EPJ Web of Conferences **287**, 05011 (2023) https://doi.org/10.1051/epjconf/202328705011
EOSAM 2023

Analysis of the recording of Fibonacci lenses using photopolymers with 3-D diffusion model

J. C. Bravo[1,2], *J. J.* Sirvent[1,2], *J. C.* García Vázquez[1,3], *A.* Pérez Bernabeu[1,2], *J.* Colomina Martínez[1,2], *R.* Fernández[1,2], *A.* Márquez[1,2], and *S.* Gallego[1,2]

[1] *Instituto Universitario de Física Aplicada a las Ciencias y las Tecnologías, Universidad de Alicante, Apartado 99, E03080 Alicante, Spain*
[2] *Departamento de Física, Ingeniería de Sistemas y Teoría de la Señal. Universidad de Alicante, Apartado 99, E03080 Alicante, Spain*
[3] *Departamento de Óptica, Farmacología y Anatomía. Universidad de Alicante. Apartado 99, E03080 Alicante, Spain*

Abstract. In the present work, a 3-Dimensional diffusion model is proposed to predict the main properties of Diffractive Optical Elements (DOEs), recorded in photopolymers, including refractive index modulation and the evolution of the transverse intensity distribution. The model enables the selection of appropriate material characteristics based on the intended application of the DOE. Specifically, a PVA/AA photopolymer based on acrylamide is simulated using the proposed model, considering coverplating and index matching systems to mitigate the effects of thickness variation. In order to compare its properties using the suggested model, the simulation focuses on a Fibonacci Lens and the dependece of the intensity on the polymerization rate. Accordingly, axial intensity pattern is represented to prove the bifocal-behaviour of these diffractive lenses.

1 Introduction

DOEs have numerous practical applications in diffractive microoptics, medical laser treatments, solar energy concentrators, etc. Due to their unique properties and versatility, DOEs are commonly used to shape light in working environments for customized illumination. Photopolymeric materials provide us with an ideal scenario to register DOEs, due to their good phase modulation properties, which we have successfully modeled recently. This model describes the diffusion process inside the photopolymer [1, 2]. Towether with non-local polymerization, light attenuation with depth and variations of the polymerization rate, such as is dealt with in [3]. The present work can be divided into three distinct stages. Firstly, the intensity pattern of a complex DOE is simulated, in particular the Fibonacci Lenses (FL) [4]. Secondly, the refractive index modulation due to the polymerization process is obtained using the theoretical diffusion model. Finally, by means of Fresnel propagation, effects on the evolution of the axial transverse intensity distribution produced by a zone plate constructed by FL are shown.

2 Theorical Diffusion Model

In general, the DOE formation in photopolymers depends on the polymerization rate, F_R, and the molecules diffusion inside the recording media, D, due to Fick's Law. Therefore the equations that govern this model are:

$$\frac{\partial M(x,z,t)}{\partial t} = D(t)\nabla^2 M(x,z,t) - F_R(x,z,t)M(x,z,t)$$

$$\frac{\partial P(x,z,t)}{\partial t} = F_R(x,z,t)M(x,z,t) \qquad (1)$$

,where M and P are the concentrations of the monomer and the polymer respectively and D is the diffusivity of the monomer in the material. Diffusion model presented solves this equations (1) using finite difference method (FDM). In this work, diffusivity in the polymerization process is consider as a constant. On the other hand, F_R is the rate of polymerization, wich depends on the rate of reaction and the recording intensity. This dependence is given by

$$F_R(x,z,t) = k_R(t)I(x,z,t)^{\gamma} = k_R(t)\left[I(x,y)e^{-\beta(t)z}\right]^{\gamma} \qquad (2)$$

$$k_R(t) = k_{R0}e^{-\alpha_T t}$$

where $I(x,z)$ is the recording intensity, k_{R0} is the rate constant, γ is the relationship between intensity and polymerization rate, $\beta(t)$ is the intensity depth attenuation coefficient due to light absorption and finally α_T is the attenuation of the polymerization due to the Trommsdorff's effect.

3 Fibonacci Lenses

The Fibonacci sequence is a recursive set of numbers that obeys:

$$F_{n+1} = F_n + F_{n-1} \qquad \lim_{n\to\infty}\frac{F_{n+1}}{F_n} = \frac{1+\sqrt{5}}{2} \equiv \varphi \qquad (3)$$

,where φ is the golden ratio. We can define a binary generating function, $\Phi_n(\zeta)$ for a π-phase of the Fibonacci Lens (FL) like.

$$\Phi_n(\zeta) = \begin{cases} 0 \quad \text{if} & (\lfloor l\varphi \rfloor) - 1)d \le \zeta < (\lfloor l\varphi \rfloor))d \\ \pi \quad \text{other case} & \end{cases} \qquad (4)$$

,where $l = 1, 2, 3, ..., F_n$, and $\zeta = (r/a)^2$ is the normalized radial coordinate, with a the radius of the lens and $\forall\ \zeta \in [0, 1]$, plus this interval is segmented into F_{n+1} subintervals of length $d = 1/F_{n+1}$. On the other hand, $\lfloor x \rfloor$, denotes the floor function, giving as its value the largest integer less than or equal to x.

4 Results and discussion

Fibonacci lens has been chosen to study the precision of this model. Specifically, FL profile will be simulated after 200 s of exposure time. As it is shown in [4], FL produce two focal points. In order to verify the aforementioned fact, an analysis will be conducted on the variation of axial intensity distribution, considering the influence of the parameter γ from Eq (2).

4.1 Refractive index modulation and Fresnel Propagation

Once the FL intensity pattern is incident onto the photopolymeric material, the diffusion process begins. This leads to a modulation of the refractive index during the exposure time as it is presented in **Figure 1**.

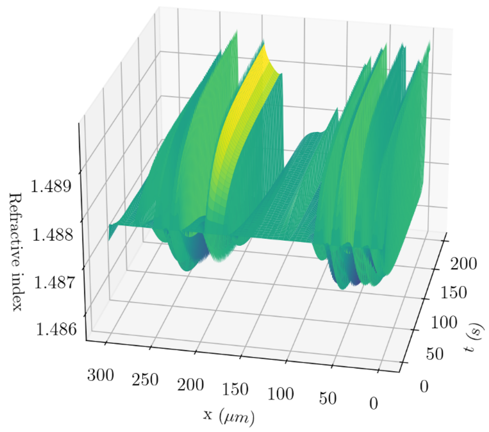

Figure 1. Refractive index modulation due to the diffusion process after recording the phase of an FL into the photopolymer, with size $300 \times 300\,\mu\text{m}^2$.

Once the refractive index modulation, Δn, has been obtained, a plane wave is propagated through the material to simulate its reconstruction. The axial irradiance distribution produced by an FL and its associated Fresnel Zone Plate (FZP) can be calculated with the Fresnel-Kirchhof integral

$$I(u) = 4\pi^2 u^2 \left| \int_0^1 \exp\left(-i2\pi u\zeta\right) \exp\left(\frac{i2\pi d\Delta n(\zeta)}{\lambda}\right) d\zeta \right|^2 \quad (5)$$

,where $u = a^2/2\lambda z$ is the axial reduced coordinate, $\lambda = 633$ nm, d is the thickness, z the axial distance from the material and ζ is the input plane spatial coordinate.

4.2 Influence of the parameter γ

The parameter γ represents the non-linearity of the polymerization rate with the incident light into the material [5]. According to **Figure 2**, the influence of non-linearity in the diffraction orders generate a deviation between successive focal points, which depends on the increase of the exposure time. We see that the, diffusion model is able to predict the two first-order foci shows that the second lobe is broader and reaches higher *DE* than the first one for a linear relationship $\gamma = 1$.

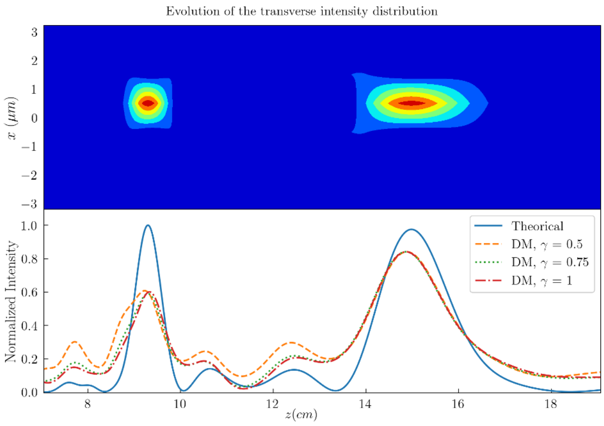

Figure 2. Evolution of the axial and transverse intensity distribution produced by a FL for different values of γ by the Diffusion Model (DM) and the theorical result. $\Phi_9(\zeta)$ lens is used with $a = 2$ mm.

Focal distances for different values of γ are shown and the following relationship between them is $f_1/f_2 = F_n/F_{n-1} = \varphi$. Therefore, foci approaches the axial positions $f_1 = a^2/2\lambda F_8 = 15$ cm and $f_2 = a^2/2\lambda F_9 = 9.29$ cm.

5 Conclusions

The properties of Fibonacci lenses have been characterized. Upon obtaining the theoretical results by the diffusion model, a value of γ can be proposed by utilizing the relationship between the intensity of recording and polymerization rate. To sum up, this study serves as a prelude to a forthcoming experimental investigation. This bifocal diffractive lens can be applied to numerous fields, like in ophthalmology or X-ray microscopy.

Acknowledgments

Funded by the “Generalitat Valenciana” (Spain) (IDIFEDER/2021/014, cofunded by EU through FEDER Programme; PROMETEO/2021/006 and INVEST/2022/419 financed by Next Generation EU), “Ministerio de Ciencia e Innovación” (Spain) (PID2021-123124OB-I00).

References

[1] C. Bowley, G. Crawford, Applied Physics Letters **76**, 2235 (2000)

[2] R. Fernández, S. Gallego Rico, A. Márquez, J. Francés, C. Neipp, D. Puerto, E.M. Calzado, I. Pascual Villalobos, A. Beléndez, *3-dimensional modelling of the DOEs formation in PVA/AA photopolymers*, in *Photosensitive Materials and their Applications* (SPIE, Online Only, France, 2020), p. 49

[3] R. Fernández, S. Gallego, A. Márquez, C. Neipp, E. Calzado, J. Francés, M. Morales-Vidal, A. Beléndez, Polymers **11**, 1920 (2019)

[4] J.A. Monsoriu, A. Calatayud, L. Remon, W.D. Furlan, G. Saavedra, P. Andres, IEEE Photonics Journal **5**, 3400106 (2013)

[5] S. Gallego, A. Marquez, F.J. Guardiola, M. Riquelme, R. Fernández, I. Pascual, A. Beléndez, Optics Express **21**, 10995 (2013)