ce/papers Proceedings in civil engineering

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ORIGINAL ARTICLE

EUROSTRUCT

Strain-based autoregressive modelling for system identification of railway bridges

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Abstract

Vehicular traffic represents the most influential loads on the structural integrity of railway bridges, therefore the design on dynamic criteria. This work explores the use of strain dynamic measurements to characterize the health condition of railway bridges under moving train loads. Specifically, the approach proposed in this work exploits the implementation of auto-regressive (AR) time series analysis for continuous damage detection. In this light, continuously extracted AR coefficients are used as damage-sensitive features. To automate the definition of the order of the AR model, the methodology implements a model selection approach based on the Bayesian information criterion (BIC), Akaike Information Criterion (AIC) and Mean Squared Error (MSE). In this exploratory investigation, the suitability and effectiveness of strain measurements against acceleration-based systems are appraised through a case study of a simply supported Euler-Bernoulli beam under moving loads. The moving loads problem in terms of vertical accelerations and normal strains is solved through modal decomposition in closed form. The presented numerical results and discussion evidence the effectiveness of the proposed approach, laying the basis for its implementation to real-world instrumented bridges.

Keywords

Autoregressive modelling, Railway bridges, SHM, Strain monitoring

1 Introduction

Structural health monitoring (SHM) represents a growing are of research [1,2]. SHM advocates the analysis of continuous monitoring data and non-destructive testing to conduct condition-based maintenance [3], either allowing to determine the performance of new structures or assessing the service conditions of civil infrastructures approaching the end of their lifespan. In general, the SHM process is organised around four stages [4]: (i) case determination and operational assessment; (ii) collection of recorded data; (iii) extraction of damage-sensitive features; (iv) damage identification. Structural damage is defined in the literature as a change in material, geometric properties and/or boundary conditions from the initial reference (undamaged) condition of the structure [5]. The damage identification problem is in turn defined according to four levels of increasing complexity: damage identification, location, quantification, and prognosis [6]. In this context, the extraction of damage-sensitive features from monitoring records is a key element in the damage identification process to link signals to the decision-making. In the realm of railway bridges, most research efforts have been devoted to the theoretical/numerical analysis of the moving load problem (see e.g. [7,8]). Nonetheless, the number of research works in the literature dealing with the implementation of SHM systems to real-world railway bridges is still scarce. Among the few experiences reported in the literature, most applications based upon the extraction of modal features through Operational Modal Analysis (OMA) (see e.g. [9,10,11]). Although OMA has the advantage of providing global damage-sensitive features [12,13,14], it can only be applied when the structure is subjected to environmental white noise excitation and its effectiveness to localise damage is related to its ability to identify high-order modes [15]. Other time series analysis approaches, such as auto-regressive modelling, offer a more flexible framework to address the problem of damage identification [16,17]. Autoregressive modelling techniques are capable of extracting damage-sensitive features through the direct use of the response time series under the passage of trains. In particular, most research works in the literature have focused on the analysis of acceleration records (see e.g. [18,19]). It is worth highlighting the work of Meixedo and co-authors [18], who developed a damage identification approach based on the evaluation of anomalies in the time series of continuously acquired AR coefficients from acceleration signals. Their

https://doi.org/10.1002/cepa.2118

results evidenced the correlation between the AR coefficients and the intrinsic stiffness of the monitored structure. This allows one to identify structural damage in a timely and accurate manner by inspecting the appearance of anomalies in the time series of continuously acquired AR coefficients. In this context, recent developments in distributed strain sensing and fiber-optics technologies may offer an ideal framework for the development of new damage identification systems. Such sensing technologies offer the possibility of conducting distributed monitoring applications with superior damage localisation capabilities. Studies such as those conducted by Pier F. Giordano [20] emphasise the importance of having the availability of real-world monitoring data to conduct sound comparisons of time-invariant vibration-based damage localisation methods. But they involve a considerable computational effort due to the need to simultaneously update model parameters [21]. To overcome this problem, various approaches can be found in the literature, such as substructure methods that allow defining dense meshes only in the vicinity of the damage, thus alleviating the computational burden (see e.g. X. Kong [22]). It is also important to remark damage detection through methodologies based on dynamic harmonic regression models. In the literature, these types of harmonic models are implemented to use prediction intervals as statistical control limits as in the work conducted by Tadhg Buckley and co-authors [23]. In addition, the latest developments in artificial intelligence (AI) have broadened the scope of engineering applications. This is the case of the contribution by Hieu Nguyen-Tran and co-authors [24], who developed a hybrid method combining AR modelling and artificial neural networks (ANN), emphasising the AR model's ability to be combined with innovative AI techniques .However, the great potential of this technology, the development of efficient damage identification techniques is yet to be fully addressed. In this light, this paper presents and exploratory analysis of the use of strain measurements and AR modelling to extract damage-sensitive features for railway bridges under passing trains. The research is framed within a research project on the design and exploitation of a SHM system installed in a steel truss railway bridge in Alicante (Spain), the Mascarat viaduct. With the aim of developing a damage identification approach exploiting the strain time series recorded by a fiber optics system, the present work analyses the effectiveness of AR modelling of strain time series against acceleration data in a simplified benchmark case study consisting of a simply supported Euler-Bernoulli beam. The moving load problem is solved through modal decomposition in terms of vertical accelerations and normal strain in closed form. Detailed parametric analyses are presented to evaluate the influence of vehicle speeds on the AR coefficients extracted from the acceleration and deformation time series. The presented results and discussion evidence that, unlike acceleration time series, time series built on deformation datasets allow the generation of low to moderate order AR models, thus providing a compact set of more robust features (AR coefficients) suitable for damage diagnostics. Overall, the conducted research aims to provide a sound theoretical basis for the use of AR modelling as an efficient damage identification technique to promote and support the use of fiber-optic sensors.

2 Theoretical framework

2.1 Autoregressive modelling

The AR(m) model, with m denoting the order or number of parameters in the model, is developed from a discrete response time-series data x_i, $i = 1, \dots, n$ and can be written as:

$$x_j = \sum_{i=1}^{m} a_i x_{j-i} + \varepsilon_j, \quad (1)$$

in such a way that x_j is defined as a linear combination of the m previous response values multiplied by AR constant parameters a_i. The quantity ε_j corresponds to the residual error in the signal value x_j. In matrix form, the AR model can be represented by:

$$\begin{pmatrix} x_{m+1} \\ x_{m+2} \\ \vdots \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} x_1 & x_2 & \cdots & x_m \\ x_2 & x_3 & \cdots & x_{m+1} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ x_{m-n} & x_{n-m+1} & \cdots & x_{n-1} \end{pmatrix} \begin{pmatrix} a_m \\ a_{m-1} \\ \vdots \\ \vdots \\ a_1 \end{pmatrix} + \begin{pmatrix} \varepsilon_{m+1} \\ \varepsilon_{m+2} \\ \vdots \\ \vdots \\ \varepsilon_n \end{pmatrix}. \quad (2)$$

The process of fitting the AR model a_i can be conducted by solving the typically overdetermined set of equations in (2) through the least-squares method or the Yule-Walker approach [25].

2.2 Definition of optimal model order of AR expansions

The model order, initially unknown, determines number of past observations required to reproduce the time series. This conditions in turn the size of the observation matrix in Eq. (2) and, consequently, the computational cost of the feature extraction. A high parameterisation of the model would, in general, tend to overestimate the noise in the training dataset, thus losing generality, while a too low order number would fail to accurately represent the physical mechanisms governing the system under investigation [26]. It is therefore desirable to adopt an automated approach to optimally select the model order. This may be conducted through parametric analyses with increasing model orders, in such a way that the optimal model order is selected through a certain error quality metric [27,28]. Some of the most widely used metrics in the literature include the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) [29]. These metrics provide an excellent trade-off between the error in AR predictions and the complexity of the model, thus avoiding problems of over-fitting, i.e. over-parametrisation of the model. The BIC and AIC metrics are defined as [30]:

$$BIC(m) = n \ln\left(\frac{RSS(m)}{n}\right) + m \ln(n), \quad (3)$$

$$AIC(m) = n \ln\left(\frac{RSS(m)}{n}\right) + 2m, \quad (4)$$

where RSS(m) represents the sum of squared residuals of the AR model of order m (i.e. $\sum_{j=m+1}^{n} \varepsilon_j^2$).

3 Numerical results and discussion

3.1 Application case study

The investigated case study of a simply supported Euler-

Bernoulli beam is sketched in Fig. 1. This bridge configuration is presented in the Spanish railway bridge design code IAPF-07 [31]. The model consists of a simply supported Euler-Bernoulli girder of length L=15 m, mass per unit length $\rho = 15\,\mathrm{t/m}$, flexural stiffness $EI = 7694.081\,\mathrm{MPa}$, and constant modal damping ratio $\xi = 2\%$. It is assumed that the beam has a rectangular cross-section with a height equal to h_s. In the moving load problem, one single point load P is considered moving at a constant speed v across the bridge and initially located at a distance d from the origin. The load as a function of time and distance can be described as $p(x, y) = P\delta(x - v\tau - d)$, where δ is the Dirac delta function and x and t denote the longitudinal coordinate along the load path and the time variable, respectively.

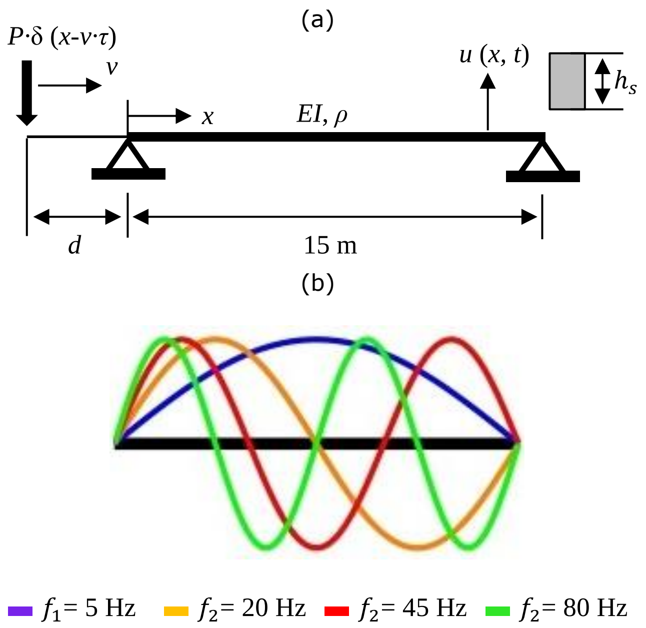


Figure 1. Geometry and mechanical properties of a simply supported beam with constant cross-section and traversed by a single moving load (a), and modal properties of the beam (b).

3.2 Analytical solution of the vibration of simply supported beams under moving loads

The moving load problem is solved through modal superposition in closed form. In the first place, the differential equation of vertical displacements $u(x,\tau)$ of the Euler-Bernoulli beam under a moving load disregarding damping effects can be written as:

$$\rho(x)\frac{\partial^2 u(x.\tau)}{\partial\tau^2} + \frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 u(x.\tau)}{\partial x^2}\right] + P\delta(x - v\tau) = 0, \qquad (5)$$

with $\tau = t - d/v$ being the relative time of the moving load on the beam. Assuming that the system behaves linearly, the solution of Eq. (5) can be obtained by applying modal variable separation $u(x,\tau) = \Phi(x)y(\tau)$, with $\Phi(x)$ being the modal matrix containing the n-th mode shapes $\Phi_n(x)$ by columns, and $y(\tau)$ the vector of modal displacements. In virtue of the orthogonality property of the mode shapes, Eq. (5) can be decoupled into modal coordinates $y_n(\tau)$ as:

$$\int_0^L \Phi_n(x)\left[\rho(x)\frac{\partial^2 y_n(\tau)}{\partial\tau^2}\right]\Phi_n(x)dx + \int_0^L \frac{\partial^2\Phi_n(x)}{\partial x^2}\mathrm{EI}(x)\frac{\partial^2\Phi_n(x)}{\partial x^2}\mathrm{d}x = 0. \qquad (6)$$

From Eq. (6), the generalized mass M_n and stiffness K_n values associated with the n-th mode can be identified as:

$$M_n = \int_o^L \Phi_n(x)\,\rho(x)\Phi_n(x)\mathrm{d}x, \ (7)$$

$$K_n = \int_0^L \frac{\partial^2\Phi_n(x)}{\partial x^2}\,\mathrm{EI}(x)\frac{\partial^2\Phi_n(x)}{\partial x^2}\mathrm{d}x. \qquad (8)$$

At this point, it is possible to include the damping effects by means of a modal damping ratio ξ_n. Using dot notation to represent time derivatives, Eq. (6) can be rewritten in a more compact form as:

$$\ddot{y}_n(\tau) + 2\zeta_n\omega_n\dot{y}_n(\tau) + {\omega_n}^2 y_n(y) + \left(\frac{P}{M_n}\right)\Phi_n(v\tau) = 0, \qquad (9)$$

where the term ω_n stands for the undamped angular frequency of the $n-th$ mode. The homogeneous solution of Eq. (9) can be readily derived as:

$$y_n^h(\tau) = e^{-\xi_n\omega_n\tau}[A_n\cos(\omega_n^d\tau) + B_n\sin(\omega_n^d\tau)], \qquad (10)$$

where terms A_n and B_n are constants to be determined by the boundary conditions. Terms ω_n and Φ_n denote the undamped angular frequency and mode shape of the $n-th$ vibration mode and are given by:

$$\omega_n = n^2\pi^2\sqrt{\frac{EI}{\rho L^2}}, \quad \Phi_n(x) = \sin\left(\frac{n\pi x}{L}\right). \qquad (11)$$

The resulting natural frequencies $f_n = \omega_n/2\pi$ and mode shapes of the first four vibration modes of the present case study are depicted in Fig. 1 (b). Defining the following non-dimensional parameters:

$$\Omega = \frac{\mathrm{n}\pi(\mathrm{v}\tau)}{\mathrm{L}}, \quad \Phi_n(x) = \sin\left(\frac{n\pi x}{L}\right), \qquad (12)$$

$$\eta = \frac{2PL^2}{EIn^4\pi^4}[(1 - S_n^2)^2 + 4(\xi_n S_n)^2]^{-1}, \qquad (13)$$

the particular solution of Eq. (5) reads:

$$y_n^p = C_n\cos(\Omega\tau) + D_n\sin(\Omega\tau), \qquad (14)$$

with C_n and D_n given by:

$$C_n = -2\zeta_n\eta L, \ D_n = (1 - S_n^2)\eta L. \qquad (15)$$

In this light, the solution of the modal coordinate $y_n(\tau)$ is obtained as $y_n(\tau) = y_n^p(\tau) + y_n^h(\tau)$, and terms A_n and B_n in Eq. (10) are found by applying the simply supported boundary conditions, which leads to:

$$A_n = -C_n, \quad B_n = \frac{\xi_n\omega_n C_n + \omega_n^d}{\omega_n^d}. \qquad (16)$$

Considering that the dynamic response is governed by m vibration modes, the solution reads:

$$\mathrm{u}(\mathrm{x},\tau) = \sum_{n=1}^{m}\{e^{-\xi_n\omega_n\tau}[A_n\cos(\omega_n^d\tau) + B_n\sin(\omega_n^d\tau)] + C_n\cos(\Omega\tau) + D_n\sin(\Omega\tau)\}\Phi_n(x) = Q(t)\,\Phi_n(x), \qquad (17)$$

with $\omega_n^d = \omega_n\sqrt{1 - {\xi_n}^2}$ denoting the damped natural angular frequency. In this light, under the assumption of linear elasticity and Navier's stress distribution for bending stresses, it is straightforward to extract the closed-form solution of the strain time-series at the top fibre of the cross-section as:

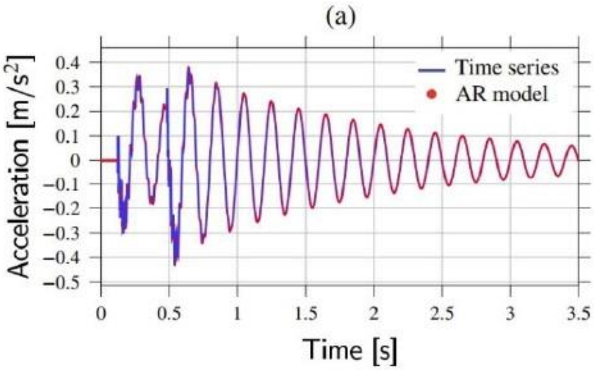
$$\varepsilon(x,\tau) = \frac{h_s}{2} Q(t) \frac{d^2\Phi_n(x)}{dx^2}. \qquad (18)$$

Since the system is assumed to be linear, the previous formulation can be readily extended to general train compositions by means of linear superposition.

3.3 Identification of the characteristic AR model

Two different load cases have been studied, namely (i) one single moving load in Fig. 2, and (ii) the ICE2 train in Fig. 3 (the train composition is given in [31]). In these analyses, the first ten vibration modes have been considered.

The first load case consists of a single moving load of 100 kN crossing the beam at 150 km/h, and originally located d = 5 m far from the origin. The acceleration and strain (top fibres) time series computed at the mid-span of the beam for the first loading case and the corresponding predictions by the AR model are shown in Figs. 2 (a) and (b), respectively. As indicated above, the model order m determines the quality of the associated autoregressive model $AR(m)$. For the definition of the optimal order of the AR models, parametric analyses considering increasing model orders have been computed as reported in Figs. 2 (c) and (d) for the acceleration and strain time series, respectively. On this basis, the optimal model order can be determined by seeking the AR model yielding the minimum error metric in terms of BIC, AIC or MSE. Alternatively, the elbow point at which the rate of variation of the considered metrics stabilizes can be also considered to extract more compact expansions. In these figures, the optimal model orders determined by the minimum value and elbow point of the BIC curves are represented in this figure with red filled dots. Finally, the AR coefficients determined for expansions with a model order of m = 26 are shown in Figs. 2 (e) and (f) for the acceleration and strain time series, respectively. It is noted in these figures that most information is provided by the first ten coefficients. It is noted in Figs. 2 (c) and (d) that the AIC and BIC metrics exhibit similar behaviours, while the MSE metric tends to achieve convergence at lower model orders.



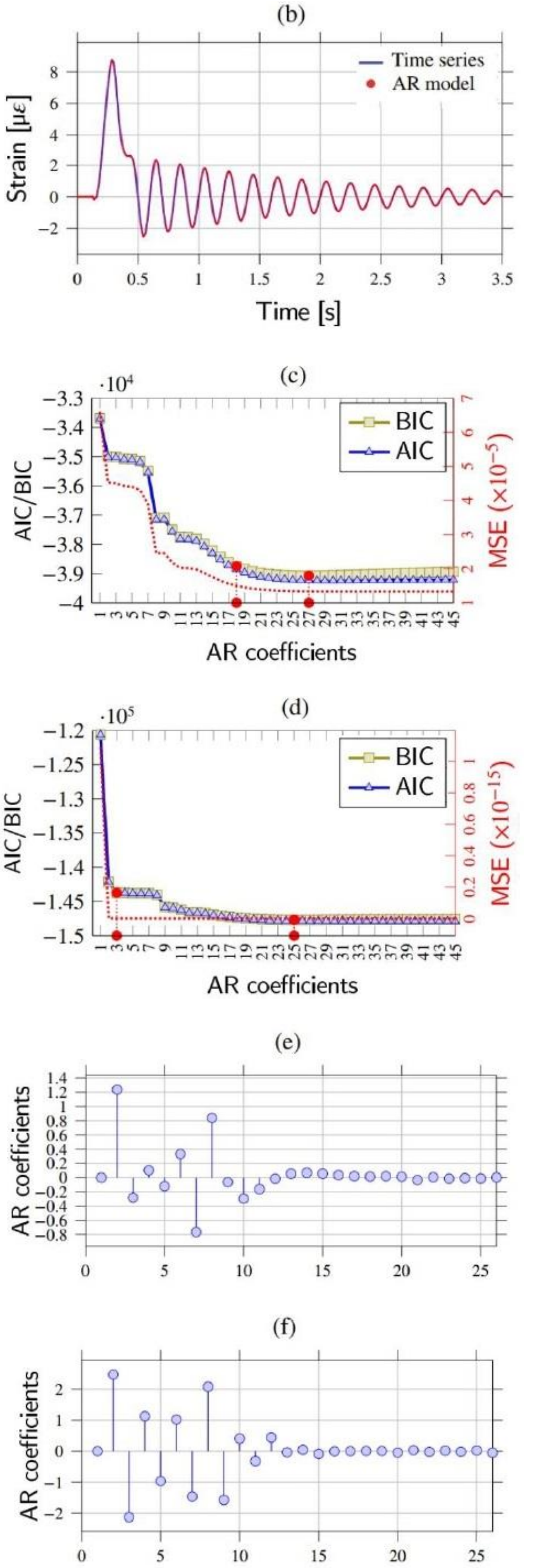
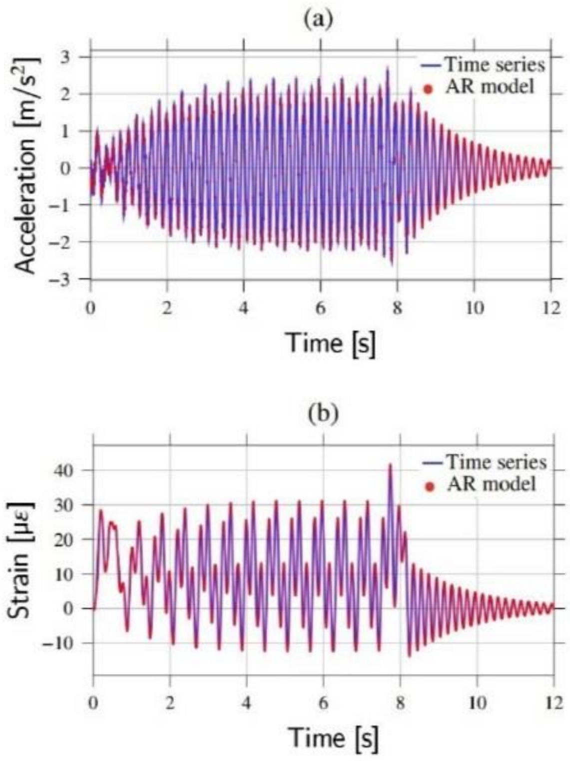


Figure 2. Acceleration (a) and strain (top surface) (b) time series and AR predictions at mid-span of a simply supported beam traversed by a single moving load of 100 KN travelling at 150 km/h and starting 5 m far from the origin of the beam ($\Delta t = 1\,ms$). Quality parametric analyses for increasing model orders for AR modelling of the acceleration

(c) and strain time series (d) (red filled dots represent the optimal model orders determined as the minimum and elbow points of the BIC curve). AR coefficients ($m = 26$) fitted with the acceleration (e) and (f) strain time series.

It is evident in these figures that the AR model requires considerably larger model orders to represent the acceleration time series. Indeed, the influence of fast oscillating terms in the strain time series is considerably lower, thus requiring lower model orders to achieve a comparable prediction accuracy. In this work, the elbow point of the BIC curves is selected to define the optimal expansion. It is noted in these figures that the elbow points are found at model orders of $m = 18$ and $m = 3$ for the acceleration and the strain time series, respectively. The predictions of the corresponding AR models are depicted in Figs. 2 (a) and (b) with red scatter points. It can be concluded from these results that strain measurements can be reproduced with low order AR models, thus providing a more compact set of AR coefficients for damage detection purposes. The same conclusions hold when considering a complete train load in Fig. 3. It this case, the elbow points are found at model orders of $m = 11$ and $m = 3$ for the acceleration and the strain time series, respectively.

Figures 4 (a, b) and (c, d) report a parametric analysis of the AR coefficients determined for the previous two loading cases, respectively. Specifically, six different train velocities are considered, namely v = 100, 150, 200, 250, 300 and 350 km/h. The analysis is performed both for acceleration and strain time series and furnished in Figs. 4 (a, c) and (b, d), respectively. Overall, only slightly larger variabilities in the amplitude of the AR coefficients for the acceleration times can be noted. From these analyses, it is clear that the variability of the AR coefficients (both in terms of acceleration and strain time series) is driven by the train composition and the passage velocity.



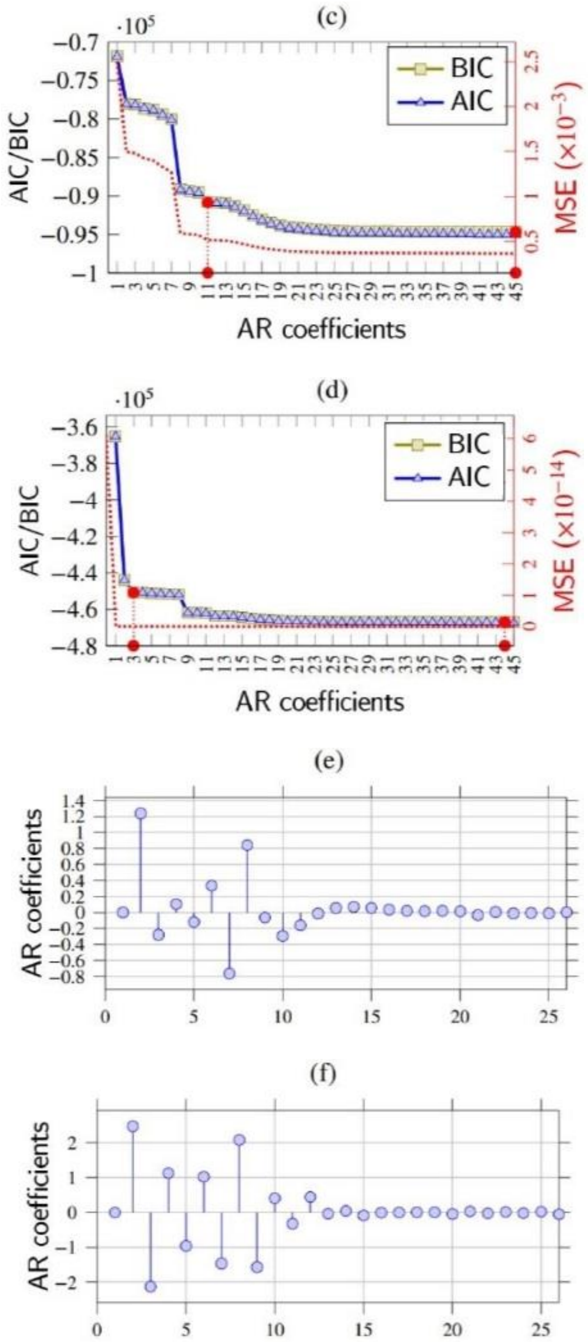
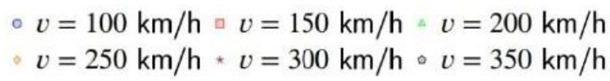


Figure 3. Acceleration (a) and strain (top surface) (b) time series and AR predictions at mid-span of a simply supported beam traversed by the ICE2 train travelling at 160 km/h ($\Delta t = 1\ ms$). Quality parametric analyses for increasing model orders for AR modelling of the acceleration (c) and strain time series (d) (red filled dots represent the optimal model orders determined as the minimum and elbow points of the BIC curve). AR coefficients ($m = 26$) fitted with the acceleration (e) *and (f) strain time series*.



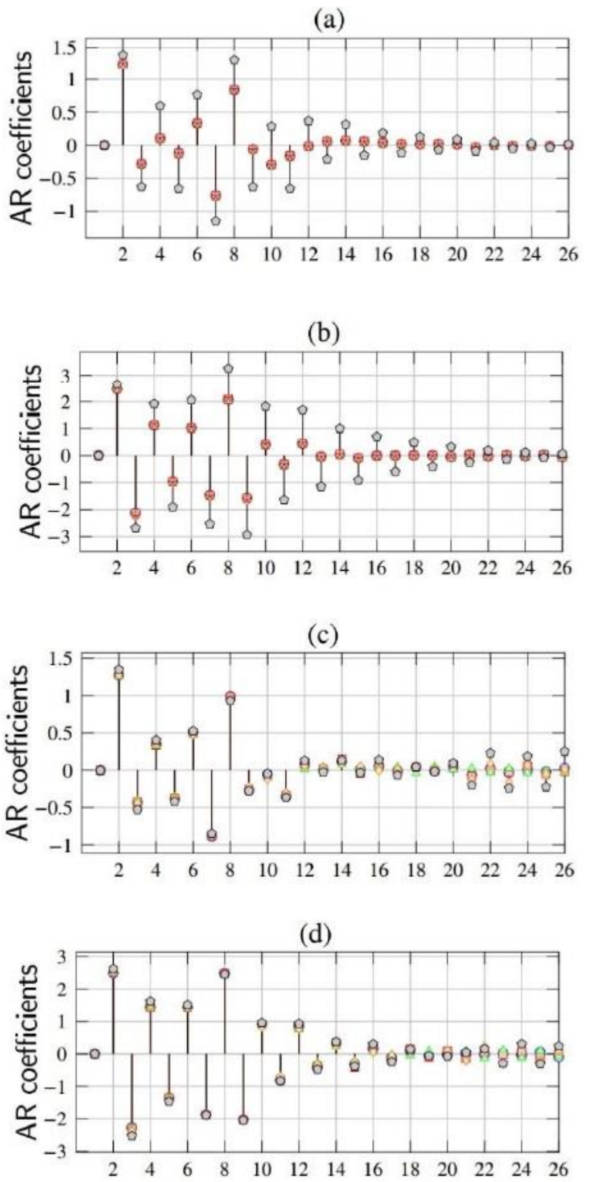


Figure 4. Parametric analysis of AR coefficients for different train velocities: (a, b) single load; (c, d) ICE2 train; (a, c) acceleration time series; (b, d) strain time series.

4 Conclusion

This paper presented a methodology for the rapid and cost-effective detection of bridge damage under moving train loads using AR signal modelling. In particular, this work analysed the advantages of using strain measurements over acceleration monitoring systems in terms of damage sensitivity. The approach proposed in this work exploits the analysis of autoregressive (AR) experimental time series for continuous damage detection, AR coefficients are extracted continuously and used as damage-sensitive features. In order to automatically select the optimal model order in the AR expansion, a systematic methodology was proposed that involves evaluating fit quality metrics for increasing model orders. This work presented a theoretical case study of a one-dimensional (1D) bridge system to investigate the potential of the proposed methodology. Detailed parametric analyses were presented to analyse the influence of train configuration and speed on the AR coefficients extracted from the acceleration and deformation time series. Overall, the results presented demonstrate the superior ability of strain measurements to provide more compact AR expansions that are less influenced by operating conditions such as train speed. This is mainly due to the less influential presence of highly oscillating terms in strain measurements compared to acceleration signals. Future research efforts will be devoted to the implementation of the proposed methodology for the continuous structural evaluation of the Mascarat Bridge (Alicante-Spain, real case) through the processing of the experimental recordings.

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