APPLIED MATHEMATICS

Testing for voter rigging in small polling stations

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Nowadays, a large number of countries combine formal democratic institutions with authoritarian practices. Although in these countries the ruling elites may receive considerable voter support, they often use several manipulation tools to control election outcomes. A common practice of these regimes is the coercion and mobilization of large numbers of voters. This electoral irregularity is known as voter rigging, distinguishing it from vote rigging, which involves ballot stuffing or stealing. We develop a statistical test to quantify the extent to which the results of a particular election display traces of voter rigging. Our key hypothesis is that small polling stations are more susceptible to voter rigging because it is easier to identify opposing individuals, there are fewer eyewitnesses, and interested parties might reasonably expect fewer visits from election observers. We devise a general statistical method for testing whether voting behavior in small polling stations is significantly different from the behavior in their neighbor stations in a way that is consistent with the widespread occurrence of voter rigging. On the basis of a comparative analysis, the method enables third parties to conclude that an explanation other than simple variability is needed to explain geographic heterogeneities in vote preferences. We analyze 21 elections in 10 countries and find significant statistical anomalies compatible with voter rigging in Russia from 2007 to 2011, in Venezuela from 2006 to 2013, and in Uganda in 2011. Particularly disturbing is the case of Venezuela, where the smallest polling stations were decisive to the outcome of the 2013 presidential elections.

INTRODUCTION

Many elections around the world end in controversies related to alleged frauds (*1*), even in mature democracies, such as the United States and Canada, where voter suppression scandals have made headlines (*2*, *3*). However, although some countries aim to ensure trust in their electoral processes by persecuting electoral malpractices, in other countries, elections may take place under regimes that lack legitimacy. Regrettably, electoral irregularities may have very serious consequences ranging from social instability to deadly violence (*4*). Certain types of irregularities seem to be characteristic of countries with extreme political polarization, with zones heavily controlled by only one political party. For example, videos filmed during recent elections in Zimbabwe that went viral on the Internet show electors that were forced to vote for President Robert Mugabe by superior officers (*5*) or allegedly transported under intimidation from rural districts to a voting station in the capital (*6*). It is an unsolved challenge to detect these abject electoral abnormalities by scrutiny of the vote counting or by audits of the electoral roll; behind every potential questionable vote, there may exist a real elector whose vote was officially counted. These abusive practices have been called voter rigging (*7*), to signal unlawful and systematic harassment of the voters themselves (as opposed to distortions of the vote counts often referred to as "vote rigging").

Reports of voter rigging are not exclusive to Zimbabwe. Most of the complaints of the 2011 Russian elections came from state employees who said they were pressured by their supervisors to vote for United Russia and from students who said they were similarly pressured by their professors, which is a similar form of rigging (*8*). The 2013 Venezuelan presidential elections gave rise to numerous accounts of voter rigging. One of the main electoral observer groups of these elections (ROE-AE) reported acts of violence or authority abuse during the election day in 9.3% of the observed voting centers. ROE-AE denounced that in the 4.7% of the cases, there were infringements to the regulation on assisted voting (*9*). The main opposition candidate claimed that these violations aimed at compelling the voters to vote for the ruling party (*10*). There were even reports that the ROE-AE's headquarters suffered a violent assault (*11*). To our knowledge, this is the first time that such an incident was reported to occur in Venezuela. Another important electoral observation organization reported pressures on voters at different times during the day. In 15.1% of the 391 observed voting centers, voter mobilization took place with public resources mainly belonging to government authorities and allied governorships and mayorships (*12*). In the opposition's appeal to the Venezuelan Supreme Court (*13*), where the outcome of the elections was contested, the inclusion of many small voting centers sensitive to voter rigging was mentioned. A point can be made that small centers might be particularly susceptible to voter rigging because it is easier to identify opposing individuals, these centers almost always lie in alleged progovernment areas, there are fewer eyewitnesses, and the centers are visited less frequently by election observers. However, beyond amateur videos and observers' testimonies at some centers, there is no sound analysis about whether these electoral irregularities were isolated occurrences or if they happened on a larger scale, to the point of determining the winner of the election.

It is important to differentiate between voter rigging and clientelism, which is understood as the distribution of benefits to citizens in exchange of electoral support. Voter rigging entails coercion, affecting the free choice of the voters, and is part of the menu of manipulation that goes beyond the limits of democratic politics (*14*). Although some clientelist strategies can also be considered as tools for electoral fraud (*15*, *16*), clientelism is voluntary most of the time (*17*). Along with vote buying and machine politics, it is viewed as a practice that perverts democratic accountability (*18*). Another difference between voter rigging and clientelism concerns the places selected for their practice. As it has been already noted (*19*, *20*), the importance of clientelism is focused on small districts, where brokers play a critical role. Nevertheless, as discussed above in the context of the Venezuelan elections, the easier targets of voter rigging are small polling stations (and not necessarily small districts).

Here, we provide a general method for testing statistical irregularities of election outcomes that are compatible with the assumption of a widespread occurrence of voter rigging. We hypothesize that the

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presence of voter rigging leads to a characteristic distortion in the election results that is especially discernible in small polling stations. The method is applied to the results of 21 elections in 10 different countries. In particular, we developed a statistical significance test that allows us to investigate whether the voting behavior in small polling stations is substantially different from those of large stations. The possibility of these irregularities in small stations is firmly rejected by our method in several elections in eight countries, including Venezuela, before the current ruling party controlled the electoral power. We observe a turning point in the size of election irregularities between 2004 and 2006 in Venezuela, culminating in an outcome-determinative effect of voter rigging–like distortions in the results of the 2013 presidential elections. In these elections, the voting behavior in small stations shows irregularities that can exceed the results of elections free of voter rigging by a factor of almost 10. The only other country where we observe anomalies of a comparable dimension is Russia, where the political landscape is dominated by United Russia.

The statistical detection of electoral fraud is not a new issue. A number of scholars have been engaged in the study of this discipline during the past decade (*21*). Among the most commonly used statistical methods are those based on Benford's laws (*22*, *23*) and other tests that also focus on the distribution of digits in vote counts (*24*, *25*). In a separate category, we can group various tools developed for the detection of anomalies in the distribution of votes and voter turnout (*26*–*29*). In a third category, we may include analyses based on exit poll (*30*) and other kinds of sampling data (*31*, *32*). Studies that adopt a statistical mechanics approach to understand statistical regularities in vote counts are also of interest (*33*). Particularly relevant are studies on the spatial correlation of turnout rates at the scale of municipalities (*34*). Now, to incorporate spatial statistics for election fraud detection, we should consider more desegregated data sets because it is well known that high levels of aggregation may mask electoral fraud (*35*). Furthermore, we have to analyze not only turnout rates but the complete vote counts.

RESULTS

Figure 1 shows the election fingerprints, as introduced by Klimek *et al.* (*27*), and the standardized election fingerprints (SEFs, see Materials and Methods) of several elections (Venezuela 1998 and 2013, Russia 2011, Austria 2008, Canada 2011, and Spain 2008). Note that the stratified standardization in the SEFs corrects multimodality (Canada) and heterogeneous voter mobilization (Austria) that has been observed in previous versions of election fingerprints. Nonetheless, there is no reason to believe that SEFs should fit an uncorrelated bivariate normal model, such as it has been suggested for the fingerprints. Strategic voting may introduce some correlation between the standardized turnout, Z_t, and the standardized number of voter for the winning party, Z_{vw}, because more turnouts may be associated with mobilization for or against the winner. If the electoral neighborhoods are homogeneous enough, asymptotic arguments can be invoked to argue that the empirical marginal distributions of the Z scores should be approximately a standard normal distribution, although with more extreme values than the ones expected for a Gaussian sample due to the complexity of the electoral processes (*29*, *36*). Therefore, the only claim that we can assert is that the joint distribution should be unimodal, centered on the origin, and roughly supported on a high confidence normal area. Additionally, we expect a particular symmetry for the SEF of an election wherein certain types of fraud can be dismissed. This becomes apparent in a contour visualization of two-dimensional (2D) histograms over all electoral units for several elections (Fig. 2). The SEFs of these elections appear to be axially symmetric, almost elliptically symmetric in many cases.

Figure 3 shows visualizations of the SEFs for small and large electoral units using contour (equidensity) lines. In this figure, small (large) units are those that have an electorate size below (above) the $p = 20$ percentile. Similar results can be observed for all other choices of p. To rule out that our results are not driven by few small electoral units with atypical vote counting, we perform a simple outlier removal procedure of Z scores based on the observed elliptical symmetry (see Materials and Methods). For Venezuela 1998, Canada 2011, and Spain 2011, the centers of the SEFs for small and large units coincide, and the shapes of the distributions are hard to distinguish. This is not the case for Venezuela 2013 and Russia 2011 and 2012. The Z scores of these elections are substantially shifted toward the upper right regions of the plot for small units. This means higher turnout values and larger

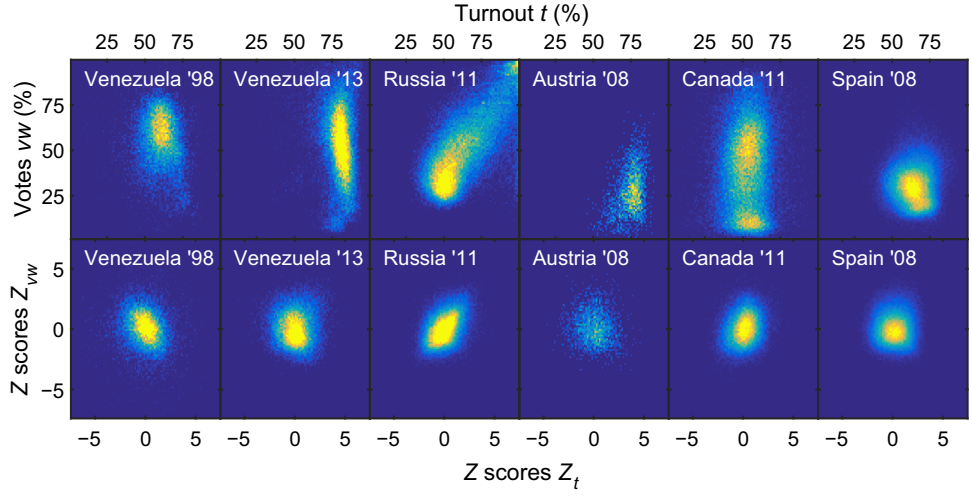

Fig. 1. Comparison of election fingerprints and their standardized variant for six different elections. The SEFs (bottom row) are compared to the election fingerprints (top row) as introduced by Klimek *et al.* (*27*) for a selected set of elections, namely (from left to right), Venezuela 1998 and 2013, Russia 2011, Austria 2008, Canada 2011, and Spain 2008. The bimodality in the fingerprint of Canada and the "smearing out" in Austria that can be associated with heterogeneous vote mobilization disappear in the SEF.

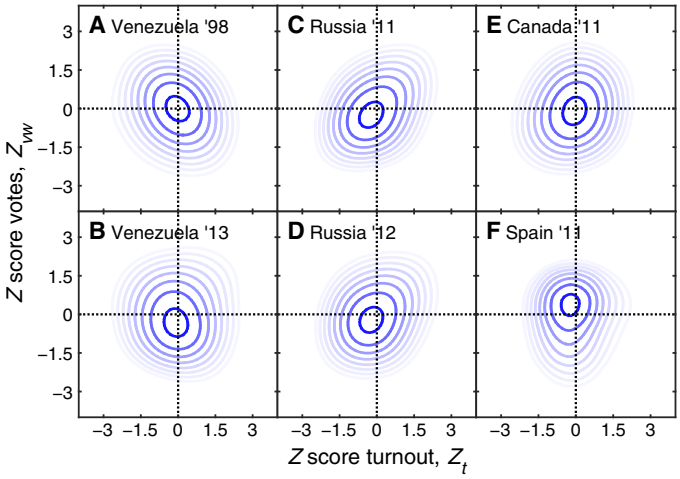

Fig. 2. Symmetry of the SEFs. The SEFs of (**A**) Venezuela 1998 and (**B**) 2013, (**C**) Russia 2011, (**D**) Russia 2012, (**E**) Canada 2011, and (**F**) Spain 2011 are shown using contour representations for the densities of data points. The SEFs appear to be axially symmetric. They are close to the elliptical symmetry.

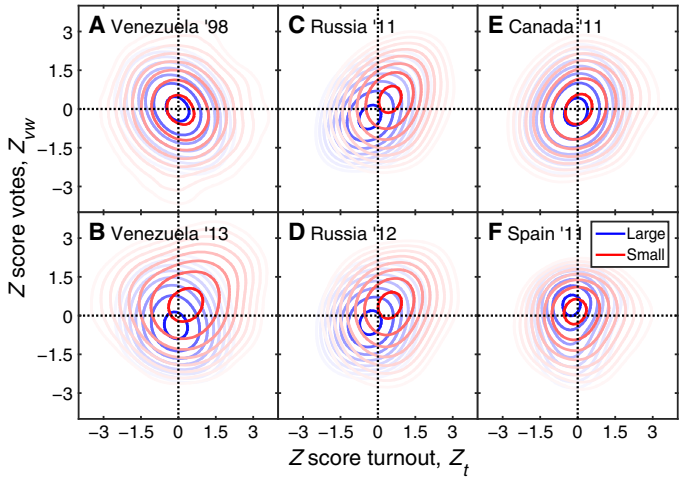

Fig. 3. SEFs for small and large electoral units. Comparison of the SEFs of large (blue) and small (red) electoral units for (**A**) Venezuela 1998 and (**B**) 2013, (**C**) Russia 2011, (**D**) Russia 2012, (**E**) Canada 2011, and (**F**) Spain 2011. The centers of the distributions of small and large centers coincide for Venezuela 1998, Canada 2011, and Spain 2011. However, there is a clear discrepancy in the SEFs of small and large centers in Venezuela 2013 and Russia 2011 and 2012. The rescaled turnout and the votes for the winner are substantially larger in small centers for these elections. This is clear evidence that the election outcomes in these small centers show systematic distortions.

numbers of votes for the winner in small electoral units as observed in their direct neighborhood, a clear evidence of a systematic distortion of the election outcomes in these units that is consistent with the effects of voter rigging in these places. To systematically quantify these distortions, we consider the standardized distance $\delta(p)$ as the effect size, which gives the number of SDs by which small and large centers are on average displaced from each other as measured over a set of non-outlier elections. Figure 4A shows results for $\delta(p)$ for the 21 elections under study. The gray region in Fig. 4A, the "accepted region," contains the results for all elections that are not classified as outliers according to the majority of outcomes of the modified Thompson tau test. Elections that lie outside of this region show results that are compatible with the assumption of widespread voter rigging in small electoral units. We observe the strongest effects in Venezuela between 2006 and 2013, with $\delta(p)$ values that almost reach a factor of 10 for size thresholds around the 5th percentile. Nevertheless, the values of $\delta(p)$ clearly lie outside of the accepted region for a wide range of choices of p. Such strong deviations are totally absent from the Venezuelan data in previous elections as well as in the last Venezuelan parliamentary election (that is, 2015). The Russian elections between 2007 and 2012 also show significant deviations, with $\delta(p)$ lying between 4 and 7. Significant results are also found for Russian 2003 and Uganda, but with substantially smaller effect sizes with values between 3 and 4. Certainly, these anomalous results for Russia, Uganda, and Venezuela may be traces of voter rigging in small electoral units and raise serious doubts regarding the integrity of the related elections. Note that the accepted region in Fig. 4A encapsulates elections that show deviations below 3 SDs, in consistency with a confidence level of $\alpha = 0.95$. The deviations in Russia from 2007 to 2012 and Venezuela from 2006 to 2013 therefore indicate truly extreme events.

To assess the robustness of the above results, we perform a leave-one-out cross-validation. Therefore, we remove each of the 21 election data sets exactly once and repeat the analysis on the reduced data. We find that only Venezuela 2006, which can already be seen to fall into the accepted regions for a wide range of choices of p in Fig. 4A, becomes insignificant in 9 of the 21 leave-one-out cross-validations. In each of these nine cases, we removed an election data set from the reference set of "trustworthy" elections. All other removals did not result in a significant result becoming insignificant. Russia from 2007 to 2012 and Venezuela from 2012 and 2013 remain significant in all 21 cross-validations.

Furthermore, we assess the robustness of the results by three randomization tests. First, for each election with significant deviations in $\delta(p)$, we randomly permute the electorate sizes on the units, keeping the percentages vw and t fixed. Thus, each unit is randomly assigned to the group of small or large units. This completely removes any correlation between electorate size and the percentages vw and t, and we would expect no significant results under this randomization. After this, we observe that for each of these elections, $\delta(p)$ falls within the acceptance region shown in Fig. 4A. For the second test we randomly permute vw, whereas for the third test we randomly permute t, keeping all other variables fixed in each test. By permuting vw, we get exactly the same set of elections with significant deviations as in the original data. By permuting t, only the results for Russia 2003 and Uganda 2011 become insignificant. Note that in second test, the correlation between electoral size and turnout is preserved by randomly permuting the percentages vw. In a similar way, the correlation between n and vw is preserved by randomly permuting t. These residual correlations are in most of the cases enough for the method to produce consistent results.

A different visualization of the atypical results reported for Russia, Uganda, and Venezuela from 2006 to 2013 is provided in Fig. 4B. Here, the electoral units are sorted according to their number of electors in a descending way. We then compute the percentages of votes for the winners over all units up to the given rank i (that is, above the corresponding number of electors). This number is denoted $cum_i(vw)$. If the voting behavior in the units is independent from the size of the unit, we expect a slope of zero for $cum_i(vw)$ for high ranks i. This means that the addition of increasingly small units does not change the overall results of the elections. For Uganda, we observe a logarithmic increase of $cum_i(vw)$ from relatively large electoral units. We observe a different pattern for Russia and Venezuela, where $cum_i(vw)$

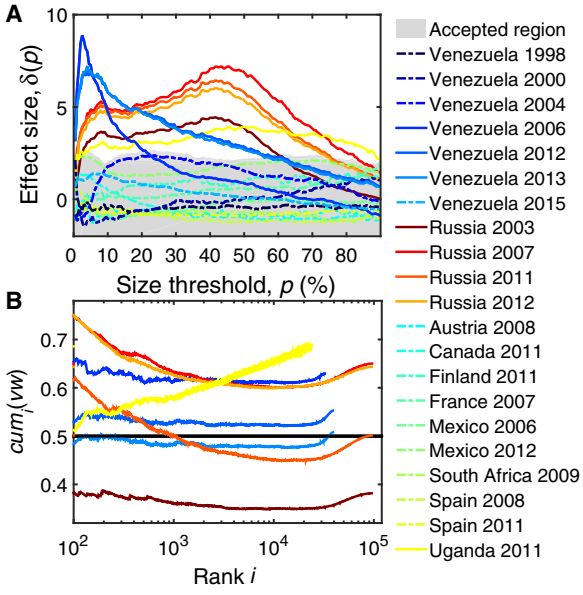

Fig. 4. Results of the test for voter rigging. (**A**) Results of the statistical test for voter rigging, quantified by the effect size (*p*), are shown for 21 different elections. The gray region contains the elections with no significant differences between small and large electoral units. These elections are also shown as dash-dotted lines. Solid lines show elections that are compatible with the voter-rigging-in-small-centers hypothesis. These include the elections in Russia, Uganda, and Venezuela from 2006 to 2013, wherein the strongest effects are observed. (**B**) We show different visualizations of the atypical results observed from Russia, Uganda, and Venezuela. In these visualizations, the electoral units are sorted in a descending way according to their number of electors, and the percentage of votes for the winner is computed using only units up to the given rank on the *x* axis (in logarithmic scale). For Venezuela 2013, it is only the addition of small units that pushes the results to determine the winner.

clearly increases at the smallest units. Therefore, the addition of these small units has a substantial impact on the election outcomes. For Russia 2011 and Venezuela 2013, we note that it is the contributions from the very small units that push the total number of votes to the barrier of 50%. In the Venezuelan case, wherein the current president (Nicolás Maduro) was elected by a plurality voting system, the irregular results of the smallest electoral units were outcome-determinative.

DISCUSSION

Here, we develop a method for testing statistical anomalies of election results in small polling stations attributable to voter rigging. In particular, we devise a comparative tool based on SEFs of different countries. Our analysis of 21 national elections in 10 countries shows significant statistical irregularities in Venezuela 2006–2013, Russia 2007–2012, and, to a lesser extent, Russia 2003 and Uganda 2011. Voter rigging could be outcome-determinative in the 2013 Venezuelan presidential elections, in particular.

The integrity of the Venezuelan electoral system has been questioned since the holding of the presidential recall referendum in 2004 (*36*). Particularly in the last elections, some tactics to influence/pressure the voter have been widely reported. The first such tactic is through assisted voting, which may be associated with voter coercion. It was detected in 6.3% of polling stations observed in 2012, 4.7% in 2013, and up to 6% in 2015, mainly on citizens that were pressured to vote for candidates of the ruling party (*37*). Second is the growth of small centers in the past decade. The electoral organism Consejo Nacional Electoral (CNE) has justified this policy on the grounds of the need to decentralize large centers and increase the number of centers in rural areas. However, these centers are more prone to irregularities and acts of intimidation/violence on election days. In these small centers, votes are mainly from citizens that are dependent on government social programs, which make them vulnerable to the modus operandi of the official machinery. In extremely competitive elections, such as the 2013 presidential elections, a manipulation in these centers may be critical to win the majority. What happened in the 2015 parliamentary elections? The types of irregularities reported 2 years before had less impact. In addition to lower mobilization of chavistas, largely due to the country's crisis, two factors seem to have played an important role: the deployment by opposition forces of activists and students, particularly in areas controlled by chavismo and therefore more vulnerable to possible fraud, and the institutional role played by the armed forces in an election more constrained by a stronger monitoring of the international community.

In Russia, on the other hand, elections have not reached the minimum standards to be considered democratic. Pressure on voters is clearly apparent. In the parliamentary elections of 2003, there is evidence of pressure on several thousands of workers who were instructed by their employers to request absentee ballots and who were subsequently bused the day of the election so that their vote could be monitored in previously designated centers under threat, in some cases, of job loss (*38*). In the parliamentary 2007 elections, there were allegations of threats against voters, of misuse of absentee ballots, and of voters being bused to designated centers (*39*). In the parliamentary elections of 2011, public officials were asked to sign letters of support for the ruling party. Owners of large companies, following instructions from local authorities, also pressured their employees to vote for United Russia (*40*). Similiar to the 2011 elections wherein electoral fraud led to widespread demonstrations, the 2012 presidential elections were marred by a large number of irregularities, although with a smaller impact on the electoral results. Compared to previous elections, it has been reported that in 2012, there was a major attempt to control the vote by such practices as massive voting using absentee voting certificates or requiring employees to vote at their workplaces (*41*). We observe greater statistical traces of pressure on voters in polling stations in the 2011 parliamentary elections wherein several types of fraud have been analyzed. A field experiment study carried out in Moscow estimates the size of fraud in voting shares for the ruling party United Russia to be 11% (*42*). More recent research has shown that the mix of electoral manipulation (electoral fraud, ballot stuffing, and voter pressure) used by incumbents varied across regions according to the competitive conditions. In particular, voter pressure was more common in competitive areas (*43*). To these findings, we add the atypical skewing electoral behavior in small polling stations to understand the victory of the incumbent party.

The 2011 elections in Uganda require a totally different assessment. This country has enjoyed a more competitive landscape since the reintroduction of multiparty elections in 2005, but it has maintained the characteristics of electoral authoritarianism, the key to keeping the National Resistance Movement (NRM) in power. In 2011, three practices common in past elections, also prevalent in Sub-Saharan Africa, are observed: intimidation, vote buying, and vote rigging (*44*). In the context of the so-called monetization of elections, pressure on some voters was exerted through bribery (warning

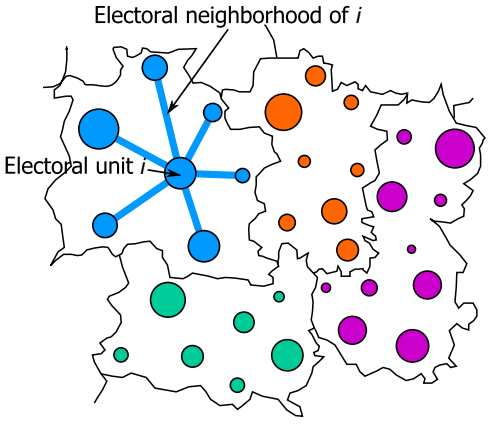

Fig. 5. Schematic overview of how to compute the *Z* scores. In this illustration, we show electoral units (circles) with different electorate sizes (indicated by the size of the circles). The colors of the units correspond to different electoral neighborhoods. The neighborhood of *i* is defined as all other units that lie in the same electoral neighborhood (blue lines). To correct for geographic heterogeneities in the data, the vote and turnout percentages of each unit *i* are rescaled by their average value and SD in the electoral neighborhood of *i*.

them of the consequences if they did not vote for the candidate who had bought their vote) and through government development programs (threatening voters with the loss of benefits if they did not vote for the NRM) (*45*). In this case, the statistical traces of bribery resemble those of voter rigging in the other two countries. It exhibits a trend wherein the smaller the center is, the larger the share of vote buying is (Fig. 4B). However, our method does not contemplate the difference between the mere purchasing of voting and the combination of buying and coercion.

Note that our method would be equally effective in diagnosing irregularities that are consistent with other forms of voter rigging as those discussed above. For instance, threatening and intimidating ethnic or racial minorities to stay absent from the polls would lead to a decreased turnout that coincides with increased win margins in small polling stations. This form of "vote denial" would effectively shift the SEFs for small stations to the upper left quadrant in Fig. 3. This effect would be picked up by the standardized distance $\delta(p)$ too, as would any other form of systematic displacement between the SEFs of small and large stations. However, one can also imagine forms of voter rigging that will not be detected by our method. This is particularly the case if voters that are targeted are "well mixed" in the population, that is, do not cluster in small or large stations. These forms of voter rigging would not lead to a displacement of the corresponding SEFs in Fig. 3 and therefore would not be detected by the test developed here. Another limitation of our method is that for the significance test to work, we need to define a reference set of "trustworthy" elections. It is a priori not clear whether a given election can be considered trustworthy because this is what, to some extent, the test should decide. However, we circumvent this issue here by constructing the reference set from election results from mature Western democracies that did not show any systematic irregularities in the vast majority of the election forensics literature. Thereby, we do not assume that no irregularities take place in these elections. Instead, we observe that their combined impact on the displacement $\delta(p)$ is typically orders of magnitude smaller than their impact in the elections flagged as irregular by our test.

MATERIALS AND METHODS

We considered vote counts at the finest level of reported results from elections in 10 countries. The countries were selected for having election data with the specifications that we require for our analysis, which are described in the Supplementary Materials. All available data regarding general elections of these countries were considered, except for Venezuela, where, between 1998 and 2015, there were 12 years with general elections or referenda. A forensic analysis of the national-level Venezuelan electoral processes held during the 1998–2012 period (*36*) shows anomalous statistical patterns, which are consistent with election fraud from 2004 onward. To balance the presence of different countries into our data set, in the present analysis, we only included Venezuelan presidential elections between 1998 and 2013, the Recall Referendum 2004, and the most recent parliamentary election (that is, 2015), wherein the ruling party lost the majority after having dominated the electoral landscape by 17 years.

Standardized election fingerprints

We denoted by *t* and *vw* the turnout percentage and the percentage of votes going to the winner per electoral unit, respectively. This is the polling station where the election results are known at the finest level. These units may be known as electoral tables, wards, or precincts, according to the election under study. The so called election fingerprint, namely, a 2D histogram for *t* and *vw*, has proved to be a valuable tool for fraud detection (*27*) and is the starting point to introduce our methodology. The key idea for taking advantage of the election fingerprints is that they must approximately fit an uncorrelated bivariate normal distribution. Nevertheless, this hypothesis may fail for many reasons. For instance, *vw* might get inflated in a fraudulent way by adding to it votes from the nonvoters (ballot stuffing). In the election fingerprints, this type of fraud will introduce a positive correlation between *t* and *vw*. Alternatively, *vw* might be increased by adding votes from opposition parties (vote stealing), which would lead to an inflation of *vw* in the election fingerprints, but not a simultaneous inflation of *t*. There are also nonfraudulent mechanisms that can explain observed discrepancies between the election fingerprint of some countries and the uncorrelated bivariate Gaussian distribution. An example is heterogeneity in the electoral population: Countries with two or more regions with different electoral behavior may correspond to Gaussian mixture models, which may provide multimodal fingerprints. This can happen if each region roughly fits a normal distribution with different means and variances among the regions. This appears to be the case of the 2011 Canadian elections wherein the fingerprint splits Quebec from English Canada (*27*). On the other hand, one may expect some sort of correlation between *t* and *vw*, especially if the voters of some electoral units are mobilized to support an option different from their first preferences to prevent an undesirable outcome (strategic vote). Strategic vote is common in several countries with proportional representation electoral systems (*46*) and could explain the aspects of the Finnish and Austrian fingerprints, among others (*27*).

To provide an alternative forensic tool to the election fingerprints that is sturdier against the effects of the nonfraudulent scenarios discussed above, we considered a stratified normalization of the percentages *t* and *vw* that we name the election *Z* scores. Therefore, we compared vote and turnout at a particular unit to the results of units in its electoral neighborhood. This is the smallest available administrative division to which the electoral unit belongs. They may correspond to different types of administrative entities depending on

the country, such as departments, parishes, counties, districts, or municipalities. Namely, the Z scores of the electoral unit i are

$$Z_t(i) = \frac{t(i) - \mu_t(i)}{\sigma_t(i)} \text{ and } Z_{vw}(i) = \frac{vw(i) - \mu_{vw}(i)}{\sigma_{vw}(i)} \quad (1)$$

where $\mu_t(i)$ and $\sigma_t(i)$ denote the average and SD, respectively, of t over the units lying in the neighborhood of unit i, and $\mu_{vw}(i)$ and $\sigma_{vw}(i)$ are the corresponding average and SD, respectivey, of vw (Fig. 5). We will refer to the 2D histograms of the Z_t and Z_{vw} values for a given election as its SEF. For an easier accessible visualization, we also represent these 2D histograms by smoothed level curves for the joint density of data points (see the Supplementary Materials); different density levels are represented by proportional color intensities.

Statistical test for voter rigging

We consider a statistical test for the detection of statistical irregularities compatible with voter rigging in small electoral units. Our approach is based on the identification of outlier differences between SEFs of small and large units. For that, we introduced a discrepancy measure, namely, $D(p)$, between the SEF corresponding to the electoral units with fewer electors than the pth percentile of the number of electors per the electoral unit (small units) and the SEF corresponding to the complementary set of electoral units (large units). We provided in the Supplementary Materials a detailed description of $D(p)$. The value $D(p)$ of each election is compared with an average taken from a set of different, trustworthy elections. To this end, we computed the values of $D(p)$ over all considered elections and identified the outliers in this set using the modified Thompson tau test at a given confidence level α (see the Supplementary Materials). This process is applied to a wide range of choices of p to obtain a reference set of (trustworthy) elections, R, by considering all elections that are not classified as outliers for at least $(1 - \alpha)$ 100% of size thresholds p. Hence, for a given election, we may rescale its value of $D(p)$ by the mean and SD of the corresponding distances of elections contained in R, obtaining a standardized discrepancy measure between the SEFs corresponding to the small and large units, which we denote by $\delta(p)$ (see equation 1 in the Supplementary Materials). Values of $\delta(p)$ far from zero imply atypical differences that indicate systematic irregular results on small electoral units. In particular, we can provide a rejection region at a given significance level for $\delta(p)$ for the hypothesis of voter rigging–like irregularities by considering the rejection region of the corresponding modified Thompson tau test. If $\delta(p)$ lies outside of this region for a wide range of small values of p and, additionally, the centers of the SEFs of small units are inside the upper right region of the plot, the outcome of the corresponding election is compatible with the hypothesis of large-scale distortion of small units with respect to their electoral neighbors, which we only explain by some type of voter rigging.

SUPPLEMENTARY MATERIALS

Supplementary material for this article is available at http://advances.sciencemag.org/cgi/content/full/3/6/e1602363/DC1

Data

Outliers removal

Visualization of the SEFs

Modified Thompson tau test

Discrepancy measures between SEFs of small and large units

table S1. List of the 21 elections under study with the numbers of electoral units that fulfill the inclusion criteria for our analysis, *N*.

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