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Competition against peer-to-peer networks

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ABSTRACT

In this paper, we consider the competition of a monopolistic provider of information products against a peer-to-peer file-sharing network that offers illegal versions of the products. We focus on the role of direct externalities caused by the P2P file-sharing technology rather than the indirect consumption externalities studied previously in the literature. In our model the market structure is endogenous and we characterize three possible scenarios where the firm uses monopoly pricing, network-deterring pricing, and network-accommodating pricing, respectively. We make a full comparative-static analysis of prices, quantities, profits, consumer surplus and total surplus for each of the scenarios as well as a comparison across scenarios. We show that in the case of network-accommodating pricing, the firm sets a higher price when facing a lower generic cost factor of downloading. Furthermore, in all scenarios, profits for the firm unambiguously decrease when the generic cost factor of downloading declines; total welfare unambiguously increases, however, a result that has implications for intellectual property rights enforcement policy.

1. Introduction

The tremendous amount of downloading and sharing of information goods on the internet in the past 10 years is largely due to the wide-spread popularity of P2P (peer-to-peer) file-sharing networks. The information content providers have, over the years, claimed that the usage of such file-sharing networks constitutes the primary cause of the steady decrease in their sales revenue, and they have resorted to law suits against the users of such networks on numerous occasions. Anti-piracy organizations have also contributed considerable work in helping the authorities reinforce intellectual property rights. Despite all their efforts, however, the file-sharing activities have, if anything, steadily increased in the past years, thanks to the increasing sophistication and non-tractability of today's file-sharing applications.

The classical literature related to piracy can be traced back to a couple of decades ago. Typical papers include for example Novos and Waldman (1984), Liebowitz (1985), Johnson (1985), where the authors study how a firm can react to piracy. Basically there are three possibilities: do not react at all if piracy poses no real threat; use piracy-deterring pricing; or use piracy-accommodating pricing. For a systematic analysis of the short- and long-run consequences of illicit copying of information goods, we refer to Belleflamme (2003).

In parallel, the literature on network externalities has been developed, represented by classical papers including, e.g. Katz and Shapiro (1985, 1986) and Farrell and Saloner (1985, 1986). Conner and Rumelt (1991) and Takeyama (1994) combine the two subjects and find that unauthorized reproduction of intellectual property in the presence of consumptive externalities can induce greater firm profits and lead to Pareto improvements.

More recently, since the rise and fall of the first generation peer-to-peer technology Napster and the controversy that ensued, a wave of new papers emerged to specifically tackle the P2P phenomenon. Gayer and Shy (2003) show

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how publishers of digitally-stored products can utilize P2P networks to enhance sales of their products sold in stores. This result follows from the positive consumptive externalities in their model. Another way in which firms can benefit from P2P activities is in set-ups with imperfect information about product characteristics. The effects of sampling possibilities due to P2P networks are studied by Peitz and Waelbroeck (2006b). The authors show that under sufficient taste heterogeneity and product diversity, the positive effect of downloading on sales due to sampling may compensate the negative effect. Peitz and Waelbroeck (2006a) provide a comprehensive review of the recent literature on piracy of digital products.

While consumption externalities are very important for software products, for the majority of illegal content shared on P2P networks (music, movies, etc.), this effect plays a role of lesser importance. We abstract from consumption externalities, but instead focus on the network externalities caused by the working principle of the P2P file-sharing network itself. A higher number of users of a network improves the availability of files and hence decreases the standard search costs and downloading time. It is precisely this network effect that we focus on in this study. This effect is not linear. The marginal benefit of an additional user joining the network is positive, but decreasing in the network size. Therefore, we implement a cost function of downloading that is decreasing and convex in the number of users of the P2P network.

Apart from the network size, the cost function of downloading is influenced by a parameter called the generic cost factor of downloading. This parameter represents a collection of factors that may affect downloading costs; for instance: population computer literacy, the availability of broadband internet infrastructure, and most importantly, the degree of legal enforcement of intellectual property rights. Observe that unlike Gayer and Shy (2003), legal buyers of the product do not benefit from the number of downloading users, which distinguishes our downloading externalities from consumption externalities. We also abstract from other factors that tend to benefit the firm selling the product like imperfect information of product characteristics, or the possibility of indirect appropriation of rents (Liebowitz, 1985). This eliminates channels through which the firm can benefit from piracy, and it is a priori unclear how total surplus is affected by piracy.

We consider a monopolistic firm that is confronted with an illegal P2P network. We refer to the legal product sold by the firm as the *physical form* and the P2P version of it as the *digital form*. These two forms, although essentially providing similar contents, differ in numerous ways. The physical form of the product is sold by the firm as CDs or DVDs, usually accompanied by appealing packaging and complementary booklets. The digital form, which is shared on the internet via P2P networks, contains largely the same content, although often of discounted quality. It does not have the nice complementary features that the physical form does, and the quality of the content is often inferior.

We model the interaction of the firm and the P2P network as a two-stage game. In the first stage, the firm sets a price for the physical form of the product. Then, in the second stage, after having observed the price set by the firm, consumers simultaneously and independently decide whether to legally buy the physical form, to download the digital form via the P2P network, or not to acquire the content at all. The consumers are heterogenous with respect to their tastes regarding the physical and the digital form, and we use the standard Hotelling model to describe their preferences. In their decision, consumers have to anticipate each others' decisions, since the resulting network size determines the actual costs of downloading. This distinguishes our model from the literature on copying (Belleflamme, 2003), where network effects are absent and decisions of other consumers are irrelevant. We analyze the model by studying its subgame-perfect Nash equilibria.

In the second stage of our game, consumers face a classical coordination problem. If they all expect that no one will join the P2P network, then it is optimal for all of them to buy the product legally from the firm. When the price set by the firm is sufficiently high, there is a critical mass (Economides and Himmelberg, 1995) such that a P2P network of that size can be supported as an equilibrium, where there is a marginal consumer that is indifferent between buying the physical form and downloading the digital form. Due to the positive downloading externalities, this critical-mass network is destabilized when slightly more consumers join it. Then, more and more consumers join the network up to a size where a new marginal consumer is indifferent between buying and downloading, or where the network contains all consumers and everybody is better off downloading.

We provide a full characterization of the subgame-perfect Nash equilibria of the model and then restrict attention to the unique subgame-perfect Nash equilibrium where the network of maximal size forms whenever possible. We characterize the equilibrium market structures, and analyze for each structure how the firm's profit, the consumers' surplus and the total welfare are affected by different parameters such as the degree of intellectual property law enforcement and the consumers' taste heterogeneity.

We find that there are three possible equilibrium market structures. Firstly, the firm may act as a traditional monopoly, either with a partially-served or a fully-served market. In particular, the P2P network does not form. For this to occur, the intrinsic value of the physical form of the product has to be substantially higher than that of the digital form. Secondly, the firm may prevent the forming of the network by means of a network-deterring pricing policy. Again, the firm either partially or fully serves the market. Finally, legal sales and the P2P network may co-exist. In this case, the market is guaranteed to be fully served.

Regarding the effect of the generic cost factor of downloading, we find that the larger this cost factor is, the less likely that a P2P network will form, and the higher the firm's profit will be. Strikingly, once the market exhibits co-existence of the two platforms, the smaller the cost factor is, the higher the price that is set by the firm. Despite this pricing behavior, the firm's profit unambiguously declines as the cost factor decreases. In the partially-served monopolistic market with network-deterring pricing and the fully-served market with multi-platform co-existence, the total welfare decreases in the cost factor. This implies that it is welfare decreasing if authorities overemphasize the protection of intellectual property rights for those contents whose creators are not significantly hurt by consumers' free-riding.

The remainder of the paper is organized as follows. In Section 2 the two-stage model is described in detail. Next, the consumers' choices in the second stage, given the price set by the firm in the first stage, are presented in Section 3. Subsequently, in Section 4, the firm's pricing decision in the first stage, and the resulting market structures, are analyzed. In order to reduce multiplicity of equilibria, we restrict attention to the case with coordination on the network of maximal size. The comparative statics of this equilibrium with respect to the model's parameters are provided in Section 5. Sections 7 and 6 conclude and discuss the results.

2. The model

We consider a monopolistic firm offering an information good and facing a P2P network. We model this situation by means of a two-stage game. In stage one, the firm sets a price p for the physical form of the product. Next, in stage two, after having observed the price set by the firm, consumers decide simultaneously and independently whether to purchase the physical form sold by the firm (S), to download the digital form via the P2P network (N), or not to acquire the product at all ($\emptyset$). We assume that there is a continuum of consumers who differ in their relative preferences of the physical form over the digital form. A strategy of a consumer with identity $x \in [0,1]$ is a function d^x that maps any price p into her choice set:

$$d^x : \mathbb{R}_+ \rightarrow \{S, N, \emptyset\},$$

and we denote the profile of strategies by $d = (d^x)_{x \in [0,1]}$.

The price p set by the firm and the profile of consumers' strategies determine the sales $s(p,d)$ by the firm and the size $n(p,d)$ of the network via

$$s(p,d) = \mu(\{x \in [0,1] | d^x(p) = S\}) \quad \text{and}$$
$$n(p,d) = \mu(\{x \in [0,1] | d^x(p) = N\}),$$

where μ denotes the Lebesgue measure.[1]

We assume, for simplicity, that the firm has zero costs in production and aims to maximize its profit, given by

$$\pi(p,d) = p \cdot s(p,d).$$

The utility of a consumer with identity $x \in [0,1]$ is given by

$$U^x(p,d) = \begin{cases} \beta - \tau x & -p & \text{if } d^x(p) = S \\ \gamma - \tau(1-x) & -C(n(p,d)) & \text{if } d^x(p) = N \\ 0 & & \text{if } d^x(p) = \emptyset, \end{cases}$$

where $\beta > 0$ and $\gamma > 0$ represent the basic utility of the physical and the digital form respectively. The identity $x \in [0,1]$ reflects the consumer's relative preference over the two forms. The consumer with identity $x = 0$ has a strong preference for the physical form, whereas the consumer with identity $x = 1$ has a strong preference for the digital form. For consumers $x \in (0,1)$, the acquisition of one of the forms generates a disutility that depends on the identity x and the parameter $\tau > 0$. The parameter τ captures the amount of heterogeneity in consumers' tastes. Finally, $C(n)$ represents the costs of downloading when the resulting network is of size $n \in [0,1]$.

We focus on the network externalities caused by the working principle of the P2P file-sharing network itself. A higher number of users of a network improves the availability of files and hence decreases the standard search costs and downloading time. The marginal benefit of an additional user joining the network is positive, but decreasing in the network size. Therefore, we implement a cost function of downloading that is decreasing and convex in the number of users of the P2P network,[2]

$$C(n) = \sigma \cdot (1-n)^2,$$

where $\sigma > 0$ represents the generic cost factor of downloading, incorporating a collection of factors that may affect downloading costs, for instance, population computer literacy, the availability of broadband internet infrastructure, and most importantly, the degree of legal enforcement of intellectual property rights. Note that σ is identical for every consumer and is independent of the network size.

We impose some assumptions on the relevant parameters.

Assumption 1. $\beta > \gamma$.

Personal preferences aside, the objective product quality of the original physical form is higher than that of the digital form (which is "ripped" from the original).

Assumption 2. $\beta > \tau$ and $\gamma > \tau$.

Every consumer prefers a free physical product and a costless download to not acquiring the product.

Assumption 3. $\sigma > \gamma$.

The generic cost factor of downloading is sufficiently high such that when the network size is zero, no consumer would like to join the network.

Although irrelevant for the economic implications resulting from the model, we impose some assumptions on the consumers' behavior in case of indifference. When a consumer is indifferent between buying from the firm and not acquiring the product or between the firm and the network, she chooses the firm. When she is indifferent between the network and not acquiring the product, she chooses the network. Thus, without loss of generality, we assume a linear order of priority of the firm (S) above the network (N), and the network (N) above not acquiring the product ($\emptyset$).

Finally, we restrict our attention to the case where the firm's price is bounded from above by the maximum that

[1] In equilibrium, the sets of consumers buying from the firm and going to the network, respectively, will be measurable.

[2] The quadratic specification of the cost function for downloading satisfies the qualitative properties just stated, while it preserves analytical tractability of the model.

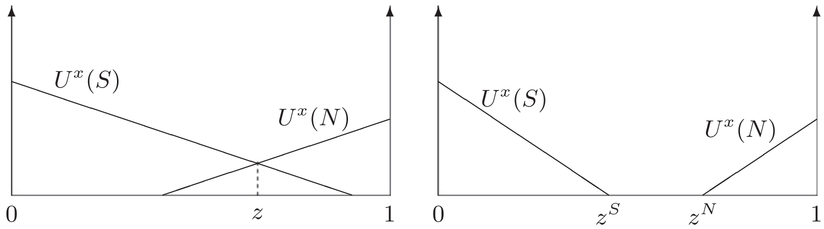

Fig. 1. Fully-served market and partially-served market.

a consumer is willing to pay for the product; i.e., $0 \leqslant p \leqslant \beta$. If $p > \beta$, all consumers would have negative utility level from buying from the firm; hence the firm's sales and profit would be zero. The firm can always do better, in equilibrium, by lowering the price to the interval $[0, \beta]$. Therefore, this assumption can be made without loss of generality and does not impose any restrictions on the results.

In the following sections we present the subgame-perfect Nash equilibria of the model. As usual, we start the analysis by determining the possible Nash equilibria for each of the subgames. That is, we determine the Nash equilibria of the games in the second stage that result from each possible first-stage price. Then, we consider the firm's pricing behavior in the first stage and find the subgame-perfect Nash equilibria.

3. Consumers' choice

Firstly, we observe that if a consumer with identity x chooses to buy from the firm, then in equilibrium all consumers with identity less than x choose to buy from the firm. Secondly, if a consumer with identity x chooses to use the P2P network, in equilibrium all consumers with identity larger than x choose to do so. This implies that in equilibrium only a few *market structures* can arise, where market structures can differ in two dimensions: the degree to which consumers are served and the platforms that are actively used. Regarding the first dimension, the market can be fully served or partially served as is displayed in Fig. 1 for a multi-platformed market.[3] Regarding the second dimension we can have a multi-platformed market or a single-platformed market with either the firm only or the P2P network only. For the second stage, we can therefore restrict our attention to the six market structures depicted in Table 1. Observe that the situation in the graph on the left of Fig. 1 is denoted by [S/N] and the one on the right by $[S/\emptyset/N]$.

Table 1
All possible market structures in stage 2.

	Single-platformed		Multi-platformed
	Firm	Network	
Fully-served	[S]	[N]	[S/N]
Partially-served	$[S/\emptyset]$	$[\emptyset/N]$	$[S/\emptyset/N]$

Our assumption that $\sigma > \gamma$ implies that the generic cost factor of downloading is sufficiently high such that when the network size is zero, no consumer can join the network. This implies that if everybody expects that nobody will join the network, we have a second-stage equilibrium without network.

Now consider the case where the price set by the firm in stage one is high enough to be consistent with the formation of a P2P network. The size of the smallest network that can be supported as a second-stage equilibrium given some price set by the firm is called the *critical-mass network* and is denoted by *c.m.* If all the consumers expect that the network is like this, such expectations are self-enforcing. There is one consumer who is indifferent between buying and downloading and whose choice is irrelevant since we have a continuum of consumers. All consumers with higher utility from downloading are strictly better off joining the network and all the other consumers are strictly better off buying from the firm.

Since we have positive downloading externalities, the critical-mass network is destabilized when slightly more consumers, located closest to the indifferent consumer, join it. If all consumers expect that the network is like this, the consumer with the highest value of $x$buying at the firm will strictly prefer to download, and these expectations are not compatible with a second-stage equilibrium. In that case, there will be a *maximum network*, denoted by *m.n.*that can be supported as a second-stage equilibrium, where the maximal network may be the one with all consumers in it.

The next proposition describes the possible second-stage equilibrium market structures conditional on the first-stage price set by the firm.

Proposition 1. *Given the first-stage price p, the possible equilibrium structures that can arise in the second stage are*

$[S]$ if $0 \leqslant p \leqslant \beta - \tau$,

$[S/\emptyset]$ *if* $\beta - \tau < p \leqslant \beta$,

$[S/N]_{c.m.}$ *if* $\beta - \gamma + \tau - \frac{\tau^2}{\sigma} \leqslant p \leqslant \beta - \frac{\tau^2}{2\sigma}\left(1 + \sqrt{1 + 4\frac{\sigma}{\tau^2}(\gamma - \tau)}\right)$,

$[S/\emptyset/N]_{c.m.}$ *if* $\beta - \frac{\tau^2}{2\sigma}\left(1 + \sqrt{1 + 4\frac{\sigma}{\tau^2}(\gamma - \tau)}\right) < p \leqslant \beta$,

$[S/N]_{m.n.}$ *if* $\beta - \gamma + \tau - \frac{\tau^2}{\sigma} \leqslant p \leqslant \beta - \gamma + \tau$, *and*

$[N]$ *if* $\beta - \gamma + \tau < p \leqslant \beta$.

[3] Notice that the vertical intercept of the utility level from the network is endogenously determined by the resulting network size. The quantities z, z^S and z^N are used for the analysis in the appendices.

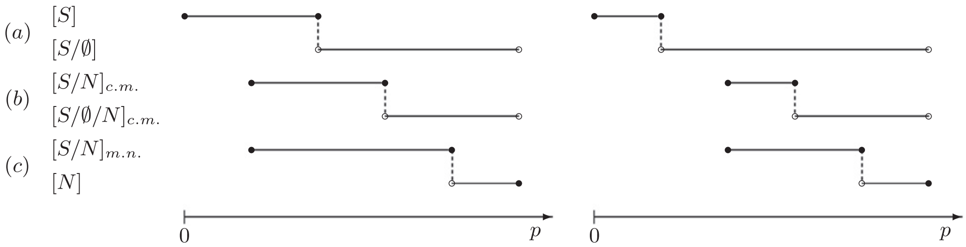

Fig. 2. Scenario A.1 (left); scenario B.1 (right).

Proof.

[∅] and [∅/N]. Observe that for these structures to occur, the consumer with identity 0 would get a negative utility from buying from the firm, which implies $p > \beta$. This case has been excluded.

[S]. Since $\sigma > \gamma$, this structure is consistent with an equilibrium if and only if the consumer with identity 1 is indifferent between not acquiring the product and buying from the firm, so $\beta - \tau - p \geqslant 0$, which is the case if and only if $0 \leqslant p \leqslant \beta - \tau$.

[S/∅]. Since $\sigma > \gamma$,for this structure to occur, the consumer with identity 1 should get a negative utility from buying from the firm, which is the case if and only if $p > \beta - \tau$.

[S/N]. See Appendix A.

[S/∅/N]. See Appendix B.

[N]. Note that for all Nash equilibria in this structure $n(p, d) = 1$,so the costs of downloading are zero. Since $\gamma > \tau$, this structure appears as an equilibrium if and only if the consumer with identity 0 prefers the network over the firm, which is the case if and only if $p > \beta - \gamma + \tau$. □

A closer look at Proposition 1 reveals that the second-stage equilibrium manifold consists of three *segments* of logically connected equilibrium structures. They are: (a) the no-network segment: [S]–[S/∅], (b) the critical-mass-network segment: $[S/N]_{c.m.}$–$[S/\emptyset/N]_{c.m.}$, and (c) the maximum-network segment: $[S/N]_{m.n.}$–[N]. A graphical representation of these segments is provided in Fig. 2. As we will explain later, the parameters give rise to five possible scenarios, two of which are illustrated in Fig. 2.

The three segments share the common property that they begin at a certain price with one of the equilibrium structures, switch to the other equilibrium structure at a higher price and end with that second structure at an even higher price. Table 2 summarizes, for each of the three segments, the starting, switching and ending prices, which are denoted by $p^{\vdash}$, $p^{\times}$ and $p^{\dashv}$ respectively.

Notice that up to a price of $p^{\vdash}_{(b,c)} = \beta - \gamma + \tau - \frac{\tau^2}{\sigma}$ there is a unique equilibrium where all consumers choose to buy from the firm, and that for higher prices there are three equilibria—one for each segment. At the price $p^{\vdash}_{(b,c)}$ there are two equilibria since the one corresponding to the critical-mass network coincides with the one of the maximum network. It is easily verified that for parameter settings satisfying Assumptions 1–3, for each segment, the switching point is strictly between the starting point and the ending point. Moreover, the switching point $p^{\times}_{(b)}$ of the critical-mass-network segment exceeds the switching point $p^{\times}_{(a)}$of the no-network segment.

Table 2
Three segments of equilibrium structures.

Segment	Starting price ($p^{\vdash}$)	Switching price ($p^{\times}$)	Ending price ($p^{\dashv}$)
(a)	0	$\beta - \tau$	β
(b)	$\beta - \gamma + \tau - \frac{\tau^2}{\sigma}$	$\beta - \frac{\tau^2}{2\sigma}\left(1 + \sqrt{1 + 4\frac{\sigma}{\tau^2}(\gamma - \tau)}\right)$	β
(c)	$\beta - \gamma + \tau - \frac{\tau^2}{\sigma}$	$\beta - \gamma + \tau$	β

At the no-network segment only one platform is active, the firm. The switching point indicates the price level at which the market switches from being fully served to partially served. At the critical-mass-network segment, the two platforms co-exist. Here, again, the switching point indicates a switch from a fully served to a partially-served market. For the maximum-network segment, the switching point indicates a switch from a multi-platformed to a single-platformed market. Along this whole segment the market is fully served.

Different parameter values of β, γ, σ and τ can lead to five *scenarios*, depending on how the starting point $p^{\vdash}_{(b,c)}$ of both network segments is related to the switching point $p^{\times}_{(a)}$ of the no-network segment and how the switching point $p^{\times}_{(c)}$ of the maximum-network segment is positioned relative to the switching point of the other two segments. In the three A scenarios, the starting point $p^{\vdash}_{(b,c)}$ of both network segments is less than or equal to the switching point $p^{\times}_{(a)}$ of the no-network segment, and the reverse holds for

Table 3
Possible scenarios and corresponding parameter settings.

Scenario	Location of switching points		Parameter values		
A.1	$p^{\vdash}_{(b,c)} \leqslant p^{\times}_{(a)}$	$p^{\times}_{(c)} \geqslant p^{\times}_{(b)}$	$\gamma \geqslant 2\tau - \frac{\tau^2}{\sigma}$	$\gamma \leqslant 2\tau$	$\gamma \geqslant \frac{2\tau^2}{\sigma} + \tau$
A.1			$\gamma \geqslant 2\tau - \frac{\tau^2}{\sigma}$	$\gamma \leqslant \frac{\tau^2}{2\sigma} + \tau$	
A.2	$p^{\vdash}_{(b,c)} \leqslant p^{\times}_{(a)}$	$p^{\times}_{(c)} \in [p^{\times}_{(a)}, p^{\times}_{(b)}]$	$\gamma \geqslant 2\tau - \frac{\tau^2}{\sigma}$	$\frac{\tau^2}{2\sigma} + \tau \leqslant \gamma \leqslant \frac{2\tau^2}{\sigma} + \tau$	
A.3	$p^{\vdash}_{(b,c)} \leqslant p^{\times}_{(a)}$	$p^{\times}_{(c)} \leqslant p^{\times}_{(a)}$	$\gamma \geqslant 2\tau$		
B.1	$p^{\vdash}_{(b,c)} > p^{\times}_{(a)}$	$p^{\times}_{(c)} \geqslant p^{\times}_{(b)}$	$\gamma < 2\tau - \frac{\tau^2}{\sigma}$	$\gamma \geqslant \frac{2\tau^2}{\sigma} + \tau$	
B.1			$\gamma < 2\tau - \frac{\tau^2}{\sigma}$	$\gamma \leqslant \frac{\tau^2}{2\sigma} + \tau$	
B.2	$p^{\vdash}_{(b,c)} > p^{\times}_{(a)}$	$p^{\times}_{(c)} \in [p^{\times}_{(a)}, p^{\times}_{(b)}]$	$\gamma < 2\tau - \frac{\tau^2}{\sigma}$	$\frac{\tau^2}{2\sigma} + \tau \leqslant \gamma \leqslant \frac{2\tau^2}{\sigma} + \tau$	

the two B scenarios. The A scenarios result when $\gamma \geqslant 2\tau - \frac{\tau^2}{\sigma}$ and the B scenarios do when $\gamma < 2\tau - \frac{\tau^2}{\sigma}$.

Since the switching point $p^{\times}_{(b)}$of the critical-mass-network segment exceeds the switching point $p^{\times}_{(a)}$ of the no-network segment, the switching point $p^{\times}_{(c)}$ of the maximum-network segment can be greater than or equal to $p^{\times}_{(b)}$, which is the case if and only if $\gamma \leqslant 2\tau$ and ($\gamma \leqslant \frac{\tau^2}{2\sigma} + \tau$ or $\gamma \geqslant \frac{2\tau^2}{\sigma} + \tau$), the switching point $p^{\times}_{(c)}$ can belong to the interval $[p^{\times}_{(a)}, p^{\times}_{(b)}]$, which is the case if and only if $\gamma \leqslant 2\tau$ and $\frac{\tau^2}{2\sigma} + \tau \leqslant \gamma \leqslant \frac{2\tau^2}{\sigma} + \tau$, and the switching point $p^{\times}_{(c)}$ can be less than or equal to $p^{\times}_{(a)}$, which is the case if and only if $\gamma \geqslant 2\tau$. In this way we obtain three subcases for each of the two scenarios, resulting in scenarios A.1, A.2, A.3, and in scenarios B.1, B.2, and B.3, where the set of parameter values giving rise to B.3 is void. Table 3 presents the parameter values that give rise to the five possible scenarios, where we have eliminated redundant constraints. Fig. 2 illustrates scenarios A.1 and B.1.

Notice that all inequalities in Table 3 only involve the parameters γ, τ and σ, and are independent of β. Moreover, notice that all inequalities are homogeneous such that we can assume $\sigma = 1$ without loss of generality. That is, whenever (γ, τ, σ) satisfies the conditions of a particular scenario, so does $(\frac{\gamma}{\sigma}, \frac{\tau}{\sigma}, 1)$. For $\sigma = 1$, Fig. 3 displays, in (τ, γ)-space, how the scenarios are related to one another.

In Fig. 3, the concave-shaped curve separates the A scenarios, with $p^{\vdash}_{(b,c)} \leqslant p^{\times}_{(a)}$, from the B scenarios, where $p^{\vdash}_{(b,c)} > p^{\times}_{(a)}$. Roughly speaking, γ and τ have to be relatively close for the B scenarios to occur, and conversely γ has to be sufficiently larger than τ for the A scenarios to occur. Intuitively, when γ is large compared to τ, the desirability of the digital form dominates the costs caused by taste heterogeneity, which makes people located closer to the physical form more prone to downloading. This leads to the A scenarios, where the firm perceives a lot of pressure from the network, i.e. even when the firm sets a low price, inducing a fully-served market, a network may form.

Finally, before moving on to the analysis of stage one, the following proposition specifies (independent of the scenarios) the behavior of the firm's profit along each of the segments. Its proof is straightforward when using the

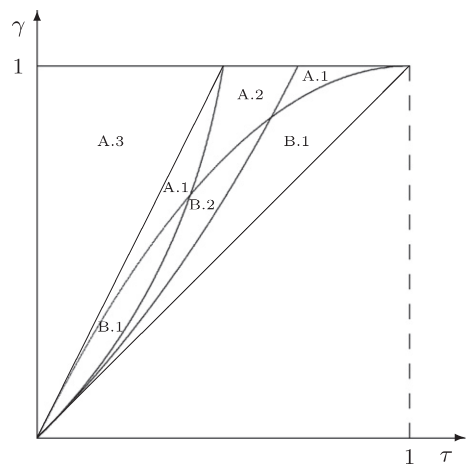

Fig. 3. Parameter settings leading to each of the scenarios.

expressions for the consumer that is indifferent between her first-best – buying from the firm – and her second-best choice, as derived in Appendices A and B.

Proposition 2. *Along each segment the profit of the firm is continuous in price. For each price, the profit on the no-network segment is larger than the profit on the critical-mass-network segment, which is in turn larger than the profit on maximum-network segment. At the ending point the profit is zero on each segment. Along the no-network segment the profit is equal to the price until the switching point and is concave afterwards. The profit at the starting point on both network segments is equal to* $\frac{\tau}{\sigma}$ *times the price. Along the maximum-network segment the profit is zero after the switching point.*

Intuitively, for a given price with multiple equilibria in the second stage, the firm always has a higher profit if the network is not formed. At the ending point of each segment the price is β, which will result in no sales, and hence zero profit. Before the switching point on the no-network segment, the sales quantity is exactly one (the full market); therefore the profit is equal to the price. After the switching point, the sales quantity decreases in price and the profit is therefore concave in price. At the starting point of both network segments, the network size is identical for the critical-mass network and the maximum network, and the sales quantity is less than one. After this starting point, the size of the critical-mass network starts to decrease and the size of the maximum network starts to increase. The profit along the maximum-network segment after the switching point is zero because there are no sales. The profits along the no-network segment (a), the critical-mass-network segment (b), and the maximum-network segment (c) are illustrated in Fig. 4. Note that in Fig. 4, the profit on the no-network segment (a) at a price equal to $p^{\vdash}_{(b,c)}$ is not guaranteed to exceed the global maximum of the profit on the maximum-network segment (c). Moreover, in the B scenarios it is possible that the global maximum of the profit on the no-network segment (a) is attained at a price below $p^{\vdash}_{(b,c)}$.

4. The firm's decision

4.1. Subgame-perfect Nash equilibria

Using the definition of a subgame-perfect Nash equilibrium and the results of the previous section, the following result is immediate.

Proposition 3. *The strategy profile* (p^*, d^*) *is a subgame-perfect Nash equilibrium if and only if, for every* $p \in [0, \beta]$, $d^*(p)$ *is consistent with one of the second-stage equilibrium structures of* Proposition 1 *and* $\pi(p^*, d^*(p^*)) \geqslant \pi(p, d^*(p))$.

In order to analyze which pairs of prices and consumer decisions $(p, d(p))$ can be supported as a subgame-perfect equilibrium outcome, it is convenient to define the firm-worst response of the consumers. Let therefore d_{LE} be the mapping that assigns to each price the equilibrium structure that generates the lowest profit for the firm, except at the starting price $p^{\vdash}_{(b,c)} = \beta - \gamma + \tau - \frac{\tau^2}{\sigma}$ of the network

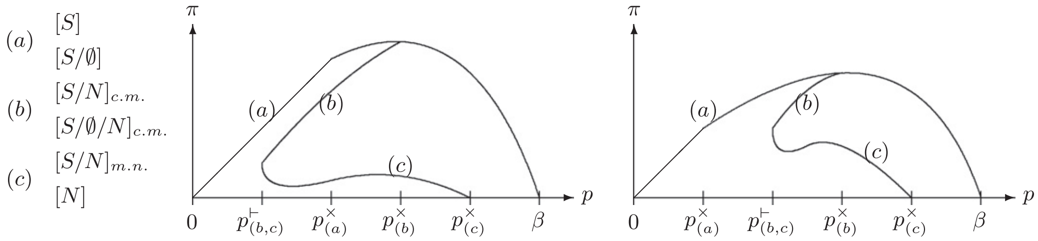

Fig. 4. Profit along segments; scenario A.1 (left); scenario B.1 (right).

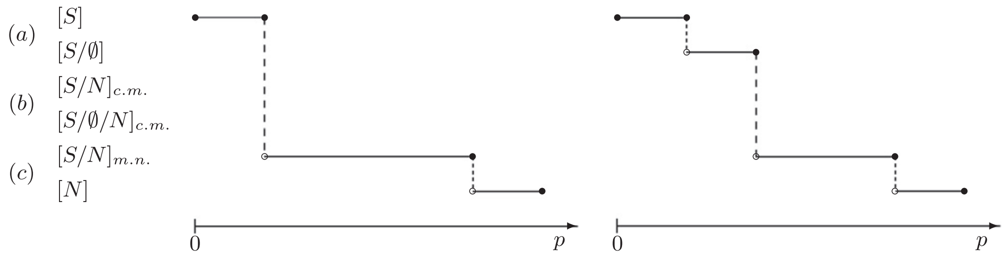

Fig. 5. The lower envelope of the different scenarios (left: A scenarios; right: B scenarios).

segments, where for later convenience we define $d_{\mathrm{LE}}(p)$ as the no-network equilibrium structure, so $d^x_{\mathrm{LE}}(p) = S$, $x \in [0, 1]$. LE stands for "lower envelope". For the five scenarios depicted in Table 3, the corresponding lower-envelope mappings are depicted in Fig. 5. Notice that the lower envelope of the three A scenarios has the structure as displayed in the left panel and the two B scenarios as displayed in the right panel. Moreover, the lower envelope only involves the no-network segment and the maximum-network segment, but not the critical-mass-network segment. We define the maximum profit along the lower envelope by $\pi^*_{\mathrm{LE}} = \max_p \pi(p, d_{\mathrm{LE}}(p))$. Proposition 2 and the definition of the lower envelope guarantees that the maximum exists, even though $\pi(p, d_{\mathrm{LE}}(p))$ is not everywhere continuous in p. We have the following characterization of subgame-perfect equilibrium outcomes.

Proposition 4. *The pair $(p^*, d^*(p^*))$ is a subgame-perfect Nash equilibrium outcome if and only if $d^*(p^*)$ is consistent with one of the second-stage equilibrium structures of* Proposition 1 *and $\pi(p^*, d^*(p^*)) \geqslant \pi^*_{\mathrm{LE}}$.*

Proof.

($\Rightarrow$) Let (p^*, d^*) be a subgame-perfect Nash equilibrium with equilibrium outcome $(p^*, d^*(p^*))$. Proposition 3 implies that $d^*(p^*)$ is consistent with one of the second-stage equilibrium structures of Proposition 1. Since, given d^* the firm has no incentive to deviate from p^*, we have, for every $p \in [0, \beta]$,

$$\pi(p^*, d^*(p^*)) \geqslant \pi(p, d^*(p)).$$

For $p \in [0, \beta] \setminus \{p^{\vdash}_{(b,c)}\}$, it holds that $\pi(p, d^*(p)) \geqslant \pi(p, d_{\mathrm{LE}}(p))$, so

$$\pi(p^*, d(p^*)) \geqslant \sup_{p \in [0,\beta] \setminus \{p^{\vdash}_{(b,c)}\}} \pi(p, d_{\mathrm{LE}}(p)) = \sup_{p \in [0,\beta]} \pi(p, d_{\mathrm{LE}}(p)) = \pi^*_{\mathrm{LE}},$$

where the first inequality uses that $\pi(p, d_{\mathrm{LE}}(p))$ is continuous from the left at $p^{\vdash}_{(b,c)}$.

($\Leftarrow$) We define the consumers' strategy profile $\tilde{d}_{\mathrm{LE}}$ by $\tilde{d}_{\mathrm{LE}}(p^*) = d^*(p^*)$ and $\tilde{d}_{\mathrm{LE}}(p) = d_{\mathrm{LE}}(p)$ for $p \neq p^*$. By Proposition 3, $(p^*, \tilde{d}_{\mathrm{LE}})$ is a subgame-perfect Nash equilibrium, and therefore $(p^*, d^*(p^*))$ a subgame-perfect Nash equilibrium outcome, if $\pi(p^*, \tilde{d}_{\mathrm{LE}}(p^*)) \geqslant \pi(p, \tilde{d}_{\mathrm{LE}}(p))$ for every $p \in [0, \beta]$. This inequality holds since, by assumption, $\pi(p^*, \tilde{d}_{\mathrm{LE}}(p^*)) \geqslant \pi^*_{\mathrm{LE}}$, and $\pi^*_{\mathrm{LE}} \geqslant \pi(p, \tilde{d}_{\mathrm{LE}}(p))$ for $p \neq p^*$ by definition of π^*_{LE}. □

4.2. Equilibrium selection

To deal with the multiplicity of subgame-perfect equilibria, we follow the convention by supposing that, once the price is known, consumers coordinate on the equilibrium continuation that they prefer, which is the one with the largest network size.[4] To avoid continuity problems, we assume coordination on the firm for the knife-edge case when the price equals $p^{\vdash}_{(b,c)}$. This corresponds to the firm-worst response of the consumers, which is captured by our lower envelope. In this subsection, we therefore study the firm's optimal pricing behavior along the lower envelope. Guided by Fig. 5, we treat the A scenarios and the B scenarios separately, and refer to them as scenario A and scenario B, respectively..

For the convenience in notation, we define $\delta \equiv 4\tau^2 - 3\sigma(\beta - \gamma + \tau)$. Roughly speaking, δ represents the attractiveness of the network relative to the firm. Indeed, δ is increasing in $\gamma - \beta$ and decreasing in σ. Without loss of generality, we assume that when different first-stage prices lead to identical second-stage profit levels, the firm selects the price that results in the smallest network size.

[4] See, for instance, Katz and Shapiro (1986) or Fudenberg and Tirole (2000).

Scenario A. In scenario A, the firm's profit along the lower envelope is given by:

$$\pi(p, d_{\mathrm{LE}}(p)) = \begin{cases} p & \text{if } 0 \leqslant p \leqslant p^{\vdash}_{(b,c)} \\ p \cdot \frac{\tau - \sqrt{\sigma(p+\gamma-\beta-\tau)+\tau^2}}{\sigma} & \text{if } p^{\vdash}_{(b,c)} < p \leqslant p^{\times}_{(c)} \\ 0 & \text{if } p^{\times}_{(c)} < p \leqslant \beta. \end{cases}$$

The profit in the interval $[N]$ is zero. The profit in the interval $[S]$ reaches its maximum at the right boundary point $p^{\vdash}_{(b,c)}$, which we denote by $\pi^*_{[S]} = \beta - \gamma + \tau - \frac{\tau^2}{\sigma}$. The profit in the interval $[S/N]_{m.n.}$ starts at $\frac{\tau}{\sigma}(\beta - \gamma + \tau - \frac{\tau^2}{\sigma})$, which is a share $\frac{\tau}{\sigma}$ of $\pi^*_{[S]}$. At this point it decreases with a slope of $-\infty$, and ends up equal to zero at the price $p^{\times}_{(c)} = \beta - \gamma + \tau$. However, before it reaches zero, it may increase and then subsequently decrease, achieving a local maximum at the price

$$p^*_{[S/N]_{m.n.}} = \frac{2}{9\sigma}(2\tau^2 - \delta + \tau\sqrt{\delta}) = \frac{2}{9\sigma}\left(2\tau - \sqrt{\delta}\right)\left(\tau + \sqrt{\delta}\right).$$

This expression is only valid if δ is non-negative, which incidentally also guarantees that the price is in the respective interval, i.e. $\delta \geqslant 0$ implies $p^{\vdash}_{(b,c)} < p^*_{[S/N]_{m.n.}} < p^{\times}_{(c)}$. In that case, the profit at this local maximum is

$$\begin{aligned}\pi^*_{[S/N]_{m.n.}} &= \frac{2}{27\sigma^2}\left(2\tau - \sqrt{\delta}\right)\left(\tau + \sqrt{\delta}\right)\left(3\tau - \sqrt{\tau^2 + \delta + 2\tau\sqrt{\delta}}\right) \\ &= \frac{2}{27\sigma^2}\left(2\tau - \sqrt{\delta}\right)^2\left(\tau + \sqrt{\delta}\right).\end{aligned}$$

Thus, the firm sets a price of $p^*_{[S/N]_{m.n.}}$ if and only if $\delta \geqslant 0$ and $\pi^*_{[S/N]_{m.n.}} > \pi^*_{[S]}$. This leads to a multi-platformed fully-served market with *co-existence* of the firm and the network. If $\delta \geqslant 0$, but $\pi^*_{[S/N]_{m.n.}} \leqslant \pi^*_{[S]}$, the firm sets a network-deterring price equal to $p^{\vdash}_{(b,c)}$, which leads to a single-platformed fully-served market with *network-deterring pricing*.[5] The firm sets the same network-deterring price in case $\delta < 0$.

Scenario B. In scenario B, the firm's profit along the lower envelope is given by:

$$\pi(p, d_{\mathrm{LE}}(p)) = \begin{cases} p & \text{if } 0 \leqslant p \leqslant p^{\times}_{(a)} \\ p \cdot \frac{\beta - p}{\tau} & \text{if } p^{\times}_{(a)} < p \leqslant p^{\vdash}_{(b,c)} \\ p \cdot \frac{\tau - \sqrt{\sigma(p+\gamma-\beta-\tau)+\tau^2}}{\sigma} & \text{if } p^{\vdash}_{(b,c)} < p \leqslant p^{\times}_{(c)} \\ 0 & \text{if } p^{\times}_{(c)} < p \leqslant \beta \end{cases}$$

Again, the profit in the interval $[N]$ is zero. In the interval $[S]$, the maximum profit is achieved at a price equal to $p^{\times}_{(a)}$. This is a global maximum on the whole lower envelope if and only if $\beta \geqslant 2\tau$. In this case, the firm optimally serves the whole market. Next, in the interval $[S/\emptyset]$, the maximum profit is achieved at the price $p = \frac{1}{2}\beta$. This is the global maximum on the lower envelope if and only if $2(\gamma - \tau + \frac{\tau^2}{\sigma}) < \beta < 2\tau$. In that case, the firm optimally serves the market partially as a monopoly, and there will be no network.

If $\beta \leqslant 2(\gamma - \tau + \frac{\tau^2}{\sigma})$, the situation is similar to scenario A. The maximum profit in the interval $[S/\emptyset]$ is achieved at the price $p^{\vdash}_{(b,c)} = \beta - \gamma + \tau - \frac{\tau^2}{\sigma}$, yielding $\pi^*_{[S/\emptyset]} = (\beta - \gamma + \tau - \frac{\tau^2}{\sigma})(\frac{\gamma}{\tau} - 1 + \frac{\tau}{\sigma})$. Thus, the firm sets the price at $p = \frac{2}{9\sigma}(2\tau - \sqrt{\delta})\left(\tau + \sqrt{\delta}\right)$ if and only if $\delta \geqslant 0$ (which guarantees $p^{\vdash}_{(b,c)} < p^*_{[S/N]_{m.n.}} < p^{\times}_{(c)}$) and $\pi^*_{[S/N]_{m.n.}} > \pi^*_{[S/\emptyset]}$. This leads to a multi-platformed fully-served market with the *co-existence* of the firm and the network. Otherwise the firm sets a network-deterring price equal to $p^{\vdash}_{(b,c)}$, which leads to a single-platformed partially-served market with *network-deterring pricing*.

Observe that, in scenario B, on top of the network-deterring pricing and co-existence cases like in scenario A, we have two cases of *monopoly pricing*, under the condition that β is sufficiently high when compared to τ. Notice also that the network-deterring pricing in scenario B occurs in a partially-served market as opposed to a fully-served market in scenario A.

We summarize the findings from our studies of the scenarios above in the following propositions.

Proposition 5. *The firm acts as a monopolist if and only if*

$$\gamma < 2\tau - \frac{\tau^2}{\sigma} \quad \text{and} \quad \beta > 2\left(\gamma - \tau + \frac{\tau^2}{\sigma}\right).$$

If in addition $\beta \geqslant 2\tau$, the market is fully served; otherwise it is partially served.

The first inequality in this proposition ensures that we are in scenario B.

Proposition 5 indicates that when the quality of the physical form of the product is sufficiently high and the one of the digital form is sufficiently low, the firm can monopolize the market. In case the gross value of the product exceeds by far the transportation costs caused by taste heterogeneity, the firm is willing to serve the whole market. When this is not the case, the firm does not attempt to sell to the consumers with strong preferences for the digital form. Notice that, here, the firm disregards the network in its pricing. This is not the case in the next proposition.

Proposition 6. *There are parameter constellations such that, in equilibrium, the firm applies network-deterring pricing. This is the case with a fully-served market if and only if*

$$\gamma \geqslant 2\tau - \frac{\tau^2}{\sigma} \quad \text{and} \quad (\delta < 0 \quad \text{or} \quad \pi^*_{[S/N]_{m.n.}} \leqslant \pi^*_{[S]}),$$

and the case with a partially-served market if and only if

$$\gamma < 2\tau - \frac{\tau^2}{\sigma}, \quad \beta \leqslant 2\left(\gamma - \tau + \frac{\tau^2}{\sigma}\right), \quad \text{and} \quad (\delta < 0 \quad \text{or} \quad \pi^*_{[S/N]_{m.n.}} \leqslant \pi^*_{[S/\emptyset]}).$$

When the digital form is above a certain quality level, or when the physical form is below a certain quality level, the network has sufficient potential to form and the firm is no longer able to apply monopoly pricing. Stated differently, the firm is disciplined by the threat of the network forming. However, when the generic cost factor of downloading σ is sufficiently large (such that δ is negative), the firm is able to price as such to deter the network from developing.

[5] Our assumption that when different first-stage prices lead to identical second-stage profit levels, the firm selects the price that results in the smallest network size, implies in this case that the firm selects the network-deterring price rather than the co-existence price when both prices lead to the same profit level.

Proposition 7. *The firm and the network co-exist if and only if the conditions in the previous two propositions are not met. This is the case if and only if*

$$\gamma \geqslant 2\tau - \frac{\tau^2}{\sigma}, \quad \delta \geqslant 0, \quad \textit{and} \quad \pi^*_{[S/N]_{m.n.}} > \pi^*_{[S]}$$

or

$$\gamma < 2\tau - \frac{\tau^2}{\sigma}, \quad \delta \geqslant 0, \quad \textit{and} \quad \pi^*_{[S/N]_{m.n.}} > \pi^*_{[S/\emptyset]}.$$

In such a case, the market is fully served.

Proof. It holds that the conditions in Propositions 5 and 6 are not met if and only if

$$\gamma \geqslant 2\tau - \frac{\tau^2}{\sigma}, \quad \delta \geqslant 0, \quad \text{and} \quad \pi^*_{[S/N]_{m.n.}} > \pi^*_{[S]},$$

or

$$\gamma < 2\tau - \frac{\tau^2}{\sigma}, \quad \beta \leqslant 2\left(\gamma - \tau + \frac{\tau^2}{\sigma}\right), \quad \delta \geqslant 0, \quad \text{and} \quad \pi^*_{[S/N]_{m.n.}} > \pi^*_{[S/\emptyset]}.$$

Now $\delta \geqslant 0$ implies

$$\beta \leqslant \frac{4\tau^2}{3\sigma} + \gamma - \tau < \frac{2\tau^2}{\sigma} + 2(\gamma - \tau),$$

so we can omit the condition on β. □

Propositions 5–7 above indicate that the parameter $\delta = 4\tau^2 - 3\sigma(\beta - \gamma + \tau)$ plays an important role in determining market outcomes. Recall that δcorresponds to the attractiveness of the network relative to the firm. For the conditions in Proposition 7 to be satisfied, δ must be positive. Thus, for the two platforms to co-exist, the two forms of the product should not be too distinct in quality and the generic cost factor of downloading should be sufficiently low. It can be shown that the inequalities in Proposition 5, where monopoly is the equilibrium market outcome, require δ to be negative.

We close this section with a graphical illustration of how the different equilibrium market structures are located in the parameter space. For the case with both β and σ equal to 1, Fig. 6 presents the areas described in the propositions in (τ, γ)-space, where 'mon', 'det' and 'co' refer to monopoly (Proposition 5), network-deterring pricing (Proposition 6) and co-existence (Proposition 7) respectively, and 'fs' and 'ps' refer to a fully-served and partially-served market respectively.

The concave curve is identical to the one in Fig. 3, and separates scenario A (to the left of the curve) from scenario B (to the right of the curve). A monopoly can only exist in scenario B, either if $\tau \leqslant \frac{1}{2}$ (fully-served market), or if $\tau > \frac{1}{2}$ and $2(\gamma - \tau + \tau^2) < 1$ (partially-served market). This suggests that for the monopoly to exist, γ should not be too high when compared to β, so the quality of the physical form of the product should be sufficiently superior to the one of the digital form. Furthermore, the ratio $\frac{\gamma}{\tau}$ should not be too high, which means that sufficient taste heterogeneity among the consumers is necessary. Co-existence, on the other hand, occurs both in scenario A and B, but only in the far upper-right corner. This requires τ to be sufficiently large, which intuitively suggests that when taste heterogeneity is large, it does not pay off for the firm to deter the network from developing. Hence, the firm will accommodate the existence of the network. Everywhere else we find monopoly with network-deterring pricing in either a fully-served (in scenario A) or partially-served market (in scenario B). This shows that there is quite some room for the firm to manipulate the price and deter the development of the network.

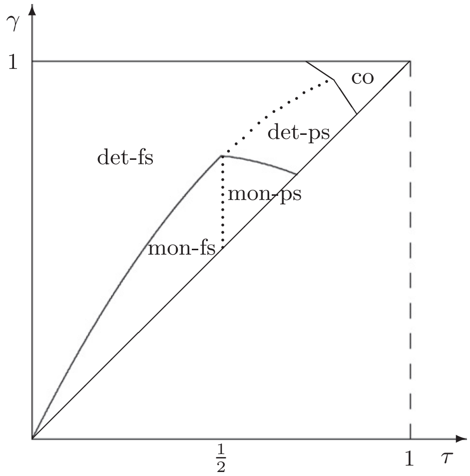

Fig. 6. A graphical illustration of the possible subgame-perfect Nash equilibrium market structures.

5. Comparative statics

In this section, we conduct some comparative-static analysis on the equilibrium outcomes of the subgame-perfect Nash equilibria discussed in the previous section. We will treat them separately according to their market structures. Recall that p^*, s^*, n^*, and π^* are the equilibrium price, the sales quantity of the firm, the network size, and the firm's profit respectively. CS^* and W^* denote the equilibrium consumers' surplus and the total welfare respectively.

5.1. Monopoly

Proposition 8. *The signs of the first-order partial derivatives of the equilibrium values with respect to the parameters in the case of the monopoly market are as shown in Table 4.*[6]

The findings here are intuitive. For example, in a fully-served monopolistic market, the consumers' surplus does not increase when the basic utility of the physical form of the product β increases, because the price increases by the same amount. The firm's profit increases and hence total welfare increases. An increase in taste heterogeneity τ decreases the price by exactly the same amount since the firm must keep the last consumer (whose transportation cost is exactly τ) on board. This results in a lower profit for the firm, while consumers benefit from the lower price.

[6] The proofs of the signs in the tables in Propositions 6–8 are provided in Appendix C.

Table 4
Monopoly pricing with fully-served market (a); monopoly pricing with partially-served market (b). The cells display the signs of the first derivatives.

	β	γ	τ	σ
(a)				
p^*	+	0	–	0
s^*	0	0	0	0
n^*	0	0	0	0
π^*	+	0	–	0
CS^*	0	0	+	0
W^*	+	0	–	0
(b)				
p^*	+	0	0	0
s^*	+	0	–	0
n^*	0	0	0	0
π^*	+	0	–	0
CS^*	+	0	–	0
W^*	+	0	–	0

Table 5
Fully-served market (a); partially-served market (b). The cells display the signs of the first derivatives.

	β	γ	τ	σ
(a)				
p^*	+	–	±	+
s^*	0	0	0	0
n^*	0	0	0	0
π^*	+	–	±	+
CS^*	0	+	±	–
W^*	+	0	–	0
(b)				
p^*	+	–	±	+
s^*	0	+	–	–
n^*	0	0	0	0
π^*	+	–	–	+
CS^*	0	+	±	–
W^*	+	+	–	–

In fact, for all but the last consumer the increase in taste heterogeneity is less than the decrease in price. Therefore, the total consumers' surplus increases in taste heterogeneity. Total welfare, however, decreases due to the larger loss in profit. The basic utility of the digital form γ and the generic cost factor of downloading σ do not influence any of the endogenous variables, since small variations in it do not lead to the establishment of the network.

In the partially-served monopolistic market, a larger basic utility β leads to more sales, higher price, higher profit, and higher welfare; whereas a larger taste heterogeneity τ leads to lower sales, lower profit, and lower welfare. Unlike in the fully-served monopoly market, the basic utility of the digital form γ and the generic cost factor of downloading σ do not influence any of the endogenous variables.

5.2. Network-deterring pricing

Proposition 9. *The comparative statics on the network-deterring pricing market structure are as shown in* Table 5.

In case the market is fully served by the firm, the price is the same as the profit. They both increase in the quality of the physical form β and decrease in that of the 'rival' digital form γ, because a lower price is needed to deter the network from forming. Moreover, for the same reason, they both increase in the generic cost factor of downloading σ. As a result, consumers' surplus increases in γ, thanks to the lower price charged for the product, but decreases in σ, due to the higher price. Total welfare increases in β, thanks to the higher profit, but does not depend on γ or σ. The reason is that in the fully-served market with network-deterring pricing, γ and σ only have an effect via the price of the product, and therefore only affect the distribution of surplus between the consumers and the firm. An increase in the taste heterogeneity τ makes consumers less likely to switch platforms, and may lead to either an increase or a decrease in price. When σ is relatively low, in particular if $\sigma < 2\tau$, an increase in τ leads to a lower price, and consequently a lower profit. However, for larger values of σ, the price and the profit increase when τ increases. The intuition is that when the taste heterogeneity increases, the digital form becomes less attractive for the consumers, an effect which is further amplified via a smaller expected network size. For larger values of the cost factor, high heterogeneity increases the expected costs of downloading sufficiently in order to shade the competition that the firm is facing from the network and allows the firm to increase the price without losing any sales. Although the effect on the consumers' surplus can be positive or negative, total welfare suffers from higher τ.

A difference we notice when the market is only partially served is that the effects of γ and σ on the the firm' sales quantity change. A decrease in γ and an increase in σ makes the network less competitive, and triggers an increase in price p^*, and leads to less sales s^*. When it comes to the profit, π^*, the price effect dominates the quantity effect. Thus, the comparative statics of prices and profits go in the same direction, similar to the case where the market is fully served. The total welfare W^* depends on γ and σ as well. It goes the opposite direction as the firm's price p^*. With a lower γ or a higher σ, leading to a higher price, the gain in profit in the partially-served market is less than that in the fully-served market, due to a lower sales quantity. The effect on the consumers' surplus being the same, total welfare decreases, which explains the positive sign for γ and the negative one for σ. An increase in τ also affects sales negatively, but the consequences for the price are again ambiguous. As with the fully-served market, the price decreases if $\sigma < 2\tau$ and increases if $\sigma > 2\tau$.[7] An increase in τ may now lead to either higher or lower consumers' surplus, depending on the quantitative significance of the decrease in sales, the increase in transportation cost, and the change in price. The change in total welfare is unambiguously negative when τ increases.

Even when the network does not form, its attractiveness (represented by a high γ and a low σ) does have a negative impact on the firm's profit and a positive impact on the total welfare. This has some implications for intellectual property rights policy, which we will discuss later.

[7] Both inequalities on parameter values are consistent with the conditions of Proposition 6.

Table 6
Co-existence of the firm and the network with a fully-served market. The cells display the signs of the first derivatives.

	β	γ	τ	σ
p^*	+	–	+	–
s^*	+	–	–	+
n^*	–	+	+	–
π^*	+	–	+	+
CS^*	–	+	–	–
W^*	+	+	–	–

5.3. Co-existence

Proposition 10. *The comparative statics of the co-existence of the firm and the network in a fully-served market are as shown in* Table 6.

One can argue that nowadays the co-existence case prevails in reality, probably most prominently so in the music industry. Here we find some very interesting results. An increase in β leads to a higher price, sales, profit, and a lower network size. Surprisingly, consumer surplus is affected negatively by an increase in β. The negative effects of a higher price and a lower network size outweigh the increase in quality β. Total welfare varies positively with β.

The comparative statics with respect to τ confirm the usual intuitions. An increase in τ leads to a higher price and profit, lower sales, and a bigger network. The consumers' surplus and total welfare are affected negatively.

The equilibrium price p^* decreases in the generic cost factor of downloading σ, despite the fact that the equilibrium network size n^* decreases in σ. This result implies that, in equilibrium, the higher the cost factor σ, the lower the firm sets its price: a counter-intuitive result, since a higher σ makes the network less strong as a competitor of the firm. The negative correlation is caused by the shape of the cost function of downloading $C(n)$, which is convex and decreasing in the network size n. In other words, since downloading costs decrease with the network size at a diminishing rate, it makes the first few consumers who join the network more vital in determining the costs (or the price, effectively) of downloading. The firm, therefore, has an incentive to "play tough" by means of a low price when the equilibrium network size is relatively small (large σ). Conversely, if σ is small, the equilibrium network size is large, and the network is stronger and will form more easily. In that case, the firm can do better by backing off from the competition and charging a high price to reap the most profit out of the customers that are more eager to buy the physical form of the product. This outcome follows because the equilibrium profit π^* is increasing in σ, despite the decreasing price. What we observe here, therefore, is a very interesting form of platform competition. Moreover, similar to the partially-served market network-deterring pricing case, total welfare W^* decreases in σ. This result implies that it maybe welfare enhancing for the internet infrastructure of the country to be improved, and for the legal enforcements of intellectual property rights to be relaxed.

5.4. Comparative statics across market structures

In the above subsections we have shown comparative statics in the three different market structures separately. In this subsection we analyze how equilibrium values change across the market structures as a function of the most important parameter in the model, σ. We show graphically how equilibrium price, profit and total welfare change in relation to σ. In order to make a clear sketch, we display the limit case where β, γ and τ are equal to *1*. We allow σ to vary on the horizontal axis and put the equilibrium values on the vertical axis. Fig. 7 shows the equilibrium price, profit and welfare responding to changes in σ.

When the generic cost factor of downloading σ is very high, the traditional monopoly setting results, as one would expect. As this cost factor decreases, the price, the profit and total welfare remain unaffected as long as the market structure remains unchanged. Once σ becomes sufficiently small (less than 2), the firm has to adapt its pricing strategy, because the network becomes a vital enemy to the firm. As long as σ is not too small (larger than $\frac{5}{4}$), the firm optimally deters the forming of the network. In this region, the price and the profit are decreasing for decreasing values of σ. The profit is decreasing at an increasing rate. Total welfare, however, increases. In case the cost factor becomes very small $\left(\sigma < \frac{5}{4}\right)$, the firm optimally tolerates the network and refrains from further deterring behavior. This results in a discontinuous increase in price and welfare as the market tips from a single-platformed structure towards a multi-platformed one. From here, the equilibrium price and welfare gradually increase as σ decreases. The profit continues to decrease, but only decreases at a diminishing rate.

The comparative statics with a decreasing σcan be applied to what happened in the last decade with the arrival of the P2P networks. We conclude that, although the society as a whole unambiguously benefits from a decrease in the generic cost factor of downloading, the firm unambiguously suffers from it despite a non-monotonic optimal pricing policy. One can, therefore, only expect every attempt possible from the firm to make file-sharing and downloading very difficult and costly indeed.

6. Discussion

The results of this paper have a bearing on intellectual property rights policy. We have shown that the total welfare unambiguously increases when the generic cost factor of downloading decreases, in particular, when there is less protection of intellectual property. To give further underpinnings for this conclusion, one would like to extend the model with a part that involves the creation of new content, and analyze how quality and quantity of new content is affected by intellectual property rights. As shown in Johnson (1985) and Bae and Choi (2006), these effects are complicated and can depend on various factors, including the nature of piracy costs and the value consumers place on product variety.

The model and its results presented in this paper can also be applied outside the context of P2P file-sharing or

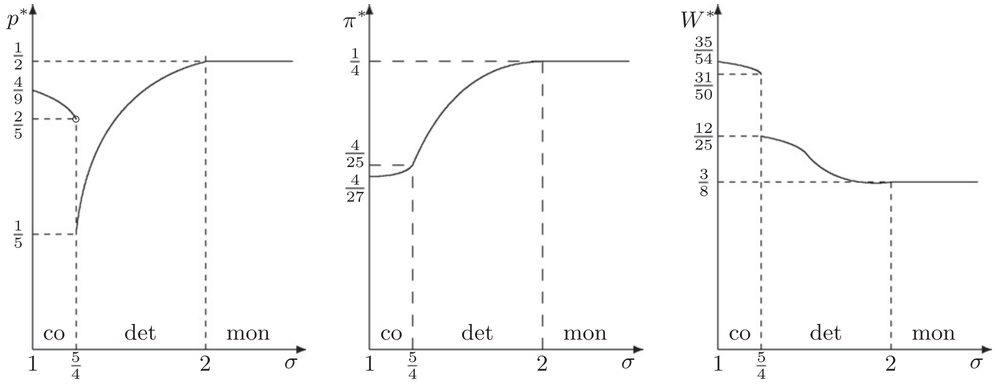

Fig. 7. Equilibrium price (left), profit (middle), and welfare (right) across markets structures.

piracy. In fact, with some modifications, it can be applied to any situation where a monopoly is faced with a free outside option that exhibits positive network externalities. An example would be the competition between commercial software against freely distributed open-source software, such as the market for operating systems, with Microsoft representing the firm and Linux representing the digital form. The externalities would in this case be related to the amount of support users of a particular platform can expect, as well as the availability of software that is compatible with the operating system chosen. Similar to the case studied in this paper, the marginal contribution of an additional user to the other users of the platform is diminishing in the size of the network. However, in this application, both platforms would create externalities for its users, which calls for an interesting extension of our model.

Another interesting application would involve an extension of the model of trading platforms as analyzed in Nocke et al. (2007). The authors study the effects of different platform ownership structures on the platform size, as well as on consumer and total surplus. In their model, buyers can either visit the trading platform or take the outside option. Regarding the outside option it is assumed that no externalities are present. Allowing for positive externalities among buyers taking the outside option, would affect the outcome of the trading platform, and it is natural to expect that these externalities interact with the platform ownership structure.

7. Conclusion

This paper investigates the competition between a monopolistic information content provider offering the physical form of the product and a P2P file-sharing network (offering the digital form), where consumers that opt for the P2P network benefit from the presence of other consumers due to decreased search costs and downloading time and increased availability. The model has four parameters that capture, respectively, the quality of the physical form of the product, the quality of the digital form, the level of consumers' taste heterogeneity, and the generic cost factor of downloading. This latter factor incorporates elements such as population computer literacy, the availability of broadband internet infrastructure, and in particular, the degree of legal enforcement of intellectual property rights.

We give a complete characterization of the subgame-perfect Nash equilibria of the model as well as of the subgame-perfect Nash equilibrium outcomes. We show that, depending on the parameter values, several market structures may result. These market structures differ not only in the number of platforms being active (one or two) and the level at which the market is served (fully or partially), but also in the firm's pricing policy. A market structure without any network is compatible both with monopoly pricing by the firm as well as with a network-deterring pricing policy. Given fixed values for the other parameters, as we gradually decrease the generic cost factor of downloading, the market experiences first the single platform (the firm) with monopoly pricing, then the single platform (the firm) with network-deterring pricing, and finally co-existence of multiple competing platforms (the firm and the network). In addition, we find that the equilibrium price moves in the opposite direction of the downloading cost factor in case the two platforms co-exist. Although, in general, the price behaves non-monotonically when the cost factor of downloading becomes smaller, the profit unambiguously decreases. The total welfare, on the other hand, unambiguously increases.

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Appendix A. The fully-served multi-platformed market

The consumer $z \in (0,1)$ that separates the consumers choosing for the firm from those choosing for the network, is herself indifferent between the two options. Moreover this consumer should weakly prefer the firm to the option not to acquire the product. So, for a second-stage equilibrium (p,d) to generate the structure $[S/N]$, it should hold that

$$0 \leqslant U^z(p,d^{-z},S) = U^z(p,d^{-z},N) \quad \Longleftrightarrow \quad 0 \leqslant \beta - \tau z - p = \gamma - \tau(1-z) - \sigma(1-n(p,d))^2,$$

where $n(p,d) = 1 - z$, since the market is fully served. Solving the inequality for z gives us two solutions:

$$z_{c.m.}(p) = \frac{\tau + \sqrt{\sigma(p+\gamma-\beta-\tau)+\tau^2}}{\sigma} \quad \text{and} \quad z_{m.n.}(p) = \frac{\tau - \sqrt{\sigma(p+\gamma-\beta-\tau)+\tau^2}}{\sigma},$$

where we call the solution corresponding to the smaller network size ($z_{c.m.}$) the critical-mass network, and the solution corresponding to the larger network size ($z_{m.n.}$) the maximum network. In order to guarantee the solution to be real, we need

$$p \geqslant \beta - \gamma + \tau - \frac{\tau^2}{\sigma}. \tag{1}$$

For the market structure $[S/N]^*$ to exist, the other conditions needed are:

$$0 \leqslant z_*(p) < 1 \quad \text{and} \quad \beta - \tau z_*(p) - p \geqslant 0.$$

The next two subsections deal with the maximum network and the critical-mass network, respectively.

A.1. The maximum network

Assume Inequality (1) holds. The maximum network solution exists for the multi-platformed and fully-served market if and only if the following two conditions are satisfied

$$0 \leqslant \frac{\tau - \sqrt{\sigma(p+\gamma-\beta-\tau)+\tau^2}}{\sigma} < 1 \quad \text{and}$$
$$\beta - \tau\frac{\tau - \sqrt{\sigma(p+\gamma-\beta-\tau)+\tau^2}}{\sigma} - p \geqslant 0. \tag{2}$$

The first condition is equivalent to

$$p \leqslant \beta - \gamma + \tau \quad \text{and} \quad \sqrt{\sigma(p+\gamma-\beta-\tau)+\tau^2} > \tau - \sigma,$$

where the second inequality is trivially satisfied owing to the assumption that $\sigma > \tau$ and using Inequality (1). The second condition is equivalent to

$$\sqrt{\sigma(p+\gamma-\beta-\tau)+\tau^2} \geqslant \tau - \frac{\sigma}{\tau}(\beta - p).$$

We will argue next that the second condition is always satisfied. In case the right-hand side of the inequality above is negative, i.e. in case $p < \beta - \frac{\tau^2}{\sigma}$, the inequality is trivially satisfied. Otherwise, $p \geqslant \beta - \frac{\tau^2}{\sigma}$, and the inequality above implies

$$\frac{\sigma}{\tau^2}(\beta-p)^2 - (\beta-p) - (\gamma-\tau) \leqslant 0,$$

which is equivalent to

$$\beta - \frac{\tau^2}{2\sigma}\left(1 + \sqrt{1 + \frac{4\sigma}{\tau^2}(\gamma-\tau)}\right) \leqslant p \leqslant \beta - \frac{\tau^2}{2\sigma}\left(1 - \sqrt{1 + \frac{4\sigma}{\tau^2}(\gamma-\tau)}\right).$$

Because $\gamma > \tau$, the lower bound on p is less than $\beta - \frac{\tau^2}{\sigma}$ and therefore satisfied since we are considering the case $p \geqslant \beta - \frac{\tau^2}{\sigma}$. The upper bound is larger than β. Hence the second condition does not impose a further restriction on p.

Hence, Inequalities (1) and (2) boil down to

$$\beta - \gamma + \tau - \frac{\tau^2}{\sigma} \leqslant p \leqslant \beta - \gamma + \tau.$$

A.2. The critical-mass network

Assume Inequality (1) holds. The critical-mass network solution exists for the multi-platformed and fully-served market if and only if the following two conditions are satisfied

$$0 \leqslant \frac{\tau + \sqrt{\sigma(p+\gamma-\beta-\tau)+\tau^2}}{\sigma} < 1 \quad \text{and}$$
$$\beta - \tau\frac{\tau + \sqrt{\sigma(p+\gamma-\beta-\tau)+\tau^2}}{\sigma} - p \geqslant 0, \tag{3}$$

Notice that the first inequality of the first condition is trivially satisfied and that the second inequality of this condition can be simplified to

$$p < \beta - \gamma - \tau + \sigma.$$

The second inequality is equivalent to

$$\sqrt{\sigma(p+\gamma-\beta-\tau)+\tau^2} \leqslant \frac{\sigma}{\tau}(\beta-p) - \tau.$$

This condition holds only if the right-hand side of this inequality is non-negative, that is $p \leqslant \beta - \frac{\tau^2}{\sigma}$, and

$$\frac{\sigma}{\tau^2}(\beta-p)^2 - (\beta-p) - (\gamma-\tau) \geqslant 0,$$

where the latter inequality is equivalent to

$$p \leqslant \beta - \frac{\tau^2}{2\sigma}\left(1 + \sqrt{1 + \frac{4\sigma}{\tau^2}(\gamma-\tau)}\right) \quad \text{or}$$

$$p \geqslant \beta - \frac{\tau^2}{2\sigma}\left(1 - \sqrt{1 + \frac{4\sigma}{\tau^2}(\gamma-\tau)}\right).$$

Since $\gamma > \tau$, the right-hand side of the second inequality exceeds β and hence cannot be satisfied—leaving the first inequality to be satisfied. Moreover, the right-hand side of the first inequality is less than $\beta - \frac{\tau^2}{\sigma}$, hence the second condition is satisfied if and only if

$$p \leqslant \beta - \frac{\tau^2}{2\sigma}\left(1 + \sqrt{1 + \frac{4\sigma}{\tau^2}(\gamma-\tau)}\right).$$

Next we show that the second condition is more restrictive than the first condition. Suppose the opposite holds true. That is, suppose that

$$\beta - \frac{\tau^2}{2\sigma}\left(1+\sqrt{1+\frac{4\sigma}{\tau^2}(\gamma-\tau)}\right) > \beta-\gamma-\tau+\sigma.$$

We will derive a contradiction. The supposition implies

$$\sqrt{1+\frac{4\sigma}{\tau^2}(\gamma-\tau)} < \frac{2\sigma}{\tau^2}(\gamma+\tau-\sigma)-1. \tag{4}$$

For Inequality (4) to be satisfied, the right-hand side has to be positive, which is the case when

$$2\sigma(\gamma+\tau-\sigma)-\tau^2 > 0. \tag{5}$$

Inequality (4) then implies that

$$\frac{\sigma}{\tau^2}(\gamma+\tau-\sigma)^2 > 2\gamma-\sigma,$$

which in turn implies that

$$2\tau^2+\sigma(\sigma-\gamma)-2\tau\sigma > 0. \tag{6}$$

We argue that Inequality (5) and (6) cannot be satisfied simultaneously. By adding Inequality (5) to twice Inequality (6) we find that $\tau > \frac{2}{3}\sigma$. Since we deal with two homogenous polynomial inequalities, we can set one of the parameters to any positive number. We set $\gamma = 1$ and realize that this induces $\tau < 1 < \sigma$. Moreover, $\tau > \frac{2}{3}\sigma$implies that $\sigma < \frac{3}{2}$. If we solve Inequality (6) for τ, we find

$$\tau < \frac{\sigma-\sqrt{2\sigma-\sigma^2}}{2} \quad \text{or} \quad \tau > \frac{\sigma+\sqrt{2\sigma-\sigma^2}}{2}.$$

The first of these inequalities is in direct conflict with $\tau > \frac{2}{3}\sigma$, which leaves the second inequality as the only feasible option. However, for this inequality to hold true, $\frac{\sigma+\sqrt{2\sigma-\sigma^2}}{2}$ has to be less than 1, so σ has to be less than 1 or to be larger than 2, which contradicts $\sigma \in (1, \frac{3}{2})$. We have obtained a contradiction to the supposition. Hence,

$$\beta - \frac{\tau^2}{2\sigma}\left(1+\sqrt{1+\frac{4\sigma}{\tau^2}(\gamma-\tau)}\right) \leqslant \beta-\gamma-\tau+\sigma.$$

Hence, Inequalities (1) and (3) boil down to

$$\beta-\gamma+\tau-\frac{\tau^2}{\sigma} \leqslant p \leqslant \beta - \frac{\tau^2}{2\sigma}\left(1+\sqrt{1+\frac{4\sigma}{\tau^2}(\gamma-\tau)}\right).$$

Appendix B. The partially-served multi-platformed market

The consumer $z^S \in (0,1)$ ($z^N \in (0,1)$) that separates the consumers choosing for the firm (network) from those choosing not to acquire any product, is herself indifferent between the two options. Moreover, this consumer prefers both the firm (network) and no acquisition to the option to acquire via the network (firm). We have a partially-served market if there is an interval of consumers that prefers no acquisition to acquisition, meaning that z^S is less than z^N. So, for a second-stage equilibrium (p,d) to generate the structure $[S/\emptyset/N]$, it should hold that

$$U^{z^S}(p,d^{-z^S},S) = U^{z^S}(p,d^{-z^S},\emptyset) \quad \text{and} \quad U^{z^N}(p,d^{-z^N},N) = U^{z^N}(p,d^{-z^N},\emptyset)$$

and

$$0 \leqslant z^S(p) < z^N(p) \leqslant 1.$$

The first two equations are equivalent to

$$\beta-\tau z^S-p=0 \quad \text{and} \quad \gamma-\tau(1-z^N)-\sigma(1-n(p,d))^2=0.$$

where $n(p,d) = 1 - z^N$. Solving the first equation for z^S and the latter for z^N gives us one solution for z^S:

$$z^S(p) = \frac{\beta-p}{\tau},$$

and two solutions for z^N:

$$z^N_{c.m.} = \frac{\tau+\sqrt{\tau^2+4\sigma(\gamma-\tau)}}{2\sigma} \quad \text{and}$$
$$z^N_{m.n.} = \frac{\tau-\sqrt{\tau^2+4\sigma(\gamma-\tau)}}{2\sigma}.$$

B.1. The maximum network

It is easily seen that $z^N_{m.n.}$ is negative, so the maximum network solution cannot co-exist with a partially-served multi-platformed market.

B.2. The critical-mass network

The critical-mass network solution exists for the multi-platformed, partially-served market if and only if

$$0 \leqslant \frac{\beta-p}{\tau} < \frac{\tau+\sqrt{\tau^2+4\sigma(\gamma-\tau)}}{2\sigma} \leqslant 1. \tag{7}$$

The first inequality is satisfied, since $p \leqslant \beta$. Owing to the assumption that $\sigma > \gamma$, the third inequality is satisfied too. Indeed,

$$\frac{\tau+\sqrt{\tau^2+4\sigma(\gamma-\tau)}}{2\sigma} < 1 \iff \sqrt{\tau^2+4\sigma(\gamma-\tau)} < 2\sigma-\tau \iff \gamma < \sigma.$$

The second inequality is equivalent to

$$p > \beta - \frac{\tau^2}{2\sigma}\left(1+\sqrt{1+\frac{4\sigma}{\tau^2}(\gamma-\tau)}\right).$$

Hence the conditions in Inequality (7) are satisfied if and only if

$$\beta - \frac{\tau^2}{2\sigma}\left(1+\sqrt{1+\frac{4\sigma}{\tau^2}(\gamma-\tau)}\right) < p.$$

Appendix C. Support for Propositions 6–8

In this Appendix we support our comparative statics results in Section 5. For each of the five different scenarios we provide the respective equilibrium values. Once a sign of the derivative of these values with respect to one of the model's parameters is not straightforward (that is, if the factor appears in multiple terms in possibly opposite directions), we provide the precise derivations.

C.1. Monopoly pricing and fully-served market

		β	γ	τ	σ
p^*	$=\beta-\tau$	+	0	–	0
s^*	$=1$	0	0	0	0
n^*	$=0$	0	0	0	0
π^*	$=\beta-\tau$	+	0	–	0
CS^*	$=\frac{1}{2}\tau$	0	0	+	0
W^*	$=\beta-\frac{1}{2}\tau$	+	0	–	0

C.2. Monopoly pricing and partially-served market

		β	γ	τ	σ
p^*	$=\frac{1}{2}\beta$	+	0	0	0
s^*	$=\frac{1}{2\tau}\beta$	+	0	–	0
n^*	$=0$	0	0	0	0
π^*	$=\frac{1}{4\tau}\beta^2$	+	0	–	0
CS^*	$=\frac{1}{8\tau}\beta^2$	+	0	–	0
W^*	$=\frac{3}{8\tau}\beta^2$	+	0	–	0

C.3. Network-deterring pricing and fully-served market

		β	γ	τ	σ
p^*	$=\beta-\gamma+\tau-\frac{\tau^2}{\sigma}$	+	–	±[1]	+
s^*	$=1$	0	0	0	0
n^*	$=0$	0	0	0	0
π^*	$=\beta-\gamma+\tau-\frac{\tau^2}{\sigma}$	+	–	±[1]	+
CS^*	$=\gamma-\frac{3}{2}\tau+\frac{\tau^2}{\sigma}$	0	+	±[2]	–
W^*	$=\beta-\frac{1}{2}\tau$	+	0	–	0

1. $\text{sign}(\frac{\partial p^*}{\partial\tau})=\text{sign}(\frac{\partial\pi^*}{\partial\tau})=\text{sign}(1-\frac{2\tau}{\sigma})=\text{sign}(\sigma-2\tau)$. We have found parameter values compatible with the conditions in Proposition 6 that support a positive sign as well as a negative one.
2. $\text{sign}(\frac{\partial CS^*}{\partial\tau})=\text{sign}(-\frac{3}{2}+\frac{2\tau}{\sigma})=\text{sign}(\frac{4}{3}\tau-\sigma)$. As in 1., both positive and negative signs are found.

C.4. Network-deterring pricing and partially-served market

		β	γ	τ	σ
p^*	$=\beta-\gamma+\tau-\frac{\tau^2}{\sigma}$	+	–	±[1]	+
s^*	$=\frac{1}{\tau}(\gamma-\tau+\frac{\tau^2}{\sigma})$	0	+	–[2]	–
n^*	$=0$	0	0	0	0
π^*	$=(\beta-\gamma+\tau-\frac{\tau^2}{\sigma})\frac{1}{\tau}(\gamma-\tau+\frac{\tau^2}{\sigma})$	+	–[3]	–[4]	+[5]
CS^*	$=\frac{1}{2\tau}(\gamma-\tau+\frac{\tau^2}{\sigma})^2$	0	+	±[6]	–
W^*	$=(\beta-\frac{1}{2}(\gamma-\tau+\frac{\tau^2}{\sigma}))\frac{1}{\tau}(\gamma-\tau+\frac{\tau^2}{\sigma})$	+	+[7]	–[8]	–[9]

1. $\text{sign}(\frac{\partial p^*}{\partial\tau})=\text{sign}(1-\frac{2\tau}{\sigma})=\text{sign}(\sigma-2\tau)$. We have found parameter values compatible with the conditions in Proposition 6 that support a positive sign as well as a negative one.
2. $\frac{\partial s^*}{\partial\tau}=-\frac{\gamma}{\tau^2}+\frac{1}{\sigma}<0$, since $\gamma\sigma>\tau^2$.
3. $\frac{\partial\pi^*}{\partial\gamma}=\frac{1}{\tau}\left(\beta-2\left(\gamma-\tau+\frac{\tau^2}{\sigma}\right)\right)<0$, since one of the conditions in Proposition 6 is $\beta\leqslant 2(\gamma-\tau+\frac{\tau^2}{\sigma})$.
4. $\frac{\partial\pi^*}{\partial\tau}=\cdots=-\frac{1}{\tau^2}(\beta-\gamma+\frac{\tau^2}{\sigma})(\gamma-\tau+\frac{\tau^2}{\sigma})+\frac{1}{\tau^2}(\beta-\gamma+\tau-\frac{\tau^2}{\sigma})(-\tau+2\frac{\tau^2}{\sigma})=\cdots=-\{\frac{1}{\tau^2}(\beta-\gamma)\gamma+\frac{1}{\sigma}\left[2\left(\gamma-\tau+\frac{\tau^2}{\sigma}\right)-\beta\right]+\left(\frac{\tau}{\sigma}-1\right)^2\}<0$, since $\beta>\gamma$ and $\beta<2\left(\gamma-\tau+\frac{\tau^2}{\sigma}\right)$.
5. $\frac{\partial\pi^*}{\partial\sigma}=\frac{\tau}{\sigma^2}(2\left(\gamma-\tau+\frac{\tau^2}{\sigma}\right)-\beta)>0$, again, using the condition in Proposition 6.
6. We have found parameter values compatible with the conditions in Proposition 6 that support a positive sign as well as a negative one.
7. $\frac{\partial W^*}{\partial\gamma}=\frac{1}{\tau}\left(\beta-\gamma+\tau-\frac{\tau^2}{\sigma}\right)>0$.
8. As a result of an increase in τ, the demand curve shifts inward. Moreover, the number of consumers decreases as a result of the increase in τ. Both effects induce a decrease in welfare. In formulas, $W=\int_{x=0}^{\frac{1}{\tau}\left(\gamma-\tau+\frac{\tau^2}{\sigma}\right)}(\beta-\tau x)dx$. As τ increases, $\frac{1}{\tau}\left(\gamma-\tau+\frac{\tau^2}{\sigma}\right)$ decreases, and for each x, $\beta-\tau x$ decreases. Since for each x in the domain of integration $\beta-\tau x$ is positive, it follows that W decreases as τ increases.
9. $\frac{\partial W^*}{\partial\sigma}=-\frac{\tau}{\sigma^2}\left(\beta-\gamma+\tau-\frac{\tau^2}{\sigma}\right)<0$.

C.5. Co-existence

	β	γ	τ	σ
$p^*=\frac{2}{9\sigma}\left(2\tau-\sqrt{\delta}\right)\left(\tau+\sqrt{\delta}\right)$	+[1]	–[2]	+[3]	–[4]
$s^*=\frac{1}{3\sigma}\left(2\tau-\sqrt{\delta}\right)$	+	–	–[5]	+[6]
$n^*=1-\frac{1}{3\sigma}\left(2\tau-\sqrt{\delta}\right)$	–	+	+	–
$\pi^*=\frac{2}{27\sigma^2}\left(2\tau-\sqrt{\delta}\right)^2\left(\tau+\sqrt{\delta}\right)$	+[7]	–[8]	+[9]	+[10]
$CS^*=\gamma-\frac{1}{2}\tau-\frac{1}{9\sigma^2}\left(2\tau-\sqrt{\delta}\right)^2(\sigma-\tau)$	–	+	–[11]	–[12]
$W^*=\gamma-\frac{1}{2}\tau-\frac{1}{27\sigma^2}\left(2\tau-\sqrt{\delta}\right)^2\cdot(3\sigma-5\tau-2\sqrt{\delta})$	+[13]	+[14]	–[15]	–[16]
where $\delta=4\tau^2-3\sigma(\beta-\gamma+\tau)$				

Notice that $CS^*=[\beta-\frac{1}{2}\tau s^*-p^*]s^*+[\gamma-\frac{1}{2}\tau n^*-\sigma(1-n^*)^2]n^*$.

Claim. *The conditions in* Proposition 7 *imply* $\sigma\in(\tau,\frac{4}{3}\tau)$ *and* $\sqrt{\delta}\in(\frac{1}{2}\tau,\tau)$.

Proof. The properties $\sigma\in(\tau,\frac{4}{3}\tau)$ and $\sqrt{\delta}<\tau$ follow directly from $\sigma,\beta>\gamma>\tau$ and $\delta\geqslant 0$. Left to be shown is $\sqrt{\delta}>\frac{1}{2}\tau$. First notice that

$$\pi^*_{[S]}=\frac{1}{3\sigma}\left(\tau+\sqrt{\delta}\right)\left(\tau-\sqrt{\delta}\right)\quad\text{and}\quad\pi^*_{[S/\emptyset]}=\frac{1}{3\sigma}\left(\tau+\sqrt{\delta}\right)\left(\tau-\sqrt{\delta}\right)\frac{1}{\tau}\left(\gamma-\tau+\frac{\tau^2}{\sigma}\right).$$

Consider the case $\gamma\geqslant 2\tau-\frac{\tau^2}{\sigma}$. Since $\pi^*>\pi^*_{[S]}$ we have

$$2\delta+(9\sigma-8\tau)\sqrt{\delta}-(9\sigma-8\tau)\tau>0.$$

Since this inequality is quadratic in $\sqrt{\delta}$ it is easily shown that it is satisfied if and only if

$$\sqrt{\delta}>-\frac{1}{4}(9\sigma-8\tau)+\frac{1}{4}\sqrt{9\sigma(9\sigma-8\tau)}.$$

Since

$$-\frac{1}{4}(9\sigma-8\tau)+\frac{1}{4}\sqrt{9\sigma(9\sigma-8\tau)} \geqslant \frac{1}{2}\tau \iff \sqrt{\sigma(9\sigma-8\tau)} \geqslant 3\sigma-2\tau \iff \sigma \geqslant \tau$$

we have shown that $\sqrt{\delta} > \frac{1}{2}\tau$.

Next, consider the case $\gamma < 2\tau - \frac{\tau^2}{\sigma}$. Since $\pi^* > \pi^*_{[S/\emptyset]}$ we have

$$2\delta + \frac{1}{\tau}(9\sigma\gamma - 9\sigma\tau + \tau^2)\sqrt{\delta} - (9\sigma\gamma - 9\sigma\tau + \tau^2) > 0.$$

This inequality is satisfied if and only if

$$\sqrt{\delta} > -\frac{1}{4\tau}(9\sigma\gamma - 9\sigma\tau + \tau^2) + \frac{1}{4\tau} \times \sqrt{9(\sigma\gamma - \sigma\tau + \tau^2)(9\sigma\gamma - 9\sigma\tau + \tau^2)}.$$

Since

$$-\frac{1}{4\tau}(9\sigma\gamma - 9\sigma\tau + \tau^2) + \frac{1}{4\tau} \times \sqrt{9(\sigma\gamma - \sigma\tau + \tau^2)(9\sigma\gamma - 9\sigma\tau + \tau^2)} \geqslant \frac{1}{2}\tau$$

if and only if

$$\sqrt{(\sigma\gamma - \sigma\tau + \tau^2)(9\sigma\gamma - 9\sigma\tau + \tau^2)} \geqslant 3\sigma\gamma - 3\sigma\tau + \tau^2,$$

which is the case if and only if $\gamma \geqslant \tau$, we have shown that $\sqrt{\delta} > \frac{1}{2}\tau$. □

1. $\frac{\partial p^*}{\partial \beta} = \frac{2}{9\sigma}\left(2\tau - \sqrt{\delta}\right)\frac{d\sqrt{\delta}}{d\beta} - \frac{2}{9\sigma}(\tau + \sqrt{\delta})\frac{d\sqrt{\delta}}{d\beta} = \frac{2}{9\sigma}\left(\tau - 2\sqrt{\delta}\right)\frac{1}{2}\frac{1}{\sqrt{\delta}}(-3\sigma) = \frac{1}{3\sqrt{\delta}}\left(2\sqrt{\delta} - \tau\right) > 0.$
2. $\frac{\partial p^*}{\partial \gamma} = -\frac{\partial p^*}{\partial \beta} < 0.$
3. $\frac{\partial p^*}{\partial \tau} = \frac{2}{9\sigma}\left(2\tau - \sqrt{\delta}\right)\left(1 + \frac{d\sqrt{\delta}}{d\tau}\right) + \frac{2}{9\sigma}\left(2 - \frac{d\sqrt{\delta}}{d\tau}\right)(\tau + \sqrt{\delta}) = \frac{2}{9\sigma}\left\{4\tau + \sqrt{\delta} + \frac{d\sqrt{\delta}}{d\tau}\left(\tau - 2\sqrt{\delta}\right)\right\} = \frac{2}{9\sigma}\frac{1}{\sqrt{\delta}}\left\{4\tau\sqrt{\delta} + \delta + \frac{1}{2}(8\tau - 3\sigma)\left(\tau - 2\sqrt{\delta}\right)\right\} = \frac{2}{9\sigma}\frac{1}{\sqrt{\delta}}\{\delta + \sqrt{\delta}(3\sigma - 4\tau) + (4\tau^2 - \frac{3}{2}\sigma\tau)\} > \frac{2}{9\sigma}\frac{1}{\sqrt{\delta}}\{\delta - \tau^2 + 2\tau^2\} > 0.$
4. Notice that $\frac{d\sqrt{\delta}}{d\sigma} = \frac{1}{2}\frac{1}{\sqrt{\delta}}[-3(\beta - \gamma + \tau)] = -\frac{1}{2}\frac{1}{\sigma}\frac{1}{\sqrt{\delta}}[4\tau^2 - \delta] = -\frac{1}{2}\frac{1}{\sigma}\frac{1}{\sqrt{\delta}}(2\tau - \sqrt{\delta})\left(2\tau + \sqrt{\delta}\right)$. Then $\frac{\partial p^*}{\partial \sigma} = \cdots = -\frac{2}{9\sigma^2}\left(2\tau - \sqrt{\delta}\right)\left(\tau + \sqrt{\delta}\right) - \frac{2}{9\sigma^2}\left(\tau - 2\sqrt{\delta}\right)\frac{1}{2}\frac{1}{\sqrt{\delta}}\left(2\tau - \sqrt{\delta}\right)\left(2\tau + \sqrt{\delta}\right) = \cdots = -\frac{1}{9\sigma^2}(2\tau - \sqrt{\delta})^2\frac{1}{\sqrt{\delta}}\tau < 0.$
5. $\frac{\partial s^*}{\partial \tau} = \frac{1}{3\sigma}\frac{1}{\sqrt{\delta}}\left(2\sqrt{\delta} - 4\tau + \frac{3}{2}\sigma\right) < \frac{1}{3\sigma}\frac{1}{\sqrt{\delta}}\left(2\sqrt{\delta} - 4\tau + \frac{3}{2}\frac{4}{3}\tau\right) = \frac{1}{3\sigma}\frac{1}{\sqrt{\delta}}2\left(\sqrt{\delta} - \tau\right) < 0.$
6. $\frac{\partial s^*}{\partial \sigma} = \cdots = -\frac{1}{3\sigma^2}\left(2\tau - \sqrt{\delta}\right) + \frac{1}{6\sigma^2}\frac{1}{\sqrt{\delta}}(2\tau - \sqrt{\delta})(2\tau + \sqrt{\delta}) = \frac{1}{6\sigma^2}\frac{1}{\sqrt{\delta}}\left(2\tau - \sqrt{\delta}\right)^2 > 0.$
7. $\frac{\partial \pi^*}{\partial \beta} = \frac{2}{27\sigma^2}\{2(2\tau - \sqrt{\delta})\left(-\frac{d\sqrt{\delta}}{d\beta}\right)\left(\tau + \sqrt{\delta}\right) + \left(2\tau - \sqrt{\delta}\right)^2\frac{d\sqrt{\delta}}{d\beta}\} = -\frac{2}{9\sigma^2}\left(2\tau - \sqrt{\delta}\right)\sqrt{\delta}\frac{d\sqrt{\delta}}{d\beta} = \frac{1}{3\sigma}\left(2\tau - \sqrt{\delta}\right) > 0.$
8. $\frac{\partial \pi^*}{\partial \gamma} = -\frac{\partial \pi^*}{\partial \beta} < 0.$
9. $\frac{\partial \pi^*}{\partial \tau} = \frac{2}{27\sigma^2}\left(2\tau - \sqrt{\delta}\right)\left\{2(\tau + \sqrt{\delta})\left[2 - \frac{d\sqrt{\delta}}{d\tau}\right] + \left(2\tau - \sqrt{\delta}\right)\left[1 + \frac{d\sqrt{\delta}}{d\tau}\right]\right\} = \cdots = \frac{2}{9\sigma^2}\left(2\tau - \sqrt{\delta}\right)\left\{\frac{3}{2}(\sigma - \tau) + \frac{1}{2}\left(2\sqrt{\delta} - \tau\right)\right\} > 0.$
10. $\frac{\partial \pi^*}{\partial \sigma} = \frac{2}{27\sigma^2}\left(2\tau - \sqrt{\delta}\right)\left\{-\frac{2}{\sigma}\left(2\tau - \sqrt{\delta}\right)(\tau + \sqrt{\delta}) - 2(\tau + \sqrt{\delta})\frac{d\sqrt{\delta}}{d\sigma} + (2\tau - \sqrt{\delta})\frac{d\sqrt{\delta}}{d\sigma}\right\} = \cdots$ (see 4.) $\cdots = \frac{2}{27\sigma^2}(2\tau - \sqrt{\delta})\left\{-\frac{2}{\sigma}\left(2\tau - \sqrt{\delta}\right)(\tau + \sqrt{\delta}) + \frac{3}{2\sigma}\left(2\tau - \sqrt{\delta}\right)(2\tau + \sqrt{\delta})\right\} = \cdots = \frac{1}{27\sigma^3}\left(2\tau - \sqrt{\delta}\right)^3 > 0.$
11. $\frac{\partial CS^*}{\partial \tau} = \frac{\partial W^*}{\partial \tau} - \frac{\partial \pi^*}{\partial \tau} < 0$ (using 9. and 15.).
12. $\frac{\partial CS^*}{\partial \sigma} = \frac{\partial W^*}{\partial \sigma} - \frac{\partial \pi^*}{\partial \sigma} < 0$ (using 10. and 16.).
13. $\frac{\partial W^*}{\partial \beta} = -\frac{1}{27\sigma^2}\left(2\tau - \sqrt{\delta}\right)\left\{2\left(-\frac{d\sqrt{\delta}}{d\beta}\right)(3\sigma - 5\tau - 2\sqrt{\delta}) + (2\tau - \sqrt{\delta})\left(-2\frac{d\sqrt{\delta}}{d\beta}\right)\right\} = \cdots = \frac{1}{3\sigma}\left(2\tau - \sqrt{\delta}\right)\frac{1}{\sqrt{\delta}}(\tau + \sqrt{\delta} - \sigma) > \frac{1}{3\sigma}\left(2\tau - \sqrt{\delta}\right)\frac{1}{\sqrt{\delta}}\frac{1}{6}\tau > 0.$
14. $\frac{\partial W^*}{\partial \gamma} = 1 - \frac{1}{27\sigma^2}\left(2\tau - \sqrt{\delta}\right)\left\{2\left(-\frac{d\sqrt{\delta}}{d\gamma}\right)(3\sigma - 5\tau - 2\sqrt{\delta}) + \left(2\tau - \sqrt{\delta}\right)(-2\frac{d\sqrt{\delta}}{d\gamma})\right\} = \cdots = 1 - \frac{1}{3\sigma}(2\tau - \sqrt{\delta})\frac{1}{\sqrt{\delta}}(\tau + \sqrt{\delta} - \sigma) = \frac{1}{3\sigma}\frac{1}{\sqrt{\delta}}\left(\tau + \sqrt{\delta}\right)[2(\sigma - \tau) + \sqrt{\delta}] > 0.$
15. $\frac{\partial W^*}{\partial \tau} = \cdots = -\frac{1}{2} - \frac{1}{9}\left(2\tau - \sqrt{\delta}\right)\left\{\frac{1}{\sigma} - \frac{2\tau}{\sigma^2} - \frac{\sqrt{\delta}}{\sigma^2} + \frac{8\tau^2}{\sigma^2\sqrt{\delta}} - \frac{11\tau}{\sigma\sqrt{\delta}} + \frac{3}{\sqrt{\delta}}\right\}$. First, let $\sigma = a\tau$ for $a \in (1, \frac{4}{3})$. Then $\frac{\partial W^*}{\partial \tau} = -\frac{1}{2} - \frac{1}{9}\left(2\tau - \sqrt{\delta}\right)\left\{\frac{1}{a\tau} - \frac{2}{a^2\tau} - \frac{\sqrt{\delta}}{a^2\tau^2} + \frac{1}{\sqrt{\delta}}\left(\frac{8}{a^2} - \frac{11}{a} + 3\right)\right\}$. Next, define $\alpha \equiv \frac{1}{a\tau} - \frac{2}{a^2\tau} - \frac{\sqrt{\delta}}{a^2\tau^2} + \frac{1}{\sqrt{\delta}}\left(\frac{8}{a^2} - \frac{11}{a} + 3\right)$. If $\alpha \geqslant 0$ then clearly $\frac{\partial W^*}{\partial \tau} < 0$. Suppose $\alpha < 0$. Then $\alpha > \frac{1}{a\tau} - \frac{2}{a^2\tau} - \frac{\tau}{a^2\tau^2} + \frac{1}{\frac{1}{2}\tau}\left(\frac{8}{a^2} - \frac{11}{a} + 3\right) = \frac{1}{\tau}\left\{\frac{13}{a^2} - \frac{21}{a} + 6\right\} > \frac{1}{\tau}\min_a\left\{\frac{13}{a^2} - \frac{21}{a} + 6\right\} = -\frac{129}{52\tau}$, where for the first inequality it has to be noted that $\frac{8}{a^2} - \frac{11}{a} + 3 < 0$ for $a \in (1, \frac{4}{3})$. Hence $\frac{\partial W^*}{\partial \tau} = -\frac{1}{2} - \frac{1}{9}\left(2\tau - \sqrt{\delta}\right) \cdot \alpha < -\frac{1}{2} - \frac{1}{9}\frac{3}{2}\tau \cdot \left(-\frac{129}{52\tau}\right) = -\frac{9}{104} < 0.$
16. $\frac{\partial W^*}{\partial \sigma} = \frac{2}{27\sigma^3}\left(2\tau - \sqrt{\delta}\right)^2\left(3\sigma - 5\tau - 2\sqrt{\delta}\right) - \frac{2}{27\sigma^2}(2\tau - \sqrt{\delta})\left(-\frac{d\sqrt{\delta}}{d\sigma}\right)(3\sigma - 5\tau - 2\sqrt{\delta}) - \frac{1}{27\sigma^2}\left(2\tau - \sqrt{\delta}\right)^2\left[3 - 2\frac{d\sqrt{\delta}}{d\sigma}\right] = \cdots = \frac{1}{27\sigma^3}\left(2\tau - \sqrt{\delta}\right)^2\frac{1}{\sqrt{\delta}}\left\{2\sqrt{\delta}(3\sigma - 5\tau - 2\sqrt{\delta}) - (2\tau + \sqrt{\delta})(3\sigma - 5\tau - 2\sqrt{\delta}) - 3\sigma\sqrt{\delta} - (2\tau - \sqrt{\delta})(2\tau + \sqrt{\delta})\right\} = \cdots = -\frac{1}{27\sigma^3}\left(2\tau - \sqrt{\delta}\right)^2\frac{1}{\sqrt{\delta}}\left\{6\tau(\sigma - \tau) + \sqrt{\delta}\left(\tau + \sqrt{\delta}\right)\right\} < 0.$

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