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# Mathematics education research on language and on communication including some distinctions: Where are we now?

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**Abstract**
In this article, we present a narrative review of mathematics education research on language and on communication over 2019–2022, but also look ahead by addressing challenges posed by the lack of distinction between language and communication. The persistence and significance of the problem of the distinction between language and communication are thus outlined in a historical moment of celebration of growth of research in the domain. Informed by the analysis of a selection of research journal articles and by our trajectories, we discuss influential topics in the recent discourse: multilingual mathematics classrooms; mathematics teacher education on language in mathematics teaching; multimodal mathematical communication; interaction and mathematics learning; mathematical language and discourse. We connect this with new emerging or old revisited concepts: instructional designing, gesturing, argumenting and languaging. We finish by further reflecting on multimodal mathematical communication and gesturing, and on the potential of expanding the notion of mathematics register towards a notion of mathematics communication register.

## 1 Introduction

Since the late 1980s with the publication of *Speaking mathematically: Communication in mathematics classrooms* (Pimm, 1987/2017), research on aspects of language and, increasingly, of communication in mathematics teaching and learning, as well as in mathematical pedagogies, has continued to grow. A question then is: Where are we now in this domain? We elaborate a response by combining lessons learned in our trajectories as researchers in the domain, and by discussing academic articles published over the period 2019–2022 in a selection of leading international journals of mathematics education research and neighbouring ones at the intersection of educational and language disciplines. Despite a proposal of articles is neither global nor neutral, we have traced works for the benefit of citational and epistemic justice. An Anglo-centric bias and absences arrive, however, with the choice of international journals.

Referring to four reviews of the same research domain before initiating this current review, we took over from two book chapters—Radford and Barwell (2016) and Planas et al. (2018)—and two research articles—Morgan et al. (2014) and Planas and Schütte (2018). From this point on, our review work was required to be focused solely on journal articles (as opposed to book chapters or conference papers). Those previous review studies reflected on the scope and achievements of the field research on language and on communication, and on newer approaches in the domain. They showed together that language remains a contested focus of research, and that work in the domain is being developed on how to conceptualise and discuss language, as well as on how to enable a more complex and visible notion of communication in research and educational processes. The diverse meanings of *language* and *communication* are still a driving force and an expression of vitality. This newer, short-period review is similar to the above-mentioned reviews in its narrative method, yet is different in the value given to specific topics and concepts, including the discussion of the fact that *language* and *communication* are not the same, so we do not study them as the same. The nature of this distinction often becomes blurred at the intersection of the broadest views of mathematics education research on language—i.e., studies in which linguistic communication is one of the many semiotic modes attended—and the narrowest views of mathematics education research on communication—i.e., studies solely attending to *linguistic* communication. We are not just doing a review, but are also drawing on review work to address the

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problem of the distinction between *language* and *communication* in mathematics education.

## 2 Method for mapping topics and concepts

Beyond the attention to different world regions, theoretical traditions and groups of authors, any identification of interesting research articles on mathematics education, language and communication is not trivial. There must be more in quantity than a total of articles chosen in a period of time (in our case 50 in 2019–2022 considering the first online publication data), if not limiting the search by citation records or other constructs upon which rankings are based. Either way, how we view language and communication has been an influence on our article identification. Interpreting Makoni and Pennycook (2007), we mostly share a broad understanding that emphasises language as primarily a communication process, and both language and communication as producers of meaning in thinking and in interaction with people, objects and practices. In all this, we challenge binary distinctions, such as people *versus* objects or interaction *versus* thinking, and argue for a nuanced approach to the multiple aspects involved in language and in communication across sites of mathematics education. Guided by this understanding, we address the following questions:

> Which are some influential topics of mathematics education research on language and on communication in the recent discourse (2019–2022), and why are they influential?
> Which are some new emerging or old revisited concepts over this period, and what do they add to or imply for the domain?
> How do some topics and concepts of the domain at present contribute to the discussion about the distinction between language and communication?

As first step, amongst prestigious research journals in mathematics education we chose the following: *Digital Experiences in Mathematics Education* (*DEME*), *Educational Studies in Mathematics* (*ESM*), *For the Learning of Mathematics* (*FLM*), *International Journal of Science and Mathematics Education (IJSME)*, *Journal of Mathematical Behavior* (*JMB*), *Journal of Mathematics Teacher Education* (*JMTE*), *Journal for Research in Mathematics Education* (*JRME*), *Mathematics Education Research Journal* (*MERJ*), *Mathematical Thinking and Learning* (*MTL*), *Research in Mathematics Education* (*RME*) and *ZDM–Mathematics Education* (*ZDM*). This alphabetic-order list was extended with journals covering wider fields: *Classroom Discourse* (*CD*), *Comparative Education Review* (*CER*), *International Journal for Lesson and Learning Studies* (*IJLLS*), *Language and Education* (*LAE*), *Linguistics and Education* (*LIE*), *Mind, Culture and Activity* (*MCA*), *Teachers College Record* (*TCR*) and *Teaching and Teacher Education* (*TATE*).

The second step was initiated individually. The final overview of topics and articles came as a result of our research experiences and conversations during the months of preparation of this review. Given the special issue guidelines about considering some 50 articles and annotating a small subset of them, we estimated that 5 topics can be illustrated by means of some 10 articles each (and one of each topic is annotated in the references). We were open to the arguments of each other in favour of or against the choices proposed, and looked at representing a variety of authors, regions and traditions. When one of us wanted to know more about what was in a topic, or in an article, and the reasons for proposing its inclusion and annotation, we started a discussion which could lead to changing the recommendation or to reinforcing the reasons for keeping either on.

The discussion of influential topics encouraged us to identify new emerging and old revisited concepts, which are explored in the second section below here. While the topics reflect lines of research in the domain, the concepts help us to understand and illustrate a variety of current challenges within and across topics. For example, at the end of producing the respective 10-lists of articles in the topics of multilingual mathematics classrooms and mathematical language and discourse, we agreed on the importance of the languaging concept, which is today central in applied linguistics and emerging in mathematics education in reference to the many challenges of understanding the potential of moves between languages and between everyday and formal registers in mathematics teaching and learning. For this, and three more concepts, we chose publications to be focused upon in the later part of this article, again considering our citational practices and acknowledging work that is not ours nor like ours.

## 3 Zooming in on influential topics

The questions guiding the first part of our review are as follows: *Which are some influential topics of mathematics education research on language and on communication in the recent discourse (2019–2022)? Why are they influential?* Topics or lines of research in the mathematics education community are embedded into a timeline of ideas and thoughts (a historical scope), and the collective practices of specific groups (a sociocultural scope). We may thus use the same words for mentioning a topic as known in the past, but its meaning will have changed. The topics in this review are: i) multilingual mathematics classrooms; ii) mathematics teacher education on language in mathematics teaching; iii) multimodal mathematical communication; iv) interaction

and mathematics learning; v) mathematical language and discourse. They differently reflect on sites of communication, in which language is a process linked to other communication processes (e.g., gestures, voice intonation, drawings) in the production of meaning in interaction with people (e.g., teachers, teacher educators, students), objects (e.g., lesson plans, computers, manipulatives) and practices (e.g., writing, explaining, argumenting). Hence, the topics are related to each other because together they represent the multiple aspects constituting language, communication and the relationship between them.

### 3.1 Topic 1: Multilingual mathematics classrooms

In the most recent period, we find a continued emphasis, as well as advances, on research regarding multilingual mathematics classrooms. Besides its historical significance, we consider this topic to be strong, growing and even more present now than in the past (e.g., Adler, 2001). Important attention remains in the study of flexible uses of languages that support mathematics learning and teaching. Part of what is different from past research is the socially more complex thinking about language as a local practice, rather than a given static structure or system (Makoni & Pennycook, 2007). Social views of language have gained decisive momentum, and all-or-nothing views of the multilingual learner participation in mathematics are being replaced by views of mathematical participation through a plurality of languages, mathematics registers, communities and cultures. Moreover, today's multilingual mathematics classroom research is specifically influential, because it is helping to refocus language diversity on any site of mathematics teaching and learning, and not just on school classrooms in contexts of poverty, migration, colonialism, ethnic difference, …, nor as a feature exclusive of the learner.

The articles in Table 1 provide examples of all this. Our review specific to the mathematics education journals shows research informed by: classroom interaction theory in analyses of patterns of multilingual talk and of mathematical generalisation (El Mouhayar, 2022a); content meta-analysis of empirical evidence of successful teaching across classroom studies with English language learners (Sharma & Sharma, 2022); learning activity theory in the discussion of how young multilingual learners' reflective actions indicate initial algebraic thinking (Eriksson & Eriksson, 2021); learning trajectory tradition for inclusion of linguistic diversity in multilingual teaching and learning of proportional reasoning and linear functions (Zahner & Wynn, 2023); sociolinguistic and dialogic theory in the exploration of multilingual mathematics learning in different second-language classroom contexts (Barwell, 2020); sociolinguistic and second language acquisition theory in the study of exploratory mathematics talk of multilingual learners (Robertson & Graven, 2019).

Various studies are published in neighbouring journals, as is the case with four articles in Table 1. These adopt 'trans–' approaches to illustrate creative language practices of mathematics teaching, assessment and learning, hence transcending bounded labels of languages, modes, registers and semiotics. Gandara and Randall (2019) revisit code-switching and language separation approaches that represent the multilingual learner as a speaker with an incomplete knowledge of the language of instruction. Tai and Wei (2021) reflect on

**Table 1** List of 10 Topic 1 studies

| Topic 1 Multilingual mathematics classrooms | |
|---|---|
| Article title, journal, year | Location(s), authorship |
| Triadic dialog in multilingual mathematics classrooms as a promoter of generalization during classroom talk. *MERJ* 2022a | Lebanon; El Mouhayar |
| Onto/epistemic violence and dialogicality in translanguaging practices across multilingual mathematics classrooms. *TCR* 2022 | Greece, Spain & Sweden; Chronaki et al. |
| Learning actions indicating algebraic thinking in multilingual classrooms. *ESM* 2021 | Sweden; Eriksson & Eriksson |
| Successful teaching practices for English language learners in multilingual mathematics classrooms. *MERJ* 2022 | New Zealand; Sharma & Sharma |
| Rethinking learning trajectories in light of student linguistic diversity. *MTL* 2021 online/2023 | USA; Zahner & Wynn |
| Co-learning in Hong Kong English medium instruction mathematics secondary classrooms. *LAE* 2021 | China; Tai & Wei |
| Teachers' beliefs and practices with respect to translanguaging university mathematics in Iraq. *LIE* 2021 | Iraq; Alhasnawi |
| Learning mathematics in a second language. *JRME* 2020 | Canada; Barwell |
| Exploratory mathematics talk in a second language. *ESM* 2019 | South Africa; Robertson & Graven |
| Assessing mathematics proficiency of multilingual students. *CER* 2019 | Democratic Republic of the Congo; Gandara & Randall |

All 5 tables are ordered by published date

the role of language across languages, modes, registers and semiotic resources to co-learn and negotiate mathematical meaning. Chronaki et al. (2022) study translanguaging in relation to pedagogical practices designed to develop mathematical and identity meaning across languages and school contexts. Alhasnawi (2021) addresses pedagogical practices with a focus on translanguaged mathematical discourse in university mathematics teaching. All these articles adopt a notion of language as a social practice and a dynamic process that can be also creative.

### 3.2 Topic 2: Mathematics teacher education on language in mathematics teaching

Topic 2 refers to studies in mathematics teacher education with pre-service and in-service mathematics teachers and a language focus which privileges the oral/written mode over spatial, visual, gestural, embodied, … modes. These others are not necessarily neglected, but are often subordinated to verbal accounts in teacher education and school teaching. We concur that current mathematics teacher education research on language in mathematics teaching—with some researchers being the teacher educators in the developmental site—is influential, because it is building a convincing case for justifying and disseminating innovative practice in initial and continuous mathematics teacher education. As argued in Morgan et al. (2021), findings from classroom research on mathematics and language in the past have not much been reflected at school-teaching levels. It may well happen that today's collaborative research with mathematics teachers on language in their or others' classroom teaching becomes reflected in progressive improvement of educational practice in the coming decades.

The articles in Table 2, with an almost entire focus on spoken and written language, show research in which the participants are: future mathematics teachers on writing for learning to teach mathematics through posing non-traditional mathematical problems (Leavy & Hourigan, 2022); future mathematics teachers on leading mathematical discussions by drawing on the linguistic resources of the learners (Shaughnessy et al., 2019); novice primary-school mathematics teachers on their use of mathematical vocabulary in linguistically diverse classrooms (Turner et al., 2019); middle-school mathematics teachers in a content-specific professional development program aimed at promoting expertise in language-responsive teaching (Prediger, 2019); secondary-school mathematics teachers in developmental workshops on word use aimed at naming and explaining meaning in algebra teaching (Planas, 2021); secondary-school mathematics teachers in a geometry lesson study mediated by a framework with a focus on word use and meaning (Adler et al., 2023).

In line with notions of dialogue, some studies focus on developmental work with teachers on word use aimed at supporting teaching talk for mathematical discussions with learners. In their theoretical article, Williams and Ryan (2020) make a claim for the dialectic nature of dialogue, and present data from a lesson study in which teachers made progress in the understanding of features of mathematical dialogue. Bergman et al. (2022 online/2023) approach dialogue through the analysis of the responses of pre-service teachers to a scripting task in which they were asked to react to a learner's conjecture regarding fractions. Ng et al. (2021) examine linguistic features and dialogic moves in the teaching of a primary-school teacher involved in a developmental teacher-intervention study. Drawing on noticing literature, Sjöblom et al. (2022) present work with four teachers in

**Table 2** List of 10 Topic 2 studies

| Topic 2 Mathematics teacher education on language in mathematics teaching | |
|---|---|
| Article title, journal, year | Location(s), authorship |
| Prospective teachers' responses to students' dialogue on fractions. *RME* 2022 online/2023 | Canada; Bergman et al. |
| Teachers' noticing to promote students' mathematical dialogue in group work. *JMTE* 2022 | Sweden; Sjöblom et al. |
| From defining as assertion to defining as explaining meaning. *IJLLS* 2022 online/2023 | South Africa & Malawi; Adler et al. |
| Balancing competing demands. *JMTE* 2022 | Ireland; Leavy & Hourigan |
| How linguistic features and patterns of discourse moves influence authority structures in the mathematics classroom. *JMTE* 2021 | China; Ng et al. |
| How specific can language as resource become for the teaching of algebraic concepts? *ZDM* 2021 | Spain; Planas |
| On the compatibility of dialogism and dialectics. *MCA* 2020 | UK; Williams & Ryan |
| A study of early career teachers' practices related to language and language diversity during mathematics instruction. *MTL* 2019 | USA; Turner et al. |
| Investigating and promoting teachers' expertise for language-responsive mathematics teaching. *MERJ* 2019 | Germany; Prediger |
| An investigation of supporting teacher learning in the context of a common decomposition for leading mathematics discussions. *TATE* 2019 | USA; Shaughnessy et al. |

design research cycles to analyse, design and evaluate teaching for engaging learners in mathematical dialogues across curricular contents.

### 3.3 Topic 3: Multimodal mathematical communication

Morgan (2021) wrote that she would generally refer to *communication* rather than *language* because, "mathematics teaching may make use of a wide range of communicational modes" (p. 102). Jewitt (2006) saw that, "all modes of communication are attended to as part of meaning making" (p. 3). Sfard (2008) wrote, "thinking can be usefully defined as an individualized version of *interpersonal communication*" (p. 81; *italics in original*). In Pimm (1987/2017), however, there was no distinction between language and communication (even at the title level), and it solely comprised an English monolinguality.

The distinction was addressed much later. Pimm (2021) disagreed with O'Halloran's (2015) extension of Halliday's linguistic notion of the mathematics register to a multimodal form; not least because O'Halloran's two additions (symbolic notation and geometric images), while not global, are translingual in the sense of these written/drawn elements transfer consistently and widely across languages, which Halliday's (1975) notion did not. For Halliday, each mathematics register was a functional subset of a single language: "the mathematical use of natural language, that is: not mathematics itself" (p. 65). Neither of these additions are lingual. We might see O'Halloran's extension as being (part of) a multimodal mathematics *communication* register. Her multimodal list does not (yet) include gestures (and gazes and …), which would also be significant elements of a mathematics *communication* register. These are not linguistic (unless in the setting of sign language), but are frequently growing connected with language and its research.

With regard to the 10 articles in Table 3, it is interesting to note the representation of gestures in the communication of data by means of transcripts. Alibali et al. (2019) focus both on student and teacher gestures and, in their production of class transcripts, they juxtapose, although separated, gestures and speech in comparable detail. Maffia and Sabena (2020) attend to the teacher's gestures and gestural repetition in class discussion, and describe gestures as a "semiotic set", one that links to others "(e.g. spoken words, mathematical symbols)" (p. 16). The gesture descriptions are marked inside the speech transcript (italics and inside square brackets), which presents them as subordinate, a not-uncommon circumstance. Oechsler and Borba (2020), by contrast, rely on a series of photographs to represent students' gestures in their mathematical videos. El Mouhayar (2022b) provides an example of representing teacher and student gestures by means of a combination of images and gesture descriptions inside the speech transcript, some of which are not associated with the images documented.

Table 2 also shows research with: grade-three students in small-group work on combinatorial problems and multimodal mathematical reasoning, including pointing and sliding (Wathne & Carlson, 2022); pairs of undergraduate students working on calculus tasks and their embodied reasoning through talk, gestures and interactions in between (Yu & Uttal, 2022); grade-six students using hand gestures in their thinking of angles relationships and the laws of exponents (Yeo & Tzeng, 2020); young students who, without the iPad present, would mimic what their contact gestures on the iPad screen when using *TouchTimes* had been (Bakos & Pimm, 2020); young students whose 'collaborative' gestures and 'gesture moves' (as an extension of 'talk moves') reveal

**Table 3** List of 10 Topic 3 studies

| Topic 3 Multimodal mathematical communication | |
|---|---|
| Article title, journal, year | Location(s), authorship |
| Third grade students' multimodal mathematical reasoning when collaboratively solving combinatorial problems in small groups. *MTL* 2022 | Norway; Wathne & Carlsen |
| Teacher's and students' use of gestures and home-language during classroom-talk to elicit a shared understanding of structure in figural pattern. *LIE* 2022b | Lebanon; El Mouhayar |
| Gestures, embodiment, and learning the rate of change. *MTL* 2022 | USA; Yu & Uttal |
| Cognitive effect of tracing gesture in the learning from mathematics worked examples. *IJSME* 2020 | Taiwan; Yeo & Tzeng |
| Beginning to multiply (with) dynamic digits. *DEME* 2020 | Canada; Bakos & Pimm |
| Mathematical videos, social semiotics and the changing classroom. *ZDM* 2020 | Brazil; Oechsler & Borba |
| On the mathematics teacher's use of gestures as pivot signs in semiotic chains. *FLM* 2020 | Italy; Maffia & Sabena |
| Collaborative gesture as a case of extended mathematical cognition. *JMB* 2019 | USA; Walkington et al. |
| Ritualisation in early number work. *ESM* 2019 | UK & Canada; Coles & Sinclair |
| Managing common ground in the classroom. *ZDM* 2019 | USA; Alibali et al. |

information out of talk—hence, once again, distinguishing language and communication (Walkington et al., 2019); one primary-school student involved in numeral-naming tasks in which the ritual (but not rote through repetition) rhythmic match of (researcher) gesturing and his responding is analysed (Coles & Sinclair, 2019). All this multimodal research continues to be important in moving the field forward. The lessons learned may have a role in mathematics teaching with linguistically disadvantaged students.

### 3.4 Topic 4: Interaction and mathematics learning

Since Cobb and Bauersfeld (1995), a book where a group of authors addressed the complexity of mathematics teaching and learning in classroom interactions by sharing videos and transcripts of lessons in an elementary school, much has been studied about the mediation of interaction in mathematics learning. In the early years, a basic orientation was around the function of the classroom language to allow mathematical discussion in interaction. Part of the research that has intensified during the past few years relates to reform contexts where mathematics learning is equated with participation in mathematical reasoning. Later in this article, we will comment on the indistinct use of or common lack of distinction between 'argumentation' and 'reasoning'—which parallels the general absence of distinction between language and communication. The fact of argumentation often being subsumed under the umbrella of reasoning is pushed out, particularly in European approaches to interaction and mathematics learning, by the influence of the notion of collective argumentation in Krummheuer (2007) that exemplifies some inseparability between reasoning mathematically and arguing with others. We see today's research on interaction and mathematics learning as influential, because of its capacity to frame the discussion of language and of communication in mathematical argumentation and reasoning in the classroom.

Table 4 mentions 6 studies which approach learners' reasoning through analyses of: interactional episodes with the teacher on the assessment and refining of reasoning products developed in lesson collaboration (Zhuang & Conner, 2022); cultural patterns arising in the human activity system around mathematical problem solving in whole groups (Sekiguchi, 2021); shared construction of mathematical explanations and meanings while discussing the enlarging of plane figures (Erath, 2021); collaboration group work in situations of minimal teacher instruction and activities of linear functions (Kämäräinen et al., 2021); conversations in mathematics lessons across different schools and teachers in which learners produce explanations (Ingram et al., 2019); small and whole-group settings of lesson work focused on the development of epistemic claims and probability reasoning (Goizueta, 2019).

We note an empirical study in the university classroom (Rasmussen et al., 2020) reporting the finding that mathematical conceptual reasoning is supported when learners engage with another's reasoning, as necessitated by interactive conversation. Also, 2 theoretical-oriented studies illustrate reflections through classroom data with younger learners. One of these studies connects together theories in the interpretation of an interactional episode in the kindergarten school with the teacher and nine children working on an addition problem (Breive et al., 2022). The other draws on two primary school classrooms to explore patterns of dialogic and non-dialogic communication in the development of the learners' participation in mathematics (Faustino & Skovsmose, 2020). One last theoretical-oriented article

**Table 4** List of 10 Topic 4 studies

| Topic 4 Interaction and mathematics learning | |
|---|---|
| Article title, journal, year | Location(s), authorship |
| Secondary mathematics teachers' use of students' incorrect answers in supporting collective argumentation. *MTL* 2022 | USA; Zhuang & Conner |
| Interpreting a kindergarten episode through three perspectives on agency. *FLM* 2022 | Norway, Australia & UK; Breive et al. |
| Enhancing students' language in collective processes of knowledge construction in group work. *ZDM* 2021 | Germany; Erath |
| Activity systems analysis of classroom teaching and learning of mathematics. *ESM* 2021 | Japan; Sekiguchi |
| Initiation and decision-making of joint activities within peer interaction in student-centred mathematics lessons. *CD* 2021 | Finland; Kämäräinen et al. |
| Ways in which engaging with someone else's reasoning is productive. *JMB* 2020 | USA & Israel; Rasmussen et al. |
| Dialogic and non–dialogic acts in learning mathematics. *FLM* 2020 | Brazil; Faustino & Skovsmose |
| The early history of the scaffolding metaphor. *MCA* 2019 | Russia & The Netherlands; Shvarts & Bakker |
| When students offer explanations without the teacher explicitly asking them to. *ESM* 2019 | UK; Ingram et al. |
| Epistemic issues in classroom mathematical activity. *JMB* 2019 | Chile; Goizueta |

traverses school ages to discuss the scaffolding metaphor, and how it is often associated with the Vygotskian zone of proximal development in the interpretations of mathematics learning in classroom interaction (Shvarts & Bakker, 2019). Aspects of all these articles cross aspects present in Topics 1, 2 and 3. The scaffolding metaphor, for example, is posed in relation to scaffolding in teaching, and how language and communication operate in facilitating the child's interaction with the others.

### 3.5 Topic 5: Mathematical language and discourse

During the period covered in this review, we have found various articles, to add to the sets of articles in a *JMB* and an *ESM* special issue, all guided by the Vygotskian-based theory of commognition. With its foundations published in Sfard (2008), this theory sees mathematical activity as practising a particular form of communication—namely, a *discourse*—with its own language, consisting of words and grammar, but also of visual mediators, routines and narratives. Moreover, we have found articles that report work guided by discursive psychology, positioning theory, ethnomethodology or functional linguistics in the study of mathematical discourse or of one of its constitutive parts, that is mathematical language. The existence of simultaneous theories makes today's mathematical language and discourse research specifically influential, because it addresses the challenge of exploiting ways of intellectual interaction and theoretical networking both inside and outside mathematics education.

As discussed in Planas and Schütte (2018) for earlier periods, this diversity of theories goes with a variety of meanings for the notions of discourse and mathematical discourse. Alongside the latter as a particular form of communication, the view of the mathematical discourse to include oral and written mathematical language in combination with academic and everyday registers and specific building practices (e.g., Moschkovich, 2021) is, for example, strong. In this scenario, studies on reading, writing, speaking and listening to mathematical language coexist with studies on participation in the mathematical discourse.

Four of the selected articles in Table 5 suggests commognition is a collective project shared by authors all over the world, although with a geographic centre, with empirical studies covering a range of mathematical topics and research participants, as well as differently focused on parts of the mathematical discourse characterised by this theory. These articles examine: types of routines of the mathematical discourse and how these are transformed from rituals to explorations in processes of learning (Lavie et al., 2019); emerging routines in the mathematical practices of describing and defining geometrical solids by undergraduate students (Fernández-León et al., 2021); discourse development of young children in tasks of classifying odd and even numbers and of reasoning about their sums (Knox & Kontorovich, 2022); teachers' narratives about unknowns and variables and on mathematics as mutable (Moustapha-Corrêa et al., 2021). Sinclair (2022) is a 5th article that interestingly comments on the advances of commognitive research, as well as on the challenges of contributing to the quandary of learning disability and to a shift towards pluralising mathematical discourse.

The other 5 articles in Table 5 illustrate the vivid moment in the research on reading, writing, speaking and listening to mathematical language in school or university mathematics. We include examples that report studies on: words, grammar and linguistic patterns of additive word problems and how prospective elementary teachers analyse the mathematical

**Table 5** List of 10 Topic 5 studies

| Topic 5 Mathematical language and discourse | |
|---|---|
| Article title, journal, year | Location(s), authorship |
| Beyond keywords. *IJSME* 2022 | USA; Kwok et al. |
| A cumulative, coherent and convincing theory that is also seductive, singular and selective. *JMB* 2022 | Canada; Sinclair |
| Growing research groves to visualize young students' learning in small groups. *MERJ* 2022 | New Zealand; Knox & Kontorovich |
| Analyzing the written discourse in calculus textbooks over 42 years. *ESM* 2022 online/2023 | Iran; Haghjoo et al. |
| Identifying routines in the discourse of undergraduate students when defining. *MERJ* 2021 | Spain; Fernández-León et al. |
| Problematizing mathematics and its pedagogy through teacher engagement with history-focused and classroom situation-specific tasks. *JMB* 2021 | Brazil & UK; Moustapha-Corrêa et al. |
| Using moment-by-moment reading protocols to understand students' processes of reading mathematical proof. *JRME* 2021 | USA; Dawkins & Zazkis |
| Identifying vernacular language to use in mathematics teaching. *LAE* 2019 | Australia & Papua New Guinea; Edmonds-Wathen et al. |
| Routines we live by. *ESM* 2019 | Israel; Lavie et al. |
| Reading mathematics text. *IJSME 2019* | South Africa; Berger |

language of these problems (Kwok et al., 2022); written mathematical languages and the related diagrammatic features in Iranian calculus textbooks (Haghjoo et al., 2022 online/2023); linguistic challenges faced by student teachers of mathematics when reading the calculus sections of a mathematics textbook (Berger, 2019); cognitive and semantic challenges faced by novice and experienced undergraduate students in their reading of mathematical proofs and noticing of the mathematical-linguistic features (Dawkins & Zazkis, 2021); identification or development, in collaboration with teachers, of mathematical terminology and grammar for counting systems, measuring and comparing location and shapes in Indigenous mathematical languages (Edmonds-Wathen et al., 2019).

## 4 Zooming back in on new emerging/old revisited concepts

In the prior section, we have differentiated 5 topics that illustrate part of the map of the current research domain. We now address commonalities by introducing some concepts that are important in two or more topics. *Which are some new emerging or old revisited concepts over 2019–2022?* and *What do they add to or imply for the domain?* As it occurs with traditional topics, even if we write about old concepts, we cannot expect them to mean the same today compared with the earlier years, and hence they are 'old' concepts revisited in or infused with 'new' contexts and meanings. In our response, we zoom in on: (i) instructional designing; (ii) gesturing; (iii) argumenting; (iv) languaging. In order to discuss each concept, we choose and draw on a few research texts. We are leaving many concepts out, as well as posing differently the emphases by referring to gesturing instead of gestures, or to languaging instead of language, in order to suggest the practices that go with the concepts across sites of mathematics education. Again, we intend to avoid binary distinctions that may suggest a separate role of the practices from the objects, processes, people, materials, events, … undergoing.

### 4.1 Concept 1: Instructional designing

Practices of instructional designing are frequent in studies of mathematics teacher education on language in mathematics teaching, but also of multilingual and multimodal mathematics teaching and classrooms. These practices generally include instructional materials, lesson plans and the teaching intervention followed by forms of validating effects of the instruction. Since the promotion of mathematical discussions in the 'old' classroom experiments, a rising body of literature studies the design of language in content teaching. The 2021 ZDM issue 2 offers examples of this literature. In the survey article, Erath et al. (2021) discuss language in mathematics teaching and learning, and state design principles of learning environments to enhance language for mathematics learning, and of "teaching practices (including teacher moves and classroom norms) involved in the enactment of those designed learning environments" (p. 245). While with no mention of the distinction between language and communication, in the focus on "discourse practices that are *means* to communicate" (p. 246; *italics in original*), or between argumenting and reasoning, in the focus on "discourse practices such as explaining, justifying, arguing, etc." (p. 246), the idea of designing language and instruction for inducing learners' into mathematical communication and reasoning is present.

Planas et al., (2022 online/2023) have expressed some caution in the consideration of instructional designing that does not address the design of mathematics teaching talk on its own as well, specifically the perils of suggesting this talk and its design as subordinated to, backgrounded or subsumed into the mathematical discourse practices, the written materials for supporting them, and their design. As the field of mathematics education has developed, and as our understanding of the tensions and the role of language in mathematics teaching has fostered further research and more attention to teacher education and professional development, other tensions have arisen. In this respect, Planas et al. point to tensions in arguing for the important place of mathematics teaching talk at the level of word use within the discourse practices in language-responsive mathematics teaching, and in arguing for the important place of talk about word use in mathematics teaching in teacher education pedagogies aimed at valuing participation in the mathematical discourse.

### 4.2 Concept 2: Gesturing

Gesturing as the practice of using gestures is present in studies on multimodal mathematical communication and increasingly considered in research on interaction and mathematics learning. Robutti et al. (2022), a chapter entitled 'Gesture in mathematics thinking and learning', recognise the work of researchers investigating language, communication and interaction in mathematics thinking, teaching and learning, with a focus to include the roles that gestures like hand movements can play in these processes. These are in a way ubiquitous co-speech gestures, receiving growing attention in field research. Gestures in many mathematics education articles are specified as hand movements that frequently adjoin with speech, but there are some larger body gestures that are also involved and may be deployed in silence and taking place in real time, as does speech. Pimm (2021) describes a university Analysis lecture with regard to the lecturer's extensive and significant pedagogic gestures, with an interest between 'conceptual' and 'pedagogic' gestures.

Concept(ual) gestures are more often than not generic—an extension to Mason and Pimm (1984).

In Pimm (1987/2017), a metaphoric adjectival extension of the notion 'triangle' is indicated by means of the constructed term 'spherical triangle' (pp. 101–104), which always seems to require the meaning of the noun to be altered (in this case, broadening to geodesic rather than straight line sides of a 'triangle'). Similarly, Sinclair and de Freitas (2014) develop the term 'digital gesture', which can be seen as a significant contemporary metaphoric term as well, in part because in order to affect an iPad item currently requires physical touch and, for some researchers, gestures are viewed as distinct from actual contact (e.g., pointing versus touching). In addition, Sinclair and de Freitas make an explicit argument for digital gestures being 'epistemic' and not just 'communicative', following the idea of Jürgen Streeck on 'tangible gesture'—which again, for some, could be viewed as an oxymoron—and they bring together the technology world's thinking about gesture and the mathematics education/linguistic one in a very profound way. It is also the case of increasing access to screen items by simply bringing one's finger close to the screen, but without having to touch it: a new notion of 'digital gesture', seen now as simply a specific adjective that does not require altering the noun 'gesture', may well be on its way.

### 4.3 Concept 3: Argumenting

The practice of argumenting is common, although of varying research interest, in studies across the 5 topics discussed in this review. Due to our paying attention to this commonality, one more distinction came to light that is important for understanding the most recent period of research on mathematics education, language and communication. We refer to the distinction between reasoning and argumenting, with several authors across the 50-list of articles seemingly equating both terms rather than associating them in one way or another. In other research domains of mathematics education, the specificities of mathematical argumenting are distinguished, not without multiplicity of distinctions from, for example, mathematical proving and mathematical conjecturing, and all these are viewed as ways of mathematical reasoning, or even of mathematical thinking or understanding (see the curricular, disciplinary and research variety of meanings for 'reasoning' in Reid, 2022, and for 'argumentation' in Reid & Knipping, 2010). Some researchers thus seem to be arguing about argumenting (and argumentation). For some, argumenting is viewed as a broadening of the term 'proving' in relation to mathematics. For others, it is available to and from learners of any age and site, and is suggested as equivalent to mathematical reasoning, and at times to thinking and understanding.

A new term for us both, which has supported our conversations about the recent discourse in the research domain examined, is 'hyponym': a word of more specific meaning than a general or superordinate term applicable to it. A hyponym is a term used to designate a particular member of a broader class. It could even be a terminological actual subset (in relation to mathematics). But one of the things that occurs regularly in school and university is that the same term is itself generalised, such as triangle (e.g., on a sphere) or multiplication (e.g., involving fractions or matrices). This becomes the result that a (plane) triangle is a hyponym of a (more general) triangle, once you have left the plane, as mentioned in Concept 2. So, is proof a hyponym of argument? Is argumenting a hyponym of reasoning? Is language a hyponym of communication? Still, what if mathematics education research use of terminology proved to be as seemingly straightforward as this?

### 4.4 Concept 4: Languaging

The practice of languaging points to one more commonality across topics, specifically present in studies about multilingual mathematics classrooms (tied to specific languages and translanguaging), mathematical language and discourse (tied to specific registers and moves between them), and multimodal mathematical communication (tied to specific modes and transmodal practices). The entrance of the languaging concept into mathematics education came hand in hand with Western sociolinguistics (e.g., Makoni & Pennycook, 2007) and is today moving the focus on systems of rules and structures towards the communicative linguistic practices, with attention to social and interactional processes, rather than products of communication. This approach is contributing to revisiting mathematics teaching and learning as practices and processes in which language is a mediator and an agent of meaning. The multilingual learners' languaging around 'aircraft' and 'minecraft', supported by hybrid everyday Hong Kong Cantonese Chinese, Mandarin/Putonghua and academic English language in Tai and Wei (2021, pp. 232–234), exemplifies the creative use of the vivid languages of the learners mediating and producing mathematical meaning and reasoning.

Some authors are importantly challenging the modes of language and of communication used in representations of data beyond the expectations of a certain academic register mostly in the language of English. Staats (2021) presents a variant of communication by challenging mathematics education research as a site of languaging around the personal voice, thinking metaphors, poetic lines, … She provides an example of opening up paths of languaging by recognising the poetic function of language in the analysis of a mathematical conversation between two undergraduate students on the algebra of polynomial, exponential and

logarithmic functions. In this way, she argues for alternative forms of seeing and representing data in order to overcome "the illusory distinction between mathematical discourse and mathematical reasoning" (p. 1). We see languaging in Staats' analyses in how she considers prose-looking data as poetic forms of spoken repetition of phrases, informing how the students understand the mathematical task around the equation for the perimeter of a string of N advanced hexagons. Part of the mathematical reasoning in the students' collaboration would have been silenced, had the researcher not searched for linguistic patterns, rhythms, similarities or dissimilarities in the pieces created by the conversational repetitions of each other's contributions to the task.

## 5 Concluding about the distinction between language and communication

The question in this concluding part is as follows: *How do some topics and concepts of the domain at present contribute to the discussion about the distinction between language and communication?* In the presentation of the topic of multimodal mathematical communication and of the concept of gesturing, we have particularly addressed the distinction between language and communication by emphasising studies and comments in which non-linguistic practices are central to processes of communication in sites of mathematics education. Several studies provide empirical evidence of communication that is not (only) language, and some of them clarify that language is not (always) communication—not only because we can communicate without language, but also because we can use language in ways that are of difficult understanding in mathematics teaching and learning, and in teacher education. In all this, we have proposed zooming forward by focusing on challenges such as the elaboration of a notion of mathematics *communication* register that necessarily expands and enriches the language-specific mathematics register.

Despite the advances in the research of different modes of communication and in the understanding of non-linguistic practices such as gesturing, the lack of distinction between language and communication persists. A basic part of the problem is that the study of this distinction does not seem to be in the research agenda, at least it is not overtly discussed in any of the 50 articles reviewed. Some of the articles reviewed do not treat language explicitly as distinct from communication, with findings that can be misleading in conclusions about the scope and nature of the communication processes. The nature of the distinction between language and communication is, nevertheless, suggested in some other articles. Some of the studies support their thinking by adding words to 'language' and to 'communication' that make any diluting of the non-language dimension behind the term 'communication' more difficult and less immediate. For example, we have seen the exploration in Ingram et al. (2019) of lessons with the teachers not 'explicitly asking' learners for the mathematical explanations that they came to produce. The addition of the term 'explicitly' to 'asking' suggests the existence of some mathematical demands to learners being communicated at a non-linguistic level. To know more about how the linguistic and the non-linguistic features of this classroom communication relate to or cross each other, the development of a notion of mathematics *communication* register seems, again, an important direction to pursue in the coming period.

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## References

Adler, J. (2001). *Teaching mathematics in multilingual classrooms*. Springer.

Adler, J., Mwadzaangati, L., & Takker, S. (2023). From defining as assertion to defining as explaining meaning: Teachers' learning through theory-informed lesson study. *International Journal for Lesson and Learning Studies, 12*(1), 38–51.

**Research and developmental collaborative work with a group of Malawian secondary-school mathematics teachers engaged in a lesson study project in plane geometry guided by a focus on language responsive teaching practices. Qualitative content analyses of cyclical lesson plans and transcripts and of reflective discussions reveal teachers' learning of forms of word use that communicate mathematical meanings for geometric properties and the exterior angle of a triangle.**

Alhasnawi, S. (2021). Teachers' beliefs and practices with respect to translanguaging university mathematics in Iraq. *Linguistics and Education, 63*, 100930.

Alibali, M., Nathan, M., Boncoddo, R., & Pier, E. (2019). Managing common ground in the classroom: Teachers use gestures to support students' contributions to classroom discourse. *ZDM Mathematics Education, 51*(2), 347–360.

Bakos, S., & Pimm, D. (2020). Beginning to multiply (with) dynamic digits: Fingers as physical–digital hybrids. *Digital Experiences in Mathematics Education, 6*(3), 145–165.

**Research on a single, half-hour involvement of two grade-three children engaging for the first time with an iPad app. In the 30-minute interview context of an iterative design experiment aimed at exploring and refining the app, the physical engagement of mathematical tactile action with the software is with their fingers. The two pairs of hands and multiple fingers are objects themselves, but they also become hybrid intermediaries among the students, the iPad and multiplication.**

Barwell, R. (2020). Learning mathematics in a second language: Language positive and language neutral classrooms. *Journal for Research in Mathematics Education, 51*(2), 150–178.

Berger, M. (2019). Reading mathematics text: A study of two empirical readings. *International Journal of Science and Mathematics Education, 17*(2), 385–404.

Bergman, A., Gallagher, K., & Zazkis, R. (2022). Prospective teachers' responses to students' dialogue on fractions: Attribute substitution and heuristic approaches. *Research in Mathematics Education, 25*(1), 85–104.

Breive, S., Goos, M., & Monaghan, J. (2022). Interpreting a kindergarten episode through three perspectives on agency. *For the Learning of Mathematics, 42*(1), 25–30.

Chronaki, A., Planas, N., & Svensson Källberg, P. (2022). Onto/epistemic violence and dialogicality in translanguaging practices across multilingual mathematics classrooms. *Teachers College Press, 124*(5), 108–126.

Cobb, P., & Bauersfeld, H. (Eds.). (1995). *The Emergence of Mathematical Meaning: Interaction in Classroom Cultures*. Routledge.

Coles, A., & Sinclair, N. (2019). Ritualisation in early number work. *Educational Studies in Mathematics, 101*(2), 177–194.

Dawkins, P., & Zazkis, D. (2021). Using moment-by-moment reading protocols to understand students' processes of reading mathematical proof. *Journal for Research in Mathematics Education, 52*(5), 510–538.

**Research on undergraduate students' involvement in tasks of reading mathematical proofs, guided through thinking-aloud and moment-to-moment protocols. The approach is comparative between students who are novice and those who are experienced in reading and writing work with mathematical proofs. Results raise important comprehensive reading challenges in both groups regarding the identification of propositions as assumptions or conclusions in the theorem statements.**

Edmonds-Wathen, C., Owens, K., & Bino, V. (2019). Identifying vernacular language to use in mathematics teaching. *Language and Education, 33*(1), 1–17.

El Mouhayar, R. (2022a). Triadic dialog in multilingual mathematics classrooms as a promoter of generalization during classroom talk. *Mathematics Education Research Journal, 34*(1), 87–112.

El Mouhayar, R. (2022b). Teacher's and students' use of gestures and home-language during classroom talk to elicit a shared understanding of structure in figural pattern: A case study in a multilingual mathematics classroom. *Linguistics and Education, 71*, 101088.

Erath, K. (2021). Enhancing students' language in collective processes of knowledge construction in group work: The case of enlarging figures. *ZDM Mathematics Education, 53*(2), 317–335.

Erath, K., Ingram, J., Moschkovich, J., & Prediger, S. (2021). Designing and enacting instruction that enhances language for mathematics learning: A review of the state of development and research. *ZDM Mathematics Education, 53*(2), 245–262.

Eriksson, H., & Eriksson, I. (2021). Learning actions indicating algebraic thinking in multilingual classrooms. *Educational Studies in Mathematics, 106*(3), 363–378.

Faustino, A., & Skovsmose, O. (2020). Dialogic and non–dialogic acts in learning mathematics. *For the Learning of Mathematics, 40*(1), 9–14.

Fernández-León, A., Gavilán-Izquierdo, J., González-Regaña, A., Martín-Molina, V., & Toscano, R. (2021). Identifying routines in the discourse of undergraduate students when defining. *Mathematics Education Research Journal, 33*(2), 301–319.

Gandara, F., & Randall, J. (2019). Assessing mathematics proficiency of multilingual students: The case for translanguaging in the Democratic Republic of the Congo. *Comparative Education Review, 63*(1), 58–78.

Goizueta, M. (2019). Epistemic issues in classroom mathematical activity: There is more to students' conversations than meets the teacher's ear. *The Journal of Mathematical Behavior, 55*, 100691.

Haghjoo, S., Radmehr, F., & Reyhani, E. (2022). Analyzing the written discourse in calculus textbooks over 42 years: The case of primary objects, concrete discursive objects, and a realization tree of the derivative at a point. *Educational Studies in Mathematics, 112*(1), 73–102.

Halliday, M. (1975). *Some aspects of sociolinguistics*. In *Final Report of the Symposium on Interactions between Linguistics and Mathematics Education* (pp. 64–73). Royal Danish School of Educational Studies.

Ingram, J., Andrews, N., & Pitt, A. (2019). When students offer explanations without the teacher explicitly asking them to. *Educational Studies in Mathematics, 101*(1), 51–66.

**Research on students' explanations in 42 mathematics lessons, with the notion of explanation as constituted of a continuum of explanation types within the practice of mathematical reasoning. The conversational analyses, with specific attention to how the turns are produced in terms of both**

**the content and the structure of the turn, reveal interactional contexts where students give explanations without being explicitly asked for one through how or why questions in the teaching.**


Jewitt, C. (2006). *Technology, Literacy and Learning: A Multimodal Approach*. Routledge.

Kämäräinen, A., Eronen, L., Björn, P., & Kärnä, E. (2021). Initiation and decision-making of joint activities within peer interaction in student-centred mathematics lessons. *Classroom Discourse, 12*(4), 299–318.

Knox, J., & Kontorovich, I. (2022). Growing research groves to visualize young students' learning in small groups. *Mathematics Education Research Journal., 2*, 2.

Krummheuer, G. (2007). Argumentation and participation in the primary mathematics classroom: Two episodes and related theoretical abductions. *The Journal of Mathematical Behavior, 26*(1), 60–82.

Kwok, M., Welder, R., Moore, J., & Williams, A. (2022). Beyond keywords: Applying Systemic Functional Linguistics to unpack the language of additive word problems. *International Journal of Science and Mathematics Education, 20*(1), 163–186.

Lavie, I., Steiner, A., & Sfard, A. (2019). Routines we live by: From ritual to exploration. *Educational Studies in Mathematics, 101*(2), 153–176.

Leavy, A., & Hourigan, M. (2022). Balancing competing demands: Enhancing the mathematical problem-posing skills of prospective teachers through a mathematical letter-writing initiative. *Journal of Mathematics Teacher Education, 25*(3), 293–320.

Maffia, A., & Sabena, C. (2020). On the mathematics teacher's use of gestures as pivot signs in semiotic chains. *For the Learning of Mathematics, 40*(1), 15–21.

Makoni, S., & Pennycook, A. (Eds.) (2007). *Disinventing and reconstituting languages*. De Gruyter.

Mason, J., & Pimm, D. (1984). Generic examples: Seeing the general in the particular. *Educational Studies in Mathematics, 15*(3), 277–289.

Morgan, C. (2021). Conceptualising and researching mathematics classrooms as sites of communication. In N. Planas, C. Morgan, & M. Schütte (Eds.), *Classroom Research on Mathematics and Language: Seeing Learners and Teachers Differently* (pp. 101–116). Routledge.

Morgan, C., Craig, T., Schütte, M., & Wagner, D. (2014). Language and communication in mathematics education: An overview of research in the field. *ZDM - the International Journal on Mathematics Education, 46*(6), 843–853.

Morgan, C., Planas, N., & Schütte, M. (2021). Developing a perspective on multiplicity in the study of language in mathematics classrooms. In N. Planas, C. Morgan, & M. Schütte (Eds.), *Classroom Research on Mathematics and Language: Seeing Learners and Teachers Differently* (pp. 3–22). Routledge.

Moschkovich, J. (2021). Learners' language in mathematics classrooms: What we know and what we need to know. In N. Planas, C. Morgan, & M. Schütte (Eds.), *Classroom Research on Mathematics and Language: Seeing Learners and Teachers Differently* (pp. 60–76). Routledge.

Moustapha-Corrêa, B., Bernardes, A., Giraldo, V., Biza, I., & Nardi, E. (2021). Problematizing mathematics and its pedagogy through teacher engagement with history-focused and classroom-situation specific tasks. *The Journal of Mathematical Behavior, 61*, 100840.

Ng, O.-L., Cheng, W., Ni, Y., & Shi, L. (2021). How linguistic features and patterns of discourse moves influence authority structures in the mathematics classroom. *Journal of Mathematics Teacher Education, 24*(6), 587–612.

O'Halloran, K. (2015). The language of learning mathematics: A multimodal perspective. *The Journal of Mathematical Behavior, 40*, 63–74.

Oechsler, V., & Borba, M. (2020). Mathematical videos, social semiotics and the changing classroom. *ZDM Mathematics Education, 52*(5), 989–1001.

Pimm, D. (1987). *Speaking Mathematically: Communication in Mathematics Classrooms*. Routledge.

Pimm, D. (2021). Language, paralinguistic phenomena and the (same-old) mathematics register. In N. Planas, C. Morgan, & M. Schütte (Eds.), *Classroom Research on Mathematics and Language: Seeing Learners and Teachers Differently* (pp. 22–40). Routledge.

Planas, N. (2021). How specific can language as resource become for the teaching of algebraic concepts? *ZDM Mathematics Education, 53*(2), 277–288.

Planas, N., & Schütte, M. (2018). Research frameworks for the study of language in mathematics education. *ZDM Mathematics Education, 50*(6), 965–974.

Planas, N., Morgan, C., & Schütte, M. (2018). Mathematics education and language: Lessons and directions from two decades of research. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven (Eds.), *Developing Research in Mathematics Education: Twenty Years of Communication, Cooperation and Collaboration in Europe* (pp. 196–210). Routledge.

Planas, N., Adler, J., & Mwadzaangati, L. (2022). What is mathematics teaching talk for? A response on three sites of practice in mathematics education. *ZDM Mathematics Education, 55*(3), 521–534.

Prediger, S. (2019). Investigating and promoting teachers' expertise for language-responsive mathematics teaching. *Mathematics Education Research Journal, 31*(4), 367–392.

Radford, L., & Barwell, R. (2016). Language in mathematics education research. In Á. Gutiérrez, G. Leder, & P. Boero (Eds.), *The Second Handbook of Research on the Psychology of Mathematics Education: The Journey Continues* (pp. 275–313). Brill.

Rasmussen, C., Apkarian, N., Tabach, M., & Dreyfus, T. (2020). Ways in which engaging with someone else's reasoning is productive. *The Journal of Mathematical Behavior, 58*, 100742.

Reid, D., & Knipping, C. (2010). *Proof in mathematics Education: Research, Learning and Teaching*. Sense Publishers.

Reid, D. (2022). 'Reasoning' in national curricula and standards. In J. Hodgen, E. Geraniou, G. Bolondi & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education* (pp. 299–306). ERME.

Robertson, S.-A., & Graven, M. (2019). Exploratory mathematics talk in a second language: A sociolinguistic perspective. *Educational Studies in Mathematics, 101*(2), 215–232.

Robutti, O., Sabena, C., Krause, C., Soldano, C., & Arzarello, F. (2022). Gestures in mathematics thinking and learning. In M. Danesi (Ed.), *Handbook of Cognitive Mathematics* (pp. 685–726). Springer.

Sekiguchi, Y. (2021). Activity systems analysis of classroom teaching and learning of mathematics: A case study of Japanese secondary schools. *Educational Studies in Mathematics, 108*(3), 493–511.

Sfard, A. (2008). *Thinking as Communicating. Human Development, the Growth of Discourses, and Mathematizing*. Cambridge University Press.

Sharma, S., & Sharma, S. (2022). Successful teaching practices for English language learners in multilingual mathematics classrooms: A meta-analysis. *Mathematics Education Research Journal., 2*, 2.

Shaughnessy, M., Ghousseini, H., Kazemi, E., Franke, M., Kelley-Petersen, M., & Hartmann, E. (2019). An investigation of supporting teacher learning in the context of a common decomposition for leading mathematics discussions. *Teaching and Teacher Education, 80*(1), 167–179.

Shvarts, A., & Bakker, A. (2019). The early history of the scaffolding metaphor: Bernstein, Luria, Vygotsky, and before. *Mind, Culture and Activity, 26*(1), 4–23.

Sinclair, N. (2022). A cumulative, coherent and convincing theory that is also seductive, singular and selective. *The Journal of Mathematical Behavior, 67*, 100983.

Sinclair, N., & de Freitas, E. (2014). The haptic nature of gesture: Rethinking gesture with new multitouch digital technologies. *Gesture, 14*(3), 351–374.

Sjöblom, M., Valero, P., & Olander, C. (2022). Teachers' noticing to promote students' mathematical dialogue in group work. *Journal of Mathematics Teacher Education., 2*, 2.

Staats, S. (2021). Mathematical poetic structures: The sound shape of collaboration. *The Journal of Mathematical Behavior, 62*, 100846.

Tai, K., & Wei, L. (2021). Co-learning in Hong Kong English medium instruction mathematics secondary classrooms: A translanguaging perspective. *Language and Education, 35*(3), 241–267.

**Research on transmodal and translanguaging practices enhanced in teaching to encourage learner participation in mathematical interactions in Hong Kong classrooms with English as the language of instruction. Analyses of ethnographic lesson data and video-stimulated-recall interviews show the mediational role of translanguaging as well as multimodal and multisensorial mathematics communication in the promotion of spaces of learning in interaction with peers and with the teacher.**

Turner, E., McDuffie, A., Sugimoto, A., Aguirre, J., Bartell, T., Drake, C., Foote, M., Stoehr, K., & Witters, A. (2019). A study of early career teachers' practices related to language and language diversity during mathematics instruction. *Mathematical Thinking and Learning, 21*(1), 1–27.

Walkington, C., Chelule, G., Woods, D., & Nathan, M. (2019). Collaborative gesture as a case of extended mathematical cognition. *The Journal of Mathematical Behavior, 55*, 100683.

Wathne, U., & Carlsen, M. (2022). Third grade students' multimodal mathematical reasoning when collaboratively solving combinatorial problems in small groups. *Mathematical Thinking and Learning., 2*, 2.

Williams, J., & Ryan, J. (2020). On the compatibility of dialogism and dialectics: The case of mathematics education and professional development. *Mind, Culture and Activity, 27*(1), 70–85.

Yeo, L., & Tzeng, Y. (2020). Cognitive effect of tracing gesture in the learning from mathematics worked examples. *International Journal of Science and Mathematics Education, 18*, 733–751.

Yu, Y., & Uttal, D. (2022). Gestures, embodiment, and learning the rate of change. *Mathematical Thinking and Learning, 24*(3), 203–229.

Zahner, W., & Wynn, L. (2023). Rethinking learning trajectories in light of student linguistic diversity. *Mathematical Thinking and Learning, 25*(1), 100–114.

Zhuang, Y., & Conner, A. (2022). Secondary mathematics teachers' use of students' incorrect answers in supporting collective argumentation. *Mathematical Thinking and Learning., 2*, 2.