markowitz (1959): pages 97-102, especially the footnotes.

(5) Treynor (1965) was also working on a similar model at this time. However, as Sharpe notes in footnote 7 of his 1964 paper, Treynor's work was at that time still unpublished. Treynor's first published work in this area was Treynor (1965).

(6) The equation is given here as it will be referred to later.

(7) See Sharpe (1966).

(8) This follows directly from the mathematical formulae defined in section 2.3.3.

(9) Or, in other words, the partial derivative of (R_y) with respect to $E(R_y)$ is equal to the partial derivative of (R_x) with respect to $E(R_p)$.

(10) Recall from equation (3.13) that $b_i = \frac{r_{im}\sigma_i}{\sigma_m}$

(11) The proof of this is identical to that in section 3.4.5, using the CML as shown in equation (3.15).

CHAPTER 4

THE RELATIONSHIP BETWEEN THE EFFICIENT MARKETS HYPOTHESIS AND THE CAPITAL ASSET PRICING MODEL

4.1 INTRODUCTION

In the previous chapter the derivation of the CAPM was discussed. In addition, the assumptions on which the model is based were examined. This chapter will explain, in an anecdotal rather than a mathematical manner, the Efficient Markets Hypothesis (EMH). The relationship between the EMH and the CAPM will also be investigated. The development of the EMH will not be discussed in detail.(1)

4.2 THE EMH

4.2.1 Background

The development of the EMH was as a result of empirical tests carried out on the price movements of securities quoted on major stock exchanges, in particular the New York Stock Exchange (NYSE). The theory was developed in order to explain the results of these empirical tests, the majority of which were carried out in the 1950's and 1960's.

The tests mentioned above were actually tests of the Random Walk Model, in which it is assumed that successive one-period price changes are independent, and that these price changes have identical distributions.(2)

The Random Walk Model is in fact a sub-set of the broader 'fair-game model', otherwise known as the 'Expected Return' model, which has as its sole primary assumption the idea that conditions of market equilibrium, and hence the prices of securities, can somehow be stated in terms of expected returns.

As Fama has said:

> "In general terms,....such theories would posit that conditional on some relevant information set, the equilibrium expected return on a security is a function of its "risk". And different theories would differ primarily in how "risk" is defined."
>
> Fama (1970: 384).

The major implication of this assumption that market equilibrium conditions can be stated in terms of expected returns based on some underlying information set which affects the securities' assessed risk, is that the expected profits of an investor in excess of the equilibrium expected (and predicted) profits, are zero. This is the so-called 'fair-game' mentioned above.

4.2.2 The Hypothesis

The EMH, in simple terms, proposes that:

> A market is efficient if prices always 'fully reflect' the available information.

The above definition was put forward by Fama (1970), and is the accepted form of the EMH. The definition is so general that no empirical tests can be carried out on it to determine whether the statement is valid or not. The definition can also become circular if information is defined as being that which is reflected in prices. If this were the case, all markets would be efficient. Beaver (1980), has suggested that Fama used the definition in order to give an intuitive description of the concept, and that he never intended it to become a rigorous definition.

4.3 FORMS OF THE EMH

In order to carry out empirical studies on the validity of the EMH, the information set which is considered to be fully impounded into share prices has been split into three subsets. Each subset is considered to encompass a greater proportion of the complete information set available to investors.

The three forms of the model, which were initially described by Roberts (1959) and have subsequently been refined to their present format, are:

1. Weak Form: The only information reflected in current prices is the past price history.

2. Semi-Strong Form: In addition to the historical prices reflected in prices in the weak form, other publicly available information is fully assimilated into, and reflected in, current share prices.

3. Strong Form: Information known only to certain groups (commonly termed 'insider information') is also reflected in the share price. The information set impounded in share prices is thus considered to be all information.

The relationship between the various forms of the EMH and the information set underlying each form, is shown in Figure 4.1 below.

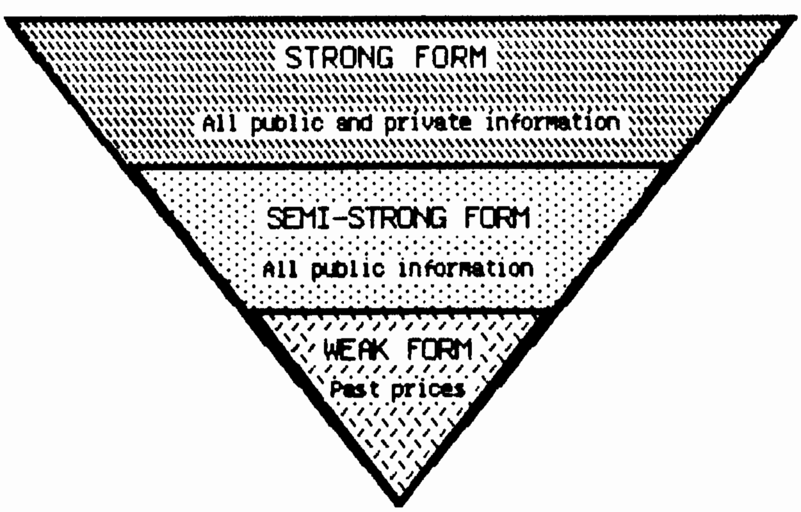


Figure 4.1: Forms of the EMH

4.4 EMPIRICAL EVIDENCE ON THE VALIDITY OF THE EMH

The trichotamization of the EMH allows empirical tests and analyses to be carried out on the hypothesis. Fama (1970) and Gonedes and Dopuch (1974), inter alia, present literature reviews of empirical studies relating to the EMH. It is beyond the scope of this study to repeat these reviews here.(3) Instead, the general results will be stated.

4.4.1 Weak Form tests

Empirical studies of the weak forms of the EMH are the most voluminous of any of the three categories. This is to be expected as it is easiest to test an information set consisting only of past prices.

The evidence is strongly in support of the EMH contention that share prices fully reflect historical share prices.

4.4.2 Semi-Strong Form tests

The empirical studies of the semi-strong form of market efficiency have in the main been tests which examine the relationship between accounting information and share prices, or that examine the impact of share issues or share split announcements on share prices.

Again, the evidence that is available points towards the existence of the semi-strong form of the EMH.

4.4.3 Strong Form tests

Testing the strong form of the EMH presents difficulties, as the existence of insider dealing is difficult to assess, as is the result of this dealing on share prices. The empirical work in testing the strong form EMH has thus concentrated on highlighting deviations from strong form efficiency. Two such deviations, both anecdotal rather than empirical, are noted by Fama (1970)(page 415), however he does not appear to regard these as being of particular importance.

A conclusion on the existence of strong-form market efficiency is thus less clear than the conclusions that can be drawn regarding the other forms of the hypothesis.

4.4.4 Conclusion on empirical validity of the EMH

The EMH would certainly appear to hold true for both the weak form and the semi-strong form. The evidence in favour of the strong form EMH existing is sparse, primarily due to the difficulty in testing the theory.

4.5 THE EMH AND THE CAPM

The fact that the EMH and the CAPM both deal with share prices in a market context suggests that there must be some link between the two models. This section will examine that relationship.

The CAPM, based as it is on expected return and risk, is a recommended strategy for investors. It recommends that all investors should hold a certain proportion of the market portfolio, combined with an appropriate positive or negative holding of a risk-free asset. The CAPM thus takes the individual risk and return of a security as given, and concentrates on the optimal risk-return profile of an investor's portfolio. An investment strategy based on this type of analysis presumes that a fair return is expected to be earned, commensurate with the degree of risk exposure which the investor adopts.

Fama (1970) has shown that if the following conditions are operating in the market, market efficiency (in the semi-strong form at least) will be present.

- Transactions costs, if they exist, are taken into account in price determination by the market. This would probably only be the case if transaction costs were reasonable.

- A sufficient number of investors, not necessarily all of them, have access to available information.

- Some disagreement amongst investors about the implications of given information may occur. If this does happen, no investor will consistently be able to make a better evaluation of the available information than the aggregate evaluation as reflected in the share prices.

The assumptions underlying the CAPM, as noted in 3.4.2, were those which related to portfolio analysis _per se_, and those which were introduced in order to develop the capital market theory.

The additional capital market theory assumptions are those which are of importance here.

Recall from 3.4.2 that, _inter alia_, the following assumptions were made:

- there are no transactions costs for buying and selling securities
- all available information is freely available to everyone
- all investors have homogeneous expectations and have the same one-period investment horizon.

Comparing these assumptions with the conditions necessary for semi-strong form market efficiency, it will readily be seen that the efficient market conditions are merely relaxations of the strict capital market assumptions, which were described in 3.4.2 as being properties of the overall assumption that capital markets are perfect.

The implication of the preceding discussion is that the Capital Asset Pricing Model actually assumes efficiency of the market, and then, having made this assumption, predicts the equilibrium pricing of assets within the perfect market.

Roll (1977) has suggested that this actually precludes any meaningful test of the CAPM being carried out, as an empirical study in fact tests a joint hypothesis: the efficiency of the market and the CAPM predicted relationship between the risk of any asset and its return. This criticism is examined in detail in Chapter 12.

Two terms which are often used are market efficiency and portfolio efficiency. The first deals with the EMH as it has been presented in this chapter, and is thus concerned with the degree of information assimilated into share prices, and the speed with which that information has been assimilated. Portfolio efficiency, although sounding similar, should not be confused with the above. Portfolio efficiency is used to describe the efficient portfolio, for which no higher returns can be earned for the chosen level of risk, whilst for a given return, no less risk need be incurred. The two terms are clearly not interchangeable, however there are elements of common ground underlying them.

The EMH can thus be considered to be a relaxation of the perfect capital market assumptions. Perfect market conditions would certainly be sufficient for the EMH to hold, but would also not be necessary conditions.

4.6 CONCLUSION

This chapter has highlighted the relationship between the Efficient Markets Hypothesis, and the Capital Asset Pricing Model.

The relationship between the CAPM and the EMH is that the EMH is implied by some of the assumptions which lead to the CAPM. The reverse is however not necessarily true. Conditions in which the EMH existed would not necessarily imply the existence and validity of the CAPM, however it may hold.

However, because of the empirically tested and proved existence of the semi-strong form EMH on some capital markets, even if it has not been proved to exist on all markets, there is a chance that the CAPM also exists in reality. It is for this reason that empirical tests of the CAPM have been carried out. These will be reviewed in Chapter 6.

Before reviewing tests of the CAPM however, it is considered necessary to examine the evidence in South Africa relating to market efficiency. The reasoning for this is as follows. Market efficiency has been shown to be a relaxation of the strict Capital Market Theory assumptions under which the CAPM is expected to operate. Thus, if the EMH, at least in the semi-strong form, is not present in South Africa, it is very unlikely that the CAPM will prove to be a valid model. If the SA evidence shows the efficiency of the market to be of an acceptable level, then a test of the actual CAPM in South Africa will be a valuable and meaningful exercise.

CHAPTER 4 - FOOTNOTES

(1) See Fama (1970) for an excellent mathematical review of the EMH and its evolution, together with a literature review.

(2) Fama has pointed out that this terminology is rather loose. In his words

> "prices will only follow a random walk if price changes are independent, identically distributed; and even then we should say 'random walk with drift' since expected price changes can be non-zero. If one-period returns are independent, identically distributed, prices will not follow a random walk since the distribution of price changes will depend on the price level."
>
> Fama (1970: 386).

(3) The major reason for not repeating the voluminous evidence of tests on the EMH carried out overseas is that the results, although indicative of what may happen on the South African market, are not directly transferable. Tests of the JSE have to be carried out to ascertain whether the market is efficient.

CHAPTER 5

TESTING THE CAPITAL MARKETS: SOUTH AFRICAN EVIDENCE

5.1 INTRODUCTION

The previous chapter presented the EMH in its various forms, and gave a brief summary of the results of the evidence relating to the existence of the various forms of the EMH, as found on the NYSE. These results are however not transferable to the South African market. They may be indicative of the situation which would prevail were the market to be efficient, however, they do not give any evidence towards the actual efficiency of the Johannesburg Stock Exchange (JSE).

Roll (1977) has pointed out that a test of the CAPM is in fact a test of the joint hypothesis of the efficiency of the market portfolio and a test of the CAPM itself. It was shown in the previous chapter that portfolio efficiency is not necessarily the same as market efficiency, nevertheless, the two are interrelated, and thus a review of the evidence relating to tests of both market efficiency and portfolio efficiency in a South African context is considered important.

The literature review which follows is thus a complete review of the tests on market efficiency and portfolio efficiency that have been performed in South Africa.

5.2 TESTS OF THE EMH

5.2.1 Introduction[1]

The split of the EMH into weak form, semi-strong form and strong form tests provides a meaningful umbrella under which the evidence can be presented.

5.2.2 Weak Form tests

As the weak form EMH is concerned only with past share prices, the majority of tests take the form of tests on the Random Walk Model. If the model holds, it is usually held as being sufficient for weak form efficiency to be present.

Such tests have taken the form of serial correlation tests and runs tests, both of which test the proposition that a lack of correlation between successive share prices is a sufficient condition for the acceptance of weak form efficiency.

The first test of any form on the JSE was that carried out by Affleck-Graves (1974).[2] This was a non-parametric Wald-Wolfowitz test in which the null hypothesis that returns were random was rejected for a number of the shares. Nevertheless, the overall conclusion was that weak form efficiency was an accurate description of the market. In addition to this test, a runs test was performed, the conclusion reached here (at a 95% confidence level), was in agreement with the serial correlation test, namely that weak-form efficiency of the market appeared to be in existence.

The same data was used in the first published test in South Africa. Affleck-Graves and Money (1975), performed a serial correlation test on the returns of 50 shares quoted on the JSE. Ten lag categories were used to analyse weekly data. Thirty three of the five hundred correlation coefficients generated in the sample were found to be greater than two standard deviations from zero. Of these, 42% were in the first and second weeks of lagging. Their conclusion was thus that no auto-correlation existed for periods greater than two weeks, and that for 80% of the market their results were consistent with weak form efficiency.

They also suggested that the slight dependence from one week to the next would be useless to an investor trying to earn an abnormal return by analysing price histories. This second test, being of a parametric nature, did however assume that the distribution of the returns was normal, with a finite variance. The advantage of the Wald-Wolfowitz test mentioned earlier was that it was a non-parametric test and thus made no inferences about the underlying population characteristics.

Hadassin (1976), carried out both runs tests and serial correlations tests for 30 shares on very short lag periods of one day and four days. His results for the runs tests were that non-independence of prices appeared to exist for 24 of the shares on a one day interval, and for 12 on a four day interval. This thus indicated non-acceptance of weak form EMH behaviour. However, in carrying out serial correlation tests, no significant evidence disputing random behaviour could be found.

Gilbertson and Roux (1977) also found evidence of non-random behaviour using runs tests. However, using serial correlation tests no such evidence could be found. In addition to the runs tests and serial correlation tests noted, Gilbertson and Roux analysed the distribution pattern of the share returns, and found evidence of a strongly peaked, long-tailed (i.e. leptokurtic) distribution. This is thus not consistent with the (usually presumed) normal distribution. Similar results were reported by Ozen (1977).

Other tests of the distributions, notably those by Schlosberg (1976) and Strebel (1977), have also shown evidence of a leptokurtic distribution of the returns. Knight (1983) has however advised caution in drawing general inferences from these studies, notwithstanding their internal validity.

Strebel (1978) has argued that linear regression tests using an ordinary least squares (OLS) technique would be worthless due to the leptokurtic distribution noted. The rejoinder to this warning is that the OLS technique does give the best linear unbiased estimate of the risk-return relationship, and that the evidence seems to disprove the existence of autocorrelation, even taking Hadassin's and Gilbertson and Roux's tests into account.

Even without analysing any potential problems in the methodologies used by Hadassin and Gilbertson and Roux, it is considered that the evidence in favour of the existence of a weak-form efficient market is sufficient to draw a general conclusion that the JSE conforms to this condition. This conclusion is reached based mainly on the excellent studies performed by Affleck-Graves (1974) and Affleck-Graves and Money (1975).

This conclusion is given marginally greater support by the final phase of the Gilbertson and Roux (1976) study, which analysed the effect of applying four different 'trading rules' on the market. Weak form efficiency would imply the futility of such an exercise.

Their findings were that a trading rule did not consistently outperform a buy-and-hold strategy. This method of testing for market efficiency has been the subject of intense debate in the literature, the general consensus now being that the application of a trading rule test does not necessarily prove efficiency, it is merely consistent with the concept of efficiency. In South Africa, the results are not inconsistent with efficiency.

The general conclusion is that the JSE conforms to weak-form efficiency conditions, however, further work could, and possibly should, be carried out.

5.2.3 Semi-Strong form tests

The evidence of semi-strong form efficiency, in which all available information is expected to be reflected in the share price, is very scarce.

Knight (1983), carried out a number of tests designed to examine the speed of adjustment of the market to new information.[(4)] He investigated the market reaction to annual earnings releases using the Ball and Brown (1968) API approach, whereby an abnormal performance index (API) is constructed for the aggregate of all the announcements. This method aggregates results across periods and abstracts from the market factors at the time of the announcement. In addition to this, Knight used the absolute residual approach (ARA) used by Beaver (1968). The advantage of the ARA approach is that it makes no assumptions as to the expected investor reaction to earnings announcements, and is thus not dependent on a maintained hypothesis.

The results of both these studies were mutually supportive in their findings. These were that the market reacted to both 'good' news (i.e. positive unexpected earnings) and 'bad' news (negative unexpected earnings), however, the magnitude of the reaction was twice as great for the positive forecast errors as it was for the negative ones. The ARA approach indicated significant information content in the following three announcements by companies. First, the preliminary report, second, the interim report, and third, and least important, the annual report.

Further analysis on the half-yearly results indicated a certain inefficient (i.e. non-instantaneous) reaction of shares to this information.

The second major item which Knight studied was the information content of dividends. He attempted to determine whether the simultaneous announcement of dividends confounded the results mentioned above. The conclusion was that dividends appeared to have little informational value, and thus would not have confounded the previous results.

The final test which Knight carried out was a replication of that performed by Sunder (1973) on the market reaction to a change to LIFO. This result used a variant of the API approach, termed Cumulative Abnormal Residual (CAR) analysis. The results here were that the announcement did have informational content, however the adjustment period was slow and thus the JSE appears inefficient. Some evidence of a 'learning phenomenon' was reported, as the negative reaction appeared to be lesser in more recent changes to LIFO, and the speed of adjustment appeared to be shorter.

Knight's general conclusion, which was conditional on the validity of the market model; the research designs; and the earnings and dividends models employed, was that the market appeared to be inefficient to a certain extent, and that the characteristics of the JSE were not the same as those shown to exist on the NYSE.

The above findings are consistent with a very tentative and exploratory study performed by this researcher (Stewart (1982) using Sunder's techniques) on the effects of the introduction of deferred taxation on share prices. The sample used in this study was very small (26 companies) and interpretation of the results was hampered by the fact that the unexpected negative reaction noted could have been due to an inefficiency in the market, or to information content in the announcement. However, the slow reaction of shares to this announcement appeared to disclose an inefficient market.

The overall conclusion on semi-strong form efficiency is thus that the market appears to be inefficient, however, much research needs to be carried out before this can be stated with any degree of finality.

5.2.4 Strong Form tests

The over-riding problem with tests of strong form market efficiency is that the tester is attempting to find out whether insider information is assimilated into the share price. This obviously poses design problems, as insider information is often difficult to ascertain, let alone test. The solution to this has been to approach the problem from a 'side-ways' perspective, and then test the condition indirectly. For example, if the researcher posits that mutual fund managers will have access to insider information, he would then test to ascertain whether mutual funds generally outperformed the market.

Certain problems do arise with this approach, the major one being the validity of the premise that mutual fund managers have access to insider information. If this is incorrect, abnormal performance may negate semi-strong form efficiency, as abnormal performance is being achieved based on publicly available information. Alternatively, if no abnormal performance is noted and the premise that the managers have access to insider information is incorrect, this may only imply semi-strong form efficiency.

Notwithstanding the above interpretational problems of tests of this type, tentative conclusions have been drawn regarding studies carried out in the USA.(5) In South Africa, two studies, namely du Plessis (1974) and Gilbertson and Roux (1976) have tested strong form efficiency in this manner. Methodological problems regarding these works have been highlighted by other authors, and in view of these, only tentative conclusions vis-a-vis efficiency have been drawn. These have not indicated strong form efficiency on the JSE.

5.2.5 Conclusion on Market Efficiency on the JSE

The evidence for market efficiency on the JSE is, when compared to that available on the NYSE, very sparse indeed. The only conclusion that can be stated with any degree of assurance is that weak-form efficiency appears to be in existence. The evidence, sparse as it is, on semi-strong form efficiency and strong-form efficiency, points towards the lack of these efficiency conditions prevailing. However, in view of the fact that testing the CAPM is in fact a test of market efficiency as well (see Roll (1977)), the present study is considered worthwhile.

5.3 TESTS OF PORTFOLIO EFFICIENCY

Affleck-Graves and Money (1976) compared the Markowitz (1959) portfolio selection procedure with that proposed by Sharpe (1963), known as his Diagonal Model. The data on which they carried out their tests was the prices of 175 shares quoted on the JSE for the period 1962 to 1973. The overall conclusion of the study was that the Markowitz procedure produced results which were far superior to the one-index model suggested by Sharpe. The authors did note that if a large amount of money was to be invested, thus forcing a low ceiling on the total amount to be invested to be in any one security, the index model performed accurately.

Carter (1983), found that the market portfolio was consistently inefficient ex-post. He concluded that this was not surprising, as this had already been pointed out by Sharpe (1970). Carter, on page 2.7 quotes Sharpe as follows:

> "The values of capital market theory are ex ante (before-the-fact) estimates. Observed values are ex post (after-the-fact) results. The portfolios that do, in fact, turn out to be efficient will lie along some line, but not necessarily the ex ante capital market line. In fact, the market portfolio invariably proves to be inefficient ex post."

The above studies have both concentrated on the efficiency of portfolios. Mention was made in Chapter 4 of the point that Roll made, that tests of the CAPM are invariably tests of the efficiency of the market portfolio also, and for this reason the above studies were reviewed.

The present study is thus considered to be the first of its kind in South Africa, in that it purports to test the CAPM, and (according to Roll) market efficiency at the same time. The importance of the CAPM to finance in general has already been noted in section 3.6, thus it is vital to determine whether or not the model is valid in the South African securities market.

5.4 CONCLUSION

This chapter has examined the evidence for market efficiency in South Africa. Numerous works have been reviewed. An overall conclusion can only be drawn to the extent that weak form efficiency appears to be present. Other, stronger forms of efficiency appear unlikely to exist, however further research is needed. The following chapter details the results of the major tests of the CAPM that have been published to date.

With regard to portfolio efficiency, very little work has been carried out in South Africa. The importance of the present study lies in its test of the CAPM and portfolio efficiency.

CHAPTER 5 - FOOTNOTES

(1) Knight (1983) has provided an excellent review of the South African evidence for the existence of the EMH. This section borrows heavily from this review. It has, of course, been brought up to date.

(2) This test was part of an unpublished M.Sc. thesis. Although it is possible that other unpublished works had already been carried out at other South African (or overseas) universities, these have not come to the knowledge of this researcher. In view of this, it is considered correct to regard this test as the first in South Africa.

(3) This implies correlation between successive prices of thinly-traded shares. Refer Chapter 8, section 8.8 as to how the intervalling effect was overcome in this study.

(4) The interested reader is referred to part 2 of Knight (1983) for the full study.

(5) Knight (1983: 48) refers to Jensen (1969) and Friend, Blume and Crockett (1970) in this regard.

CHAPTER 6

EMPIRICAL TESTS OF THE CAPM

6.1 INTRODUCTION

The CAPM, as derived in Chapter 3, presents a model for market equilibrium in which there is a linear relationship between the return on an asset in excess of the risk-free rate of return and the excess return on the market portfolio. This model is of an ex-ante nature which, for the following reasons, is impossible to test directly:(1)

- investor's expectations often differ from the ex-post results, thus empirical data cannot give an entirely satisfactory test. In addition, investor's expectations cannot be tested directly.
- the theory is probably oversimplified due to the strict assumptions underlying it.
- not all parts of the model may be supported empirically. For example, the capital market line (CML) may be achieved equally through naive diversification as through Markowitz diversification. If this were the case, then the CML would be derived, however the SML would not be derived and would not be supported by it.

- the risk-measure commonly used, viz. the variability of the return-measures, is only a surrogate for the actual risk, and may not be the best measure there is.
- there is a downwards bias in the regression coefficient and correlation coefficients found by the usual vast squares technique, owing to errors that enter into both the expected-return estimates and the expected risk estimates.
- the true market portfolio (i.e. that portfolio consisting of all assets in the capital market) has never been precisely defined. The market index is merely used as a surrogate for the market portfolio.

In spite of the above problems inherent in testing the CAPM theory, significant tests of the model have been carried out. These will be reviewed in this chapter. However, before reviewing these, it is necessary to re-examine the CAPM in terms of its testability. This is because of the fact that it is primarily an ex-ante, or expectations, model.

6.2 THE EX-POST FORM OF THE CAPM

In order to move from the ex-ante CAPM to an empirically testable ex-post form, it is necessary to assume fair-game properties(2) for the assets within the CAPM framework. In addition, it is necessary to assume bivariate normality of the underlying security returns, so that the beta-coefficient in the CAPM is the same as the beta-coefficient in the fair-game. This is explained below.

The fair game assumed above, in which the expected rate of return on an asset is, on average, equal to the realised rate of return, can be written as follows:

$$R_{jt} = E\ (R_{jt}) + \beta_j(R_{mt} - E(R_{mt}) + \varepsilon_{jt} \qquad (6.1)$$

where $E\ (R_{mt} - E\ (R_{mt})) = 0$

ε_{jt} = a random term.

$E\ (\varepsilon_{jt}) = 0$

$cov\ (\varepsilon_{jt};\ R_{mt} - E\ (R_{mt}) = 0$

$cov\ (\varepsilon_{jt};\ \varepsilon_{j,t-1}) = 0$

$$\beta_{jt} = \frac{cov\ (R_{jt};\ R_{mt})}{var\ (R_{mt})}$$

Equation (6.1) can be seen to be a fair game because if the expectation of both sides of the equation is evaluated, the average realised return is equal to the expected return.

Recall from Chapter 3, equation (3.11), that

$$E(R_j) = P + b_j\,(E(R_m) - P))$$ (6.2)

where $E(R_j)$ = expected return on asset j in time t = $E\,(R_{jt})$

P = riskless rate of return

b_j = risk coefficient = β_j

$E(R_m)$ = expected rate of return on the market in time t = $E(R_{mt})$

Equation (6.2) can also be rewritten as:

$$E\,(R_{jt}) = P + \beta_j(E(R_{mt}) - P)$$ (6.3)

Now substituting the RHS of (6.3) into (6.1) yields:

$$\begin{aligned} R_{jt} &= P + \beta_j\,(E(R_{mt}) - P) + \beta_j(R_{mt} - E(R_{mt})) + \varepsilon_{jt} \\ &= P + \beta_j\,(R_{mt} - P) + \varepsilon_{jt} \end{aligned}$$ (6.4)

and, subtracting P from both sides,

$$R_{jt} - P = \beta_j(R_{mt} - P) + \varepsilon_{jt}$$ (6.5)

which is the ex-post form of the CAPM. Equation (6.5) is an ex-post form because it is expressed in terms of ex-post observations of data, instead of ex-ante expectations. This can readily be seen by noting that there is no expectations term on either side of equation (6.5).

The derivation of an ex-post form of the CAPM is important, as it means that tests on the validity of the predictions of the CAPM can be carried out. The major tests of the CAPM that have been performed are reviewed below.

6.3 LITERATURE REVIEW

6.3.1 Introduction

The previous chapter presented a major literature review of the tests on market efficiency and other aspects of capital market theory that have been carried out in South Africa. In view of the extremely large number of articles that have been published and which have contributed to the evidence on the validity of the CAPM on the NYSE and other major markets, and because of the breadth of their scope, the literature review which is presented below is very restrictive.

It was considered far more beneficial to present the major, most significant articles which have been published, which relate closely to the 'pure' CAPM, than to detail the vast spread of published material. This enables the development of empirical research into the CAPM to be traced in a broad, easy to follow manner that highlights the major development in research methodology, together with the findings.

With the above in mind, the brief review is presented below, its objective being to present the major tests on CAPM validity that have been carried out in various countries.

6.3.2 Initial studies

Lintner (1965) was probably the first person to carry out any empirical tests on the CAPM. This is hardly surprising as he was, together with Sharpe, responsible for much of the development of the model. It is perhaps only in retrospect that Lintner's work can be seen to be a test of the CAPM, nevertheless, it is the first.

Lintner regressed the annual rates of return of 301 securities over the period 1954 - 1963 against the market return in the same period. Using this time-series regression technique he estimated the systematic risk (that is, the beta-coefficient) and then regressed the mean rate of return on both the systematic risk and on the estimate of the residual variance. (3) Lintner found that the returns on his sample of stocks were positively and significantly related to their variance.

Lintner's results are contrary to the predictions of the CAPM, which posits that the beta-coefficient is the only significant risk measure. The beta coefficient is presumed to show the systematic portion of the total risk of a security, which cannot be diversified. The variance on the other hand represents total risk, a portion of which can be removed through diversification.

A study that was very similar in design to Lintner's, as described above, was carried out by Douglas (1969), on the NYSE. He used annual and quarterly share price data, and regressed the returns from a large cross-sectional sample of shares against both their own variance, and their own beta estimates. Douglas' results showed that the return on a security is positively and statistically significantly related to its own variance, but not to its beta-coefficient. Thus increased returns are being earned for bearing unsystematic risk. This would appear to be a violation of the model, as it predicts that variance should have no impact on a security's return, once its beta value has been taken into account.

Both Douglas' and Lintner's works were thus contradictory to the predictions of the CAPM. For this reason their methodology was re-examined by Miller and Scholes (1972), who found that the mis-specification of the model could have been due to skewness in the distribution of returns, together with random error measurements in the beta calculations. These two factors were shown to be capable of producing Douglas' and Lintner's results. Miller and Scholes concluded that the CAPM appears to be a valid model.

One of the major problems with the methodology employed by Lintner and by Douglas was the fact that they did not attempt to abstract from the correlation amongst residual errors in various stocks. This factor has been discovered by King (1966).

Friend and Blume (1970), in their article "Measurement of Portfolio Performance Under Uncertainty", attempted to analyse

> "the extent to which the risk-adjusted rates of return successfully abstract from risk".
>
> Friend and Blume (1970 :561)

Their results were somewhat surprising, as they found that the risk-coefficient they used appeared to be biased downwards. Friend and Blume's methodology was as follows. They derived performance and risk measures for two hundred random portfolios. Friend and Blume then regressed the three different performance measures they had highlighted with each of the two measures of portfolio risk identified. Their results were that risk-adjusted performance is dependent on risk. This relationship is inverse and highly significant. Friend and Blume concluded that the reason for their results was probably due to the restrictive assumption of an unrestricted risk-free rate of return and of borrowing. Their conclusion was that it was more desirable to use the two-stage parameters, risk and rate of return, to measure portfolio performance, than to use the risk coefficient alone.

6.3.3 The first proper test of the CAPM

Probably the first major test of the CAPM was carried out by Black, Jensen and Scholes (1972), hereafter referred to as BJS. BJS estimated the share market line (SML) by first estimating the beta-coefficients and average rates of return for each stock in their sample, using time-series regression. They termed these the first-pass regressions. BJS were reluctant to estimate the SML merely by regressing the sampled stocks' average return onto its beta, as King (1966) had shown that the residual errors from the market model are correlated amongst many stocks. This non-independence between stocks could introduce bias and inefficiencies that would confound the second-pass regressions (see below).

In order to reduce the risk of measurement errors in their work, BJS formed ten portfolios of stocks based on each individual stock's risk ranking. In order to eliminate selection bias, the stocks were grouped by their rankings based on betas calculated over a previous 5 year period. BJS then used the mechanism to measure each of the portfolio's returns in the sixth year against the betas generated in the preceding five years.

The so-called 'second-pass' regressions were now carried out. A cross-sectional estimate of the SML was obtained by regressing the portfolio return in the sixth year against the risk-coefficient of the particular portfolio, as generated in the previous regression.

BJS found that, particularly in the long term, the CAPM appeared to be a valid model, as the SML derived by their 'second-pass' regressions was an upwards-sloping line. This was thus the first evidence that the CAPM might apply in the real world.

6.3.4 More sophisticated tests emerge

In 1973, in an article entitled "A new look at the Capital Asset Pricing Model", Blume and Friend (1973) re-examined their findings noted above (Friend and Blume:1970). Their extended tests confirmed the findings by BJS that the relationship between risk and return for New York Stock Exchange assets is linear. Again, however, Blume and Friend did find abberations in their results. Their interpretation of these abberations was that the market for stocks is partially segmented from that for bonds and other assets.

In the same year, Blume and Husic (1973), again using a cross-sectional regression technique, showed that the CAPM did hold. Their findings were also interesting however, in that they suggested that for equivalent risks, the returns of stocks on the American Stock Exchange could be different to those on the NYSE, at the same point in time. They did not attempt to explain the economic rationale behind this observation.

The abovementioned studies were all, in their own way, valid tests of aspects of the CAPM, if not tests of the whole model. The methodology certainly improved with the passage of time, however, no researcher had yet come up with a methodology which tested all the assumptions of the CAPM.

Fama and MacBeth (1973), presented a technique which satisfied the above shortcomings, in that it examined each aspect of the CAPM individually.

Fama and MacBeth (1973), studied the relationship between average return and risk for NYSE common stocks, and they examined the coefficients and residuals of the cross-sectional regression techniques they used. In view of the fact that the methodology they used is, to date, the best approach to an empirical evaluation of the CAPM, and thus it is the one used in this study, a detailed analysis of their methodology will not be presented in this chapter, but will be presented in Chapter 7. Briefly, what they did was to expand on the cross-sectional technique used by BJS, incorporating into it various other factors which tested the underlying assumptions of the CAPM. This was therefore the first thorough test of the CAPM.

The results of the Fama and MacBeth study were that

> "We cannot reject the hypothesis of these models (of market equilibrium) that the pricing of common stocks reflects the attempts of risk-averse investors to hold portfolios that are "efficient" in terms of expected value and dispersion of return. Moreover, the observed "fair-game" properties of the coefficients and residuals of the risk-return regressions are consistent with an "efficient capital market" - that is, a market where prices of securities fully reflect available information".
>
> Fama and MacBeth (1973: 607)

As the research into the CAPM intensified, so the breadth of the empirical work expanded. For example, Basu (1977), examined the relationship between the performance of common stocks and their price-earnings ratios. Basu's findings appeared to reject the EMH with regard to the speed of assimilation of new information into the market. Basu postulated that the CAPM is probably based on an untrue assumption (viz. perfect markets) and may thus not be valid itself. Basu's findings seemed to indicate that the price-earnings ratios of firms possessed an information content that was not assimilated into the share price.

6.3.5 Mis-specification of the CAPM

The study by Fama and MacBeth (1973) did find that the intercept term of the cross-sectional regression equation was not equal to zero, and that low-beta securities earn more than the CAPM would predict, while for high-beta securities the opposite is often the case. Further research in the late 1970's and early 1980's highlighted this, and, other possible mis-specifications of the model.(4)

Litzenberger and Ramaswamy (1979), studying the relationship between personal taxes and dividends on capital asset prices, found

"...a strong positive relationship between before tax expected returns and dividend yields of common stocks...Evidence is also presented for a clientele effect: that is, that shareholders in higher tax brackets choose stocks with low yields, and vice versa."

Litzenberger and Ramaswamy (1979:190)

The literature in the late 1970's and the 1980's is filled with tests which are concerned mainly with testing applications of the CAPM to other areas of finance. Nevertheless, there have been some direct tests of the CAPM. Banz (1981), tested the empirical relationship between the return and the total market value of NYSE stocks. Again, a cross-sectional regression technique was adopted. His findings indicate a definite mis-specification in the CAPM on the NYSE over a forty-year period from 1936-1977. The small NYSE firms, have, on average, had significantly larger risk-adjusted returns than larger NYSE firms have had in this period. Thus the size of the firm would appear to be a factor in determining its overall return. This size-effect is generally known as 'the small-firm effect'.

Reinganum (1981b), returning to the area examined by Litzenberger and Ramaswamy (1979) examined the CAPM in terms of both earnings yields and market values. The initial results showed that depending on both the size of the firm and on the earnings yield, an abnormal return could be achieved. However, further examination showed that when the firm-size effect was controlled, leaving only the earnings yield, the abnormal returns disappeared.

The converse was not the case, thus Reinganum concluded that a mis-specification of the CAPM existed, which appeared to be due to the small-firm effect.

Since 1981, empirical studies have tended to concentrate on factors such as the small-firm effect.[5] It is however, beyond the scope of this review to analyse these findings.

Roll (1977; 1978) has suggested that it is impossible to test the CAPM directly without testing the efficiency of the market at the same time. His work, together with the mis-specifications of the CAPM noted by many researchers, has prompted the formulation of Arbitrage Pricing Theory (APT), a testable alternative to the CAPM. APT is briefly mentioned in Chapter 13.

6.3.6 Evidence on other markets

The studies above all concentrated on either or both of the NYSE or the American Stock Exchange. Although not nearly as voluminous, research has been carried out on some of the other markets. Hawawini, Michel and Viallet (1983), credit Modigliani, Pogue and Solnik (1972) with the first, limited, test on the Belgian Stock Exchange. Guy (1977) has researched the German equities market, whereas the Israeli market has been examined by Levy (1980). All of the above studies used the BJS (1972) methodology, which has since been superseded by the Fama and MacBeth (1973) techniques.

Firth (1977) has studied the relationship between risk and return on the UK unit trusts market.

Hawawini, Michel and Viallet (1983) tested the CAPM in relation to French Common stocks, using the Fama and MacBeth methodology. They did however incorporate some more variables into the cross-sectional regression technique in order to test extensions to the CAPM theory which had been published since the Fama and MacBeth study. These were Black's Zero-beta model, and Levy's Generalised CAPM. These extensions have been included in the current study, as outlined in Chapter 7.

The general consensus of the above studies was that, with some slight anomalies, there appeared to be a positive linear relationship between the risk and the expected return of different types of stocks.

To the knowledge of this researcher, no comparable study has been carried out on the JSE.

6.4 SUMMARY OF MAJOR FINDINGS

The overall conclusion that can be drawn from the brief literature review in section 6.3 is that:

- a linear relationship exists between the expected return and the risk of different securities. In the long-term, the return on the market portfolio appears to be greater than the risk-free rate of interest

- low beta securities appear to earn more than the CAPM predicted earnings, while high beta securities appear to earn less
- the dominant risk-measure is beta
- other factors explain a portion of returns not 'captured' by the beta-coefficient. Examples of these factors are the earnings-yield and the size of the firm.

Thus, generally, the purely theoretical form of the CAPM does not agree well with reality, although the CAPM appears to possess some validity.

Roll (1977;1978), has criticised tests on the CAPM on the grounds that they do not actually test what they purport to be testing. His critique will not be examined here, as it is examined fully in Chapter 12.

CHAPTER 6 - FOOTNOTES

(1) This sub-section draws heavily on the reasons hampering direct testing of the CAPM put forward by Francis and Archer (1979).

(2) Refer to Chaper 4, section 4.2.1 for a brief description of a 'fair-game'.

(3) Residual variance was the term used by Lintner (1965) to describe what is now commonly known as the unsystematic risk.

(4) Roll (1977) published a critique on the testability of the CAPM. This is mentioned at the end of the current chapter and reviewed in detail in Chapter 12.

(5) See for example: Roll (1981)
Reinganum (1982)
Basu (1983)
Fowler and Rorke (1983)
Keim (1983)
Reinganum (1983)
Schultz (1983)
and Barry and Brown (1984) amongst others.

CHAPTER 7

THE RESEARCH MODEL

7.1 INTRODUCTION

The previous chapter presented an overview of the major empirical work that has been carried out in testing the CAPM.

This chapter presents, in a general sense, the model used in this study. Testable hypotheses which will be empirically evaluated in Chapter 9, are also developed.

7.2 SOURCE OF THE MODEL

Mention was made in section 6.3.3 of the major contribution to testing the CAPM that was made by Fama and MacBeth (1973), hereafter referred to as FM, and the subsequent testing of the CAPM on French common stocks using FM's technique as carried out by Hawawini, Michel and Viallet (1983) - HMV.

This study uses the techniques presented by FM, as extended and modified by HMV.

It is worth mentioning at the outset of the study that the FM approach is designed to investigate the relationship between future returns and estimates of risk based on current information. The test is thus not a contemporaneous association test(1), but a test involving lagged relationships.

The decision to use the FM and HMV technique instead of other techniques which have been developed (for example: Miller and Scholes (1972) or Black, Jenson and Scholes (1972)), was based on an evaluation of the various methodologies employed. The FM/HMV technique as presented below represents the best method yet developed as a direct test of the CAPM, as explained in Chapter 6, section 6.3.4.

It is acknowledged that Roll's criticism (see Chapter 12) has some validity, however in view of the extent of tests carried out on the NYSE, and the absence of such tests on the JSE, the tests do have validity in a South African context.

7.3 THEORETICAL BACKGROUND

Recall from 3.4.5 that there exists, in equilibrium, a linear relationship between the expected return of any asset and the risk of that asset. The risk is measured by the ratio of the covariance between that asset's returns and those of the market portfolio to the variance of the market portfolio's returns. In this linear relationship the constant term is the return of the risk free asset R_F and the slope is the expected return on the market portfolio in excess of the risk free rate.

This can be written as follows:

$$E(\tilde{R}_j) = R_F + (E(\tilde{R}_M) - R_F) \frac{\sigma_{jM}}{\sigma_M^2} \qquad (7.1)$$

where $E(\tilde{R}_j)$ = expected return on asset j

R_F = expected risk free rate

$E(\tilde{R}_M)$ = expected return on the market

σ_{jM} = covariance between returns of asset j and the market

σ_M^2 = variance of the market portfolio returns

$\sim$ = a random variable.

The ratio $\frac{\sigma_{jM}}{\sigma_M^2}$ is often referred to as the systematic risk of asset j, or its beta coefficient. This is, as mentioned earlier, the innovation of the CAPM, in that risk is expressed in terms of the covariance of an asset's returns with those of the market rather than by measuring the total variability of these returns.

The effect of equation (7.1) is that it splits the return on any asset into two components: the return of an asset that is riskless in relation to the market (R_F) and a risk premium, that is β_j times the difference between $E(R_M)$ and R_F.

In order to develop testable hypotheses regarding equation (7.1), it is necessary to review the underlying assumptions of the CAPM. These were discussed fully in 3.4.2 thus they are only briefly mentioned here.

Assumptions underlying the CAPM

1. Capital markets are perfect: investors are price-takers (i.e. they cannot set the price themselves), and there are neither transaction costs nor information costs. In addition, assets are infinitely divisible.

2. Distributions of the one-period percentage returns of all assets and portfolios are assumed normally distributed, or to conform to some other two-parameter symmetric-stable distribution class.

3. Investors are assumed to be risk averse and to behave as if their choice amongst portfolios is based on maximum expected utility.

Equation (7.1) is in a sense a 'snap-shot' view of an investor's beliefs, as it reveals nothing about the behaviour of returns through time. This behaviour is implied by the assumptions of the two parameter model that the capital market is perfect with regard to the cost of transactions. In order to test the CAPM it is necessary to choose a model that gives period-by-period returns which enables the researcher to test the implications(2) of the one-period expectations model.

Such a stochastic[3] generalisation of (7.1) was suggested by FM[4] as follows:

$$\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{1t}\beta_j + \tilde{\gamma}_{2t}\beta_j^2 + \tilde{\gamma}_{3t}s_j + \tilde{\mu}_{jt} \qquad (7.2)$$

where subscript t refers to time period t and $\tilde{\gamma}_i$ are stochastic coefficients varying from period to period.

HMV extended the suggested process noted in (7.2) above by introducing two more variables into the equation. These were introduced as a result of articles published by Black (1972) and Levy (1978) respectively. Black's article showed that if the risk free asset specified in (7.1) does not exist then it can be replaced by an asset known as a zero-beta asset, i.e. one whose returns are uncorrelated with the market portfolio's returns. Levy's contribution to the literature was an examination of the extreme diversification assumption. He assumed that any investor holds only a few securities, instead of investing in each and every security. If this is the case, Levy shows that the dominant measure of risk is no-longer β_j, the systematic risk, but the variance, which is the total risk.

If these factors are added into the stochastic process suggested by FM and shown by equation (7.2) the process becomes:

$$\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{1t}\beta_j + \tilde{\gamma}_{2t}\beta_j^2 + \tilde{\gamma}_{3t} s_j + \tilde{\gamma}_{4t}\sigma_j^2 + \tilde{\gamma}_{5t} S_j + \tilde{\mu}_{jt} \qquad (7.3)$$

where: β_j = systematic risk

s_j = unsystematic risk

σ_j^2 = total risk

S_j = relative-skewness of the return-distribution

where all of the above relate to asset j.

and $\tilde{\gamma}_i$ = stochastic variables $i = 0...5$.

This equation will be referred to later, in section 7.5.

7.4 IMPLICATIONS OF THE STOCHASTIC PROCESS AND THE EXPECTATIONS MODEL

The expectations model specified in (7.1) has certain testable implications. These are as follows:(5)

1. The relationship between the expected return on a security and its risk in any efficient portfolio is linear. This can be rephrased by saying that the relationship between an asset's expected return and its systematic risk is linear. This is exactly what equation (7.1) posits.

2. β_j is a complete measure of the risk of security j within an efficient portfolio, as no other measure of risk appears in equation (7.1). In other words,

> "investors are compensated only for the systematic portion of the risk of an asset since the unsystematic portion can be costlessly diversified away."(6)

3. Higher risk should be associated with higher return in a market of risk-averse investors. This means that in the expected return - systematic risk relationship posited by (7.1), the slope is positive.

4. Investors in the market invest and make decisions based on the assumption that the distribution of asset returns is symmetrical. This implication is a direct result of the second assumption underlying the CAPM, as mentioned in 7.3 above.

5. Unrestricted riskless lending and borrowing exists at a unique risk free rate. This is a specific characteristic of the Sharpe-Lintner version of the CAPM.(7)

6. If investors are not extreme diversifiers, but rather invest only in small portfolios containing a few securities, then total-risk is a better measure of their risk-exposure than the beta. This was Levy's generalised CAPM, referred to in 7.3 above.

7.5 TESTING THE STOCHASTIC MODEL FOR RETURNS

Testing the two parameter model presents a problem, which is unavoidable, of 'errors-in-the-variables' existing.(8) This is because the expected return - risk equation (7.1) is expressed in terms of the true (investors') values of the β_j characteristic, whereas any empirical test can only be carried out using the estimates ($\hat{\beta}_j$). The problem centres on the fact that if a proxy explanatory variable is used in a least-squares regression, the computed coefficients do not have the same properties as if the true explanatory variable were used. It follows that, to the extent that the estimates differ from the true values of the explanatory variables, errors in interpretation of results may occur. This is however not considered to limit the validity of the tests, as the problem is avoided, or at least minimised, by the technique noted below.

Blume (1970) has shown that the $\hat{\beta}$'s of portfolios are far more precise estimates of true β_j than the $\hat{\beta}$'s for individual shares. (Note that a ' ^ ' denotes an estimated variable). This is the solution to the 'errors-in-the-variables' problem noted above. The statistical sampling variance of $\hat{\beta}_j$ as an estimate of β_j is

$$\sigma^2(\hat{\beta}_j) = \frac{\sigma^2(\hat{\varepsilon}_j)}{\sum_{t=1}^{T} (\tilde{R}_j - \bar{R}_j)^2}.$$

The value of $\sigma^2(\hat{\beta}_j)$ can be reduced either by making the denominator large, by estimating $\hat{\beta}_j$ over a long time-period, or by reducing the numerator. The first solution depends too heavily on the assumption that β_j is stationary over time, thus the second solution is favoured. The $\sigma^2(\hat{\beta}_j)$ of portfolios are significantly less than those for individual securities, due to the fact that the value of $\sigma^2(\hat{\varepsilon}_j)$ for a portfolio is merely the sum of the values relating to the underlying securities. These securities will not be perfectly correlated, thus by forming portfolios some diversification is introduced, which lowers the $\sigma^2(\hat{\varepsilon}_j)$ term. It is thus preferable to use portfolio returns. This limits the bias and inconsistency caused by the 'errors-in-the-variables' problem.

In order to limit the loss of information in the risk-return tests caused by using portfolios instead of individual securities a larger range of values of portfolios $\hat{\beta}_p$'s is obtained by ranking the $\hat{\beta}_i$ of the individual securities and then forming the portfolio based on these rankings. The loss of information referred to above is caused by the fact that a random allocation of securities to portfolios is likely to leave each portfolio with a risk-measure fairly close to one (i.e. the market return). By forming high-beta portfolios and low-beta portfolios, the beta characteristics of individual securities are maintained. This in turn leads to another problem, as such a procedure can seriously damage the regression results. As Fama and MacBeth put it:

"But such a procedure, naively executed could result in a serious regression phenomenon. In a cross-section of $\hat{\beta}_i$, high observed $\hat{\beta}_i$ tend to be above the corresponding true β_i and low observed $\hat{\beta}_i$ tend to be below the true β_i. Forming portfolios on the basis of ranked $\hat{\beta}_i$ thus causes bunching of positive and negative sampling errors within portfolios. The result is that a large $\hat{\beta}_p$ would tend to overstate the true β_p, while a low $\hat{\beta}_p$ would tend to be an underestimate".

Fama and MacBeth (1973: 615)

This problem is overcome by using data in one period to estimate the $\hat{\beta}_i$ from which to form the portfolios, and then using data from a different period to obtain the $\hat{\beta}_p$ for these portfolios used to test the two parameters model. By recalculating the betas of the portfolio in the next period, the over- and under-estimations of the individual securities within the portfolios become random instead of pre-determined.

Recall the stochastic model developed in section 7.3.

$$\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{1t}\,\hat{\beta}_j + \tilde{\gamma}_{2t}\,\hat{\beta}_j^2 + \tilde{\gamma}_{3t}\,\hat{s}_j + \tilde{\gamma}_{4t}\,\hat{\sigma}^2_j + \tilde{\gamma}_{5t}\,\hat{S}_j + \tilde{\mu}_{jt} \qquad (7.4)$$

where $\hat{\beta}_j$ = systematic risk

$\hat{s}_j$ = unsystematic risk

$\hat{\sigma}_j^2$ = total risk

$\hat{S}_j$ = relative skewness of the return distribution, all of asset j

$\sim$ = a random variable

$\hat{}$ = an estimated variable.

This model is applied in testing the CAPM.

The methodology involved is rather complex to explain. It was thus decided that it would be best described by detailing a general procedure in this chapter, followed by a detailed review of the actual procedures carried out, which is given in the following chapter. In order to simplify the description given here, no rationale is given for the various problems involved, and the methodology used to overcome them. This explanation is given in Chapter 8.

The general procedure is as follows:

Preliminary

The total time for which security returns are available, and over which the analysis is to be carried out, is divided into various periods, usually of a minimum of one year in duration. These periods are then used in the various stages of the test, as described below.

Step One: Initial Beta Calculation

In the first period of the security returns being examined, calculate the $\hat{\beta}_j$ of the individual securities using the standard market model:(9)

$$\tilde{R}_j = \alpha + \beta_j \tilde{R}_M + \tilde{\varepsilon}_j$$

Step Two: Rank the Shares by Beta and Allocate into Portfolios

Rank the individual $\hat{\beta}_j$ estimates thus obtained, from highest to lowest, and allocate each underlying security to a portfolio, based on the rankings.

The procedure to achieve this is as detailed below:

a) Let N = total number of securities to be allocated

Let X = desired number of portfolios

Let X' = the largest integer equal to or less than $N/_X$.

b) Allocate the individual securities to portfolios based on their rankings:

thus Portfolio 1 : the first $X' + 1/_2$ (N-(X.X'))securities

Portfolio 2 : the next X' securities

Portfolio X-1: the following X' securities

Portfolio X : has $X' + 1/_2$ ((N-(X.X')) securities if N is even, or $X' + 1/_2$ (N-(X.X'))+1 if N is odd.

An example will explain this procedure more clearly:

Assume there are 15 securities, and 4 portfolios desired. Assume further that security 1 has highest $\hat{\beta}$, and security 15 the lowest $\hat{\beta}$.

The allocation is as follows:

N = 15
X = 4
$N/_X$ = 3.75
X' = 3

thus Portfolio 1 has: 3 + 1/2 (15-3,75.3)
= 4 securities

Portfolios 2 and 3 have X' or 3 securities each

and Portfolio 4 has: 3 + 1/2 (15 - 3,75.3) + 1
= 5 securities

Total allocated = 4 + 3 + 3 + 5 = 15 securities.

At the completion of this stage there are thus X portfolios, each portfolio containing shares having similar risk rankings.

Step Three: Portfolio Parameters Obtained

Obtain portfolio parameters for the period immediately following that used to form the securities into portfolios (viz. the second year of returns). The parameters to be obtained are those as detailed in (7.4) above. However, they are obtained for portfolios by obtaining the individual securities' characteristics and then averaging them across the portfolio. The rationale for this is given in section 8.4.

Equation (7.4) can thus be restated as:

$$\tilde{R}_{pt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{1t}\hat{\beta}_p + \tilde{\gamma}_{2t}\hat{\beta}_p^2 + \tilde{\gamma}_{3t}\hat{s}_p + \tilde{\gamma}_{4t}\hat{\sigma}_p^2 + \tilde{\gamma}_{5t}\hat{S}_p + \tilde{\mu}_{pt} \qquad (7.5)$$

$$p = 1 \ldots X$$

$$t = 1 \ldots T$$

where:

$\hat{\beta}_p$ = systematic risk

$\hat{s}_p$ = unsystematic risk

$\hat{\sigma}_p^2$ = total risk

and $\hat{S}_p$ = relative skewness of the return distribution, all for portfolio p.

The five parameters in (7.5) are obtained as follows:

1. <u>Systematic risk</u> $(\hat{\beta}_j)$

This is again estimated using the market model,

$$\tilde{R}_{jt} = \alpha + \beta_j \tilde{R}_{mt} + \tilde{\varepsilon}_{jt} \qquad (7.6)$$

from which: $$\hat{\beta}_j = \frac{\sum_{t=1}^{T} (R_{jt} - \bar{R}_j)(R_{mt} - \bar{R}_m)}{\sum_{t=1}^{T} (R_{mt} - \bar{R}_m)^2}$$

where a bar indicates an arithmetic mean, a hat signifies an estimated variable and T equals the number of weekly observations.

This is in fact the estimated slope coefficient of the regression equation specified by (7.5).

2. <u>Alternative risk estimate</u> $(\hat{\beta}^2_j)$

This parameter is defined as the square of the first parameter obtained.

3. <u>Unsystematic risk</u> $(\hat{s}_j)$

This follows directly from (7.5), and is expressed by

$$\hat{s}_j = \left[\frac{\sum_{t=1}^{T} (R_{jt} - \hat{\alpha}_j - \hat{\beta}_j \tilde{R}_{mt})^2}{T - 1} \right]^{\frac{1}{2}}$$

where $$\hat{\alpha}_j = \bar{R}_j - \hat{\beta}_j \bar{R}_m$$

Unsystematic risk is thus the standard error of the regression equation.

4. <u>Total risk</u> ($\hat{\sigma}_j^2$)

Total risk is measured by the variance (σ_j^2) of the return distribution of asset j, and is calculated without reference to the market model. It is not considered necessary to specify the equation for the variance here as this is a standard statistical term.

5. <u>Relative skewness</u> ($\hat{S}_j$)

This parameter is measured by the ratio of the third moment around the mean of asset j's return distribution to that distribution's standard deviation cubed.

The above factors are obtained for each security and then the simple average across securities within the portfolio is calculated to obtain the portfolio parameter.

<u>Step Four: Portfolio Parameters Regressed against Portfolio Returns</u>

Obtain the portfolio return (R_p) in the period following that used in step 3 above, and, using the parameters obtained in step 3, regress the R_p for each week against the parameters, using various different combinations of equation (7.5).[10]

<u>Step 5: Assess the Statistical Significance of the Results</u>

Assess the statistical significance of the regression results obtained by using the student t-test to evaluate the various hypotheses.

Step 6: Roll-forward the Process

Finally, steps 3 to 5 above are repeated, by 'rolling forward' the entire process one year at a time.

7.6 SCHEMATIC MODEL OF THE METHODOLOGY

The following simple flowchart (Figure 7.1) details the research methodology described in 7.5. Note that the numbers on the right-hand side of the diagram correspond to the steps described in 7.5 above.

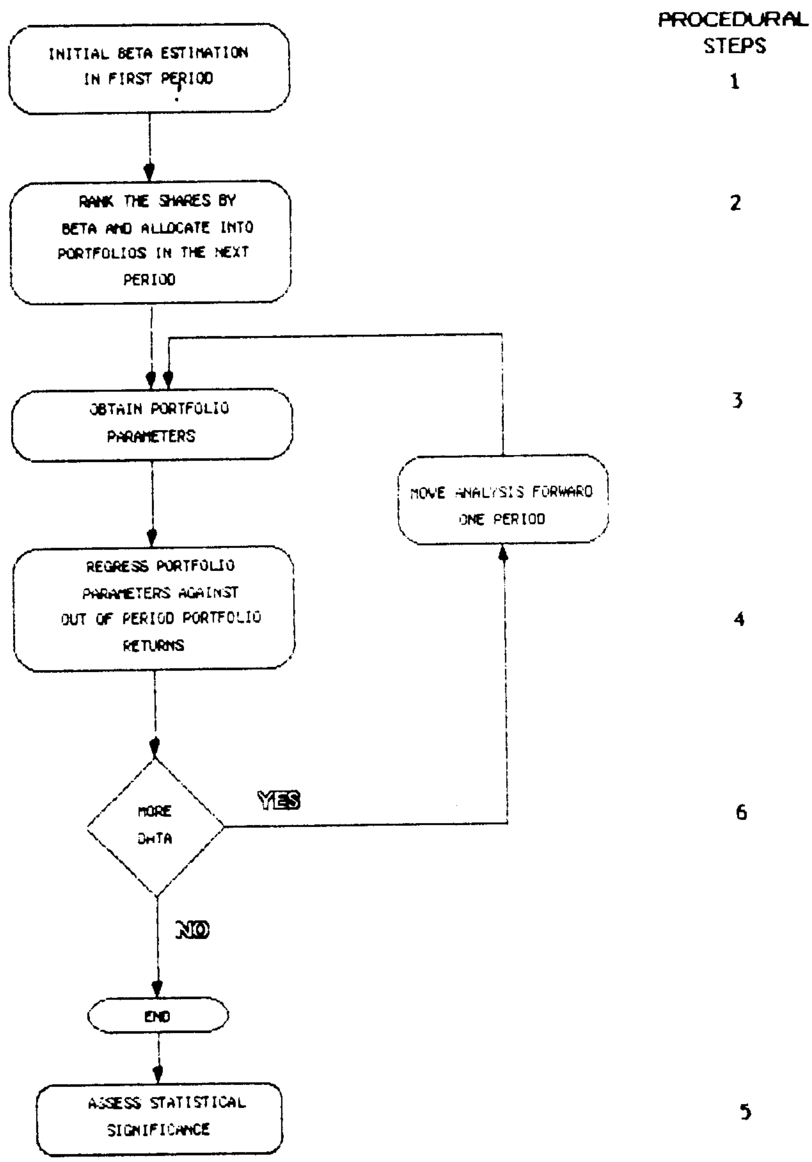


Figure 7.1: Flowchart of the research model

The model can also be presented in a time-frame as shown in Figure 7.2 below. Again, the circled numbers correspond to the steps described in section 7.5 above.

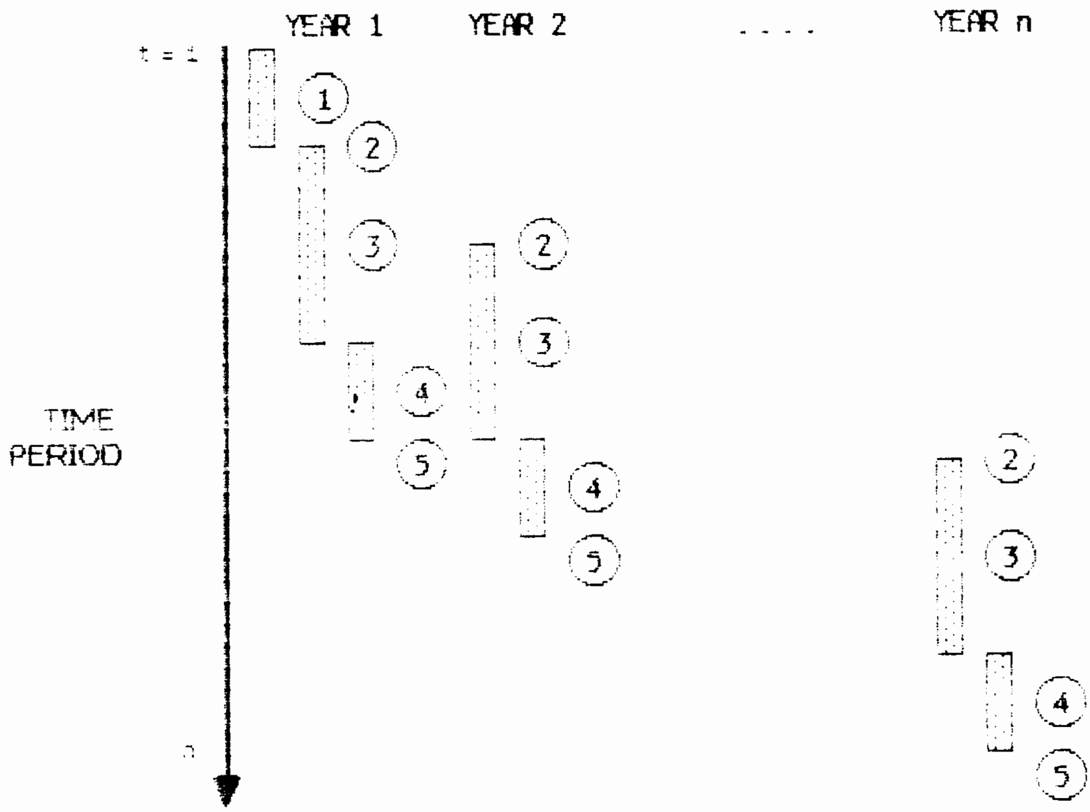


Figure 7.2: A time-frame explanation of the research model

7.7 HYPOTHESIS SETTING

The stochastic process developed and expressed in 7.3 (see also 7.5) was employed by HMV to formulate a set of six testable hypotheses, labelled A through F, which are detailed below. These hypotheses directly test the underlying assumptions of the CAPM, as well as testing findings in other studies of the CAPM.

Hypothesis A:

Null Hypothesis (H_oA):

The relationship between a security's expected return and its systematic risk is linear.

To test this hypothesis, it is necessary to examine the random coefficients of the following two equations.[11]

$$\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{1t}\hat{\beta}_j + \tilde{\gamma}_{2t}\hat{\beta}_j^2 + \tilde{\mu}_{jt}$$

$$\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{2t}\hat{\beta}_j^2 + \tilde{\mu}_{jt}$$

The two competing hypotheses to be tested are:

H_oA : $E(\tilde{\gamma}_{1t}) \neq 0$
and $E(\tilde{\gamma}_{2t}) = 0$

versus

H_aA : $E(\tilde{\gamma}_{1t}) = 0$
and $E(\tilde{\gamma}_{2t}) \neq 0$

The null hypothesis is that the expected value of $\tilde{\gamma}_{1t}$ is not equal to zero, whereas $\tilde{\gamma}_{2t}$ is equal to zero. The alternative hypothesis is H_aA, that the expected value of $\tilde{\gamma}_{1t}$ equals zero and that of $\tilde{\gamma}_{2t}$ is not equal to zero.

If $E(\tilde{\gamma}_{1t}) \neq 0$ and $E(\tilde{\gamma}_{2t}) = 0$, then H_oA cannot be rejected.

Hypothesis B:

Null Hypothesis (H_oB):

Investors should bear only systematic risk, as this is all they are compensated for.

The two equations to be used are:

$$\tilde{R}_{jt} = \tilde{\Upsilon}_{ot} + \tilde{\Upsilon}_{1t} \hat{\beta}_j + \tilde{\Upsilon}_{3t} \hat{s}_j + \tilde{\mu}_{jt}$$

and $\tilde{R}_{jt} = \tilde{\Upsilon}_{ot} + \tilde{\Upsilon}_{3t} \hat{s}_j + \tilde{\mu}_{jt}$

where $H_oB : E(\tilde{\Upsilon}_{3t}) = 0$

versus

$H_aB : E(\tilde{\Upsilon}_{3t}) \neq 0$

Hypothesis B will be rejected if $E(\tilde{\Upsilon}_{3t}) \neq 0$.

Hypothesis C:

Null Hypothesis (H_oC):

Increasing risk brings increasing returns. In other words, the trade-off between risk and return is positive.

In this case only one equation is necessary:

$$\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{1t} \hat{\beta}_j + \tilde{\mu}_{jt}$$

and H_oC : $E(\tilde{\gamma}_{1t}) > 0$

versus

H_aC : $E(\tilde{\gamma}_{2t}) < 0.$

Thus hypothesis C will be rejected if $E(\tilde{\gamma}_{2t}) < 0.$

Hypothesis D:

Null Hypothesis (H_oD):

The perceived return-distribution of securities (by investors) is symmetrical; thus the expected value of $\tilde{\gamma}_{5t}$ in the next two equations must be zero.

$$\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{1t} \hat{\beta}_j + \tilde{\gamma}_{5t} \hat{S}_j + \tilde{\mu}_{jt}$$
$$\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{5t} \hat{S}_j + \tilde{\mu}_{jt}$$

and H_oD : $E(\tilde{\gamma}_{5t}) = 0$

versus

H_aD : $E(\tilde{\gamma}_{5t}) \neq 0$

Hypothesis D will be rejected if $E(\tilde{\gamma}_{5t}) \neq 0.$

Hypothesis E:

Null Hypothesis (H_oE):

Levy's generalised CAPM model is not valid.

This hypothesis is in direct response to Levy's findings (Levy: 1978) that the variance of a security's return emerges as a dominant risk measure, as opposed to the beta-coefficient (systematic risk), when investors do not hold all available assets.

The two equations to be used are:

$$\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{1t} \hat{\beta}_j + \tilde{\gamma}_{4t} \hat{\sigma}^2_j + \tilde{\mu}_{jt}$$
$$\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{4t} \hat{\sigma}^2_j + \tilde{\mu}_{jt}$$

and H_oE : $E(\tilde{\gamma}_{4t}) > 0$

versus

H_aE : $E(\tilde{\gamma}_{4t}) < 0$

If $E(\tilde{\gamma}_{4t}) < 0$, hypothesis E can be rejected.

The final hypothesis is as follows:

Hypothesis F:

Null Hypothesis (H_oF):

Unrestricted riskless borrowing and lending at a unique risk-free rate R_F exists.

Here, use: $\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{1t} \hat{\beta}_j + \tilde{\mu}_{jt}$

H_oF : $E(\tilde{\gamma}_{ot}) = R_F$

versus

H_aF : $E(\tilde{\gamma}_{ot}) \neq R_F$

This hypothesis is a test of the Sharpe-Lintner form of the CAPM. As noted in Chapter 6, certain empirical findings have tended to support Black's zero-beta version of the CAPM and to reject the Sharpe-Linter form.(12)

Numerous problems exist in testing the 'pure' Sharpe-Lintner form of the CAPM, due to the problem of isolating and quantifying the risk-free rate. Hawawini, Michel and Viallet (1983) used as a proxy the weekly average of the day-to-day lending rate given by the major French banks. This, it is respectfully submitted, is not a true risk-free rate, as it is in essense a commercial lending rate. A truer risk-free rate would be the interest-rate on Government bonds. However, the problems that arise with this measure are that there is in fact a term structure to interest rates; in addition, the risk free rate is not really the nominal rate of the coupon, it is the rate at which that particular coupon changes hands in the period under review. In other words, the risk free rate should really be the 'effective' rate, and not the nominal rate. In view of these problems, and because the distinction between the Black model and the Sharpe-Lintner model is not large, this hypothesis was not tested in the current study. No loss of validity of the study arises as a result of this.

The hypotheses are summarised in Table 7.1. In each case the null hypothesis is followed by the alternative hypothesis.

Table 7.1: Testable Hypotheses of the CAPM

A	H_oA:	The relationship between a security's expected return and its systematic risk is linear
	H_aA:	The relationship is not linear
B	H_oB:	Investors bear only systematic risk
	H_aB:	Investors bear more risk than only systematic risk
C	H_oC:	The risk-return trade-off is positive
	H_aC:	The risk-return trade-off is negative
D	H_oD:	Investors perceive securities' return distributions as symmetrical
	H_aD:	The perceived return distribution is asymmetrical
E	H_oE:	Levy's Generalised CAPM is not valid
	H_aE:	Levy's Generalised CAPM is valid

7.8 SUMMARY

In this chapter the model used to test the CAPM has been developed. Following on from this, the broad basis used to test the model has been discussed. Finally, five testable hypotheses have been drawn up and expanded upon.

The next chapter will explain the actual research methodology used in this study.

CHAPTER 7 - FOOTNOTES

(1) Contemporaneous association tests have been used extensively to test the efficiency of markets, and also to test the degree of association between different factors e.g. accounting methods and risk of stocks. In this regard see Fama, Fisher, Jensen and Roll (1969), who were the first to develop the method. A literature review of tests of this nature will be found in Fama (1970) and Gonedes and Dopuch (1974) amongst others.

(2) These implications are discussed later, in section 7.4.

(3) A stochastic process is defined as "a process consisting of a number of steps having a random variable, the successive values of which are not independent".

(4) Refer Fama and MacBeth (1973), page 611.

(5) Implications 1 to 3 and 5 were shown by FM, whereas 4 and 6 were, to the knowledge of this researcher, developed by HMV.

(6) Hawawini, Michel and Viallet (1983), page 335.

(7) Refer Chapter 3.

(8) This problem was highlighted first by Blume (1970).

(9) See Chapter 3, section 3.3.2 for a description of the market model.

(10) These combinations of parameters will be explained later when the hypotheses are set.

(11) The equations listed in the hypotheses are all subsets of the stochastic process referred to in 7.5.

(12) Black's zero-beta model is described in section 3.5 and represented graphically in section 12.2.

CHAPTER 8

RESEARCH METHODOLOGY

8.1 PRELIMINARY WORK

The Johannesburg Stock Exchange (JSE) can be split into two main sectors: the Mining sector and the Industrial sector. In view of the close relationship between the prices of mining shares and the world market price of the particular commodity being mined, this study has concentrated only on the Industrial sector of the market.[1]

Weekly share price data for the eight year period 2 February 1973 to 14 November 1980 was available on the University of Cape Town Univac 1106 Computer. This data consisted of prices for 107 individual securities, and for the Rand Daily Mail 100 Index (RDM-index). See Appendix A for a complete list of the companies used in this study. The share price data was however, stored on four different data files.

The return on a share is defined as the change in the price from one period to the next, plus the dividend in that period, all divided by the previous period price. It was thus necessary to obtain the dividend relating to each security, ascertain the last day to register (LDR) for that dividend, and add this amount to the appropriate price in the week in which the LDR fell. This would thus yield the correct return when the return was calculated.

The information on dividends was manually extracted from another data file kept on the Univac, chronologically summarised and then entered into the original price data file by means of a Fortran program written especially for this purpose. The data on the files was manipulated, yielding weekly log-returns. This file will henceforth be known as the 'returns file'.(2)

The choice of the time period over which to calculate the returns is a difficult one, as there is a trade-off involved between the amount of the data to be included in the study and the accuracy of the same data. The use of weekly returns can lead to biased estimates of beta, particularly for firms whose shares are thinly traded.

This has been pointed out by Dimson (1979) as a problem with weekly returns. In the South African context, Carter (1983) has said that monthly returns should be used if possible, however if the number of return periods drops to less than fifteen, then weekly returns should be used (Carter (1983): 8.7).

The use of monthly returns would have led to only 101 estimates in total, whereas weekly returns left 404 returns for investigation in the study. At this initial phase of the study it was felt that the presence of thinly-traded shares, otherwise known as the intervalling effect, was less likely to be a problem than the potential error in estimates if monthly returns were used. It was thus decided to use weekly returns in the study.

The decision to use log returns instead of simple returns was made because of the additive nature of log returns. Consider the following example: The price of a security in week 1 = 100, in week 2 = 200, and in week 3 = 100 again. The simple return is calculated as follows:

$$\text{Week 2} \quad R_2 = \frac{200 - 100}{100} = 1,0$$

$$\text{Week 3} \quad R_3 = \frac{100 - 200}{200} = 0,5$$

Overall, the return is clearly zero, yet adding the above returns yields a net return of + 0,5. Using log returns solves the problem:

$$\text{Week 2} \quad R_2 = \log_e P_2/P_1 = 0,69$$

$$\text{Week 3} \quad R_3 = \log_e P_3/P_2 = -0,69$$

and, overall return = 0,69 - 0,69 = 0, which is the correct answer.

Carter (1983) has shown that the choice of log return or simple return does not have a significant impact on the results of the regression carried out when using the market model, however he does advocate its use wherever possible. For this reason, the log return method was adopted in the current study. Fama (1965) has shown that this method is in essence equivalent to a continuously compounded rate of return.

At the end of the preliminary phase of the work there was thus one file covering 404 weeks from 9 February 1973 to 14 November 1980.(3) This file contained log returns on a weekly basis for 107 industrial-sector shares, together with weekly log returns for the RDM-index. All the shares were quoted for the full period of the study, and share splits have all been taken into account in determining the return.

The next stages of the study follow the procedures described in 7.5.

8.2 STEP ONE: INITIAL BETA CALCULATION

8.2.1 Methodology

The first 100 weeks of the period were used to generate the beta-coefficient for each share, using the standard market model described in chapter 3, section 3.2.2.

Recall:

$$R_{it} = \alpha_i + \beta_i R_{RDM.t} + \varepsilon_t$$

Where R_{it} = return on i'th security in time t

$i = 1 \ldots 107$

$R_{RDM.t}$ = return on RDM-index in time t

$t = 1 \ldots 100$

α_i = y-intercept of the regression equation

β_i = risk coefficient

= slope of the regression equation.

The result of the above was 107 beta-coefficients, one for each security.

8.2.2 Defence of the methodology used

The use of the market model for generating beta-coefficients is a generally accepted method. There are however a number of problems underlying it. These will be addressed now.

1. General comments on the use of the Market Model (MM)

King (1966) has noted that the use of the MM does explain a large extent of the variance of a securities returns, thus it would appear to be a valid model. The linearity assumption of the model has been shown to be adequately satisfied by Fama, Fisher, Jensen and Roll (1969), who used it to test market efficiency and stock splits. The fact that beta-coefficients do provide a good estimate of the inherent risk of a security was shown by Beaver, Kettler and Scholes (1970). These results, although not carried out on the JSE, will be accepted as being equally valid in the South African context, as there has, to date, been no evidence to disprove these.

2. <u>The use of Ordinary Least Squares Regression</u>

It is generally acknowledged that the use of an ordinary least squares regression technique (OLS) requires the satisfaction of four assumptions about the error term in the MM. It is also however, the most extensively used method for calculating the beta coefficient. The assumptions regarding the error terms are:

- the mean of the residuals is zero: this is achieved by construction, as the OLS technique 'forces' this result.

- the covariance between the error terms relating to any security, from one period to another, is zero: this zero-autocorrelation assumption holds if the Random Walk Model is a valid assumption (refer section 4.2.1). This has been shown to be valid on the JSE by Affleck-Graves and Money (1975), thus no further work is considered necessary here. Fama <u>et al</u> (1969) showed it to be true for the NYSE also.

- homoscedasticity of the error term: This refers to the fact that the variance of the error term should be constant over time, and that the value of the error term should be independent of the return on the market. Carter (1983) has referred to Affleck-Graves (1977) as finding that some 30% of the industrial securities on the JSE exhibited significant heteroscedasticity. Dimson (1979) has said that this could be due to thin-trading of the shares of certain firms. Affleck-Graves (1977) does however say that the problem is not significant if the 'fit' between the security return and the market return is not good. This study has thus assumed that the homoscedasticity assumption is not a problem, even if it is not met.

- the final assumption is the normality in the distribution of the error terms: This has been shown not to be the case on the JSE, as Affleck-Graves (1974) has found the distribution to be a member of the stable paretian family of distributions. Carter (1983) has found that this does not affect the results of the OLS and is thus not considered a problem in the current study.

The above discussion has shown that the procedure adopted, namely the use of OLS and the market model, is a valid exercise. Step two of the analysis can now be performed.

8.3 STEP TWO: RANK THE SHARES BY BETA AND ALLOCATE INTO PORTFOLIOS

8.3.1 Methodology

The 107 securities were ranked from highest to the lowest beta coefficient. The shares were then allocated into portfolios as follows (Note β_1 = highest beta value; β_{107} = lowest beta value).

$\beta_1 \ldots\ldots \beta_{13}$ = Portfolio 1 : 13 shares

$\beta_{14} \ldots\ldots \beta_{23}$ = Portfolio 2 : 10 shares

etcetera

$\beta_{84} \ldots\ldots \beta_{93}$ = Portfolio 9 : 10 shares

$\beta_{94} \ldots\ldots \beta_{105}$ = Portfolio 10 : 14 shares

107 shares

Ten portfolios were chosen in order to give enough inputs for the cross-sectional regression described in step four below.

Note that the procedure as described in 7.5 is used here to calculate the number of securities desired per portfolio. Using the same symbols as in section 7.5.

$N = 107$

$X = 10$

$X' = 10$

Thus, Portfolio 1 = $X' + 1/2\ (N - (X\ X'))$

$= 10 + 1/2\ (107 - 10.10)$

$= 10 + 3$

= 13 securities

Portfolio 2 ...9 = 10 securities

Portfolio 10 = 14 securities

This stage of the analysis was carried out using the BMDP statistical packages available on the Univac.

Full details of the share names, β -rankings and allocations to portfolios are given in Appendix B.

8.3.2 Reasons for the ranking of securities into portfolios

Reference was made in section 8.1 to the intervalling effect. Blume (1970) has shown that this problem can be reduced by using portfolios instead of individual securities. This procedure was explained in section 7.5. By forming portfolios arbitrarily, the beta measure of the portfolios will tend towards one. This might lead to a loss of information. The solution is thus to ensure the spread of betas between portfolios is large by ranking the shares and forming portfolios having a large spread of beta values. These are then more consistent with the beta characteristics of individual securities.

8.4 STEP THREE: PORTFOLIO PARAMETERS OBTAINED

The portfolio parameters are obtained as follows:

For each share, the 'returns file' is used to generate the five parameters noted in section 7.5 This was done for 104 weeks at a time. The five parameters could not all be obtained from one statistical package, thus two packages were used.(4)

In this way the five parameters viz. beta, beta-squared, standard error, variance and relative skewness for each share were obtained. The shares were then grouped into the portfolios and the simple average of the individual security parameters was calculated in order to obtain the portfolio parameter.(5)

Initially it was felt that the use of the portfolio characteristics would yield the same results as if the average of the characteristics of the individual shares within the portfolios was used. However, a detailed analysis of the mathematical formulae involved in calculating these parameters showed that the values would be different depending on how they were calculated. The approach chosen is considered to be the correct one, and to yield the most accurate results.

This procedure was repeated four times, each time moving the analysis forward by one year (52 weeks).

Four different portfolio parameter sets were thus obtained as detailed in Appendix C. The parameters, and the weeks used to derive them are noted below:

Table 8.1: Parameter sets used in the Analysis

Parameter set	Obtained on returns in periods
One	101 - 204
Two	153 - 256
Three	205 - 308
Four	257 - 360

8.5 STEP FOUR: PORTFOLIO PARAMETERS REGRESSED AGAINST PORTFOLIO RETURNS

The fourth stage of the technique involves calculating the portfolio return, and then regressing this return against the various parameters (see step 3 above) as obtained from the two year period immediately prior to the one containing the portfolio returns to be used in the regression.

The 'returns file' was manipulated using a specially written Fortran program to calculate portfolio returns. The program calculated the overall portfolio return in each week for each of the ten portfolios, by aggregating the returns for all the shares in any one portfolio.(6)

In this way the 'returns file', containing 108 variables in each week was converted to a 'portfolio returns' file containing 11 variables in each week.(7)

For 52 weeks, the ten portfolio parameter sets were cross-sectionally regressed against the ten portfolio returns, using the regression equations as developed in 6.6, in order to test the various hypotheses.

In the next 52 week period the second parameter set was used in the regressions, and so on.

Table 8.1 can now be expanded as follows:

Table 8.2: Full parameter-regression period used in the analysis.

Parameter set	Obtained on returns in periods	Regressed on out of period returns	No of periods	Year
One	101 - 204	205 - 256	52	1977
Two	153 - 256	257 - 308	52	1978
Three	205 - 308	309 - 360	52	1979
Four	257 - 360	361 - 400	40	1980

Note that the years as labelled in the table above refer to the out of period returns, and run from mid-January in one year to mid-January in the next year, except for 1980, which runs from mid-January to mid-November, a forty week period only. Although the classification by year alone is not strictly speaking correct, the period does cover the greatest part of the calendar year, and thus for the sake of brevity the periods are referred to as 1977, 1978 and so forth.

Step four of the technique and step six, the roll-forward, were thus completed, and step five, the assessing of the statistical significance of the results was performed as detailed below.

8.6 STEP FIVE: ASSESSING THE STATISTICAL SIGNIFICANCE

Up until this stage the majority of the computations had been carried out on the mainframe computer. In order to give greater flexibility in manipulating the data, the regression results from step four were entered onto a specially created Lotus 123 'speadsheet' on an IBM mini-computer.

The variances for each year's cross-sectional regression were calculated, and, using the hypotheses developed in section 7.7 above, t-Test statistics were calculated and analysed for significance. In addition to analysing results by year, the results were analysed in two two-year periods, and for the four year period overall.[8]

Formally, the t-test is described:

$$t(\tilde{\gamma}_i) = \frac{\bar{\gamma}_i}{\sigma(\gamma_i)/\sqrt{n}}$$

where $\bar{\gamma}_i = \frac{1}{n} \Sigma \gamma_i$

$\tilde{\gamma}_i$ = regression coefficients generated using the cross-sectional regression equation presented in section 7.3 $\quad i = 1...5$

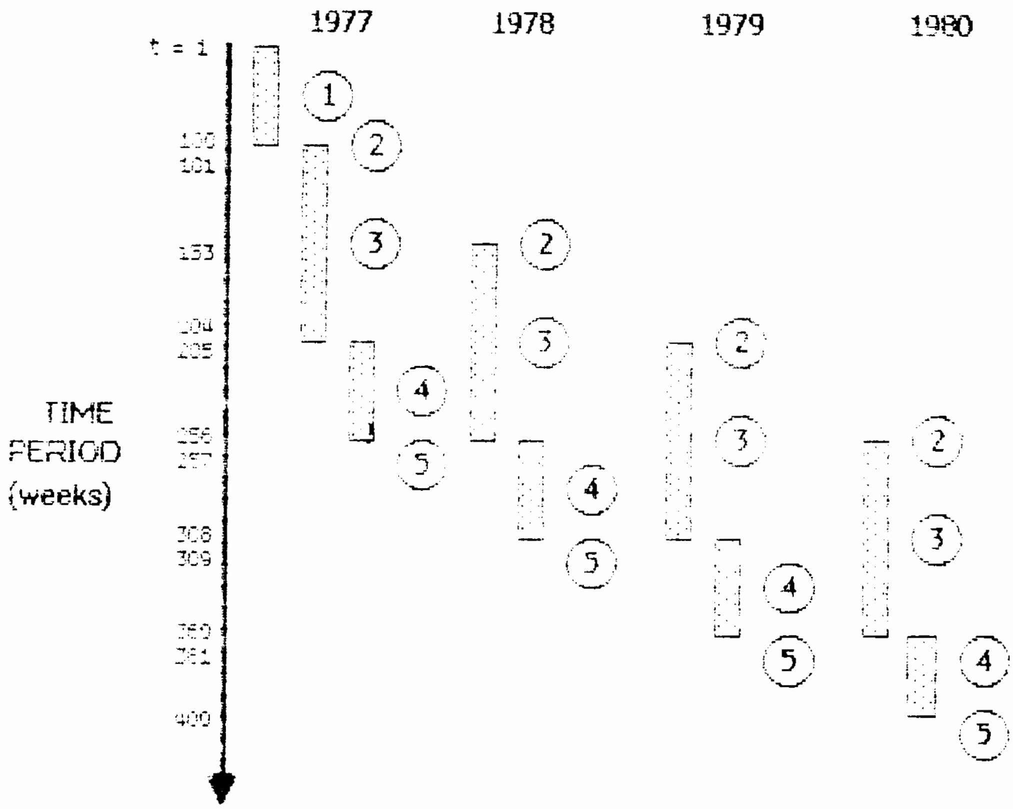
$\sigma(\gamma_i)$ = standard deviation of $\tilde{\gamma}_i$

n = number of periods used in the regression.

8.7 SUMMARY OF THE PROCEDURE

The whole procedure is summarised below, in the time-framework shown in Figure 7.2 in section 7.6.

Figure 8.1: Schematic illustration of the methodology



Stages: 1. Initial betas estimated over weeks 1 to 100

2. Rankings used to create portfolios

3. Portfolio parameters calculated

4. Portfolio returns regressed against portfolio parameters obtained in the immediately preceding period

5. Results interpreted

8.8 A GENERAL DEFENCE OF THE METHODOLOGY

The methodology for the study has been defended at length within the body of this chapter, and also, to a lesser extent, in Chapter 7. It is however felt necessary, in this section, to defend the major criticisms that may be levelled at the study.

The use of the market model has been defended in 8.2.2, wherein it was concluded that the market model presents the best and most often used, method to calculate the risk of a specific security. The use of OLS has also been defended. There is a problem with the heteroscedasticity of the variance of the error terms, however this was not considered to be a major limiting factor to the results.

Carter (1983) has outlined the best method of beta estimation for the JSE. This is as follows:

"(i) Choose the length of historical period from which the beta estimate is to be made equal to the length of the period for which it is required.

(ii) Using an overall market surrogate such as the JSE All Share Index, calculate monthly continuously compounded returns (log price relatives) for the security and the market. If the historical period is less than fifteen months, weekly data should be used to provide sufficient sampling points. Dividends, which in any event are excluded from the calculation of the JSE Actuaries Indices, may be ignored (Sharpe and Cooper (1972)).

(iii) Using ordinary least squares (OLS), obtain estimates of the alpha and beta parameters using the MM and the returns calculated above. Zero return weeks should be retained, whether traded or not."

Carter (1983: 8.3-8.4)

The current study has followed essentially the same method as outlined by Carter, with the following deviations. With regard to point (i), the length of time used to estimate the beta is two years, whereas the time period over which it is used is one year. The extra period was chosen in view of the fact that the beta coefficient is used on out of period returns, and thus becomes in a sense predictive. Carter has said that in this case as long a period of estimation as possible is required. The use of two years estimation for one year of returns is considered to be an adequate mixture of the two approaches.

Although the historical estimation period is greater than fifteen months, weekly returns have been used in order to give greater accuracy. The intervalling effect may be a problem here.[9] Dividends have been included in the current study, as they do represent an actual return on the share. It is acknowledged that the index does not include dividends, thus the beta coefficient may be slightly mis-specified. The affect of this is unlikely to be large however, because of the fact that dividends would usually only be paid, at most, twice a year, and even then the share price is normally significantly greater than the dividend, thus it does not affect the return to a large degree. The use of the RDM-100 index instead of the JSE All Share Index is considered to be more appropriate because no gold or mining shares have been included in this study.

All other facets of the proposed 'best' methodology have been followed. It is suggested that the estimates obtained will therefore be accurate and descriptive of the actual market in the time period under examination.

The intervalling effect has also, it is hoped, been minimised by the use of portfolio returns instead of individual security returns.

The above discussion has concentrated on internal characteristics of the study, and its internal validity. There remains, of course, the external validity of the study.

This is, as already mentioned, the first test of the CAPM in South Africa. Every effort has been made to ensure that the best techniques developed have been used in the study. These techniques relate both to the JSE (e.g. Carter's suggestions noted above) and internationally (e.g. the use of the Fama and MacBeth (1973) techniques in place of those of Black, Jensen and Scholes (1972)). The study does cover a large number of shares in the industrial sector of the JSE, for a period of four years, and can thus, it is submitted, be representative of the industrial sector of the JSE as a whole, certainly during the time period being studied.

In summary, it is submitted that the study presented in this thesis has both external and internal validity.

8.9 SUMMARY

This chapter has explained the specific methodology used in this study. In addition, the methodology has been linked to the general methods described in Chapter 6. The study has also been defended on methodological grounds. The results of the study are presented next, in Chapter 9.

CHAPTER 8 - FOOTNOTES

(1) This partitioning of the JSE and concentration on the industrial sector only is in keeping with numerous other studies on the JSE. See for example Affleck-Graves and Knight (1983).

(2) Professor Barr's assistance in writing Fortran programmes to manipulate the data was greatly appreciated. Without his help the task would have been extremely difficult.

(3) In calculating the returns, the first week of the file is lost, hence the 9 February start of the 'returns file'.

(4) The BMDP (1982 version) packages used were: P1R for beta and standard error and P2D for skewness and total risk.

(5) HMV were ambiguous in stating how their portfolio parameters were obtained. Whether directly from the portfolio returns, or whether they were obtained as above was not clear. Close analysis of the FM technique revealed that the simple average was used, thus this method has been used in this study.

(6) Once again, Professor Barr's help in this regard was invaluable.

(7) That is, the ten portfolio returns plus the market returns as reflected by the change in the RDM-100 index.

(8) This is because of the fact that, although the CAPM predicts an upward-sloping linear relationship between risk and return, the change from ex-ante to ex-post, based on actual returns, means that a negative relationship may exist in the short-term. By aggregating the data into longer periods any short-term abberation should be removed.

(9) In fact, this is a problem, as suggested in Chapter 10.

CHAPTER 9

RESULTS OF THE TESTS ON WEEKLY RETURNS

9.1 INTRODUCTION

The previous chapter introduced the actual methodology used in the study together with a defence of the methodology. The hypotheses to be tested were developed in Chapter 7, section 7.7. In this chapter the results for each hypothesis are presented, period by period. Finally, a summary of the results is made, and initial conclusions are drawn. It is stressed that, in view of the results that were obtained, significant re-analysis of the methodological issues had to be carried out. This is noted in Chapter 10, while the results of the refined methodology are presented in Chapter 11. In view of this, the discussion on the results presented in this chapter is of a fairly curtailed nature. The results using the two different methods are compared in Chapter 11 where detailed analysis as to the cause of the findings is carried out.

9.2 HYPOTHESIS A: LINEARITY OF THE RISK-RETURN RELATIONSHIP

The random coefficients of the following two equations are tested in this section:

Regression One: $\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{1t}\hat{\beta}_j + \tilde{\gamma}_{2t}\hat{\beta}_j^2 + \tilde{\mu}_{jt}$, and

Regression Two: $\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{2t}\hat{\beta}_j^2 + \tilde{\mu}_{jt}$

The results are presented in Table 9.1 below.

Table 9.1: Linearity of the risk-return relationship

Calculated Value	Year	Regression One: Beta γ_1	Regression One: Beta-squared γ_2	Regression Two: Beta-squared γ_2
t-TEST STATISTIC	1977	-1.96 **	1.77 **	0.43
	1978	0.500	-0.660	-1.13
	1979	0.146	0.134	0.940
	1980	-1.49 *	1.29	0.011
	1977-1978	-0.488	0.133	-0.528
	1979-1980	-1.47 *	1.36 *	0.353
	1977-1980	-1.44 *	1.14	0.150
AVERAGE REGRESSION COEFFICIENT	1977	-.021	.009	.001
	1978	.010	-.007	-.002
	1979	.002	.001	.002
	1980	-.040	.020	.0001
	1977-1978	-.005	.001	-.0001
	1979-1980	-.020	.011	.001
	1977-1980	-.012	.006	.0002
VARIANCE OF REGRESSION COEFFICIENTS	1977	.006	.003	.0001
	1978	.019	.006	.0001
	1979	.146	.134	.0002
	1980	.032	.010	.002
	1977-1978	.013	.004	.0001
	1979-1980	.019	.007	.001
	1977-1980	.016	.005	.0006

* Statistically significant at the 10% level
** Statistically significant at the 5% level

In Table 9.1, the t-Test statistics are given first, as these are the most important results. In addition, the average regression coefficient and the variance of the regression coefficient are shown.

Recall from section 7.7 that the hypothesis would be rejected if

$$E(\tilde{\gamma}_{1t}) = 0, \text{ and}$$
$$E(\tilde{\gamma}_{2t}) \neq 0.$$

The most important result is that for the complete four year period. In Regression One the t- statistic is statistically not equal to zero, for the overall period and for certain sub-periods. In some of the individual years the beta-coefficient is however not significantly different from zero.

What is perhaps more surprising is the fact that in four of the seven sub-periods the overall beta-result, although statistically not equal to zero, was negative. This would appear to be contrary to the primary assumption of the CAPM that increasing risk yields increasing return. However, this result is possible in an ex-ante situation, as noted in Chapter 8. In general the beta-squared coefficient is not significantly different from zero. Further insights into the slope of the line are given in testing hypothesis C. Perusing the average regression statistics, and the variances, it is clear that no significant deviations or abnormalities are present.

The results are not what would be expected were the CAPM to be valid. The test is designed to ascertain whether the risk-expected return relationship is linear, and also whether it is described by one factor, namely beta, as posited by Sharpe (1964) and others since then. The inclusion of beta-squared into the regression line was intended to ascertain whether the risk-measure as denoted by beta is sufficient to explain the relationship.

The results of the test indicate that beta is sufficient, and that the relationship is linear, however it is negative.

Possible reaons for this negative relationship are explored in Chapter 11. In the meanwhile, the linearity (but not the slope) of the equation has been satisfactorily shown to be in accordance with the CAPM predictions. This implies that Hypothesis H_aA will be rejected, thus the null hypothesis is accepted, and the risk-return relationship is expected to be linear in beta.

The second hypothesis will now be examined.

9.3 HYPOTHESIS B: SYSTEMATIC RISK IS THE ONLY RISK BORNE BY INVESTORS

To test this hypothesis the random coefficients of following two equations are compared.

Regression Three: $\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{1t}\,\hat{\beta}_j + \tilde{\gamma}_{3t}\,\hat{s}_j + \tilde{\mu}_{jt}$

Regression Four: $\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{3t}\,\hat{s}_j + \tilde{\mu}_{jt}$

If the expected value of $\tilde{\gamma}_3$ in both equations is not equal to zero, the null hypothesis will be rejected. The results are shown in Table 9.2 overleaf.

Table 9.2: Systematic risk only is borne by investors

Calculated Value	Year	Regression Three: Beta γ_1	Regression Three: Std-error γ_3	Regression Four: Std-error γ_3
t-TEST STATISTIC	1977	0.087	1.14	1.19
	1978	-1.06	-0.220	-0.272
	1979	1.01	-0.248	-0.658
	1980	-0.053	-0.466	-0.577
	1977-1978	-0.655	0.702	0.692
	1979-1980	0.402	-0.582	-0.953
	1977-1980	0.016	0.188	-0.040
AVERAGE REGRESSION COEFFICIENT	1977	.0002	.261	.267
	1978	-.003	-.045	-.009
	1979	.003	-.040	-.110
	1980	-.0004	-.103	-.116
	1977-1978	-.001	.108	.106
	1979-1980	.002	-.077	-.123
	1977-1980	.0000	.019	-.004
VARIANCE OF REGRESSION COEFFICIENTS	1977	.0004	2.73	2.58
	1978	.0004	2.18	2.21
	1979	.0005	1.35	1.46
	1980	.003	2.14	1.78
	1977-1978	.0004	2.48	2.42
	1979-1980	.001	1.69	1.59
	1977-1980	.0009	2.10	2.02

None of the t-Test statistics is statistically significant at the 5% level or at the 10% level.

As shown above, neither the beta-coefficient nor the standard error-coefficient is significantly different to zero. In Regression Three and in Regression Four the standard error term is equal to zero.

The standard error is a measure of the unsystematic risk, which should theoretically be diversified away by investors. If the standard error term was shown to be significantly reflected in, and correlated with, the return of the portfolio, this would indicate a non-efficient situation in which investors could not diversify away this element of total risk. The expected result in this case is thus that the standard error term will be equal to zero.

The actual result is in fact the expected result, as the standard error term is never significantly different from zero. Admittedly, neither is the beta-coefficient, but this has already been indicated in testing hypothesis A.

Hypothesis B can thus be accepted, as the expected value of the standard error-coefficient equals zero.

9.4 HYPOTHESIS C: THE RISK-RETURN TRADE-OFF IS POSITIVE

In testing whether the risk-return trade-off is positive, only one equation is needed. If the value of the random coefficient relating to $\tilde{\gamma}_1$ in Regression Five (below) is positive, this implies that the trade-off between risk and expected return is also positive.

Regression Five: $\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{1t}\hat{\beta}_j + \tilde{\mu}_{jt}$

The results of this test are presented in Table 9.3

Table 9.3: The risk-return trade-off is positive

	REGRESSION FIVE		
	RANDOM COEFFICIENTS RELATING TO BETA (γ_1)		
YEAR	t-TEST STATISTIC	AVERAGE REGRESSION COEFFICIENT	VARIANCE OF THE REGRESSION COEFFICIENT
1977	0.216	.0006	.0005
1978	-1.07	-.003	.0004
1979	1.09	.004	.0006
1980	-0.308	-.002	.002
1977-1978	-0.576	-.001	.0004
1979-1980	.205	.0008	.001
1977-1980	-0.123	-.0002	.0009

No values are statistically significant at the 5% or at the 10% level.

The t-statistics given above indicate that, for the full four year period, the risk-return relationship was negative, however not statistically significantly so. In 1977 and 1979 the trade-off was positive, again not significantly so, as it was in the two year period 1979-1980.

This result was suggested by the results of the tests on Hypothesis A.

This finding is surprising, although not unknown in tests of the CAPM. In moving from an ex-ante model to an ex-post, testable model, it is theoretically possible to get a negatively sloping risk-return line. Hawawini, Michel and Viallet (1983) did find instances of this in the majority of the periods they studied.

It is all too easy to explain away this negative ex-post reaction by reasoning that the market return in the period tested was negative. This is however unlikely, as the expectation is that, over a sufficient period of time, the market portfolio, being riskier than the risk-free asset, will outperform the risk-free asset. The observed results are thus possibly due to methodological factors. These are examined in Chapter 10, and again in Chapter 11.

Hypothesis H_oC must thus be rejected, so the alternative hypothesis (H_aC) is accepted. In the period studied, it would appear that the relationship between risk and return is linear, but negative i.e. low beta portfolios would have earned higher returns in this period than high beta portfolios or, stated in another way, increased risk results in decreased returns.

9.5 HYPOTHESIS D: RETURN DISTRIBUTIONS ARE PERCEIVED AS BEING SYMMETRICAL

In testing whether the return distributions of the portfolios are perceived as being symmetrical and thus that skewness is ignored in making decisions, the following two equations were examined, and the random coefficients of the regression equations tested for significance.

Regression Six: $\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{1t}\,\hat{\beta}_j + \tilde{\gamma}_{5t}\,\hat{S}_j + \tilde{\mu}_{jt}$

Regression Seven: $\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{5t}\,\hat{S}_j + \tilde{\mu}_{jt}$

These results are presented in Table 9.4.

Table 9.4: Investors perceive symmetrical return-distributions

		Regression Six		Regression Seven
Calculated Value	Year	Beta γ_1	Skewness γ_3	Skewness γ_3
t-TEST STATISTIC	1977	0.25	-0.12	-0.003
	1978	-1.03	0.56	0.83
	1979	0.20	-1.62 *	-2.25 **
	1980	-1.01	-1.13	-0.44
	1977-1978	-0.50	0.07	0.29
	1979-1980	-0.98	-1.89 *	-1.42
	1977-1980	-1.10	-1.10	0.50
AVERAGE REGRESSION COEFFICIENT	1977	.0008	-.0007	-.0002
	1978	-.0003	.001	.002
	1979	.0008	-.003	-.003
	1980	-.010	-.006	-.002
	1977-1978	-.001	.0002	.0009
	1979-1980	-.005	-.005	-.003
	1977-1980	-.003	-.002	-.0009
VARIANCE OF REGRESSION COEFFICIENTS	1977	.0005	.002	.002
	1978	.0002	.0003	.0002
	1979	.0009	.0002	.0001
	1980	.005	.001	.0007
	1977-1978	.0005	.001	.0009
	1979-1980	.002	.0007	.0004
	1977-1980	.001	.0009	.0007

* Statistically significant at the 10% level
** Statistically significant at the 5% level

If the returns were not symmetrical, the expected value of $\tilde{\gamma}_3$, the cross-sectional coefficient relating to the skewness of the return-distribution of the portfolios, would not be zero. As can be seen in Table 9.4, in the majority of the periods tested this was the case. Certainly for the overall four year period, the distribution is symmetrical. In 1979, and again in the two-year period 1979-1980, the value of the skewness co-efficient is significantly negative. This is the case (in 1979) for both Regression Six and Regression Seven. In the aggregate of the two periods, the significance noted in Regression Six is no longer present in Regression Seven.

Again, in general, the expected result has been shown to be the actual result. The hypothesis that skewness is ignored by investors in making their decisions can be accepted, as the empiricial results indicate this to be the case.

Hypothesis D can thus be accepted, the return-distribution is perceived as being symmetrical when decisions are made by investors, they show no particular preference for skewness, either positive or negative.

The results of the final hypothesis will now be examined.

9.6 HYPOTHESIS E: LEVY'S GENERALISED CAPM IS NOT VALID

This test is, as mentioned in section 7.7, a test of Levy's (1978) findings that variance is a risk measure that dominates beta, when total diversification is not possible. The two equations to be tested are thus:

Regression Eight: $\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{1t} \hat{\beta}_j + \tilde{\gamma}_{4t} \hat{\sigma}_j^2 + \tilde{\mu}_{jt}$

Regression Nine: $\tilde{R}_{jt} = \tilde{\gamma}_{ot} + \tilde{\gamma}_{4t} \hat{\sigma}_j^2 + \tilde{\mu}_{jt}$

Table 9.5. shows the results of this test.

Table 9.5: Investors are extreme diversifiers

Calculated Value	Year	Regression Eight: Beta γ_1	Regression Eight: Variance γ_4	Regression Nine: Variance γ_4
t-TEST STATISTIC	1977	-0.122	0.895	0.920
	1978	-0.107	-0.144	-0.107
	1979	1.101	0.109	-0.181
	1980	-0.063	-0.785	-0.736
	1977-1978	-0.775	0.499	0.495
	1979-1980	0.463	-0.360	-0.594
	1977-1980	-0.034	0.242	0.091
AVERAGE REGRESSION COEFFICIENT	1977	-.0004	1.481	1.334
	1978	-.003	-.256	-.190
	1979	.004	.140	-.235
	1980	-.004	-.670	-.652
	1977-1978	-.002	.609	.568
	1979-1980	.002	-.287	-.480
	1977-1980	-.00007	.179	.065
VARIANCE OF REGRESSION COEFFICIENTS	1977	.0006	141.51	108.08
	1978	.0004	165.80	165.51
	1979	.0006	86.21	87.51
	1980	.002	32.02	34.53
	1977-1978	.0005	154.53	137.15
	1979-1980	.001	60.98	62.78
	1977-1980	.0009	109.30	101.24

None of the t-Test statistics is significant at either the 5% level or at the 10% level.

Again, no t-statistics are statistically significant, implying that the variance is not a better measure of the risk, and thus that investors do diversify in the extreme. The average values of the coefficient are low, but in both Regression Eight and Regression Nine, the variance is very high. This indicates the wide dispersion of the coefficients relating to total risk.

The implications of this are that diversification is practised on the JSE. Levy had shown that if investors were limited to holding only a few securities, they were unable to diversify and thus the variance of the returns, rather than the beta-coefficient, became the dominant risk measure. The findings have however indicated that this strict restriction on diversification does not appear to be the case on the JSE.

Hypothesis E is accepted, investors appear to be extreme diversifiers.

9.7 SUMMARY OF THE RESULTS

The results of the tests that have been carried out using weekly log-returns of share-prices during the years 1977 to 1980 are summarised below, in Table 9.6. The expected result, as predicted by the CAPM, is given first, followed by the actual results obtained.

Table 9.6: Summary of the results

Hypothesis	Predicted CAPM result	Actual empirical result	Agrees with CAPM
A	The Share Market line (SML) is linear	The SML is linear	✓
B	Systematic risk is the only relevant risk measure	Systematic risk is the only relevant risk measure	✓
C	The risk-return relationship is positive	The risk-return relationship is negative	X
D	Investors regard the return-distributions as symmetrical	Investors regard the return-distributions as symmetrical	✓
E	Systematic risk is the relevant risk measure, rather than total risk.	Systematic risk is the relevant risk measure, rather than total risk.	✓

In all cases other than Hypothesis C, the null hypothesis, H_0, has been accepted.

As can be seen from the above table, in all cases except the slope of the share market line, the empirical result is in agreement with the predicted CAPM result. The evidence certainly points towards the validity of the CAPM on the JSE. However, the negative sloping SML is a worrying factor, as Fama and MacBeth did not find evidence of this on the NYSE, and it is not the predicted result.

As a result of this finding in particular, the underlying methodology was re-examined and extensive further work was carried out. This is detailed in the following chapter.

CHAPTER 10

THE RESEARCH METHODOLOGY REFINED

10.1 INTRODUCTION

The initial research methodology and the results of this have already been presented in Chapters 8 and 9 respectively. The overall conclusion that was reached in Chapter 9 was that the CAPM appeared to be a valid model of the equilibrium pricing of assets on the JSE in the years 1977 to 1980, however in the ex-post model tested, the Share Market Line (SML) had a negative slope. This was contrary to the fundamental assumption, going right back to Markowitz's pioneering works in Portfolio Theory, that increasing risk yielded an increased return.

In this chapter possible causes of the negative-sloping SML highlighted in Chapter 9 are analysed and refinements that were made to the initial methodology are detailed. The results of these additional tests are noted in Chapter 11. In addition, an interesting statistical comparison between the RDM-index and an internally generated index is presented, which could aid other researchers in their use of an index in studies of this sort.

10.2 POSSIBLE CAUSES OF THE NEGATIVE-SLOPING CAPM

Although the CAPM does predict an upwards sloping line, it is possible to obtain a downwards sloping line as was found here, due to the change from an ex-ante model to an ex-post test of the model.[1] It follows that the results are not definitely incorrect, however they may be incorrect, thus necessitating further testing and further analysis of the methodology used.

In view of this, the methodology was re-examined in order to ascertain possible weaknesses in the preliminary research reported on in Chapters 8 and 9.

This examination pinpointed two possible areas of concern.
These were:

- the choice of weekly returns instead of a longer period of a month, as used by Fama and MacBeth (1973). This was discussed in section 8.1, however the decision reached there may have been the incorrect one.

- the use of the RDM-index as a surrogate for the market-return in the periods being examined, instead of using the true market return.

10.3 SOLUTIONS TO THE POSSIBLE PROBLEMS

10.3.1 Use of monthly returns

Hawawini, Michel and Viallet (1983) used weekly returns instead of the monthly returns used by FM (1973) in their study. Their argument for the use of weekly returns was that the number of observations they had available to include in their test design would have been significantly reduced had they used returns over a period longer than a week.

HMV do however note (HMV (1983:336)) that the use of weekly data may produce estimates of systematic risk that are biased, particularly for thinly-traded securities.[(2)] Use of a longer return period helps to overcome this bias.

In view of the above, and the fact that the original FM study used monthly returns, it was decided to replicate the study using returns generated over a four week period instead of weekly. The effect of this was to reduce the total number of observations from 404 weekly observations to 101 monthly values. It is felt that the benefit to be gained from using monthly returns to test the CAPM will outweigh the disadvantages inherent in using fewer actual results.

Recall from 8.1 that Carter (1983) indicated that in a South African context, monthly returns should be used except where the number of returns is less than fifteen, in which case weekly returns should be used. The use of monthly-returns here means that in the period used to estimate the various parameters [(3)], twenty-six returns are being used, thus the critical minimum level recommended by Carter is exceeded by a factor of 1.73, indicating no loss of information in using monthly returns. In all likelihood increased, more accurate information will result. The period (13 months) over which these parameters are regressed is irrelevant to the fifteen month period mentioned by Carter, as no estimation of paramaters such as beta is being made. The already-estimated parameters are being fitted to the actual results.

10.3.2 Use of an internal index

The 'pure' market-model suggests that the index used in obtaining the beta coefficient and any other necessary statistics should be the complete market of the shares being examined. It is quite likely that whilst some of the 107 shares included in the ten portfolios would be included in the computation of the RDM-index, many of them would not be included. This implies that the index used in the analysis may lead to a mis-specification of some of the parameters used in the study.

Sharpe (1963) has said that the market index used in the regression calculation to obtain the beta of the share should be the complete set of securities being examined, excluding that one security whose returns are being regressed against the market return.(4)

Few studies have even gone so far as to construct an 'internal index' let alone construct an index specific to each share, which is what Sharpe suggests is the technically correct solution.

In view of the difficulties involved in calculating an index for each share individually, it was decided that an internal index for all 107 shares would be constructed, and used in the regressions to obtain the various parameters and results.

Although not in agreement with Sharpe's ideal index, this decision is considered to be valid in terms of the research design, as only a small portion of the actual return of the individual share being used in any one regression against the market return will be incorporated into the market return. This is less than 1% of the components of the total return and is thus unlikely to cause a serious auto-correlation problem.

A procedure comparing the two indices is presented in section 10.6.

10.4 RESEARCH REDESIGN: PRELIMINARY WORK

10.4.1 Introduction

In view of the fact that the beta-estimates used in calculating the initial values by which portfolios were ranked had been calculated over a 100 week period, which is a relatively long time-span, it was decided not to recalculate these values, nor to reorganise the portfolios. The portfolios and their constituent shares are thus as used in the initial study, and are as shown in Appendix A. Note that the parameters will change however (refer section 10.5.3).

Another advantage of this decision is that comparability between the results can be maintained, as the portfolios will be the same as used before. It is however extremely unlikely that the portfolios would have changed anyway, had monthly returns been used to calculate beta values for ranking purposes.

10.4.2 Monthly returns and internal index calculated

The 'returns-file' obtained in the initial study was recalculated using a specially-written Fortran program. This yielded a file of the monthly log-returns for the 107 individual securities.(5) In addition, a monthly return for the RDM-index was calculated. This file will henceforth be called the 'monthly returns' file.

Once the monthly security returns had been calculated, a portfolio index (hereafter called the Internal Index) was obtained. A Basic program was written which aggregated the monthly returns on all 107 securities used in the analysis, and calculated a new index. This new index was then inserted into the 'monthly returns' file using another program written in Basic.

At this stage there existed monthly returns covering 101 weeks for 107 securities, and two different indices, the RDM-index and the Internal Index, each giving monthly 'market' returns.

10.5 STAGES OF THE METHODOLOGY

This section will detail the various stages in the methodology. As before, the stages will relate to the general discussion in Chapter 7 of the research methodology.

10.5.1 Step 1: Initial beta calculation

This step was not reperformed in the refined methodology as it was felt that the beta-values of each security would be unlikely to change significantly when calculated using monthly returns from those values calculated using weekly returns.

10.5.2 Step 2: Rank the shares by beta and allocate into portfolios

Again, this step was not reperformed. This reason for this is noted in section 10.4.1 above.

10.5.3 Step 3: Portfolio parameters obtained

In order to obtain the parameters, the 'monthly returns' file was used. The two different market indices (RDM and Internal) would yield different beta values, beta-squared values and different standard error terms, however the values of the variance and the relative skewness terms would not be affected by the different market indices used.

The five parameters were thus calculated for 26 months at a time, for each security. The individual security's values were then aggregated and simple-averaged across the portfolio, for the reasons described in section 8.3.2.

This procedure was repeated four times, each time moving the analysis forward by one year (13 months). The procedure was repeated for the Internal index (Int-index).

10.5.4 Step 4: Portfolio parameters regressed against portfolio returns

Monthly portfolio returns were obtained using the Fortran program previously used (see section 8.5), which resulted in a portfolio monthly returns file containing monthly returns for 12 variables.(6)

For both the RDM-index and the Int-index, the respective portfolio parameter sets are cross-sectionally regressed against the ten out of period portfolio returns. Different regression equations are evaluated in order to test the various hypotheses.

In the following 13 week period the two second parameter sets are used in the regressions, and so on. The various parameters as obtained using first the RDM-index and then the Int-index are shown in Appendix D.

The procedure can be shown as in Table 10.1 overleaf:

Table 10.1 Full parameter regression periods used in the analysis

Parameter sets*	Obtained on return in periods	Regressed on out of period returns	No of periods	Year
One	26-51	52-64	13	1977
Two	39-64	65-77	13	1978
Three	52-77	78-90	13	1979
Four	65-90	91-100	11	1980

*Two sets of parameters for each period exist, one for RDM-index, one for Int-index.

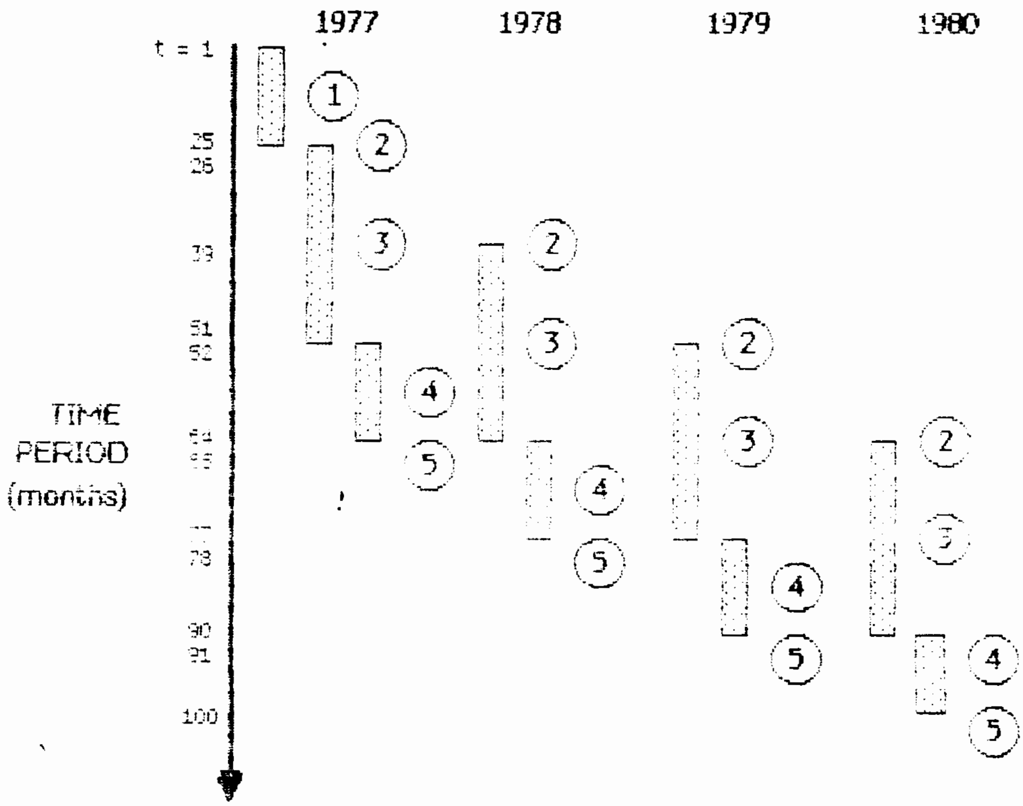
The years as labelled again run from mid January in one year through to mid January the following year, with the exception of 1980, which ends in November.

Steps 4 and 6 of the methodology were thus completed, leaving only step 5, the assessing of the statistical significance of the results, to be performed.

10.5.5 Step 5: Assessing the statistical significance

Again, the results were manually entered onto a LOTUS 123 spreadsheet, for final computations. Results for the 4 sub-periods, plus results for the combined periods were obtained once again. These are presented in Chapter 11. Prior to presenting the results however, the schematic illustration of the model (Table 10.3) is presented overleaf. This gives a brief outline of the revised methodology.

Table 10.3 Schematic Illustration of the methodology



Stages: 1. Initial betas estimated over months 1 to 25

2. Rankings used to create portfolios

3. Portfolio parameters calculated

4. Portfolio returns regressed against portfolio parameters obtained in the immediately preceding period

5. Results interpreted

10.6 COMPARING THE TWO INDICES

10.6.1 Introduction

As noted in section 10.2, a possible cause of the negative sloping CAPM line was the use of the RDM-index instead of a true market-index representative of all the securities being examined. In section 10.3 it was noted that the solution found for this particular problem was to calculate an internal index consisting only of the returns of the securities used in the study.

It was felt that it would be interesting and relevant to compare these two indices statistically. The way in which this was done and the results are presented below.

10.6.2 Comparison of the indices

The comparison being made here is a by-product of the current research, which is presented out of interest, thus a detailed test was not performed. The method by which the RDM-index was compared to the Int-index was fairly simple, however, it is submitted, still valid.

The values of the Int-index in each week were regressed against the equivalent value of the RDM-Index in the same week, and a scatter plot, together with the correlation coefficient and the regression equation, was produced. This is presented overleaf.

The expected slope of the relationship between the Int-index and the RDM-index, were they to be interchangeable as measures of the market return, would be upwards at 45° to the V-axis (i.e. a slope of 1).

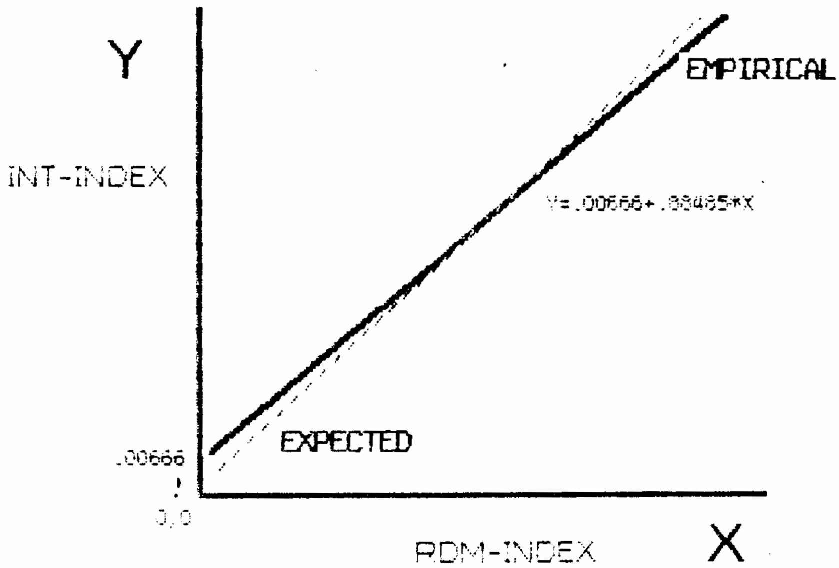


Figure 10.1: Int-index regressed against RDM-index

As can be seen, the actual result is very similar to the expected result.

The correlation of the regression is .9231. Had the two indices been identical the correlation would have been 1.00, thus the two are shown to be highly correlated.

The residual mean squares of the regression is $4,66 \times 10^{-4}$, showing how low the variance of the regression line is, and again indicative of the excellent 'fit' of the line.

The actual equation, where Y equals Int-index and X equals RDM-index is:

$Y = .00666 + .88485 * X.$

This slope factor (.88485) implies that the returns of the shares used in this study are in general slightly lower than those used in calculating the RDM-index, thus the Int-index will usually be lower than the corresponding value of the RDM-index.

10.6.3 Implications of the comparison

The implications of the above comparisons are that the RDM-index would appear to be a valid surrogate for the actual market return, thus it is unlikely that the results will change significantly due to the use of a 'total' market-index. The RDM-index does however appear to be overstated in relation to the return of the sample shares, as evidenced by the slope of less than one, however whether this is significant or not remains to be seen. This will be evaluated in the following chapter, where different results for the two indices will be shown up if they are significantly dissimilar.

It thus appears that the use of the RDM-index as a surrogate for the true market return by past researchers (e.g. Knight (1983)) and by future researchers is a valid assumption which will be unlikely to affect their results adversely.

10.7 CONCLUSION

Possible causes for the negative slope of the CAPM line found in Chapter 9 were highlighted in this chapter. It was felt that these could have been either the use of weekly returns instead of monthly returns, or the use of the RDM-index instead of constructing a 'true' market-index. These changes were incorporated into the methodology and the tests reperformed. The results of these tests are presented in the following chapter.

Finally, a statistical comparison between the RDM-index and the Int-index was carried out. The result of this was that the Int-index is very similar to the RDM-index, although usually slightly lesser in amount. This implies that the use of the RDM-index by other researchers has been a valid assumption.

CHAPTER 10 - FOOTNOTES

(1) HMV (1983) say in their introduction:

> "The empirical findings indicate that the lagged relationship between the average returns and the risk of French common stocks was generally negative...Despite these seemingly startling empirical results one cannot reject the hypotheses that the pricing of the sample of French common stocks conforms to the Capital Asset Pricing Model over the 1969-1979 decade."
>
> HMV (1983: 333)

(2) This is the so-called intervalling effect.

(3) See section 10.5.3.

(4) In other words, the market return should consist of a weighted return of all securities being examined except the particular one used in the regression.

(5) These returns are strictly-speaking 4-weekly returns, however for the sake of brevity they have been termed monthly-returns hereafter.

(6) That is, the ten portfolio returns plus the RDM-index and the Int-index.

CHAPTER 11

RESULTS USING MONTHLY DATA AND TWO INDICES

11.1 INTRODUCTION

This chapter presents the results of the tests carried out using the parameters calculated on monthly returns, for both the RDM-index and the Int-index. As before, the results are presented hypothesis by hypothesis, with the t-statistic shown first, followed by the average of the cross sectional regression random coefficients, and the variance of these coefficients. In addition, the average multiple correlation coefficient (R^2) for each equation is presented, as this gives an indication of the correlation between the dependent and independent variables. Results for both the RDM-index and the Int-index are given.

In view of the fact that the regression equations being tested have already been given twice they will not be repeated in the hypothesis evaluation in this chapter. Note however that the tables do give details of the coefficients being tested.

As mentioned above, the results are given hypothesis by hypothesis. No attempt is made to present an economic reason or any other reason for the results as presented, until all six hypotheses have been presented and discussed. Once this has been done, the results are compared with those presented in Chapter 9, the preliminary study, and possible reasons for the findings are advanced.

11.2 HYPOTHESIS A: THE RISK-RETURN RELATIONSHIP IS LINEAR

The results of the tests on Regressions One and Two involving beta and beta-squared are shown in Table 11.1 below.

Note that for each year the results using the RDM-index are presented first, and that these are then followed by the results obtained using the Int-index. As before, the t-statistic is presented at the start of the table, as this is the most important result. For ease of presentation the R^2 value is presented in the same 'block' as the t-test results. Note however that it is a true R^2 value, and not a t-test result.

TABLE 11.1: Linearity of the risk-return relationship

CALCULATED VALUE	YEAR	REGRESSION ONE: Beta $(\tilde{Y}_1)$ RDM	Beta $(\tilde{Y}_1)$ INT	Beta-squared $(\tilde{Y}_2)$ RDM	Beta-squared $(\tilde{Y}_2)$ INT	R^2 RDM	R^2 INT	REGRESSION TWO: Beta-squared $(\tilde{Y}_2)$ RDM	Beta-squared $(\tilde{Y}_2)$ INT	R^2 RDM	R^2 INT
t-TEST STATISTIC	1977	-0.620	-0.213	0.979	0.505	.160	.177	1.034	0.589	.078	.105
	1978	-1.423	-0.319	1.227	0.079	.224	.258	-0.049	-0.582	.124	.107
	1979	-0.244	-0.129	0.400	0.293	.258	.367	0.483	0.337	.147	.271
	1980	-1.740*	-2.428**	2.666**	2.803	.235	.269	0.934	0.258	.127	.120
	1977-1978	-1.301	-0.380	1.461	0.489	.192	.218	0.769	0.196	.101	.106
	1979-1980	-1.665	-1.703	2.318**	2.062*	.247	.322	1.066	0.433	.138	.202
	1977-1980	-2.075**	-1.602	2.666**	1.829*	.218	.268	1.309	0.450	.119	.152
AVERAGE REGRESSION COEFFICIENT	1977	-0.017	-0.006	0.014	0.008			0.006	0.004		
	1978	-0.024	-0.008	0.009	0.001			-0.0003	-0.003		
	1979	-0.005	-0.004	0.006	0.003			0.004	0.002		
	1980	-0.116	-0.095	0.090	0.043			0.016	0.015		
	1977-1978	-0.017	-0.007	0.009	0.004			0.002	0.0008		
	1979-1980	-0.056	-0.046	0.047	0.022			0.009	0.002		
	1977-1980	-0.037	-0.026	0.027	0.013			0.006	0.001		
VARIANCE OF REGRESSION COEFFICIENTS	1977	0.010	0.009	0.003	0.003			0.0005	0.0006		
	1978	0.004	0.008	0.0007	0.001			0.0004	0.0002		
	1979	0.005	0.015	0.003	0.002			0.0007	0.0006		
	1980	0.048	0.017	0.013	0.003			0.003	0.0004		
	1977-1978	0.006	0.008	0.002	0.002			0.0004	0.0004		
	1979-1980	0.027	0.018	0.009	0.003			0.002	0.0004		
	1977-1980	0.016	0.013	0.005	0.002			0.001	0.0004		

* Statistically significant at the 10% level
** Statistically significant at the 5% level

In no single period other than in 1980 was the t-statistic for both beta and beta squared significantly different to zero, either when calculated using the RDM-index, or when calculated using the Int-index. In 1980, the beta t-statistic calculated using the RDM index (-1.749) was significant at the 10% level. The t-result for the Int-index was however also significantly negative at the 5% level. In the same year the beta-squared was significantly positive, for both indices.

Looking next at the complete four year span, the beta coefficient is significantly negative at the 5% level when using the RDM-index, but not significant, although still negative, when using the Int-index. The beta-squared statistic in both cases is positive, at the 10% and 5% levels respectively. The above comments all relate to Regression One.

Turning now to the Regression Two results, it is apparent that the beta-squared term alone is not a significant risk-measure. Not in any single year, nor in any of the combined periods is it significant.

It is thus apparent that the beta and beta-squared risk factors explain a greater amount of the actual return than does beta-squared alone. This is borne out by the multiple correlation coefficient (R^2) which for the full four year period is equal to .218 for the RDM-index and .268 for the internal index, compared with values of .119 and .152 for the two indices respectively. In the case of Regression One the R^2 for the internal index is invariably higher than it is for the RDM index. This is similarly so for Regression Two, except for 1980, where the values are very similar (.127 for RDM and .120 for Int). The Int-index would thus appear to be a better predictor of the actual return than the RDM index. This is not surprising, as the Int-index is composed of the returns of the securities in this study only, whereas the RDM-index comprises other shares' returns also.

Returning to the t-statistics for $\tilde{\gamma}_1$, it can be seen that the value here is invariably negative. This appears to suggest a downward-sloping CAPM. This will be investigated further under hypothesis C.

The results for hypothesis A are not in agreement with those found by Fama and MacBeth (1973). FM found a positive value for the $\tilde{\gamma}_1$ coefficient in all but one of the numerous periods they examined. In addition, they found no evidence for any additional power being added by the $\tilde{\gamma}_2$ coefficient.

Hawawini, Michel and Viallet (1983), on the contrary found a non-significant negative value for the t-statistic relating to $\tilde{\gamma}_1$ in eight of the ten years they examined, from 1969 to 1978. In the same decade the $\tilde{\gamma}_2$ coefficient was positive (but not significantly so) in 7 years. HMV did not publish results for combined periods, thus it is difficult to say whether they would have found a similar result as is shown here viz. only significantly negative in one of the years (1980, at the 10% level), but for the whole period significantly negative. These results have been checked, and the reason for this apparent anomaly is that the 1979-1980 period is very close to being significantly negative. This then becomes so in the full period. HMV found significant t-values in their equivalent to Regression Two, but did not explain the cause of these.

In view of the fact that in the full four year period the values of both beta and beta-squared are significant, the view that the risk-return relationship is not linear in beta only would appear plausible. However, because of the fact that in every year except 1980, and for the 1977-1978 period, the results are not significantly positive or negative, it is difficult to draw an absolute conclusion. In the absence of further testing, the hypothesis will be accepted, albeit tentatively so.

11.3 HYPOTHESIS B: SYSTEMATIC RISK ONLY IS BORNE BY INVESTORS

The results for the tests on Hypothesis B, which were carried out using Regressions Three and Four, are noted in Table 11.2 below.

TABLE 11.2: Systematic risk only is borne by investors

CALCULATED VALUE	YEAR	REGRESSION THREE						REGRESSION FOUR			
		Beta ($\tilde{\gamma}_1$)		Std error ($\tilde{\gamma}_3$)		R^2		Std error ($\tilde{\gamma}_3$)		R^2	
		RDM	INT	RDM	INT	RDM	INT	RDM	INT	RDM	INT
t-TEST STATISTIC	1977	0.829	0.693	0.107	0.885	.146	.206	0.005	0.995	.082	.114
	1978	-0.189	-0.018	2.231**	1.906*	.219	.259	2.060*	1.580	.093	.112
	1979	0.365	0.339	-0.192	-1.402	.452	.318	-0.071	-1.366	.352	.347
	1980	0.912	-0.271	-1.191	-0.818	.213	.334	-0.565	-0.832	.104	.195
	1977-1978	0.483	0.473	2.076**	1.712*	.183	.233	1.936*	1.746	.087	.113
	1979-1980	0.987	0.145	-1.196	-1.502	.342	.325	-0.580	-1.490	.239	.132
	1977-1980	1.096	0.404	0.538	0.611	.259	.277	0.836	0.661	.160	.122
AVERAGE REGRESSION COEFFICIENT	1977	0.010	0.008	0.001	0.348			0.0001	0.373		
	1978	-0.002	-0.0002	0.459	0.399			0.480	0.402		
	1979	0.004	0.006	-0.007	-0.226			-0.002	-0.218		
	1980	0.023	-0.004	-0.323	-0.228			-0.189	-0.231		
	1977-1978	0.003	0.004	0.023	0.374			0.240	0.387		
	1979-1980	0.013	0.002	-0.151	-0.227			-0.087	-0.224		
	1977-1980	0.008	0.003	0.047	0.086			0.083	0.094		
VARIANCE OF REGRESSION COEFFICIENTS	1977	0.002	0.009	0.002	0.003			0.002	0.0006		
	1978	0.002	0.002	0.551	0.570			0.706	0.841		
	1979	0.002	0.005	0.016	0.337			0.015	0.332		
	1980	0.007	0.002	0.808	0.852			1.226	0.851		
	1977-1978	0.002	0.002	0.320	1.239			0.400	1.279		
	1979-1980	0.004	0.003	0.385	0.546			0.550	0.544		
	1977-1980	0.003	0.002	0.381	0.980			0.490	1.003		

* Statistically significant at the 10% level
** Statistically significant at the 5% level

In order for the null hypothesis to be accepted, the expected value (and thus the t-statistic) relating to $\tilde{\gamma}_3$, the standard error term, must be zero in both equations.

The condition that the std error term must be zero is held in all periods other than in 1978, where the term is significant at the 5% level in Regression Three and at the 10% level in Regression Four, using the RDM-index, and it is significant only at 10% in Regression Three using the Int-index. This again seems to indicate that the Int-index is a better index, as it captures a greater portion of the total risk, leaving a lower standard error term (unsystematic risk) in the regression equation for the market model. In general however, the condition of a standard error term equal to zero appears to be met. In the overall four year results it certainly does hold.

HMV found, as was found in this study, both positive and negative terms for $\tilde{\gamma}_3$. Their results too showed only one year in which the standard error coefficient was significant. This was also a positive value as it is here. FM found no periods with significant values, although their results also showed both positive and negative t-values relating to $\tilde{\gamma}_3$, the standard error term.

Hypothesis B cannot be rejected, the null hypothesis is accepted, thus investors would appear to be compensated only for systematic risk. This result is an expected result, as the CAPM theory predicts that investors are compensated only for bearing systematic risk, and that unsystematic risk will be costlessly diversified away.

11.4 HYPOTHESIS C: THE RISK-RETURN TRADE-OFF IS POSITIVE

This test requires only one equation, Regression Five, the null hypothesis of which suggests that if the calculated value of the t-statistic relating to $\tilde{\gamma}_1$ (beta) is positive, then so is the risk-return relationship. These results are presented in Table 11.3 below.

Table 11.3: The risk-return trade-off is positive

			REGRESSION FIVE					
			RANDOM COEFFICIENTS RELATING TO BETA (γ_1)					
YEAR	t-TEST STATISTIC		AVERAGE REGRESSION COEFFICIENT		VARIANCE OF THE REGRESSION COEFFICIENT		R^2	
	RDM	INT	RDM	INT	RDM	INT	RDM	INT
1977	0.792	0.983	0.009	0.011	0.002	0.141	.064	.045
1978	-0.899	-0.529	-0.011	-0.007	0.002	0.002	.141	.187
1979	0.975	0.292	0.045	0.005	0.028	0.005	.067	.209
1980	-0.167	-0.307	-0.003	-0.004	0.004	0.002	.145	.129
1977-1978	0.087	0.188	0.001	0.004	0.002	0.002	.102	.134
1979-1980	0.705	0.084	0.021	0.001	0.004	0.003	.105	.194
1977-1980	0.805	0.219	0.010	0.002	0.008	0.002	.104	.163

No t-statistics are significant at either the 10% or the 5% level.

If the CAPM were to hold, the result that would have been expected in the above test was that the values of the t-statistic relating to $\tilde{\gamma}_1$, would be significantly positive. As can be seen, in two of the four sub-periods, the results are positive, although not significantly so. However, in the other two sub-periods the results are negative. This is the case for both indices. Again, not significantly so. Looking next at the combined two year periods, the results in both periods, and for both indices, are a non-significant positive relationship between risk and return. The complete four year period exhibits a similar result viz. a non-significantly positive relationship between the risk of an asset and its expected return.

Comparing the R^2 values for the two indices, it is apparent that the internal index captures more of the market movement than does the RDM index. This is reflected in the generally higher R^2 values for the Int-index when compared with those obtained for the RDM-index. This is consistent with the result for hypothesis A.

FM found a significantly positive relationship between risk and return, whereas the findings of HMV were not as obvious. In four of the ten years studied by them, their results showed a significantly negative relationship between risk and return. In no single year in the period they studied was the t-statistic significantly positive.

The results presented above thus tend towards the view that the CAPM-predicted positive linear relationship between risk and return might hold. These findings are not conclusive, however they are, it is suggested, valid pointers.

In view of the fact that the existence of a positive relationship between risk and expected return is the fundamental core of the CAPM, it is necessary to examine these results further.

The results are not conclusive, however they do appear to indicate a positive relationship between risk and return. The t-statistic for the full four year period based on the RDM-index suggests a significance level of approximately 17%, for the sample size of 50 months, using 49 degrees of freedom. The value of .219 calculated for the Internal index suggests a significance level of approximately 55%. The true level of significance is probably somewhere between these two values. The significance level for the two-year period 1979-1980, calculated on the Int-index, is marginally over 50%.

Even though the significance levels noted above are very low, it is felt that they do help to confirm the conclusion that the risk-return relationship in South Africa, is positive.

The results of hypothesis C above thus tend towards confirming the even more tentative conclusion with regard to hypothesis A above, that the relationship between the risk and the return on any asset is linear.

The conclusion of the above discussion is thus that hypothesis C will be accepted, in view of the general tendency towards positive values for the t-statistic. The risk-return relationship appears to be positive. This means that one of the fundamental predictions of the CAPM appears in reality to be valid.

11.5 HYPOTHESIS D: THE RETURN-DISTRIBUTIONS ARE PERCEIVED AS BEING SYMMETRICAL

The test which follows examines another of the assumptions of the CAPM, that the investors treat the return-distributions of securities as symmetrical, and ignore skewness. The parameter used to test this assumption is the skewness of the return-distributions of the securities involved. If the coefficients are shown not to be significant, then the assumption underlying the model is valid, and the perceived distribution is an accurate reflection of the true state.

Table 11.4 overleaf presents the results of this test, which involved examining the random regression coefficients of Regression Six, and of Regression Seven. The coefficients are $\tilde{\gamma}_1$ relating to beta, and $\tilde{\gamma}_5$ relating to skewness.

The expected result, to conform with the predictions of the CAPM, is that the skewness factor will not be significantly negative or significantly positive. This is indeed the case, and the results as shown in Table 11.4 are completely in accordance with the CAPM predictions.

The values of the t-statistic relating to beta are mainly negative for beta linked to the internal index, while there are not nearly as many negative values of $\tilde{\gamma}_1$ in the column relating to the RDM-index. Again, none of the values mentioned is significantly positive or negative.

TABLE 11.4: Investors perceive symmetrical return-distributions

CALCULATED VALUE	YEAR	REGRESSION SIX: Beta ($\tilde{\gamma}_1$) RDM	REGRESSION SIX: Beta ($\tilde{\gamma}_1$) INT	REGRESSION SIX: Skewness ($\tilde{\gamma}_s$) RDM	REGRESSION SIX: Skewness ($\tilde{\gamma}_s$) INT	REGRESSION SIX: R^2 RDM	REGRESSION SIX: R^2 INT	REGRESSION SEVEN: Skewness ($\tilde{\gamma}_s$) RDM	REGRESSION SEVEN: Skewness ($\tilde{\gamma}_s$) INT	REGRESSION SEVEN: R^2 RDM	REGRESSION SEVEN: R^2 INT
t-TEST STATISTIC	1977	0.910	1.052	0.468	0.908	.141	.161	0.400	0.671	.064	.063
	1978	-0.861	-0.287	0.120	-0.547	.200	.255	-0.291	-1.012	.058	.060
	1979	-0.212	-0.256	-0.777	-1.223	.219	.379	-0.861	-1.296	.127	.123
	1980	0.582	-0.514	-0.809	-0.914	.347	.254	0.195	-0.727	.185	.147
	1977-1978	0.018	-0.471	0.389	0.321	.170	.208	0.035	-0.097	.061	.062
	1979-1980	0.404	-0.482	-0.993	-1.557	.277	.331	-0.409	-1.465	.153	.134
	1977-1980	0.345	-0.074	-0.866	-0.844	.222	.267	-0.318	-1.150	.105	.097
AVERAGE REGRESSION COEFFICIENT	1977	0.012	0.013	0.006	0.012			0.005	0.008		
	1978	-0.012	-0.004	0.002	-0.006			-0.004	-0.009		
	1979	-0.003	-0.005	-0.169	-0.016			-0.014	-0.015		
	1980	0.017	-0.007	-0.080	-0.012			0.005	-0.010		
	1977-1978	0.0002	-0.005	0.004	0.003			0.0001	-0.0007		
	1979-1980	0.006	-0.006	-0.046	-0.014			-0.006	-0.013		
	1977-1980	0.003	-0.0006	-0.020	-0.005			-0.003	-0.006		
VARIANCE OF REGRESSION COEFFICIENTS	1977	0.002	0.002	0.002	0.002			0.002	0.002		
	1978	0.002	0.003	0.004	0.002			0.003	0.001		
	1979	0.003	0.006	0.006	0.002			0.004	0.002		
	1980	0.009	0.002	0.109	0.002			0.006	0.002		
	1977-1978	0.002	0.002	0.003	0.002			0.002	0.001		
	1979-1980	0.005	0.004	0.515	0.002			0.005	0.002		
	1977-1980	0.004	0.003	0.026	0.002			0.003	0.002		

No t-values are statistically significant at either the 10% or the 5% level

The outcome of the tests on Regressions Six and Seven, are thus perfectly in line with the expected result. The skewness of the return-distribution does not have any impact with the returns, when cross-sectionally regressed against the returns. The assumptions underlying the CAPM that the return-distribution are perceived as being symmetrical thus appears to be valid. The relative skewness of the return distributions does not seem to affect the price of securities.

The conclusion is thus that the null hypothesis is accepted, investors view the return-distributions as being symmetrical, and thus they ignore skewness when making investment decisions.

11.6 HYPOTHESIS E: LEVY'S GENERALISED CAPM IS NOT VALID

Levy (1978) found that, when relaxing the assumption that investors hold a portion of every asset, and letting them hold only a few assets, the dominant measure of risk is no longer the beta-coefficient, but the variance. This test attempts to discover whether Levy's Generalised CAPM is not valid, thus leaving the risk-exposure of an investor as being that amount show by beta, or whether the model is valid, thus yielding a dominant risk measure of the variance of the total portfolio. Regressions Eight and Nine were tested in this test, the expected outcome of which is, given the assumptions of the CAPM, a regression coefficient t-statistic value of zero where the CAPM does exist, and a value significantly different to zero if Levy's Generalised CAPM holds.

The results of the tests on this phenomenon are shown in Table 11.5 overleaf.

As can be seen, the expected result of zero significance for the variance term is present in all cases except the RDM-calculated $\tilde{\gamma}_4$ relating to 1977. This statistic is significantly positive at the 5% level. It is however an isolated case, as all the other values are not significant, and it does not appear in either the two-period result or in the four year full-period results.

FM did not test this finding, as Levy had not published his article when they performed their test. HMV's findings were that the variance of returns is not a significant feature of the Belgian market. They too

TABLE 11.5: Levy's generalised CAPM is not valid

CALCULATED VALUE	YEAR	REGRESSION EIGHT: Beta ($\tilde{\gamma}_1$) RDM	Beta ($\tilde{\gamma}_1$) INT	Variance ($\tilde{\gamma}_4$) RDM	Variance ($\tilde{\gamma}_4$) INT	R^2 RDM	R^2 INT	REGRESSION NINE: Variance ($\tilde{\gamma}_4$) RDM	Variance ($\tilde{\gamma}_4$) INT	R^2 RDM	R^2 INT
t-TEST STATISTIC	1977	0.442	-0.183	1.375	0.957	.141	.217	2.456**	1.439	.068	.080
	1978	-0.806	-0.716	-1.191	-1.535	.216	.272	-1.095	-1.335	.076	.092
	1979	0.466	0.398	-0.899	-0.974	.126	.300	-0.782	-0.809	.059	.068
	1980	0.912	0.048	-1.319	-0.704	.234	.316	-0.888	-0.749	.093	.168
	1977-1978	-0.166	-0.550	0.328	0.244	.178	.244	0.788	0.234	.072	.086
	1979-1980	1.027	0.364	-1.602	-1.144	.176	.307	-1.197	-1.096	.075	.114
	1977-1980	0.771	-0.135	-0.709	-0.386	.177	.275	-0.072	-0.535	.073	.099
AVERAGE REGRESSION COEFFICIENT	1977	0.006	-0.003	2.095	2.105			2.795	1.688		
	1978	-0.009	-0.009	-1.430	-1.504			-1.346	-1.311		
	1979	0.004	0.008	-0.678	-0.803			-0.600	-0.634		
	1980	0.029	0.001	-2.138	-1.074			-1.200	-1.037		
	1977-1978	-0.002	-0.006	0.333	0.301			0.725	0.188		
	1979-1980	0.015	0.005	-1.347	-0.927			-0.875	-0.819		
	1977-1980	0.007	-0.001	-0.474	-0.289			-0.043	-0.295		
VARIANCE OF REGRESSION COEFFICIENTS	1977	0.003	0.005	30.187	62.906			16.825	17.882		
	1978	0.002	0.002	18.726	12.478			19.643	12.529		
	1979	0.001	0.005	7.384	8.828			7.659	7.996		
	1980	0.011	0.003	28.878	25.615			20.090	21.095		
	1977-1978	0.002	0.003	26.709	39.572			21.961	16.935		
	1979-1980	0.005	0.004	16.961	15.762			12.824	13.386		
	1977-1980	0.004	0.003	22.306	27.971			17.876	15.182		

** Statistically significant at the 5% level

found only one period in which the $\tilde{\gamma}_4$ term was significant. The South African market would thus seem to be the same as the Belgian market in this respect. Levy's analysis was carried out on the NYSE, and it is thus indicative of the behaviour of that market.

The result of the test is thus that Hypothesis E is accepted, the traditional CAPM rather than Levy's Generalised CAPM appears to be valid.

11.7 COMPARISON OF THE MONTHLY WITH THE WEEKLY RESULTS

11.7.1 Introduction

The results of the tests carried out on the weekly returns and those carried out on the monthly returns are summarised in Table 11.5 overleaf. The meaning of the results is then discussed, and possible reasons for the findings analysed. Finally, an overall conclusion for the tests is drawn.

Table 11.5: Comparison of monthly results with weekly results

HYPOTHESIS	PREDICTED CAPM RESULT	ACTUAL EMPIRICAL RESULT		
			MONTHLY	
		WEEKLY	INT	RDM
A	The Share Market Line (SML) is linear	✓	✓	✓
B	Systematic risk is the only relevant risk measure	✓	✓	✓
C	The risk-return relationship is positive	X	✓	✓
D	Investors regard the return-distribution as symmetrical	✓	✓	✓
E	Systematic risk is the relevant risk measure, rather than total risk.	✓	✓	✓

✓ Empirical result conforms to CAPM
X Empirical result does not conform to CAPM

It is immediately obvious that the results carried out on the monthly returns all conform to the predicted CAPM behaviour. This is in contrast to the earlier finding in which it was found that the positive sloping risk-return relationship was not present. In the refined methodology all the null hypotheses (H_0) have thus been accepted.

11.7.2 Interpretation of the results

The results of the refined methodology are very encouraging, as none of the CAPM predictions are violated. The use of the monthly returns instead of weekly returns led to more accurate results, as shown by the change from a (non-significant) negative slope in the test of hypothesis C, to a positive slope using monthly results (again this was not significant).

However, the use of monthly returns did lead to some doubt as to the slope of the risk return equation, as the beta calculated using the RDM-index was significantly negative. A possible reason for this is the fact, reported in Chapter 10, and subsequently shown again in the results in this chapter, that the Int-index is a better measure of the actual returns than the RDM-index.

The one weakness of the study is that the expected significantly positive relationship between risk and return has been shown not to exist. Possible causes of this are:

- the use of a sample of securities instead of the whole market
- the use of the original portfolios when using monthly returns instead of recalculating beta's and reforming portfolios
- the existence of thinly-traded shares in the sample.

These are discussed in turn.

Use of a sample of securities

The returns of only 107 firms were used in this study, instead of the total number of firms quoted on the JSE. This may have caused a loss of information, however the shares chosen are representative of the industrial sector of the market. Carter (1983) has found that South African investors should not hold gold shares in their efficient portfolios, thus the exclusion of the mining sector from this study is not a significant omission.

The use of a sample of securities rather than the whole market is, it is concluded, not likely to be a cause of the results.

Use of portfolios calculated using weekly returns

Recall from Chapter 10, section 10.4.1 that a decision was taken not to recalculate the initial betas, and thus to keep the portfolios as originally formed. This could be a cause of the results not yielding a statistically significant positive relationship.

This is however unlikely to lead to significant errors when changing from weekly returns to monthly returns, due to the fact that log-price relatives (log returns) were used. As shown in Chapter 8, these are additive, thus the effect would not have been much different had the monthly return first been calculated and then used to obtain the initial beta value.

This procedure is thus also unlikely to have caused a serious problem with the results.

Thinly traded shares

Dimson (1979) has shown that, when shares are infrequently traded, seriously biased estimates of beta may result. Dimson suggests that the major source of bias is the tendency for shares recorded at the end of a period to have been the result of a transaction which occurred early in the period, or even, if the share is very seldom traded, in a previous period. This introduces positive serial correlation into the returns, and the estimated variance of returns on the index is biased downwards.

Another result of infrequent trading is the fact that the covariance of these shares with the market is substantially underestimated.

The mean beta for all securities is, by definition, equal to one, thus if the beta estimates of thinly traded shares are downwards biased, those of highly traded shares must be biased upwards. This introduces a large spread between these beta values, hence enhancing the possibly incorrect conclusions drawn from empirical research on a market with thinly traded shares quoted on it.

The existence of thinly traded shares could very likely give rise to the intervalling effect described in Chapter 8, however it is not the only cause of it. The intervalling effect is a phenomenon which arises whenever there is any error in the underlying estimates used in the market model.

Dimson developed a technique whereby the thinly-traded share effect could be overcome. He shows that this technique removes most of the bias in conventional beta estimates.

The JSE is a market on which thinly-traded shares are very likely to be found. A brief analysis of the volume of shares traded on the JSE in the four years used in this study showed that some were very thinly traded in this period.

Consider the effect of understating the betas of certain securities and overstating those of others. As noted above, the spread of beta estimates is widened by this phenomenon. In general, the beta estimates of the thinly traded securities would plot toward the left side of the risk-return space, with the converse occurring for the highly traded securities. The returns will not be effected, thus they will be unchanged. The possible situation is shown below:

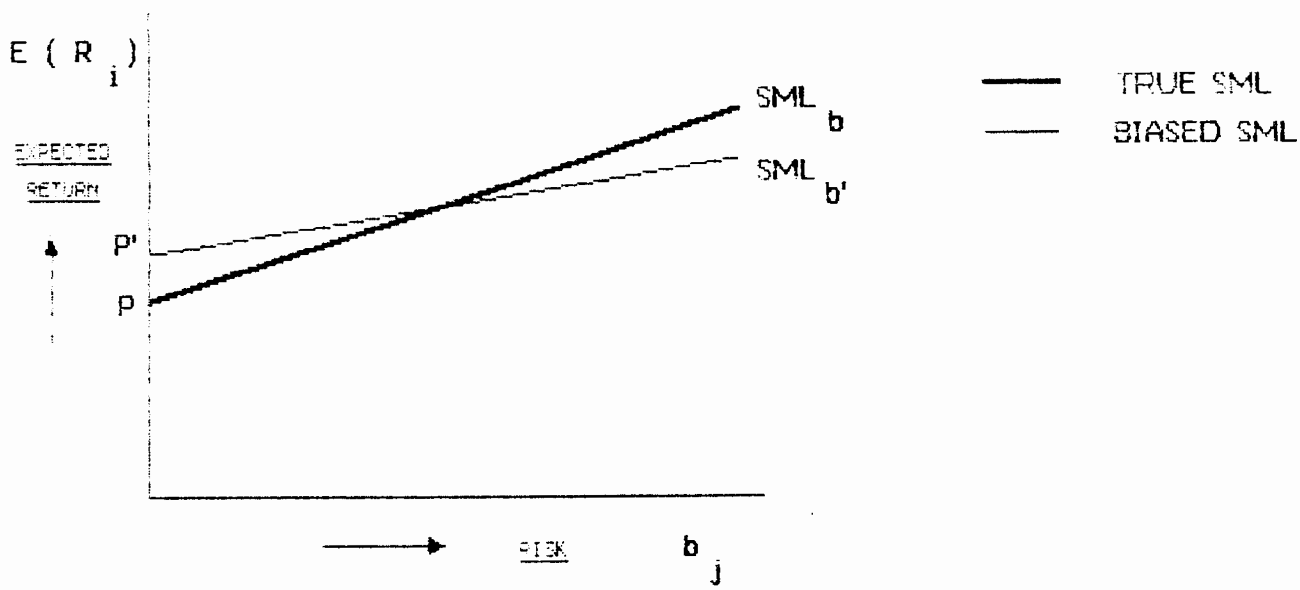


Figure 11.1: A possible empirical SML for thinly-traded securities

As shown in Figure 11.1, the SML (P'b') represented by the incorrect biased beta estimates is considerably flatter than the true SML (shown by Pb). Had the beta estimates been correctly calculated, the relationship would have been correctly described.

It is thus suggested that the results of this study may be influenced by the thin-trading effect highlighted by Dimson. Had this effect been removed, a significantly positive relationship between risk and return may have been noted. The empirical test of this supposition would however have to constitute a separate work, as the entire study would have to be repeated.

11.7.3 Conclusion

The results of the two different methodologies are not dissimilar. In each case, the majority of the expected CAPM results have been shown to be in existence on the JSE.

The replication of the study using monthly log-relative returns represented a definite improvement over the results obtained using the weekly returns, as the weekly returns had shown a negative relationship between risk and return. The use of monthly returns resulted in this becoming a positive relationship.

The two minor problems (viz. signficance of the beta-squared term as well as the beta term in Hypothesis A; and the non-significance of the positive relationship between risk and expected return) were analysed as being due to three possible factors. First, the use of a sample of securities instead of the whole market; second, the use of portfolios of securities aggregated on beta estimates calculated on weekly data; and third, the existence of thinly traded shares.

The first two factors are not considered to be a plausible cause of the results obtained, however the existence of thinly-traded shares could be a significant factor. This has been investigated by Dimson (1979), who suggests a technique for correcting the bias in estimates of thinly-traded shares. This would constitute a meaningful follow-on to the present study.

The final section of this chapter will conclude on the overall results of the tests.

11.8 CONCLUSION ON THE RESULTS

The results have been, in almost all cases, indicative of the existence of the CAPM on the JSE. Five hypotheses were set, of which four were accepted using statistics calculated on weekly returns. The use of monthly returns increased the validity of the study and resulted in the acceptance of all five hypotheses with regard to the pricing of risky assets on the JSE.

Certain methodological weaknesses have been examined, and it has been concluded that the major additional work that could be carried out would be to replicate the current study using Dimson's technique whereby the bias in estimates caused by a lack of trading of some shares would be removed. It is suggested that the non-significance of the expected positive relationship between risk and return may be caused by the existence of these thinly traded securities.

Nonetheless, the general conclusion that can be drawn is that the CAPM appears to be a valid model for the pricing of risky assets on the JSE. This is thus in agreement with the findings of Fama and MacBeth (1973) on the NYSE, and with the study on the Belgian equities market carried out by Hawawini, Michel and Viallet (1983).

In certain respects the findings are more akin to those of HMV than to the Fama and MacBeth results, particularly in so far as the relationship between risk and return has not been shown to be significantly positive or negative. HMV accepted the CAPM on a (generally) negative, but non-significant, result, thus the decision reached here is, it is considered, valid.

The results of the tests certainly suggest further work. One possible avenue of research has already been suggested, more areas are suggested in the final chapter of the thesis.

CHAPTER 12

A CRITIQUE OF TESTS ON THE CAPM

12.1 INTRODUCTION

The ex-post form of the Security Market Line, or CAPM, provides a very useful technique whereby the performance of securities can be tested. This was one of the major outcomes of the formalised CAPM, both in the Sharpe-Lintner form and in Black's zero-beta model.(1)

The technique whereby this performance is usually evaluated is as follows. Derive the risk coefficient of the particular security, then, using the SML as obtained using prior-period data, obtain the expected return on the security given its beta-coefficient. The abnormal performance of the security is then any return in excess of the predicted return or any shortfall between the predicted return and the actual return.

Roll (1977) and (1978) has presented a convincing argument wherein he takes exception to this method of interpreting abnormal performance measures, and indeed, to most tests of the CAPM.

This chapter will show how Roll reaches this conclusion, and will expand on his general criticism given in the above paragraph.

12.2 ROLL'S CRITIQUE ON THE TESTABILITY OF THE CAPM

Recall from section 3.5 that it is possible to construct a CAPM for the case in which no risk-free asset exists. This is known as Black's zero-beta portfolio. The SML in this case becomes a combination of the market portfolio and a zero-beta portfolio which is uncorrelated with the market index. If this is the case, the expected return on any asset can be written as:

$$E(\tilde{R}_i) = E(\tilde{R}_z) + \beta_i (E(\tilde{R}_m) - E(\tilde{R}_z)) \qquad (12.1)$$

The graph of the Capital Market Line relating to this zero-beta portfolio is shown in Figure 12.1 below.

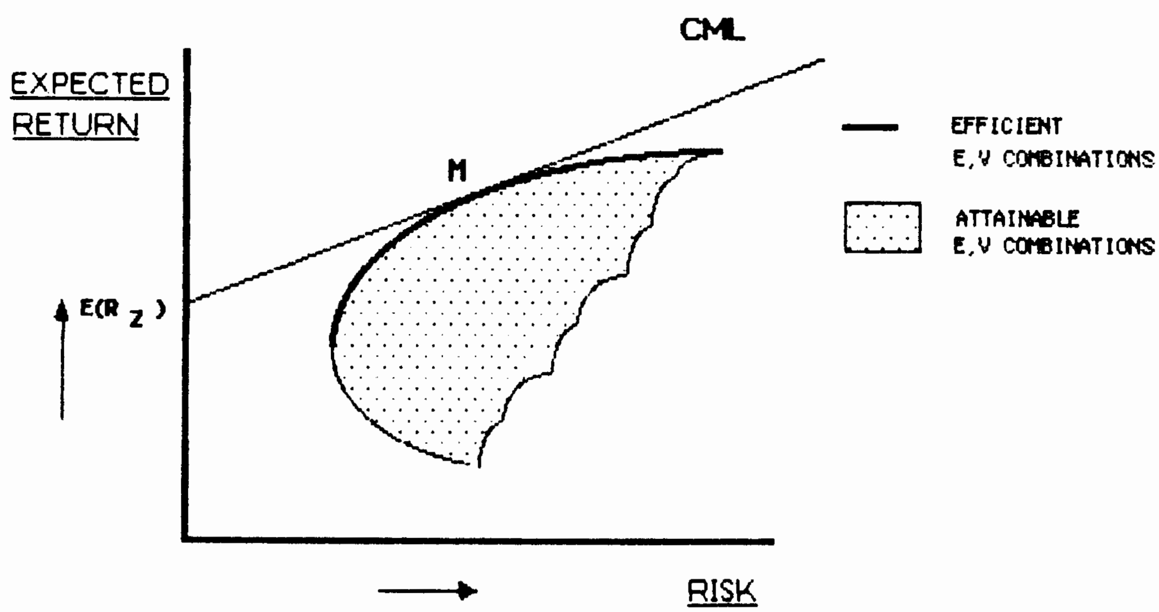


Figure 12.1: Black's zero-beta portfolio

Equation (12.1) above is the SML derived from the CML shown in Figure 12.1. The expected return on any risky asset i is shown to be the expected return on the zero-beta portfolio $E(R_z)$ plus β_i times the excess expected return on the market over the expected zero-beta return. Recall further that the beta-coefficient is a function of the covariance of asset i's returns with the returns on the market portfolio, and that beta also depends on the variance of the market returns.

The market model was shown in section 3.4.6 to be a useful method whereby the ex-post beta of securities can be calculated. In calculating the ex-post beta, it is necessary to use some market index as a surrogate for the return on the market as a whole.

It follows from the above discussion that an index relating to the market portfolio can be chosen in order to satisfy and solve equation (12.1).

Roll suggests that there is in fact nothing unique about this market portfolio or the index relating thereto, as long as the index itself is efficient. He shows that it is always possible to choose any <u>efficient</u> portfolio as an index, and then find the uncorrelated minimum variance portfolio relating to this index. This is shown in Figure 12.2 overleaf.

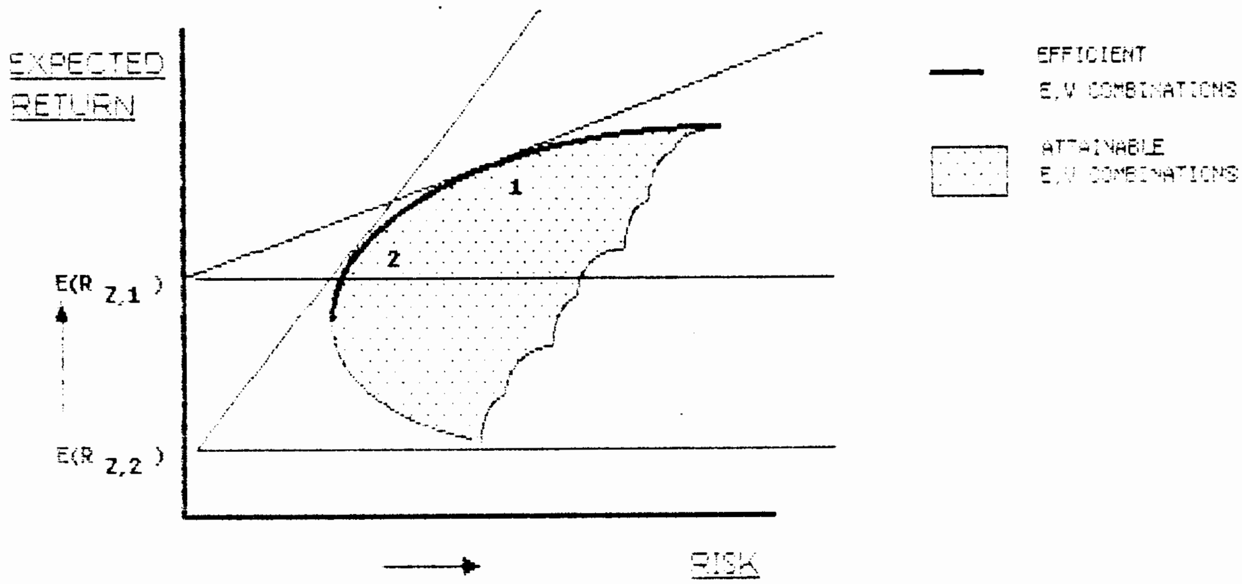


Figure 12.2: Roll's efficient portfolio

The equation of this line with any efficient portfolio chosen can be derived and written as follows:

$$E(\tilde{R}_i) = E(\tilde{R}_{z,I}) + \beta_{i,I}(\tilde{E}(R_I) - E(\tilde{R}_{z,I})) \qquad (12.2)$$

In this case the market portfolio, (R_m) has been replaced by an efficient index (R_I) and the beta is measured relative to the efficient index (I) rather than to the market. It follows from this that the expected return on any asset can be expressed as a linear function of its beta, measured relative to any efficient index. In addition, one need only know the composition of an efficient index in order to write equation (12.2) above.

Further to the above, if the index chosen turns out to be ex-post efficient, then every security will fall exactly on the SML and there will be no abnormal returns. If there are systematic abnormal returns, it means that the index chosen is simply not ex-post efficient.

12.3 ROLL'S CONCLUSIONS

The main conclusions which Roll reaches are as follows:(2)

a) The only testable hypothesis of the CAPM (Black's version) is that "the market portfolio is mean-variance efficient".

b) All the other implications of the theory, in particular the linearity of the relationship between risk and return, follow from point a) above, and are not independently testable. Linearity of the relationship follows directly from the 'efficiency' of the market portfolio.

c) The CAPM theory is not testable unless the exact composition of the true (complete) market portfolio is known, and is used in the tests. What this actually means is that <u>all</u> individual assets must be included in the test.

d) If the performance of individual securities is measured relative to an index which is ex-post <u>efficient</u>, then, it follows from the mathematics underlying the derivation of the efficient set, that no security will have abnormal performance when measured as a departure from the SML. If however, the performance of the security is measured relative to an index which is ex-post inefficient, then any ranking of portfolio performance is possible, depending on the choice of index.

12.4 IMPLICATIONS OF ROLL'S CRITIQUE

The broad result of Roll's analysis is an implication that, even if portfolios are efficient and the CAPM is valid, the cross sectional regression technique (as used in this study) cannot be used as a means of measuring the ex-post performance of portfolio selection techniques.

Roll does not however imply that the CAPM is invalid. The fact that a test shows the CAPM to be valid or not simply means that the market index chosen was either ex-post efficient, or not.

In testing the CAPM, Roll says that a joint hypothesis, namely the efficiency of the market portfolio and the validity of the CAPM, is being tested, and thus the results are almost impossible to interpret. The only way to test the CAPM directly is to see whether or not the true market portfolio is ex-post efficient. This is however virtually impossible as the true market portfolio contains all assets, marketable and non-marketable, and this, given present measurement techniques, is impossible to measure and observe.

Mayers and Rice (1979), have argued that Roll was overly critical of tests of the CAPM in the Roll (1977) and Roll (1978) papers. They have said that Roll focuses too closely on "a truly ex-ante efficient index". (Mayers and Rice (1979:3)) Roll has however taken a strong view in "A Reply to Mayers and Rice (1979)" (Roll: 1979) wherein he contends that Mayers and Rice have failed to recognise the unusual testing implications of the CAPM, and that they ignore alternative pricing models.

In conclusion, it would appear that Roll's critique is valid, and that interpretations regarding the CAPM should be drawn with care. The next obvious question then concerns the validity of the present study.

12.5 VALIDITY OF THE PRESENT STUDY

It is submitted that the present study does have validity, in that it gives valuable insights into the actual performance of securities on the JSE in the period 1977 to 1980. This is particularly important in view of the relative scarcity of tests on the JSE. In addition, as Mayers and Rice say

> "there is some information in these tests, even with imperfect proxies testing joint hypotheses. More importantly, this information is the <u>best</u> available. It does no good to ignore this information without providing some better information in its place."
>
> Mayers and Rice (1979:23)

In a direct reply to Roll's comment that the market portfolio must contain all risky assets, Ibbotson and Siegel (1983) constructed a world market wealth portfolio which probably represents the true universe of risky assets as adequately as possible. Stambaugh (1982) found that even when common stocks represented only 10% of a 'market' portfolio, inferences about the CAPM were almost identical to those obtained from a portfolio of stocks only. This shows that Roll's critique, although severe and having some merit, does not instantly nullify any tests of the CAPM, nor does it nullify CAPM theory either.

In view of this, the results of the present study are of real importance, showing, as they do, the behaviour of the JSE in the periods under review.

12.6 SUMMARY

This chapter has presented the critique which Roll has made about the testable implications of the CAPM. The rationale underlying his critique has been explored, as have his detailed conclusions. The present study has also been defended in view of Roll's comments.

The next chapter concludes the thesis.

CHAPTER 12 - FOOTNOTES

(1) Recall from section 3.5 that the major difference that Black (1972) brought to the model was his removal of the assumption of the Sharpe-Lintner CAPM that there exists a riskless asset and hence a riskless rate of return.

(2) These are listed in part 1 of his paper "A Critique of the Asset Pricing Theory's Tests" (Roll: 1977).

CHAPTER 13

CONCLUSION

13.1 RESUME OF THE THESIS

This thesis has presented a background to the historical development of the CAPM. The development was presented from first principles. Utility theory was first introduced, and this was followed by a discussion on the concept of an efficient portfolio.

This led naturally into the development of the CAPM, and thence to a discussion on the Efficient Markets Hypothesis and the CAPM. Empirical evidence relating to the Capital Markets in South Africa was presented, as was a brief review of the major tests of the CAPM.

The research model and actual research methodology were then introduced. The CAPM was tested using three different versions of the data: weekly returns, monthly returns using the RDM-index, and monthly returns using an internally generated index. The results of these tests were analysed and compared. The conclusions are discussed below. Finally, Roll's criticism of tests of the CAPM was discussed. In addition, a brief test of the appropriateness of using the RDM-index was performed in Chapter 10.

13.2 CONCLUSIONS OF THE STUDY

The CAPM in South Africa appears to be a valid description of the market. Although the results are not absolutely conclusive, they certainly indicate that the model appears to be valid.

Three different analyses were performed. The results of the first, carried out on weekly returns, indicated a probable acceptance of the CAPM, even though the CAPM line was downwards sloping. This, although contrary to the theory, has been noted by prior researchers, who did not conclude that the CAPM was an invalid model. In view of these findings, further research was carried out, using monthly returns and the RDM-index together with an internally calculated index.

The second results proved virtually conclusively that the CAPM describes the risk-return relationship of assets quoted on the JSE. The only inconclusive proof relates to the significance of the slope of the empirically found CAPM line. The probable reason for the insignificance of the upwards slope was ascribed to the impact of thinly traded shares on the study. It was suggested that a worthwhile exercise would be a replication of the study, using the Dimson (1979) technique for calculating adjusted risk measures.

Overall, the study has shown the validity of the CAPM. It is stressed however, that these results are dependent on the assumption of the existence of the (efficient) market portfolio underlying the model. In addition, the results were carried out on data covering the years 1974 to 1980, and thus apply to those years. The results are thus specific to this data. Nevertheless, the results are indicative of the probable current relationship between risk and return.

13.3 IMPLICATIONS OF THE RESULTS

The implications of these results are that the CAPM can be used by portfolio managers and other investors in evaluating various securities. In addition, the results indicate the validity of the CAPM in the cost of capital calculations, as briefly described in section 3.6.

A suggestion for further research, namely a replication of the study using Dimson's technique for adjusting risk measures of thinly traded shares, was noted in Chapter 11.

In view of the non-significant positive slope to the SML, it would be very interesting to carry out a similar study using as many shares as possible, over a very long time period. This would give an indication of the long-run relationship between risk and return, and would very likely yield a significant upwards-sloping share market line.

The above tests are valid further studies, however the most important further research, it is considered, relates to a test of a model known as Arbitrage Pricing Theory (APT), which is a valid testable alternative to the CAPM. It is beyond the scope of this thesis to examine the APT. It is a new theory, and thus empirical research has yet to prove or disprove its validity. Excellent reviews of the APT are given by both Reinganum (1981a) and Copeland and Weston (1983). Ross (1976) developed the theory, whereas the major empirical tests to date have been performed by Gehr (1975), Roll and Ross (1980), Chen (1981) and Reinganum (1981b). The APT has been shown to be a better model than the CAPM in some instances, thus this would constitute the most relevant next step from here.

13.4 CONCLUSION

The aims of the study as enumerated in Chapter 1 have, it is considered, been achieved. The research has indicated the existence of the CAPM on the JSE, which evidence will help in evaluating the market. The secondary aims, namely presentation of the historical development of the CAPM and presentation of the review of empirical studies in South Africa and overseas have been met in the theory sections of the thesis.

It is far too early to judge whether this work has achieved its final aim of stimulating further research. That is a judgement which can only be delivered with the passage of time.

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APPENDIX A

COMPANIES INCLUDED IN THE STUDY

1 Abercom Investments Limited
2 Aberdare Cables Africa Limited
3 Adcock Ingram Limited
4 AECI Limited
5 African Cables Limited
6 African Oxygen Limited
7 Anglo Alpha Limited
8 Anglo American Industrial Corporation Limited
9 Anglo Transvaal Industries Limited
10 Asea Electric South Africa Limited
11 Associated Engineering S.A. Limited
12 Associated Furniture Companies Limited
13 Barlow Rand Limited
14 Beares Limited
15 Blue Circle Limited
16 Bonmore Investments Limited
17 Bonuskor Beperk
18 Boumat Limited
19 Cadbury Schweppes Limited
20 Calan Limited
21 Carlton Paper Corporation Limited
22 Chemical Holdings Limited
23 Claude Neon Lights (S.A.) Limited
24 C.N.A. Investments Limited
25 Cullinan Holdings Limited
26 Currie Finance Corporation Limited
27 Currie Motors (1946) Limited
28 Die Afrikaanse Pers (1962) Beperk
29 Dorbyl Limited
30 Dunlop South Africa Limited
31 Dunswart Iron & Steel Works Limited
32 Edgars Consolidated Investments Limited
33 Edgars Stores Limited
34 Ellerine Holdings Limited
35 Everite Limited
36 Federale Voedsel Beperk
37 Federale Volksbeleggings Beperk

38 Foschini Limited
39 Frasers Limited
40 Gallo (Africa) Limited
41 General Tire & Rubber Co. (South Africa) Limited
42 Greatermans Stores Limited
43 Gresham Industries Limited
44 Grinaker Holdings Limited
45 Gubb & Inggs Limited
46 Highveld Steel and Vanadium Limited
47 Huletts Corporation Limited
48 Industrial & Commercial Holdings Group Limited
49 Irvin & Johnson Limited
50 Kaap Kunene Beleggings Beperk
51 Kanhym Limited
52 Kohler Brothers Limited
53 Lamberts Bay Holdings Limited
54 Lefic Limited
55 L H Marthinusen Limited
56 LTA Limited
57 Malbak Limited
58 Marine Products Limited
59 McCarthy Group Limited
60 Metal Box South Africa Limited
61 Metkor Limited
62 Metcash Limited
63 Mitchell Cotts Limited
64 Murray & Roberts Holdings Limited
65 Nampak Limited
66 National Trading Co. Limited
67 O.K. Bazaars (1929) Limited
68 Otis Elevator Co. Limited
69 Pick 'n Pay Stores Limited
70 Pep Stores Limited
71 Placor Holdings Limited
72 Plate Glass & Shatterprufe Industries Limited
73 Premier Milling Limited
74 Protea Holdings Limited
75 Rembrandt Beherende Beleggings Beperk

76 Rembrandt Group Limited
77 Rennies Consolidated Holdings Limited
78 Reunert & Lenz Limited
79 Rex Trueform Clothing Co. Limited
80 Romatex Limited
81 Russel Holdings Limited
82 S.A. Druggists Limited
83 Sappi Limited
84 Scottish Cables (South Africa) Limited
85 Seardel Investment Corporation Limited
86 Sentrachem Limited
87 South African Breweries Limited
88 South African Marine Corporation Limited
89 South West Africa Fishing Industries Limited
90 Steelmetals Limited
91 Stewarts & Lloyds of South Africa Limited
92 Television & Electrical Holdings Limited
93 The Argus Printing & Publishing Co. Limited
94 The Sterns Diamond Organisation Limited
95 The Tongaat-Hulett Group Limited
96 The Union Steel Corporation of South Africa Limited
97 Tiger Oats & National Milling Co. Limited
98 Toyota (South Africa) Limited
99 Trek Beleggings Beperk
100 Triomf Fertilizer Investments Limited
101 Truworths Limited
102 Unisec Group Limited
103 W & A Investment Corporation Limited
104 Wesco Investments Limited
105 Willem Barendz Limited
106 Williams Hunt South Africa Limited
107 Woolworths Limited.

APPENDIX B

INITIAL RISK ESTIMATES AND ALLOCATION TO PORTFOLIOS

	Company Name	Initial Risk Estimate	Portfolio
1	The Sterns Diamond Organisation Limited	4.69	1
2	Russell Holdings Limited	1.91	1
3	Beares Limited	1.88	1
4	Ellerine Holdings Limited	1.77	1
5	Imperial Cold Storage Limited	1.63	1
6	Mitchell-Cotts Limited	1.57	1
7	Abercom Investments Limited	1.53	1
8	Rennies Consolidated Holdings Limited	1.51	1
9	Placor Holdings Limited	1.48	1
10	Currie Finance Corporation Limited	1.37	1
11	Rembrandt Beherende Beleggings Beperk	1.36	1
12	Tiger Oats and National Milling Co. Limited	1.33	1
13	Boumat Limited	1.33	1
14	Rembrandt Group Limited	1.33	2
15	Woolworths Limited	1.32	2
16	Television and Electrical Holdings Limited	1.31	2
17	Anglo Transvaal Industries Limited	1.30	2
18	Kaap Kunene Beleggings Beperk	1.28	2
19	Sappi Limited	1.24	2
20	Murray and Roberts Holdings Limited	1.24	2
21	Associated Furniture Companies Limited	1.23	2
22	South African Breweries Limited	1.23	2
23	McCarthy Group Limited	1.23	2
24	Calan Limited	1.22	3
25	Edgars Consolidated Investments Limited	1.21	3
26	L H Marthinusen Limited	1.21	3
27	Premier Milling Limited	1.18	3
28	Nampak Limited	1.18	3
29	Romatex Limited	1.18	3
30	Grinaker Holdings Limited	1.16	3
31	Bonmore Investments Limited	1.13	3
32	W & A Investment Corporation Limited	1.13	3
33	Sentrachem Limited	1.12	3

	Company Name	Initial Risk Estimate	Portfolio
34	Lamberts Bay Holdings Limited	1.10	4
35	AECI Limited	1.09	4
36	LTA Limited	1.08	4
37	Wesco Investments Limited	1.07	4
38	Huletts Corporation Limited	1.06	4
39	Blue Circle Limited	1.05	4
40	Federale Volksbeleggings Beperk	1.05	4
41	Stewarts & Lloyds of South Africa Limited	1.04	4
42	Unisec Group Limited	1.02	4
43	Toyota (South Africa) Limited	1.01	4
44	O.K. Bazaars (1929) Limited	1.01	5
45	National Trading Co. Limited	1.00	5
46	South African Marine Corporation Limited	1.00	5
47	Trek Beleggings Beperk	0.99	5
48	Bonuskor Beperk	0.98	5
49	Protea Holdings Limited	0.98	5
50	Pep Stores Limited	0.97	5
51	Williams Hunt South Africa Limited	0.97	5
52	S.A. Druggists Limited	0.97	5
53	Barlow Rand Limited	0.95	5
54	Plate Glass and Shatterprufe Industries Limited	0.91	6
55	Pick 'n Pay Stores Limited	0.89	6
56	Dunlop South Africa Limited	0.89	6
57	Metal Box South Africa Limited	0.88	6
58	Marine Products Limited	0.87	6
59	Greatermans Stores Limited	0.86	6
60	Carlton Paper Corporation Limited	0.84	6
61	Lefic Limited	0.83	6
62	Gresham Industries Limited	0.82	6
63	Steelmetals Limited	0.81	6
64	Frasers Limited	0.81	7
65	Willem Barendz Limited	0.79	7
66	Anglo American Industrial Corporation Limited	0.78	7
67	The Tongaat-Hulett Group Limited	0.77	7
68	African Oxygen Limited	0.77	7
69	Highveld Steel and Vanadium Limited	0.75	7
70	Dorbyl Limited	0.74	7
71	Asea Electric South Africa Limited	0.73	7

	Company Name	Initial Risk Estimate	Portfolio
72	Metkor Limited	0.71	7
73	C.N.A. Investments Limited	0.71	7
74	Seardel Investment Corporation Limited	0.70	8
75	Kanhym Limited	0.69	8
76	Reunert & Lenz Limited	0.69	8
77	Malbak Limited	0.68	8
78	Associated Engineering S.A. Limited	0.68	8
79	General Tire and Rubber Co. (South Africa) Limited	0.65	8
80	Metcash Limited	0.65	8
81	Edgars Stores Limited	0.65	8
82	Die Afrikaanse Pers (1962) Beperk	0.64	8
83	The Union Steel Corporation of South Africa Limited	0.61	8
84	Irvin and Johnson Limited	0.61	9
85	Adcock Ingram Limited	0.57	9
86	Cadbury Schweppes Limited	0.57	9
87	Dunswart Iron and Steel Works Limited	0.51	9
88	Rex Trueform Clothing Co. Limited	0.51	9
89	Kohler Brothers Limited	0.51	9
90	South West African Fishing Industries Limited	0.48	9
91	Currie Motors (1946) Limited	0.48	9
92	Triomf Fertilizer Investments Limited	0.47	9
93	Otis Elevator Co. Limited	0.46	9
94	Claude Neon Lights (S.A.) Limited	0.45	10
95	Anglo Alpha Limited	0.45	10
96	Gallo (Africa) Limited	0.39	10
97	Industrial and Commercial Holdings Group Limited	0.38	10
98	Truworths Limited	0.36	10
99	Foschini Limited	0.34	10
100	Everite Limited	0.34	10
101	The Argus Printing and Publishing Co. Limited	0.33	10
102	Aberdare Cables Africa Limited	0.28	10
103	Scottish Cables (South Africa) Limited	0.22	10
104	African Cables Limited	0.19	10
105	Chemical Holdings Limited	0.17	10
106	Gubb and Inggs Limited	0.13	10
107	Cullinan Holdings Limited	-1.05	10

APPENDIX C

PORTFOLIO PARAMETERS CALCULATED USING WEEKLY RETURNS

Period*	Portfolio	Beta	Beta-squared	Standard-error	Variance	Skewness
1977	1	1.33	1.81	.044	.003	.33
	2	1.11	1.65	.045	.002	.29
	3	1.08	1.29	.041	.002	.23
	4	1.22	1.56	.046	.003	.32
	5	0.84	0.79	.039	.002	.18
	6	0.91	0.88	.041	.002	.30
	7	0.76	0.65	.038	.002	.18
	8	0.73	0.59	.046	.002	.40
	9	0.60	0.43	.043	.002	-.06
	10	0.52	0.31	.045	.003	.29
1978	1	1.06	1.18	.040	.002	.23
	2	1.33	1.86	.048	.003	-.08
	3	1.02	1.15	.040	.002	.12
	4	0.99	1.17	.046	.002	-.01
	5	0.82	0.86	.038	.002	.08
	6	0.88	0.84	.042	.002	-.42
	7	0.82	0.83	.446	.003	-.78
	8	0.48	0.27	.045	.003	.19
	9	0.58	0.44	.045	.003	.05
	10	0.50	0.32	.043	.002	-.02
1979	1	1.05	1.32	.042	.002	.22
	2	1.23	1.55	.043	.002	-.19
	3	1.05	1.15	.041	.002	.12
	4	0.88	0.85	.042	.002	.10
	5	0.87	1.22	.041	.002	.06
	6	0.95	1.02	.046	.002	-.27
	7	0.89	0.91	.047	.003	-.58
	8	0.60	0.48	.045	.002	.40
	9	0.57	0.43	.055	.003	.48
	10	0.59	0.46	.042	.002	.32
1980	1	0.64	0.46	.045	.002	.01
	2	0.59	0.38	.044	.002	.10
	3	0.49	0.24	.041	.002	.31
	4	0.61	0.43	.046	.004	.39
	5	0.53	0.43	.046	.003	.23
	6	0.56	0.37	.047	.002	.17
	7	0.46	0.26	.041	.002	.57
	8	0.30	0.11	.042	.002	.49
	9	0.42	0.19	.051	.003	.45
	10	0.37	0.19	.042	.002	.52

* The parameters are actually calculated using returns for the two years prior to this.

APPENDIX D

PORTFOLIO PARAMETERS CALCULATED USING MONTHLY RETURNS*

Period**	Portfolio	Beta		Beta-squared		Standard-error		Variance	Skewness
		RDM	Int	RDM	Int	RDM	Int		
1977	1	1.28	1.28	1.69	1.73	.075	.075	.011	-.06
	2	1.21	1.22	1.64	1.67	.072	.072	.011	.14
	3	1.02	1.01	1.12	1.08	.069	.070	.008	.16
	4	1.12	1.15	1.32	1.38	.072	.071	.009	.05
	5	0.92	0.91	0.97	0.95	.062	.062	.007	.06
	6	1.04	1.05	1.15	1.17	.068	.067	.008	-.20
	7	0.80	0.82	0.74	0.74	.066	.067	.006	.25
	8	0.96	1.04	1.00	1.16	.087	.084	.011	.20
	9	0.86	0.89	0.94	1.01	.078	.077	.009	-.18
	10	0.63	0.68	0.54	0.60	.076	.074	.008	.27
1978	1	1.17	1.27	1.42	1.71	.074	.073	.008	.16
	2	1.33	1.40	2.05	2.30	.071	.071	.009	.12
	3	1.07	1.19	1.19	1.48	.068	.067	.007	.04
	4	0.98	1.06	1.05	1.22	.070	.069	.007	-.15
	5	0.88	0.88	0.96	0.95	.065	.066	.014	.25
	6	0.87	1.05	0.83	1.23	.078	.075	.008	-.30
	7	0.89	0.90	0.88	0.95	.077	.078	.009	-.27
	8	0.66	0.83	0.47	0.75	.085	.083	.009	-.26
	9	0.80	0.83	0.70	0.77	.096	.095	.011	.05
	10	0.52	0.64	0.42	0.55	.076	.075	.007	-.15
1979	1	1.09	1.05	1.48	1.45	.074	.074	.008	.07
	2	1.11	1.11	1.32	1.36	.065	.065	.006	-.36
	3	1.14	1.19	1.40	1.51	.073	.070	.007	-.14
	4	0.69	0.87	0.64	0.98	.072	.076	.007	-.09
	5	0.94	0.96	1.24	1.25	.064	.065	.006	.08
	6	1.29	1.46	2.04	2.77	.096	.092	.012	-.11
	7	0.97	0.93	1.02	0.96	.082	.077	.010	-.20
	8	0.72	0.85	0.69	0.94	.083	.080	.008	.01
	9	0.78	0.88	0.73	0.98	.105	.104	.013	.38
	10	0.67	0.80	0.78	1.09	.079	.078	.008	.22
1980	1	0.84	1.21	0.79	1.65	.083	.076	.010	.24
	2	0.72	1.13	0.62	1.47	.077	.067	.008	-.02
	3	0.65	0.97	0.44	0.99	.071	.065	.007	.11
	4	0.62	1.01	0.53	1.30	.084	.076	.009	.15
	5	0.68	1.01	0.51	1.10	.067	.060	.006	-.24
	6	0.94	1.33	0.99	2.05	.093	.085	.012	.13
	7	0.66	1.03	0.49	1.14	.081	.074	.008	.29
	8	0.54	0.81	0.40	0.88	.076	.072	.007	.15
	9	0.63	0.89	0.41	0.83	.093	.091	.011	.14
	10	0.49	0.67	0.32	0.66	.080	.078	.078	.36

* Parameters are calculated using both the RDM-index and the internal-index. The variance and skewness parameter calculations are not affected by the index used, thus only one value is given for each.

** The parameters are actually calculated using returns for the two years prior to this.