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An aggregate Laffer curve – a multi-peak case

In the current paper we propose to distinguish between micro and aggregate Laffer functions. We prove that in spite of the fact that a Laffer curve of any representative individual may have one peak point where tax revenue is at its maximum, the aggregate Laffer curve is more likely to have multiple peaks. We show this for the case where there is a high degree of wage distribution inequality along with a backward bending supply curve of labor, which appears to be the case for many Western countries. Since this scenario is typical of many countries, the welfare implications of the multi-peak Laffer curve should be considered by the policy maker.

INTRODUCTION

On a cool autumn evening in Washington in 1974, *Art Laffer,* (then 35) had one of those moments that end up defining someone for the rest of his life. Watergate and stagflation gripped the country. Ford wanted to WIN – Whip Inflation Now! – with a five-percent tax surcharge, which was supposed to re-ignite the American economy by taking big bites out of it. Today raising tax rates in a recession seems silly. Back then it was accepted folklore. In the fall of 1974, Rumsfeld and Cheney were looking for alternatives. Happy to oblige was Laffer, who pointed to a mandala sketched on a cocktail napkin-two perpendicular lines and an arc-as the answer to the complex problems plaguing the nation. The Laffer Curve, one of the icons of supply-side economics, was born (see the *American Spectator,* Jan/Feb 2002).

It has been said that one of the great advantages of the Laffer curve is that you can explain it to a congressman in half an hour and he can talk about it for six months. But levity aside, since its arrival on the public scene literally hundreds of journal articles have analyzed, dissected, rejected, accepted, objected, executed, and rehabilitated the Laffer curve. But all this literature has one thing in common; it implicitly assumes a single-peaked Laffer curve. In most of these articles the underlying assumption is that tax revenue approaches zero when the (average) tax rate is either zero or close to 1 (100%), and at some intermediate tax rates there is one peak point where tax revenue is at its maximum.

Somewhat surprisingly, the issue of a multi-peaked Laffer curve has not been properly discussed in the literature. All journal articles and all text books in microeconomics, macroeconomics and public finance that we are aware of take the one-peaked curve for granted [e.g. Borgas (2000), Ulbrich (2003), pp. 168–169, Rosen (2002), pp. 383–384], Stiglitz (1999), pp. 699–700).[1]

Moreover, most authors do not even bother to distinguish between the micro and the aggregate Laffer curves. The typical approach to the aggregate Laffer curve is similar to that of the individualistic Laffer curve (see, for example, Canto et al., 1983; Stuart, 1981; Fullerton, 1982; Lindsey, 1985). Although *Sutter* and *Weck-Hannemann* (2003) have shown in an experimental study that the Laffer curve can have multiple peaks, they have no theoretical framework or formal proof for this possibility. They state that:

"A noteworthy result of our experiment, however, is the fact that the tax revenue curve can have more than one peak. Econometric estimations of the 'real' Laffer curve have, so far, implicitly assumed a concave shape of the Laffer curve. This need not necessarily be the case....

Nevertheless, presumably due to the acute lack of a formal proof, the literature persists in assuming that the aggregate Laffer curve is simply a vertical summation of the individualistic Laffer curves of all heterogeneous individuals in the society (in terms of hourly wage rate) at each average tax rate. This paper closes the gap in the literature by presenting the required formal proof for the likelihood that the aggregate Laffer curve is indeed multi-peaked. We show that:

a The Laffer curve tends to be multi-peaked.

b Tax revenues do not necessarily tend to approach zero as tax rates approach 100% (see Spiegel and Templeman (2004)).

Economists such as Nobel Prize winner *James Mirrlees* and many others have dealt with the problem of finding an optimal tax structure that would maximize social welfare. The solution to this problem will always depend on the nature of the social welfare function selected and the nature of the utility function proposed to describe the labor market behavior of the population . But a principal-agent problem could well exist between the government and its citizens. This could well mean that the government is not even remotely attempting to maximize some social welfare function but is simply attempting to maximize its revenues from taxation. It is to this type of taxation that our analysis is addressed.

In this we go one step further to show theoretically and with illustrative examples that even if each individualistic Laffer curve has one peak, the aggregate economic Laffer curve is more likely to have multiple peaks.

This is based on three presumptions which are supported by the labor supply curve in most western countries. The presumptions are as followings.

1 The wage distribution demonstrates a very high degree of inequality. The distribution is one-tailed asymmetric with a narrow margin approaching very high wage rates and most of the population have a comparatively low wage rate [see Chinhui Murphy and Pierce (1993)]. Furthermore, recent evidence of this income inequality can be found in *Borjas* (2000) p. 277, who bases his analysis on the US current population survey of 1997. The data show that the top 10% of USA households get 28.5% of total income, while the bottom 10% get only 1.5% of the total income.

2 To make our task even more difficult and challenging we assume that on the individual level each individual has a peak point of tax payment at some tax rate, which can be different for different income groups [see a panel study by Martin Feldstein (1995)]. Even when the individuals are homogeneous in tastes and differ in wage rates, they are likely to have different peak points of revenue maximizing tax rates.

3 The individualistic supply curve of labor exhibits at the lower wage rates, for most individuals in society, a positive relationship between labor supply and the wage rate. This demonstrates that the negative substitution

effect of wage leisure is dominant in comparison to the positive income effect. However, at relatively high wage rates the phenomenon of a backward bending labor supply occurs, indicating the dominancy of the income effect (see Link and Settle, 1981).

Based on these three presumptions we show that in spite of one-peak individualistic Laffer curves, the aggregate Laffer curve may have a number of peaks. It should be emphasized that the theoretical results of our paper show that a multi-peaked aggregate Laffer curve is possible, whereas the empirical test findings of Sutter and Weck-Hannemann (2003) show that it is likely to be empirically true. Those results should be treated as a serious warning to policy makers that their preconceptions concerning the aggregate Laffer curve are probably false.

The importance of such a phenomenon is crucial for public finance theory as well as for policy makers. Suppose that an increase in the income tax rate, starting from the status quo, yields higher tax revenue. Then it is nevertheless not safe to assume, as many policy makers appear to do, that further increases in the tax rate will yield progressively lower increases in tax revenue.

- Suppose that an increase in the income tax rate leads to lower tax revenue. Then it is nevertheless not safe to suppose that further increases in the tax rate will lead to even greater declines in the tax revenue.
- Various different tax rates could yield the same amount of tax revenue. But the work incentive effects of these alternative tax rates will be different, leading to different labor participation rates.
- The previous point implies that the government has latitude in deciding how the tax burden should be distributed over the population.

A lower tax rate with a peak reflects a tax imposition on the relatively poor, whereas the tax burden on the rich is relatively small. The degree of progressivity or regressivity of the tax burden should, of course, be an important factor for policy makers to take into account.

In the next section we discuss a general aggregate Laffer function. We have previousy demonstrated [Spiegel, U., and Templeman, J., (2004)] that this can be readily shown for two individuals. Our goal now is to develop a more general model. To illustrate our approach, we introduce in the third section a special utility function characterized by a backward bending supply of labor. Based on this function we derive in the fourth section the Laffer curves of individuals who differ by wage rate. In the fifth section we combine individual Laffer curves with respect to a given wage distribution into an aggregate Laffer function and formally prove that this function can have multiple peaks. Based on the actual rough wage distribution of the U.S.A., a numerical example then demonstrates how the aggregate Laffer curve with three peak points is derived. In the last section we discuss some implications and conclusions.

PRELIMINARY STUDIES

Our model is based only on taxes on wages, while for simplicity we ignore tax revenues from capital gains, rental, and profit income. As a result the Laffer curve shows the relationship between tax revenue, *(T)*, and the tax rate *t* for any level of basic gross wage rate *(W)*, i.e., $T=f(t,W)$, i.e. we are developing the Laffer curve within the framework of a simple micro model. Let wage *(W)* distribution be characterized by a continuous (exogenous, for simplicity) density function, $r(W)$, so that

$$\int_0^\infty r(W)dW = 1$$

Then a straightforward single tax rate aggregation, $T^\alpha(t)$, of the tax revenue function $T=f(t,W)$ over wage distribution is:

$$T^\alpha(t) = \int_0^\infty r(W)f(t,W)dW$$

The idea behind the single-peak individual Laffer curve is that once the tax rate reaches a certain level, the motivation to work declines so sharply that tax revenues begin to fall. Further increases in tax rates eventually result in a tax rate which is so high that the worker refuses to supply any labor at all, that is $T=0$. Since the Laffer curve is continuous, beyond that maximum rate the curve becomes negative which corresponds to a negative tax revenue, i.e., income subsidies (or the related cost of a social security system). In this paper we assume that the unemployment rate is low. Therefore work avoidance (voluntary unemployment) can easily be distinguished from joblessness and thus not compensated with income subsidies. Consequently, the high tax rate will result in a zero actual tax revenue rather than the negative tax revenue of the continuous Laffer curve. Thus, the part of the Laffer curve which corresponds to negative tax revenues becomes non-feasible. That is to say, if there exists a positive ξ, such that for $t>\xi$, taxes as well as subsidies are not paid while the continuous individual curve T may be negative, then a more precise definition of the individual Laffer curve is:

$$T = \begin{cases} f(t, W), & \text{if } 0 \le t \le \xi \\ 0, & \text{otherwise.} \end{cases}$$

The presence of the negative areas does not affect revenue maximization over a given individual Laffer curve, and therefore a more precise definition is not required even when unemployment rates are low. However, if one combines workers with different wages in order to introduce an aggregate Laffer curve, the unfeasible areas will clearly affect the above straightforward aggregate curve. Indeed, if for a single high tax rate there is a well-paid worker who is still willing to work and an ordinary worker who is no longer motivated to work, then the total tax revenue will include a positive value from the former and a negative (non-feasible) one from the latter. Thus the above, more accurate definition of T, removes unfeasible areas of individual Laffer curves to ensure correct aggregation over appropriate tax intervals. In fact, it is because of the interaction of the two factors, wage distribution and worker motivation (some workers are more motivated and some are less for each and every given tax rate interval), that we obtain an "explosion" of peaks in an aggregate Laffer curve as we intend to demonstrate.

Thus, proper aggregation of the individual Laffer curves leads to considering tax rate intervals which are applicable to only certain wage groups (classes) as is typically the case in reality. That is, in order to model the cases where the tax rates are such that lower wage earners are no longer tax payers while higher level wage groups are still paying taxes, we need to select wage groups and associate them with corresponding tax rates. This work has already been accomplished by the tax authorities in the form of standardized tax tables. Similarly, let us assume that tax rates $0 \le t \le 1$ can be divided into K (not necessarily equal) progressive tax intervals, $t_{i-1} \le t < t_i$, $(t_{i-1} < t_i)$ $i=1,2,..,K$, $t_0 = 0$ and $t_K \le 1$, which correspond to K wage groups. Each tax rate interval $t_{i-1} \le t < t_i$ applies to a group of individuals characterized by a wage interval $[Z_{i-1}, Z_i]$. Specifically, the first tax interval, $0 \le t < t_1$, contains all $i=1,2,..,K$ tax payer groups, the second tax interval contains $i=2,..,K$ classes and so forth through the highest wage level $i=K$ which is the only group still paying taxes at the highest tax rate interval $t_{K-1} \le t < t_K$. If there is only one tax interval $t_{i-1} \le t < t_i$, then the revenue function for this tax interval is

$$T^i = \begin{cases} \int_{Z_{i-1}}^{\infty} r(W) f(t, W) dW, & \text{if } t_{i-1} \le t < t_i; \\ 0, & \text{otherwise} \end{cases}$$

Thus the aggregate Laffer function over all K tax intervals (which takes into account only those wage earners who are paying taxes at each tax rate t) is as follows:

$$T^a(t) = \sum_{i=1}^{K} T^i \quad .$$

Using the first order optimality conditions we find:

$$\frac{\partial T^a(t)}{\partial t} = \sum_{i=1}^{K} \int_{Z_{i-1}}^{\infty} r(W) \frac{\partial f(t,W)}{\partial t} dW = 0 \quad t_{i-1} \le t < t_i \ ,$$

By denoting tax rate $t^0 i$ that satisfies the following equation:

$$\sum_{i=1}^{K} \int_{Z_{i-1}}^{\infty} r(W) \frac{\partial f(t,W)}{\partial t}\Bigg]_{t=t_i^0} dW = 0 \ ,$$

We conclude that the aggregate Laffer function can have multiple peaks including boundary points ti and stationary points t^0_i for which $t_{i-1} < t^0_i < t_i$ holds. Furthermore, from the last equation we observe that the number of peaks depends on the form of the underlying revenue function. Therefore, to elaborate on the number of peaks and their conditions, we need to select a specific tax revenue function. In what follows, we illustrate the analysis with the Laffer curve derived from the case of a backward-bending labor supply. This Laffer curve is then aggregated so that unfeasible areas do not affect the optimization.

Note, the theoretical wage distribution $r(W)$ is difficult to obtain, while statistical data on wage classes is regularly published in the form of histograms with average wages, w_i, and corresponding weights, r_i, $i=1,2,..,K$. Consequently, in our model we replace the continuous distribution (an integral) with the corresponding discrete distribution (a summation). This replacement however, has no effect on the final results which can be readily transformed into a continuous form by replacing the summations with integrals, if a continuous distribution is known.

THE CASE OF BACKWARD-BENDING LABOR SUPPLY

Let us assume that each individual has an additive utility function, U, which is a positive function of the daily share of leisure, l, i.e., $0 < l < 1$, and daily consumption, C, that is measured in \$ terms. The utility function that is maximized is as follows:

$$U = \alpha C - \beta \frac{C^2}{2} + \gamma l \qquad (1)$$

where α β, and γ are positive parameters and the budget constraint is

$$W(1-t)(1-l) = C^2 . \qquad (2)$$

The F.O.C. are

$$\frac{MU_l}{MU_c} = W(1-t) \ ,$$

thus,

$$\frac{\gamma}{\alpha - \beta C} = W(1-t) \qquad (3)$$

or

$$C = \frac{\alpha}{\beta} - \frac{\gamma}{\beta W(1-t)} \qquad (3')$$

From (2) and (3') we can derive the demand for leisure as:

$$l = 1 - \frac{\alpha}{\beta W(1-t)} + \frac{\gamma}{\beta [W(1-t)]^2} \qquad (4)$$

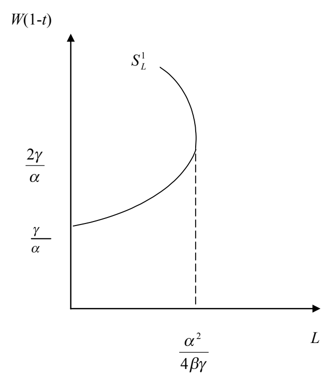
Because $L + l = 1$, we get the supply function of labor L as follows:

$$L = 1 - l = \frac{\alpha}{\beta W(1-t)} - \frac{\gamma}{\beta [W(1-t)]^2} \qquad (5)$$

The curve of L with two regions as a function of $W(1–t)$ is introduced in *Chart 1.*

Chart 1

THE SUPPLY FUNCTION OF LABOR *L*



From (5) we find that for a net wage rate per hour of

$$W(1-t)\leq \frac{\gamma}{\alpha},\ L=0.$$

At a net wage rate of $W(1-t)=\frac{2\gamma}{\alpha}$, $L=\frac{\alpha^2}{4\beta\gamma}$ is at a maximum.[3]

For any increase in the net wage above, $(\frac{2\gamma}{\alpha})$, the daily labor supply decreases.[4]

THE SHAPE OF THE INDIVIDUAL LAFFER CURVE

The Laffer curve shows the relationship between tax revenue, T, and the tax rate t for any level of basic gross wage rate, W, and labor supply, L.

In our specific case, where, according to equation (5) above, we obtain

$$T=t\cdot W\cdot\left[\frac{\alpha}{\beta W(1-t)}-\frac{\gamma}{\beta[W(1-t)]^2}\right]=\frac{t}{\beta(1-t)^2}\left[\alpha(1-t)-\frac{\gamma}{W}\right] \quad (6)$$

For t = 0, and $t=1-\frac{\gamma}{\alpha W}$ there is no tax revenue.

For $0<t<1-\frac{\gamma}{\alpha W}$ tax revenue is positive.

For

$$t=\frac{1-\frac{\gamma}{\alpha W}}{1+\frac{\gamma}{\alpha W}} \quad (7)$$

tax revenue is at its maximum (peak point of the Laffer curve).

The backward bending supply curve yields another interesting characteristic. In the backward bending part of the supply curve shown in Chart 1 an increase in t leads to an increase in labor supply. Therefore, the Laffer curve increases at an increasing rate (first and second derivatives are positive). At some point the curve changes its form and continues to increase at a diminishing rate up to the peak point and afterward it starts diminishing until it reaches zero tax revenue.

In the next section we prove that in spite of the fact that the micro Laffer curve of private earners demonstrates a single peak, the aggregate curve derived from a whole distribution of wage rates may have more than one peak.

PROPERTIES OF THE AGGREGATE LAFFER CURVE

Let wage distribution be characterized by K average wages (classes) w_i ($w_{i-1}<w_i$, $i=2,..,K$) with corresponding weights r_i, $i=1,2,..,K$, $\sum_{i=1}^{K} r_i =1$. Without loss of generality assume that tax rates $0\leq t\leq 1$ can be divided into K (not necessarily equal) progressive tax intervals, $t_{i-1}\leq t<t_i$, ($t_{i-1}<t_i$) $i=1,2,..,K$, $t_0=0$ és $t_K\leq 1$, corresponding to K wage groups. Specifically, the first tax interval, $0\leq t<t_1$ contains all $i=1,2,..,K$ tax payer groups. The second tax interval contains $i=2,..,K$ wage groups and so forth through the highest wage class $i=K$ which is the only wage group being taxed at the highest tax rate interval $t_{K-1}\leq t<t_K$. If there is only one tax interval, ($t_{i-1}\leq t<t_i$), then equation (6) for this tax interval is straightforwardly generalized as:

$$T=\sum_{n=i}^{K} r_n\frac{t}{\beta(1-t)^2}\left[\alpha(1-t)-\frac{\gamma}{w_n}\right]=\frac{\alpha t}{\beta(1-t)^2}\sum_{n=i}^{K} r_n\left[\left(1-\frac{\gamma}{\alpha w_n}\right)-t\right] \quad (8)$$

Consequently, by denoting the aggregated revenue over each separate interval of tax rate, ($t_{i-1}\leq t<t_i$, $i=1,2,..,K$) as:

$$T^i=\begin{cases}\frac{\alpha t}{\beta(1-t)^2}\sum_{n=i}^{K} r_n\left[\left(1-\frac{\gamma}{\alpha w_n}\right)-t\right], \text{if } t_{i-1}\leq t<t_i; \\ 0, \text{otherwise}\end{cases} \quad (9)$$

we obtain the aggregate Laffer function over all K tax intervals,

$$T^a(t)=\sum_{i=1}^{K} T^i \quad (10)$$

which is similar to the original Laffer function, $T^\alpha=0$ when $t=0$ and $t=1-\frac{\gamma}{\alpha w_K}$. As the ultimate goal is to maximize tax revenue, the meaningful choice of tax intervals is such that T^α does not become negative, i.e.,

$$t_i \le \frac{\sum_{n=i}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)}{\sum_{n=i}^{K} r_n}, \; i=1,2,..,K \quad (11)$$

where a new constant, $\lambda = \frac{\gamma}{\alpha}$, is introduced to simplify the further presentation. As shown in the following proposition, such a choice is always possible.

Proposition 1.

There always can be selected such sequence of progressive tax rate intervals, $t_{i-1} \le t < t_i$, $i=1,2,..,K$ that $T^{\alpha}(t) \ge 0$, for $0 \le t < t_K$.

Proof: To prove the proposition, we need to show that the maximum meaningful length of tax interval

$$i, \; t_i = \frac{\sum_{n=i}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)}{\sum_{n=i}^{K} r_n} \qquad t_{i+1} = \frac{\sum_{n=i+1}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)}{\sum_{n=i+1}^{K} r_n}$$

is always feasible, that is the right-hand side of inequality (11):

$$R_i = \frac{\sum_{n=i}^{K} r_n\left(1-\frac{\gamma}{\alpha w_n}\right)}{\sum_{n=i}^{K} r_n} = 1 - \frac{\lambda \sum_{n=i}^{K} r_n \frac{1}{w_n}}{\sum_{n=i}^{K} r_n}$$

is an increasing function of i.

Consider the difference Δ_i between two consecutive values $R_{i+1} - R_i$

$$\Delta_i = \frac{\lambda \sum_{n=i}^{K} r_n \frac{1}{w_n}}{\sum_{n=i}^{K} r_n} - \frac{\lambda \sum_{n=i+1}^{K} r_n \frac{1}{w_n}}{\sum_{n=i+1}^{K} r_n} = \frac{\sum_{n=i+1}^{K} r_n \lambda \sum_{n=i}^{K} r_n \frac{1}{w_n} - \sum_{n=i}^{K} r_n \lambda \sum_{n=i+1}^{K} r_n \frac{1}{w_n}}{\sum_{n=i+1}^{K} r_n \sum_{n=i}^{K} r_n} =$$

$$= \frac{\lambda r_i \left(\sum_{n=i+1}^{K} r_n \frac{1}{w_i} - \sum_{n=i+1}^{K} r_n \frac{1}{w_n} \right)}{\sum_{n=i+1}^{K} r_n \sum_{n=i}^{K} r_n}$$

Combining similar terms and using the definition of the tax intervals, $w_i < w_j$ for $j > i$, we immediately observe that

$$\sum_{n=i+1}^{K} r_n \frac{1}{w_i} - \sum_{n=i+1}^{K} r_n \frac{1}{w_n} = \sum_{n=i+1}^{K} r_n \left(\frac{1}{w_i} - \frac{1}{w_n} \right) > 0,$$

always holds, which ensures $\Delta_t > 0$, and, therefore, R_i strictly increases in i.

To study the aggregate function (10), consider its derivative with respect to t:

$$\frac{\partial T^{a}(t)}{\partial t} = \frac{\alpha}{\beta(1-t)^3} \sum_{n=i}^{K} r_n \left[\left(1-\frac{\lambda}{w_n}\right) - t\left(1+\frac{\lambda}{w_n}\right) \right] \quad (12)$$

From equation (12) it follows that the aggregate Laffer function's stationary points are determined by

$$t_i^0 = \frac{\sum_{n=i}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)}{\sum_{n=i}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)}, \; i=1,..,K \quad (13)$$

This results in a straightforward condition for the aggregate function to have a peak along a particular tax interval.

Condition 1: Aggregate Laffer function has a local peak at tax interval $t_{i-1} \le t < t_i$, if

$$t_{i-1} < \frac{\sum_{n=i}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)}{\sum_{n=i}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)} < t_i$$

With respect to Proposition 1 and inequality (11), the choice of

$$t_i = \varepsilon_i \frac{\sum_{n=i}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)}{\sum_{n=i}^{K} r_n}$$

is feasible if $\varepsilon_i \le 1$. Then

$$t_{i+1}^0 = \frac{\sum_{n=i+1}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)}{\sum_{n=i+1}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)} < t_{i+1} = \varepsilon_{i+1} \frac{\sum_{n=i+1}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)}{\sum_{n=i+1}^{K} r_n}$$

holds if

$$\varepsilon_{i+1} > \frac{\sum_{n=i+1}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)\sum_{n=i+1}^{K} r_n}{\sum_{n=i+1}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)\sum_{n=i+1}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)} = \frac{\sum_{n=i+1}^{K} r_n}{\sum_{n=i+1}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)} \quad (14)$$

Furthermore, if (14) holds, then according to Condition 1, we have a peak when

$$t_{i+1}^0 = \frac{\sum_{n=i+1}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)}{\sum_{n=i+1}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)} > t_i = \varepsilon_i \frac{\sum_{n=i}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)}{\sum_{n=i}^{K} r_n}$$

Based on these observations, we obtain the following condition:

Condition 2. Aggregate Laffer function has a local peak at tax interval $t_i \leq t < t_{i+1}$ if

$$\varepsilon_i < \frac{\sum_{n=i+1}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)\sum_{n=i}^{K} r_n}{\sum_{n=i+1}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)\sum_{n=i}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)} \quad \text{and} \quad \varepsilon_{i+1} > \frac{\sum_{n=i+1}^{K} r_n}{\sum_{n=i+1}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)}$$

Clearly if Condition 2 is met for M tax intervals, then the Laffer function has M peaks as stated in the next condition.

Condition 3. Aggregate Laffer function has $M \leq K$ local peaks at intervals $t_i \leq t < t_{i+1}$, $i=i_1, i_2,..,i_M$, $i_M < i_K$, if

$$\varepsilon_i < \frac{\sum_{n=i+1}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)\sum_{n=i}^{K} r_n}{\sum_{n=i+1}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)\sum_{n=i}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)} \quad \text{and} \quad \varepsilon_{i+1} > \frac{\sum_{n=i+1}^{K} r_n}{\sum_{n=i+1}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)}$$

for $i=i_1, i_2,..,i_M$.

Note that with respect to Proposition 1, there always exists a feasible choice of tax interval bounds so that the aggregate Laffer function will have a number of peaks. So far we have considered only the cases of internal peaks, i.e., when the stationary points (13) of the aggregate tax revenue are within their corresponding tax rate intervals, which is ensured by the proper choice of $\varepsilon_i \leq 1$. The wage distribution is normally highly skewed and has its own peaks, which can induce different choices for the tax rate intervals. In such a case, tax revenue peaks may extend out of the interval bounds. That is to say, that the maximum revenue will be observed at the bounds of some of the intervals rather than inside of them. Therefore a straightforward peak condition would be true if T^{i-1} increases for $t \leq t_{i-1}$ and T^i decreases for $t \geq t_{i-1}$. This, in terms of the corresponding stationary points, implies that $t^0_i > t_{i-1} > t^0_i$. The following proposition shows that this condition never holds.

Proposition 2.

Peak condition $t^0_{i-1} > t_{i-1} > t^0$ never holds.

Proof: Using (13) the condition $t^0_{i-1} > t_{i-1} > t^0$ of a local peak at point $t_i - 1$ transforms into

$$\frac{\sum_{n=i-1}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)}{\sum_{n=i-1}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)} > t_{i-1} > \frac{\sum_{n=i}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)}{\sum_{n=i}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)}$$

Thus, to show that this condition is not feasible, we need to prove that

$$\frac{\sum_{n=i-1}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)}{\sum_{n=i-1}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)} - \frac{\sum_{n=i}^{K} r_n\left(1-\frac{\lambda}{w_n}\right)}{\sum_{n=i}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)} < 0$$

is true. After combining similar terms of this inequality we find

$$\sum_{n=i}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)\sum_{n=i-1}^{K} r_n\left(1-\frac{\lambda}{w_n}\right) - \sum_{n=i-1}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)\sum_{n=i}^{K} r_n\left(1-\frac{\lambda}{w_n}\right) =$$

$$\sum_{n=i}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)\left[\sum_{n=i}^{K} r_n\left(1-\frac{\lambda}{w_n}\right) + r_{i-1}\left(1-\frac{\lambda}{w_{i-1}}\right)\right] -$$

$$\left[r_{i-1}\left(1+\frac{\lambda}{w_{i-1}}\right) + \sum_{n=i}^{K} r_n\left(1+\frac{\lambda}{w_n}\right)\right]\sum_{n=i}^{K} r_n\left(1-\frac{\lambda}{w_n}\right) =$$

$$2 r_{i-1}\lambda\sum_{n=i}^{K} r_n\left(\frac{1}{w_n} - \frac{1}{w_{i-1}}\right)$$

which is always negative, as $w_{i-1} < w_n$, for $n=i, i+1,..,K$.

From Proposition 2 it immediately follows that $t^0_{i-1} < t^0$ holds and, therefore, the only feasible boundary peak condition is due to a jump of $T^\alpha(t)$ at ti when $T^\alpha(t)$ increases at two consecutive intervals as stated in the following condition.

Condition 4. Aggregate Laffer function has a local peak at point t_i, if $t^0_i \leq t_i$ and

$$\lim_{t \to t_i - 0} T^\alpha(t) > T^\alpha(t_i).$$

Similar to Conditions 2 and 3, if Condition 4 is met for M tax intervals, then the Laffer function has M peaks at corresponding tax rate upper bounds as stated in the next condition.

Condition 5. Aggregate Laffer function has M local peaks at points t_i, $i=i_1, i_2,..,i_M$, $i_M<i_K$ if $t_i^0 \geq t_i$ and

$$\lim_{t \to t_i - 0} T^{\alpha}(t) > T^{\alpha}(t_i).$$

for $i=i_1,i_2,..,i_M$.

Note, that Conditions 1 and 4 are self exclusive implying that there can be only one peak at a tax interval either in between or on the upper bound of the interval, while any combination of both types of peaks at different tax intervals is possible.

NUMERICAL EXAMPLE

Consider the utility function (1) with parameters $\lambda = \frac{\gamma}{\alpha} = 200, \ \frac{\alpha}{\beta} = 10$

and typical for the US wage distribution over three standard classes:

$i=1$, lower class, with weight 80% (r_i=0.8) and wages from 0 to \$600, i.e., the average wage w_i=\$300;

$i=2$, middle class, with weight 15% (r_i=0.15) and wages from \$600 to \$1000, i.e., the average w_i=\$800; and

$i=3$, upper class, with r_i=0.05 and the average w_i=\$1500.

With respect to this distribution we select three tax rate interval bounds as

t_1=0.28, t_2=0.5 and t_3=0.8667.

Therefore the first tax rate interval to which all classes of individuals contribute will be $0 \leq t < 0.28$, the second, to which only middle and upper classes contribute is $0.28 \leq t < 0.5$ and finally $0.5 \leq t < 0.8667$ is the tax interval relevant only for upper class. Note, though t_3=0.8667 is obtained from non-negativity condition (11), i.e., $t_3 = 1 - \frac{200}{w_i}$, the results below would be still the same if we simply set t_3=1.

First, with the aid of equations (13), we find the following stationary points $t_1^0 = 0.267829$, $t_2^0 = 0.638225$ and $t_3^0 = 0.764706$.

Next, by comparing these points with the corresponding tax intervals we find that Condition 1 is met twice, yielding two internal peaks in tax revenues. Namely, since t_i^0= 0.267829< t_1=0.28, we conclude that t_i^0 = 0.267829 is a local peak of the aggregate Laffer curve at the first tax rate interval. According to (9) and (10) this peak is equivalent to 0.772754 tax revenue (tax revenues are calculated in billions dollars).

Similarly we observe that $t_2=0.5<t_3^0$= 0.764706 < t_3=0.8667 is met at the third tax interval, that is, we have a local peak at the stationary point t_3^0 = 0.764706 with tax revenue equal to 0.704167. Note, this peak provides less tax revenue than that found in the first tax interval which is based on contributions from all classes.

Next, comparing t_2^0 = 0.638225 and t_2=0.5, we observe that Condition 1 is not met and therefore the aggregate Laffer curve monotonously increases over the second tax rate interval with no peak in between. However, it is easy to verify that Condition 4 is met. Indeed, there will be a local peak when t tends to the bound, t_2=0.5 because t_2^0 = 0.638225 > t_2=0.5, and from (9) and (10) we determine that $\lim_{t \to t_2 - 0} T^{\alpha}(t)$ =1.11667> $T^{\alpha}(t_2)$=0.366667. Note that this is the third local peak and it is globally optimal with tax revenue of 1.11667. This result sustains a well-known fact that the major tax revenue is typically obtained from the maximum tax burden that the middle class is willing to bear. The lower wage group does not contribute to found maximum revenues, while the upper class contributes insignificantly.

It is also worth mentioning that since the classical Laffer function can take into account wage distributions only in a very rough manner, it may not provide any of the found local peaks especially that of a jump form which turns out to be globally optimal.

Indeed, using the example data we can calculate average wage $\widehat{w}$ as:

$$\widehat{w} = \sum_{i=1}^{K} r_i w_i = 0.8*300 + 0.15*800 + 0.05*1500 = \$435$$

Then equation (6) presents a straightforward, aggregate Laffer function which roughly takes into account wage distribution by setting $W = \widehat{w}$. Consequently, from equation (7) we find the tax rate of the single peak for this curve, t_0=0.37, which is different from all locally optimal peaks found above with the aggregate Laffer function.

Let us assume that the current average tax rate is high, t=0.45. Then, with respect to the straightforward aggregate curve (6), the tax rates could be cut dramatically towards the peak point t_0=0.37, for example by 10%, from 0.45 to 0.405. As a result, one would expect an increase in the tax revenue from 1.342 to 1.547, i.e., by 15.27%. However, more careful consideration of the wage distribution with the aggregate Laffer function (10) shows that the result will be the opposite! Namely, using equations (9) and (10) we find that the real tax revenue will dramatically decrease from 0.9793 to 0.856, which is 12.59%.

IMPLICATIONS AND CONCLUSIONS

The individualistic Laffer curve shape normally has a single peak point as illustrated by many economists.

The transformation from the individual curve to the aggregate Laffer curve does not necessarily lead to the same shape. Furthermore, using a backward bending labor supply curve, and a skewed wage distribution, we prove that under certain conditions the vertical summation of Laffer curves of different individuals will generate an aggregate curve with dual or even multiple peak values of tax revenue with respect to the number of tax rate intervals imposed on the taxpayers. These peaks can be observed both in between and on the upper bounds of the tax rate intervals. Thus, they can be of continuous and jump-wise form.

The scope of the results demonstrated in this paper is however much broader than the issue of the average tax rate and its effect on tax revenues. For example, it might occur that either a reduction or an increase in the tax rate at every marginal rate might yield an increase in tax revenue. This adds an additional dimension to the famous controversy between Martin Feldstein and Laffer in the mid- 80's regarding President Reagan's tax reduction policy. If the issue were merely how to collect tax revenues, we see that there is really a choice among a variety of tax rates that can yield the identical revenue, and not merely between two such rates as under the traditional single-peaked Laffer curve. Various tax rates will of course have varying affects on members of a population heterogeneous in wage and in potential earning ability. Some may pay more tax, and some may have a higher tax burden. Some will be encouraged to work more, and some will work less. Some will gain from the marginal tax change and others will be hurt.

Moreover, it is likely that the government may gain more tax revenue in both directions of the tax change, since at a given low tax revenue either a positive or a negative change in the tax rate may result in a move to one of two different local peaks. Even if the tax revenues at both peaks are equal, the government should not be indifferent amongst the two options. The issue may shift from that of tax revenues to that of the tax burden, i.e., on whom should the tax burden be imposed, on the middle income or even low-income groups or on the wealthy. The discussion on this issue must consider different aspects including political, psychological, and social issues such as equality and fairness, and not necessarily the simple fiscal economic question of how to finance the government's budget.

Last but not least is the issue of how changes in the tax rate actually affect the work-leisure tradeoff. Again we can show that a positive change in the tax rate may encourage wealthy people to work more, while middle or low wage rate individuals may react differently. This may occur if the supply curve of labor demonstrates a positive relationship between net wage and labor supply at low net wage rates, and the inverse effect for high wage rates.

NOTES

1 Since the Laffer curve first appeared on the scene scholars have based their research on the concept a single peaked Laffer curve. Examples of this abound throughout the literature. A few representative samples of this misguided belief include: Stuart (1981) who concludes that "Sweden is currently on the downward-sloping portion of its Laffer Curve"; similarly, Feige, and McGee (1983) "Sweden seems to have passed its Laffer peak"; Paulson and Adams (1987) who conclude that "imposing only an excise tax is worse than taxing on the down side of a Laffer curve"; Aasim (1997) who refers to the downward side of the Laffer curve as the wrong side of the tax Laffer curve; and similarly for Sanyal, Gang and Goswani (2000).

2 Contrary to recent articles that discuss more general models in which the use of the tax revenue is considered either to finance the supply of a public or private good (see Gahvari (1998)) or transfer payments, we for simplicity ignore these issues and assume that the government acts as a revenue maximizing firm and the consumer perceives his/her tax burden as simply that-a burden.

3 This we get by taking the derivative of (5)

$$\frac{dL}{d(W(1-t))} = \frac{-\acute{a}}{\hat{a}[W(1-t)]^2} + \frac{\breve{a}}{\hat{a}[W(1-t)]^3} = 0$$

4 The backward bending of the labor supply also exists for a specific value when marginal utility from leisure is increasing (and not necessarily constant as we assume in equation (1) above). The proof will be provided by the author upon request.

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