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Job security, asymmetric information, and wage rigidity

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Abstract

We consider a labour market with risk averse workers, directed search and asymmetric information in which firms can commit to wage contracts but not to retain workers. The model predicts that in downturns i) there is equal treatment of incumbents and new hires, ii) wages are insensitive to the severity of the downturn, iii) this leads to an amplified employment effect, and iv) wages are determined by forecasts of labour market conditions rather than actual values. By contrast in upswings, new-hire wages are more attuned to actual conditions than forecasts, whilst incumbent wages remain relatively rigid. We find that these novel predictions are well supported in German administrative data.

JEL Codes: E32, J41.

Keywords: Labour contracts, business cycle, unemployment, equal treatment, cross-contract restrictions.

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1 Introduction

The behaviour of real wages over the business cycle is critical to understanding the mechanisms that drive employment and output fluctuations. The procyclicality or otherwise of real wages was the subject of considerable debate following the publication of Keynes's (1936) *General Theory*, and it remains a subject of considerable interest.¹ In this paper, we develop a model that has implications for the cyclicality of real wages and for output volatility, but one that emphasises *asymmetric* wage responses to different phases of the business cycle. Our model exhibits equilibria in which partial "equal treatment" is at play. Here, the wages of new hires are equal to those of existing (incumbent) workers in recessions even though it would benefit firms *ex post* to pay new hires less. The implication is that if there is a reason for wages of incumbents to be rigid — here, risk aversion — this will be transmitted to the wages of new hires in recessions.

This paper starts out with a baseline model — building on the approach of Menzio and Moen (2010) — with the above characteristics, but then goes on to develop an extension that allows for asymmetric information about the state of nature (productivity). This extended model is a key innovation of our paper: it generates novel implications for wages, implications that find support in our data. The extended model assumes that firms are better informed than workers about the aggregate state so that contracts cannot be conditioned on aggregate variables. It results in wages that may be fully rigid downward (to be precise: wages may fall, but the rate of fall will be independent of the severity of negative shocks), thus further amplifying the variability of unemployment and vacancies. We show that it is the *interplay* between equal treatment in bad states and asymmetric information that leads to this result; without equal treatment, introducing asymmetric information has no impact on allocations.

A rough intuition for the results is as follows. Firms are assumed to be risk neutral while workers are risk averse. Contracts in which wages for ongoing hires are constant are thus efficient. Let us first consider the case of full information. Firms can commit to future wages for both incumbent workers and new hires. Importantly, in this setup there is a 'no-undercutting' constraint, whereby new hires *cannot be paid less than incumbents* (why is discussed below). Now consider a downturn. If the firm were unconstrained, it would respond to this by cutting

¹See Galí (2013) for a comparison of the cyclicality of real wages in the General Theory and in New Keynesian Models and, e.g., Pissarides (2009) for a discussion of more recent empirical evidence in the context of the "unemployment volatility puzzle" (e.g., Shimer, 2005; Costain and Reiter, 2008).

the new-hire wage. However, the existence of the no-undercutting constraint combined with the desire to stabilise incumbent wages limits the extent to which firms may lower the new-hire wage. Therefore, the desired wage rigidity for incumbent workers in recessions is transmitted to some extent to new hires. In contrast, in booms, when the optimal hiring wage is above the incumbent wage, the constraint does not bind, and there is no corresponding trade-off to dampen new-hire wage increases. In this case, the hiring wage is fully flexible upwards.So, in downturns, equal treatment implies that the new-hire wage will be above what firms would otherwise pay.

Now suppose that there is asymmetric information, so that the state of the market is not known to incumbent workers. If equilibrium wages were to fall with productivity across downturn states, firms would have an incentive to exaggerate the severity of downturns; this would allow them to lower wages (common to both new hires and incumbents). This reduces the wage bill for incumbents and, for new hires, brings the wage closer to what would be the optimal wage in the absence of any constraints. Therefore, the only incentive-compatible contract may involve a (large) range of shocks in downturns for which wages of incumbents and new hires are not only equal but also do not vary with the severity of the shock.

The wages of new hires are allocational in our two-period model. Therefore, both the dampened downward wage changes in recessions in the full information case and the fully downwardly rigid wages in the asymmetric information case affect hiring and increase the variability of both unemployment and vacancies in response to productivity shocks.

The no-undercutting constraint may be imposed, or it may arise endogenously from the desire to insure incumbents against job loss. In Menzio and Moen (2010), it is endogenous and arises to protect incumbents from the risk of being replaced by cheaper outsiders. The costs incurred by firms in compensating workers for this risk may more than offset any ex-post benefits from violating it.² The approach we take in this paper is to impose the constraint but also to analyse those parameter configurations in our model where it is endogenously satisfied in equilibrium. We find that it is satisfied for a wide range of parameter values.

The outline of the paper is as follows. Following a review of the literature,

²This type of argument was also made in Snell and Thomas (2010) in the context of a perfectly competitive labour market. Menzio and Moen's (2010) model, however, concerns a frictional labour market, and we follow their approach.

Section 3 outlines our baseline model. We characterise equilibrium wage contracts assuming that optimal wage contracts always satisfy the no-undercutting constraint. Subsection 3.2 analyses the case where workers and firms have symmetric information about the state. Subsection 3.3 contains the key innovation in our paper. There, we extend the model to allow workers to be asymmetrically informed about the state. Section 4 analyses the conditions under which it is optimal to satisfy the no-undercutting constraint if it is not imposed. In Section 5, we test certain predictions of the model using German administrative data. As we have noted, the predictions for the asymmetric information version of the model are novel and striking. Its key implication is that in downturns, not only is there equal treatment of new hires and incumbents, but also wages should be better related to the forecast severity of the recession rather than its actual severity, a feature similar to what would obtain under staggered Taylor contracts. By contrast in upturns new-hire wages are attuned to actual rather than forecast labour market conditions whilst the wages of incumbents remain comparatively rigid. Empirical analysis of German administrative wage data provides support for these empirical predictions. Section 6 contains concluding comments.

2 Relationship to the Literature

The baseline, symmetric information, version of our model is based on and follows the logic of Menzio and Moen (2010). We expand on the main differences in Section 3 below.

Equal treatment can lead to amplified fluctuations in unemployment in competitive models (e.g., Thomas, 2005; Snell and Thomas, 2010). See Gertler and Trigari (2009) for a somewhat related mechanism within a search-matching model with staggered Nash bargaining rather than optimal contracting, as employed here.

Our emphasis is on situations where it is optimal to avoid replacement, and we consider conditions under which this holds. Our empirical results also suggest that firms do not exploit downswings to undercut incumbents. In the model with asymmetric information there is no undercutting so that workers do not get replaced in recessions and unemployment rises only via a fall in hiring. The findings of Bachmann et al. (2021) for Germany suggest that replacement hiring, as defined by our theory, does not seem to be significant in Germany. If it exists, it would imply that worker churn, due to separations and hires into and out of

nonemployment, increases in recessions. However, Bachmann et al. (2021) show that cyclical variations in worker churn, "which are actually procyclical", are accounted for almost wholly by job-to-job transitions rather than by transitions to and from nonemployment.

We show that asymmetric information amplifies fluctuations beyond any amplification that equal treatment in downturns implies. Menzio (2005) considers an asymmetric information bargaining model in which firms are informed about the current state of productivity and workers are not. It exhibits equal treatment, with amplification of shocks. Kennan (2010), develops a model of procyclical information rents to firms: wages are again relatively rigid, and procyclical rents to the employer mean that employment fluctuations are magnified. Moen and Rosen (2011) analyse a model of moral hazard (unobservable worker effort) and competitive search and show that it introduces a countercyclical element to rents accruing to workers relative to a standard search-and-matching model, enhancing fluctuations in employment over the cycle. However, see also Guerrieri (2007) for a model in which workers have private information about match characteristics but which exhibits little amplification. Bruegemann and Moscarini (2010) derive a bound on the additional amplification of employment that can arise in frictional labour markets when there is acyclicality in the *rents* of workers. Another explanation for higher employment fluctuations can be found in Mercan and Schoefer (2020). They analyse a matching model in which quits lead to vacancies, which in turn lead to further vacancies through replacement hiring. Amplification arises as incumbents hold on to jobs during recessions, shortening the chain of job vacancies and employment opportunities for the unemployed and increasing unemployment. On the other hand, in upswings, the labour market tightens and workers leave their matches, creating additional jobs for the unemployed.

For the empirical results, we attempt to identify asymmetric responses of real wages to business cycle up- and downswings. This is in contrast to the empirical literature on wage stickiness, which typically has looked for evidence of downward real (and also nominal) rigidity by comparing empirical wage-change distributions with notional distributions, i.e., an attempt to capture how wage changes will be distributed in the absence of downward rigidities (see, e.g., Dickens et al., 2007; Basu and House, 2016). Evidence points to the existence of some real downward rigidity in individual wage changes in ongoing employment relationships. Our approach differs in that we focus on the real wages of new hires and incumbents separately (the former are omitted by construction in the usual approach) and look at how these wages respond to different phases of the cycle.

Recent evidence from a study of 15 European Union countries by Galuscak et al. (2012) suggests that new-hire wages are intimately related to wage structures that already exist in the firm; moreover, this relationship is stronger in periods of labour market slack, which is a feature of the equilibrium we derive here (see also Bewley, 1999). Gertler and Trigari (2009) estimate the cyclicality of hiring wages in the U.S. by using Survey of Income and Program Participation data and argue that wages of new hires do appear to be more procyclical than those of ongoing employees. However, using the same data, Gertler et al. (2020) find that it is the composition of match quality that explains the greater wage flexibility for new hires from unemployment. In their empirical analysis of nominal wage rigidity, Grigsby et al. (2021) examine inter alia the differential cyclicality of new hires versus incumbent workers and find that when they control for worker quality, the excess cyclicality of new-hire wages often found in empirical studies disappears.³ In our work we are also careful to control for worker (match) quality — we do so via the use of match fixed effects. Similarly to Grigsby et al. (2021) we do not find excess cyclicality of new-hire wages in downswings, but in upswings things are different: new-hire (real) wages move flexibly in response to current conditions, while incumbent wages are relatively sticky.

In Snell et al. (2018), we also examined evidence of downward real rigidity in German data. The model tested in that paper does have worker insurance but no search frictions; the labour market there is competitive. It predicts downward rigidity for both incumbents and new hires in bad states, but — contrary to the current paper — also predicts equal treatment in upswings. We return to this earlier empirical work in Section 5 where we compare and reconcile it with the findings of the current article.

³For Germany, e.g., Stüber (2017) shows that the wages of newly hired workers are slightly but not significantly more procyclical.

3 Model with No-Undercutting Condition Imposed (Restricted Model)

3.1 Preliminaries

Our approach builds on Menzio and Moen (2010). There, overlapping generations of two-period lived firms interact with infinitely lived workers in the context of a frictional labour market but where employment dynamics are driven by firm entry (each firm is of a fixed size in terms of jobs, with constant productivity per filled job, and free entry of firms). In our model, rather than firm entry being the driver of employment fluctuations, we assume a fixed number of firms operating subject to decreasing returns to scale. The supply of jobs then varies not with variations in the number of firms entering the market, but with firms' choices about how many jobs to create in each period. This allows us to impose a finite horizon and restrict our exposition to a simple two-period model. We follow the logic of their approach so that a wage contract in which new-hire wages are set no lower than incumbent wages will guarantee incumbents job security.⁴

There are two periods t = 1, 2, and a large number of identical firms and workers.⁵ Each firm and worker lives for both periods and the ratio of workers to firms equals S. We identify each firm with the entrepreneur who owns it; entrepreneurs do not supply labour. In each period, each firm operates a decreasing returns technology that produces a perishable good, with production function f(n; x), where n is the current number of workers employed at the firm, which we treat as a continuous variable, $x \in X$ is a productivity shock observable to the firm at the start of the period (and to the worker in the symmetric information version), and the first and second derivatives with respect to the first argument are, respectively, f' > 0, f'' < 0, with f(0; x) = 0. Hours per worker are not variable. We assume that $x = x_0$ is fixed at t = 1, but at t = 2, x is a random variable, common across firms, with finite support. Henceforth, xwithout a subscript 0 will refer to the second period productivity shock. Each worker has a per-period utility of consumption function v(c), with v' > 0 and

⁴Our assumption of a fixed cost per job opening replaces Menzio and Moen's (2010) assumption of a fixed cost incurred per firm that enters. Overall our model admits more tractability; in particular we are able to evaluate the model's response to standard productivity shocks, whilst Menzio and Moen's (2010) set up only readily admits analysis of responses to MIT shocks. This facilitates the extension to asymmetric information, the main contribution of the paper.

⁵Formally, we will treat these as measures.

v'' < 0. Workers cannot borrow or save, so they consume all their current income; for simplicity, we assume that there is no discounting of the future by workers. Entrepreneurs, on the other hand, are risk neutral, but they also do not discount the future (nothing depends on this, provided that discounting is symmetric). A worker who is unemployed in any period receives an income of b.

A firm has a wage policy $\sigma = (w_1, (w_{2,i})_{i=I,N})$ to which it commits, where $w_{2,I}$ is the second period wage paid to incumbents, $w_{2,N}$ that paid to new hires in period 2, and $w_{2,i}$ may be random (state contingent). For the moment we assume that it is optimal ex-ante to satisfy the *no-undercutting condition* $w_{2,N}(x) \geq w_{2,I}(x)$ and treat it as an exogenous constraint. We refer to this as the *restricted model*.⁶ We relax this below, where we analyse circumstances in the unrestricted model under which it is optimal to satisfy the condition and those where it is not; to avoid further cluttering the exposition, we defer details of this part of the model (modelling the costs of violating the condition) until Section 4. Given this condition, a worker who accepts a contract at t = 1 suffers only the exogenous risk of separating from the firm at the end of the first period, with probability δ . In this case, they will be in the same position as a worker who did not gain employment in the first period; in the second period, these unattached workers seek work.

At the start of each period (in period 2, after x is observed), search and matching occur (see Figure 1). We assume directed search (see Moen, 1997; Acemoglu and Shimer, 1999; Rudanko, 2009). Briefly, an unemployed worker can apply for one job at a single firm in each period.⁷ We rule out on-the-job search so that at t = 2, a worker cannot apply for a job if the worker is already employed. We identify the 'type' of a job with the utility V a successful applicant obtains from it. The application succeeds with probability $p(\theta(V))$, where $\theta(V)$, "the expected queue length for the job," is the ratio of applicants to jobs of type

⁶The assumption that firms can commit in period 1 to the future (state-contingent) wages not only of period 1 hires but also of period 2 hires seems strong. In fact, it is enough that they can commit both to the future wages of period 1 hires, and also to not undercut: in any state where the no-undercutting condition is slack, the firm will (ex-post) optimally choose the ex-ante optimal new-hire wage. (This also holds in the asymmetric information extension.) In a repeated version of the model, we conjecture that reputation arguments could be used to justify both the commitment to incumbent wages and the maintenance of no undercutting. At the same time, a reputation for not replacing incumbents by cheaper new hires may be much harder to establish if the reason for separations is unobservable.

⁷We do not consider search intensity on the worker side to be a choice variable. See, e.g., Choi and Fernández-Blanco (2018), who consider optimal policy in a two-period directed search model with contract posting, as here, where search intensity depends on unemployment risk amongst other things.

V, that is, the inverse of labour market tightness.⁸ The function $p(\cdot)$ is assumed to be strictly decreasing, differentiable and such that p(0) = 1, $p(\infty) = 0$. Similarly, the firm fills a job of type V with probability $q(\theta(V))$ where $q(\cdot)$ is strictly increasing and satisfies $q(\theta) = p(\theta)\theta$, q(0) = 0, and $q(\infty) = 1$. Moreover, denoting the elasticity of q with respect to θ by $\epsilon_q(\theta)$, $q(\theta) \epsilon_q(\theta) / (1 - \epsilon_q(\theta))$ is assumed to be a decreasing function of θ .⁹

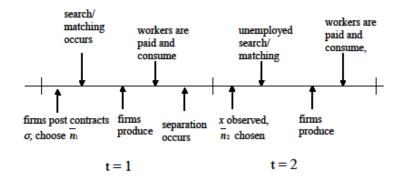


Figure 1: Timeline

Simultaneously with committing to a wage policy at the start of t = 1, firms choose how many new jobs \overline{n}_i to create in periods i = 1, 2, at a cost of k > 0per job; \overline{n}_2 depends on shock x. Unfilled jobs from the first period "die" at the end of the period, along with filled jobs in which exogenous separation occurred. The implication is that the employment in the firm in period i will increase by $q(\theta(V))\overline{n}_i$.

Let Z_1 be the lifetime utility of a worker at the search stage in period 1 and $Z_2(x)$ be that of a worker in period 2 searching for work in state x. Z_1 and Z_2 are the endogenous variables that determine the economic environment the firm faces. Define $Z = (Z_1, (Z_2(x))_{x \in X})$. The value to a worker at t = 1 of being employed by a firm with a wage policy σ is then

$$V_1(\sigma; Z) := v(w_1) + E[\delta Z_2(x) + (1 - \delta)v(w_{2,I}(x))],$$
(1)

where E denotes the expectation.¹⁰

⁸For the moment, we suppress other arguments of $\theta(\cdot)$ corresponding to the economic environment. The determination of $\theta(V)$ is discussed below.

⁹Menzio and Moen (2010), who also assume this, point out that many standard matching processes satisfy these assumptions.

¹⁰To avoid complicating the exposition, we will ignore the possibility that at the optimal

Let U_1 be the lifetime utility of a worker in t = 1 who fails to get a job:

$$U_1(Z) = v(b) + E[Z_2(x)],$$

as currently, the worker receives b and is able to search the next period. Given U_1 and Z_1 , it is assumed that the expected queue length for a job offering V_1 satisfies:

$$\theta_1(V_1, Z_1, U_1) = \begin{cases} \theta : p(\theta)V_1 + (1 - p(\theta))U_1 = Z_1, & \text{if } V_1 > Z_1 \\ 0, & \text{if } V_1 \le Z_1 \end{cases}.$$
 (2)

The idea is that if the value of the job for a successful applicant, V_1 , is greater than the value of search, Z_1 , the expected queue length is driven up to the point where workers are indifferent between applying for the job and searching somewhere else, and vice versa. The expected queue length for the job will be zero if the value of the job is less than (or equal to) the value of search.

For a worker seeking work at t = 2, the value of being employed at the wage $w_{2,N}$ is $v(w_{2,N})$, so the expected queue length for period 2 firms and workers for a job with wage $w_{2,N}$ is

$$\theta_2(w_{2,N}, Z_2) = \begin{cases} \theta : p(\theta)v(w_{2,N}) + (1 - p(\theta))v(b) = Z_2, & \text{if } v(w_{2,N}) > Z_2 \\ 0, & \text{if } v(w_{2,N}) \le Z_2 \end{cases}$$
(3)

A firm's profit is

$$F(\sigma; \overline{n}_{1,}(\overline{n}_{2}(x))_{x \in X}; Z) = (f(n_{1}; x_{0}) - w_{1}n_{1} - k\overline{n}_{1}) +$$

$$E[(f((1-\delta)n_{1} + n_{2}; x) - w_{2,I}(1-\delta)n_{1} - w_{2,N}n_{2} - k\overline{n}_{2})]$$
(4)

where n_i is the number of new hires in period *i* and is given by $n_i = q(\theta_i) \overline{n}_i$, i = 1, 2, where θ_i depends on σ , as given by $\theta_1(V_1(\sigma, Z), Z_1, U_1(Z))$ in (2) and $\theta_2(w_{2,N}, Z_2(x))$ in (3) above.

period 2 wage, the firm will prefer not to hire at all, and to dismiss some of its incumbents. This situation will arise if $w_{2,I} > f'((1-\delta) n_1; x)$. In our simulations, parameters are chosen so that this scenario does not arise. We will assume throughout that positive hiring occurs in equilibrium. Given average annual turnover rates of around 30% in the U.S., e.g., this assumption is not restrictive for any reasonable parameterisation.

3.2 Symmetric Information

In this section, we assume that workers can observe the state in period 2 (or equivalently that courts can enforce state-contingent wage contracts). In equilibrium, firms choose wages and employment to maximise profits, and Z_1 and $Z_2(x)$ must be consistent with the resulting queue lengths.¹¹

Because of the no-undercutting condition, incumbents who are not exogenously churned do not face a replacement risk. It is nevertheless useful to compare the equilibrium wage dynamics of the current model with the standard case where firms *can commit* to retain incumbents who are not churned exogenously, even when $w_{2,N} < w_{2,I}$. Eliminating the replacement risk means that $w_{2,N}$ can be set optimally, i.e., without regard to $w_{2,I}$. We refer to the equilibrium of this alternative model as the *full commitment* or *FC equilibrium*. In that model, firms would completely ensure risk-averse period 1 hires, so $w_{2,I} = w_1$ in each state, and $w_{2,N}$ would be set (in combination with job creation) so as to minimise the total cost of hiring each worker.

We show that in any period 2 state where the new-hire wage $w_{2,N}$ is below $w_1, w_{2,N}$ will be above the corresponding FC level: Because the no-undercutting condition binds so $w_{2,I} = w_{2,N}$, firms have to trade off offering insurance to period 1 hires with cutting $w_{2,N}$ as far as they would like. Hence, the no-undercutting condition leads to a reduction in downward wage flexibility. By contrast when $w_{2,N}$ is above the period 1 wage, it will be at the corresponding FC level as there is no corresponding trade-off to dampen wage increases.

We can illustrate the argument using a supply-demand analysis for period 2 taking n_1 and w_1 as given. The wage $w_{2,N}$ that corresponds to the cheapest way of hiring n_2 workers (taking into account the number of jobs that must be created) traces out the *FC quasi-supply* curve of labour (FC, because this ignores the no-undercutting constraint). A point on this locus equates two ways of increasing employment by one unit: The firm could open more jobs (1/q) jobs at a cost of k/q). Alternatively increasing wages, holding the number of jobs constant, accomplishes the same result by increasing the queue length and, hence, the probability that each existing job is filled.¹² The two must be equal in equilibrium, and this relationship defines the locus. The locus is positively sloped: When equilibrium n_2 is higher, it is more difficult to fill each job because

¹¹We postpone formal conditions until Subsection 3.3.

¹²The cost of this is q/\tilde{q}' , where \tilde{q}' is the increase in the job-filling probability for a unit increase in the wage. The locus is then defined by $q^2 (\tilde{q}')^{-1} = k$.

the labour market is tighter (θ_2 is lower, so $k/q(\theta_2)$ is higher). This makes wage increases, as a way to fill jobs, more attractive than creating additional jobs, and $w_{2,N}$ increases until the two methods cost the same. The locus is independent of the revenue generated from a filled job.

We can combine this with the downward-sloping labour demand curve, which is standard except that the unit cost of increasing employment $(k/q(\theta_2))$, itself increasing as n_2 increases) is added to the wage.¹³ The intersection yields a unique equilibrium for each value of x.¹⁴ As x varies, only the labour demand curve shifts. Denote the crossing point by $(w_{2,N}^{FC}(x, w_1, n_1), n_2^{FC}(x, w_1, n_1))$.

Consider now the restricted model. If $w_{2,N} \ge w_{2,I}$ is binding at the optimum (when productivity is sufficiently low), the intersection of demand and supply occurs at a wage below w_1 , but the wage can be shown to be above $w_{2,N}^{FC}(x, w_1, n_1)$, while employment is below $n_2^{FC}(x, w_1, n_1)$. In the proof, it is shown that the unit cost of increasing employment through creating extra jobs is lower than that through increasing wages, so it would be cheaper to cut wages and increase jobs; however, this is not done because the wage cut has a negative externality on incumbents' wage smoothing.

This is illustrated in Figure 2. The restricted model quasi-supply curve coincides with the FC curve above w_1 , but below w_1 , the curve lies above the FC curve (taking w_1 as given). The equilibrium again occurs at the intersection with the labour demand curve. In the figure, a situation is illustrated where the crossing point occurs below w_1 . If x is high enough that the intersection occurs above w_1 , then the equilibrium will be at the FC solution, $(w_{2,i}^{FC}(x, w_1, n_1), n_2^{FC}(x, w_1, n_1))$. The proposition summarises the relationships between equilibrium outcomes in our model compared with what would hold in the FC case.

Proposition 1 (a) If the equilibrium hiring wages in any state in period 2 are below period 1 wages, $w_{2,N} < w_1$, we have $w_{2,N} > w_{2,N}^{FC}(x;w_1,n_1)$ and $n_2 < n_2^{FC}(x;w_1,n_1)$: the wage for new hires is higher and employment is lower than they would be in the FC model;¹⁵ and, in addition, $w_{2,I} = w_{2,N} < w_1$. Otherwise,

¹³ It satisfies $f'(n) = w_{2,N} + k/q$. As n_2 increases, $p(\theta_2)$ must increase from $n_2 = p(\theta_2) S_2$, and therefore θ_2 has fallen as p' < 0; thus $q(\theta_2)$ falls, given that q' > 0. Thus the demand curve traces out, for each n_2 , the wage such that firms would choose to employ n_2 workers taking into account the hiring cost.

¹⁴The positions of these two curves depend only on n_1 , which implies the value of S_2 , and x.

¹⁵ If firms were not constrained by the no-undercutting condition in this state, unless the state had a negligible probability, then the equilibrium two-period contract may be different, that is, w_1 and n_1 may differ. The proposition concerns the implied values of $w_{2,N}^{FC}$ and n_2^{FC} in a hypothetical equilibrium that has the same period 1 values.

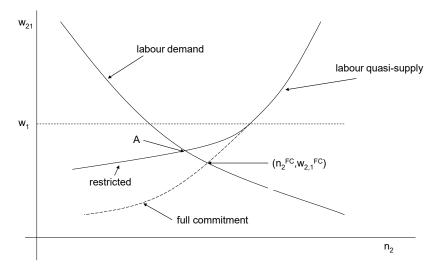


Figure 2: Restricted model (symmetric information) quasi-supply

(b) wages and employment are at the FC levels: $w_{2,N} = w_{2,N}^{FC}(x; w_1, n_1)$ and $n_2 = n_2^{FC}(x; w_1, n_1)$, with $w_{2,I} = w_1$. Case (a) occurs when the labour demand curve intersects the FC quasi-supply curve below w_1 ; otherwise, case (b) occurs.¹⁶

Wages are allocational¹⁷ in period 2 so that the flatter quasi-supply in the region where there is downward pressure on wages will also imply more variable employment.^{18,19} The result is unchanged if there is symmetric discounting. If discounting is asymmetric, then the reference wage in period 2, which determines the regime (and $w_{2,I}$ when the no-undercutting condition does not bind), differs from w_1 , but otherwise the proposition extends.

3.3 Asymmetric Information

In this section, we introduce asymmetric information over the period 2 state x into the restricted model, treating the no-undercutting condition as an exogenous constraint as in Section 3.1.

We will assume that in period 2, ongoing hires in a firm can observe only wages $w_{2,N}$ and $w_{2,I}$, but cannot observe x (nor Z_2 so they cannot infer x).

¹⁶Formal proof is provided in online Appendix B.1.

 $^{^{17}}$ I.e., firms hire until the marginal product net of the hiring cost (k/q) is equal to the new-hire wage.

 $^{^{18}}$ See Section 3.3.2 for some simulations.

¹⁹If there are multiple periods (with long-lived firms and workers), Proposition 1 readily extends, where now undercutting is defined in terms of discounted wage costs rather just than the current wage. Details are available on request.

Additionally, they cannot observe the total employment or job openings in the firm (we relax this in the online appendix A). Equivalently, we assume that such variables are not contractible. By the revelation principle, we can restrict attention to state-contingent wage/employment plans as in Section 3.1, but now with the added constraint that the firm must prefer to announce the true state. The resultant incentive compatibility constraints on the contract imply that the equilibrium contract exhibits more wage rigidity and greater employment and job opening fluctuations than induced by equal treatment alone. It is important to stress that we assume that the firm commits to wages, *including the new-hire wage*, in each state. In the asymmetric information setting, this is equivalent to committing to a menu of wages $(w_{2,N}(x), w_{2,I}(x))$ across states x, from which the firm will choose. Or equivalently, the firm "announces" the state, and we require that it cannot do better than announce the true state. Our claim is that across any two states x' and x'' with binding no-undercutting constraints (i.e., $(w_{2,N}(x) = w_{2,I}(x)))$, the menu items will be the same so that $(w_{2,N}(x'), w_{2,I}(x')) = (w_{2,N}(x''), w_{2,I}(x'')).$

The basic intuition is that if wages vary across states, there will be an incentive to announce a state with a lower wage, at least when the no-undercutting constraint is binding. Therefore, to prevent this, wages must be constant at a "wage floor". In more detail: In Section 3.1, we saw that when the constraint binds in period 2 - in "bad" states — the firm would like to cut the new-hire wage to reduce the cost of new hires, but limits this because of the desire to offer ex-ante insurance to period 1 hires (now incumbents). Under asymmetric information, and if there are multiple states where the constraint binds, there cannot be a different wage across such states. If the wage did vary, the firm would choose to claim that the current state was the lowest wage state. The key point is that by doing this the firm would benefit both with respect to incumbents, to whom it pays less, and also with respect to new hires, because it pays a wage closer to that which would minimise the per-worker cost of hiring. As employment is not observable/contractible, it is not constrained in its hiring decision, so the firm would prefer the lowest new-hire wage.

This argument would only fail if the wage in some state was below the optimal new-hire wage in another state and *sufficiently* below it to offset any gain the firm would make by reducing its incumbent wage bill. Such a scenario can only arise if the productivity shocks across these two states are very different and the number of incumbents is very small. Suppose instead that there is a state such that the optimal new-hire wage is above the incumbent wage. Then no similar issue arises: the firm can pay the optimal new-hire wage and has no incentive to announce a worse state with lower wages, since the incumbent wage would be set low enough to ensure that announcing the lower state would not lead to a gain on the incumbent wage bill sufficient to offset any loss on an inappropriate new-hire wage.

We remark that it is the combination of no undercutting with asymmetric information that leads to rigid wages. Asymmetric information in the FC model would not have an impact: In the FC case, the firm would set period 2 incumbent wages to w_1 and new-hire wages at the optimal level for hiring, and under asymmetric information, this is incentive compatible, as misrepresenting the state would not impact the incumbent wage and would only lead to a suboptimal new-hire wage.

3.3.1 Equilibrium

Let $F^{(x)}$ be period 2 profits in state x (the term inside the expectation in (4)). Given $(Z_2(x))_{x \in X}$, a wage policy σ and a job creation plan $(\overline{n}_1, (\overline{n}_2(x))_{x \in X})$ satisfy the *incentive compatibility constraints* if for all $x \in X$

$$F^{(x)}(\sigma; \overline{n}_1, \overline{n}_2(x); Z) = \max_{x', \overline{n}'_2} \left\{ (f((1-\delta)n_1 + n'_2; x) - w_{2,I}(x')(1-\delta)n_1 - w_{2,N}(x')n'_2 - k\overline{n}'_2) \right\}$$
(5)

where $n'_2 = q(\theta_2) \overline{n'_2}$ and $\theta_2 = \theta_2(w_{2,N}(x'), Z_2(x))$. That is, in state x the firm could misrepresent the state and announce x' and it will optimise job openings, $\overline{n'_2}$.²⁰ Incentive compatibility requires that it cannot increase wages by announcing any $x' \neq x$.

We define an equilibrium as follows:

Definition 1 A symmetric equilibrium in the restricted asymmetric information model with positive hiring consists of search values $Z = (Z_1, (Z_2(x))_{x \in X})$, a wage policy σ and a job creation plan $(\overline{n}_1, (\overline{n}_2(x))_{x \in X})$ with the following properties:

²⁰These are ex-post (after the period 2 state is observed) constraints; for simplicity, we assume that n_1 is contractible. Otherwise, the incentive compatibility constraints should be expressed in terms of an ex-ante constraint that requires that should the firm deviate at date 1 (i.e., possibly changing n_1) and in any period 2 state, it cannot increase its discounted expected profit. Since in the latter case, the ex-post constraints will also hold, the results will be very similar.

(i) $(\sigma; (\overline{n}_1, (\overline{n}_2(x))_{x \in X})$ satisfies $w_{2,N}(x) \ge w_{2,I}(x), x \in X$, and (5); (ii) *Profit maximisation*: For all $(\sigma'; \overline{n}'_1, (\overline{n}'_2(x))_{x \in X})$ satisfying $w'_{2,N}(x) \ge w'_{2,I}(x), x \in X$, and (5),

$$F((\sigma;\overline{n}_1,(\overline{n}_2(x))_{x\in X});Z) \ge F(\sigma';\overline{n}_1',(\overline{n}_2'(x))_{x\in X};Z);$$

and

(iii) Consistency: $\theta_1(V_1(\sigma, Z), Z_1, U_1) = S/\overline{n}_1$, and, for all $x, \theta_2(w_{2,N}, Z_2(x)) = S_2/\overline{n}_2(x)$ where $S_2 := ((1 - p(S/\overline{n}_1)) + \delta p(S/\overline{n}_1))S$ is the number of workers (per firm) seeking work in period 2.

Property (ii) requires there to be no other feasible contract that generates higher profits; property (iii) requires that queue lengths consistent with equilibrium values correspond to the equilibrium vacancy and unemployment rates.

Before we analyse the equilibrium, consider the asymmetric information model without the no-undercutting condition, that is, with full commitment on the part of the firm. In fact, the introduction of asymmetric information does not affect the FC equilibrium. The firm will offer a non-contingent period 2 contract wage to period 1 hires (equal to w_1) and hence there is no benefit from deviating from the optimal hiring wage to period 2 workers.²¹

However, when the no-undercutting condition is imposed, we can establish the following. Assume that $X \subset R_+$, and that f is differentiable and increasing in x;

Proposition 2 (Asymmetric information) (i) [wage floor] Suppose in the restricted asymmetric information model with a single period 2 productivity state \hat{x} , that there is an equilibrium with no undercutting and the no-undercutting condition binds strictly. Then, in a perturbed version of this model where this state is replaced with two different equal-probability states, $\hat{x} - \varepsilon$ and $\hat{x} + \varepsilon$ (i.e., with expected value \hat{x}), and assuming that there exists $\overline{\varepsilon}$ such that for $\varepsilon \in [0, \overline{\varepsilon})$, the equilibrium is unique and continuous in ε , then period 2 wages are constant across these states, provided that the perturbation ε is sufficiently small.²² Period 2 wages are allocational. (ii) [upward flexibility] In the restricted asymmetric

²¹That is, $w_{2,I}$ is independent of the period 2 state x, and $w_{2,N}$ is chosen independently of $w_{2,I}$ to minimise the cost of hiring a new worker in state x. With asymmetric information, the firm has no incentive to misreport since the wage paid to non-separated period 1 hires is constant, while any different $w_{2,N}$ can only increase new-hire costs.

²²For ease of presentation, the proposition considers the case where there is a single period 2 state \hat{x} in the initial situation. If there are other states in which the no-undercutting condition is not binding, the argument can be extended straightforwardly. The argument also extends readily to non-equi-probability perturbations.

information model, at the highest $w_{2,I}$, i.e., for $x \in \arg \max_{x'} w_{2,I}(x')$, $w_{2,N}(x) = w_{2,N}^{FC}(x, w_1, n_1)$ if the no-undercutting condition is not binding, and $w_{2,N}(x) \ge w_{2,N}^{FC}(x, w_1, n_1)$ otherwise. (iii) $w_{2,I}(x) \le w_1$, all x.²³

Part (i) considers what happens in the restricted model, where asymmetric information now matters: if there are two states close to each other and the noundercutting condition is binding, then wages are non-contingent, i.e., constant across these states; this has direct implications for hires.

Part (ii) says that in the state with the highest $w_{2,I}$, if the no-undercutting condition is not binding, the new-hire wages are in the FC solution. Part (iii) says that it is never optimal for the incumbent wage to exceed w_1 .

The result in part (i) leads to a central and novel empirical hypothesis that wages in downturn states are equal to a wage floor, independent of realised shocks. This begs the question of what determines the wage floor: *ceteris paribus* the ex-ante distribution of shocks will determine how low the floor is. When the shocks are, as in part (i), close together, the wage floor will approximately be at the crossing point of the labour demand functions with the restricted quasisupply curve. In comparative statics where both shocks worsen, the wage floor moves down the quasi-supply curve, and unemployment rises. This leads to a relationship between average unemployment in recession states and the wage floor. We equate average unemployment with the predicted level of unemployment in downswings for our empirical analysis in Section 5. In simulations, even with a much larger variance of shocks, provided the coefficient of variation of the shocks is held constant, a clear negative relationship between average unemployment and the wage floor always holds.²⁴ Thus we hypothesise that in downswings wages are more related to predicted unemployment than realised unemployment. We also test a hypothesis based on part (ii).

In part (i), we require the variance of the shocks to be small: As explained above, if the wage varies with the state, say if $w_{2,N}(x_1) = w_{2,I}(x_1) < w_{2,N}(x_2) = w_{2,I}(x_2)$, then in state x_2 the firm will prefer to "announce" state x_1 : it benefits from paying a lower wage to its existing employees. In addition, because the no-undercutting condition is binding, the optimal wage for new hires (i.e., ignoring the no-undercutting condition) would be lower than at the restricted model

²³Formal proof is provided in online Appendix B.2.

 $^{^{24}}$ In the simulations using matching function A (see Section 3.3.2), semi-elasticities of the wage floor with respect to average unemployment vary between approximately -0.33 and -0.8 (increasing in the separation rate and decreasing in risk aversion). While these are only indicative, they encompass our downswing semi-elasticity estimate of wage with respect to predicted unemployment of -0.55.

solution in each state considered separately. Because the two states are close, the wages will also be close, and in particular $w_{2,N}(x_1)$ will be above the optimal wage in x_2 . Hence, the cost per new-hire would be reduced in x_2 by lowering the wage to $w_{2,N}(x_1)$. Consequently, period 2 profits increase by announcing state x_1 , thus ruling out wage variation.

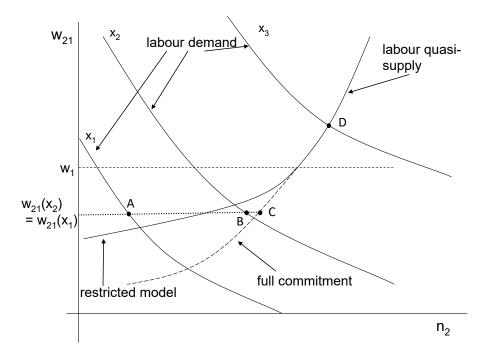


Figure 3: A rigid wage under asymmetric information

What if the shocks are very different? For a very wide variation in shocks, the lower $w_{2,N}$ in the restricted model symmetric information equilibrium might be so low — below the optimal level in the other state — that switching to it reduces profits from new hires. This fall in profits is unlikely to outweigh the gains from cutting $w_{2,I}$ though, as the latter are first-order and large, while around the optimal hiring wage the change in profits on cutting $w_{2,N}$ will be second-order.²⁵ The same forces exist when there are multiple states that imply a wage floor \underline{w} in which the wages of both new hires and incumbents are equal across a wide range of states. An incentive compatible contract is illustrated by

 $^{^{25}}$ For very high rates of turnover (such that incumbents become a very small fraction of the workforce) and for large negative shocks such that wages are not very close together in the restricted model solution, the latter solution *will* satisfy incentive compatibility. We discuss this in further detail in Section 3.3.2 below.

points A and B in Figure 3, assuming that there are only two states, x_1 and x_2 .²⁶ Intuitively, continuing the previous discussion, suppose that there is a third state $x_3 > x_1, x_2$, such that $w_{2,N}(x_3) = w_{2,I}(x_3) = \underline{w}$; now suppose that this state of nature improves (i.e., consider perturbing the model by increasing x_3 holding all else constant). As x_3 increases, the new-hire wage that is optimal in the FC model, $w_{2,N}^{FC}(x_3, w_1, n_1)$, that is, ignoring the constraints on no-undercutting and incentive compatibility in that state, rises above \underline{w} . This happens when the demand and FC quasi-supply curves intersect above point C in Figure 3. It is incentive compatible to have $w_{2,N}(x_3)$ at the optimal level (see point D) with $w_{2,I}(x_3) = \underline{w}$: announcing a lower state from state x_3 will reduce profits $(w_{2,N})$ will be at a suboptimal level, while $w_{2,I}$ will be the same). In fact, the firm can do even better: $w_{2,I}$ will be slightly higher than \underline{w}^{27} . For sufficiently favourable x_3 , $w_{2,I}$ can increase all the way to w_1 without violating incentive compatibility, but as shown in general in (iv), it is never optimal to exceed w_1 . Nevertheless, due to the incentive constraints incumbent wages are procyclical — though within the restricted interval of wages $[\underline{w}, w_1]$ — even over a range of "positive" shocks (i.e., such that $w_{2,N}^{FC}(x, w_1, n_1) > w_1$) in contrast to the symmetric information case, something that may accord better with empirical evidence.²⁸

When there is just one state in which wages exceed a wage floor, the latter logic also implies that the restricted model quasi-supply curve under asymmetric information coincides with the FC one for a range of wages below w_1 , down to the "wage floor" \underline{w} (in contrast to the symmetric information case). Therefore, the region of "flexibility" for new-hire wages extends further (i.e., wages are initially more flexible downward, but then fully rigid) than in the symmetric information case. Consider point C in Figure 3: if there is a state with demand curve passing through this point, the fact that incentive compatibility lowers the incumbent wage even in such a state implies that the no-undercutting condition first binds

²⁶The level of the wage floor will depend on the severity of the distribution where the constrained regime applies, as, roughly speaking, the wage floor averages across the wages on the restricted model quasi-supply curve in this region. In the empirical section, we proxy for productivity in this region with forecast unemployment conditional on the latter being above its long run mean.

²⁷There will now be a cost of deviating by announcing a lower state, given that the newhire wage will fall below the optimal level, so $w_{2,I}(x_3)$ can increase towards w_1 , increasing incumbent wage costs by a corresponding amount (recall that moving $w_{2,I}$ towards w_1 will improve ex-ante profits). Hence, $w_{2,I}(x_3)$ will be set to exactly satisfy the incentive compatibility constraint subject to not exceeding w_1 . Initially, this scenario is a comparison between a second-order cost and a first-order gain, so the increase in $w_{2,I}$ is itself second-order to avoid violating incentive constraints.

²⁸In the simulations, incumbent wages increase up to the point where the new-hire wage is approximately 10% higher than w_1 .

only at lower levels of the new-hire wage so that $w_{2,N}$ will be set at this level.

3.3.2 Employment Elasticity Simulations

In Table 1 we present results of (two-state) simulations of the elasticity of employment with respect to productivity changes in each of the three regimes we have considered, for varying degrees of risk aversion (α) and labour turnover (δ), and for the matching function²⁹ $m(u, v) = uv/(u^l + v^l)^{1/l}$, l = 0.5, where u is the number of workers searching and v is the number of vacancies and mthe total number of matches (as in, e.g., Hagedorn and Manovskii, 2008). The parameters are chosen so that period 2 wages are below the period 1 wage, and we target similar replacement ratios and similar average unemployment rates in period 2.³⁰

Consider first a turnover rate of 10% ($\delta = 0.1$). Under full commitment there is a low elasticity which is not responsive to risk aversion; this is intuitive as the new-hire wage is set without respect to insurance considerations; in each case, wages respond sufficiently to demand to imply a low elasticity. When no undercutting is imposed (i.e., with symmetric information), and risk aversion is high $(\alpha = 2)$, the insurance motive is enough to stabilise wages sufficiently that the elasticity is much higher, and even under asymmetric information (so, with constant wages) the elasticity is similar. However, for lower α the motive to insure period 1 hires is weaker, so that there is sufficient variability in the period 2 wage across states to imply a higher elasticity under asymmetric information, considerably so for $\alpha = 0.5$. For higher δ , $\delta = 0.3$, the no-undercut elasticity falls — higher turnover means that the incentive to insure incumbents is lower, as they now comprise a smaller proportion of the period 2 workforce, so that wages are more flexible. Under asymmetric information, elasticities are somewhat higher — to be expected given the level of new hires is higher — but vary little with risk-aversion as in each case wages are again constant. They are, however, considerably higher than in the no-undercut case: For $\alpha = 0.5$, the asymmetric information elasticity is four times that with the no-undercutting constraint only.

These elasticities are computed for small shock differences. As mentioned in the discussion below Proposition 2, the logic underlying the wage floor result suggests that wages will be constant over a wide range of shocks for which the

²⁹See online Appendix B.5 for the analysis of an alternative matching function.

³⁰We concentrate on downswings in the simulations as this is our primary interest.

	AI			NU			FC		
α	0.5	1	2	0.5	1	2	0.5	1	2
$\delta = 0.1$	0.25	0.27	0.26	0.14	0.23	0.26	0.04	0.04	0.04
$\delta = 0.3$	0.33	0.31	0.28	0.08	0.11	0.16	0.05	0.05	0.06

Table 1: Elasticity of employment with respect to productivity changes

Note: AI = asymmetric information; NU = no-undercutting constraint imposed; FC = full commitment; production function is logarithmic, matching function as in text; constant relative risk-aversion utility with risk aversion coefficient of α ; exogenous separation rate δ ; unemployment benefit b targets an average replacement rate of 43%, and for given δ we calibrate the vacancy cost parameter k to yield an average period 2 unemployment rate for log utility in the NU case of 8.5%.

no-undercutting constraint binds. To get an idea of how wide the range is, we start from the equilibria calculated in the case of asymmetric information in Table 1, and reduced the magnitude of the lower shock x_1 .³¹ The shock can be reduced until job creation falls arbitrarily close to zero. The reason why the wage stays constant even down to this level is that the restricted model quasi-supply curve is "U-shaped", eventually rises as hiring falls far enough. New hires form such a low fraction of the workforce at this level that the insurance of incumbents dominates keeping the hiring wage $w_{2,N}(x_1)$ low, and therefore in the absence of incentive constraints, $w_{2,N}(x_1) (= w_{2,I}(x_1))$ would eventually increase toward w_1 as x_1 falls. Recall from the above discussion that an incentive compatible nonconstant wage would require a $w_{2,N}(x_1)$ sufficiently far below the optimal good-state hiring wage that it would make hiring in the good state x_2 very expensive. This does not occur because of the U-shaped restricted model quasi-supply curve.

For very low shocks, it is the upward incentive constraint — that in state x_1 the firm prefers not to announce x_2 — that is binding. This prevents $w_{2,N}(x_1)$ from rising above $w_{2,N}(x_2)$. The logic is similar to that underlying Proposition 2: announcing state x_2 when the state is x_1 would lead to savings on the incumbent wage bill, and as there are few new hires $w_{2,N}(nx_1) > w_{2,N}(x_2)$ would not be incentive compatible.³²

³¹In the simulations for Table 1 x_1 and x_2 are close together and each occurred with probability 0.5. In these simulations, we reduce the probability on x_1 to 0.1 when varying it, to avoid substantial changes in the first period values and average period 2 unemployment (likewise for the probability of x_2 when increasing it).

³²In fact for $\delta = 0.3$ and $\alpha = 0.5$, the downward constraint alone leads to constant wages

Thus, for separation rates that are not too high, simulations indicate that the constant wage across states result applies whenever the no-undercutting constraint binds. For very high separation rates, however, this will not be the case. Not only are there few incumbents, so little to be gained in terms of wage cuts, but hiring is high, so being close to the optimal hiring wage is much more important. Moreover, the restricted model quasi-supply curve will be close to the full commitment one, meaning that announcing incorrectly a state with a lower wage will probably increase hiring costs unless x_1 and x_2 are very close together. If not, the wage differentials will be incentive compatible. For example, if $\delta = 0.97$, $\alpha = 2$ (and targeting a similar level of labour market tightness as in the original simulations) the constant wage range corresponds to around a 1% change in tightness θ^{-1} ; any further spread of shocks leads to $w_{2,N}(x_1) < w_{2,N}(x_2)$.

3.3.3 Discussion

It is useful to contrast our result with earlier models in the asymmetric information implicit contracting literature, such as Grossman and Hart (1981). A firm employs risk-averse workers with a decreasing returns to scale production function, as here, and likewise with asymmetric information where the firm knows the state. If the firm is risk neutral, then the first-best contract can be implemented, but if the firm is risk averse, it would prefer to lay off some workers in some productivity states where it would be efficient to employ them (in that their marginal products exceed their reservation wage). A risk-averse firm would optimally set the contract to shift some risk to workers, and to implement this under asymmetric information incentive compatibility requires the firm to employ fewer workers than is efficient in some states. This model differs from the current one, aside from having a risk averse firm, in that it is effectively a one-

until job creation falls close to zero, so that market tightness (θ^{-1}) is 2.4% of its level in the better state (for $\alpha = 2$, the corresponding ratio is 2.5%.). By contrast, for $\delta = 0.1, \alpha = 0.5$, the number is 16.3%, while for $\alpha = 2$ it is 83.3%. Because in the latter case turnover is low and risk aversion is high, insuring incumbents becomes important relative to hiring at a lower wage, so the restricted model quasi-supply curve rises above $w_{2,N}$ at a high level of x_1 . For x_1 below this level, we would have $w_{2,N}(x_1) > w_{2,N}(x_2)$ if only the downward incentive compatibility constraint is imposed. However, once the upward constraint is added, wages stay constant across states, even for arbitrarily low x_2 .

When $\delta = 0.1$ wages stay constant across states as we *increase* x_2 starting again from the equilibria in Table 1. For $\alpha = 0.5$ (2), wages remain constant up to a point where the tightness in state x_2 is almost twice (respectively, four times) that in x_1 . For any higher x_2 the no-undercutting constraint in state x_2 ceases to bind and $w_{2,N}(x_1) < w_{2,N}(x_2)$ (see the discussion of Proposition 2 (ii)). For $\delta = 0.3$, this inequality occurs much closer to the equilibria in Table 1.

period setting in which a firm has a pool of workers associated with it with which it contracts (the firm and workers enter into a contract before the state is known, but workers may be immobile once contracted).

Our base assumption is that firm employment is unobservable to workers or not contractible, as in, e.g., Grossman and Hart (1983).³³ This contrasts with work such as Grossman and Hart (1981), Chari (1983), and Green and Kahn (1983). In practice, however, the level of employment in a firm can be difficult to define precisely. For example, if the relevant employment level is at the plant, the firm may be able to move production to other plants within the same company, making it difficult to condition on employment (as argued by Stiglitz, 1986). We also consider an extension in the online Appendix A in which we allow contracts to depend on employment levels, and we show that (when shocks are not too far apart) a similar logic applies and that wages are essentially constant.

How do these results extend to a longer horizon? We do not have a formal analysis, but we claim that the broad picture would carry over. First, suppose that there is a third period, so that firms and workers are three-period lived, with period 2 and period 3 shocks denoted by x_2 and x_3 respectively. Let us assume for simplicity that shocks are independent and identically distributed (iid), so workers do not need to update their beliefs about period 3 shocks. The above analysis for the final (second) period applies *mutatis mutandis* to period 3. The labour demand curve will be as before, and across states where the no-undercutting constraint binds, the same logic will imply a constant wage (subject to the same caveat as before that the variance of shocks and exogenous separation rate are not both very high). Next, consider period 2: labour demand should take into account not only the marginal product of labour but also the fact that an extra hire, if not exogenously separated at the end of the period, will save on a hire in period 3 (as before assuming that turnover is assumed to be high enough that new hires are needed in both periods). If the no-undercutting constraint is expected to be binding in period 3, there is no wage difference between a new-hire and an incumbent, so the only saving is the difference in the hiring cost; hence labour demand should very similar as a function of x^{34} The

 $^{^{33}}$ Grossman and Hart (1983) consider a single-worker model in which a worker is either employed or unemployed, or equivalently, a firm with many workers but where the level of employment is, as here, not contractible.

 $^{^{34}}$ On the other hand, if it is likely that the period 3 new-hire wage will be above the incumbent wage, then this implies that there is an additional future saving that needs to be added to the value of a hire.

FC quasi-supply curve will also be similar to before.³⁵ The argument that wages should be constant across states where the no-undercutting constraint binds can be repeated, as it does not depend on marginal conditions. For longer horizons, similar considerations would apply, with wages for new hires constant across states where the no-undercutting constraint binds.³⁶

4 Unrestricted Model: When is it Optimal to Satisfy the No-Undercutting Condition?

Here we drop the restriction that wage contracts are such that no undercutting occurs and flesh out the implications for wages when undercutting can occur. As in our baseline model, we continue to assume that the firm cannot commit *not* to replace workers by cheaper new hires. We then analyse the circumstances under which a firm will want to satisfy the no-undercutting condition $w_{2,N} \ge w_{2,I}$; in short, we outline the circumstances in which no undercutting is a feature of the optimal wage contract.

The discussion in the following assumes symmetric information; however, it will also apply with few changes to the asymmetric information version of the model. Consider a two-state version of the model as in Section 3.3.2, where the no-undercutting constraint is binding. When the variance of shocks is not too large, an undercutting deviation from a no-replacement equilibrium in the asymmetric information model will have approximately the same benefit or cost as in the symmetric information version, since the wage floor will be close to an average of the wages in the downturn states in the symmetric information model. For larger variances, the wage in the better state will be closer to the optimal hiring wage, and that in the worse state further away. This is likely to lead to a

³⁵A minor difference is that to increase queue length by raising the utility offered may now involve wage changes in both periods because of optimal wage smoothing.

³⁶The main difference with our two-period analysis in this respect would be that workers hired at a low wage in one period, say t_1 , may be paid lower than wages for hiring in constrained states at a later date, t_2 ; for such workers in such states the constraint wouldn't bind. This weakens the argument for why wages should be constant across such states at t_2 since a lower wage in one state may not lead to a saving on incumbent wages for those incumbents employed up to t_1 . However, this is true only for small wage changes across constrained states at t_2 which do not take wages below the earlier wage, and so we conjecture that it is unlikely to lead to wages varying across states. Such state contingent wages are only likely to be incentive compatible when considering new hires alone if there are large changes in wages which take the wage in a bad state well below the optimal hiring wage in better states, as argued earlier. But then there would be a savings on the incumbents hired earlier, making the lower wage attractive in the better states.

larger benefit from a deviation since the firm has more incentive to exploit the weak labour market (our simulations in the symmetric information case suggest this is where the largest gains arise). Hence we might expect a somewhat smaller parameter space such that no undercutting is an equilibrium.

We suppose that employment is "at will", so during the matching stage of the second period (after observing x), the firm can dismiss a worker without compensation; that is, the firm can dismiss a worker after matching with an applicant who can replace the original worker, and the dismissed worker will be unemployed.³⁷ Specifically, at t = 2, suppose that unemployed workers can apply for jobs that are already filled; if there is a successful applicant, the firm can, by at-will contracting, choose whether to replace the incumbent or not. If $w_{2,N} \geq w_{2,I}$ firms have no incentive to do this (and unemployed workers have no incentive to apply for such positions), but for $w_{2,N} < w_{2,I}$ there is an incentive to replace. In the latter case, to the extent that the matching process succeeds in selecting a successful applicant for this position, the incumbent is at risk of losing her position. We assume that there is no cost associated with receiving applicants for filled jobs and that the new-hire wage $w_{2,N}$ applies to any new hire. An incumbent's position is on a par with all other created positions; a filled job is as attractive as an unfilled one from the point of view of an applicant when $w_{2,N} < w_{2,I}$ and is equally likely to be filled by a new entrant.³⁸

The expression for profits $F(\sigma; \overline{n}_{1,}(\overline{n}_{2}(x))_{x \in X}; Z)$ is generalised as follows. In the expression for the value to a worker at t = 1 of being employed by a firm with wage policy σ , if replacement occurs in some states, that is, if $w_{2,N} < w_{2,I}$, then in such states, the term inside the square brackets in (1) must be replaced by

$$\delta Z_2(x) + (1 - \delta)q(\theta_2)v(b) + (1 - \delta)(1 - q(\theta_2))v(w_{2,I}(x))$$

This expression reflects the additional risk $q(\theta_2)$ for a surviving worker of being replaced by a successful applicant.

Similarly, in any state where replacement occurs, the expression for the

³⁷Less relevant is the decision of the worker to quit if we assume a worker can quit without penalty, but will remain unemployed in the second period. This situation implies that the only participation constraint that matters for period 1 hires is the period 1 constraint. An alternative assumption that leads to this implication is that a worker who changes jobs incurs a high mobility cost. In either case we will ignore the worker quit decision.

³⁸To be clear, and following Menzio and Moen (2010), in this case, a filled job will attract the same number of applicants as any newly created unfilled job and will have the same probability of a successful applicant being found and, hence, of the incumbent losing his/her position.

second-period profit in (4) is replaced by

$$f\left((1-\delta)n_{1}+n_{2};x\right)-w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-q\left(\theta_{2}\right))(1-\delta)n_{1}-w_{2,N}\left(n_{2}+q\left(\theta_{2}\right)\left(1-\delta\right)n_{1}\right)-k\overline{n}_{2},w_{2,I}(1-\delta)n_{2}-w_{2,I}(1-\delta)n_$$

where $q(\theta_2)(1-\delta)n_1$ is the number of incumbents who are replaced by new hires, and $n_2 = q(\theta_2)\overline{n_2}$ is the number of new hires *into newly created jobs*.

4.1 No-Replacement Equilibria

We define a no-replacement equilibrium with positive hiring as an equilibrium of the unrestricted model in which replacement does not occur in any state, or equivalently in which $w_{2,N} \ge w_{2,I}$, and $\overline{n}_2 > 0$, in each state. The definition is as in Definition 1, but without the condition $w'_{2,N}(x) \ge w'_{2,I}(x), x \in X$ in condition (i). That is, if $Z, \sigma, \overline{n}_1, (\overline{n}_2(x))$ is a symmetric equilibrium in the restricted model — where the condition is *imposed* — it remains an equilibrium provided $F(\sigma; \overline{n}_1, (\overline{n}_2(x))_{x \in X}; Z) \ge F(\sigma'; \overline{n}'_1, (\overline{n}'_2(x))_{x \in X}; Z)$ where σ' now includes all replacement deviations, wage policies with $w'_{2,N}(x) < w'_{2,I}(x)$ for some x.

We ran simulations based on the parameterisation in Section 3.3.2 with matching function A, and $\delta = 0.1$, $\alpha = 2$, to see where replacement deviations do not improve profits, i.e., where the no-replacement equilibrium exists.

As the coefficient of variation of shocks increases — in particular, the severity of the bad shock worsens — undercutting becomes relatively more attractive. The optimal new-hire undercut wage falls substantially (and hiring increases) as Z_2 falls, whereas in the putative no-replacement equilibrium $w_{2,N}$ falls much less. This allows the undercutting firm to exploit the state of the labour market in the bad state to a greater extent.

Increasing b, and hence the replacement rate, makes it more likely (i.e., for a wider range of other parameter values) that it is optimal to satisfy the noundercutting condition. This is somewhat counter-intuitive in that the downside of undercutting is the risk of replacement, with income falling to b, so a lower risk could make undercutting less costly in terms of the period 1 risk premium. However, an offsetting factor is how much the new-hire wage can be cut. With bhigher, the increase in \mathbb{Z}_2 makes the optimal new-hire wage in the undercutting deviation higher, more than offsetting any benefit from a reduced risk premium.

Similarly, increasing δ , the rate of turnover, also makes it more likely that it is optimal to satisfy the no-undercutting condition. This is again counter-intuitive

in that the relative importance of new hires in period 2 increases, and so in a bad state the benefit from lower new-hire wages, i.e., undercutting, should increase. However, this is offset by the fact that a smaller survival probability reduces the value to stabilising wages, so $w_{2,N}$ will fall in the absence of undercutting. Furthermore, in equilibrium, additional replacement hiring pushes Z_2 up as θ falls and the labour market tightens. The optimal undercutting wage is then higher when δ is higher, and so there is less to be gained from cheaper new hires.

Reducing job creation costs, k, decreases the likelihood that it is optimal to satisfy the no-undercutting condition. In fact, the replacement deviation will dominate for k small enough. Intuitively, as the job opening cost decreases, θ and $q(\theta)$ fall as more jobs are created. Then, the firm is better off setting $w_{2,N} < w_{2,I}$ and offering full insurance to an incumbent if he/she remains in the firm, but with a small risk of replacement $q(\theta)$. The benefit from a lower new-hire wage is greater than the (very small) risk premium that has to be offered to period 1 hires.

However, consider the limiting case of a competitive labour market, as in Snell and Thomas (2010). In this case, if $w_{2,N} < w_{2,I}$ in some state, all incumbents will be replaced, provided that $w_{2,N}$ is not below the supply price of unemployed workers, as the firm can then hire as many new hires as it wants. Since the supply price of an unemployed worker in period 2 will be at least as high as what a replaced worker would obtain from unemployment, changing the contract so that $w_{2,I} = w_{2,N}$ clearly does not leave the firm worse off, as it faces the same costs in period 2. Period 1 hires will weakly prefer this contract because they are not replaced. Thus, satisfying the no-undercutting condition is weakly dominant (and strictly so if the supply price of the unemployed exceeds what a replaced worker obtains). The reason for this apparent discontinuity at the limit is that although as $k \to 0$ the market in the frictional case becomes competitive (both $p \to 1$ and $k/q \to 0$ in an undercutting equilibrium), and the firm can approximately hire at a going wage,³⁹ the firm can only find a replacement for an incumbent with a probability tending to zero (rather than a probability of one in the competitive case); in this case, the no-undercutting condition is (optimally) violated.

The parameter space where there is a no-replacement equilibrium is illustrated in Figure 4. We find that there are usually three local maxima to profits for a firm in a putative no-replacement equilibrium. These are the putative equi-

³⁹The wage elasticity of employment $\partial(\tilde{q}(w_{2,N},x)\overline{n}_2)w_{2,N}/\partial w_{2,N}\tilde{q}(w_{2,N},x)\overline{n}_2 \to \infty$.

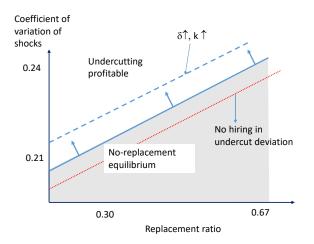


Figure 4: No-replacement equilibrium

librium plan where the condition $w_{2,N} \ge w_{2,I}$ is imposed, an undercutting plan as described above where the firm benefits from bringing in new hires at a wage below that of incumbents, and an undercutting plan where the wage in the bad state falls to match Z_2 which means that the firm cannot hire. The latter is an alternative way of committing to retention (q = 0); however, it can never dominate profits when the condition is imposed, which allows hiring, albeit at a higher wage. The fine dotted line shows the border between areas where either undercutting strategy yields higher profits.⁴⁰

4.1.1 Equilibria with Replacement

Next, we characterise outcomes when replacement *does* occur in some states in equilibrium.⁴¹ Here we find that, in contrast to the case where replacement does not occur, in downturns new-hire wages are *more* rather than less flexible than the wage from the FC model, and, moreover, the incumbent wage will be completely rigid downwards.⁴² The consistency condition for equilibrium must be generalised, so that in any state for which replacement occurs,

⁴⁰One strategy we have not considered is for the firm to dismiss all incumbents and replace them by new hires. However, this is dominated by a contract where $w_{2,I}$ is set equal to $w_{2,N}$ and all incumbents are retained, which reduces period 2 costs for the same employment (it saves on hiring costs), and period 1 hires would be better off as $v(w_{2,N}) \ge Z_2$.

⁴¹The proof of Proposition 1 assumed that there is no replacement in period 2 in *any* state; even with replacement in some states, the statement still holds for non-replacement states x: if there is replacement in some state $x' \neq x$, it modifies the expectation term in (B.1) and (B.4), but they cancel).

 $^{^{42}}$ We are able to test for these implications against alternatives in Section 5.

$$\theta_2(w_{2,N}(x), Z_2(x)) = S_2/(\overline{n}_2(x) + (1-\delta)q(S/\overline{n}_1)\overline{n}_1).$$

Proposition 3 Suppose that replacement occurs in state x in equilibrium. Then, for a given w_1 and n_1 , the wage for new hires is lower (and employment is higher) than it would be in the FC model,⁴³ $w_{2,N} < w_{2,N}^{FC}(x; w_1, n_1) < w_1$; in addition, $w_{2,I} = w_1$.⁴⁴

Intuitively, cutting the new-hire wage makes a job less attractive and therefore the risk of replacement decreases; this positive externality to incumbents makes a wage that is lower than the FC wage $w_{2,N}^{FC}$ optimal. The firm should stabilise the wages of first-period hires because there is no cost of doing this, since the replacement probability is independent of $w_{2,I}$ whenever $w_{2,N} < w_{2,I}$.

The above analysis also applies in the asymmetric information case when the variance of shocks is not too large, as an undercutting deviation from a noreplacement equilibrium in the unrestricted model will have approximately the same benefit or cost. (Details available on request.)

5 Testing the Model's Predictions

In this section, we present tests of the salient features of our model. Our main focus is on the equilibrium predictions in the restricted model under asymmetric information (henceforth RAI) as laid out in Proposition 2. However, we are also able to examine how this model compares with the versions where undercutting occurs and the extent to which asymmetric information is important. We use panel data from the IAB Beschäftigten-Historik to extract composition-free estimates of the annual aggregate wages of new hires and incumbents in Germany from 1978 to 2014.

Referring to high- and low-productivity states as up- and downswing periods, respectively, the RAI model implies three broad stylised facts about new-hire and incumbent wages.

Implication A: In downswings, new-hire and incumbent wages are equal and relatively sticky.

Implication B: Wages in downswings are more closely related to forecasted economic conditions rather than ex-post current economic conditions (in par-

 $^{^{43}}$ See note in Footnote 15.

⁴⁴Formal proof is provided in online Appendix B.3.

ticular, they are better related to the forecast of unemployment in downswings rather than productivity itself).

Implication C: In upswings, the response of incumbent wages is damped relative to those of new hires, with the latter fully adjusting to current conditions.

Implication B is the most important of the three. It relates directly to the main innovation of this article, the analysis of wage contracts under asymmetric information based on part (i) of Proposition 2, and implies distinctive wage behaviour, which to our knowledge is novel (see the discussion after the proposition). Explicitly, the proposition implies that in downswing states, wages are constant across states, so it is only the severity of the downswing states as a whole, which maps to the forecasted unemployment rate, that will matter for the wage. If asymmetric information was not important, i.e., if workers were fully informed about the current state, then regardless of undercutting, Implication B would not hold; we would find instead that ex-post economic conditions in downswings matter more than their forecasts.⁴⁵

Implication A relates directly to undercutting. If undercutting *does* occurs, then implication A would fail: In downswings, incumbent wages would be rigid while new-hire wages would fall (Proposition 3). We are also able to test for this.

For Implication C, the discussion of part (ii) of Proposition 2 implies that incumbent wages increase to a limited extent in upswings, in contrast to the symmetric information model in which they are constant.⁴⁶

In our empirics, we adopt the traditional approach of using (demeaned) unemployment as an indicator of the aggregate state rather than productivity itself. Following the empirical literature in this area, we define upswing (downswing) years as those with below (above) average unemployment. We perform two analyses: one using a single "aggregate" wage series and another where we examine the behaviour of wages in each of six broad sectors⁴⁷ of the economy. Doing so allows for the possibility that each sector approximates a segregated labour market. If this is so, even in only a subset of sectors, then drilling down to the sector level will offer greater power to our tests, as we argue in the following.

⁴⁵Recall from Proposition 1 that the wage would lie at the intersection of the labour-restricted model quasi-supply and demand curves, so the wage response would be muted relative to upswings but depend on ex-post demand, and not on the average level of unemployment.

⁴⁶At the other extreme, in a competitive model where workers can costlessly move to higher paying firms incumbent wages would move with new-hire wages in upswings.

⁴⁷1) Mining, Agriculture, etc., 2) Manufacturing, 3) Power, 4) Construction, 5) Retail, and 6) all other activities. Please refer to Table D.5 in the online Appendix D for more detailed information.

5.1 The Data

For our empirical exercises, we use the IAB Beschäftigten-Historik (BeH, version 10.01), the Employee History File of the Institute for Employment Research (IAB) of the German Federal Employment Agency. The BeH covers all workers who were at least once employed subject to social security in Germany since 1975. Not covered are self-employed, civil servants (Beamte), family workers assisting in the operation of a family business, and regular students. The BeH includes roughly 80% of the German workforce. To protect data privacy, we are not allowed to work with the universality of BeH. Therefore, we use a 20% random sample of all workers that worked full-time during at least one year since 1975.⁴⁸

The BeH is organised by employment spells. A *spell* is a continuous period of employment within an establishment in a particular calendar year. Hence, the maximum spell length is 366 days. For each identified full-time worker of our sample, we observe all existing employment spells, including part-time employment, apprenticeships, etc. These spells are needed to clearly identify new-hire spells.

We define a new-hire spell as the first spell of a worker in the establishment.⁴⁹ Thus, a worker's tenure in an establishment that spans more than one calendar year will consist of multiple spells, with the first being classified as a new-hire spell. For new hires, we focus exclusively on workers transitioning to employment from unemployment in our analyses, for reasons we explain in Section 5.2. We define these hires as workers who were without a job for more than four weeks prior to arriving at the firm.

Our dependent variable is the real average daily wage of a worker over any employment spell. As the earnings data are right-censored at the contribution assessment ceiling ("Beitragsbemessungsgrenze"), only non-censored wage spells

⁴⁸More precisely, we focus on "regular workers" according to the definition used in the Administrative Wage and Labor Market Flow Panel (see Stüber and Seth, 2019): a regular worker is employed full time and belongs to person group 101 (employee s.t. social security without special features), 140 (seamen) or 143 (maritime pilots). Therefore, all (marginal) part-time employees, employees in partial retirement, interns, etc., are not considered regular workers.

⁴⁹Re-hires are therefore not identified as new hires. Our decision to treat returning workers as incumbents is due to the relatively short time of absence; 70% of returners returned after an absence of less than one year, and the average returner time away is approximately 20 months. This suggests that these spells are for workers who have long-term relationships with the establishment and whose absences were temporary (for reasons such as paternity/maternity leave).

are considered in the analyses.⁵⁰ To calculate the average daily real wage and real output per capita in 2010 prices, we use the German Consumer Price Index (CPI). As a proxy for the state of the business cycle (aggregate productivity in our model), we follow the literature (e.g., Bils, 1985; Solon et al., 1994) and use the demeaned aggregate unemployment rate, which we obtained from the Federal Unemployment Agency. The CPI and unemployment series are displayed in Table D.3 in the online Appendix D.

For our analyses, we restrict our attention to the employment spells of fulltime workers⁵¹ between 16 and 65 years of age from West Germany for the period 1978 to 2014. We do not use the first few years of the dataset, as we use workers' establishment tenure as an independent variable in our analyses.⁵² We further keep employment spells only if the workers are employed on December 31^{st} of the respective year.⁵³

The final data set used in our analyses contains more than 97.8 million employment spells for nearly 9 million workers working for more than 2.8 million establishments (see Table D.4 in the online Appendix D). The BeH contains an establishment identifier, but from now on we refer to establishments as "firms" according to the phrasing used in the discussion of the theory.⁵⁴

⁵⁰We drop spells with wages ≥ 0.98 * the contribution assessment ceiling. Dropping topcoded spells leads to an under-representation of highly qualified workers, making the results somewhat less generalisable. Because highly qualified workers' wages are less likely to be covered by a collective bargaining agreement (see, e.g., Düll, 2013) and because uncovered wages are more flexible than covered wages (see, e.g., Devereux and Hart, 2006), we are likely slightly underestimate the cyclicality of the wages. For a quantitative evaluation of the effect of dropping censored spells, see, e.g., Appendix A of Stüber and Beissinger (2012).

 $^{^{51}}$ The BeH documents only total spell earnings, not hours worked in that spell. Therefore, we consider only full-time workers, as these workers' hours are likely to be acyclical. In earlier work that is available upon request, we analyse the time-series properties of an extraneous estimate of the average hours worked in a year by full-time employees in Germany. We find that cyclicality — in the sense of having a significant correlation with output — is relatively weak

 $^{^{52}}$ We drop all spells for which we cannot calculate establishment tenure, i.e., spells that started on (or before) January 1^{st} , 1975.

⁵³This specification implies that we only ever have a maximum of one spell per worker per year, so when we compute yearly averages over spells, we do not more heavily weight those workers with multiple within-year spells. It also excludes most short-lived spells in the data, particularly temporary summer work.

⁵⁴The main results of this paper hinge on estimates that control for match fixed effects, with the underlying assumption being that matches are with establishments, not firms. However, even if matches are formed at the firm level, then using worker-establishment fixed effects will absorb them in any event; their use in this case may be inefficient but will not bias the estimated year effects.

5.2 Extracting Composition-Bias-Free Estimates of New-Hire Wages

We wish to test the model's predictions concerning the cyclical behaviour of new-hire wages relative to those of incumbents. To do this, one must extract estimates of these wages from the panel data, controlling for composition bias. Following Solon et al. (1994), this can be achieved with a two-step method. In the first stage, year effects are extracted from the panel using year dummies, while controlling for worker-firm characteristics. In the second stage, the year effects are treated as composition-controlled estimates of the average new-hire and average incumbent wage in each year. In the two-period asymmetric information model, new hires come from unemployment, not from other firms. Hence, the wage year effects that we would like to identify are those for new hires arriving directly from unemployment — so called UE transitions. We also use match fixed effects (MFE) to control for match quality⁵⁵ in the most general way possible. However, controlling for match quality in this way is not without its problems. If the amount by which new-hire wages are above/below that of incumbents (the "new-hire premium") is permanent during the workers' tenure with the firm, then MFE will absorb them and the measured excess cyclicality of new-hire wages will be zero. By contrast, if these premia are temporary, then they will show up in the estimates — at least to some extent.⁵⁶ Our view is that it is highly unlikely that new-hire premia will be fully persistent; for one thing, on the job search will limit the time which a newly hired worker may be paid below her marginal product. In fact, a number of papers have argued along these lines, most notably Hagedorn and Manovskii (2013). In any event even if

⁵⁵Gertler et al. (2020) argue that the quality composition of the UE transitions is acyclical because they eliminate procyclical job ladder moves, but this is not obvious. Using CPS data, Mueller (2017) argues that the quality of the unemployed pool is countercyclical. With this as a stylised fact, both procyclical and countercyclical match quality are conceivable. For example, it is possible that in upswings when the number of vacancies is growing, a (imperfect) screening process of applicants for jobs results in higher quality workers being over-represented in UE transitions, and UE match quality would be procyclical. Alternatively, if matching out of unemployment was random, then UE match quality may be countercyclical. In the former (latter) case, estimates of new-hire wage cyclicality would be biased away from (towards) zero.

⁵⁶It is easy to show that using MFE will cause downward bias to new-hire premia but whilst this may affect small sample power of a significance (from zero) test it will not drive the estimate to zero asymptotically. We return to this issue when we present our empirical estimates below. We should also note that dropping MFE and controlling for worker fixed effects instead (we follow workers across different firms, so these effects are distinct from MFE) does not qualitatively change the results. The fact that dropping match controls does not alter the results for U-to-E transitions arguably offers some support to Gertler et al.'s (2020) claim that match quality in such transitions is not very cyclical.

MFE did eradicate new-hire premia entirely this would be a double-edged sword in terms of support for our theory; one of the theory's key predictions is that there is a new-hire premium in upswings and if the cyclical new-hire premia are in fact permanent, then using MFE would result in us finding no support for this prediction of our model. In this respect, at least, the use of MFE is conservative because it works against finding in favour of our model.

In the first stage, the primary specification to be estimated is the panel regression

$$w_{ijt} = m_{ijt} + \sum_{\tau=1}^{T} \beta_{\tau}^{I} I_{t}^{\tau} + \sum_{\tau=1}^{T} \beta_{\tau}^{E} E_{t}^{\tau} + \sum_{\tau=1}^{T} \beta_{\tau}^{N} N_{t}^{\tau} + \sum_{k=1}^{2} \lambda_{k} age_{it}^{k} + \sum_{k=1}^{4} \phi_{k} ten_{ijt}^{k} + v_{ijt},$$
(6)

where w_{ijt} is the log of the real average daily wages of the worker *i* in the firm *j* during year *t*, and v_{ijt} is an error term.

The equation allows for three distinct sets of year effects written in the first three summation terms. The first consists of dummies I_t^{τ} ($\tau = 1, \ldots, 37$) with coefficients β_{τ}^I where I_t^{τ} equals one if $t = \tau$ and the worker is an incumbent, but is zero otherwise. An incumbent is currently defined as a worker with more than 365 days of tenure. The β_t^I coefficients are the incumbents' year effects. The second and third sets of dummies E_t^{τ} and N_t^{τ} take the value of one if the wage is from an EE or UE new-hire, respectively.⁵⁷ Otherwise, $t = \tau$ is equal to zero. The β_t^E and β_t^N are the corresponding year effects. In the following analyses, we focus on the year effects of incumbents (β_t^I) and UE new hires (β_t^N). The variable age_{it} is the worker's age in years and ten_{ijt} is the worker's firm tenure measured in days at the end of the spell. Finally, m_{ijt} is a MFE. Note that MFE's control for the sum of a firm j's effect plus a worker i's effect plus a match quality effect.

5.3 Testing the Model

5.3.1 Tests Based on Correlations of Wages with Unemployment

Here, we examine the empirical support for the three implications of the model outlined in Section 5 above. As noted — and as is now standard in this literature — we use the unemployment rate as a proxy for the state of the business cycle (i.e., a proxy for the model's aggregate productivity). We categorise the data

 $^{^{57}}$ We count all transitions into employment that are not EE transitions as UE transitions. Therefore, our UE transitions also include transitions from nonemployment into employment.

into up- and downswing years according to whether demeaned unemployment is, respectively, negative or positive.⁵⁸ We adopt this "levels" approach rather than using changes in unemployment as we argue that it better identifies when wages will be either at the wage floor or dependent on realised shocks. Suppose that the economy moves into recession in period t, so that the no-undercutting constraint binds. This implies that the wage floor at t is above the optimal hiring wage. Suppose that the economy stays around the same level of unemployment. Because the optimal hiring wage at t+1 is below the current wage, the no-undercutting constraint will continue to bind and wages will be at the relevant wage floor for t+1. This argument should hold even at somewhat higher productivity shocks, so with lower unemployment. On the other hand, when sufficiently good shocks happen, the constraint does not bind. In the period after a good shock, if there is a small increase in unemployment, the wage is likely not to be at the wage floor as there is a range for wages below the lagged wage such that wages are not on the floor, but respond to realised shocks (see Figure 3). In summary, at higher unemployment rates, even when the rate falls, the economy is likely to stay at the wage floor; at lower rates, even if the rate goes up, the economy is likely to remain above the floor; consequently, unemployment changes are unlikely to be as important as levels in determining whether wages are on the wage floor or not. The main caveat to this would be if the economy remained at a high unemployment level for a number of periods as the wage should gradually fall and eventually become responsive to ex-post shocks.

We start with a traditional exercise of examining the comovement between unemployment and wages over the business cycle for new hires and incumbents, extended to allow for asymmetric responses in upswings and downswings. We will progress to an analysis more attuned to our theory later on.

Explicitly, we start with the model

$$\beta_t^i = \gamma_d^i \widetilde{u}_{dt} + \gamma_u^i \widetilde{u}_{ut} + \varepsilon_t \quad i = N, I,$$
(7)

where, denoting the demeaned unemployment rate by \tilde{u}_t , \tilde{u}_{ut} (\tilde{u}_{dt}) equals \tilde{u}_t when $\tilde{u}_t < 0$ (> 0) and is zero otherwise. Superscripts N and I denote new hires from unemployment and incumbents, respectively. In keeping with the literature, we refer to the wage cyclicality coefficients (the γ 's) as semi-elasticities. We should emphasise at this point that the γ 's are not structural parameters but are

 $^{^{58}}$ Defining downswings (upswings) as years when unemployment is above (below) its full sample mean gives us 15 upswing years and 22 downswing years (see Table D.3).

merely the (normalised) sample covariances between wages and unemployment in upswings and downswings, respectively.

To proceed to estimation we first difference (7). Both unemployment and our composition free annual wage measures are highly persistent while their common cyclical components (by definition) are not. Therefore, the first differencing here serves to focus inference on the common cyclical components of these two series.⁵⁹ Of course, estimation in first differences is quite common and is usually implemented to remove fixed effects — such as match quality — in a one-step procedure. However, here we control for match quality by adding fixed effects to a first stage in levels. The first differencing occurs in the second stage and is purely a device to sharpen the estimates of cyclicality.

First-differencing (7) gives

$$\Delta \beta_t^i = \gamma_d^i \Delta \widetilde{u}_{dt} + \gamma_u^i \Delta \widetilde{u}_{ut} + \Delta \varepsilon_t \quad i = N, I.$$
(8)

We estimate (8) using composition-controlled wages — the β_t^i — from (6). The results for new hires from unemployment and incumbents are given in the second column of Table 2 below with t-ratios (which here and throughout the paper are computed using Newey West standard errors that are asymptotically robust with respect to heteroscedasticity and autocorrelation) given in brackets.

Table 2: Estimates of Upswing and Downswing Semi-Elasticities

γ_u^N	γ_u^I	γ_d^N	γ_d^I	γ^N_{fd}	γ^{I}_{fd}	$\gamma_u^N - \gamma_u^I$	$\gamma^N_{fd} - \gamma^I_{fd}$
-1.300	-0.985	-0.618	-0.507	-0.527	-0.410	-0.319	-0.117
(5.01)	(3.08)	(1.84)	(1.57)	(2.25)	(1.70)	(2.59)	(0.78)

Note: γ_u^i (γ_d^i) is semi-elasticity in upswings (downswings) of new hires from unemployment (i = N) and incumbents (i = I), respectively. Subscript f indicates the use of forecasted unemployment instead of actual unemployment for downswings.

Both, new-hire and incumbent upswing semi-elasticities are highly significant and correctly signed. Downswing estimates are small relative to their upswing counterparts and are roughly the same for new hires and incumbents. Their

⁵⁹In the extreme case where wages are the sum of a nonstationary component (productivity say) and a stationary component containing the business cycle then it is easy to show that the regression coefficient of wages on unemployment converges to a random variable not a constant. If the innovation in the nonstationary and cyclical components are uncorrelated, then the estimate has a correct mean. However, inference based on the usual t-ratio would obviously be hazardous in such a context. Of course, match-fixed effects in the first stage make within spell wage deviations stationary. However, where the within firm job spells are long as they are in Germany, then the within spell deviations can be highly persistent.

significance is borderline. Taken together, these outcomes support implication A (in downswings, new-hire and incumbent wages are equal and relatively sticky). However, the large and significant upswing semi-elasticity for incumbents jars somewhat with implication C; the model predicts a muted response of incumbent wages to upswings. Nevertheless, the new-hire upswing elasticity is larger than that for incumbents and significantly so, as the penultimate column indicates. It is possible that some degree of on-the-job search — not allowed for in our model — is driving the incumbent semi-elasticity upwards. Overall, the results appear to offer some support for implication C above.

Finding nonzero semi-elasticities with respect to actual unemployment in downswings does not reject the hypothesis that it is forecasts rather than actual values that matter in this context (Implication B). The two series are very highly correlated, so if forecasts were the relevant explanatory variables, we would still expect outcomes to explain wages quite well. Put another way, it could be that once forecasts are controlled for, the outcomes in downswings are not relevant. We now turn to analyse this possibility specifically and the role of forecasts versus outcomes more generally.

Implication B says that in downswings, wages for both classes of workers are more closely related to the forecast of the state of the economy (\hat{x} in theory) than to its actual state. The predictions here in downturns have the flavour of Taylor (1980) contracts where forecasts rather than actual labour market conditions determine wages. However, this is not the case in upturns where new-hire wages appear more attuned to actual conditions rather than forecasts, with new hires experiencing a greater sensitivity to the state of the business cycle relative to incumbents.

To examine Implication B, we estimate a simple forecasting model for unemployment in "bad" states. Explicitly, we estimate an AR(2) model for unemployment using only the years in which unemployment was above its long-term mean ($\tilde{u}_t > 0$). We denote this forecast as \tilde{u}_{dt}^f . If we call the forecast error for these downswing years e_{dt} then we have

$$\widetilde{u}_{dt}^f = \widetilde{u}_{dt} - e_{dt} \quad \text{when } \widetilde{u}_t > 0$$

and where $\widetilde{u}_{dt}^f = 0 \quad \text{when } \widetilde{u}_t < 0$

Implication B says that in downswings, the wages of incumbents and new hires should respond to \tilde{u}_{dt}^{f} rather than \tilde{u}_{dt} . To test this, we amend (7) to give

$$\beta_t^i = \gamma_{fd}^i \widetilde{u}_{dt}^f + \gamma_{fu}^i \widetilde{u}_{ut} + \varepsilon_t \quad i = N, I.$$

Once again we first difference to get

$$\Delta \beta_t^i = \gamma_{fd}^i D x_{dt} + \gamma_{fu}^i D x_{ut} + \Delta \varepsilon_t \quad i = N, I \tag{9}$$

$$Dx_{ut} = \begin{cases} 0 \text{ if } \widetilde{u}_t > 0 \& \widetilde{u}_{t-1} > 0 \\ \widetilde{u}_t \text{ if } \widetilde{u}_t < 0 \& \widetilde{u}_{t-1} > 0 \\ \widetilde{u}_t - \widetilde{u}_{t-1} \text{ if } \widetilde{u}_t < 0 \& \widetilde{u}_{t-1} < 0 \\ -\widetilde{u}_{t-1} \text{ if } \widetilde{u}_t > 0 \& \widetilde{u}_{t-1} < 0 \end{cases},$$
$$Dx_{dt} = \begin{cases} 0 \text{ if } \widetilde{u}_t < 0 \& \widetilde{u}_{t-1} < 0 \\ \widetilde{u}_{dt}^f \text{ if } \widetilde{u}_t > 0 \& \widetilde{u}_{t-1} < 0 \\ \widetilde{u}_{dt}^f \text{ if } \widetilde{u}_t > 0 \& \widetilde{u}_{t-1} < 0 \\ \widetilde{u}_{dt}^f - \widetilde{u}_{dt-1}^f \text{ if } \widetilde{u}_t > 0 \& \widetilde{u}_{t-1} > 0 \\ -\widetilde{u}_{dt-1}^f \text{ if } \widetilde{u}_t < 0 \& \widetilde{u}_{t-1} > 0 \end{cases}$$

We estimate (9) and compare the t-ratios of the downswing semi-elasticities with those from (8). If forecasted rather than actual unemployment is the relevant correlate of wages in downswings, then the significance of the downswing γ 's would be greater in the forecast model and this would be a finding in favour of asymmetric information.⁶⁰

Before proceeding, we note that the AR(2) coefficients for the downswing unemployment forecast are highly significant (the p-value is less than 0.0001) despite the scarcity of data points; unemployment has clearly defined dynamic momentum in the annual frequency during downswings.

The results for γ_{fd}^i are also given in Table 2.⁶¹ We see that for both incumbents and new hires the unemployment forecast in downswings is statistically more important than its actual value. Interestingly, $\gamma_{fd}^N - \gamma_{fd}^I$ is insignificant. (The estimates of γ_{fu}^i and their t-ratios were very close to those for γ_u^i and so we do not report them here).

 $^{^{60}}$ It is tempting to say that in this scenario the actual value is subject to classical measurement error and that the estimates should be downward biased. However, the first differencing implies that this is not the case; the "measurement error" is correlated with the measure in the first-difference estimation.

⁶¹Note that the forecast is a generated regressor. However, the size of regular significance tests — where the null is zero — is not affected (Pagan, 1984). The power of these tests may be affected, however, especially if attenuation bias results. Note also that we do not report γ_u^i again — they are the same as before due to orthogonality of the regressors.

The superior explanatory power of forecasted unemployment over actual unemployment in downturns suggests that the significance of the latter could be due to its correlation with the forecasts. To approach this from another angle, we could add forecast errors to the regression in $(9)^{62}$. If in downswings wages responded to actual recession severity rather than forecasts of it, then these errors should be significant. One problem with this is that this requires us to estimate three coefficients from 36 data points so that any results would be indicative rather than definitive. Nevertheless, executing these tests should at least provide some supplementary evidence about the role of forecasts versus actuals. The $F_{1,32}$ tests for incumbents and new hires were 1.47 and 1.23, respectively, indicating that once forecasts are controlled for actual values are not relevant in downswings. This adds some weight to the evidence in favour of Implication B.

We summarise the results so far. We find good support for implications A and B above of the RAI model and partial support for implication C. On the contrary, there is little support for the symmetric information version of the model. The fact that there appears to be equal treatment in downswings is evidence against the undercutting equilibrium in the model.

5.3.2 Tests Based on Sectoral Data

Empirical exercises such as the one in the previous section now abound in the literature; a large panel data set on wages is used to synthesise composition-free estimates of aggregate wages in order to ascertain (some aspect of) the cyclicality of the economy's wage. Despite the huge dimension of the panel data from which the annual aggregate is derived, the fact remains that the results here and in the literature as a whole rest on a small number of time series observations. Equivalently, there are rarely more than a handful of business cycles on offer from which to draw inferences about wage cyclicality.⁶³ Here we try and bring more data to bear on our empirics by drilling down to the sector level. Explicitly, we obtain composition-corrected wages for new hires and incumbents as we did above, but now for six broad economic sectors. These are

⁶²An interesting suggestion by a referee was to run a "horse race" between forecasts and actuals by including both actual and forecast variates in the regression. However, this set of regressors are highly collinear, and with so few annual data points the power of any test would be extremely questionable. By contrast, the first differenced forecasts and forecast errors used here are far less correlated (largely because their levels are by definition orthogonal). Hence, the approach adopted here.

 $^{^{63}}$ For any given time span of the panel, having a large number of workers within each time period obviously helps improve the precision of the estimates. However, that process is bounded when there are common unobserved macro (time t) components in the errors.

1) Mining, Agriculture, etc., 2) Manufacturing, 3) Power, 4) Construction, 5) Retail, and 6) all other activities.⁶⁴ If we were to obtain measures of sectoral unemployment rates, then we could repeat our estimation exercises sector by sector and see if the predictions of our model hold in each case. Unfortunately, there are no data on sectoral unemployment rates. Instead, we assume that each sector's unemployment rate u_{it} co-moves with the aggregate u_t — at least to some extent. In particular, suppose that

$$u_{it} \approx \delta_i u_t + error_t \quad i = 1, \dots, 6.$$

Repeating the above estimations for each sector separately would then yield semi-elasticities that were each scaled up by δ_i .⁶⁵ Clearly model implications A, B, and C could be assessed exactly the same way as before. If the sectors were segregated labour markets (or if it were approximately so), then the potential heterogeneity in the cyclical responses could add power to our tests. Alternatively, if sectors were not segregated labour markets, all estimates should be close to those obtained for the aggregate. Put another way, if the aggregate estimates are masking some sectoral cyclical responses that are at odds with our model, sector-by-sector estimation may expose this.

We reestimate (8) and (9) for each of the six sectors. The results — the analogues of those for the aggregate given in Table 2 — are given in Table 3.

In all sectors, except sector 2, using a one-tailed test, we find significantly higher upswing elasticities for new hires from unemployment than for incumbents. The amounts by which the new-hire upswing semi-elasticity exceeds that of the incumbents in these sectors varies between 0.39 and 0.71 with an average of around 0.54.⁶⁶ However, in sector 2 (manufacturing) we do not find a significant difference in upswing elasticities. This, combined with the borderline significance of the difference in downswing elasticities in this sector, is broadly consistent with equal treatment. Here, the equal treatment models of Snell and Thomas (2010) and Snell et al. (2018) — models without search frictions — appear to be more relevant. We speculate that manufacturing is likely dominated by high-skilled workers with well-defined jobs, so that getting workers in post

⁶⁴Please refer to Table D.5 in the online Appendix D for more detailed information.

⁶⁵Of course estimates would be subject to downward attenuation bias but as we stated earlier significance tests (from zero) would have the correct size.

⁶⁶Recall that semi-elasticities with respect to sectoral unemployment are unknown; the estimates here are a factor δ_i times these unknown values. However, the significance of the difference in upswing semi-elasticities that is important for validating the model, and for five out of six sectors we find this.

Sector	1	2	3	4	5	6
γ_u^N	-1.120	-1.197	-1.359	-1.018	-1.072	-1.317
	(3.48)	(3.98)	(3.04)	(2.01)	(3.73)	(5.17)
γ^I_u	-0.568	-1.195	-0.646	-1.330	-0.687	-0.810
	(2.53)	(5.74)	(1.60)	(2.95)	(1.97)	(2.01)
γ_d^N	-0.613	-0.862	-0.291	-0.603	-0.773	-0.754
	(1.68)	(3.09)	(1.06)	(2.25)	(2.16)	(2.38)
γ_d^I	-0.654	-0.783	-0.002	-0.701	-0.447	-0.290
	(1.62)	(2.46)	(0.00)	(2.13)	(1.56)	(0.88)
γ_{fd}^N	-0.621	-0.658	-0.624	-0.589	-0.595	-0.759
9	(2.20)	(3.55)	(1.68)	(1.78)	(1.69)	(3.76)
γ^{I}_{fd}	-0.574	-0.459	-0.236	-0.107	-0.393	-0.454
J	(1.63)	(1.81)	(0.84)	(0.42)	(1.61)	(1.69)
$\gamma_u^N - \gamma_u^I$	-0.581	0.021	-0.719	0.341	-0.373	-0.509
	(3.50)	(0.11)	(3.19)	(1.82)	(3.93)	(2.78)
$\gamma_{fd}^N - \gamma_{fd}^I$	-0.047	-0.199	-0.388	-0.482	-0.202	-0.305
	(0.15)	(1.76)	(0.75)	(1.23)	(1.27)	(1.72)

Table 3: Sectoral Estimates of Upswing and Downswing Semi-Elasticities

Note: γ_u^i (γ_d^i) is semi-elasticity in upswings (downswings) of new hires from unemployment (i = N) and incumbents (i = I), respectively. Subscript f indicates the use of forecasted unemployment instead of actual unemployment for downswings.

may be less costly than in other sectors. The wedge between new-hire and incumbent wages is driven mainly by search frictions, so if these search frictions are relatively small, we would expect something approaching equal treatment.

Using a two-tailed test the downswing elasticities are not significantly different across the two types of workers in any sector although a one-tailed test where the alternative is that new-hire wages are more flexible yields marginally significant values in sectors 2 and 6. These findings offer broad support for implication A of the RAI version of the model (in downswings, new-hire and incumbent wages are equal and relatively sticky). There is also some support for implication B, that in downswings, forecasted unemployment is a better correlate of wages than its actual value. Forecasts are better correlates in eight out of the twelve cases. All 12 of the upswing semi-elasticities are significant, and ten of the twelve downswing ones are likewise. Finally, note that the finding that here and in the aggregate results (see Table 2) the downswing semi-elasticities are similar for both incumbents and new hires is at odds with the FC model which predicts that in downswings real wages will be constant for incumbents but falling for new hires (see Section 3.2). Finally, we repeat the (indicative) exercise above, in which forecast errors are added to the specification containing forecasts. This gives us 12 $F_{1,32}$ tests and they are presented in Table 4 below. Ten of the 12 tests are insignificant. However, both of the statistics for manufacturing are significant — a sector we argued above may have a different labour market structure than the other five. Notwithstanding these rejections, we may conclude that there is not strong evidence that the severity of recessions matters for wages once the effects of forecasts have been controlled for.

Table 4: The Significance of Forecast Errors after Controlling for Forecast Effects

Sector	1	2	3	4	5	6
$F_{1,32}^{N}$	0.29	5.24	0.06	1.95	3.40	1.88
$F_{1,32}^{I}$	1.37	7.25	0.85	2.98	1.28	0.02

Notes: $F_{1,32}^N$ and $F_{1,32}^I$ are *F*-tests for significance of forecast errors added to Equation (9).

Robustness Analysis

In our analysis, we chose to use demeaned unemployment as our cyclical indicator. As noted, this choice was partly inspired by our model, but also because it is a standard choice of cyclical indicator in this context. However, unlike much of the other papers in this literature, our results also hinge crucially on how we split the sample into upswings and downswings. In the absence of a more formal mechanism to effect this split (such as a model for structural unemployment), it would be comforting to obtain some kind of external validation for our classification of up- and downswing years. To do so, we examine estimates of Germany's output gap produced by the International Monetary Fund (IMF, see De Masi, 1997). The correlation of demeaned unemployment with this series is -0.6. More importantly in 28 of the 35 years⁶⁷ of the sample, the two series "agree" on the classification of data into up- and downswing years. In seven of the eight years where there is a conflict in this classification, demeaned unemployment is very close to zero.

The fact that in most cases forecasts of unemployment perform as well or better than ex-post unemployment may to some extent be due to the fact that unemployment lags swings in productivity by one or two quarters. To assess

⁶⁷The IMF series begins in 1980 so we lose one data point relative to our core sample.

whether spurious factors such as this could be at play, we estimate a counterfactual model — one where the upswing unemployment variable in (9) is replaced with its forecasts⁶⁸. We do this for the six sectors and for the aggregate. Neither the baseline nor the asymmetric models imply that forecasts are the correct wage correlate in upswings, so we would expect forecasts to be less statistically important than ex-post values. In these counterfactual regressions the upswing elasticities in the six sectors fall markedly in value and significance, with eight of the 12 becoming wholly insignificant whilst the other four are borderline. We should note that the results are equally stark for the aggregate case.

We summarise by saying that the co-movement displayed between compositioncontrolled aggregate wages and unemployment offers broad support for the noreplacement equilibrium in the asymmetric information version of our model. This support is reinforced by sector-level co-movements. There we see a lot of heterogeneity and many estimates are quite different from their aggregate counterparts. However, very few of these differences are in directions that undermine the model's predictions.

5.3.3 Comparing our Empirical Findings with those in the Recent Literature

There is now a vast literature testing the hypothesis that new hires have the same wage cyclicality as incumbents (which we refer to as "equal treatment"⁶⁹). Here, we compare our empirical findings with important recent articles in this literature. Key papers in this context are those by Martins, Solon, and Thomas (2012), Gertler, Huckfeldt and Trigari (2020) and Grigsby, Hurst and Yildirmaz (2021), henceforth MST, GHT and GHY respectively.

Perhaps the first thing to point out is that our results on equal treatment are nuanced compared with those in the literature; in downswings we find equal treatment (although wage changes depend on the forecasted rather than the actual severity of the recession), but in upswings there are relatively small but significant new-hire wage premia. These are complex data features that to our knowledge have not been explored or found before — ones which our model led us to investigate. But the finding of new-hire premia in upswings deserves some discussion in relation to the three papers cited above.

 $^{^{68}}$ As before we use an AR(2) model this time estimated from upswing years only and again we find its coefficients to be highly jointly significant with a p-value of 0.022.

⁶⁹The strongest form of equal treatment is that workers with the same productivity at the same firm receive identical pay. But this is virtually untestable.

MST look at "entry jobs" — jobs that arguably have homogeneous productivity. Although they do not present a statistical test they find wage cyclicality in entry jobs is slightly bigger than existing estimates for stayers (an absolute difference in semi-elasticities of around 0.4). GHT do explicitly test for equal treatment and cannot reject it. Using the SIPP and new hires who have arrived from unemployment, their estimates in first differences (the specification closest to ours) show excess semi-elasticities (henceforth, "ESE's") in the region of -0.5 but these are insignificant. GHY argue that base pay (contracted hourly wages) is the cyclical component of wages. Using accurate payroll data from a large sample of US workers and (inter alia) by comparing the base pay growth of matched pairs of incumbents and new hires they find new ESE's of around -0.2 but again find them to be insignificant.

In our analysis for the whole economy, we find the ESE to be -0.31 in upswings only. We obtain a similar number by averaging estimates across our six sectors. These estimates are quantitatively and statistically important. They are also close to that found for the business cycle as a whole in the papers detailed above. Furthermore, a 95% confidence interval around our upswing ESE estimates includes the values typically found in these other studies, and the converse is also true; a 95% confidence interval around the ESE's found by others includes our estimates. All in all then in terms of point estimates our results on ESE's are not so far away from those found previously. However, we argue that our data offer more precision and test power than is typically found in other work. 70

A distinctive feature of our data is that we observe a large number of workers over a very long time span — something that is likely to yield high test power in the context of analysing wage cyclicality. There are two factors limiting test power in this context; the number of new hires in the sample and the amount of business cycle variation during the sample period. In panel data, macroeconomic observables, including the rate of unemployment, account for only a tiny fraction of the variation in a worker's wage. Therefore, precise estimates of the newhire premium require a large number of new hires to average out idiosyncratic "noise" in the wage ⁷¹. Here is where the first factor is important. The second

⁷⁰GHT obtain standard errors by clustering by worker. However, if there are other macroeconomic variates besides unemployment impacting wages, this could cause standard errors to be biased. By using composition-controlled yearly averages, we have avoided this potential pitfall.

⁷¹Of course when controlling for composition effects incumbent worker observations help to obtain good estimates of tenure, experience etc. But when one is solely concerned with

factor matters for obvious reasons; aggregate wage cyclicality is a pure time series phenomenon and precise estimates of new-hire ESE's also require a fair amount of cyclical variation; typically this means that we need a reasonably large "T".

In terms of the first factor, our 20% random sample of BeH workers contains more than 18 million new-hire spells (see Table D.4 in the online Appendix D) compared to GHT's $6,000^{72}$ or so. In the Appendix, we present indicative simulations to estimate the power of GHT's test using the structure and size of their data. Taking the point estimates in their first differenced specification as true values (for the new-hire ESE their point estimate is -0.44) we find that there is a less than one in three chance of finding significant new-hire excess cyclicality with their sample size. Even when the sample size is doubled, the chance only rises to just below one in two.

With respect to the second factor, GHY have only one business cycle over 9 years (2008–2016). On the contrary, our data span nearly 40 years and encompass four of the post-war business cycles. Of course, GHY are fully aware of the cyclical variation issue. To counter it, they drill down to the state level and use local monthly unemployment rates. However, it is not obvious that state unemployment rates represent the appropriate outside option for many workers, particularly those who are mobile and have high skill levels. Additionally, the state unemployment data are themselves very noisy — they are estimated from a small number of CPS data points per state (the Bureau of Labor Statistics reports 90% confidence intervals for monthly unemployment rates in each state, and in 2023 the average width of these was around 1.2%). These measurement errors may cause serious attenuation bias. One final point here relates to the data period used by these authors. There is suggestive evidence that the Great Recession was a period of generally muted real wage responses to unemployment — at least relative to earlier epochs (see, e.g., the discussion in Elsby et al., 2016). If so, we may expect that all wage cyclicality measures, including those of new hires, will also be muted⁷³.

We summarise by saying that our estimates of new-hire ESE in upswings are quantitatively well in line with those found by others for the entire cycle. The

the comovement of new-hire wages and unemployment, then this is effectively a time series regression of (composition-controlled) average new-hire wages in the year on unemployment in that year. Having a large number of new-hire wages in each year will reduce the "noise" component of their year averages.

⁷²Using GHT's online data file we count 9,404 "U to E" wages. However, in the first differenced specification, this number falls to less than 6,000.

⁷³For example GHY's stayer semi elasticities (using their preferred base pay measure) are around -0.3 which is lower than other numbers that have typically been found in US data.

fact that we find significance of these estimates where others do not may be down to the power that our large sample yields plus the fact that unlike others, we analyse up- and downswings separately.

Finally, we mention two other papers that use BeH data: In Snell et al. (2018) we tested for equal treatment in up- and downswings. An important difference there was that we defined upswings (downswings) as periods of positive (negative) GDP growth rather than above (below) average unemployment. In that context we found evidence of equal treatment in both up- and downswings, but in the case of the former the evidence was marginal. Bauer and Lochner (2020) find no significant new excess cyclicality once attention is confined to workers arriving from unemployment (and also controlling for certain history dependence variates). However, they only have a 2% random sample of BeH workers (the SIAB) with fewer than 950,000 workers (only a small proportion of whom will be new hires) over 15 years (2000 to 2014 — effectively only one business cycle). As noted above, we observe more than 18 million new-hire spells from a 20% random sample of BeH workers over nearly 40 years. Again, the difference between our results and theirs may be down to power.

6 Concluding Comments

We have considered a simple frictional model of the labour market, which has equilibrium wage contracts where incumbents are not undercut and displaced by new hires, leading to a degree of downward wage rigidity for new hires. Rigidity arises from worker risk aversion and a desire to limit temporal wage variation for incumbent workers, which also transmits to new hires in downturns. Because period 2 new-hire wages are allocational, the response of unemployment and job openings to negative shocks is amplified. In an important extension, we show that the interplay with asymmetric information can substantially enhance downward wage rigidity and increase the responsiveness of unemployment and job openings to productivity shocks. We find that empirical results from the German BeH panel data are broadly supportive of the predictions of the asymmetric information version of the model; new-hire wages respond to upswings more aggressively than those of incumbents and in downswings both classes of wage are relatively sticky and respond more to forecasts of the downswing state than the actual state itself.

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Online Appendices

A Extension in Asymmetric Information Model to Employment-Contingent Contracts

The analysis in Section 3.3 of the paper concerned the case in which no variables that are observable to both parties can be contracted upon. While in a model which features a frictional labour market, it is plausible to suppose that it may be difficult to condition contracts on aggregate labour market variables such as wages offered by other firms, employment at the firm in which the worker is employed may be a variable that could be conditioned upon. Intuitively, in a low-productivity state, employment could be specified to be inefficiently low to discourage the firm from under-reporting productivity in better states to avail itself of lower wages, given that such inefficiency harms profits more in the better state. Here, we consider how matters change if employment-contingent contracts are possible; for small variations in productivity, in fact, it does not affect the constant wage result.

Proposition A.1 (Employment-contingent contracts) In the restricted asymmetric information model where period 2 employment is contractible and with a single period 2 productivity state \hat{x} , suppose that for given parameter values, there is a unique equilibrium and that the no-undercutting condition binds strictly. Then, in a perturbed version of this model where this state is replaced with two different equal probability states, $x' = \hat{x} - \varepsilon$ and $x'' = \hat{x} + \varepsilon$ (i.e., with expected value \hat{x}), and assuming the differentiability of equilibrium values,⁷⁴ equilibrium period 2 wages are approximately constant across these states, provided that the perturbation ε is sufficiently small; formally, $\lim_{\varepsilon \to 0_+} (w_{2,N}(x'') - w_{2,N}(x'))/2\varepsilon = 0.^{75}$

A rough intuition for this result is as follows: Given that for a small perturbation in both states x' and x'', the no-undercutting condition continues to bind, and wages for incumbents and new hires are equal. If in the lower-productivity state, wages are lower by more than a second-order amount, there will be, as earlier, a first-order incentive for the firm in x'' to announce x', as there is a benefit both in terms of lower wages for period 1 hires and in terms of reducing

⁷⁴That is, assuming that Z_2 is a differentiable function of ε in a neighbourhood of 0.

⁷⁵Formal proof is provided below in section B.4.

the hiring cost for new hires. To prevent this, hiring can be reduced in x', which would be costly in the state x'', but it must be reduced by a large amount, given that hiring is initially (in the unperturbed equilibrium) optimal; this cut in hiring will also impose *first-order* costs in x', swamping any benefit from the lower wages (which are second-order).

B Proofs and Further Simulations

B.1 Proof of Proposition 1

Proof. We derive the necessary conditions by considering the following Lagrangian, assuming that there is an interior solution.

$$\mathcal{L} = (f(\tilde{q}_{1}(V_{1})\overline{n}_{1}) - w_{1}\tilde{q}_{1}(V_{1})\overline{n}_{1} - k\overline{n}_{1}) + E_{x'}[(f((1-\delta)\tilde{q}_{1}(V_{1})\overline{n}_{1} + \tilde{q}(w_{2,N}, x')\overline{n}_{2}; x') - w_{2,I}(1-\delta)\tilde{q}_{1}(V_{1})\overline{n}_{1} - w_{2,N}\tilde{q}(w_{2,N}, x')\overline{n}_{2} - k\overline{n}_{2}] + E_{x'}[\lambda_{x'}(w_{2,N} - w_{2,I})],$$

where $\tilde{q}(w_{2,N}, x) := q(\theta_2(w_{2,N}, Z_2(x)))$ and we write $\tilde{q}' \equiv \partial \tilde{q}/\partial w_{2,N}$, and where $\tilde{q}_1(V_1)$ is defined analogously to $\tilde{q}(w_{2,N}, x)$, $\lambda_{x'}$ is the multiplier on the $w_{2,N} \geq w_{2,I}$ constraint in state x' and recall $V_1 = v(w_1) + E[\delta Z_2(x') + (1-\delta)v(w_{2,I}(x'))]$. This expression leads to the first-order conditions:

$$\tilde{q}_{1}'v'(w_{1})\,\overline{n}_{1}(f'(n_{1}) - w_{1} + E_{x'}[f'(n;x')(1-\delta) - w_{2,I}(x')(1-\delta)]) - \tilde{q}_{1}(V_{1})\,\overline{n}_{1} = 0$$
(B.1)

$$f'(n;x)\tilde{q}(w_{2,N},x) - w_{2,N}\tilde{q}(w_{2,N},x) - k = 0$$
(B.2)

$$f'(n;x)\,\tilde{q}'\overline{n}_2 - \tilde{q}\left(w_{2,N},x\right)\overline{n}_2 - w_{2,N}\tilde{q}'\overline{n}_2 + \lambda_x = 0 \tag{B.3}$$

$$\tilde{q}_{1}'v'(w_{2,I}(x))(1-\delta)\overline{n}_{1}(f'(n_{1})-w_{1}+E_{x'}[f'(n;x')(1-\delta)-w_{2,I}(x')(1-\delta)]) - \lambda_{x} - (1-\delta)\tilde{q}_{1}(V_{1})\overline{n}_{1} = 0$$
(B.4)

together with the complementary slackness conditions. Note that (B.2) im-

plies the labour demand equation

$$f'(n) = w_{2,N} + k/q.$$
 (B.5)

From (B.1) and (B.4),

$$\frac{v'(w_1)}{v'(w_{2,I})}\left(q_1 + \frac{\lambda_x}{\overline{n}_1(1-\delta)}\right) = q_1.$$
(B.6)

Using this to eliminate λ_x in (B.3):

$$f'(n;x)\,\tilde{q}'\overline{n}_2 - \tilde{q}\,(w_{2,N},x)\,\overline{n}_2 - w_{2,N}\tilde{q}'\overline{n}_2 + q_1\overline{n}_1\,(1-\delta)\left(\frac{v'(w_{2,I})}{v'(w_1)} - 1\right) = 0.$$
(B.7)

There are two cases:

A. If $\lambda_x = 0$, then (B.6) $w_1 = w_{2,I}$, and (B.7) implies

$$f'((1-\delta)n_1 + n_2; x) q'\overline{n}_2 - w_{2,N}q'\overline{n}_2 - q\overline{n}_2 = 0,$$
(B.8)

and hence, we get the (full commitment, FC) quasi-supply locus:

$$q^2 \left(\tilde{q}' \right)^{-1} = k.$$
 (B.9)

We characterise points that satisfy (B.9). For clarity, we let $\tilde{w}_{2,N}$ and $\tilde{\theta}_2$ denote the individual firm's values. Then

$$\tilde{q}' = \frac{dq}{d\theta_2} \frac{d\tilde{\theta}_2}{d\tilde{w}_{2,N}} \mid_{Z_2 \text{ constant}} .$$

From (3),

$$\frac{d\tilde{\theta}_2}{d\tilde{w}_{2,N}} \mid_{Z_2 \text{ constant}} = -\frac{pv'(w_{2,N})}{\frac{dp}{d\theta_2} \left(v(w_{2,N}) - v(b)\right)},$$

and differentiating $q = p \cdot \theta_2$ to eliminate $\frac{dp}{d\theta_2}$, we obtain

$$\tilde{q}' = -\frac{dq}{d\theta_2} \frac{p\theta_2 v'(w_{2,N})}{\left(\frac{dq}{d\theta_2} - p\right) \left(v(w_{2,N}) - v(b)\right)}.$$
(B.10)

After rearrangement,

$$\frac{q^2}{\tilde{q}'} = q^2 \frac{\left(1 - \frac{\theta_2}{q} \frac{dq}{d\theta_2}\right)}{\theta_2 \frac{dq}{d\theta_2}} \frac{v\left(w_{2,N}\right) - v\left(b\right)}{v'\left(w_{2,N}\right)}.$$
(B.11)

From our assumption on q, q^2 is increasing in θ_2 , and the second term in the product is also increasing in θ_2 by assumption (it is the inverse of $\frac{q(\theta)\epsilon_q(\theta)}{(1-\epsilon_q(\theta))}$) while the final term is increasing in $w_{2,N}$. Thus, the locus of values of θ_2 and $w_{2,N}$ such that (B.9) holds is negatively sloped. Recall that $n_2 = p(\theta_2) S_2$, and as p' < 0, there is a one-to-one negative relationship between n_2 and θ_2 . Therefore, (B.9) can be solved to give a positively sloped locus of values for n_2 and $w_{2,N}$ that is compatible with equilibrium.

Next, (B.2) is negatively sloped in $n_2 - w_{2,N}$ space by f'' < 0 and $q(\theta_2) = q(p^{-1}(n_2/S_2)), q' > 0, p' < 0$. Therefore, $(w_{2,N}, n_2)$ is at the unique intersection point, denoted by $(w_{2,N}^{FC}(x; w_1, n_1), n_2^{FC}(x; w_1, n_1))$ in the text. Since $w_{2,N} \ge w_1$ implies $\lambda_x = 0$ (see next line), claim (b) is established.

B. If $\lambda_x > 0$, then $w_{2,I} = w_{2,N}$ and from (B.6) $w_1 > w_{2,I} = w_{2,N}$, and (B.7) implies

$$(1-\delta)n_1 - (f'(n;x)\,\tilde{q}'\bar{n}_2 - w_{2,N}\tilde{q}'\bar{n}_2 - q\bar{n}_2) = n_1\,(1/v'(w_1))\,((1-\delta)\,v'(w_{2,N}))\,.$$
(B.12)

Thus, eliminating f' using (B.2), and using $n_2 = q\overline{n}_2$,

$$1 + \frac{(1 - k\tilde{q}'/q^2) n_2}{n_1 (1 - \delta)} = \frac{v'(w_{2,N})}{v'(w_1)},$$
(B.13)

so that as $w_{2,N} < w_1$, $k\tilde{q}'/q^2 < 1$, i.e., $k < q^2/\tilde{q}'$. (The locus of points satisfying (B.13) is the quasi-supply curve below w_1 .) Holding n_2 (and hence θ_2) constant, q^2/\tilde{q}' increases in $w_{2,N}$, so the locus of points $(n_2, w_{2,N})$ satisfying (B.13) must lie above — $w_{2,N}$ is higher — that defined by (B.9). At $w_{2,N} = w_1$ we have $k\tilde{q}'/q^2 = 1$, so the two loci coincide. Thus, the downward sloping (B.2) must intersect (B.13) at a higher wage and a lower value for n_2 than it would intersect (B.9). Thus, claim (a) is established.

Since $\lambda_x > 0$ if and only if $w_{2,N} < w_1$, the final claim of the proposition follows.

B.2 Proof of Proposition 2

Proof. (i) Let $x' := \hat{x} - \varepsilon, x'' := \hat{x} + \varepsilon$. Consider an arbitrary sequence $\{\varepsilon_s\}_{s=0,1,\ldots}, \varepsilon_s > 0, \varepsilon_s \to 0$; we show that there is some \bar{s} such that for $s \geq \bar{s}$, wages are

equal in both states: $w_{2,I}(x') = w_{2,I}(x'') = w_{2,N}(x') = w_{2,N}(x'')$.⁷⁶ By the assumptions of continuity and the binding no-undercutting condition at \hat{x} ,

$$\lim_{s \to \infty} w_{2,I}(x') = \lim_{s \to \infty} w_{2,I}(x'') = \lim_{s \to \infty} w_{2,N}(x') = \lim_{s \to \infty} w_{2,N}(x'') = \hat{w}_{2,I} = \hat{w}_{2,N},$$
(B.14)

where the original equilibrium corresponding to \hat{x} is denoted by $\hat{}$. In what follows, we will deal with the case where $w_{2,I}(x') \leq w_{2,I}(x'')$ infinitely often as $s = 0, 1, \ldots$, so we consider below the circumstances in which this is true; the arguments apply equally to the opposite case. To consider this case, we define

$$C(w_{2,N}, x'') := (k/q(\theta_2(w_{2,N}, Z_2(x''))) + w_{2,N})$$

and

$$w^{**}(x'') \in \arg\min_{w_{2,N}} \left(k/q \left(\theta_2(w_{2,N}, Z_2(x'')) \right) + w_{2,N} \right)$$
(B.15)

where $\theta_2(w_{2,N}, Z_2(x''))$ is as defined in (3); $C(w_{2,N}, x'')$ is the cost per period 2 hire in state x''(k/q + w) is the total cost of a new hire), while $w^{**}(x'')$ is the wage that minimises this cost. It is independent of the number of hires, and the cost is strictly convex in $w_{2,N}$ (hence, $w^{**}(x'')$ is unique).

To see this, as earlier, write $q\left(\theta_2(w_{2,N}, Z_2(x''))\right) \equiv \tilde{q}(w_{2,N}, x'')$, so

$$\frac{dC(w_{2,N}, x'')}{dw_{2,N}} = -\frac{k\tilde{q}'}{\tilde{q}^2} + 1$$

$$= -\frac{k}{\tilde{q}^2} \frac{\theta_2 \frac{dq}{d\theta_2}}{\left(1 - \frac{\theta_2}{q} \frac{dq}{d\theta_2}\right)} \frac{v'(w_{2,N})}{v(w_{2,N}) - v(b)} + 1,$$
(B.16)

using (B.10). Given that $\tilde{q}' > 0$ (a higher wage increases the job-filling rate), the second term in the product is $q(\theta_2) \epsilon_q(\theta_2) / (1 - \epsilon_q(\theta_2))$ and therefore is decreasing in θ_2 (by assumption) and, hence, also decreasing in $w_{2,N}$, while the final term in the product is also decreasing in $w_{2,N}$, we have

$$\frac{d^2 C\left(w_{2,N}, x''\right)}{dw_{2,N}^2} > 0. \tag{B.17}$$

Additionally, given the assumption that the no-undercutting condition is

 $^{^{76} {\}rm The}$ dependence of values on ε_s will mostly be left implicit to avoid the notation becoming more cluttered.

strictly binding initially, we have $\hat{w}_{2,N} > w^{**} := w^{**}(x)$ (the value for $w^{**}(x'')$ when $\varepsilon = 0$, being equal to the optimal hiring wage in the unperturbed model), and therefore, by (B.14) and the continuity of $w^{**}(x')$ and $w^{**}(x'')$ in ε (by the Theorem of the Maximum, as they are both unique by the strict convexity of C and C is continuous in Z and hence in ε),

$$\lim_{s \to \infty} w^{**}(x') = \lim_{s \to \infty} w^{**}(x'') = w^{**} < \hat{w}_{2,N}.$$
 (B.18)

Profits in period 2, in state x'', are

$$\max_{n_2} \left(f\left((1-\delta)n_1 + n_2; x'' \right) - w_{2,I}(x'')(1-\delta)n_1 - C\left(w_{2,N}(x''), x'' \right) n_2 \right).$$

In state x'', the firm can claim that x' occurred and make non-negative savings in wages paid to incumbents because $w_{2,I}(x') \leq w_{2,I}(x'')$. It follows that we must have

$$C(w_{2,N}(x''), x'') \le C(w_{2,N}(x'), x'')$$
(B.19)

since otherwise, by announcing x', hiring costs are also reduced.

There are three possibilities to consider, and at least one of which must occur infinitely often along the sequence $s = 0, 1, \ldots$ First, $w_{2,N}(x') < w_{2,N}(x'')$. From (B.19), $w_{2,N}(x') < w^{**}(x'')$ by (B.17). But, as $s \to \infty$, a contradiction occurs in view of $\lim_{s\to\infty} w_{2,N}(x') = \hat{w}_{2,N}$ and (B.18).

On the other hand, if $w_{2,N}(x') > w_{2,N}(x'')$, then by (B.19) and (B.17), $w_{2,N}(x') > w^{**}(x'')$. However, we have

$$w_{2,N}(x') > w_{2,N}(x'') \ge w_{2,I}(x'') \ge w_{2,I}(x')$$

where the second inequality follows from no-undercutting and the final inequality by hypothesis. However, consider a change where $w_{2,N}(x')$ is cut to $w_{2,N}(x'')$ and $w_{2,I}(x')$ increases to $w_{2,I}(x'')$ if it is initially below this value. This changed contract satisfies no-undercutting and (trivially) incentive compatibility. The decrease in $w_{2,N}(x')$ reduces hiring costs by (B.14) and (B.18), which implies $w_{2,N}(x') > w^{**}(x')$ for a large s. Additionally, for s large enough, $w_{2,I}(x'')$ $< w_1(\varepsilon_s)$ by the binding non-undercutting condition in problem A (from Proposition 1, this implies $\hat{w}_{22} < \hat{w}_1$), (B.14) and, by assumption, $\lim_{s\to\infty} w_1(\varepsilon_s) = \hat{w}_1$ using an obvious notation. Then, v'' < 0 implies that a small reduction in w_1 leaving V_1 constant will reduce expected wages while leaving the hiring constant. Therefore, for a large enough s, the contract is not optimal, contrary to the assumption. The final possibility has $w_{2,N}(x') = w_{2,N}(x'')$. By no-undercutting, then,

$$w_{2,N}(x') = w_{2,N}(x'') > w_{2,I}(x'') = w_{2,I}(x'),$$

where the final equality follows by incentive compatibility (otherwise, x' would be announced because the incumbent wages would be lower), and the inequality is strict under the assumption that it is not a constant wage contract. Similar to the previous case, both $w_{2,I}(x'')$ and $w_{2,I}(x')$ can be increased by the same small amount without violating incentive compatibility or no-undercutting, which is compensated by a small reduction in $w_1(\varepsilon_s)$, reducing expected wages paid to period 1 hires. Thus, again, the equilibrium contract is not optimal, contrary to assumption.

(ii) Period 2 profits from the contract for state x in state x' can be written as

$$\pi(x, x') := \max_{n_2} \{ f\left((1-\delta)n_1 + n_2; x' \right) - w_{2,I}(x) \left(1-\delta \right) n_1 - C\left(w_{2,N}(x), x' \right) n_2 \}.$$

We proceed in a number of steps. (a) Suppose that there is a binding incentive compatibility constraint between states x' and x'' such that $\pi(x', x') = \pi(x'', x')$ and $C(w_{2,N}(x'), x') > C(w_{2,N}(x''), x')$, so the firm benefits from announcing x'' in state x' from the point of view of new-hire costs. Incentive compatibility implies $w_{2,I}(x') < w_{2,I}(x'')$. Then consider replacing the x' contract by that at x'' (holding n_1 constant). This must trivially satisfy incentive compatibility and no-undercutting and leave ex-post profits unchanged. However, since $w_{2,I}$ is increased in state x', ex-ante utility V_1 rises, which reduces period 1 hiring costs; hence, profits increase, contrary to optimality. We conclude that $\pi(x', x') =$ $\pi(x'', x')$ implies $C(w_{2,N}(x'), x') \leq C(w_{2,N}(x''), x')$, and hence, by incentive compatibility, $w_{2,I}(x') \geq w_{2,I}(x'')$ (and if the first inequality is strict or an equality, so is the second and vice versa).

(b) Let $X' \subseteq X$ be such that for $x \in X'$, $w_{2,I}(x) > w_1$. We show that $X' = \emptyset$. For $x' \in X'' := X \setminus X'$, $x \in X'$, we cannot have $\pi(x', x') = \pi(x, x')$, since $w_{2,I}(x') < w_{2,I}(x)$, contradicting (a). Therefore, $\pi(x', x') > \pi(x, x')$ (incentive compatibility is slack). Hence, we can find (by X finite) an $\eta > 0$ such that $\pi(x',x') \geq \pi(x,x') + \eta$ for all $x' \in X''$, $x \in X'$. Next, cut $w_{2,I}(x)$ by $\varepsilon < \eta((1-\delta)n_1)^{-1}$ for all $x \in X'$; this does not affect the incentive compatibility between $x, x'' \in X'$ as the profits change by the same amount in each state, and by construction of ε , $\pi(x',x') > \pi(x,x')$, $x' \in X''$, $x \in X'$. As $\pi(x,x)$ is increased for each $x \in X'$ by $\varepsilon(1-\delta)n_1$, $\pi(x,x) > \pi(x',x)$, $x' \in X''$, as the RHS is unchanged and a weak inequality held before the change. Thus, the (global) IC is satisfied. No undercutting is satisfied because only $w_{2,I}$ is cut. If $X' \neq \emptyset$, for a sufficiently small ε , this uniform cut in $w_{2,I}$ in all states where $w_{2,I} > w_1$ and a corresponding increase in w_1 to leave V_1 unchanged increases profits by standard consumption smoothing arguments (hold n_1 constant), i.e., a profitable deviation that is contrary to the assumption. We conclude that $X' = \emptyset$, i.e., $w_{2,I}(x') \leq w_1$ all $x' \in X$.

(c) Let $\hat{X} := \arg \max_{\hat{x}} w_{2,I}(\hat{x})$. If this is a singleton, $\{x\}$, then by part (a), there is no other state x' with $\pi(x', x') = \pi(x, x')$. It follows that provided that the no-undercutting constraint is slack in state $x, w_{2,N}(x) = w^{**}(x)$ and, hence, $w_{2,N}(x) = w_{2,N}^{FC}(x, w_1, n_1)$, as otherwise if $w_{2,N}(x) \neq w^{**}(x)$ a sufficiently small change in $w_{2,N}$ towards w^{**} increases profits in state x (by the strict convexity of $C(\cdot, x)$), satisfies no-undercutting, violates no $\pi(x', x') \geq \pi(x, x')$ constraint for all $x' \neq x$, and relaxes $\pi(x, x) \geq \pi(x', x)$ for $x' \neq x$. If no-undercutting binds in state x, this argument implies $w_{2,N}(x) \geq w^{**}(x)$, as $w_{2,N}$ can be increased if $w_{2,N} < w^{**}$ and, hence, $w_{2,N}(x) \geq w_{2,N}^{FC}(x, w_1, n_1)$.

If \hat{X} is not a singleton, by a similar argument, consider $x \in \hat{X}$ such that $w_{2,N}(x) \neq w^{**}(x)$. If no-undercutting is not binding in state x, change $w_{2,N}(x)$ towards $w^{**}(x)$ by an amount ε such that $C(w_{2,N}(x), x)$ falls. Again, by part (a) for all $x' \notin \hat{X}$, we have $\pi(x', x') > \pi(x, x')$, and provided that ε is small enough, these incentive compatibility and no-undercutting constraints are not violated. If any incentive compatibility constraint for $x'' \in \hat{X}$ is violated, replace $w_{2,N}(x'')$ by the new value of $w_{2,N}(x)$; this increases ex-post profits in x'' and does not affect period 1, as $w_{2,I}$ is unchanged. Profits increase by this change, contrary to the assumption. Therefore, $w_{2,N}(x) = w^{**}(x)$ for all $x \in \hat{X}$. If no-undercutting binds at the lowest $w_{2,N}(x), x \in \hat{X}$, again, $w_{2,N}(x) \ge w^{**}(x)$.

(iii) Follows from (ii) (b) above. \blacksquare

B.3 Proof of Proposition 3

Proof. If replacement occurs, as in Section 3.2, the firm must locally maximise profits plus weighted incumbent utility:

$$f((1-\delta)n_1 + n_2; x) - w_{2,I}(1-\delta)(1-q)n_1 - w_{2,N}(q(1-\delta)n_1 + n_2) - k\overline{n}_2 + n_1(1/v'(w_1))((1-\delta)(1-q)v(w_{2,I}) + \delta Z_2 + (1-\delta)qv(b)),$$

where \overline{n}_2 is again the number of *new* jobs created, and $n_2 = q\left(\theta\left(w_{2,N}, Z_2\left(x\right)\right)\right)\overline{n}_2$. Note that the probability of replacement q is accounted for in the composition of period 2 workers and workers' period 1 utility. Then, differentiating with respect to $w_{2,I}$,

$$(1-\delta)(1-q)n_1 = n_1 (1/v'(w_1)) ((1-\delta) (1-q) v'(w_{2,I}))$$

so that $w_1 = w_{2,I}$, as expected. Differentiating with respect to $w_{2,N}$, we obtain

$$f'(n;x) \tilde{q}' \overline{n}_2 + (1-\delta)n_1(w_{2,I} - w_{2,N})\tilde{q}' - w_{2,N}\tilde{q}' \overline{n}_2 - q\left((1-\delta)n_1 + \overline{n}_2\right) + n_1\left(1/v'(w_1)\right)\left(1-\delta\right)\left(q'\right)\left(v\left(b\right) - v\left(w_{2,I}\right)\right) = 0$$

where the last term on the left hand side is the extra cost of compensating period 1 hires for their increased likelihood of replacement (defining \tilde{q}' as before). Differentiating with respect to \bar{n}_2 ,

$$f'(n;x)q = w_{2,N}q + k.$$
(B.20)

Thus, employment is on the labour demand curve, as in Footnote 13. We can combine these latter two equations to obtain

$$(k/q) \,\tilde{q}'\overline{n}_2 + (1-\delta)n_1\tilde{q}' \left((w_{2,I} - w_{2,N}) + (1/v'(w_1)) \left(v(b) - v(w_{2,I}) \right) \right) = q \left((1-\delta)n_1 + \overline{n}_2 \right)$$

or

$$k\tilde{q}'/q^{2} = 1 + (1-\delta)n_{1}\tilde{q}'\left((w_{2,N} - w_{2,I}) + (1/v'(w_{1}))\left(v(w_{2,I}) - v(b)\right)\right)/q\overline{n}_{2} + (1-\delta)n_{1}/\overline{n}_{2}$$
(B.21)

Both the second and third terms on the right-hand side (henceforth RHS)

of (B.21) are positive, the second as v is concave, $w_{2,I} = w_1$ from the above, $w_{2,I} > w_{2,N}$ (as replacement occurs) and $b \le w_{2,N}$. Recall from the proof of Proposition 1 that \tilde{q}'/q^2 is decreasing in θ and $w_{2,N}$. Thus, compared to the quasi-supply of FC given by (B.9), at fixed θ , or equivalently fixed n_2 given $n_2 = p(\theta_2) S_2$ as in Figure 2, $w_{2,N}$ must be lower to satisfy (B.21). Thus, the intersection with the downward sloping labour demand curve $f'(n) = w_{2,N} + k/q$ (see Footnote 13) must occur at a lower wage and higher employment than in the FC solution.

Finally, $w_{2,N}^{FC}(x; w_1, n_1) < w_1$ because otherwise, the commitment solution could be implemented, which would be superior.

B.4 Proof of Proposition A.1

Proof. Incentive compatibility in state x'' requires that

$$(f((1-\delta)n_{1}+n_{2}(x'');x'')-w_{2,I}(x'')(1-\delta)n_{1}-C(w_{2,N}(x''),x'')n_{2}(x'')) \geq (f((1-\delta)n_{1}+n_{2}(x');x'')-w_{2,I}(x')(1-\delta)n_{1}-C(w_{2,N}(x'),x'')n_{2}(x')),$$
(B.22)

where $C(\cdot, \cdot)$ is the total cost of a new period 2 hire as defined in the proof of Proposition 2, and the hire in the state x' is denoted $n_2(x')$, etc. We will write $w_{2,N}(x')$ as $w'_{2,N}$ etc. to simplify the notation below.

We start by assuming that the optimal contract is differentiable (from the right) at $\varepsilon = 0$. Consider ε small and take a first-order approximation for (B.22) around the initial equilibrium⁷⁷ at \hat{x} , where (B.22) holds trivially with equality (and where as in the proof of Proposition 2 we use a $\hat{}$ to denote the corresponding initial equilibrium contract) and define deviations as $\Delta w'_{2,I} := w'_{2,I} - \hat{w}_{2,I}$ etc., and where $\Delta x''(= -\Delta x') := x'' - \hat{x} = \varepsilon$: $f'(\Delta n''_2 - \Delta n'_2) - (1 - \delta)n_1(\Delta w''_{2,I} - \Delta w'_{2,I}) - \frac{\partial C}{\partial w}n_2(\Delta w''_{2,N} - \Delta w'_{2,N}) - C(\Delta n''_2 - \Delta n'_2) \geq 0$, with the reverse inequality implied by incentive compatibility in state x', so given that f' = C in the initial equilibrium (\hat{n}_2 is chosen efficiently given $\hat{w}_{2,N}$ in the absence of incentive compatibility constraints), we get

$$-(1-\delta)n_1\left(\Delta w_{2,I}'' - \Delta w_{2,I}'\right) - \frac{\partial C}{\partial w}n_2\left(\Delta w_{2,N}'' - \Delta w_{2,N}'\right) = 0.$$
(B.23)

⁷⁷That is, we omit terms of order smaller than ε in the expressions that follow. We assumed that the equilibrium of the model is differentiable in ε on an interval $[0, \bar{\varepsilon})$ (from the right at 0), so that, in particular, C is also differentiable in x. In the approximation $\partial C/\partial x$ cancels.

Suppose that $\Delta w_{2,I}'' < \Delta w_{2,I}'$; we will establish a contradiction. Since $\frac{\partial C}{\partial w} > 0$ (at initial equilibrium), (B.23) implies $sgn\left(\Delta w_{2,I}'' - \Delta w_{2,I}'\right) = -sgn\left(\Delta w_{2,N}'' - \Delta w_{2,N}'\right)$. Hence $\Delta w_{2,N}'' > \Delta w_{2,N}'$; thus $w_{2,I}'' < w_{2,I}'$ and $w_{2,N}'' > w_{2,N}'$ and

$$w_{2,I}'' < w_{2,I}' \le w_{2,N}' < w_{2,N}'',$$

where the weak inequality follows by no undercutting in state x'.

Consider the following change in the contract (use to denote this new contract): Set wages in x'' to equal those in x': increase $w''_{2,I}$ to $\tilde{w}''_{2,I} = w'_{2,I}$ and reduce $w''_{2,N}$ to $\tilde{w}''_{2,N} = w'_{2,N}$; keep n_1 constant, set n_2 in each state to maximise period 2 profits given $w'_{2,N}$ and $\tilde{w}''_{2,N}$, and change w_1 to \tilde{w}_1 to keep V_1 constant. The cut in $w''_{2,N}$ reduces hiring costs by, for ε small enough, $w''_{2,N} > w^{**}(x'')$ (the latter being the cost minimising wage of the new-hire in the state x'', using notation and the argument in the proof of Proposition 2 above), and as \tilde{n}''_2 is chosen optimally, profits on new hires in x'' must rise. Similarly, as \tilde{n}'_2 is chosen optimally, profits in x' cannot fall. Incentive compatibility is satisfied trivially. From V_1 constant (which implies constant period 1 job opening costs),

$$v(\tilde{w}_1) - v(w_1) + 0.5\beta (1 - \delta) \left(v(w'_{2,I}) - v(w''_{2,I}) \right) = 0.$$
 (B.24)

By $w_{2,I}^* < w_1^*$, $w_{2,I}' < w_1$; also $w_{2,I}' < \tilde{w}_1$ for ε small enough, so

$$w_1 > \tilde{w}_1 > w'_{2,I} > w''_{2,I}.$$

It follows from (B.24) and by v'' < 0 that

$$w_1 - \tilde{w}_1 > 0.5 (1 - \delta) (w'_{2,I} - w''_{2,I});$$

thus the change in costs of period 1 hires is

$$n_1 \left(\tilde{w}_1 - w_1 + 0.5 \left(1 - \delta \right) \left(w'_{2,I} - w''_{2,I} \right) \right) < 0$$

Thus, the new contract is more profitable than the putative equilibrium one, a contradiction. This establishes that $\Delta w_{2,I}'' < \Delta w_{2,I}'$ is not possible. Similarly $\Delta w_{2,I}'' > \Delta w_{2,I}'$ yields a contradiction. Thus $\Delta w_{2,I}'' = \Delta w_{2,I}'$ and so by (B.23) $\Delta w_{2,N}'' = \Delta w_{2,N}'$. It follows that $(\Delta w_{2,N}'' - \Delta w_{2,N}) / (2\varepsilon) = 0$, which establishes the claim.

Now we allow the contract to be non-differentiable in ε (from the right) at

 $\varepsilon = 0$. It must be (right) continuous at $\varepsilon = 0$, as otherwise profits would also be discontinuous, while a simple constant wage contract would be continuous so would do better.⁷⁸ Consider a sequence for $\varepsilon \equiv (x'' - x')/2$: $\{\varepsilon_{\nu}\}, \varepsilon_{\nu} \to 0$ as $\nu \to \infty$. Assume that the no-undercutting constraint binds (so that $w_{2,I} = w_{2,N}$ $=: w_2$ say) in both states along the sequence (cf. the proof of Proposition 2) and that only the downward incentive constraint binds (i.e. (B.22)). Then by standard arguments $w_2'' \ge w_2'$ and n_2'' is at the optimal level given w_2'' .⁷⁹ The other possibilities can be dealt with in an analogous manner. Again, we suppress the explicit dependence of the optimal contract on ε_{ν} for simplicity of notation. We suppose, contrary to hypothesis, that

$$0 < \lim \sup_{\nu \to \infty} \left| w_2'' - w_2' \right| / \varepsilon_{\nu}. \tag{B.25}$$

Rearranging (B.22):

$$f\left((1-\delta)n_{1}+n_{2}'';x''\right)-f\left((1-\delta)n_{1}+n_{2}';x''\right)-C\left(w_{2,N}\left(x''\right),x''\right)n_{2}\left(x''\right)+C\left(w_{2,N}\left(x'\right),x''\right)n_{2}\left(x'\right)-(1-\delta)n_{1}\left(w_{2}''-w_{2}'\right)\geq0.$$
 (B.26)

By (B.25) we can take a subsequence such that $\lim_{\nu\to\infty} (w_2'' - w_2') / \varepsilon = a$ where |a| > 0, and where n_1 converges to say \tilde{n}_1 , we get after dividing (B.26) by ε_{ν} and taking the limit:

$$\lim \inf_{\nu \to \infty} \left[(f((1-\delta)n_1 + n_2''; x'') - f((1-\delta)n_1 + n_2'; x'') - C(w_2'', x'') n_2'' + (B.27) \right]$$

$$C(w_2', x'') n_2') / \varepsilon_{\nu} \geq (1-\delta)\tilde{n}_1 a.$$

By $w_2'' - w_2' \ge 0$, a > 0. In other words, assuming for small ε we have lower wages in state x' than in x'' by a first-order amount, implies that the RHS of (B.27) is positive, that is, there is a (first-order) incentive in state x'' to underreport x to benefit from lower wage costs; to offset this (i.e., to preserve

⁷⁸Profits are bounded above by a contract which ignores the incentive constraint, which would be continuous, so any discontinuity must imply profits jump down for $\varepsilon > 0$. Holding wages constant across states and setting period 2 employment efficiently at those wages as in the construction in the proof of Proposition 2 would satisfy incentive constraints and lead to profits varying continuously; hence, this would be a profitable deviation.

⁷⁹I.e., it maximises $f((1-\delta)n_1 + n''_2; x'') - C(w''_2, x'')n''_2$.

incentive compatibility) the level of new hires in state x' needs to be sufficiently different (below in this case) that in x'' to lead to a fall in profits from new hires that is also first-order. We show that such a difference in hires would also imply, contrary to optimality, that a deviation contract is profitable which avoids the costs of distorted employment, where wages are constant and employment in state x' is set at an efficient level given wages.

Consider then the following possible deviation contract. In state x' set $w_{2,N} = w_{2,I}^{\prime\prime} = w_2^{\prime\prime}$, and set n_2 at the profit maximising level in state x' for $w_2^{\prime\prime}$, say \tilde{n}_2' . Change w_1 to leave V_1 unchanged (and leave hiring in period 1 the same). In period 2 this contract differs only in state x', satisfies no-undercutting, and is incentive compatible as wages are the same across states and n_2 is chosen optimally in each state. Considering only incumbents the wage increase from w_2' to w_2'' must increase profits once the reduction in w_1 is taken into account $(w_2' < w_1 \text{ implies that more smoothing reduces wage costs})$. As overall profits cannot be improved by any deviation, the change in profits in state x' ignoring incumbents must be nonpositive, i.e.,

$$0 \ge$$

$$(B.28)$$

$$(f((1-\delta)n_1 + \tilde{n}'_2; x') - C(w''_2, x') \tilde{n}'_2) - (f((1-\delta)n_1 + n'_2; x') - C(w'_2, x') n'_2) \ge$$

$$(f((1-\delta)n_1 + n''_2; x') - C(w''_2, x') n''_2) - (f((1-\delta)n_1 + n'_2; x') - C(w'_2, x') n'_2),$$

where the second inequality follows by definition of \tilde{n}'_2 yielding at least as much profit as n''_2 at w''_2 . Dividing the RHS of (B.28) by ε_{ν} , note that this differs from the term in square brackets in (B.27) only by the argument in x, so that given differentiability of f and C in x the two expressions differ by a term of order less than ε_{ν} .⁸⁰ So taking the limit as $\nu \to \infty$, we get the same value, which is a contradiction as from (B.27) it is at least $(1 - \delta)\tilde{n}_1 a > 0$, whereas from (B.28) it is nonpositive.

B.5 Further Simulations

We consider an alternative matching function to that in Section 3.3.2: $m(u, v) = \mu u^{0.5} v^{0.5}$, $\mu = 0.18$, where again u is the number of workers searching and v is

⁸⁰I.e., by a term $h(\varepsilon) = o(\varepsilon)$ so that $h(\varepsilon)/\varepsilon \to 0$ as $\varepsilon \to 0$. This follows as the derivative of the RHS of (B.28) with respect to x at the limit contract, i.e., the initial ($\varepsilon = 0$) contract, equals zero. Recall that by continuity n'_2 , n''_2 , converge to the same value, etc.

the number of vacancies and m the total number of matches.⁸¹ We repeat the exercise underlying Table 1.

	AI			NU			FC		
α	0.5	1	2	0.5	1	2	0.5	1	2
$\delta = 0.1$									
$\delta = 0.3$	-0.64	-0.62	-0.57	-0.53	-0.55	-0.54	-0.45	-0.46	-0.47

Table B.1: Elasticity of employment with respect to productivity changes with Cobb-Douglas matching

Note: AI = asymmetric information; NU = no-undercutting constraint imposed; FC = full commitment; production function is logarithmic, Cobb-Douglas matching function; constant relative risk-aversion utility with coefficient of risk aversion of α ; exogenous separation rate δ ; unemployment benefit b targets an average replacement rate of 43%, and we calibrate the vacancy cost parameter k to yield an average period 2 unemployment rate of 8.5%.

This Cobb-Douglas matching function has a higher (constant) elasticity of matches with respect to vacancies of 0.5 (compared to approximately 0.28 at the equilibrium θ for the earlier matching function); this leads to higher employment elasticities but broadly similar qualitative results, although there is substantially less variability across the different scenarios, and the elasticities are much higher for the full-commitment case in particular. A mechanical explanation for this large difference is that in the Cobb-Douglas case the absolute value of $p'(\theta)$ is much higher around our calibrated equilibria (by a factor of around 8 in our simulations for the FC case). This implies that for a given change in n_2 , θ will vary much less: recall that $n_2 = p(\theta_2) S_2$. Then the change in the product of the first two terms on the right-hand side of (B.11) will be much smaller in the current case as n_2 is varied. In fact, the derivative of this product with respect to θ is approximately double that of the case considered earlier, but the net effect is still a much smaller impact on the product, so a much smaller change in $w_{2,N}$ is needed to keep the expression constant. This implies a much flatter FC quasi-supply curve, and hence much higher elasticities. A similar argument then applies for the NU case.

⁸¹Neither matching function satisfies our sufficient condition (assumed in Section 3.1) that $q(\theta) \epsilon_q(\theta) / (1 - \epsilon_q(\theta))$ is a decreasing function of θ ; nevertheless in both cases $q^2 \frac{\left(1 - \frac{\theta_2}{q} \frac{dq}{d\theta_2}\right)}{\theta_2 \frac{dq}{d\theta_2}}$ is increasing in θ so by (B.11) the FC quasi-supply curve is upward sloping.

C Power Computations in Gertler, Huckfeldt and Trigari's Scenario

Here we offer some indicative power computations using Gertler, Huckfeldt, and Trigari's (2020, henceforth GHT) first-differenced data scenario as a baseline. Explicitly we adopt the specification explaining wage changes in terms of unemployment changes with separate coefficients for U to E and E to E new hires (labelled "ENE" and "EE" by GHT).

We use their data as follows. We downloaded the series of worker-time identifiers together with their status as an incumbent, ENE or EE, plus the change in the unemployment rate associated with each worker-time observation. Then, taking GHT's point estimates reported in the first column of their Table 2, we generated the following fitted values for each worker-time observation

$fit_{it} = -0.426\Delta u_t - 1.868\Delta u_t.I.EE - 0.437\Delta u_t.I.ENE + 0.045I.EE - 0.047I.ENE$

where following their notation I.EE and I.ENE are EE and ENE dummies, respectively. We then added an independent and identically distributed (iid) mean zero normal shock with standard deviation 0.25 (the root mean square error of GHT's regression) to each fitted data point. Then we regressed the resulting series on each of the above regressors and recorded the t-ratio on $\Delta u_t.I.ENE$. Following GHT, the standard errors for this t-ratio were obtained by clustering by individual. In addition, we used their analytical weights. Repeating this 1,000 times and counting the proportion of one-tailed rejections gives us an estimate of the power of the t-test at GHT's point estimates. This is reported in the table below under "n = 1". To get an idea of power in larger samples than the GHT baseline, we drew n datasets each with identical fitted values to the above but of course augmented with iid normal errors as before. The results for n = 2, ..., 5are tabulated below.

Our estimates indicate that GHT had a less than one in three chance of rejecting the false null of no excess ENE cyclicality at their point estimate of -0.47. This chance grows to slightly less than one in two if they had had twice as many data points and to over 80% if they had had a data set five times bigger.

Table C.2: Estimated Power of GHT's Tests for Excess New Hire Cyclicality

n	1	2	3	4	5
Power	30.3	48.6	69.5	76.1	83.0

D Further Tables

Year	Nominal GDP	CPI	Population	Unemployment
rear	(in Mill. Euros)	011	(in 1,000)	rate (in %)
1978	678,940	47.6	61,322	4.3
1979	737,370	49.5	61,439	3.8
1980	788,520	52.2	61,658	3.8
1981	825,790	55.5	61,713	5.5
1982	860,210	58.4	61,546	7.5
1983	898,270	60.3	61,307	9.1
1984	942,000	61.8	61,049	9.1
1985	984,410	63.0	61,020	9.3
1986	1,037,130	63.0	61,140	9
1987	1,065,130	63.1	61,238	8.9
1988	1,123,290	63.9	61,715	8.7
1989	1,200,660	65.7	$62,\!679$	7.9
1990	1,306,680	67.5	63,726	7.2
1991	1,415,800	70.2	64,485	6.2
1992	1,485,759	73.8	65,289	6.4
1993	1,503,858	77.1	65,740	8.0
1994	1,556,575	79.1	66,007	9.0
1995	$1,\!606,\!164$	80.5	66,342	9.1
1996	$1,\!625,\!847$	81.6	66,583	9.9
1997	1,664,512	83.2	$66,\!688$	10.8
1998	1,711,722	84.0	66,747	10.3
1999	1,751,665	84.5	66,946	9.6
2000	1,799,706	85.7	$67,\!140$	8.4
2001	1,856,557	87.4	65,323	8.0
2002	1,879,896	88.6	65,527	8.5
2003	1,888,205	89.6	$65,\!619$	9.3
2004	1,933,051	91.0	$65,\!680$	9.4
2005	1,960,396	92.5	$65,\!698$	11
2006	2,038,803	93.9	$65,\!667$	10.2
2007	2,142,032	96.1	$65,\!664$	8.3
2008	2,180,829	98.6	65,541	7.2
2009	2,088,073	98.9	65,422	7.8
2010	$2,\!191,\!138$	100.0	65,426	7.4
2011	$2,\!298,\!449$	102.1	64,429	6.7
2012	2,345,295	104.1	$64,\!619$	6.6
2013	2,401,853	105.7	$64,\!848$	6.7
2014	$2,\!483,\!514$	106.7	65,223	6.7

Table D.3: GDP, CPI, Population, and Unemployment Rate

Note: Identified downswing years are indicated in bold year numbers. Real GDP per capita calculated using nominal GDP, CPI and population. Sources for nominal GDP for West Germany: German Federal Statistical Office & the Federal Statistical Offices of the Federal States. Source of German CPI: Federal Reserve Bank of St. Louis (FRED Economic Data). Source West German population: German Federal Statistical Office. Source of the West German unemployment rate (in % of the total civilian workforce): Sachverständigenrat.

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	Year	New Hires	Incumbents	Year	New Hires	Incumbents
	1978	$536,\!480$	860,131	1997	481,019	2,405,614
	1979	580,482	1,070,423	1998	524,318	2,392,430
	1980	562,231	1,254,231	1999	580,765	2,385,722
	1981	472,966	1,423,195	2000	601,915	2,445,300
	1982	383,748	1,535,036	2001	$558,\!655$	$2,\!454,\!149$
	1983	384,038	1,607,852	2002	471,745	2,444,711
	1984	421,761	1,650,744	2003	424,415	2,505,278
	1985	433,296	1,703,623	2004	395,014	$2,\!473,\!805$
	1986	480,197	1,829,471	2005	391,361	2,443,718
	1987	467,208	1,925,379	2006	441,206	2,449,759
	1988	501, 192	2,008,610	2007	487,477	2,465,401
	1989	580,223	2,080,315	2008	474,157	2,506,474
	1990	$674,\!453$	2,164,259	2009	400,230	2,502,328
	1991	$651,\!557$	2,284,766	2010	462,299	2,502,616
	1992	569,494	$2,\!394,\!251$	2011	444,522	2,409,295
	1993	482,607	2,431,712	2012	430,893	$2,\!480,\!722$
	1994	496,822	$2,\!428,\!188$	2013	418,203	2,519,325
	1995	$516,\!571$	$2,\!416,\!687$	2014	432,368	2,521,718
	1996	$481,\!872$	$2,\!408,\!716$	Total	18,097,160	79,785,954

Table D.4: Number of Spells of Incumbent and Newly Hired Workers

Note: New hires identified using the first employment spell in an establishment.

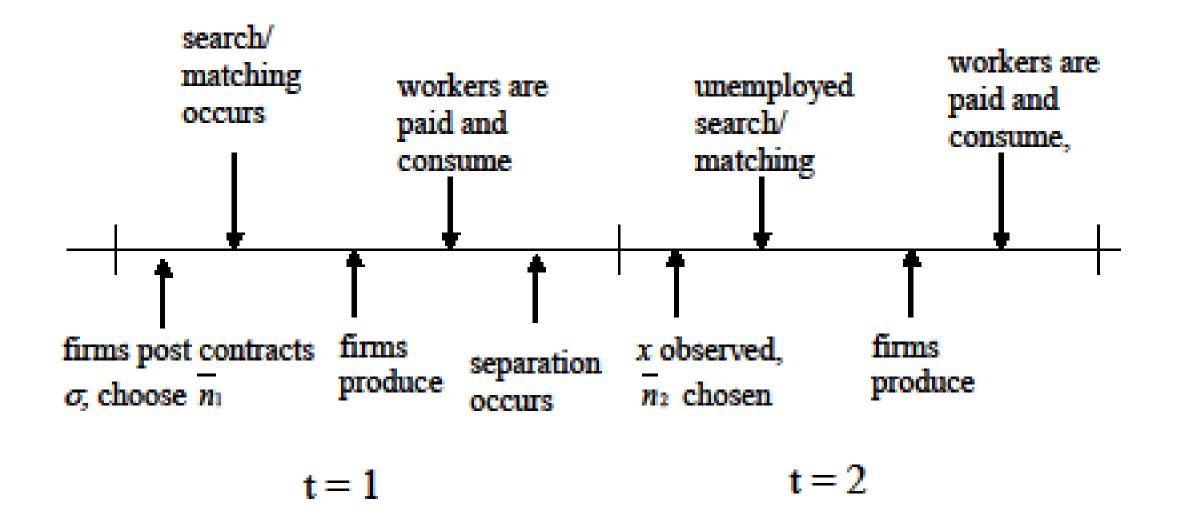
Table D.5: Classification of Economic Activities, Edition 1993 (WZ 93)

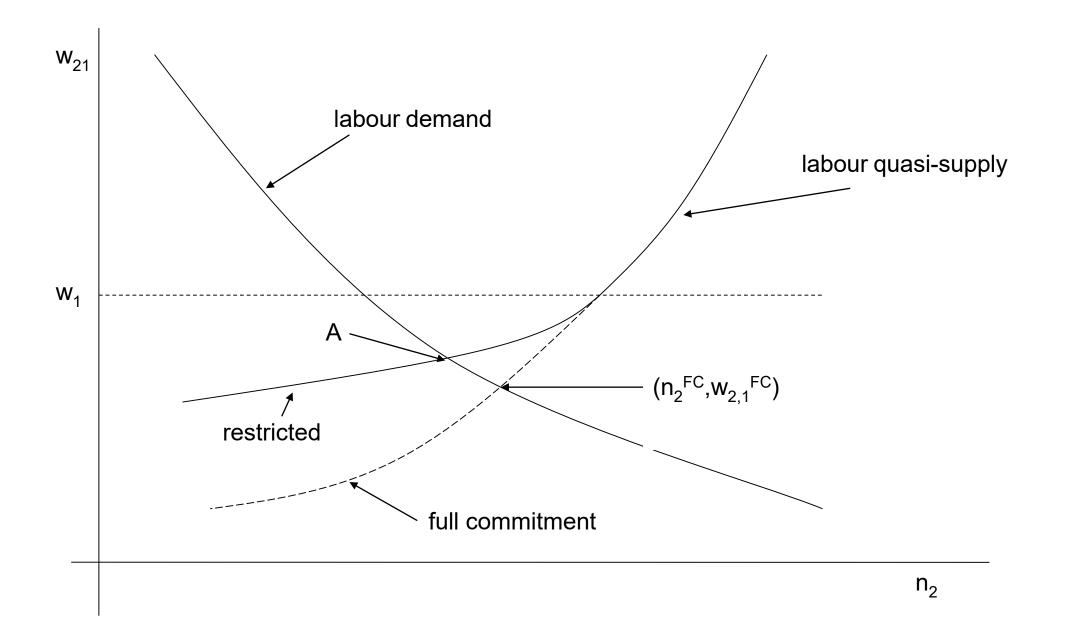
Section	Description
А	Agriculture, hunting and forestry
В	Fishing
\mathbf{C}	Mining and Quarrying
D	Manufacturing
E	Electricity, Gas and water supply
\mathbf{F}	Construction
G	Wholesale and retail trade; repair of motor vehicles, motorcycles and personal and household
	goods
Н	Hotels and restaurants
Ι	Transport, storage and communication
J	Financial intermediation
Κ	Real estate, renting and business activities
\mathbf{L}	Public administration and defence; compulsory social security
Μ	Education
Ν	Health and social work
О	Other community, social and personal service activities
Р	Private households with employed persons
\mathbf{Q}	Extra-territorial organisations and bodies

Note: For some analyses, we examine the behaviour of wages in each of six broad sectors. The sectors are: Sector 1 (Mining, Agriculture, etc.) includes WZ 93 sections A to C. Sector 2 (Manufacturing) equals section D, Sector 3 (Power) equals section E, Sector 4 (Construction) equals section F, and Sector 5 (Retail) equals section G. Sector 6 (all other activities) includes sections H to Q. Source: Statistisches Bundesamt(Ed.) (2003).

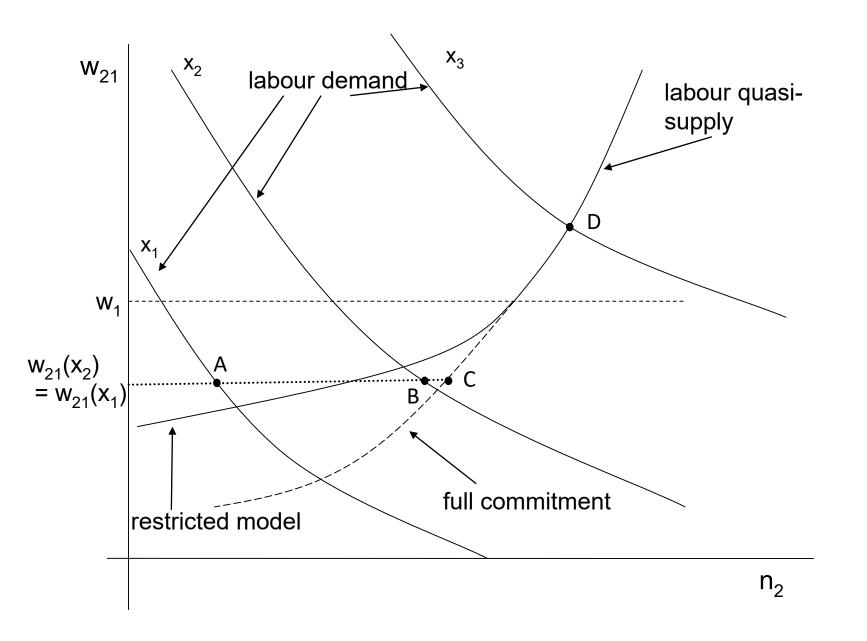
References

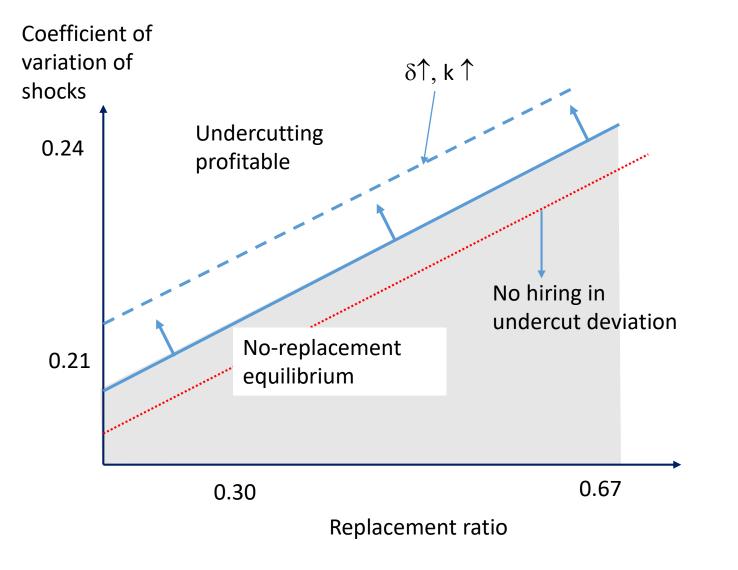
Gertler, M., C. Huckfeldt, and A. Trigari (2020). Unemployment fluctuations, match quality, and the wage cyclicality of new hires. *The Review of Economic Studies* 87(4), 1876–1914. Statistisches Bundesamt(Ed.) (2003). German Classification of Economic Activities, Edition 2003 (WZ 2003). Wiesbaden.











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