SINGLE VENDOR MULTI BUYER PRODUCTION REMANUFACTURING SYSTEM

BY

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This Thesis work is dedicated to my father, my mother and my wife.

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LIST OF ABBREVIATIONS

EOQ	:	Economic Order Quantity
RL	:	Reverse Logistics
SC	:	Supply Chain
CLSC	:	Closed Loop Supply Chain
JELS	:	Joint Economic Lot Size
EPQ	:	Economic Production Quantity
OEM	:	Original Equipment Manufacturer
IRRD	:	Individually Responsible and Rational Decision
MINLP	:	Mixed Integer Non Linear Programming
RFI	:	Ready For Issue
NRFI	:	Non Ready For Issue
SVMB	:	Single Vendor Multi Buyer
EU-ETS	:	European Union Emission Trading System
GHG	:	Green House Gas
GSCM	:	Green Supply Chain Management

ABSTRACT

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For many years used items are remanufactured and used along newly procured items to fulfill demand. These systems are known as closed loop supply chain (CLSC) systems and models are developed to study them. Economic lot sizing system is such system that received a lot of attention both in single stage and in a supply chain context. In recent years these models also consider the environmental aspect of remanufacturing/manufacturing activities in an attempt to make supply chains greener.

In this thesis, we deal with lot sizing decisions for a closed loop supply chain involving a single vendor and multi buyer, a situation that has not been considered so far in the literature. In particular, we propose two models. The first model, we develop a cost minimization model that determine the optimal manufacturing and remanufacturing lot sizes produced by the vendor and the size and number of shipment sent to each buyer. We develop such a model, present numerical results and conduct a sensitivity analysis to investigate the impact of key model parameters. The second model is an extension of the first model that take into account the carbon emission of production and transportation activities. A sensitivity analysis is considered as well.

ملخص الرسالة

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في سلاسل التوريد المغلقة يتم إعادة تصنيع المنتجات المستخدمة لعدة سنوات مع المنتجات الجديدة لتلبية الطلب .هنالك نماذج يتم تطويرها لدراسة هذه السلاسل. نظام تحديد حجم الدفع الاقتصادية هو أحد الأنظمة التي استرعت كثيرا من الانتباه في سلاسل التوريد. مؤخرا, أصبحت هذه النماذج تأخذ في الاعتبار العنصر البيئي في أنشطة التصنيع وإعادة التصنيع في محاول لجعل أنشطة سلاسل التوريد صديقة للبيئة.

في هذه الرسالة, نتعامل مع قرارات تحديد حجم دفعة الانتاج لسلسلة توريد مغلقة تتضمن بائعا واحدا وعدة مشترين, وهذه الحالة لم يتم التعامل معها من قبل في الدراسات السابقة.

على وجه التحديد, في هذه الرسالة نقترح نموذجين. النموذج الأول هو نموذج لتقليل التكاليف يحدد حجم الدفعات المثالية للتصنيع وإعادة التصنيع من قبل المصنع وكذلك حجم وعدد الشحنات المرسلة للبائع. نقدم هذا النموذج كما نعرض نتائجه ونقوم بدراسة الحساسية لمعرفة أثر العناصر الرئيسية في النموذج.

النموذج الثاني هو امتداد للنموذج الأول يأخذ بالاعتبار انبعاث الكربون الناتج من أنشطة الإنتاج والمواصلات . ويتم دراسة الحساسية لهذ النموذج أيضا.

CHAPTER 1

INTRODUCTION

1.1 Background

Supply Chains are complex as they involve many entities such as suppliers, manufacturers, distributors, and retailers. In a typical supply chain, raw materials are transferred from suppliers to a manufacturer where products are made. Products are produced in batches and then transferred from the manufacturer to retailers and then sold to buyers. This is also known as the forward supply chain. But when we introduce remanufacturing, it becomes backward supply chain also known as reverse logistics (RL). Integration of forward and reverse supply chain forms closed loop supply chain (CLSC). One of the most important issue in managing supply chains is inventory management

The concept of reverse logistics is not new. In the past reverse logistics was primarily managed considering the profit motive but now environmental legislative committees have also put some restrictions on waste disposals and impact on the environment such as green house gas emissions. With the introduction of RL researchers proposed quantitative models to study and analyze it. Most of the work in RL focused on deterministic models where the demand is known using EOQ-based models are the basic approach. Joint economic lot size (JELS) systems have gained much importance as they integrate both vendors and buyers. Finding batch sizes that are best for both vendor and buyer is a difficult task and that's why managing closed loop supply chains is more complex than forward supply chain. Moreover authors have used different approach and different assumptions for quantitative models that will be presented in the literature review chapter.

1.2 Closed loop supply chain

By definition RL is the reuse of products taken from the final destination for the purpose of capturing value and proper disposal. As discussed previously RL was not considered in the past. Industries, factories didn't pay attention to it but with the passage of time RL gained importance in terms of profit. More revenue generation was the prime objective of the interest in RL when first considered.

One of the important issues examined under the context of CLSC is remanufacturing. Remanufacturing is a process of transforming used products into 'like new' products that have the same or higher warranty and quality performance [1].

It is a valued-adding process that has economic and environmental benefits. From the environmental point of view, remanufacturing is predicted to conserve energy and raw materials. From economic and social perspectives, remanufacturing can reduce cost, create jobs, and boost company image.

Remanufacturing involves collection, disassembly, cleaning, sorting, repairing, reassembling, and testing. Quantitative models can be used to optimize remanufacturing activities and evaluate the benefits of various policies.

Many quantitative models have been made describing different scenarios and modeling assumptions. Modeling assumptions include demand rate, production rate, production rate, return rate, repair rate, maximum remanufacturing percentage allowed, number of times a product can be repaired, single or multiple buyers and single or multi item products etc. In this thesis, we will present our approach and methodology to deal with our remanufacturing activities.

1.3 Objectives

The main objective of the proposed work is to present mathematical models that can be used to optimize remanufacturing activities. In our case we consider a single vendor and multi buyers and their integration in a manufacturing/remanufacturing context.

The purpose of this thesis is to propose a single-vendor multi-buyer closed loop supply chain system operating under a centralized consignment stock policy. We develop appropriate mathematical models for this problem and illustrate them with several numerical examples followed by a sensitivity analysis to explore the effect of key model parameters.

The specific objectives of the proposed work are:

1. Development of a single-vendor multi-buyer closed loop supply chain system using different batch sizes for produced and remanufactured lots.
2. Consideration of a special case of the previous model using equal batch sizes for produced and remanufactured lots.
3. Extend the first model with the inclusion of carbon footprints cost to incorporate the environmental impact.
4. Conduct a sensitivity analysis of both models to study the impact of key model parameters.

1.4 Motivation and justification

Importance and practicality of supply chain models cannot be denied as first inventory model of CLSC was a real life case study of US Naval Supply Systems that stocks a number of moderate to high cost items. Further these inventory models gained importance when US Navy Inventory Control Program of repairable items was launched. CLSC model considering end life of vehicle treatment in Germany, problem of redesigning recycling system for liquid petroleum gas tanks in Netherland, case study of Llyn Beef producers Cooperative in Wales related to short food supply chains, consideration of supply chain optimization models and methods for Sodra forest company of Sweden and making a CLSC model due to abundant quantity of used ink cartridges in Hong Kong are the examples of remarkable importance of supply chain modeling in real life.

Remanufacturing models have been largely considered in a single stage context. Recently some researcher started considering remanufacturing in a supply chain context. However, to the best of our knowledge, only single vendor single buyer has been considered recently under the remanufacturing context. To fill this gap we deal in this thesis with the single vendor multi-buyer problem. We also consider the environmental impact of remanufacturing activities given the importance of green supply chain and the wide interest in the environmental impact and sustainability in general.

1.5 Research methodology

The above stated objectives will be achieved by performing following tasks. (1) by making closed loop diagram of whole CLSC system to understand the complete supply chain (2) by defining variables and notations of our system (3) we will draw the inventory profile at various stages where needed to determine inventory costs. Mathematical models will be developed for the two scenarios mentioned in the objectives. A sensitivity analysis will be conducted to study and evaluate the impact of key model parameters on the results of the developed models.

1.6 Organization of thesis

In chapter 2 we will discuss the relevant literature review of CLSC systems. We will focus on the deterministic quantitative models. The main points of each model will be highlighted variables will be discussed and analysis strategy will be presented. The proposed models are derived and presented in Chapter 3 including notation, decision variable, assumptions, and derivation of objective function component. In chapter 4 results and analysis of the proposed models will be presented and discussed. In chapter 5 we will conclude the work with discussion and provide directions for future research.

CHAPTER 2

LITERATURE REVIEW

2.1 Remanufacturing

The return of products from the customers to remanufacturing facility for the purpose of repair is known as remanufacturing. As discussed earlier remanufacturing was started because of environmental issues and economic benefits. Industries that started doing remanufacturing were considered under RL systems. Mathematical modeling of such systems in not new at this point and have gained huge focus of the authors. These quantitative models provided a better understanding of the inventory behavior and assisted in decision making processes.

In literature we will present the economic production quantity (EPQ), economic order quantity (EOQ) and joint economic lot size (JELS) models and mathematics involved in them. In the coming sections literature references are provided in three aspects i.e. firstly we will present references of vendor and buyer models, secondly we will present mathematical models of pure reverse logistics systems, thirdly environmental impact in vendor buyer models is discussed.

2.2 Review of vendor/buyer and RL models

There are many studies that presented vendor buyer models in past years. [2] presented a joint economic lot size model to find optimal costs for both vendor and buyer. [3] developed a model for establishing an optimal quantity discount model favoring vendor. [4] presented a good model focusing on manufacturer and its customer. Batches were produced by finite rate and then delivered to buyer. Products were consumed at fixed rate and main objective was to minimize the total average cost per unit time. The assumption taken was known demand to manufacturer or vendor. [5] found the limitation in the previous models that capacity was not once considered, so he found the optimal policy for single vendor and single buyer model with capacity constraint considered. [6] proposed philosophical approach for managing inventories in supply chains. The study was conducted to outline benefits by proper coordination of company and its suppliers. After proper study and analysis suppliers were allowed to stock inventory at warehouses and assure minimum and maximum level at them. [7] considered single vendor single buyer integrated production inventory problem where assumption of deterministic demand was relaxed. They assumed lead time is varying linearly with the lot size. Solution procedure with numerical example was also illustrated to show the benefits of integration of vendor and buyer. Sensitivity analysis was significant that explored the effect of key parameters on total cost. [8] considered integrated vendor buyer inventory system with controllable lead time and developed a heuristic solution to minimize total cost. [9] presented a model in which vendor and buyer collaborated to share benefits. Objective was to find out production and shipment policy to minimize overall cost. [10] removed the conception of greater holding cost of vendor or buyer. It could be greater for anyone and they concluded by finding optimal policy for production and shipment. One important thing in their work was that shortage at the buyer's side was allowed for the first time in modeling. [11] wrote a review paper indicating the benefits of coordination between vendor and buyer. Emphasize on supply chain management was linked with vendor buyer relationship.

The authors have put their work on single vendor and multi buyer models too. [12] presented quantity discount model in which price did not alter demand. Price was not only the decision making factor but pricing scheme was developed where buyer was attracted to order more. [13] again presented quantity discount model in which range of order sizes and prices were characterized which reduced costs for seller and buyer. [14] presented more practical approach named individually responsible and rational decision (IRRD) approach that is more economical than their previous joint economic lot size (JELS). First JELS model was formulated and then compared with IRRD model with same deterministic conditions. [15] considered stochastic conditions for a vendor and multi buyers dealing with single product. They showed implementation of this model is economical for both parties. [16] considered single vendor multi buyer model with the aim to minimize overall cost of vendor. For this purpose only thing that should be known to vendor is buyer's annual demand and order frequency. Optimal solution of this single vendor multi buyer model is obtained through heuristic approach. [17] analyzed the benefits of coordinating supply chain inventories through use of common replenishment epochs or time periods. For this purpose one vendor and multi buyer supply chain of a product is analyzed. A coordinated supply chain model of one wholesaler and multiple retailers is analyzed by [18] in which consignment policy was introduced to gain maximum benefits. The benefits were gained by combined pricing and lot sizing schemes. They also showed that separate pricing and lot sizing decisions were near optimal when the demand was very huge. [19] proposed new methodology to obtain joint economic lot size where multiple buyers were demanding one type of item from one vendor. Shipment policy to minimize cost of both vendor and multi buyers was also identified. Numerical example showed the illustration of the model and significant savings were obtained in joint settings. Sensitivity analysis also verified the durability of the model. Joint economic production, procurement delivery policy for multiple items in a single manufacturer, multi retailer system was presented by [20]. The model was formulated as lot scheduling problem in which objective was to find production sequence of multiple items, production cycle length, delivery frequencies and batch sizes that minimizes the total average cost. Heuristic approach was again used to find results of the model. [21] presented an interesting analytical model of single vendor and multiple buyer to find optimal replenishment decisions for both vendor and buyers. Results showed that consignment stock policy worked better than uncoordinated system of vendor and buyer. [22] identified issues in reverse supply chain after analysis of journal papers.

[23] expanded the previous approach to new one by introducing two shops in their model. First shop was for remanufacturing and production whereas second shop was for collection of used items which are to be supplied to first shop for remanufacturing. They showed none of the extreme strategies are perfect to minimize cost i.e. pure production and no remanufacturing, pure remanufacturing and no production. A mixed strategy of producing and remanufacturing items gave the optimal cost. [24] reviewed the closed loop supply chain (CLSC) literature in detail and identified pros and cons of it. The authors presented the parameters that affect the CLSC more severely and identified the opportunities where research can be done. Paper that discussed opportunity of recovery parts was presented by [25]. Two stage spare parts supply chain was analyzed where independent repair shops are responsible for handling repair process. Original equipment manufacturer (OEM) preferred the use of new products and repair shops achieved larger profit by repairing. This paper contributed simple deterministic framework to answer whether buyback option should be

offered by OEM to repair shops or not. [26] proposed remanufacturing model with the introduction of learning costs. According to the authors the previous models are not always true when the labor costs and learning costs are high. For this mathematical models were developed, numerical examples were done and results were provided. [27] expanded the literature by presenting a single product case where constant demand is satisfied by producing new or repairing old ones. But they indicated a limitation in the previous models that sub-assemblies are not treated separately. When the used products are collected they are disassembled first then returned to repair facility where subassemblies are again assembled to make a product. They are regarded as good as new. They showed the results of their model which indicated that managing subassemblies separately is sound. Model in which produced and repaired components are dealt separately means the quality was not considered the same rather incompatible was proposed by [28]. That resulted in lost sales situation and author also gave permission to manufacturer for fully or partially backordering. Mathematical models of this situation were developed and numerical examples provided the results. A more general model was developed by [29] that divided the RL system into three segments. The first segment is for the production of new items, second segment is for collecting the returned products while third segment is for the remanufacturing of returned products. In the model it is assumed that stored item gets deteriorate during storage period and has some salvage value. The model is solved by minimizing the inventory cost of deteriorating items where demand and deterioration are both function of time. Single manufacturer and single supplier model was suggested by [30] in which sustainability is treated as quality attribute and could be investigated through investing in production process. It was shown at the end that profit was higher and quality was lower in the integrated model than nonintegrated model. A model in which product may have two standards was proposed by [31]. They considered closed loop supply chain and according to them repaired items are not as good as new. Due to this assumption shortage occurred and demand is completely backlogged. Numerical example with sensitivity analysis is provided at the end. [32] presented a RL model with concept of two inventories. One of the serviceable stock and other of the collected items. Objective was to minimize the overall cost of system with the addition of dynamic policy. The dynamic optimal policy that varies with time was shown better than static optimal policy with the help of numerical example. [33] presented model with time decay and shortages introduction. They extended approach of Wagner-Whitin to determine lot sizes, replenishment cycle and schedules. Sensitivity analysis was conducted to justify the advantages. A product recovery model was discussed by [34] in which demand rate for the new and remanufactured products is deterministic and constant. Objective was to find the minimum total cost of this system. They first developed lower bound among all classes of policies for the problem and also discovered that optimal integer ratio policy obtained a solution whose cost is at most 1.5% more than lower bound. [35] discussed inventory of repairable items is important as it affects the decisions of whole supply chain. It used continuous review inventory policy plus production and remanufacturing in basic inventory model. Dynamic programming approach was used for solving the model and analysis was provided to see the behavior of model. Continuous time varying demand model with finite planning horizon was proposed by [36]. In this model it is assumed that new and repaired items are used to fulfill the demand and objective is to find joint policy for raw materials that are procured, new items that are fabricated and used items that are repaired such that total cost of model is minimized. A consignment stock policy model was discussed by [37] in which a two level model is made by adding retailer to it. Model was formulated for two cases when batch sizes of production and remanufacturing are equal and when they are different. The results showed that repair rate and collection rate have significant effect on total cost of system. [38] proposed a simple reverse logistics model by presenting a case study of plastic bottles. The theme of paper is that used plastic bottles are collected and are inspected for repair. The repairables are repaired and the other ones are sold to industries that use low grade plastic. Mathematical model is developed for the above scenario. [39] presented a review paper of RL models after analyzing 382 papers between 2007 and 2013. Another literature review paper was proposed by [40]. Total of 242 papers were considered in it and it tried to identify the gaps in literature to suggest for future research. [41] has proposed a VMI model with fuzzy demand for single vendor and multiple retailers in which the centroid defuzzification method defuzzifies trapezoidal fuzzy numbers of demand. Number of orders and warehouse space are considered as constraints whereas replenishment rate of retailers, maximum backordered quantities of retailers and items price are considered as decision variables. [42] reviewed literature in detail and discussed inventory systems that are based on EOQ, EPQ and JELS settings. Literature is divided and classified according to issues faced and assumptions of models. Special attention is given to the idea of green supply chain in inventory models.

2.3 Review of mathematics of RL systems

2.3.1 Schrady's model

All the work that has been done till today traces its idea from the work of [43]. Model of Schrady was the first one that quantized the RL in supply chain. It is a simple model that determines the economic order quantity for production and repair jointly. Model is described in such a way that one stock point has RFI (ready for issue) inventory that is sold to customers. Used items are brought back to one stock pint that has NRFI (not ready for issue) inventory. They are repaired here and supplied to RFI point.

Notations

Q_p Procurement quantity

Q_r Repair batch size

d Demand rate

$1-r$ Scrap rate

r Recovery rate

τ_p Procurement lead time

τ_r Repair lead time

A_p Fixed procurement cost per batch

A_r Fixed repair cost per batch

h_1 RFI holding cost per unit per unit time

h_2 NRFI holding cost per unit per unit time

T Cycle time

Total cost function

Total cost of the system is given by;

$$TC = \frac{A_p d(1-r)}{Q_p} + \frac{A_r rd}{Q_r} + \frac{h_1 r}{2}\left(Q_r + \left(\frac{1-r}{r}\right)Q_p\right) + \frac{h_2 r}{2}\left(Q_p + Q_r\right)$$

The decision variables are Q_r and Q_p. Total cost is function of procurement order cost, induction cost, holding cost of RFI items and NRFI items. Numerical example is provided to see the working behavior of model.

2.3.2 Richter's model

[44] presented a two stage deterministic EOQ model in which 1st shop is for manufacturing and remanufacturing and the 2nd shop is for the collection of used products. Assumption is first remanufacture products and then produce the new ones to fulfill demand. Manufacturing and remanufacturing setups are different and different cost is associated with them. In the 2nd shop items are collected all the time during a period.

Notations

x Lot size

r Repair setup cost

k Unit cost of repaired item

s Manufacturing setup cost

b Unit cost of manufactured item

h Holding cost of repaired and manufactured items

μ Holding cost of used products

e Unit cost of wasted item

β Repair rate

α Disposal rate i.e. $\alpha + \beta = 1$

Total cost

$$TC = (mr + ns) + \frac{h}{2d}\left(\frac{\alpha^2 x^2}{n} + \frac{\beta^2 x^2}{m}\right) + \frac{\mu\beta Tx}{2} + \frac{\mu\beta^2 x^2 (m-1)}{2dm}$$

Where total cost is summation of fixed cost, holding cost of new and used products and holding cost of used products in first and second shop. The decision variables are x, m, n and α. The objective of model is to trace behavior of optimal solution by changing waste disposal rates.

2.3.3 Zavanella, Zanoni's model

[21] presented a single vendor multi buyer model with the concept of different batch sizes for each buyer. Reverse logistics is not considered in the model but coordination scheme is between vendor and buyers is discussed. Optimal solutions are found using integrated and sequential approach.

Objective of model is to minimize the stock held at vendor by shipping all the stocks whenever a delivery is ready for transportation. Shipments being made are of equal sizes for a particular buyer but different for Y buyers. Batches are produced for first buyer and delivered to him, then for second buyer and it goes on till last buyer gets his all shipments.

Notations

A_1 Setup cost of vendor

$A_{2,i}$ Ordering cost of ith buyer

h_1 Holding cost of vendor per item per unit time

$h_{2,i}$ Holding cost of ith buyer per item per unit time

P Vendor production rate

d_i Demand rate of ith buyer

Y Number of buyers

T Ordering or production cycle time

n_i Number of shipments of ith buyer

q_i Quantity delivered to ith buyer per shipment

TC Average total cost

Assumptions of the model include single product case where $P > D$, $h_{2,i} < h_1$ and $D = \sum_{i=1}^{Y} d_i$

Total cost of system is summation of vendor and all buyers cost.

Total cost

$$TC = \frac{1}{T}\left(A_1 + \sum_{i=1}^{Y} n_i A_{2,i} \right) + \frac{T}{2}\sum_{i=1}^{Y}\left(\left(h_1 + h_{2,i}\right)\frac{d_i^2}{n_i P} + h_{2,i} d_i \left(1 - \frac{d_i}{n_i P}\right)\right)$$

Sequential solution suggests that consider only vendor's cost and find optimal cycle time by taking derivative of it. Find optimal number of shipments for *ith* buyer by taking derivative of ith buyer total cost. Sensitivity analysis showed that joint optimal solution is better while varying the ratio of holding costs. The results also showed that lower the D/P larger the benefits for the supply chain.

2.3.4 Jaber, Zanoni's model

[37] presented a consignment stock model in which RL system is discussed with the addition of buyer in the model. Model is made for the two cases i.e. when the production and remanufacturing batches have equal sizes and when they have different sizes. Model is also expanded by considering transportation, inspection and sorting costs. Produced and remanufactured items are gathered at one stock point at vendor side. They all are shipped to buyer's side and are consumed according to demand. Some of the used items are collected and inspected. Wasted items are disposed and remaining are considered as recoverable, which are then finally sent for remanufacturing. This is how the model works and cycle continues. Assumptions of the model include that remanufactured items are considered as good as new, shortages are not allowed, demand rate is constant, lead time is zero for a single product case.

Notations

S_i Setup cost for process *i;* r = remanufacturing, p = production, s = sorting, u = used, w = disposal

h_u Holding cost for used items

c_i Unit cost for processes

c_u Cost for collecting used item

c_w Cost for disposing an item

h_r Holding cost for remanufactured item

h_p Holding cost for manufactured item

h_b^b Buyer's holding cost of item at buyer's side

h_b^v Vendor's holding cost of item at buyer's side

h_b Total holding cost of item at buyer's side

D Demand rate

P_i Production, remanufacturing or inspection rate for process *i*

q_i Shipment size for process *i*

τ_j Unit transportation cost

m Number of remanufactured batches

n Number of production batches

T Length of time interval

ρ Collection percentage

α Disposal percentage

β Repairable percentage

Decision variables are m, n, q_r, q_p and β.

Total cost: Case: $q_r \neq q_p$

$$TC = \frac{D(1-\rho\beta)}{nq_p}\left[mS_r + nS_p + nq_p\left(c_r \frac{\rho\beta}{1-\rho\beta} + c_p \right) + \frac{q_p^2}{2}\left\{ h_r \frac{m}{P_r}\left(\frac{n\rho\beta}{m(1-\rho\beta)} \right)^2 + h_p \frac{n}{P_p} \right\} \right]$$
$$+h_b \frac{n\rho^2\beta^2 D^2 q_p}{2m(1-\rho\beta)P_r^2} + h_b \frac{(n-1)(1-\rho\beta)D^2 q_p}{2nP_p^2} + h_b \frac{n(m+1)\rho^2\beta^2 D q_p}{2P_r m(1-\rho\beta)}\left(1-\frac{D}{P_r}\right)$$
$$+h_b \frac{n(n-1)\rho^2\beta^2 D q_p}{m(1-\rho\beta)P_r}\left(1-\frac{D}{P_r}\right) + h_b \frac{(n-1)(1-\rho\beta)\rho D q_p}{2P_p}\left(1-\frac{D}{P_p}\right) +$$
$$\frac{h_b \rho(1-\rho\beta)q_p}{2n}\left[n\frac{\rho\beta}{1-\rho\beta}\left(1-\frac{D}{P_r}\right) + n - (n-1)\frac{D}{P_p} \right]^2 + \frac{S_s D(1-\rho\beta)}{nq_p}$$
$$+(c_u + c_s)\rho D + e_w \rho\alpha D + h_u \frac{\rho}{2}\left(1-\frac{D\rho}{P_s}\right)\left(1+\frac{D\rho}{P_s}\right)\frac{nq_p}{1-\rho\beta} +$$
$$(m+n+1)\frac{AD(1-\rho\beta)}{nq_p} + \tau_F D + \tau_R \rho\beta D + h_u \frac{\rho^2\beta^2 n q_p}{2(1-\rho\beta)}$$

Where total cost is integration of vendor costs, buyer's holding cost, inspection cost, transportation cost and holding cost of repairable items. Vendor's costs include setup cost, production cost, remanufacturing cost and holding cost. The total cost function for the other case can be found by putting $q_r = q_p = q$.

2.4 Review of carbon footprint references

Carbon emissions are increasing day by day and are affecting the climate severely. Carbon emissions are even gaining public interest due to future health and climate problems.

Environmental legislative committees have put strict conditions and penalties on industries and business sectors on exceeding carbon emission limit. The European Union Emission Trading System (EU-ETS) is mainly active in this particular scenario since 2005 and putting its best efforts to reduce carbon emissions. The studies are conducted to factorize the cost of greenhouse gas (GHG) emissions. Generally carbon emissions in supply chain are through production and transportation.

Bogaschewsky in 1995 provided the relationship for estimating carbon emission from per item produced which is used by many authors recently. According to [45] organizational benefits and environmental concerns are main issues of the era. Current practices and future requirements of environment friendly organizations are discussed in this work. With the increasing interest in carbon free environment [46] provided a decision framework based on literature and practices. Focus of the work is on elements of green supply chain and their impact on decision framework. [47] used econometric techniques to develop price of emission permits. The results showed that price is affected by trading rules and regulations. Relationship between the green supply chain management and practice was discovered by [48]. It also investigated that how optimization techniques of manufacturing affect the above relationship. Phenomena of resemblance between production systems and physical systems was presented by [49]. The authors concluded that entropy of system can be reduced through first and second law of thermodynamics. For illustration economic order quantity model was used. Evaluation of impact of carbon tax on manufacturing sectors in Greece was done by [50]. The results showed that carbon tax on Greek industries was costly but beneficial for the environment. [51] presented a study showing the behavior of certain costs when the objective is to minimize the overall cost of system. The optimal inventory policy was also compared before and after introducing tradable permits. [52] discussed the adoption of green supply chain management (GSCM) practices in Chinese organizations. Survey was conducted with some propositions and results were compared with previous findings. Analysis showed that Chinese organizations had not properly adopted GSCM practices. Different pollution reduction schemes were highlighted in the study by [53] and the author discussed that most efficient pollution reduction technique cannot be determined because it depends on the ambition of target. [54] focused on collaboration between the companies and their suppliers in order to promote the importance of environmental behavior for each entity of supply chain. Their work provided unique methodology to help managers in evaluating supplier's performance in terms of green supply chain. A study to help policy makers in selecting the method of carbon reduction was presented by [55]. Authors have showed cost curves indicating cost of each available approach. [56] presented a case study in which industry can obtain aluminium in liquid or solid form. Two things were under consideration i.e. transportation of aluminium and its environmental aspects. The study proposed a model for evaluating economic and environmental effects. [57] organized and presented the literature on green supply chain management. The motivation of this work was to help researchers and practitioners to get classified review on the basis of approach and methodology. [58] greatly emphasized the significance of integration of environmental aspect into logistics and inventory systems. [59] presented a neutral study in favor of carbon tax and argued the approaches against tax. [60] used Lagrangian and Eulerian transport methods and showed that carbon emissions are great threat if not considered while designing supply chain. Again [61] examined the importance of inventory planning to the environment in detail. [62] presented work to help supply chain managers in integrating carbon footprint cost in supply chain. It presented its study after continuous interaction with the Hyundai Motor Company (HMC). [63] investigated that how firms manage carbon footprints in inventory management under carbon emission trading scheme. Environmental impact is incorporated in two level supply chain by [64]. Fixed and variable carbon costs were considered while finding optimal production and shipment policy. Sustainable order quantity model was formulated by [65]. This was a multi-objective model and results showed the effectiveness of different carbon emission schemes. [30] proposed a two level quantitative supply chain model in which sustainability was measured in terms of carbon emissions. The behavior of model was analyzed by varying the production rate and results showed that more customers can be attracted by controlling emissions. [66] designed mix integer linear programming model for the sustainable supply chain and suggested that environmental legislative committees must be strengthened for productive environmental mechanism. [67] presented a two level supply chain considering the carbon emission cost from manufacturing. The study is helpful for those who want to reduce inventory and carbon costs of system. [68] used second law of thermodynamics to calculate entropy cost of supply chain. They also compared some previous models when entropy cost was included. A single product case with two level supply chain was considered by [69]. All typical costs were considered by the authors with the addition of carbon footprint cost. They found optimal order quantity, optimal number of shipments between vendor and buyer and production rate that minimized total cost. [70] considered a carbon aware company that would like to reselect transportation mode for the delivery of items with the introduction of carbon emission cost. Different carbon emission techniques were observed and results showed that actual decision still based on other parameters such as lead time variability. A good study of cold items with emission functions was done by [71]. Optimal order quantity was found using set of algorithms for cost function and emission function optimization. Finally the analysis was performed to gain insights. [72] dealt with the change in price of fuel and its impact on total supply chain cost. A function was developed to find fuel price in future, which was then used to calculate transportation cost of future. Overall analysis showed that organizations having suppliers nearby are at advantageous positions and will be effective in future. [73] presented a model with an approach to highlight the environmental issues that arise from production and transportation of products in RL system. Main idea of the study revolved around evaluating supply chain implications presented in RL setting. [74] aimed to estimate greenhouse gas emissions by formulating mathematical models that consider delivery scheduling for time dependent demand of multi temperature foods. The paper finally analyzes the emissions to highlight influence of delivery scheduling on emissions. [75] discussed multi echelon production inventory model with carbon emissions and lead time constraints. Items that are not delivered to customer on time are lost sales. Carbon emissions are considered from manufacturing process, transportation and storage of inventory at different stages of SC. The model provided interesting insights for decision makers. [76] paid attention to carbon emissions from cooking systems. This work revealed that choosing an environmental type of cooking methods, fuel and cookware are beneficial in reducing carbon emissions from cooking unit. [77] developed inventory model in which demand depends on length of credit period offered by retailer to its customers. Sensitivity analysis is conducted at the end to see features of model. [78] provided a concise and readable summary of latest research in CLSC field. The emphasis throughout this book is on business practices that are environmentally friendly and profitable. A mixed integer programming model is presented in [79] with the inclusion of capacity, transportation, inventory costs and global warming impact. Lagrangian decomposition principle is used and at the end some insights are drawn from analysis. [80] developed multi product inventory model for cold items and determined the inventory levels that minimize costs of emissions. [81] presented a two level closed loop supply chain scenario and discussed the behavior of two models. Energy usage, carbon emissions and number of times to remanufacture were the three main attributes of this paper. [82] emphasized on designing for environment. According to the authors industries ignoring the environmental aspect still at this stage will find it increasingly difficult to compete in the coming century.

CHAPTER 3

MODELS DESCRIPTION

3.1 Introduction

In this chapter we derive the proposed mathematical inventory models. We will present a CLSC scenario with single vendor and multi buyers. Two models are proposed with two sets of costs. The first model considers setup cost, production cost, holding cost of vendor, ordering cost, holding cost of buyers and holding cost of used items. The second model is the extension of first model with the consideration of carbon emission costs

3.2 Contribution of our models

As discussed in the literature review closed loop supply chain lot sizing with remanufacturing was considered for the single vendor single buyer case only. A major contribution of our work is to extend this line of research to the single vendor multi buyer case. Furthermore a thorough sensitivity analysis is conducted to investigate the effect of key parameters for both proposed models. Some insights are drawn as well.

3.3 Notations

D Annual rate of demand where $D = \sum_{i=1}^{I} d_i$

d_i Annual rate of demand of ith buyer

P_p Annual rate of production

P_r Annual rate of remanufacturing

A_p Production setup cost

A_r Remanufacturing setup cost

A_i Ordering cost of ith buyer per batch

h_v Holding cost of produced or remanufactured item at vendor

h_i Holding cost per item at ith buyer

q_i Batch size of ith buyer

$q_{i,r}$ Remanufacturing batch size for ith buyer

$q_{i,p}$ Production batch size for ith buyer

Q Production/remanufacturing batch size where $Q = \sum_{i=1}^{I} q_i$ (only for equal Q case)

Q_p Production batch size where $Q_p = \sum_{i=1}^{I} q_{i,p}$

Q_r Remanufacturing batch size where $Q_r = \sum_{i=1}^{I} q_{i,r}$

u_r Unit cost of remanufactured item

u_p Unit cost of produced item

n Number of production batch sizes

m Number of remanufacturing batch sizes

α Return percentage

SC Setup cost per unit of time

PC Production cost per unit of time

HC_v Holding cost of vendor per unit of time

HC_u Holding cost of used products per unit of time

OC_{ib} Ordering cost of *ith* buyer per unit of time

HC_{ib} Holding cost of *ith* buyer per unit of time

TC_v Total cost of vendor per unit of time

TC_{ib} Total cost of *ith* buyer per unit of time

TC Total cost of system per unit of time

T Cycle time

3.4 Assumptions

- P_r and P_p are greater than D.
- Remanufactured items are considered to be as good as new.
- Shortages are not allowed
- Demand rate is constant over time
- Single product case

- Product recovery case
- Single vendor, single market and multi buyers
- Remanufacturing and production processes are always in control and no defective items are produced.

3.5 Single vendor multi-buyer remanufacturing model

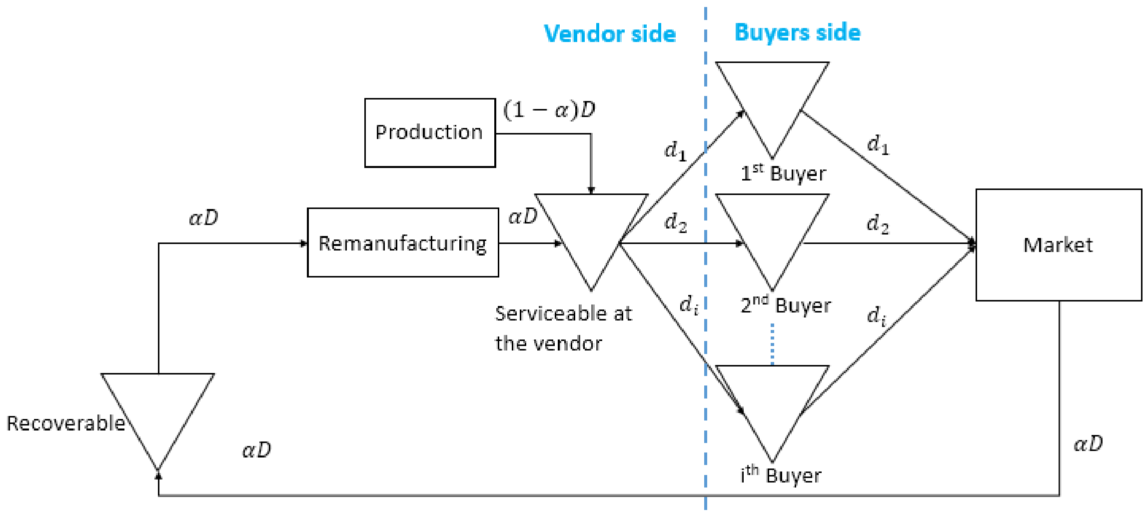

Figure 1 Single vendor multi buyer production remanufacturing model

Single vendor multi buyer closed loop supply chain system is presented in Figure 1. Demand is fulfilled from produced and remanufactured items at vendor side. Each batch of production and remanufacturing is delivered to all buyers at the same time and buyers sell them in market. Some portion of used items is collected and are stored at recoverable point. All of the collected items are considered recoverable. These items are then sent to remanufacturing facility. The vendor first satisfies buyer demand by remanufacturing and then by production. There are m batches of remanufacturing and n batches of production where $P_r \neq P_p$. Remanufacturing is always done prior to production according to the literature as it is more economical.

When vendor prepares a batch size of Q_r then *ith* buyer gets a batch size of $q_{i,r}$ and when vendor prepares a batch size of Q_p then *ith* buyer gets a batch size of $q_{i,p}$. We will discuss a special case of equal batch size for vendor during calculation of each cost i.e. $Q_r = Q_p = Q$. For special equal *Q* case of vendor, *ith* buyer gets a batch size of q_i.

3.5.1 Vendor's cost functions

3.5.1.1 Setup cost

Setup cost of production and remanufacturing is incurred only once per cycle. So setup cost per unit of time is;

$$SC = \frac{A_r + A_p}{T} \tag{3.1}$$

Case: $Q_r = Q_p = Q$

For this case the above cost remains same.

3.5.1.2 Production cost

Q_r is produced *m* times and Q_p is produced *n* times. When cost of remanufactured item is u_r and cost of produced item is u_p then production cost per unit of time is;

$$PC = \frac{u_r m Q_r + u_p n Q_p}{T} \tag{3.2}$$

Case: $Q_r = Q_p = Q$

$$PC = \frac{u_r mQ + u_p nQ}{T} = \frac{Q}{T}(u_r m + u_p n) \tag{3.3}$$

3.5.1.3 Holding cost of vendor

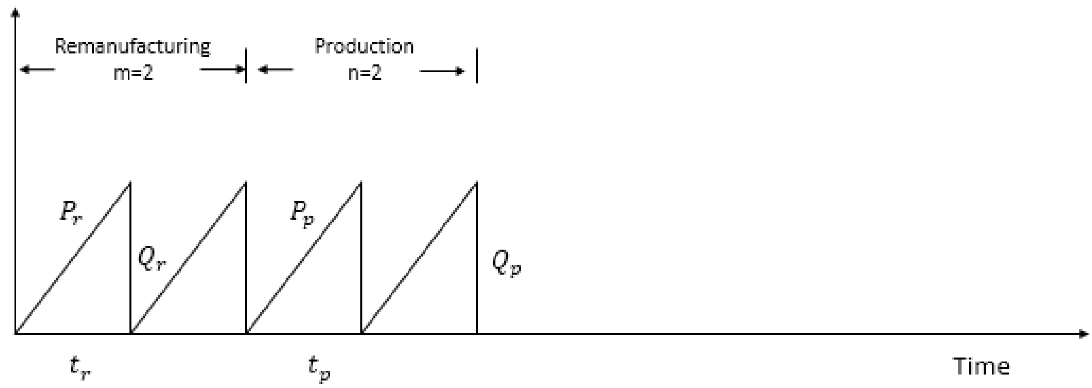

Figure 2 Vendor's inventory profile

For holding cost keep following things in mind. There are m triangles of remanufacturing and n triangles of production (see Figure 2). Each remanufacturing triangle has base t_r and height Q_r, so area of m such triangles will be $m\frac{1}{2}t_rQ_r$. Similarly area of n triangles will be $n\frac{1}{2}t_pQ_p$. Now vendor's holding cost per unit of time is;

$$HC_v = \frac{h_v}{T}\left(\frac{mt_rQ_r}{2} + \frac{nt_pQ_p}{2}\right) = \frac{h_v}{2T}\left(\frac{mQ_r^2}{P_r} + \frac{nQ_p^2}{P_p}\right) \tag{3.4}$$

Case: $Q_r = Q_p = Q$

$$HC_v = \frac{h_v}{T}\left(\frac{mt_rQ}{2} + \frac{nt_pQ}{2}\right) = \frac{h_vQ}{T}\left(\frac{mt_r}{2} + \frac{nt_p}{2}\right) = \frac{h_vQ^2}{2T}\left(\frac{m}{P_r} + \frac{n}{P_p}\right) \tag{3.5}$$

3.5.1.4 Total cost of vendor

$$TC_v = SC + PC + HC_v$$

$$TC_v = \frac{A_r + A_p}{T} + \frac{u_r m Q_r + u_p n Q_p}{T} + \frac{h_v}{2T}\left(\frac{mQ_r^2}{P_r} + \frac{nQ_p^2}{P_p}\right) \tag{3.6}$$

Case: $Q_r = Q_p = Q$

$$TC_v = \frac{A_r + A_p}{T} + \frac{Q}{T}(u_r m + u_p n) + \frac{h_v Q^2}{2T}\left(\frac{m}{P_r} + \frac{n}{P_p}\right) \tag{3.7}$$

3.5.2 *ith* buyer cost functions

3.5.2.1 Ordering cost

Ordering cost for the *ith* buyer occurs only once during a cycle for m batches of remanufacturing and n batches of production. Ordering cost per unit time is;

$$OC_{ib} = \frac{(m+n)A_i}{T} \tag{3.8}$$

Case: $Q_r = Q_p = Q$

Above equation also holds for this case.

3.5.2.2 Holding cost of *ith* buyer

Each time a batch of Q_r is ready at vendor it is distributed between all buyers with each buyer receiving a batch size of $q_{i,r}$ (remanufacturing batch size for *ith* buyer), similarly when a batch size of Q_p is ready it is distributed between all buyers with each buyer receiving a batch size of $q_{i,p}$. Buyer starts consuming each batch by demand rate d_i and his inventory is increased upon arrival of next batch as shown in Figure 3. This saw tooth curve continues until buyer gets the last batch. Cycle for buyer is finished when he has consume the last batch and its inventory level gets zero.

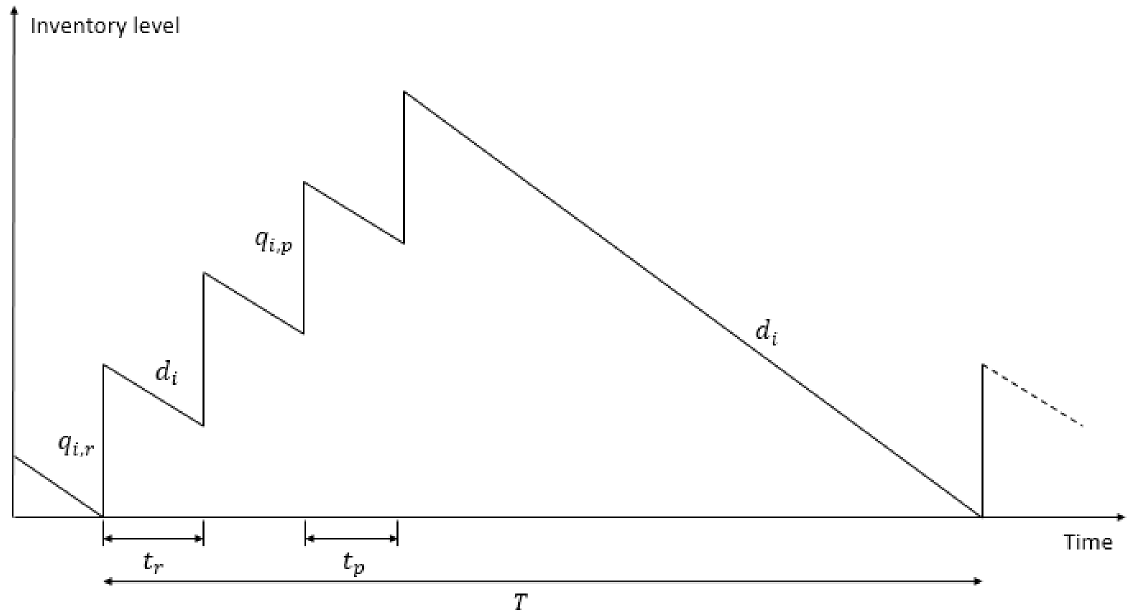

Figure 3 Inventory profile of *ith* buyer

There are m trapezoids of remanufacturing, $n-1$ trapezoids of production and one big triangle. While calculating holding cost of buyer some authors have miscalculated the *mth* trapezoid area of remanufacturing. Notice that *mth* trapezoid has base t_p so holding cost of *ith* buyer can be calculated by finding area under the curve in Figure 3. Holding cost per unit time is;

$$HC_{ib} = \frac{h_i d_i}{2DT}\left(\frac{\left(mQ_r + nQ_p\right)^2}{D} - \frac{Q_r(m-1)\left(mQ_r + 2nQ_p\right)}{P_r} - \frac{n(n+1)Q_p^2}{P_p}\right) \quad (3.9)$$

Case: $Q_r = Q_p = Q$

$$HC_{ib} = \frac{h_i d_i Q^2}{2DT}\left(\frac{(m+n)^2}{D} - \frac{(m-1)(m+2n)}{P_r} - \frac{n(n+1)}{P_p}\right) \quad (3.10)$$

3.5.2.3 Total cost of buyer

$$TC_{ib} = OC_{ib} + HC_{ib}$$

$$TC_{ib} = \frac{A_i(m+n)}{T} + \frac{h_i d_i}{2DT}\left(\frac{\left(mQ_r + nQ_p\right)^2}{D} - \frac{Q_r(m-1)\left(mQ_r + 2nQ_p\right)}{P_r} - \frac{n(n+1)Q_p^2}{P_p}\right) \tag{3.11}$$

Case: $Q_r = Q_p = Q$

$$TC_{ib} = \frac{A_i(m+n)}{T} + \frac{h_i d_i Q^2}{2DT}\left(\frac{(m+n)^2}{D} - \frac{(m-1)(m+2n)}{P_r} - \frac{n(n+1)}{P_p}\right) \tag{3.12}$$

3.5.3 Holding cost of repairable items

All collected used items are considered repairable. The repairable items inventory decreases at a constant rate of $P_r - \alpha D$ during remanufacturing and increases at a constant rate of αD during production.

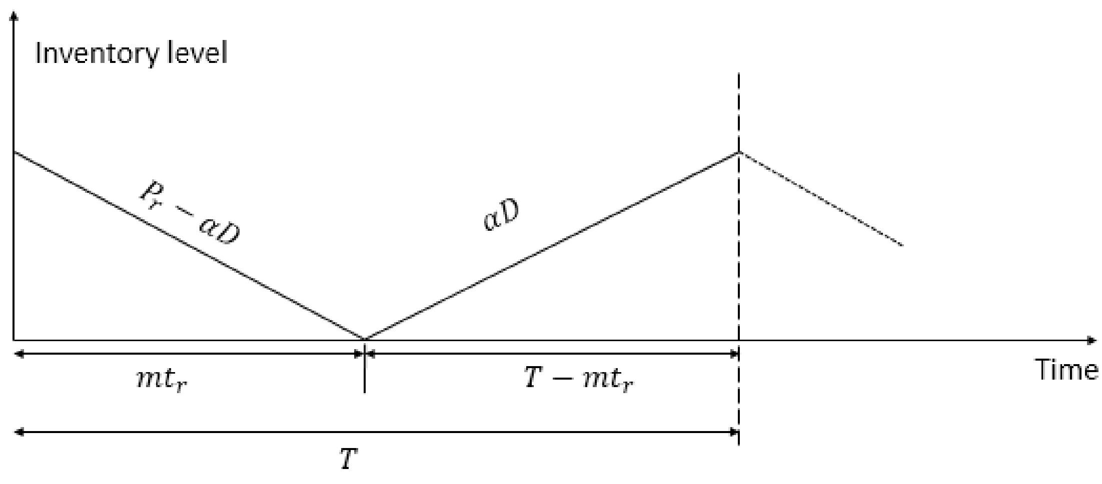

Figure 4 Inventory profile of repairable items

Inventory behavior of repairable items can be seen in Figure 4 and holding cost of these items per unit of time is;

$$HC_u = h_u\left(\frac{mQ_r(P_r - \alpha D)}{2P_r}\right) \tag{3.13}$$

Case: $Q_r = Q_p = Q$

$$HC_u = h_u\left(\frac{mQ(P_r - \alpha D}{2P_r}\right) \tag{3.14}$$

3.5.4 Total cost of system

$$TC = TC_v + HC_u + \sum_{i=1}^{I} TC_{ib}$$

$$\begin{aligned} TC = {} & \frac{A_r + A_p}{T} + \frac{u_r mQ_r + u_p nQ_p}{T} + \frac{h_v}{2T}\left(\frac{mQ_r^2}{P_r} + \frac{nQ_p^2}{P_p}\right) + \frac{h_u mQ_r(P_r - \alpha D)}{2P_r} + \\ & \sum_{i=1}^{I}\left(\frac{A_i(m+n)}{T} + \frac{h_i d_i}{2DT}\left(\frac{\left(mQ_r + nQ_p\right)^2}{D} - \frac{Q_r(m-1)\left(mQ_r + 2nQ_p\right)}{P_r} - \frac{n(n+1)Q_p^2}{P_p}\right)\right) \end{aligned} \tag{3.15}$$

Where $T = \dfrac{mQ_r}{\alpha D}$ and $nQ_p = mQ_r\left(\dfrac{1-\alpha}{\alpha}\right)$ so

$$\begin{aligned} TC = {} & \frac{(A_r + A_p)\alpha D}{mQ_r} + (u_r\alpha + u_p(1-\alpha))D + \frac{h_v DQ_r}{2}\left(\frac{\alpha}{P_r} + \frac{m(1-\alpha)^2}{P_p n\alpha}\right) + \frac{h_u mQ_r(P_r - \alpha D)}{2P_r} \\ & + \sum_{i=1}^{I}\left(\frac{A_i\alpha D(m+n)}{mQ_r} + h_i d_i Q_r\left(\frac{m}{2\alpha D} - \frac{(m-1)(2-\alpha)}{2P_r} - \frac{m(n+1)(1-\alpha)^2}{2P_p n\alpha}\right)\right) \end{aligned} \tag{3.16}$$

For optimal Q_r putting $\dfrac{dTC}{dQ_r} = 0$

$$Q_r^* = \sqrt{\frac{\frac{(A_r + A_p)\alpha D}{m} + \sum_{i=1}^{I} \frac{A_i \alpha D(m+n)}{m}}{\frac{h_v D}{2}\left(\frac{\alpha}{P_r} + \frac{m(1-\alpha)^2}{P_p n\alpha}\right) + \frac{h_u m(P_r - \alpha D)}{2P_r} + \sum_{i=1}^{I} h_i d_i \left(\frac{m}{2\alpha D} - \frac{(m-1)(2-\alpha)}{2P_r} - \frac{m(n+1)(1-\alpha)^2}{2P_p n\alpha}\right)}} \tag{3.17}$$

Putting Q_r^* back in equation (3.16) to get $TC(Q_r^*)$.

$$TC(Q_r^*) = 2\sqrt{\left(\frac{(A_r + A_p)\alpha D}{m} + \sum_{i=1}^{I} \frac{A_i \alpha D(m+n)}{m}\right) * \left(\frac{h_v D}{2}\left(\frac{\alpha}{P_r} + \frac{m(1-\alpha)^2}{P_p n\alpha}\right) + \frac{h_u m(P_r - \alpha D)}{2P_r} + \sum_{i=1}^{I} h_i d_i \left(\frac{m}{2\alpha D} - \frac{(m-1)(2-\alpha)}{2P_r} - \frac{m(n+1)(1-\alpha)^2}{2P_p n\alpha}\right)\right)}$$
$$+ (u_r \alpha + u_p(1-\alpha))D \tag{3.18}$$

Case: $Q_r = Q_p = Q$

$$TC = \frac{A_r + A_p}{T} + \frac{Q}{T}(u_r m + u_p n) + \frac{Q^2 h_v}{2T}\left(\frac{m}{P_r} + \frac{n}{P_p}\right) + \frac{h_u m Q(P_r - \alpha D)}{2P_r}$$
$$+ \sum_{i=1}^{I}\left(\frac{A_i(m+n)}{T} + \frac{h_i d_i Q^2}{2DT}\left(\frac{(m+n)^2}{D} - \frac{(m-1)(m+2n)}{P_r} - \frac{n(n+1)}{P_p}\right)\right) \tag{3.19}$$

Where $T = \dfrac{mQ}{\alpha D}$ and $n = \dfrac{m(1-\alpha)}{\alpha}$

$$TC = \frac{(A_r + A_p)\alpha D}{mQ} + \left(u_r \alpha + u_p(1-\alpha)\right)D + \frac{h_v Q D}{2}\left(\frac{\alpha}{P_r} + \frac{(1-\alpha)}{P_p}\right) + \frac{h_u m Q(P_r - \alpha D)}{2P_r}$$
$$+ \sum_{i=1}^{I}\left(\frac{A_i D}{Q} + h_i d_i Q\left(\frac{m}{2\alpha D} - \frac{(m-1)(2-\alpha)}{2P_r} - \frac{(1-\alpha)(m - m\alpha + \alpha)}{2P_p \alpha}\right)\right) \tag{3.20}$$

For optimal Q putting $\dfrac{dTC}{dQ} = 0$

$$Q^* = \sqrt{\frac{\frac{(A_r + A_p)\alpha D}{m} + \sum_{i=1}^{I} A_i D}{\frac{h_v D}{2}\left(\frac{\alpha}{P_r} + \frac{(1-\alpha)}{P_p}\right) + \frac{h_u m(P_r - \alpha D)}{2P_r} + \sum_{i=1}^{I} h_i d_i \left\{\frac{m}{2\alpha D} - \frac{(m-1)(2-\alpha)}{2P_r} - \frac{(1-\alpha)(m - m\alpha + \alpha)}{2P_p \alpha}\right\}}} \quad (3.21)$$

Putting Q^* back in equation (3.20) to get $TC(Q^*)$.

$$TC(Q^*) = 2\sqrt{\left(\frac{(A_r + A_p)\alpha D}{m} + \sum_{i=1}^{I} A_i D\right) * \left(\frac{h_v D}{2}\left(\frac{\alpha}{P_r} + \frac{(1-\alpha)}{P_p}\right) + \frac{h_u m(P_r - \alpha D)}{2P_r} + \sum_{i=1}^{I} h_i d_i \left\{\frac{m}{2\alpha D} - \frac{(m-1)(2-\alpha)}{2P_r} - \frac{(1-\alpha)(m - m\alpha + \alpha)}{2P_p \alpha}\right\}\right)} + \left(u_r \alpha + u_p(1-\alpha)\right) D \quad (3.22)$$

3.6 The SVMB remanufacturing model considering emissions

The second model is the extension of the 1st model as it includes environmental impact of production and transportation. Due to the intense pressure from environmental legislative committees it is not possible to ignore the environmental issue in supply chain. The environment related costs are included in supply chain in terms of carbon emission from supply chain. Amount of carbon released in environment determines how much tax should be implemented. Authors have identified that carbon emission is generally through production processes and transportation. We will also follow this trend and below are the notations;

3.6.1 Notations

C_{ec} Emission tax $(\$ / ton)$

E Greenhouse gas (CO_2) emission from production $(ton / unit)$

a_p Production emission function parameter $(ton.year^2 / unit^3)$

b_p Production emission function parameter $(ton.year / unit^2)$

c_p Production emission function parameter $(ton / unit)$

a_r Remanufacturing emission function parameter $(ton.year^2 / unit^3)$

b_r Remanufacturing emission function parameter $(ton.year / unit^2)$

c_r Remanufacturing emission function parameter $(ton / unit)$

$C_{GHG,e}$ Carbon emission cost from production/remanufacturing $(\$ / year)$

E_{tr} Greenhouse gas emission (CO_2) from transportation $(ton / year)$

g Fuel required per truck $(gallons)$

e_t Amount of CO_2 emitted from fuel per gallon $(ton / gallon)$

η_r Number of trucks needed to deliver each remanufacturing batch

η_p Number of trucks needed to deliver each production batch

η Number of trucks needed to deliver each production or remanufacturing batch for the case $Q_r = Q_p = Q$

t_c Fixed capacity of one truck $(units)$

$C_{GHG,t}$ Carbon emission cost from transportation $(\$ / year)$

3.6.2 Carbon emission cost from production/remanufacturing

Amount of carbon emission from production process depends on production rate and use the relation provided by *Bogaschewsky* (1995);

$$E = aP^2 - bP + c$$

The above relation gives us the amount of carbon emission per item produced. Now in our case we have two different production rates i.e. production rate of remanufactured items P_r and production rate of newly produced items P_p. So amount of carbon emission per item will be;

$$E = \alpha\left(a_r P_r^2 - b_r P_r + c_r\right) + \left(1-\alpha\right)\left(a_p P_p^2 - b_p P_p + c_p\right) \tag{3.23}$$

Now carbon emission cost per unit of time from production/remanufacturing is;

$$C_{GHG,e} = EDC_{ec} = \left(\alpha\left(a_r P_r^2 - b_r P_r + c_r\right) + \left(1-\alpha\right)\left(a_p P_p^2 - b_p P_p + c_p\right)\right)DC_{ec} \tag{3.24}$$

Case: $Q_r = Q_p = Q$

Above equation is also valid for this case.

3.6.3 Carbon emission cost from transportation

For calculating this cost we need (1) the number of trucks to be used during each shipment (2) number of gallons to be used by each truck during each shipment. In this way we can find the carbon emission from transportation.

Vendor produces a batch size of newly produced items or remanufactured items and transport each batch to buyer separately. Batch sizes for production and remanufacturing are different which means same number of trucks cannot be attributed to them. Our carbon emission cost from transportation is based on this idea. So if g is the number of gallons required by each truck to deliver m shipments of remanufactured items through η_r trucks and n shipments of produced items through η_p trucks then carbon emission per unit time is given as;

$$E_{tr} = \frac{ge_t(m\eta_r + n\eta_p)}{T} \tag{3.25}$$

Carbon emission cost per unit of time from transportation is;

$$C_{GHG,t} = E_{tr}C_{ec} = \frac{ge_t(m\eta_r + n\eta_p)\alpha DC_{ec}}{mQ_r} \tag{3.26}$$

Case: $Q_r = Q_p = Q$

$$C_{GHG,t} = E_{tr}C_{ec} = \frac{ge_t(m+n)\eta\alpha DC_{ec}}{mQ} \tag{3.27}$$

3.6.4 Total carbon footprint cost

$$CFC = C_{GHG,e} + C_{GHG,t}$$

$$CFC = \left(\alpha\left(a_rP_r^2 - b_rP_r + c_r\right) + (1-\alpha)\left(a_pP_p^2 - b_pP_p + c_p\right)\right)DC_{ec} + \frac{ge_t(m\eta_r + n\eta_p)\alpha DC_{ec}}{mQ_r} \tag{3.28}$$

Case: $Q_r = Q_p = Q$

$$CFC = \left(\alpha\left(a_rP_r^2 - b_rP_r + c_r\right) + (1-\alpha)\left(a_pP_p^2 - b_pP_p + c_p\right)\right)DC_{ec} + \frac{ge_t(m+n)\eta\alpha DC_{ec}}{mQ} \tag{3.29}$$

3.6.5 Total cost of system

Total cost of system is summation of all previous costs of 1st model and carbon footprint costs.

$$
\begin{aligned}
TC=&\frac{(A_r+A_p)\alpha D}{mQ_r}+\left(u_r\alpha+u_p(1-\alpha)\right)D+\frac{h_v DQ_r}{2}\left(\frac{\alpha}{P_r}+\frac{m(1-\alpha)^2}{P_p n\alpha}\right)+\frac{h_u mQ_r(P_r-\alpha D)}{2P_r}+\\
&\sum_{i=1}^{I}\left(\frac{A_i\alpha D(m+n)}{mQ_r}+h_i d_i Q_r\left(\frac{m}{2\alpha D}-\frac{(m-1)(2-\alpha)}{2P_r}-\frac{m(n+1)(1-\alpha)^2}{2P_p n\alpha}\right)\right)\\
&+\left(\alpha\left(a_r P_r^2-b_r P_r+c_r\right)+(1-\alpha)\left(a_p P_p^2-b_p P_p+c_p\right)\right)DC_{ec}+\frac{ge_t(m\eta_r+n\eta_p)\alpha DC_{ec}}{mQ_r}
\end{aligned}
\qquad (3.30)
$$

As we have so many variables i.e. m, n, Q_r, Q_p, η_r and η_p so we cannot adopt the same strategy as adopted in 1st model. The reason is η_r and η_p are dependent on Q_r and Q_p i.e. $\eta_r = Q_r / t_c$ and $\eta_p = Q_p / t_c$. Now we are going to formulate our model as mixed integer nonlinear programming (MINLP) model with some constraints. As we know that we have only some of the constraints or restrictions to be followed as demand satisfaction is already made by putting value of T . Now we have to make sure that Q_r and Q_p are non-negative whereas m, n, η_r and η_p are integer variables. Finally we have to put conditions for η_r and η_p i.e.

$$(\eta_r - 1) < \frac{Q_r}{t_c} \le \eta_r \qquad (3.31)$$

$$(\eta_p - 1) < \frac{Q_p}{t_c} \le \eta_p \qquad (3.32)$$

Putting value of Q_p in equation (3.32)

$$(\eta_p - 1) < \frac{mQ_r(1-\alpha)}{n\alpha t_c} \leq \eta_p \tag{3.33}$$

Case: $Q_r = Q_p = Q$

$$\begin{aligned} TC = {} & \frac{(A_r + A_p)\alpha D}{mQ} + \left(u_r\alpha + u_p(1-\alpha)\right)D + \frac{h_v QD}{2}\left(\frac{\alpha}{P_r} + \frac{(1-\alpha)}{P_p}\right) + \frac{h_u mQ(P_r - \alpha D)}{2P_r} \\ & + \sum_{i=1}^{I}\left(\frac{A_i D}{Q} + h_i d_i Q\left(\frac{m}{2\alpha D} - \frac{(m-1)(2-\alpha)}{2P_r} - \frac{(1-\alpha)(m - m\alpha + \alpha)}{2P_p\alpha}\right)\right) \\ & + \left(\alpha\left(a_r P_r^2 - b_r P_r + c_r\right) + (1-\alpha)\left(a_p P_p^2 - b_p P_p + c_p\right)\right)DC_{ec} + \frac{ge_t(m+n)\eta\alpha DC_{ec}}{mQ} \end{aligned} \tag{3.34}$$

Similarly for minimizing equation (3.34) we have same restrictions for Q, m, n with the following constraint;

$$(\eta - 1) < \frac{Q}{t_c} \leq \eta \tag{3.35}$$

CHAPTER 4

RESULTS AND DISCUSSIONS

We have initially considered three buyers for analysis and numerical data required for both models is presented below.

Table 1 Numerical data for both models

Parameter	Value	Parameter	Value	Parameter	Value
A_r	600	A_1	100	a_r	0.000000833
A_p	1200	A_2	100	b_r	0.002
P_r	1300	A_3	100	c_r	1.4
P_p	2600	d_1	300	a_p	0.0000003
h_v	10	d_2	300	b_p	0.0012
h_u	5	d_3	300	c_p	1.4
u_r	30	h_1	7	g	100
u_p	50	h_2	7	e_t	0.01008
α	0.6	h_3	7	t_c	80
D	900	C_{ec}	18		

4.1 Analysis of 1st model

Numerical examples are solved and sensitivity analysis is performed by varying some important parameters and ratios to see their impact on the model and draw some insights.

4.1.1 Effect of alpha on TC

Figure 5 shows the relationship between total cost and return percentage. We have varied return percentage from 0.1 to 1 to get the behavior of TC . Increasing α decreases the TC to a significant level. The TC is reduced by 31.37% from 48463.5 to 33259.3 indicating that increased remanufacturing benefits organization.

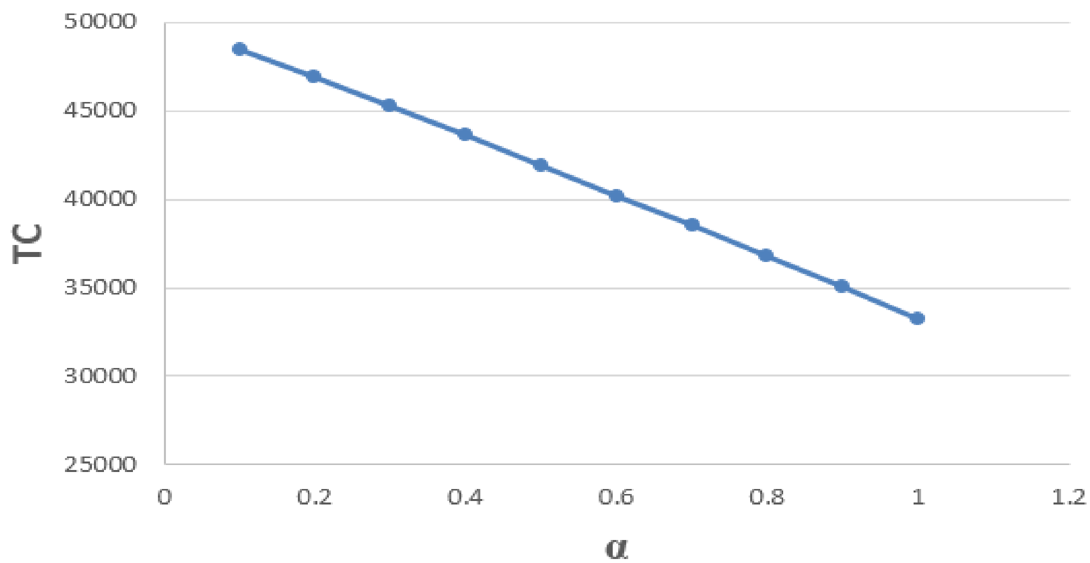

Figure 5 Effect of alpha on total cost

Increasing α means increasing return percentage of used items. As costs associated with remanufacturing are lower so when more demand is fulfilled from used items then costs go down as seen in Figure 5. Although TC has been reduced but not all cost components are reduced with increasing α. Behavior of all cost components with changing α will be discussed later in next chapter when we look at the effect of number of buyers. But production cost is the only cost that is totally dependent on α and shows a definite decreasing trend as expected. At $\alpha = 0.1$ $TC = 48463.5$ and $PC = 43200$ which means PC is 89.1% of TC, at $\alpha = 1$ $TC = 33259.5$ and $PC = 27000$ which means PC is 81.2% of TC. So decreasing trend of TC is because of PC as it is the major component of TC. The reason behind the decrement of PC is the significant difference in the per unit cost of produced and remanufactured item i.e. $u_p = 50$ and $u_r = 30$.

Two things can be recommended that if a firm is already doing remanufacturing then it should try to collect maximum of the used products to increase its return percentage. Secondly if a firm is in the process of decision making that whether it should go for remanufacturing or not then the above figure clearly shows that it is profitable.

4.1.2 Effect of u_p/u_r ratio

Figure 6 shows the behavior of TC at various per unit cost of produced item to per unit cost of remanufactured item ratio by changing α.

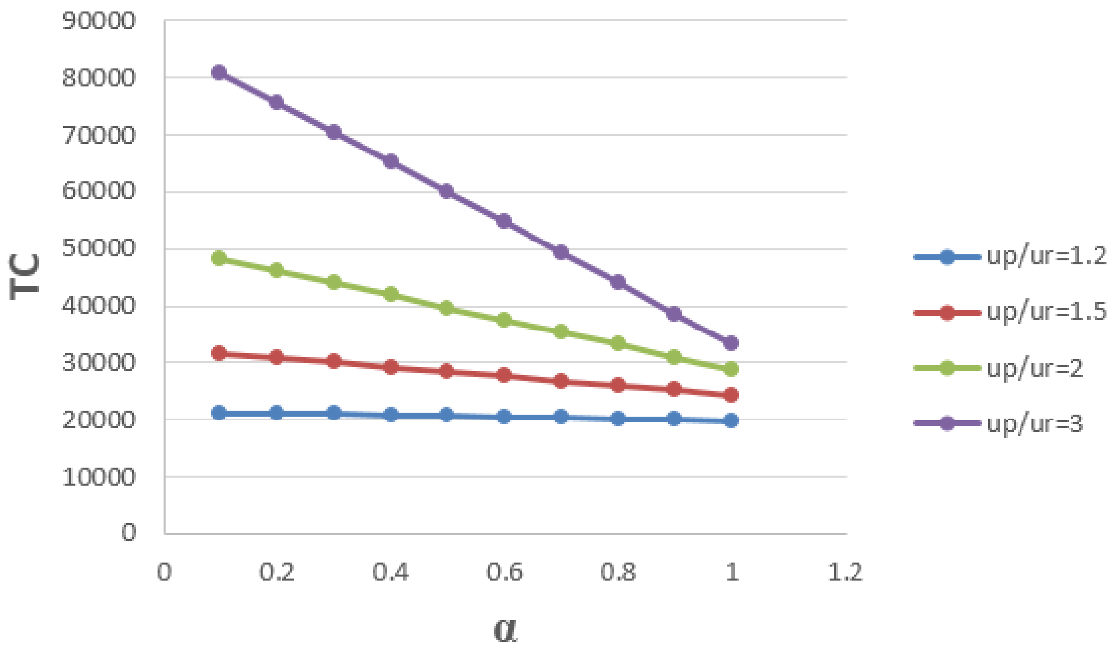

Figure 6 Effect of u_p/u_r ratio

Ratio $u_p / u_r = 1.2$ is found by selecting $u_p = 18$ and $u_r = 15$, $u_p / u_r = 1.5$ is found by selecting $u_p = 30$ and $u_r = 20$, $u_p / u_r = 2$ is found by selecting $u_p = 50$ and $u_r = 25$ and $u_p / u_r = 3$ is found by selecting $u_p = 90$ and $u_r = 30$.

As PC is the cost that is affected by this ratio and we have mentioned earlier that it is the major component of total cost of system. Increasing this ratio means more increment in u_p than u_r. This increment gives us higher value of PC which in turn increases TC.

The greater the ratio greater the savings are observed in TC. Cost savings for the ratios 1.2, 1.5, 2 and 3 are 6.77%, 22.65%, 40.1% and 58.87% respectively. A firm that is having great difference in the cost of produced and remanufactured item must adopt practices to increase α.

4.1.3 Effect of h_v/h_i ratio

Figure 7 shows the behavior of holding cost of vendor to holding cost of *ith* buyer ratio by changing α. The ratio $h_v / h_i = 1.2$ is found by selecting $h_v = 12$ and $h_i = 10$, $h_v / h_i = 1.5$ is found by selecting $h_v = 18$ and $h_i = 12$, $h_v / h_i = 2$ is found by selecting $h_v = 28$ and $h_i = 14$ and $h_v / h_i = 3$ is found by selecting $h_v = 48$ and $h_i = 16$. Now increasing this ratio means increasing holding cost of vendor more than holding cost of buyer. At $\alpha = 0.6$ when the ratio is 1.2 then $HC_v = 664.9$ and $\sum HC_{ib} = 2006.9$, when the ratio is 3 then $HC_v = 1798.45$ and $\sum HC_{ib} = 2307.82$. HC_v has increased by 170.5% whereas $\sum HC_{ib}$ has increased by 15%. By increasing ratio TC is increased due to significant increment in

HC_v. Second thing is these costs have gone down by increasing α which confirms the purpose of remanufacturing.

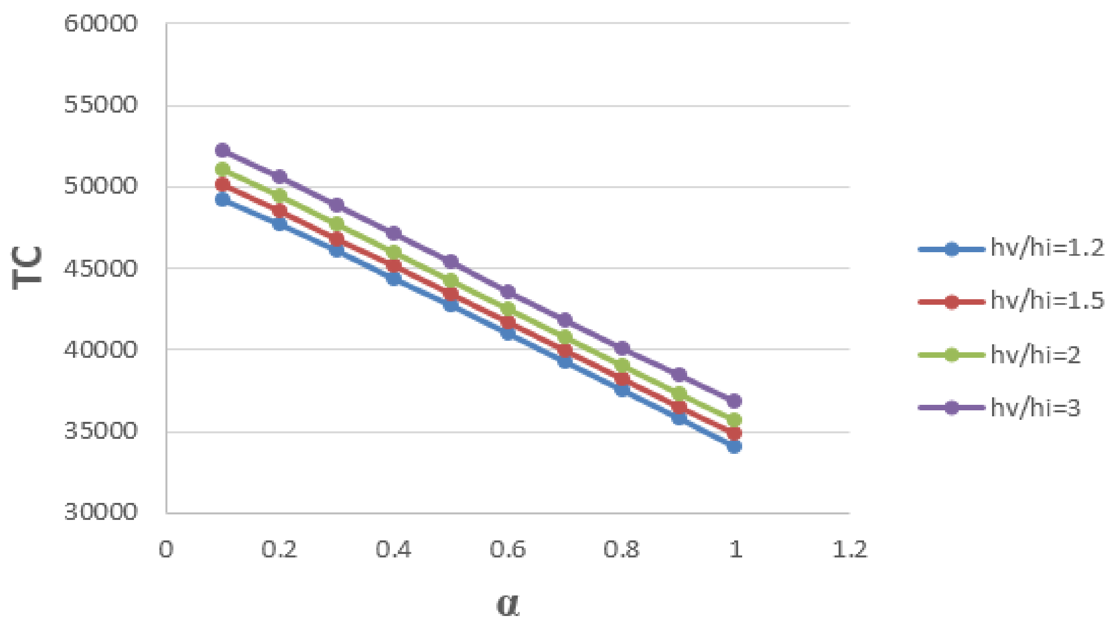

Figure 7 Effect of h_v/h_i ratio

Cost savings for the ratios 1.2, 1.5, 2 and 3 are 30.79%, 30.49%, 30.15% and 29.49% respectively. Smaller the ratio greater the savings are observed in TC that means greater difference in holding cost between vendor and buyer is not profitable. Vendor and buyer should coordinate in order to mitigate the huge holding cost difference between them.

4.1.4 Effect of A_p/A_r ratio

Behavior of TC with increasing A_p / A_r ratio is depicted in Figure 8. $A_p / A_r = 1.2$ is found by selecting $A_p = 480$ and $A_r = 400$, $A_p / A_r = 1.5$ is found by selecting $A_p = 750$ and $A_r = 500$, $A_p / A_r = 2$ is found by selecting $A_p = 1200$ and $A_r = 600$, $A_p / A_r = 3$ is found by selecting $A_p = 2100$ and $A_r = 700$.

Increasing this ratio means increasing production setup cost more than remanufacturing setup cost. Both of these costs are related to vendor. Increment in above mentioned input parameters increases setup cost of vendor SC. At $\alpha = 0.1$, $SC = 1228.83$ at $A_p / A_r = 1.2$ and $SC = 2579.63$ at $A_p / A_r = 3$. This is 109.93% increase in SC which is the reason in increment of TC.

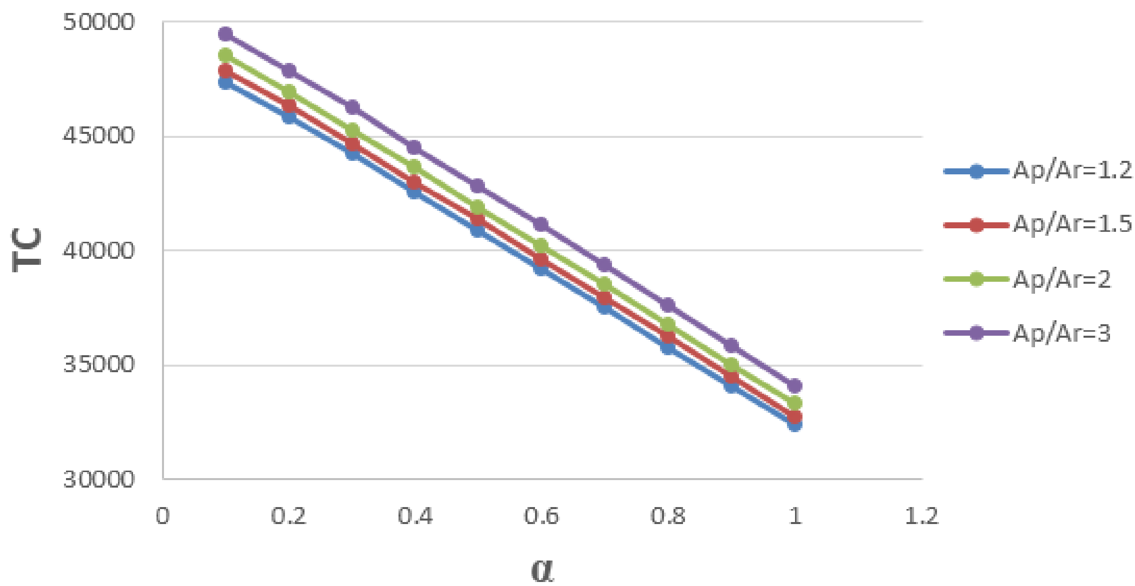

Figure 8 Effect of A_p/A_r ratio

Cost savings obtained for the ratio 1.2, 1.5, 2 and 3 are 31.69%, 31.56%, 31.37 and 31.14% respectively. Analysis shows that more cost savings can be obtained at lower ratio. There is no question of profitability of remanufacturing but at higher ratios even remanufacturing cannot bring the costs down as compared to lower ratios. Again it is clear that greater difference between A_p and A_r is not suitable for the supply chain system.

4.2 Analysis of 2nd model

We have discussed earlier that our 2nd model includes all previous costs of 1st model with the addition of carbon costs. Detailed analysis is performed and behavior of decision variables and total cost is analyzed in next sections.

4.2.1 Effect of alpha

Starting the analysis with our basic parameter α . By increasing α we would like to see that what impact it puts on our total cost of system and other decision variables. We are varying α when $C_{ec} = 18$ and $t_c = 80$. Above solutions are obtained using Lingo14.0. From Table 2 we can see that increasing α decreases TC by 27.53% from 53496.58 to 38769.69.

Table 2 Effect of alpha

α	m	n	η_r	η_p	Q_r	Q_p	TC
0.1	1	1	1	9	80	720	53496.58
0.2	2	1	1	8	80	640	51803.09
0.3	2	1	2	8	133.95	625.11	49983.63
0.4	2	1	2	6	160	480	48162.04
0.5	3	1	2	6	158.03	474.09	46286.20
0.6	4	1	2	5	150	400	44448.8
0.7	4	1	2	4	160	274.29	42559.1
0.8	4	1	3	3	201.29	201.29	40679.28
0.9	4	1	3	2	223.46	99.32	38769.69

It is better for a system to operate with only one production batch and more than one batch of remanufacturing as indicated by Table 2. With lower value of α we cannot produce huge batch size of remanufacturing so at $\alpha = 0.1$ our $Q_r = 80$ and $Q_p = 720$. To transport a single low quantity batch size of Q_r we need less trucks i.e. $\eta_r = 1$. To transport a single high quantity batch size of Q_p we need more trucks i.e. $\eta_p = 9$. As we increase the value of α to 0.9 then our Q_r increases as we have huge return percentage of used items. As α is changed from 0.1 to 0.9, Q_p is decreased by 86.2% from 720 to 99.32. Due to decreased value of Q_p and increased value of Q_r at $\alpha = 0.9$ number of trucks needed to transport each batch size of production and remanufacturing have also changed dramatically i.e. $\eta_r = 3$ and $\eta_p = 2$.

Above all variations obtained by changing α suggest more remanufacturing as increased value of α decreases total cost of system. The reason behind this is costs associated with remanufacturing are low as compared to production. For example unit cost to produce new item is more as compared to unit cost to remanufacture used item. The above values of TC with increasing α confirm the expected behavior of system. Organizations should take serious steps to collect more and more used items. At high α balance can be seen in the values as difference in the batch size quantity of production and remanufacturing is lower as compared to difference at low α. Similar balance can be seen with number of trucks as difference between the number of trucks at high alpha is very low as difference obtained at low alpha. All values of decision variables at high α seem logical whereas all values at low α seem illogical.

4.2.2 Effect of fixed capacity of truck

Fixed capacity of singe truck t_c is changed from 20 to 200 by keeping $C_{ec} = 18$ and $\alpha = 0.6$ and all other parameters constant. Values of decision variables are presented in Table 3. Total cost of system is decreased because reduction in carbon emission from transportation is observed as it is the cost that is affected by capacity of truck. At $\alpha = 0.6$, $C_{GHG,t}$ is 816.82 when t_c is 20 and $C_{GHG,t}$ is 89.32 when t_c is 200. The depletion of 89.1% in $C_{HG,t}$ is responsible for the decrement in TC. At first glimpse we see that number of production batches are again restricted to 1 as increased value of them at various values of t_c will increase the total cost of system. Number of remanufacturing batches follow random trend but are more than 1. This indicates that more of the demand is to be satisfied from remanufacturing which is the basic theme of doing remanufacturing.

Table 3 Effect of fixed capacity of truck

t_c	m	n	η_r	η_p	Q_r	Q_p	TC
20	3	1	9	18	180	360	45024
40	3	1	5	9	180	360	44642.97
60	3	1	3	6	180	360	44479.68
80	4	1	2	5	150	400	44448.80
100	3	1	2	4	185.61	371.22	44386.06
120	3	1	2	3	180	360	44370.81
140	4	1	1	3	140	373.33	44367.72
160	4	1	1	3	150.81	402.16	44350.74
180	3	1	1	2	180	360	44298.24
200	3	1	1	2	182.94	365.88	44297.44

As t_c is changed from 20 to 200 TC decreases by only 1.6% from 45024 to 44297.44. This little reduction in TC shows that t_c does not affect too much. But the case of reduction shows that high capacity of single truck allows to ship more quantity on it and hence less number of trucks are needed. At $t_c = 20$, $\eta_r = 9$ and $\eta_p = 18$ whereas at $t_c = 200$, $\eta_r = 1$ and $\eta_p = 2$. Percentage reduction is 88.9% for both η_r and η_p.

Although t_c has not affected TC much but decision makers would like to take benefit from the significant reduction in the number of trucks for both production and remanufacturing. Managing 1and 2 trucks for remanufacturing and production instead of 9 and 18 is really easy. Sometimes changing t_c also means changing type of truck. Organization may have to arrange some petrol or hybrid trucks for changing truck capacity. If type of truck is changed then fuel consumption and carbon emission of truck will also be changed. So based on analysis presented shipment policy must be revisited if t_c is to be changed.

4.2.3 Effect of carbon emission tax

During our analysis we are changing the values with a realistic purpose as done in the previous sections. So in this section we are going to change the value of carbon emission tax C_{ec} from 10$/ton to 200$/ton. The reason of doing this is C_{ec} changes in every country depending on country's policy. Countries or regions that are very conscious about the environment have put huge tax whereas in some countries the tax is low to facilitate manufacturers too. For example in UK C_{ec} is 15.75$/ton, in USA C_{ec} is 19.6$/ton, in Ireland C_{ec} is 26.57$/ton, in Australia C_{ec} is 20.1$/ton, in Saudi Arabia C_{ec} is 30$/ton, in Switzerland C_{ec} is 68$/ton and in Sweden C_{ec} is 168$/ton. Table 4 presents the values of decision variables with $t_c = 80$ and $\alpha = 0.6$. We have got some interesting facts from Table 4. It shows that increasing C_{ec} is not in favor of the production/remanufacturing system. Increasing C_{ec} from 20 to 200 has increased TC by 93.95% from 44918.54 to 87117.85. Increased carbon tax magnified total carbon footprint cost which in turn increased total cost. At $C_{ec} = 20$, CFC is 4697.5 and at $C_{ec} = 200$, CFC is 46884.1. So reason behind the inflation of TC is the 898.1% increment in CFC.

Table 4 Effect of carbon emission tax

C_{ec}	m	n	η_r	η_p	Q_r	Q_p	TC
20	4	1	2	5	150	400	44918.54
40	3	1	2	4	160	320	49611.32
60	3	1	2	4	160	320	54299.64
80	3	1	2	4	160	320	58987.96
100	3	1	2	4	160	320	63676.27
120	3	1	2	4	160	320	68364.59
140	3	1	2	4	160	320	73052.90
160	3	1	2	4	160	320	77741.22
180	3	1	2	4	160	320	82429.54
200	3	1	2	4	160	320	87117.85

No improvement in number of shipments can be made as cost will be increased by slight change in m and n. We have got fixed values of above mentioned decision variables because of the fact that high quantity batches will need increased number of trucks.

Increased number of trucks will emit more carbon. More carbon emission means more tax to be implemented which in turn increases TC .

Countries must cooperate with its manufacturing industries in order to select the best environmental policy. Just increasing C_{ec} make conditions difficult for organizations to operate. Even if there is a need of increasing C_{ec} then it should be done with proper collaboration so industries can think of redesigning their supply chain in terms of production and transportation.

4.2.4 Effect of number of buyers

Effect of number of buyers on 2nd model are discussed in Table 5 when $\alpha = 0.3$, in Table 6 when $\alpha = 0.6$ and in Table 7 when $\alpha = 0.9$. All costs show definite increment in them except some costs that showed random behavior when number of buyers are increased from 1 to 4. From Table 5 to 7 it can be easily observed that percentage increased in TC has decreased by increasing α . Though majority of the costs are increased with number of buyers but again remanufacturing provides the advantage in terms of cost savings.

Cost components along with m, n, η_r and η_p for each buyer with changing values of α are clearly stated in tables. Q_r and Q_p are again random as they heavily depend on m, n and α . With changing m, n and α batch sizes vary but neither in increasing or decreasing order. $SC, \sum HC_{ib}$ and HC_u are other costs that showed random behavior with increased buyers due to the same reason. $PC, \sum OC_{ib}, HC_v$ and CFC are the costs that have increased so our focus will be on them.

Table 5 Effect of number of buyers when $\alpha = 0.3$

Costs	With 1 buyer $m = 2, n = 1$ $\eta_r = 1, \eta_p = 4$	With 2 buyers $m = 2, n = 1$ $\eta_r = 2, \eta_p = 6$	With 3 buyers $m = 2, n = 1$ $\eta_r = 2, \eta_p = 8$	With 4 buyers $m = 2, n = 1$ $\eta_r = 2, \eta_p = 10$	Percentage increase
SC	1317.07	1649.19	1814.11	2025	53.75
PC	13200	26400	39600	52800	300
HC_v	137.19	438.26	896.43	1427.69	940.67
HC_u	286.21	423.14	530.65	578.46	102.11
$\sum OC_{ib}$	219.51	549.73	907.06	1350	515.01
$\sum HC_{ib}$	1188.29	1503.93	1513.46	1165.95	Random
CFC	1580.52	3167.96	4722.01	6289.20	297.92
TC	17928.8	34132.2	49983.72	65636.3	266.09

At $\alpha = 0.3, 0.6$ and 0.9, PC is increased by 300% with increased number of buyers. Increased buyers mean more demand to be satisfied so more items to be produced and hence PC is increased. HC_v and $\sum OC_{ib}$ are the costs that have increased dramatically by increasing buyers from 1 to 4. HC_v has increased to 940.67%, 1110.16% and 530.82 % at $\alpha = 0.3, 0.6$ and 0.9 respectively. This increment is in continuation with production cost. With increased buyers more items are produced so more are stored at vendor's side. Combination of m, n, α and Q_r also affects HC_v but demand has the major impact.

Table 6 Effect of number of buyers when $\alpha = 0.6$

Costs	With 1 buyer $m = 3, n = 1$ $\eta_r = 1, \eta_p = 2$	With 2 buyers $m = 3, n = 1$ $\eta_r = 2, \eta_p = 4$	With 3 buyers $m = 4, n = 1$ $\eta_r = 2, \eta_p = 5$	With 4 buyers $m = 4, n = 1$ $\eta_r = 3, \eta_p = 7$	Percentage increase
SC	1372.82	1647.41	1620	1542.86	Random
PC	11400	22800	34200	45600	300
HC_v	90.77	302.57	588.46	1098.46	1110.16
HC_u	508.33	711.05	876.92	936.92	84.31
$\sum OC_{ib}$	305.07	732.18	1350	1714.29	461.93
$\sum HC_{ib}$	1147.98	1532.03	1585.77	1326.77	Random
CFC	1407.67	2843.04	4227.74	5649.43	301.33
TC	16232.65	30568.28	44448.89	51868.73	219.53

We have mentioned earlier that our system predicts so much vulnerability to vendor as he suffers most with changes in values of decision variables and input parameters. Again vendor has to look at its profit to cost ratio while selecting more buyers. $\sum OC_{ib}$ is associated with buyers and has increased with increased number of buyers. It has increased to 515.01%, 461.93% and 688.74% at $\alpha = 0.3, 0.6$ and 0.9 respectively. $\sum OC_{ib}$ also depends on each decision variable but directly proportional to demand and summation of m and n.

Table 7 Effect of number of buyers when $\alpha = 0.9$

Costs	With 1 buyer $m = 2, n = 1$ $\eta_r = 2, \eta_p = 1$	With 2 buyers $m = 4, n = 1$ $\eta_r = 2, \eta_p = 1$	With 3 buyers $m = 4, n = 1$ $\eta_r = 3, \eta_p = 2$	With 4 buyers $m = 8, n = 1$ $\eta_r = 3, \eta_p = 3$	Percentage increase
SC	1540.41	1635.70	1631.16	1012.5	Random
PC	9600	19200	28800	38400	300
HC_v	165.84	316.17	713.35	1046.15	530.82
HC_u	624.93	868.50	842.27	812.31	Random
$\sum OC_{ib}$	256.74	908.72	1359.30	2025	688.74
$\sum HC_{ib}$	1083.96	1508.33	1665.16	1427.28	Random
CFC	1253.75	2500.62	3758.53	4980	297.21
TC	14525.63	26938.04	38769.78	49703.25	242.18

From tables it is clear that with increased buyers, number of shipments of production and remanufacturing together with demand have increased hence $\sum OC_{ib}$ is increased. But $\sum OC_{ib}$ is equally distributed between buyers so there is not much problem for one buyer. Look at the case in Table 7 at $\alpha = 0.9$, $\sum OC_{ib}$ with one buyer is 256.74 and for four buyers it is 2025. Now 2025 is equally distributed between four buyers so cost of each buyer is 506.25. Now cost increment for each buyer is 97% which is much less as compared to all other increased costs. CFC is the cost that is specially attributed to 2nd model and it has increased to 297.92%, 301.33% and 297.21% at $\alpha = 0.3, 0.6$ and 0.9 by increasing number of buyers. CFC is the summation of carbon emission cost from production/ remanufacturing $C_{GHG,e}$ and carbon emission cost from transportation $C_{GHG,t}$ Both of these costs have increased with increased buyers and as a result CFC is also increased. $C_{GHG,e}$ is not dependent on any of the decision variable m, n and Q_r but only on D. As D is increased with increased buyers so $C_{GHG,e}$ is ballooned. $C_{GHG,t}$ is majorly dependent on m, n, η_r and η_p. $C_{GHG,t}$ is increased due to following sequential increment of decision variables presented in Table 8.

Two special decision variables which are attributed to CFC are η_r and η_p. At $\alpha = 0.3, 0.6$ and 0.9 total number of trucks needed for all the shipments have increased with increased buyers. The reason behind is the increment in the produced and remanufactured items. At $\alpha = 0.3$, $mQ_r + nQ_p$ is 410.6, 654.87, 893 and 1066.67 for one, two, three and four buyers respectively. $m\eta_r + n\eta_p$ have also increased in the order 6,10,12 and 14. This increasing trend with increased buyers can also be seen for $\alpha = 0.6$ and 0.9 in Table 8.

Table 8 Effect of number of buyers on $mQ_r + nQ_p$ and $m\eta_r + n\eta_p$

α	Number of buyers	m	n	Q_r	Q_p	η_r	η_p	$mQ_r + nQ_p$	$m\eta_r + n\eta_p$
0.3	1	2	1	61.59	287.42	1	4	410.6	6
	2	2	1	98.23	458.41	2	6	654.87	10
	3	2	1	133.95	625.1	2	8	893	12
	4	2	1	160	746.67	2	10	1066.67	14
0.6	1	3	1	78.67	157.34	1	2	393.35	5
	2	3	1	131.12	262.24	2	4	655.6	10
	3	4	1	150	400	2	5	1000	13
	4	4	1	210	560	3	7	1400	19
0.9	1	2	1	157.75	35.01	2	1	350.56	5
	2	4	1	148.56	66.03	2	1	660.27	9
	3	4	1	223.46	99.32	3	2	993.16	14
	4	8	1	240	213.33	3	3	2133.33	27

The above all values have determined that $C_{GHG,t}$ has increased by inflating number of buyers. The other components of CFC i.e. $C_{GHG,e}$ has also increased by increasing buyers.

$C_{GHG,e}$ depends only on demand in particular which has ballooned with more buyers. Hence CFC has increased due to increased demand and number of trucks.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

Our work extended the previous work by considering CLSC with single vendor and multi buyers. Demand from market is fulfilled through multiple buyers where demand rate of each buyer is same. So this assumption allowed to have same inventory profile of each buyer. Integration of these multi buyers is done with single vendor with inclusion of remanufacturing. With this scenario we presented two models, one model considered traditional supply chain costs whereas other model considered environmental related costs with all previous costs of initial model. For both models α has paramount importance as it affects total cost of system a lot. We also revealed that PC accounted for 80% of TC. From increment in u_p / u_r we got huge value of TC but we got cost savings from increment in α. Ratios h_v / h_i and A_p / A_r provided more insights about behavior of model. With the analysis of 2nd model we discussed impact on each decision variable by varying carbon emission related parameters. Increased t_c reduced TC which is a logical result as number of trucks reduced. Gradual rise in C_{ec} gave static optimal shipment policy. Increased number of buyers made some costs inflated and some random.

5.2 Recommendations

Despite of vast contribution our models have limitations that can be resolved in future. We have considered produced and remanufactured items have same quality and it can be challenged. It will be interesting to consider secondary market for remanufactured items.

Further we have considered single product case that can be altered for multiple products. Assumption of ‘no failure in production and remanufacturing processes’ can be changed by allowing failure in the processes. Moreover we have considered all used products are repairable which is not always true. It can be adjusted by allowing some percentage of used items repairable. Supply chain network presented by us can be modified by introducing distribution center between vendor and buyer. Researchers can reshape coordination mechanism and interesting results could be seen in terms of number of shipments and batch sizes. We have covered the gap of multi buyers in CLSC but it will also be useful to consider multiple vendors to fulfill demand from multiple buyers.

Environmental issues are covered in our 2^{nd} model but again this work can be refined by resolving limitations. As we have considered only trucks for delivering shipments so transportation mode can be reviewed. Other mode can be adopted with different fixed capacity, fuel consumption and carbon emission. Penalty cost for excess carbon emissions can be incorporated in our carbon footprint model. We have focused on just carbon emissions but there are further environmental factors. Chemical waste, toxic waste, noise pollution etc. should be considered while modeling for green supply chain. Though it would be very difficult but a very good future dimension.

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