ANALYSIS AND SYNTHESIS OF RESET CONTROL SYSTEMS WITH TIME DELAYS

BY

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To My Parents

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This thesis is a milestone on my path to obtaining the Master degree at the University of King Fahd University of Petroleum and Minerals. At the end of my Master program, I would like to thank the guidance of my supervisor, help from colleagues, and support from my family.

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THESIS ABSTRACT

NAME:	Bilal Jafar Karaki
TITLE OF STUDY:	Analysis and Synthesis of Reset Control Systems with Time Delays
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This dissertation considers reset control theory for dynamical systems with and without delay. Reset mechanisms in controller design could affect the performance and stability of control systems. Stability of reset systems can be investigated based on a similar theorem of Lyapunov theory. In this thesis, novel analyses of reset systems scheme are proposed for certain and uncertain dynamical systems.

Firstly, reduced order reset controllers with new reset mechanisms are established to stabilize unstable plants. The developed reset mechanism assumes that the base dynamics might be unstable on the contrary of other proposed design methodologies that lack this property. The order of the reset controller is reduced to be less than or equal to the rank of the plant's input matrix. Moreover, it has

been shown that the controller forces a number of states to reach the origin in a finite time while the remaining states behavior depends on the state feedback which can be chosen by any appropriate classical method.

A delay-dependent stability analysis of reset control systems is proposed for linear time invariant case. The conditions of stability are given in terms of linear matrix inequalities. In addition, extending results are given to investigate stability of reset systems with uncertainty. The stability conditions of the time delay reset systems are less conservative than those presented in the literature. By using the same LMI's, extending the results to obtain controller gains can be easily obtained using some modifications.

Finally, sufficient stability conditions are derived in terms of multiple Lyapunov functions in order to study reset systems in general. Consequently, this increases the flexibility of LMI's and decreases their conservativeness. Simulation results using numerical examples demonstrate the ability of the proposed methods to investigate good performance thereof.

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تدرس هذه الاطروحة أساليب إعادة ضبط نظرية التحكم للنظم الديناميكية في وجود تأخير زمني. ومن المعروف ان أساليب ضبط نظرية التحكم في تصميم الحاكمات تؤثر على أداء واتزانيه نظم التحكم. وتم اقتراح عدة طرق لتحليل وتصميم النظم الديناميكية مع إعادة ضبط نظرية التحكم في مواجهة تغيرات المعاملات. وقد أفردت الدراسة لوسائل متنوعة لتصميم الحاكمات مع ضمان اتزانيه نظم تحكم العروة المغلقة في زمن محدود وصغير نسبيا.

ثم انتقلت الدراسة الى معالجة اتزانيه نظم التحكم الخطية في وجود تأخير زمني حيث قامت باستنباط شروط الاتزان في صورة مجموعة من المصفوفات الخطية وتعتبر هذه الشروط أفضل من الشروط المتوفرة في الأبحاث المنشورة.

واختتمت الدراسة باستخدام بعض الدوال الرياضية لتعميم نتائج اتزانيه نظم تحكم العروة المغلقة مع إعادة ضبط المحكمات لزيادة مرونة التصميم وقد استخدام اساليب المحاكاة لإيضاح قوة أداء النظم المصممة.

CHAPTER 1

INTRODUCTION

In this chapter, we give a glimpse into reset control systems. Recent decade has witnessed a substantial interest in reset control that emerged 70 years ago to overcome fundamental drawbacks of the linear control systems. First, we address a review of research activities of reset control systems from its beginning up to date. Finally, motivation and general objectives of the thesis are provided.

1.1 An Overview and Literature Survey

Linear control system theory has been extensively used in industrial applications for decades, and has proved to be the most widely applied control technique. However, linear control methodologies have inherent limitations. Such limitations emerge when the process of designing a control system introduces competing user demands. This situation involves losing one performance quality in return for gaining another quality. For example, it can be argued which is the best performance due to the trade-off between fast response and robustness of control systems.

Dynamical systems usually demand the control theory to carry out user specifications in terms of stability and performance. Most of the control strategies that has been studded over the decades devoted to study the controllers that are similar to the nature of the plant. However, the use of controllers that contains dynamics different from the controlled process increases the analysis complexity and makes the implementation more sophisticated. On a particular point of view, technological advancements allow us to use controllers comprise mixed dynamics, i.e. continuous and discrete behavior. Such controllers provide advantages over the classical one due to the rich dynamics.

Based on the numerous research of control community we can examine how the linear controllers without hybrid dynamics limit the achievable control performance. Despite the fact that closed loop stability is the main issue, many user demands (in terms of performance) must be achieved. The main objective of closed loop control is the stability with desirable performance. The main limitations in standard feedback systems occurs when multiple performance objectives are required to be satisfied. Good disturbance rejection requires large open loop gain. However, noise suppression needs small open loop gain which conflicts the disturbance rejection. In addition, the open loop gain must be large enough for perfect tracking of the reference signal. But usually large gains produces saturation or may burn the actuators of the plant[1]. These conflicting requirements can be solved using loop shaping and $\mathcal{H}_\infty$-loop shaping methods. However, loop shaping for multi-input-multi-output systems (MIMO) needs sophisticated work[2, 3].

In many practical applications, many conflicting performance specifications are crucial to be satisfied. This constitutes a substantial challenge in linear control theory. Consequently, more complex control techniques might be sued to overcome these limitations. Nonlinear control is a possible solution that outperform all linear control methods. This motivates researchers to introduce a new class of nonlinear controllers called reset controllers. This class can also be classified as a particular class of hybrid dynamical systems [4].

In many practical systems, the physical state can not be solely characterized by continuous dynamics. Systems with mixture dynamics are classified as a hybrid dynamical systems. Reset control system is similar in some sense to switched system, is a special type of hybrid systems [5]. Obviously, both aforementioned types of control systems are hybrid but are intrinsically different. The trajectories of switched systems losses its differentiability at switching instants [6], while reset systems states are not continuous. With this in mind, reset control systems represents a discontinuous behavior in which the system states reset at the instant of crossing a predefined surface. This class of controllers have attracted researchers attention [1, 5, 7, 8, 9, 10, 11, 12].

The origin of reset control systems dates back to the nonlinear Clegg integrator in 1958 [13]. The Clegg integrator was considered to overcome the time lag introduced by the linear integrator. The reset condition for Clegg integrator usually occurs when the controller input crosses the zero. Another important improvement of reset systems, first order reset element (FORE), is introduced by Horowitz [9]. FORE is simply modeled by a first order dynamics with a zero crossing reset mechanism.

Although the reset controllers show more advantages over conventional linear controllers, several drawbacks and challenges could arise. On one hand, reset controller may destabilize a stable base control system if some reset sequence is applied. This means that reset systems must be designed carefully in order to improve the performance and robustness [14]. On the other hand, it might be there exists linear controller that performs better than the reset one. However, linear controllers have limitations that makes the design very challenging or impossible [15].

Reset control systems have proved to be indispensable when it comes to guaranteeing performance specification in presence of limitations. But the theory of reset control is much more difficult than linear systems theory. But it performs better and provides satisfactory results in a large number of delayed systems. The methodology of designing a rest control law usually makes more demands than other classical control methods. These demands emerge in selecting the reset laws and the states to be reset as well as the dynamics of the reset controller.

As mentioned in literature, reset control systems represents a jump behavior in which the system state resets at the instant it crosses a defined reset surface. As a consequence of the sudden jump of the behavior, it can be classified as a particular class of hybrid dynamical systems [4]. When the reset sequence is demonstrated as a function of time, the dynamics is considered as an impulsive system[16].

This motivates us to use non-classical control in order to overcome some of these limitations. The reset control is one of the methodologies that can be used to achieve good performance when the classical control fails. The main idea behind the advent of reset control and in particular the first order reset element is the attempt to achieve fast response and robust control. Lyapunov stability is a strong tool to investigate the stability of reset systems. Lyapunove based methods [17, 18, 19, 11] and also passivity based approaches [20, 21, 22] are used to address the reset systems stability. Recently some mathematical tools that treat the delay reset systems appeared in [23] as an extension result of Lyapunov-Krasoviskii theorem. The design of control systems may improve the performance, but it has been clear that the design must to be done with care because it may destabilize the process [23].

Reset control systems framework is based on the theory of Impulsive Dynamical Systems (IDS). Similar to the Clegg integrator, impulsive systems are mainly characterized by three elements, continuous, discrete dynamics, and jump law. The continuous-time dynamics is described by a differential equation in order to characterize the behavior of the impulsive system between reset events. The discrete-time part is used to demonstrate the instantaneous change in the states of the continuous part. Finally, reset law is used to govern the switching between the continuous and discrete dynamics.

Recently reset control systems have been studded through several works in an attempt to provide flexible tools to achieve improvement in terms of performance [24, 14, 25, 26, 15, 27, 28, 29]. The purpose of this introduction is to report some fundamental contributions in reset control systems. For example, a series works by Horowitz [30, 31] where reset mechanisms were implemented by the first order element. Several reset controllers where implemented through either experiments or simulation [32, 33, 34, 35, 36]. Bounded-input bounded-output stability was established for reset control systems using first order element in the thesis [37]. In addition, this thesis studied the asymptotic stability and transient response for second order systems, see [38].

The stability analysis of stable linear base system has been studied in [39] by using quadratic Lyapunove functions. The results constitute a sufficient and necessary conditions that can be checked using the well know H_β-condition. H_β condition relies on the positive realness of a transfer function matrix or similarly by checking a pair of LMI conditions. This was achieved by eliminating the direct use of reset times in the analysis. A more general result obtained a less conservative pair of LMIs [40].

Mechanical vibration of flexible was suppressed using reset actions as proposed in [41] by Bobrow et. al.. This is the first experimental application of reset control. Moreover, the results were not related to the previous work due to the lack of literature to the authors. In 1997, new control research on reset control appeared [42] by Hu H. et. al.. The advancement of theoretical results for the general framework of reset systems can be considered as one of the major contributions in the field of reset systems, see [39]. They established series of works exploring performance analysis [43, 44, 45], stability analysis [46, 47, 48], and experimental works [49].

There is another different research line regarding reset control, the work [50, 51], that focuses on the design of reset systems using the dissipation in energy of the closed loop system. This work considers a more general frame work of reset systems where the reset mechanism maps the state to a non-zero value. In addition, the considered reset surface is time varying and depends on the states as well. This framework can be considered by the theory of impulsive differential equations [52, 53, 54].

During the last two decades, a multitude of practical and experimental research in the field of reset control systems, see [55, 56, 57, 58, 59, 60, 61]. Another line of research based on the stability analysis of hybrid model approach in [62, 63, 64, 65, 66, 67, 68]. Reset control with saturation has been developed in [69, 70]. Moreover, experimental applications of reset control systems has been established in [69, 70].

Despite a multitude of research has brought many advantages of reset systems over the LTI systems, few works have investigated reset compensators in the presence of time delay ([71],[72], [73],[74],[75]). Several other works have considered reset control for real processes with time delay [76, 77, 78, 79]. A design methodology to obtain a reset controller for systems with time-delay can be taken from the proposed approach in [80, 75].

Control design using proportional-Integral control plus Clegg-Integrator (PI+CI) is proposed with tuning methods [81, 82, 83, 84]. This methodology was successfully applied to many experimental applications [85, 86, 84]. Reset adaptive observer is motivated by its appealing properties in [87], with the goal of obtaining a simple reset mechanism to reduce the overshot and settling time of the estimation process. It is worth mentioning that optimal reset observers are analyzed in [88]. Further advancements in reset observer theory are applied on multi-input-multi-output systems in [89] and to nonlinear systems in [90, 91].

In most practical applications, analog control systems have been replaced by discrete controllers. However, reset controllers can be easily implemented using digital computers. Reset controllers were applied on hard disc drives using discrete reset controllers in [58, 92, 59]. Moreover, networked control systems have been established using a formal representation of discrete time reset systems, see [93, 94, 95, 96].

1.2 Application and Examples of Impulsive Systems

The invention of reset control scheme leads to a new viewpoint of controlling plant that has changeable state variables. Moreover, it competes with the other control methods. We give basic examples of plants whose state variables change swiftly [97].

1. The impacts in physical systems can cause instant changes of velocities.

2. The population of a kind of bacterium by releasing its natural enemies at proper instants. In this case, state variables of population could be changed instantly.

3. In a nano-scale electronic circuits, the change of single-electron-tunneling-junctions charge can cause impulsive behavior.

4. When deep-space spacecraft has limited fuel supply, it can not leave its engine on continuously. It should be controlled by using reset control systems.

Some physical plants are difficult to be controlled by continuous inputs. In many cases reset control systems provide an efficient paradigm to deal with plants that can not afford continuous control laws.

In spite of the sound of the previous examples and the interesting behavior of the plants, reset control systems have been applied to an enormous number of non-impulsive practical applications. Using reset controllers, hard disc drive control is investigated in [98, 99, 100, 101, 102, 103]. Control of pH-processes are implemented using reset methodology in [104, 105]. Moreover, it is applied to control the thruster of marines in [106].

1.3 Reset Control Systems

Consider a feedback interconnection of linear time invariant plant $P(s)$ and a reset controller as illustrated in Figure 1.1. The broken box is used to represent the reset controller. Its continuous operation is given by a transfer function $G(s)$.

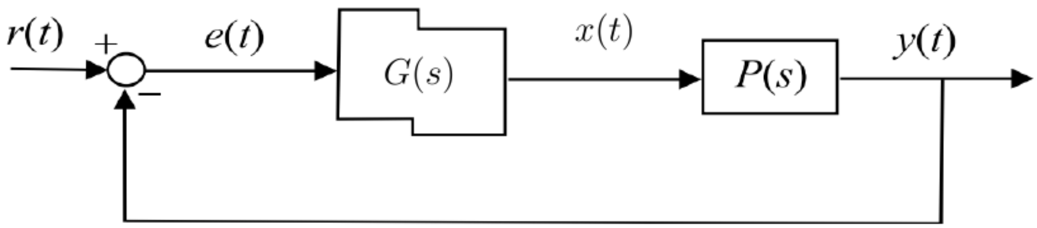

Figure 1.1: Standard linear reset control systems.

1.3.1 Clegg Integrator

Clegg integrator is a one of the simplest nonlinear controllers. It comprises a single linear integrator and a simple reset surface which causes a nonlinear-behavior. For the sake of simplicity, suppose the base system is linear and given by a transfer function:

$$G(s) = \frac{1}{s}$$

which is the transfer function of an integrator. This equation describes the continuous dynamics where its input is the error $e(t)$. Assume that a reset action changes its initial condition when the input is zero, i.e. $e(t) = 0$. The reset action of Clegg integrator is the zero reset of the state, i.e. $x(t^+) = 0$. Therefore, the reset state space equations that describe the reset controller (Clegg-type) are:

$$\begin{cases} \dot{x}(t) = e(t), & e(t) \neq 0 \\ x(t^+) = 0, & e(t) = 0 \end{cases}$$

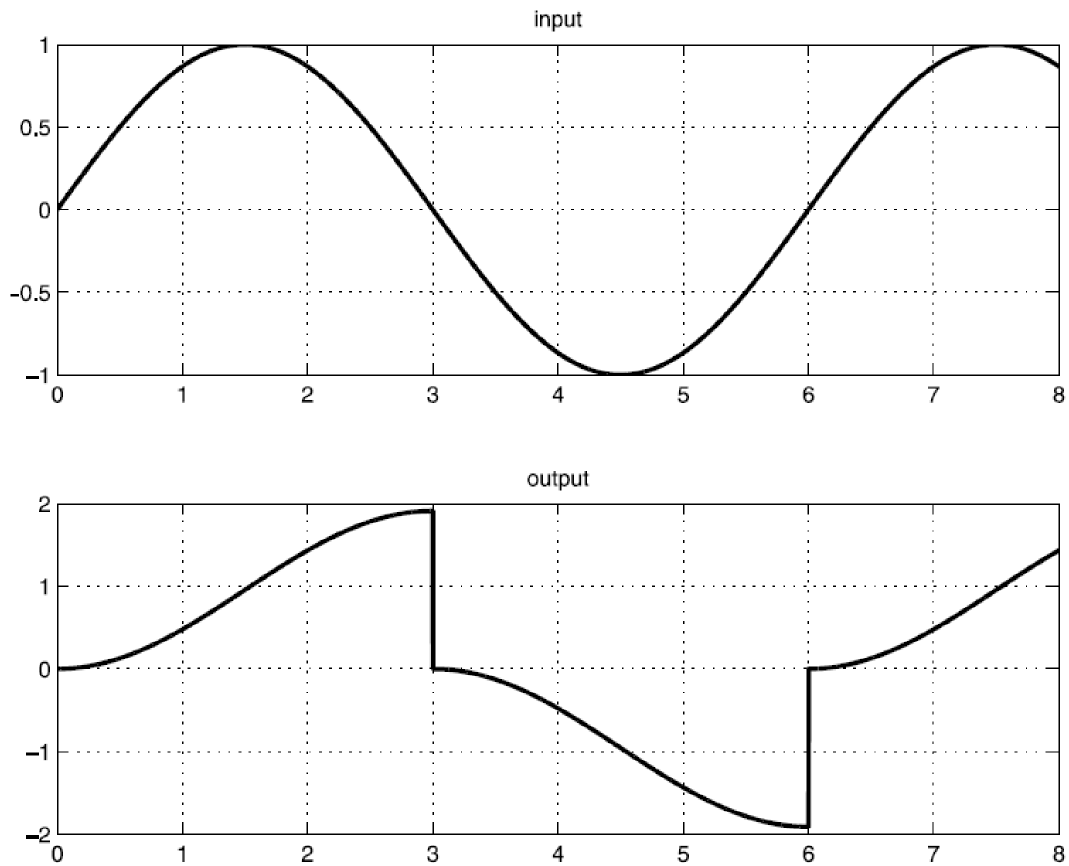

Figure 1.2: Clegg integrator response for a sinusoidal signal.

The first equation defines the continuous flow of the linear integrator. The last equation describes the discrete event or impulsive dynamics which is called the jump behavior. Figure 1.2 plots the response of the Clegg integrator to an input of sinusoidal behavior. The reset instants in this example are t_k such that $e(t_k) = sin(\omega t_k) = 0$, looking at Figure 1.2, when $t_k = 0, 3, 6...$ Normally the after reset value for the output is zero.

1.3.2 First Order Reset Element

As an illustration of the basic principle of reset control system, consider a first order reset element that its state vanishes when some condition holds. The reset rule in this type of reset controllers is activated when its input crosses the zero. Similar to Clegg integrators, two main equations are used to describe the first order reset controller. The first equation defines the base dynamics which is a linear first order differential equation, i.e. $\dot{x}(t) = -ax(t) + Ke(t)$. The second equation describes the after reset value of state, i.e. it illustrates the impulsive jump mode. Finally, the dynamics of the system changes between the continuous evolution and the discrete event depending on the zero crossing of the input.

$$\begin{cases} \dot{x}(t) = -kx(t) + Ke(t), & c(t) \neq 0 \\ x(t^+) = 0, & c(t) = 0 \\ u(t) = x(t) & \end{cases}$$

where $u(t)$ is the output of the controller or the control law, and $c(t)$ represents the reset condition where a jump to the value of $a(t)$ must be accomplished when $c(t) = 0$. Figure 1.3 demonstrates the response of the first order element controller to an input-error of sinusoidal behavior when $k = K = 1$. The reset instants in this example are $t = t_k$ such that $c(t) = 0$ where $a(t)$ is a saw-tooth signal. Figure 1.3 shows that the control law $u(t)$ is set to zero when $c(t) = 0$. The behavior of the response still the same between every two consecutive resets.

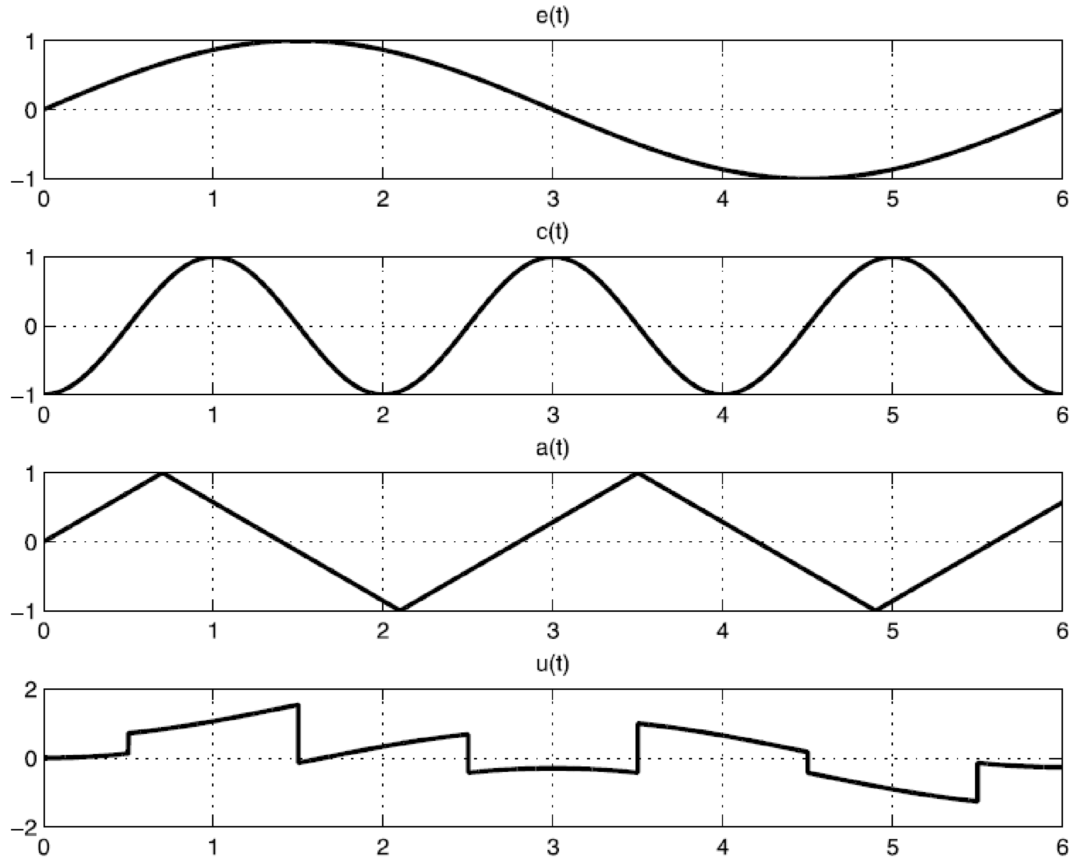

Figure 1.3: First-order-reset-element response for a sinusoidal signal.

1.4 Aims and Objectives of the Thesis

Our main objective in this thesis aims at providing an advancement in the field of reset control theory. Although the reset controllers show more advantages over conventional linear controllers, several drawbacks and challenges could arise. Hence, reset controller may destabilize a stable base control system if inappropriate reset sequence is applied. This means that reset systems must be designed carefully in order to improve the performance and robustness. The theory of reset control is much more difficult than linear systems theory. But it performs better and provides satisfactory results in a large number of delayed systems.

In impulsive dynamical systems, time delay occurs for different reasons depending on the nature of the system. The existence of time delay in the plant model is one of the major source of limitations of LTI systems. This motivates us to focus on reset systems to overcome the limitations of the LTI controller. In this regard, we are also interested in the development of stability for impulsive systems with time delay.

In this thesis, the desired progress in the field of reset systems theory will be accomplished by fulfilling the following research objectives:

1. To review the stability of reset control systems for plants without time delay. In addition, to study the stability of reset systems in the presence of time delay.

2. Our particular viewpoint is centered on analysis and synthesis of time delay systems using reset methodologies. Hence, several methods will be investigated to construct sufficient conditions for stability in terms of linear matrix inequalities.

3. To design reduced order reset controllers with new reset surfaces to stabilize unstable base dynamics. In addition, to construct new reset surfaces that enable us to stabilize linear time invariant systems with/without time delay.

4. To apply the developed theory on real processes in order to demonstrate its applicability.

1.4.1 List of Publications

The proposed methods and analysis results in this thesis have been accepted to be published or submitted to several journals and international conferences:

- M. Mahmoud, and B. Karaki, Chapter 8: "Reset Control Systems with Time Delays" in Time delay systems: Concepts, Design, and Stability Analysis, Nova Publisher 2018.
- M. Mahmoud, and B. Karaki, "Novel Results on Reduced Order Reset Controllers" (under review).
- M. Mahmoud, and B. Karaki, "Stability and Control Design of Reset Systems" (under review).

1.5 The Structure of The Thesis

This thesis comprises six chapters.

Chapter 2 looks at general principles regarding stability analysis of reset systems. It addresses the characteristics and aims of the terms reset mechanism and impulsive behavior, and assesses whether they are equivalent or not. It also explores the definition of the reset control system and time delay control systems, and which type of impulsive systems has relevant characteristics.

Chapter 3 deals with synthesis of reset controllers. Reduced order controllers for reset systems are presented in this chapter. Moreover, a simple design procedure is presented to construct reset controllers for stable and unstable base dynamics.

Chapter 4 focuses on the general reset control systems. General theorems are derived for the stability of state dependent reset surfaces and a multitude of design improvements are analyzed. We consider the $\mathcal{H}_\infty$ optimization problem for linear time invariant reset system. Finally, simple formulas are provided to calculate the maximum number of reset instants during the control process.

Chapter 5 addresses the stability of time delay systems. Similar to the Lyapunov-Krasoviskii theory, a theorem is proposed to prove a sufficient condition to check the stability of time delay reset systems. It is done using Krasoviskii functionals to produce LMIs condition. Stability of uncertain rest system with delay is also analyzed using the obtained LMIs.

Chapter 6 provides some ideas and future work in the reset control systems field.

1.5.1 Notations

In the sequel, $\mathbb{R}^n$ denote the n-dimensional vector space equipped with the Euclidean norm. We use W^T and W^{-1} to denote the transpose and the inverse of any square matrix W, respectively, and W^{-T} is the inverse transpose of the matrix. We use $W > 0$ to denote a symmetric positive definite matrix W and I to denote the $n \times n$ identity matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. In symmetric block matrices or complex matrix expressions, we use the symbol $\bullet$ to represent a term that is induced by symmetry.

CHAPTER 2

RESET CONTROL SYSTEMS

We begin this chapter by presenting a number of mathematical tools and notions of control theory to formally introduce the reset control theory in a consistent manner. This should give the reader a glimpse into how reset control system may be treated. To make the presentation more clear, we use impulsive differential equations in order to characterize reset system behavior. Moreover, stability analysis of reset systems with and without delays are presented in the sequel.

2.1 Control Theory and Mathematical Preliminaries

Before we state our analysis, we need a number of essential theorems and definitions. This section provides basic definitions and concepts on the stability of control systems. The stability definitions play a central role in dynamical systems. There are several types of stability problems that arise in the study of reset control systems.

2.1.1 Main Definitions and Theorems

Lyapunov analytical tools provide sufficient conditions for stability. Consider the following continuous system:

$$\dot{x}(t) = f(t, x(t)), \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $f(t, x(t)) : \mathbb{R} \times \mathbb{D} \rightarrow \mathbb{R}^n$ is a locally Lipschitz map, $\mathbb{D} \subset \mathbb{R}^n$, and t is a continuous time variable. A point x_e is said to be an equilibrium point of system (2.1) if $f(t, x_e) = 0$ for any $t \geq 0$. This implies that the trajectory remains at the equilibrium point if there is no external action on it. Without loss of generality, we assume the origin to be the equilibrium point because any non zero equilibrium point can be shifted easily. Using the change of variable $y = x - x_e$, then

$$\dot{y} = f(t, y + x_e) = g(t, y)$$

has an equilibrium at the origin, i.e. $y_e = 0$, where $g(t, 0) = 0$. If there is an external input moves the trajectory slightly from the origin, the trajectory might remain near the equilibrium or it move farther and farther away. The following definitions are defined below for system (2.1) when the $x_e = 0$.

Definition 2.1 *:*

1. *If for any* $\epsilon > 0$, *there is a positive real number* $\delta(\epsilon) > 0$, *such that*

$$\|x(t_0)\| < \delta(\epsilon) \implies \|x(t)\| < \epsilon$$

   then the system (2.1) is stable in the Lyapunov sense.

2. *If the system is stable and* δ *can be chosen such that:*

$$\|x(t_0)\| < \delta \implies \lim_{t\to\infty} x(t) = 0$$

   then the system (2.1) is asymptotically stable.

3. *If there exist constants* $\beta > 0$ *and* $\lambda > 0$ *such that:*

$$\|x(t_0)\| < \delta \implies \|x(t)\| \leq \beta\|x(t_0)\|e^{-\lambda(t-t_0)}$$

   then the system is exponentially stable.

If δ is independent of the initial time t_0 then a stability terms are called uniformly stable. Moreover, global-stability is referred to any one of the stability terms in Definition 2.1 when its corresponding condition is satisfied for all $x(t_0) \in \mathbb{R}^n$.

The same definition can be used to define the stability of delay systems. Let $\mathcal{C} = \mathcal{C}([-h, 0], \mathbb{R}^n)$ be the space of continuous functions that map the time interval

$[-h, 0]$ into a subspace of the Euclidean space $\mathbb{R}^n$. This space of functions is a normed space with a norm defined by $\|\phi\|_c = \max_{-h\leq\theta\leq 0} \|\phi(\theta)\|$. Then the general form of dynamical systems with delay can be described by a retarded functional differential equation with the initial function $\phi_t(\theta)$:

$$\dot{x}(t) = f(t, x_t)) \tag{2.2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $f : \mathbb{D} \times \mathcal{C} \to \mathbb{R}^n$, and $\phi_t(\theta) = \phi(t - \theta)$ for all $-h \leq \theta \leq 0$. The following definition of class $\mathcal{K}$ functions is used extensively to prove the stability of time varying and time delay systems.

Definition 2.2 *A continuous function $\alpha(.)$ is said to be a class $\mathcal{K}$-function if it is strictly increasing and $\alpha(0) = 0$. If it is not bounded, then it is called a class $\mathcal{K}_\infty$ function.*

Now we are in a position to present one of the main theorems in time delay systems, Lyapunov-Krasovski theorem:

Theorem 2.1 *Assume $f(t, x_t)$ in (2.2) maps $\mathbb{R} \times \mathcal{C}$ into a bounded set in $\mathcal{R}^n$. Let there exist class $\mathcal{K}$ functions $\alpha_1(.), \alpha_2(.)$, and a nonnegative and nondecreasing continuous function $w(.)$. If there exists a continuously differentiable functional $V(t, \phi(t))$ that satisfies the following:*

$$\alpha_1(\|\phi(0)\|) \leq V(t,\phi) \leq \alpha_2(\|\phi\|_c) \tag{2.3}$$

such that

$$\dot{V}(t,\phi(t)) \quad \leq \quad -w(\phi(0)) \tag{2.4}$$

$\forall x(t) \in \mathbb{R}^n$. *Then the time delay system (2.2) is uniformly stable. If* $w(.)$ *belongs to the class* $\mathcal{K}$*-functions, then the time delay system is asymptotically stable. Moreover, if* $u(.)$ *belongs to the class* $\mathcal{K}_\infty$*-functions, then the system is globally asymptotically stable.*

Definition 2.3 *The solution* $x(t)$ *of the dynamical system (2.1) is uniformly bounded if for every constant* $\alpha \in (0,a)$ *for some constant* $a > 0$*, there exists* $\beta(\alpha) > 0$ *independent of* t_0 *such that*

$$\|x(t_0)\| \leq \alpha \implies \|x(t)\| \leq \beta$$

Definition 2.4 *The space* $\mathcal{L}_2$ *contains all measurable functions* $v(.) : \mathbb{R}^+ \rightarrow \mathbb{R}^n$ *such that:*

$$\|v(t)\|^2_{\mathcal{L}_2} = \int_0^\infty v^T(t)v(t)dt$$

is finite.

Definition 2.5 *The* $\mathcal{L}_2$*-gain is defined as the maximum energy gain from an input* *signal $w(t)$ to system's output signal $z(t)$, which is given by the following quantity*

$$\mathcal{L}_2\text{-}gain = \sup_{w(t)\in\mathcal{L}_2} \frac{\|z(t)\|_{\mathcal{L}_2}}{\|w(t)\|_{\mathcal{L}_2}}$$

.

Fact 1 *For any positive definite scalar λ and any real matrices X and Y, with appropriate dimensions,*

$$X^TY + Y^TX \leq \lambda X^TX + \lambda^{-1}Y^TY$$

2.2 Impulsive Systems

The essential mathematical tool for analyzing reset control systems is the impulsive differential equations theory. Several books on the theory of impulsive differential equations are available [97, 107, 108, 109, 110, 111, 112]. The reader might refer to these books for the detailed mathematical analysis. In this section, we present the basic impulsive control paradigm based on impulsive deferential equations.

Impulsive systems comprises at least one state variable with impulsively effect. Hence, not all plants can be modeled by impulsive scheme. However, non-impulsive plant could be controlled by impulsive controller. Impulsively state variables change instantly at discrete events. These events can be generated whenever some conditions are satisfied.

When the reset sequence is demonstrated as a function of time, the dynamics is considered as an impulsive system [15]. Reset control systems framework is based on the theory of Impulsive Dynamical Systems (IDS). Impulsive Dynamical Systems are mainly characterized by three elements:

1. Continuous flow dynamics.

2. Discrete flow dynamics.

3. Reset surfaces or reset law.

The continuous-time dynamics is described by a differential equation (2.5) that represents the behavior of the impulsive system between consecutive reset events.

$$\dot{x}(t) = f(t, x(t)). \tag{2.5}$$

The discrete dynamics (2.6) is used to govern the instantaneous change in the states.

$$\Delta x(t) = x(t^+) - x(t) = J(t, x(t)), \tag{2.6}$$

where $x(t^+) = \lim_{\delta \to 0, \delta \geq 0} x(t + \delta)$ is the new state after the impulsive event. $J(t, y(t))$ represents the impulsive law that maps the state at a specific instant.

Finally, The reset law is used to govern the switching between the continuous and discrete events. The reset law is characterized by a surface $\mathcal{S}$ in which the reset event occurs when $(t, x(t)) \in \mathcal{S}$ [113]. Then the overall impulsive system can be written in its general form as follows:

$$\begin{cases} \dot{x}(t) = f(t, x(t)), & (t, x(t)) \notin \mathcal{S} \\ x(t^+) = x(t) + J(t, x(t)), & (t, x(t)) \in \mathcal{S} \\ y(t) = h(t, x(t), u(t)), & \end{cases} \tag{2.7}$$

2.2.1 Types of Impulsive Systems

Let $\mathbb{T} = \eta_k(t),\ \eta_k(t) < \eta_{k+1}(t), k = 1, 2, ...$ be a given set of jump instants that governs the reset law. Then, based on the characteristics of the control law, impulsive systems can be classified into three types [97]:

Type-I: In this type, $J(t, y(t))$ represents the impulsive control law which is an impulsive implemented by instant jumps. Note that the plant is controlled by a pure sudden jumps.

$$\begin{cases} \dot{x}(t) = f(t, x(t)), & t \neq \eta_k(x) \\ \Delta x(t) = J(t, y(t)), & t = \eta_k(x) \\ y(t) = h(t, x(t)) & \end{cases} \tag{2.8}$$

Type-II: The plant is controlled by the impulsive control law $J(t, y(t))$ and the continuous control law $u(t)$.

$$
\begin{cases}
\dot{x}(t) = f(t, x(t), u(t)), & t \neq \eta_k(x) \\
\Delta x(t) = J(t, y(t)), & t = \eta_k(x) \\
y(t) = h(t, x(t), u(t)),
\end{cases}
\tag{2.9}
$$

Type-III: In this case, the control law is pure continuous $u(t)$ where the plant itself is impulsive but there is no impulsive control law.

Based on the choice of the resetting law, three primary types of impulsive systems :

1. Impulsive systems with fixed time-events: In this type, the sequence η_k is independent of the state vector $x(t)$ and the initial condition x_0.
2. Impulsive systems with variable time-events: The resetting surface depends on both time and the state vector. This type is more difficult than the impulsive system with fixed time-events.
3. Impulsive systems with state dependent events: In this case, the continuous flow dynamics and the resetting surface is time independent.

2.3 Stability of Reset Systems

Consider the unforced linear reset system:

$$\begin{cases} \dot{x}(t) = A_{cl}x(t) & x(t) \notin \mathcal{M} \\ x(t^+) = A_\rho x(t) & x(t) \in \mathcal{M} \\ y(t) = Cx(t) \end{cases} \tag{2.10}$$

where the reset surface $\mathcal{M} = \{x(t) \in \mathbb{R}^n : Cx(t) = 0 \text{ and } (A_\rho - I)x(t) \neq 0\}$.

Theorem 2.2 *[114] If there exists a continuously differentiable, positive definite, and radially unbounded function $V(x) : \mathbb{R}^n \to \mathbb{R}$ such that*

$$\dot{V}(x) = \frac{\partial V}{\partial x} A_{cl} < 0 \quad x \in \mathcal{M}$$

$$\Delta V(x) = V(A_\rho x) - V(x) \leq 0, \quad x \notin \mathcal{M}$$

Then the equilibrium state is globally asymptotically stable.

The dynamical system (2.10) is called quadratically stable if there exists a quadratic function $V(x) = x^T(t)Px(t)$ that satisfies the conditions in Theorem 2.2 where P is a positive definite matrix.

Theorem 2.3 *[114] There exist a positive definite matrix P and a scalar β such that the following transfer function matrix*

$$H_\beta(s) = \begin{bmatrix} \beta C_p & 0 & P \end{bmatrix} (sI - A_{cl})^{-1} \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \tag{2.11}$$

is positive real if and only if the reset control system in (2.10) is quadratically stable.

This condition is called the H_β condition which can be checked using linear matrix inequalities. It is reduced to a simple version when the order of reset control system is one. It is worth to mention that not all stable reset control systems are quadratically stable.

2.4 Stability of Time Delay Reset Systems

It is of great importance to study the idea of delayed reset systems. The purpose of this section is to give a glimpse of the problem time delay reset controller by presenting its representation and structure. As it is well known in the literature that plants having right half-plane zeros and plants that have time-delays are generally hard to control. If the delay is relatively large, it is difficult to obtain a linear controller that provide a fast and a robust performance. Designer has to compromise between a fast response with bad stability margin and a large stability margin with poor response. Time-delay is one of the main sources of control limitations, while reset control is one of the simplest nonlinear methodology that is used to overcome this limitation.

In the sequel, we closely review the stability problem for the single delay case and aim at presenting LMI-based stability conditions. We look at a control system constructed by a plant P and a reset controller G:

$$
\begin{aligned}
&(P) \quad \begin{cases} \dot{x}_p(t) = A_p x_p(t) + B_p u_p(t) \\ y(t) = C_p x_p(t) \end{cases} \\
&(G) \quad \begin{cases} \dot{x}_r(t) = A_c x_r(t) + B_c u_r(t) & x(t) \notin \mathcal{M} \\ x_r(t^+) = A_\rho x_r(t) & x(t) \in \mathcal{M} \end{cases}
\end{aligned}
\tag{2.12}
$$

where the first equation defines the plant dynamics with the state $x_p(t) \in \mathbb{R}^{n_p}$, the second equation represents the reset controller with the state $x_r(t) \in \mathbb{R}^{n_c}$, t^+ is the after reset time, and $\mathcal{M}$ is the reset surface. The reset surface defies the set of trajectories that activate the discrete event of the reset controller.

The closed loop control between the plant (P) and the reset controller (G) might affected by a delay $\tau(t)$. Let the closed loop connection is given by $u_p(t) = C_c x_r(t - \tau(t)) - KC x_p(t - \tau(t))$ and $u_r(t) = -y(t)$, then the closed loop system is easily written as

$$
\begin{cases}
\dot{x}(t) = Ax(t) + A_d x(t-\tau(t)), & \Big\} \quad (x(t), x(t-\tau), \Delta) \notin \mathcal{M} \\
\\
x(t^+) = A_r x(t), & \Big\} \quad (x(t), x(t-\tau), \Delta) \in \mathcal{M}
\end{cases}
\tag{2.13}
$$

where $x(t) = \begin{bmatrix} x_p^T(t) & x_r^T(t) \end{bmatrix}^T$, $A_r = \text{diag}\left\{I_{n_p}, A_\rho\right\}$, $A = \begin{bmatrix} A_p & 0 \\ -B_c C_p & A_c \end{bmatrix}$, and $B = \begin{bmatrix} -B_p K C_p & B - pC_c \\ 0 & 0 \end{bmatrix}$.

Since the main focus of this section will be devoted on the time delay reset systems independently of the structure of the plant and the controller matrices, they are assumed to be arbitrary for the sake of generality. Moreover, time regulation is added to the time delay system in order to avoid the Zeno solutions. A linear time invariant delay reset system with time regulation may be described the following impulsive functional differential equation:

$$
\begin{cases}
\left.\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-\tau(t)), \\ \dot{\Delta}(t) &= 1, \end{aligned}\right\} & (x(t), x(t-\tau), \Delta) \notin \mathcal{M} \\
\left.\begin{aligned} x(t^+) &= A_r x(t), \\ \Delta(t^+) &= 0, \end{aligned}\right\} & (x(t), x(t-\tau), \Delta) \in \mathcal{M} \\
\left.\begin{aligned} x(t) &= \psi(t), \\ \Delta(t) &= 0, \end{aligned}\right\} & t \in [-h, 0]
\end{cases}
\tag{2.14}
$$

where $\Delta \in \mathbb{R}^+$, $\tau(t) \leq h$ and $\psi(t)$ is the initial condition.

The reason for using reset controllers over time-delay systems is that they are capable of overcoming the limitation in linear systems and the complexity of non-linear systems. In spite of being the main reason for using this kind of controllers, the research activities that study the design of time delay reset controllers are very rare.

2.4.1 Sufficient Stability Conditions

Stability of delayed reset systems is addressed in this section. It is important to distinguish between the initial value $x(0)$ for ordinary differential equations and the initial value function $\psi(t)$ for functional differential equations. This initial function is a distributed state $x_t(\theta) = x(t+\theta)$, $\theta \in [-h, 0]$, and defined by the piecewise continuous space $\mathcal{C}([-h, 0], \mathbb{R}^n)$.

Different Lyapanov-based stability results for delayed time reset control systems have been established by several authors (see, e.g., [115, 116, 117, 118]).

Proposition 2.1 *[119] Consider a continuously differentiable functional* $V(x_t) : \mathcal{C} \to \mathbb{R}$ *with possibility of discontinuities when* $x_t(0) \in \mathcal{M}$*, and for some* $\epsilon > 0$ *it satisfies:*

$$V(x_t) \geq \epsilon \|x_t(0)\|^2,$$

and its time derivative along the trajectory of (2.14) satisfy:

$$\dot{V}(x_t) \leq -\epsilon \|x_t(0)\|^2, \qquad x \notin \mathcal{M},$$

and the difference of the functional between reset instances is:

$$\Delta V(x_t) = V(x_{t^+}) - V(x_t) \leq 0, \quad x \in \mathcal{M}$$

where $\mathcal{M} := \{x \in \mathbb{R}^n : Cx = 0\}$*. Then the zero equilibrium* $x_t = 0$ *is globally asymptotically stable.*

One possibility to use this proposition is to consider several Lyapunov-Krasovskii functoinals:

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) \tag{2.15}$$

this Lyapunov-Krasovskii functional is defined by the following integral-quadratic forms:

$$V_1(x_t) = x^T(t)\mathcal{P}x(t)$$

$$V_2(x_t) = \int_{-h}^{0} x^T(t+\theta)\mathcal{S}x(t+\theta)d\theta$$

$$V_3(x_t) = \int_{-h}^{0}\int_{\theta}^{0} f^T(t+\xi+\theta)\mathcal{Z}f(t+\xi+\theta)d\xi d\theta$$

with $\mathcal{P}, \mathcal{S}$, and $\mathcal{Z}$ are positive definite matrices, and $f(t) = Ax(t) + A_d x(t)$.

Suppose the reset matrix A_ρ is zero and $n = n_1 + n_2 + n_3$, then $A_r = diag(I_{n12}, 0_{n3})$ where $n_{23} = n_2 + n_3$ and $n_{12} = n_1 + n_2$. The first n_{12} states of $x(t)$ are not resettable while the last n_3 states vanish at any reset instant. In this way, by using the Lyapunov-Krasovskii functionals we arrive to stability conditions defined by linear matrix inequalities. The time derivative requirement of proposition (2.1) has been already treated in [120] and the proof of the incremental condition can be found in [119]. The combination of these conditions leads to the following proposition:

Proposition 2.2 *[119] Let a partitioned matrix $\{\mathcal{P} = (P_{i,j}), i, j = 1, 2, 3\}$ be positive definite. The control system (2.14) with maximum delay h is quadratically stable for some functional $V(x_t)$ as stated in Proposition 2.1, if and only if there exist matrices $\mathcal{X} = \mathcal{X}^T$, $\mathcal{Y}$, and $\mathcal{Q}$, with*

$$(0, 0, I_{n3})\mathcal{P} = (\beta(C_1, C_2), \mathcal{P}_{33}), \tag{2.16}$$

where the output matrix is partioined as $C = [C_1 \quad C_2 \quad 0]$ *and*

$$\begin{bmatrix} \mathcal{N} & \mathcal{P}A_d - \mathcal{Y} & -A^T\mathcal{Y}^T \\ A_d^T\mathcal{P} - \mathcal{Y}^T & -\mathcal{Q} & -A_d^T\mathcal{Y}^T \\ -\mathcal{Y}A & -\mathcal{Y}A_d & -\frac{1}{h}\mathcal{X} \end{bmatrix} < 0 \tag{2.17}$$

for any appropriate matrix β, *and*

$$\mathcal{N} = \mathcal{P}A + A^T\mathcal{P} + \mathcal{Q} + h\mathcal{X} + \mathcal{Y} + \mathcal{Y}^T$$

Another useful proposition that uses the basic tools of Lyapunov-Krasovskii theory to address the stability of delayed reset systems:

Proposition 2.3 *[121] Let* $V(x_t) : \mathcal{C} \to \mathbb{R}$ *be a continuously differentiable radially unbounded such that it is positive-definite and its time derivative along (2.14)*

$$\dot{V}(x_t) < 0 \quad x_t \neq 0.$$

and its increment satisfy

$$\Delta V(x_t) = V(x_{t^+}) - V(x_t) \leq 0, \quad x(t) \in \mathcal{M}.$$

Then the reset system (2.14) is globally asymptotically stable.

In general, the choice of Lyapunov-Krasovskii functional is very important for deriving LMI conditions. Different forms of the functional produces different classes of conditions. The classification of these condition can be made based on the dependency of the time delay. The following proposition provides delay-independent conditions:

Proposition 2.4 *The delayed reset system in (2.14) is quadratically stable if and only if there exist positive definite matrices $\mathcal{P}$, $\mathcal{Q}$, such that*

$$\begin{bmatrix} A^T\mathcal{P} + \mathcal{P}A + \mathcal{Q} & \mathcal{P}A_d \\ \bullet & -\mathcal{Q} \end{bmatrix} < 0 \tag{2.18}$$

$$\Theta^T(A_r^T\mathcal{P}A_r - \mathcal{P})\Theta \leq 0 \tag{2.19}$$

for any matrix Θ with Im $\Theta =$ Ker C.

The reset mechanism introduces another condition to the standard stability of delay systems. Since the reset matrix A_r has a very special form, it can be considered as a restriction over the LMI. We note that $A_r = \text{ diag } (I, A_\rho)$ has a diagonal structure that allows us to use partitioning in order to simplify the condition. Let $A_\rho = 0$, then

$$A_r^T\mathcal{P}A_r - A_r = A_r^T \begin{bmatrix} \mathcal{P}_1 & \mathcal{P}_2 \\ \bullet & \mathcal{P}_3 \end{bmatrix} A_r = \begin{bmatrix} \mathcal{P}_1 & 0 \\ 0 & 0 \end{bmatrix}$$

wher $\mathcal{P}_1 > 0$, then condition (2.19) becomes

$$\Theta^T(A_r^T\mathcal{P}A_r - \mathcal{P})\Theta = \begin{bmatrix} \Theta_1^T & \Theta_2^T \end{bmatrix} \begin{bmatrix} 0 & -\mathcal{P}_2 \\ -\mathcal{P}_2 & -\mathcal{P}_3 \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix} \tag{2.20}$$

where $\Theta = \begin{bmatrix} \Theta_1^T & \Theta_2^T \end{bmatrix}^T$. Since $C = (C_1, 0)$ and the output is scalar, the following proposition holds.

Proposition 2.5 *The delayed reset system in (2.14) is quadratically stable if and only if there exist positive definite matrices $\mathcal{P}$, $\mathcal{Q}$, with*

$$\mathcal{P} = \begin{bmatrix} \mathcal{P}_1 & C_1^T\beta^T \\ \bullet & \mathcal{P}_3 \end{bmatrix} \tag{2.21}$$

such that

$$\begin{bmatrix} A^T\mathcal{P} + \mathcal{P}A + \mathcal{Q} & \mathcal{P}A_d \\ \bullet & -\mathcal{Q} \end{bmatrix} < 0 \tag{2.22}$$

where β is some matrix with appropriate dimension.

Another less conservative LMI stability condition is presented in Proposition 2.6.

Proposition 2.6 *[122] If there exists symmetric matrices $\mathcal{P} > 0$, and $\mathcal{V} > 0$ such* *that*

$$\Gamma = \begin{bmatrix} A^T\mathcal{P} + \mathcal{P}A + \mathcal{Q} & \frac{1}{2}(A_d^T\mathcal{P} + \mathcal{P}A) \\ \bullet & -\mathcal{Q} \end{bmatrix} \tag{2.23}$$

and

$$\Theta^T(A_r^T\mathcal{P}A_r - \mathcal{P})\Theta \leq 0 \tag{2.24}$$

for some $\Theta \in \mathbb{R}^{n\times m}$*, such that Proj*$(\mathcal{Z}) \subseteq \mathcal{R}(\Theta)$*.*

Remark 1 *The matrix* Θ *must be chosen appropriately for the success of the stability criterion. When* Θ *is the identity matrix, the projection is satisfied for all* $\mathcal{Z}$*. But the appropriate election of may lead to a reduced dimension of the LMI condition. This fact has been disscused in [121] and [123] for time delay reset systems.*

The aforementioned results and propositions focus on delay-independent stability conditions for rest-delay systems. The idea to develop delay-dependent stability condition is to add double integral terms to the Lyapunov-Krasovskii functional. In the following, the time delay is assumed to be constant with $\tau(t) = h$ and three propositions [121, 122] about delay-dependent stability are given.

Proposition 2.7 *If there exist matrices* $\mathcal{P}, \mathcal{Q}, \mathcal{R} > 0$*,* $X^T = X$*, and* $Y \in \mathbb{R}^{n\times n}$

such that

$$\begin{bmatrix} \mathcal{P}A + A^T\mathcal{P} + hX + Y + Y^T + hA^T\mathcal{R}A + \mathcal{Q} & \mathcal{P}A_d - Y + hA^T\mathcal{R}A_d \\ \bullet & -\mathcal{Q} + hA_d^T\mathcal{R}A_d \end{bmatrix} < 0$$

$$\begin{bmatrix} X & Y \\ \bullet & -\mathcal{R} \end{bmatrix} < 0$$

$$Y\big(A_r - I\big)$$

$$\Theta^T(A_r^T\mathcal{P}A_r - \mathcal{P})\Theta \leq 0$$

for some $\Theta \in \mathbb{R}^{n\times m}$*, such that Proj*$(\mathcal{Z}) \subseteq \mathcal{R}(\Theta)$*.*

In comparison with the results of Proposition2.2, the main difference is the delay-dependent LMI condition. On the other hand, more general Lyapunove-Krasovskii functional is needed to provide the delay dependent criteria. Moreover, choosing the matrices X and Y as follows:

$$X = (W^T + \mathcal{P})Z^{-1}(W + \mathcal{P})$$

$$Y = (W^T + \mathcal{P})A_d$$

where $Z > 0$ and W is an appropriate matrix, then the results in [121] is obtained as stated in proposition 2.8.

Proposition 2.8 *[121] Let a partitioned matrix $\{\mathcal{P} = (P_{i,j}), i, j = 1, 2, 3\}$ be positive definite. The control system (2.14) is asymptotically stable, if there exist matrices $\mathcal{Q} > 0$, $\mathcal{V}^T > 0$, and $\mathcal{W}$, with*

$$(0, 0, I)\mathcal{P} = (M_\beta(C_1, C_2), \mathcal{P}_{33}), \tag{2.25}$$

and

$$\begin{bmatrix} \mathcal{N} & -\mathcal{W}A_d & A^T A_d^T \mathcal{V} & h\big(\mathcal{W} + \frac{1}{2}\mathcal{P}\big) \\ \bullet & -\mathcal{Q} & A^T A_d^T \mathcal{V} & 0 \\ \bullet & \bullet & -\mathcal{V} & 0 \\ \bullet & \bullet & \bullet & -\mathcal{V} \end{bmatrix} < 0$$

for some matrix M_β of adequate dimension. where

$$\mathcal{N} = \frac{1}{2}\big((A + A_d)^T \mathcal{P} + \mathcal{P}(A + A_d)\big) + \mathcal{W}A_d + A_d^T \mathcal{W} + \mathcal{Q}$$

Introdusing a new parameter r to split the integrands of the Lyapunove-Krasovskii functional implies a reduction of the conservativeness. Hence, the maximum time delay is very close to the actual value. This improvement is due to the use of gap reduction in Jensen's inequality using the reduction of the integration time (see [124] for a detailed analysis).

Proposition 2.9 *The trivial solution of the reset system (2.14) is globally asymptotically stable, if there exists an integer* $r > 0$ *and matrices* $\mathcal{P} > 0, \mathcal{R} > 0$ *and* $\mathcal{Q}$ *such that*

$$M_{\mathcal{P}}^T \begin{bmatrix} 0 & \mathcal{P} \\ \mathcal{P} & 0 \end{bmatrix} M_{\mathcal{P}} + M_{\mathcal{R}}^T \begin{bmatrix} \frac{h}{r}\mathcal{R} & 0 \\ 0 & -\frac{h}{r}\mathcal{R} \end{bmatrix} M_{\mathcal{R}} + M_{\mathcal{Q}}^T \begin{bmatrix} \mathcal{Q} & 0 \\ 0 & \mathcal{Q} \end{bmatrix} M_{\mathcal{Q}} < 0.$$

$$\mathcal{R}(A_r - I) = 0$$

$$\Theta^T (A_r^T \mathcal{P} A_r - \mathcal{P}) \Theta \leq 0$$

for some $\Theta \in \mathbb{R}^{n \times m}$*, such that Proj*$(\mathcal{Z}) \subseteq \mathcal{R}(\Theta)$*, where*

$$M_{\mathcal{P}} = \left[\begin{array}{ccc} A & 0 & A_d \\ \hline I_n & & 0 \end{array}\right], \quad M_{\mathcal{R}} = \left[\begin{array}{ccc} A & 0 & A_d \\ \hline -I_n & I_n & 0 \end{array}\right], \quad M_{\mathcal{Q}} = \left[\begin{array}{cc} I_{rn} & 0 \\ \hline 0 & I_{rn} \end{array}\right].$$

CHAPTER 3

REDUCED ORDER RESET CONTROLLERS

3.1 Preliminaries

This chapter establishes a new mechanism to stabilize plants using reduced order reset controllers. The proposed method uses state feedback to change the dynamics of plant. We show that the base system could be unstable while the reset mechanism drives the states to the equilibrium point. The order of the reset controller equals to the rank of the plant's input matrix. It has been shown that the controller forces a number of states to reach the origin in a finite time while the remaining states behavior depends on the state feedback which can be chosen by any appropriate classical method. Simulation results are used to illustrate the effectiveness of the proposed approach.

Although the reset controllers show more advantages over conventional linear controllers, several drawbacks and challenges could arise. On one hand, reset controller may destabilize a stable base control system if inappropriate reset sequence is applied. This means that reset systems must be designed carefully in order to improve the performance specifications. On the other hand, it might be there exists linear controller performs better than the reset controller. Although reset control theory is much more difficult than linear systems theory, we show that the our reduced-order technique gives a relatively simple procedure that can stabilize the system while decreasing the settling time efficiently.

3.2 Problem Formulation

Consider a plant described by the following linear time invariant system

$$\dot{x}(t) = A_p x(t) + B_p u(t) \tag{3.1}$$

where $x_p(t) \in \mathbb{R}^n$ is the state vector of the plant, $u_p(t) \in \mathbb{R}^m$ is the input of the plant, $A_p \in \mathbb{R}^{n\times n}$, and $B_p \in \mathbb{R}^{n\times m}$. The input matrix B_P is assumed to be a full column rank matrix.

The reset controller that is used to stabilize the plant (3.1) is represented by the following dynamics.

$$\begin{aligned} \dot{x}_r(t) &= A_{r1} x_p(t) + A_{r2} x_r(t) \\ x_r(t^+) &= A_{\rho 1} x_p(t) + A_{\rho 2} x(t) \end{aligned} \tag{3.2}$$

where $x_r(t) \in \mathbb{R}^m$ is the state vector of the controller, $A_{r1} \in \mathbb{R}^{m\times n}$, and $A_{r2} \in \mathbb{R}^{m\times m}$. Since $x_r(t)$ is the controller state, it can be used as a fully resettable state. $x_r(t^+)$ represents the after reset state, which is updated when some predefined condition holds. The dynamics of the reset controller and the plant can be written as:

$$\begin{cases} \left.\begin{aligned} \dot{x}(t) &= A_p x_p(t) + B_p u(t) \\ \dot{x}_r(t) &= A_{r1} x_p(t) + A_{r2} x_r(t), \\ \dot{\tau}(t) &= 1, \end{aligned}\right\} (x_p, x_r, \tau) \notin \mathcal{M} \\ \\ \left.\begin{aligned} x_p(t^+) &= x_p(t), \\ x_r(t^+) &= A_{\rho 1} x_p(t) + A_{\rho 2} x_r(t) \\ \tau(t^+) &= 0, \end{aligned}\right\} (x_p, x_r, \tau) \in \mathcal{M} \end{cases}$$

where $\mathcal{M}$ is the reset surface and $\tau(t)$ is a time regulation parameter. The parameter $\tau(t)$ is used to regulate the reset instants in order to avoid Zeno behavior. The reset mechanism is activated immediately when the trajectory of the reset system hits the surface $\mathcal{M}$. Let $u(t) = Kx_p(t) + Lx_r(t)$, then the overall reset control system becomes

$$\begin{cases} \dot{x}(t) = Ax(t), \dot{\tau}(t) = 1, & (x_a, x_b, \tau) \notin \mathcal{M} \\ x(t^+) = A_R x(t), \tau(t^+) = 0, & (x_a, x_b, \tau) \in \mathcal{M} \end{cases} \tag{3.3}$$

where $x(t) = \begin{bmatrix} x_P(t) \\ x_r(t) \end{bmatrix}$, $A = \begin{bmatrix} A + B_pK & B_pL \\ A_{r1} & A_{r2} \end{bmatrix}$, and $A_R = \begin{bmatrix} I & 0 \\ A_{\rho 1} & A_{\rho 2} \end{bmatrix}$.

The first equation in (3.3) represents the base system or the so called continuous flow dynamics while the second equation describe the jump behavior at reset instants. However, the asymptotic stability of the closed loop reset system (3.3) depends greatly on the structure of its base system. If A is Hurwitz, then it is enough for asymptotic stability of the closed loop system to ensure that A_R is Shure, see [125]. The stability of the reset system (3.3) is equivalent to the stability of the base system if the reset mechanism is not activated. Reset systems with an inappropriate reset sequence may become unstable even if the base dynamics is stable. On the other hand, a reset mechanism must be applied if the base system is unstable.

To our best knowledge, most of results in the literature study reset control theory when the base system is stable. In this thesis, new results are proposed to stabilize reset systems with unstable base dynamics. In addition, a reduced order controller is derived based on the aforementioned stabilization criterion.

3.3 Theoretical Results

Consider the following special type of reset systems, $A_{\rho 1}$ and $A_{\rho 2}$ are both zero matrices:

$$
\begin{cases}
\begin{bmatrix} \dot{x}_a \\ \dot{x}_b \end{bmatrix} = \begin{bmatrix} 0 & I \\ \Lambda_1 & \Lambda_2 \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix} \\
\dot{\tau}(t) = 1,
\end{cases} \quad (x_a, x_b, \tau) \notin \mathcal{M}
\\
\begin{cases}
\begin{bmatrix} x_a(t^+) \\ x_b(t^+) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix} \\
\tau(t^+) = 0,
\end{cases} \quad (x_a, x_b, \tau) \in \mathcal{M}
\tag{3.4}
$$

where $x_a(t) \in \mathbb{R}^n$ represents the continuous state without reset effect, $x_b(t) \in \mathbb{R}^n$ is the resettable state, $\tau(t) > 0$, and $\Lambda_i = \text{diag}(\lambda_{i1}, \lambda_{i2}, ..., \lambda_{in}), i = 1, 2$ with $\lambda_{ij} \in \mathbb{R}$. Let $x_a(t)$ and $x_b(t)$ be partitioned as $[x_{a1}(t), ..., x_{an}(t)]$ and $[x_{b1}(t), ..., x_{bn}(t)]$, respectively. It is obvious that the system is uncoupled and becomes oscillatory if $(\lambda_{2i}^2 + 4\lambda_{1i})$ is negative for all $i = 1, 2, ..., n$.

Lemma 3.1 *Let the reset surface in (3.4) be defined as* $\mathcal{M} = \{ [x_a^T(t), x_b^T(t), \tau(t)]^T \in \mathbb{R}^{2n+1} : \tau(t) > \bar{\tau}$ *and for any* $i = 0, 1, ..., n,\ x_{ai}(t) = 0$ *such that* $x_{bi}(t) \neq 0 \}$ *where* $\bar{\tau} = \min_{1 \leq i \leq n} \{\tau_i\}$ *and* $\tau_i = \frac{1}{\sqrt{-\lambda_{2i}^2 - 4\lambda_{1i}}}$. *If* $\lambda_{2i}^2 + 4\lambda_{1i} < 0,$ *for all* $i = 1, 2, ..., n,$ *then the states fall exactly into the origin at most within* $T = n\tau^\star$ *seconds, where* $\tau^\star = \max_{1 \leq i \leq n} \{3\tau_i\}$.

Proof. Since $x_a(t)$ and $x_b(t)$ are completely decoupled, then every pair x_{ai} and x_{bi} represent a second order subsystem for all $i = 1, 2, ..., n$. Obviously, from linear systems theory, each subsystem has complex eigenvalues since $(\lambda_{2i}^2 + 4\lambda_{1i})$ is negative, with the angular frequency $\omega_i = \sqrt{-\lambda_{2i}^2 - 4\lambda_{1i}}/2$. Regardless of the real part sign of the eigenvalues, this oscillation guarantees that $x_{ai}(t)$ crosses the zero before time reaches the half-period $\frac{\pi}{\omega_i}$ and then $x_{bi}(t)$ becomes zero after the reset.

When some element of $x_{ai}(t)$ crosses the zero, $x_{bi}(t)$ vanishes due to its reset. Consequently, both $x_{ai}(t)$ and $x_{bi}(t)$ stay at the origin even if other states do not reaches the origin because the system is completely decoupled. However, $x_{ai}(t) = 0$ can not activate the resetting because $x_{bi}(t) = 0$ as stated in $\mathcal{M}$. Every subsystem behaves in similar manner but with different settling time depending on its angular frequency. Finally, the required time for each state is the half period $\frac{\pi}{\omega_i}$ in addition to the regulation time $\bar{\tau}$, i.e. $T_i = \frac{\pi}{\omega_i} + \bar{\tau}$. With straight forward calculations, it is easy to show that $T = n\tau^\star$ is the maximum required time to ensure that all trajectories settled at the origin. ∎

Now, let $A_{\rho 1} = 0$ and $A_{\rho 2} = \text{diag}(1 - \delta_{1i}, 1 - \delta_{2i}, ..., 1 - \delta_{ni})$, where δ_{ji} is the Kronecker-delta function with $\delta_{ji} = 1$ if $j = i$, and zero otherwise. This structure of $A_{\rho 2}$ is used to activate resetting only for the state that hits the reset surface.

Lemma 3.2 *Let the reset surface in (3.4) be defined as* $\mathcal{M} = \{\, [x_a^T(t), x_b^T(t), \tau(t)]^T \in \mathbb{R}^{2n+1} : \tau(t) > \bar{\tau}$ *and for any* $i = 0, 1, ..., n$, $x_{bi}(t) = \alpha_i x_{ai}(t)$ *such that* $x_{bi}(t) \neq 0 \,\}$ *where,* $\bar{\tau} = \min_{1 \le i \le n} \{\tau_i\}$ *and* $\tau_i = \frac{1}{\sqrt{-\lambda_{2i}^2 - 4\lambda_{1i}}}$. *If there exists s such that*

$$|e^{\beta_i s} \cos(\omega_i s + \tan^{-1}(\frac{\beta_i}{\omega_i}))| < \sqrt{(\frac{\beta_i}{\omega_i})^2 + 1} \tag{3.5}$$

$$\alpha_i = \frac{x_{bi}(s)}{x_{ai}(s)}, \quad i = 1, 2, ..., n,$$

and

$$\lambda_{2i}^2 + 4\lambda_{1i} < 0$$

where, $\beta_i = \frac{\lambda_{2i}}{2}$ *and* $\omega_i = \sqrt{-\lambda_{2i}^2 - 4\lambda_{1i}}/2$, *then the close loop system (3.3) is asymptotically stable.*

Proof. As mentioned previously, every pair $x_{ai}(t)$ and $x_{ai}(t)$ of second-order-decoupled-subsystem becomes oscillatory if $(\lambda_{2i}^2 + 4\lambda_{1i})$ is negative for all $i = 1, 2, ..., n$. Then the general solution of $x_{ai}(t)$ can be written in the following form:

$$x_{ai}(t) = C_1 e^{\beta_i t} \cos(\omega_i t + C_2) \tag{3.6}$$

Without loss of generality, since the system is linear time invariant systems between reset instants, the initial time t_0 is assumed to be zero. Moreover, the initial condition of the resettable state can be assumed to be zero, i.e. $x_{bi}(t_0) = 0$ for $i = 1, ..., n$. Now it is easy to show that the constants C_1 and C_2 are given by $\omega_i x_{ai}(0)/\sqrt{\beta_i^2 + \omega_i^2}$ and $\tan^{-1}(\beta_i/\omega_i)$, respectively.

Since $\lambda_{2i}^2 + 4\lambda_{1i} < 0$, the solution is oscillatory. Hence, the trajectory $x_{ai}(t)$ crosses the line $x_{bi}(t) = \alpha_i x_{ai}(t)$ within a finite time s, for any α_i. When this occurs, $x_{bi}(t)$ vanishes because $x_{bi}(s^+) = (1-\delta_{ii})x_{bi}(s) = 0$ while $x_{aj}(s^+) = x_{aj}(s)$, $j \neq i$. To ensure the stability of the trajectories then the following condition must be satisfied:

$$|x_{ai}(s^{+})| = |x_{ai}(s)| < |x_{ai}(0)| \tag{3.7}$$

The stability of the system is guaranteed because Equation (3.7) is equivalent to $|x_{ai}(t_{k+1})| < |x_{ai}(t_k)|$, where $t_{k+1} - t_k = s$. Substitute the solution in Equation (3.6) at time $t = s$ in Equation (3.7) gives

$$|\frac{\omega_i x_{ai}(0)}{\sqrt{\beta_i^2 + \omega_i^2}} e^{\beta_i t} \cos(\omega_i t + \tan^{-1}(\beta_i/\omega_i))| < |x_{ai}(0)|$$

Straight forward calculations shows that this condition is equivalent to Equation (3.5). This shows that the point s is independent on the initial condition $x_{ai}(0)$ but depends on the dynamics of the system β_i and ω_i. Then α_i is given at $t = s$ using $\alpha_i = \frac{x_{bi}(s)}{x_{ai}(s)}$.

∎

Remark 2 *It is worth mentioning that the reset surface in Lemma 3.2 uses linear equation with a finite slope α_i instead of the zero crossing method in Lemma 3.1 with infinite slope. This finite slope impose a new condition that the decoupled components should have a separate reset rule from the other ones (that is, it is not true that all controllers are reset at the same time, but rather one by one when the corresponding condition is met). This is ensured by the reset matrix $A_{\rho 2}$ that is defined by the Kronecker-delta function as mentioned previously.*

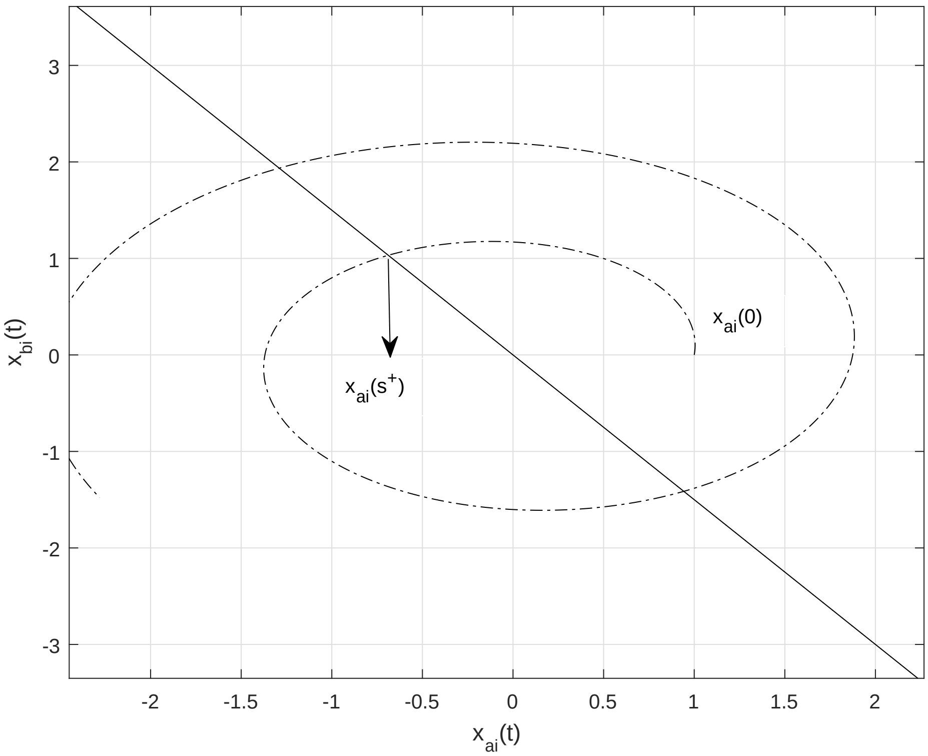

Figure 3.1: Unstable base system with the reset surface $\mathcal{M}$ presented in Lemma 3.1.

Figure 3.1 demonstrates how the state $x_{ai}(t)$ is contracted by an appropriate resetting surface. It is clear that the reset surface is linear equation and its slope is very crucial. In addition, the reset system is still stable even the trajectory reverses its direction. Hence, the linear equation of the reset surface $x_{bi}(t) = \alpha_i x_{ai}(t)$ and $x_{bi}(t) = -\alpha_i x_{ai}(t)$ are equivalent in terms of stability. Consequently, both of these linear equations can be constructed in the reset surface $\mathcal{M}$ to guarantee fast response. The fast response comes from the increasing number of resets from both sides of the trajectories where the shrinking events increases.

3.4 The Main Results

Now, we consider the closed loop reset system (3.3) to be stabilized by a reduced order reset system. The following theorem provides sufficient condition for asymptotic stability of the control system (3.3).

Theorem 3.1 *Let $A_{\rho 1}$ and $A_{\rho 2}$ be zero matrices. If there exist matrices $\Lambda_i = diag(\lambda_{i1}, \lambda_{i2}, ..., \lambda_{im}), i = 1, 2$ with $\lambda_{ij} \in \mathbb{R}$, $K, A_{r1}, A_{r2}, \mathcal{X}$ and L and an invertible matrix T such that the following equations are satisfied*

$$T(A + B_p K)T^{-1} = \begin{bmatrix} 0 & 0 \\ \mathcal{X} & A_{stable} \end{bmatrix} \tag{3.8}$$

$$TB_pL = \begin{bmatrix} I_m \\ 0 \end{bmatrix} \tag{3.9}$$

$$A_{r1}T^{-1} = \begin{bmatrix} \Lambda_1 & 0 \end{bmatrix} \tag{3.10}$$

$$A_{r2} = \Lambda_2 \tag{3.11}$$

$$\Lambda_2^2 + 4\Lambda_1 < 0, \tag{3.12}$$

then the reset control system (3.3) is asymptotically stabilized using the reset surface $\mathcal{M}$ defined in Lemma 3.1.

Proof. Applying a similarity transformation $T_1 = \begin{bmatrix} T & 0 \\ 0 & I_m \end{bmatrix}$, with $z(t) = T_1 x(t)$ to the reset systems in (3.3). The instantaneous change of the state at reset instances does not change under the similarity transformation T_1:

$$\begin{aligned} z(t^+) &= \begin{bmatrix} T & 0 \\ 0 & I_m \end{bmatrix} x(t^+) \\ &= \begin{bmatrix} T & 0 \\ 0 & I_m \end{bmatrix} A_R x(t) \\ &= \begin{bmatrix} T & 0 \\ 0 & I_m \end{bmatrix} \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T & 0 \\ 0 & I_m \end{bmatrix}^{-1} z(t) \\ &= \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix} z(t) \end{aligned} \tag{3.13}$$

which is equivalent to the original jump dynamics.

Using the same transformation, the continuous dynamics becomes

$$\dot{z}(t) = \begin{bmatrix} T(A + B_p K)T^{-1} & TB_p L \\ A_{r1}T^{-1} & A_{r2} \end{bmatrix} z(t) \tag{3.14}$$

Using Equations (3.8),(3.9), (3.10), and (3.11), Equation (3.14) can be written as:

$$\dot{z}(t) = \begin{bmatrix} 0 & 0 & I_m \\ \mathcal{X} & A_{stable} & 0 \\ \Lambda_1 & 0 & \Lambda_2 \end{bmatrix} z(t) \tag{3.15}$$

Using Equation (3.13) and another rearranging transformation for continuous dynamics $T_2 = \begin{bmatrix} I_m & 0 & 0 \\ 0 & 0 & I_m \\ 0 & I_{n-m} & 0 \end{bmatrix}$, with $\zeta(t) = T_2 z(t)$, the continuous and the

discrete dynamics of (3.3) becomes

$$\dot{\zeta}(t) = \begin{bmatrix} 0 & I & 0 \\ \Lambda_1 & \Lambda_2 & 0 \\ \mathcal{X} & 0 & A_{stable} \end{bmatrix} \zeta(t) \tag{3.16}$$

and

$$\dot{\zeta}(t^+) = \begin{bmatrix} I_m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_{n-m} \end{bmatrix} \zeta(t) \tag{3.17}$$

respectively Now, partition $\zeta(t)$ to be

$$\zeta(t) = \begin{bmatrix} \zeta_a^T(t) & \zeta_b^T(t) & \zeta_c^T(t) \end{bmatrix}^T \tag{3.18}$$

It is obvious that $\zeta_c(t)$ is decoupled from the dynamics of $\zeta_a(t)$ and $\zeta_b(t)$ which means that the stability of $\zeta_a(t)$ and $\zeta_b(t)$ is independent of the stability of $\zeta_c(t)$.

The continuous flow of $\zeta_a(t)$ is written as

$$\dot{\zeta}_c(t) = A_{stable}\zeta_c(t) + \mathcal{X}\zeta_a(t) \tag{3.19}$$

Since $\zeta_c(t)$ in Equation (3.19) is not resettable as can be seen from the discrete event in (3.17), the asymptotic stability of $\zeta_c(t)$ is guaranteed if $\zeta_a(t)$ is asymptotically stable. And $\zeta_a(t)$ and $\zeta_b(t)$ are asymptotically stable by Lemma 3.1, this implies that $\zeta_c(t)$ is asymptotically stable. This completes the proof. ∎

Remark 3 *The number of resettable states can be at most equals to the rank of B_p as can be concluded from Equation (3.9), i.e. $m \leq rankB_p \leq n$. Hence, m states of the plant vanish during a finite time as described in Lemma 3.1.*

It is easy to conclude that TB_p has a left inverse because B_p has full column rank while T is nonsingular. From Equation (3.9), the following must be satisfied

$$L = \left[(TB_p)^T(TB_p)\right]^{-1}(TB_p)^T \begin{bmatrix} I_m \\ 0 \end{bmatrix} \tag{3.20}$$

The following steps provide a systematic method to obtain the required variables for reduced order controller using Theorem 3.1. First of all we determine the rank of the input matrix B in order to select the reset controller order. Let the order of the reset controller be m. Then, choose a matrix K such that the eigenvalues of $(A + B_pK)$ are m-zeros and $n - m$ stable eigenvalues. The stable eigenvalues could be chosen using any method like pole-placement or LQR method

... etc. Now we are ready to obtain the transformation T. Construct A_{stable} using an upper triangle matrix such that it contains the same stable eigenvalues of $(A+B_pK)$. Finally, we solve the equations in Theorem 3.1, keeping in mind that $\Lambda_2 = A_{r2}$. L can be calculated using Equation (3.20).

Theorem 3.2 *Let $A_{\rho 1}$ be a zero matrix and $A_{\rho 2} = diag(1-\delta_{1i}, 1-\delta_{2i}, ..., 1-\delta_{ni})$. And the reset surface in (3.4) is defined as in Lemma 3.2. If there exists s satisfies*

$$|e^{\beta_i s}\cos(\omega_i s + \tan^{-1}(\frac{\beta_i}{\omega_i}))| < \sqrt{(\frac{\beta_i}{\omega_i})^2 + 1} \tag{3.21}$$

with

$$\alpha_i = \frac{x_{bi}(s)}{x_{ai}(s)}, \quad i = 1, 2, ..., n,$$

and there exist matrices $\Lambda_i = diag(\lambda_{i1}, \lambda_{i2}, ..., \lambda_{im}), i = 1, 2$ with $\lambda_{ij} \in \mathbb{R}$, $\mathcal{X}$, L and an invertible matrix T such that the following equations are satisfied

$$\lambda_{2i}^2 + 4\lambda_{1i} < 0$$

$$T(A + B_pK)T^{-1} = \begin{bmatrix} 0 & 0 \\ \mathcal{X} & A_{stable} \end{bmatrix} \quad (3.22)$$

$$TB_pL = \begin{bmatrix} I_m \\ 0 \end{bmatrix} \quad (3.23)$$

$$A_{r1}T^{-1} = \begin{bmatrix} \Lambda_1 & 0 \end{bmatrix} \quad (3.24)$$

$$A_{r2} = \Lambda_2 \quad (3.25)$$

$$\Lambda_2^2 + 4\Lambda_1 < 0, \quad (3.26)$$

where, $\beta_i = \frac{\lambda_{2i}}{2}$*,*

then the reset control system (3.3) is asymptotically stable.

Proof. Similar to the proof of Theorem 3.1, but instead of using Lemma 3.1 we use Lemma 3.2. ∎

3.5 Simulation Results

In this section, a second order linear system is used to illustrate that the presented strategy in this note is effective. Consider the mass spring system shown in Figure (3.2). Let the spring-constant and damping-parameter values be $k = c = m = 1$, respectively. The state space dynamics of the mass-spring systems becomes:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} F(t) \quad (3.27)$$

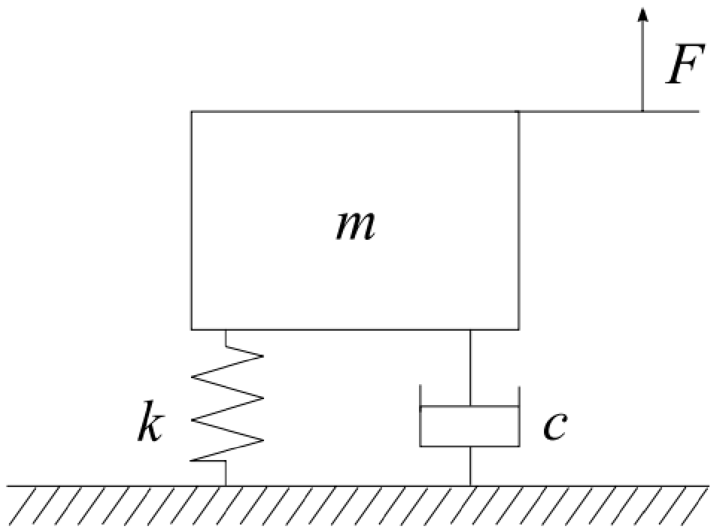

Figure 3.2: Simple mass-spring system

Since the rank of B_p is unity, the reset controller is of order one.

According step 2, we use the pole placement method to obtain a state feedback gain $K = [-1, -2]$ in order to get $(0, -1)$ eigenvalues. Solving the equations in Theorem (3.1) using Procedure 1 gives $L = -1$, $A_{stable} = -1$. Note that $\mathcal{X}$ is arbitrary, the similarity transformation T is given by

$$T = \begin{bmatrix} -0.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix} \tag{3.28}$$

when $\mathcal{X} = 1$.

Let $\lambda_1 = -10$ and $\lambda_2 = 2$ are chosen according to Lemma (3.1), so we can obtain the controller parameters $A_{r1} = \begin{bmatrix} 5 & 5 \end{bmatrix}$ and $A_{r2} = 2$. Then the angular frequency $\omega = 2$ that restricts the regulation time to be less than 0.785 seconds.

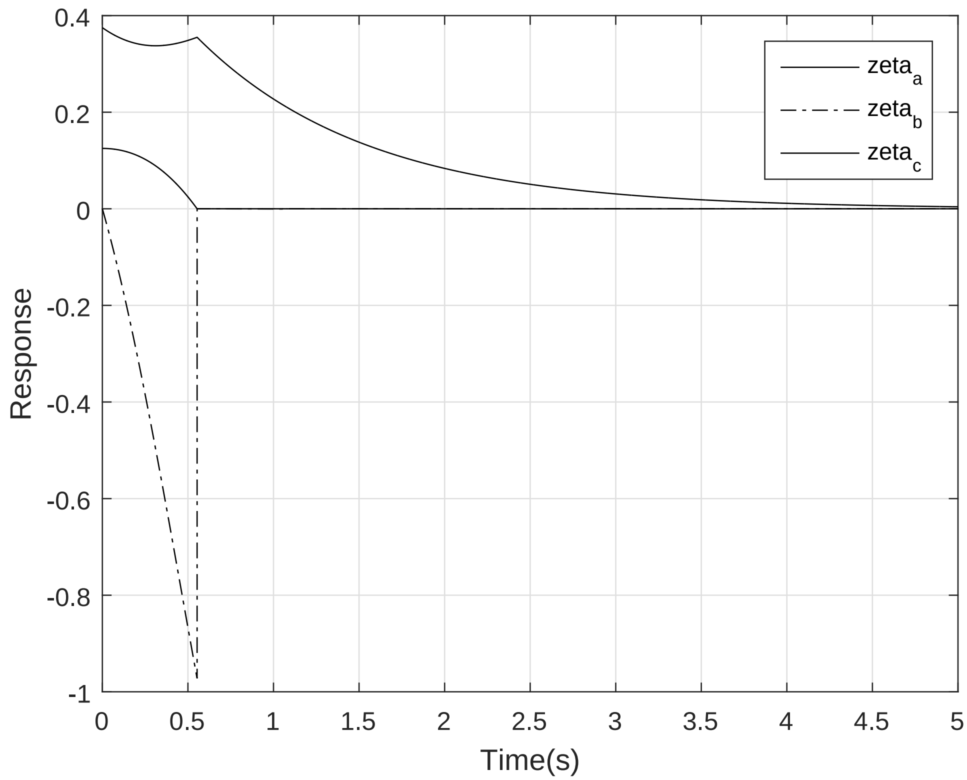

Figure 3.3: State response of the reset control system in the transformed form when $\bar{\tau} = 0.7$

Figure 3.3 demonstrates the state response of the reset control system in the transformed form (3.16), when the regulated parameter $\tau = 0.7$. It is obvious that the controller state $\zeta_b(t)$ is reset to zero when the other state $\zeta_a(t)$ vanishes. Moreover, the decoupled state $\zeta_c(t)$is slower than the state $\zeta_a(t)$ that is intimately related to the reset state $\zeta_b(t)$. The settling time of the decoupled state can be decreased by changing its closed loop eigenvalues, i.e. eigenvalues of A_{stable}.

For a larger values of $\bar{\tau}$ the system will postpone the reseting event which might cause a non-acceptable increase in the state magnitude as can be seen in Figure 3.4 when $\bar{\tau} = 2$. It is apparent that the zero crossing occurs before the first second but the reset controller ignores it since the time progress does not

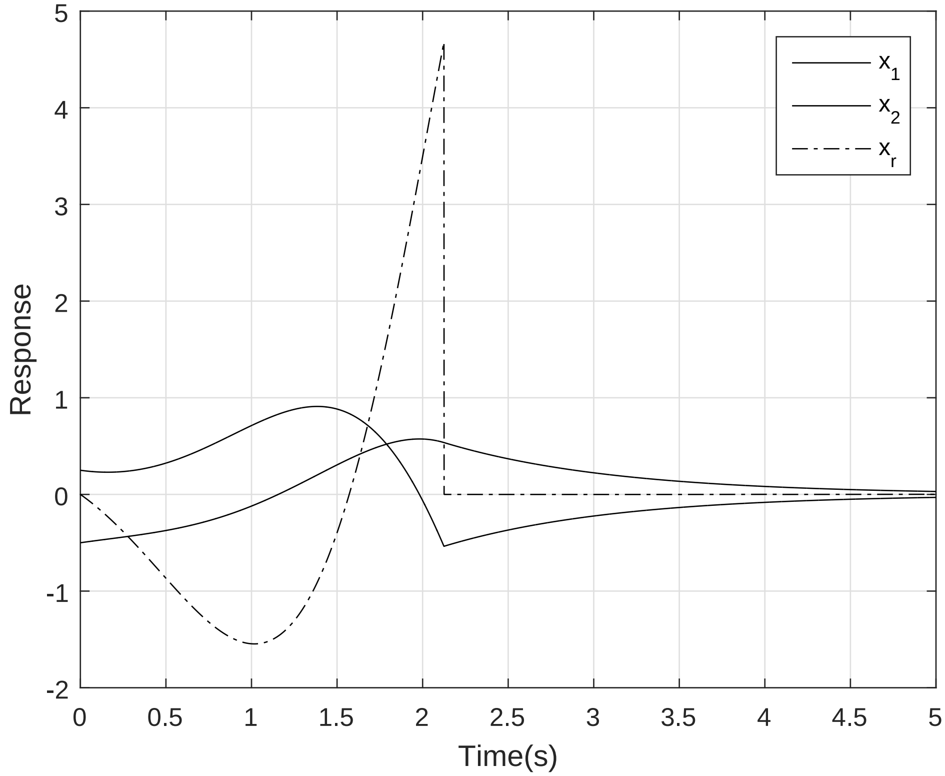

Figure 3.4: State response of the reset control system in the original form when $\bar{\tau} = 2$

exceed the maximum of the regulated variable. This situation is demonstrated using Figure 3.5 where the focus continues to cross the zero state but this cause an increase in the state magnitude which must be eliminated. The response of the transformed reset control system is shown in Figure 3.4.

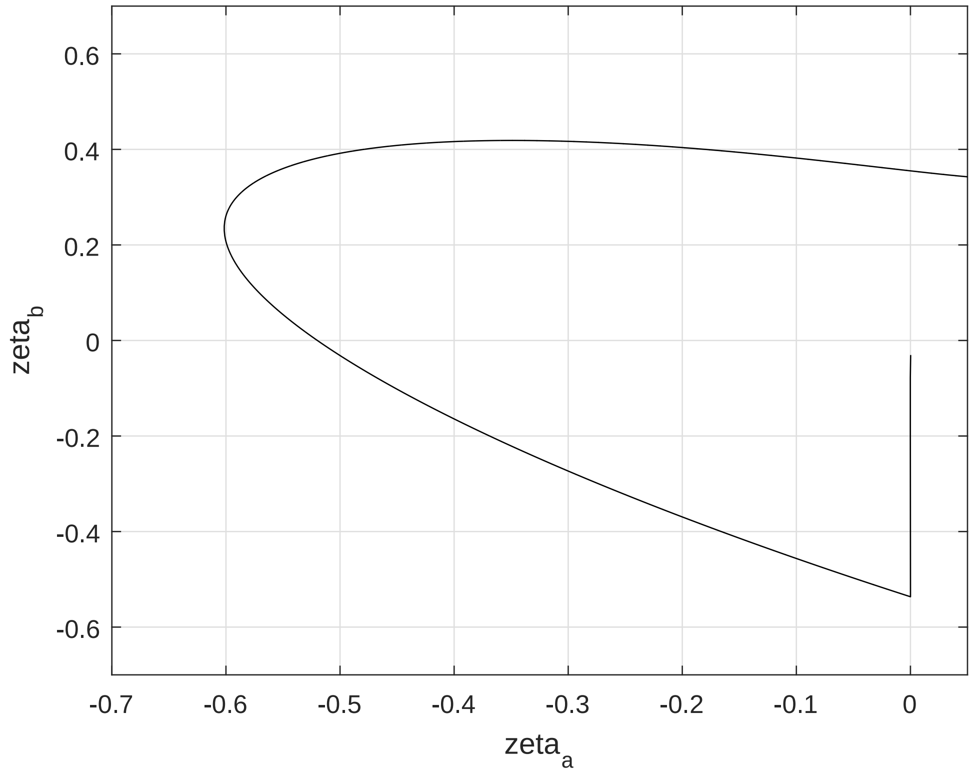

Figure 3.5: The ossilation behavior of the designed reset system when $\tau \bar{=} 2$

3.6 Summary

In this chapter, new reset controller strategies are proposed to control linear time invariant systems. The methods use state feedback to change the eigenvalues and then plug the reset controller to generate a new set of complex eigenvalues. It is demonstrated that unstable base systems can be stabilized using reset mechanisms with very fast response. The methodologies allow the user to combine reset controller for a set of states and the remaining states can be designed by classical methods. Consequently, a reduced order reset controller can be tuned.

While concentrating on developing reset systems methods, the main contribution of this article are: The developed reset mechanism assumes that the base dynamics might be unstable on the contrary of most proposed design methodologies that lacks this property. Our methodology forces a number of states to reach the equilibrium point in a finite time whereas the reset controllers in the literature require infinite time to drive the states to the origin.

The proposed method adds a substantial contribution in terms of a reset control design method rather than on reset analysis and hence our approach can be remarkably used to design more effective reset controllers to meet a predefined performance objective.

CHAPTER 4

STABILITY ANALYSIS OF RESET CONTROL SYSTEMS

A reset methodology in controller design can affect the performance and stability of control systems. Stability of reset time delay systems can be addressed based on a similar theorem of Lyapunov-Krasoviskii theory. In this paper, a robust delay-dependent stability analysis of reset control systems is proposed for linear time invariant case. The conditions of stability are given in terms of linear matrix inequalities. In addition, an extending results are given to investigate stability of nonlinear reset systems. Numerical examples are used to illustrate the effectiveness of the proposed approaches.

4.1 Introduction

Reset control systems framework is based on the theory of Impulsive Dynamical Systems (IDS). Similar to the Clegg integrator, reset systems are mainly characterized by three elements, continuous, discrete dynamics, and reset law. The continuous-time dynamics is described by a differential equation that describes the behavior of the impulsive system between reset events. The discrete-time part is used to demonstrate the instantaneous change in the states of the continuous part. Finally, reset law is used to govern the switching between the continuous and discrete dynamics.

In this chapter, new analysis tools of reset systems are proposed. We note that the existing methods consider simple reset surfaces. However, new reset surfaces are investigated to ensure the asymptotic stability for reset systems with and without time delay. We give a nice formula to bound the maximum number of reset instants. Moreover, the introduced disturbance rejection method leads to qualitatively new results for reset control systems, allowing a good rejection under reset mechanism. The presented methodologies provide efficient tools for various design problems in the reset control framework.

4.2 Problem Statement

Consider a non-autonomous reset system given by the general framework of impulsive differential equation

$$\begin{cases} \dot{x}(t) = f(t, x(t)), & x(t) \notin \mathcal{S} \\ x(t^+) = J(t, x(t)), & x(t) \in \mathcal{S} \end{cases} \tag{4.1}$$

where $t \geq t_0 \geq 0$, $x(t) \in \mathbb{R}^n$ is the system state, $f(t, x(t)) : \mathbb{R}^n \to \mathbb{R}^n$ is continuous in t and globally Lipschitz in x, $x(t^+)$ is the new state after the reset takes place, $J(t, x(t))$ represents the jump behavior at reset instants, and $\mathcal{S}$ is the reset surface. A reset action takes place when the state vector $x(t)$ hits the reset surface $\mathcal{S}$. Let $t_0, t_1, ..., t_k, ...$ be a sequence of time-instants that represents the time t at which the reset action is performed, i.e. $t_k = t$ whenever $x(t) \in \mathcal{S}$.

The reset system is modified by adding time regulation variable to prevent the existence of beating and Zeno solutions. This time variable represents the minimum time between two consecutive reset actions. The modified reset system can be represented as follows:

$$\begin{cases} \left.\begin{aligned} \dot{x}(t) &= f(t, x(t)), \\ \dot{\tau}(t) &= 1, \end{aligned}\right\} & x(t) \notin \mathcal{S} \vee \tau \leq \rho \\ \\ \left.\begin{aligned} x(t_k^+) &= J(t_k, x(t_k)), \\ \tau(t_k^+) &= 0, \end{aligned}\right\} & x(t_k) \in \mathcal{S} \wedge \tau \leq \rho \end{cases} \tag{4.2}$$

Since the time regulation imposes a lower bound on the reset intervals, then every reset instant t_k is performed at least ρ seconds after the last reset event t_{k-1}, i.e. $t_k - t_{k-1} \geq \rho$. This bound is used in the sequel to obtain the maximum number of reset actions if it is finite.

It is obvious that this class of systems encounter a number discontinuities at reset events t_k. The formal definition of $x(t_k^+)$ is given by $x(t_k^+) = J(t_k, x(t_k)) =$

$lim_{\epsilon>0,\epsilon\to 0}x(t+\epsilon)$. The solution of the impulsive system (4.1) with an initial condition $x(t_0) = x_0$ has an absolutely continuous solution $x(t)$ on every interval $(t_{k-1}, t_k]$ and it is left continuous at all reset instants.

A possible extension of the reset system is to include input or/and output signals. In a specific parts of this thesis we consider disturbance rejection criterion for linear reset systems. Consider the following time invariant linear reset system with input signals

$$\begin{cases} \left.\begin{array}{l} \dot{x}(t) = Ax(t) + Bu(t) \\ \quad +\Gamma w(t), \\ \dot{\tau}(t) = 1, \end{array}\right\} x(t) \notin \mathcal{S} \vee \tau \le \rho \\ \\ \left.\begin{array}{l} x(t_k^+) = A_R x(t_k), \\ \tau(t_k^+) = 0, \end{array}\right\} x(t_k) \in \mathcal{S} \wedge \tau \le \rho \\ \\ z(t) = Cx(t) + Du(t) \\ \quad +\Phi w(t) \end{cases} \tag{4.3}$$

where $u(t)$ is the input, $w(t) \in \mathcal{L}_2$ is the disturbance signal and $z(t)$ is the controlled output. When the input $u(t)$ is assumed to be zero, we define the following performance index for a prescribed positive scalar $\gamma > 0$:

$$J = \int_{t_0}^{\infty} [z^T(t)z(t) - \gamma^2 w^T(t)w(t)]dt \tag{4.4}$$

If $J < 0$ then we ensure that $\sup \frac{\|z(t)\|_{\mathcal{L}_2}}{\|w(t)\|_{\mathcal{L}_2}} \leq \gamma$.

4.3 Main Results

In this section, we address several stability tools to the reset systems in their general representation. The following theorem is used to check the bounded stability of the reset systems (4.2) in terms of Definition (2.3).

Theorem 4.1 *Let $V(t, x(t))$ be a continuously differentiable function such that:*

$$\alpha_1(\|x(t)\|) \leq V(t, x(t)) \leq \alpha_2(\|x(t)\|) \tag{4.5}$$

$$\begin{aligned} \dot{V}(x(t)) &< -W(x(t)), & \forall \|x(t)\| \geq \mu > 0 \\ \Delta V x(t_k)) &< -W(x(t_k), & \forall \|x(t_k)\| \geq \mu > 0 \\ \Delta V x(t_k)) &< \xi, & \forall \mu \geq \|x(t_k)\| > 0 \end{aligned} \tag{4.6}$$

$\forall t, t_k \geq 0$ and $x(t) \in \mathbb{D}$, where $\alpha_1(.)$ and $\alpha_2(.)$ are class $\mathcal{K}$ functions and $W(x)$ is a continuous positive definite function. Take $r > 0$ such that $\{x(t) \in \mathbb{R}^n : \|x(t)\| < r\} \subset \mathbb{D}$ and $\mu < \alpha_2^{-1}(\alpha_1(r))$. If ξ satisfies

$$\xi < \alpha_1(\alpha_2^{-1}(\alpha_1(r)))) - \alpha_2(\|\mu\|), \tag{4.7}$$

then for any initial condition $\|x_0\| \leq \alpha_2^{-1}(\alpha_1(r))$, *the solution* $x(t)$ *satisfies*

$$\|x(t)\| \leq \alpha_2^{-1}(\alpha_1(r)) \tag{4.8}$$

In addition, if $\mathbb{D} = \mathbb{R}^n$ *and* $\alpha_1(x(t)) \in \mathcal{K}_\infty$ *then the reset control system in (4.2) is globally stable.*

Proof. Let $\beta = \alpha_1(r)$ and $\eta = \alpha_2(\mu)$, then $\eta = \alpha_2(\mu) < \beta$ and $\alpha_2(\|x(t_0)\|) \leq \beta$. Define $\Omega_\eta = \{x(t) \in \mathbb{R}^n : \|x(t)\| < r \text{ and } V(x(t)) \leq \eta\}$ and $\Omega_\beta = \{x(t) \in \mathbb{R}^n : \|x(t)\| < r \text{ and } V(x(t)) \leq \beta\}$. Then

$$\begin{aligned}
\{x(t) \in \mathbb{R}^n : \|x(t)\| < \mu\} &\subset \Omega_\eta \\
&\subset \{\alpha_1(\|x(t)\|) \leq \eta)\} \\
&\subset \{\alpha_1(\|x(t)\|) \leq \beta)\} \\
&\subset \{x(t) \in \mathbb{R}^n : \|x(t)\| < r\} \\
&\subset \mathbb{D}
\end{aligned} \tag{4.9}$$

and

$$\Omega_\eta \subset \Omega_\beta \subset \{x(t) \in \mathbb{R}^n : \|x(t)\| < r\} \subset \mathbb{D}$$

Any solution starting in any of the set Ω_η or Ω_β can not leave it because $\dot{V}(x(t))$ and $\Delta V(x(t_k))$ are negative on the boundary. And $\alpha_2(\|x(t_0)\|) \leq \beta$ implies that $x(t_0) \in \Omega_\beta$. Consequently, the solution enters Ω_η in a finite time because

$$\dot{V}(x(t)) \leq -l < 0, \quad \forall \mu \leq \|x\| \leq \beta \tag{4.10}$$

where $l = min_{\mu \leq \|x\| \leq \beta} \{W(x(t))\}$

Given $N >> 1$ of reset events, we integrate Equation (4.10) from 0 to t_N. Tacking into account $\Delta V(t_k) < -l$. We find

$$\begin{aligned} V(t_N) - V(t_{N-1}^+) + V(t_{N-1}) - ... - V(t_1^+) + V(t_1) \\ -V(t_0) \leq -l(t_N - t_0) \end{aligned} \tag{4.11}$$

Since $V(t_k^+) - V(t_k) < 0$ for $k = 2, 3, ..., N$, we have

$$V(t_N^+) < V(t_0) - l(t_N - t_0) \tag{4.12}$$

This shows that $x(t_N)$ enter Ω_η within the time interval $[t_0, t_0 + \frac{\beta-\eta}{l}]$.

Now the trajectory stays in Ω_η unless a reset action takes place when $\|x(t_k)\| < \mu$ for some integer k. However, at this situation we have $\Delta V x(t_k)) = V(x(t_k^+)) - V(x(t_k)) < \xi$, using Equation (4.7), we have

$$\begin{aligned} \alpha_1(\|x(t_k^+)) &\leq V(x(t_k^+)) \\ &\leq V(x(t_k)) + \xi \\ &\leq \alpha_2(x(t_k)) + \xi \\ &\leq \alpha_2(\mu) + \xi \\ &\leq \alpha_1(\alpha_2^{-1}(\alpha_1(r)))) \end{aligned}$$

This implies that $\alpha_2(\|x(t_k^+)\|) \leq \beta$ and the solution satisfies Equation (4.8). Consequently, the solution does not leave Ω_β. This completes the proof. ∎

Remark 4 *It is worth mentioning that ξ in Equation (4.6) might be chosen to be positive. It is used to ensure that the jump to be finite and does not leave Ω_β set. However, there is no restriction on the value of $\dot{V}(t, x(t))$ when $\|x(t)\| \leq \mu$ for $t \in (t_{k-1}, t_k]$. If $\dot{V}(t, x(t)) > 0$, the continuous behavior of the solution guarantees that $x(t)$ must pass through the region defined by $\|x(t)\| \geq \mu$ at which $\dot{V}(t, x(t))$ becomes negative.*

In the following Lemma, we restrict ξ to be negative in order to decrease the the ultimate bound of the state $x(t)$. Another interesting feature of this is that the number of reset actions are finite and can be upper bounded.

Lemma 4.1 *Let the conditions of Theorem 4.1 be satisfied. If* ξ *in Equation (4.7) is negative then for all initial conditions satisfying* $\|x_0\| \leq \alpha_2^{-1}(\alpha_1(r))$*, we find*

$$\|x(t)\| \leq \alpha_2^{-1}(\alpha_1(\mu)) \tag{4.13}$$

Moreover, the solution enters this region within a number of reset events N *given by*

$$N \leq \frac{\alpha_2(x(t_0))}{\rho l} \tag{4.14}$$

Proof. The proof is similar to Theorem 4.1. Since ρ represents the minimum time between two consecutive events, then $t_N - t_n > N\rho$. Using Equation (4.12), we have

$$\begin{aligned} 0 &\leq V(t_N^+) \\ &\leq V(t_0) - l(t_N - t_0) \\ &\leq \alpha_2(x(t_0)) - lN\rho \end{aligned}$$

this completes the proof.

∎

Now asymptotic stability is addressed for autonomous reset systems in the next Theorem.

Theorem 4.2 *Let the reset surface $\mathcal{S}$ be defined as follows:*

$$\begin{aligned} \mathcal{S} &= \{x(t) \in \mathbb{R}^n : \tau > \rho \text{ and} \\ &\quad \alpha_2(\|x(t)\|) - \alpha_1(\|x(t_k)\|) < -W(x(t_k))\} \end{aligned} \tag{4.15}$$

where $W(x(t_k)) > 0$, $\alpha_1(.)$ and $\alpha_2(.)$ are class $\mathcal{K}_\infty$ functions. If there exist continuous and discrete Lyapunov functions $V_1(x(t))$ and $V_2(x(t))$ satisfy the following:

$$\alpha_1(\|x(t)\|) \leq V_1(x(t)) \leq \alpha_2(\|x(t)\|) \tag{4.16}$$

$$\alpha_1(\|x(t)\|) \leq V_2(x(t)) \leq \alpha_2(\|x(t)\|) \tag{4.17}$$

such that

$$\dot{V}_1(x(t)) < 0 \tag{4.18}$$

$$\Delta V_2(x(t_k)) = V_2(x(t_k^+)) - V_2(x(t_k)) \leq 0 \tag{4.19}$$

$\forall x(t) \in \mathbb{R}^n$. *Then the autonomous version of reset control system in (4.2) is globally asymptotically stable.*

Proof. Without loss of generality, assume the initial condition $x_0 = x(t_k) \in \mathcal{S}$. Since $\Delta V_2(x(t_k)) < 0$, it is well known that for every $\epsilon^\star > 0$ there exists $\delta > 0$ (see Figure 4.1.) such that:

$$x(t_k^+) = J(x(t_k)) \in B_{\epsilon^\star} = \{x \in \mathbb{R}^n : \|x\| < \epsilon^\star\}$$

whenever

$$x(t_k) \in B_\delta = \{x \in \mathbb{R}^n : \|x\| < \delta\} \tag{4.20}$$

It is obvious that $x(t) \notin \mathcal{S}, t \in (t_k, t_{k+1}]$ with the new initial condition $x(t_k^+)$ for the continuous dynamics. The state value before the last jump becomes $x(t_k) = x_0$. Now since $\dot{V}_1(x(t)) < 0$, for every $\epsilon > 0$ there exists $\delta^\star > 0$ such that:

$$x(t) \in B_\epsilon = \{x \in \mathbb{R}^n : \|x\| < \epsilon\}, \quad \forall t \in (t_k, t_{k+1}]$$

whenever

$$x(t_k^+) \in B_\delta^\star = \{x \in \mathbb{R}^n : \|x\| < \delta^\star\}$$

A trivial case occurs if $x(t) \in B_\epsilon$ does not hit $\mathcal{S}$, i.e. $t_{k+1} \to \infty$, then $x(t) \to 0$ by Equation (4.18). On the contrary, if $x(t) \in \mathcal{S}$ for some t such that $t - t_k > \rho$ and $\alpha_2(\|x(t)\|) - \alpha_1(\|x(t_k)\|) < -W(x(t_k))$ then $t_{k+1} = t$, and since $x(t_k) \in B_\delta$, we have

$$\begin{aligned}
\|x(t_{k+1})\| &< \alpha_2^{-1}(\alpha_1(\|x(t_k)\|) - W(x(t_k))) \\
&< \alpha_2^{-1}(\alpha_1(\|x(t_k)\|) \\
&< \alpha_2^{-1}(\alpha_1(\delta)) \\
&< \delta \qquad (4.21)
\end{aligned}$$

This implies that $x(t)$ stays in B_ϵ for $t \in (t_k, t_{k+1}]$ and $x(t_{k+1}) \in B_\delta$ as a new initial condition. In other words, $x(t_k), x(t_{k+1}) \in B_\delta \quad \forall k = 0, 1, 2, ...$ Because $\epsilon^\star$ is an arbitrary and dummy variable, it can be restricted to be sufficiently small such that $\epsilon^\star < \delta^\star$. From the aforementioned argument, given $\epsilon > 0$, we can find a sufficiently small δ such that $\alpha_2(\delta) < \alpha_1(\epsilon^\star)$ and $\alpha_2(\delta^\star) < \alpha_1(\epsilon)$ then it is easy to show that $\delta < \alpha_2^{-1}(\alpha_1(\alpha_2^{-1}(\alpha_1(\epsilon)))$. Hence, for any initial condition $\|x_0\| < \delta$ implies that $\|x(t)\| < \epsilon$.

To prove asymptotic stability, we know from the reset condition occurs when $\alpha_2(\|x(t_{k+1})\|) < \alpha_1(\|x(t_k)\|) - W(x(t_k))$ is satisfied, and using Equations (4.16), (4.17), (4.18) and (4.19) we find:

$$
\begin{aligned}
\alpha_1(\|x(t_{k+1}^+)\|) &\leq V_2(\|x(t_{k+1}^+)\|) \\
&\leq V_2(\|x(t_{k+1})\|) \\
&\leq \alpha_2(\|x(t_{k+1})\|) \\
&< \alpha_1(\|x(t_k)\|) - W(x(t_k)) \\
&< V_1(\|x(t_k)\|) - W(x(t_k)) \\
&< \alpha_2(\|x(t_k)\|) - W(x(t_k)) \qquad (4.22)
\end{aligned}
$$

we can split it into two parts

$$\alpha_1(\|x(t_{k+1}^+)\|) < \alpha_1(\|x(t_k)\|) - W(x(t_k))$$

and

$$\alpha_2(\|x(t_{k+1})\|) < \alpha_2(\|x(t_k)\|) - W(x(t_k))$$

then

$$\|x(t_{k+1})\| < \|x(t_k)\|$$

and

$$\|x(t_{k+1}^+) < \|x(t_k)\|$$

These equations imply that the state at reset events $x(t_k) \to 0$ and $x(t_k^+) \to 0$ as $t_k \to \infty$. From Equation (4.19) the following is also true for the state $x(t)$

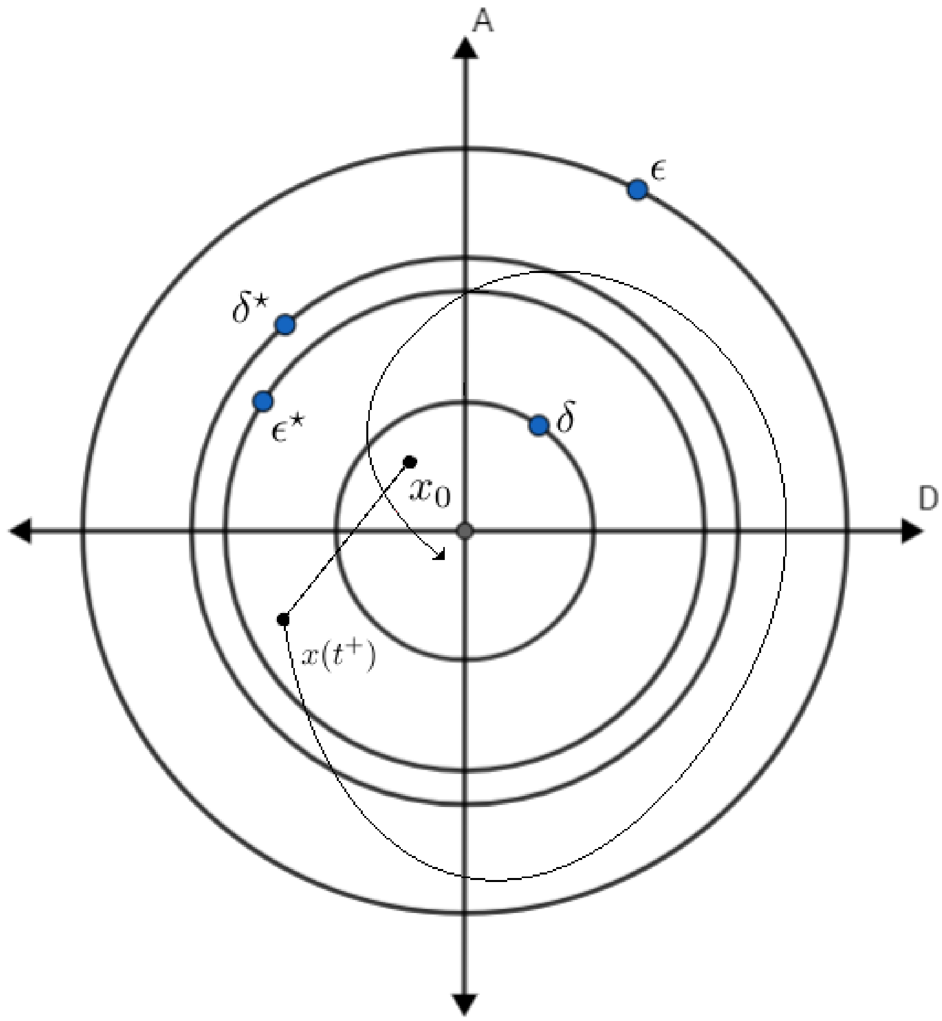

Figure 4.1: Geometric representation of the sets in the proof of Theorem 2.

$t \in (t_k, t_{k+1}]$:

$$\alpha_1(\|x(t)\|) \leq V_1(x(t)) \leq V_1(x(t_k^+)) \leq \alpha_2(\|x(t_k^+)\|)$$

Consequently, $x(t) \to 0$ as $t_k \to \infty$ because $x(t_k^+) \to 0$. This completes the proof.

∎

Remark 5 *Theorem 4.2 provides a less conservative tool for proving asymptotic stability of reset systems, which relies on multiple Lyapunov functions.*

Lemma 4.2 *Let the conditions of Theorem 4.2 be satisfied but $W(x) = -l$ for some positive scalar $l > 0$ then the maximum number of reset events N is bounded by*

$$N < \frac{\alpha_2(\|x(t_0)\|)}{l\rho}. \tag{4.23}$$

Proof. Form Equation (4.22),

$$\alpha_2(\|x(t_{k+1})\|) - \alpha_2(\|x(t_k)\|) < -l \tag{4.24}$$

Take the summation for both sides of (4.24) from t_0 till $t = t_N$, we find

$$\alpha_2(\|x(t_{N+1})\|) - \alpha_2(\|x(t_0)\|) < -l(t_N - t_0) \tag{4.25}$$

The system is asymptotically stable by Theorem 4.2. Since ρ represents the minimum time between two consecutive events, then $t_N - t_0 > N\rho$. The reset events is deactivated when $x(t)$ can not satisfy the reset surface $\alpha_2(\|x(t)\|) - \alpha_1(\|x(t_N)\|) < -l$ for some $t - t_N > \rho$ this occurs when $\|x(t_N\| = 0$. To obtain the maximum events substitute $\|x(t_{N+1}\| = 0$ in Equation (4.25). Simple manipulation of Equation (4.25) gives the upper bound in of N in Equation (4.23)

Remark 6 *Lemma 4.2 gives the total number of reset events that are performed for all* $t \in [t_0, \infty)$*. On the other hand, Lemma 4.1 gives the upper bound of the required reset events such that the solution* $x(t)$ *is guaranteed to satisfy* $\|x(t)\| \leq \alpha_2^{-1}(\alpha_1(\mu))$*. Hence, Lemma 4.1 is not related to the total number of events since its assumptions don not lead to asymptotic stability.*

4.3.1 $\mathcal{H}_\infty$- Design

In what follows, we consider the $\mathcal{H}_\infty$ optimization problem for linear time invariant reset system (4.3). The main result is summarized in the following Lemma:

Lemma 4.3 *Under the assumption that the stability conditions of Theorem 4.2 are satisfied, if along (4.3)*

$$\dot{V}_1(t) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0 \tag{4.26}$$

for almost all t*. For zero initial conditions, the unforced reset system (4.3) has a finite* $\mathcal{L}_2$*-gain* γ *from* $w(t)$ *to* $z(t)$*.*

Proof. Take the number of reset-events $N > 1$ and integrate Equation (4.26) from t_0 till t_N. Taking into account the discontinuties of $V_2(t)$ at $t_0, t_1, ..., t_N$, we find

$$V_2(t_N) - V_2(t_{N-1}^+) + V_2(t_{N-1}) - V_2(t_{N-2}) - ... + V_2(t_1)$$
$$-V_2(t_0) + \int_{t_0}^{t_N} [z^T(t)z(t) - \gamma^2 w^T(t)w(t)]dt < 0 \tag{4.27}$$

Since $V_2(t_k^+) - V_2(t_k) < 0$, $V(t_0) = 0$, and $V(t_N) > 0$ we have

$$J = \int_{t_0}^{t_N} [z^T(t)z(t) - \gamma^2 w^T(t)w(t)]dt < 0$$

Thus, for $t_N \to \infty$ we arrive at:

$$\|z(t)\|_{\mathcal{L}_2}^2 < \gamma^2 \|w(t)\|_{\mathcal{L}_2}^2$$

this completes the proof. ∎

It follows from Lemma 4.3 that the solution of this problem corresponds to determining the controller parameters that guarantees the feasibility of Equation (4.26).

We direct attention to alternative method for computing state feedback controller $u(t) = Kx(t)$. The closed loop reset system is described by

$$
\begin{cases}
\left.\begin{aligned} \dot{x}(t) &= A_s x(t) + \Gamma w(t), \\ \dot{\tau}(t) &= 1, \end{aligned}\right\} & x(t) \notin \mathcal{S} \vee \tau \le \rho \\
\\
\left.\begin{aligned} x(t_k^+) &= A_R x(t_k), \\ \tau(t_k^+) &= 0, \end{aligned}\right\} & x(t_k) \in \mathcal{S} \wedge \tau \le \rho \\
\\
z(t) = C_s x(t) + \Phi w(t)
\end{cases}
\tag{4.28}
$$

where $A_s = A + BK$ and $C_s = C + DK$.

The design result is summarized in the following theorem.

Theorem 4.3 *The reset system (4.28) is asymptotically stable with γ-disturbance rejection if there exist positive definite matrices $\mathcal{Z} > 0$, $\mathcal{X} > 0$, and a scalar $\gamma > 0$ such that the following LMI's are satisfied:*

$$
\begin{bmatrix} -\mathcal{Z} & \mathcal{Z} A_R^T \\ \bullet & -\mathcal{Z} \end{bmatrix} < 0 \tag{4.29}
$$

$$
\begin{bmatrix} \Psi_0 & \Gamma & \Psi_c \\ \bullet & -\gamma^2 I & \Phi^T \\ \bullet & \bullet & -I \end{bmatrix} < 0 \tag{4.30}
$$

$$\Psi_0 = A\mathcal{Z} + \mathcal{Z}A^T + B\mathcal{X} + \mathcal{X}^T B^T < 0 \tag{4.31}$$

$$\Psi_c = \mathcal{Z}C^T + \mathcal{X}^T D^T \tag{4.32}$$

Moreover, the controller gain is given by $K = \mathcal{X}\mathcal{Z}^{-1}$.

Proof. Suppose $V_1(x) = V_2(x) = x(t)^T P x(t)$,

we express the inequality (4.26) in the following form

$$\begin{aligned} x^T[A_s^T P + PA_s]x &+ [C_s x + \Phi w]^T[C_s x + \Phi w] \\ &+ 2x^T P\Gamma w - \gamma^2 w^T w < 0 \end{aligned} \tag{4.33}$$

Inequality (4.33), by Schur complements, is equivalent to

$$\begin{bmatrix} [A_s^T P + PA_s] & P\Gamma & C_s^T \\ \bullet & -\gamma^2 I & \Phi^T \\ \bullet & \bullet & -I \end{bmatrix} < 0 \tag{4.34}$$

for any $[x^T, w^T]^T \neq 0$. Now, apply the congruent transformation diag$[\mathcal{Z}, I]$, $\mathcal{Z} = P^{-1}$ to Equation (4.34) and using $K\mathcal{Z} = \mathcal{X}$ we obtain LMI (4.30). Now, it is easy to show that the LMI's (4.30) and (4.31) are sufficient to satisfy the conditions presented in Theorem4.2, which concludes the proof. ∎

The following theorem provides less conservative LMI's when the feedback gain is given or the system is unforced $u(t) = 0$.

Theorem 4.4 *The reset system (4.28) is asymptotically stable with γ-disturbance rejection if there exist positive definite matrices $P_1 > 0$, $P_2 > 0$, and a scalar $\gamma > 0$ such that the following LMI's are satisfied:*

$$\begin{bmatrix} A^T P_2 + P_2 A & P_2 \Gamma & C^T \\ \bullet & -\gamma^2 I & \Phi^T \\ \bullet & \bullet & -I \end{bmatrix} < 0 \tag{4.35}$$

$$A_R^T P_2 A_R - P_2 < 0 \tag{4.36}$$

$$A^T P_1 + P_1 A \leq 0 \tag{4.37}$$

Proof. The proof is similar to the proof of Theorem 4.4, but here we use different Lyapunov functions $V_1(x(t) = x^T(t) P_1 x(t)$ and $V_2(x(t) = x^T(t) P_2 x(t)$. ∎

4.4 Simulation Results

Consider the following nonlinear dynamical system:

$$f(t,x) = \begin{bmatrix} -x_1[1+x_1^2] + x_2(t) + \frac{1}{2}\cos(\omega t) \\ -x_2(t)[1+x_1^4(t)] - x_1(t) + \cos(\omega t) \end{bmatrix}$$

$$J(x(t_k)) = \frac{1}{2} \begin{bmatrix} [x_1(t_k) - x_2(t_k)] \\ [x_1(t_k) + x_2(t_k)] \end{bmatrix}$$

Let a candidate Lyapunov function be defined as $V(x(t)) = x_1^2(t) + x_2^2(t)$ with class $\mathcal{K}_\infty$-functions $\alpha_1(v) = \frac{1}{2}v^2$ and $\alpha_2(v) = 2v^2$. Then

$$\begin{aligned} \dot{V}(x(t)) &= -2\|x(t)\|_2^2 - x_2^4(t) - 2x_2^2(t)x_2^4(t) \\ &+ [x_1(t) + 2x_2(t)]\cos(\omega t) \\ &\leq -(2-\theta)\|x(t)\|_2^2 - x_2^4(t) - 2x_2^2(t)x_2^4(t) \\ &- \theta\|x(t)\|_2^2 + \sqrt{5}\|x(t)\|_2 \end{aligned} \tag{4.38}$$

where $0 < \theta < 2$. The following inequality ensures that the time derivative of the proposed Lyapunov function is negative

$$\dot{V}(x(t)) \leq -(2-\theta)\|x(t)\|_2^2 - x_2^4(t) - 2x_2^2(t)x_2^4(t) \tag{4.39}$$

for all $\|x(t)\|_2 \geq \frac{\sqrt{5}}{\theta}$.

Figure 4.2 shows the state response $x_1(t)$ of with and without the reset mechanism. It can be seen that the reset mechanism maintains the state $x_1(t)$ to be near the origin whereas the non resettable one is settled away from the origin. Moreover, Figure 4.3 illustrates the state response $x_2(t)$ of with and without the reset mechanism. Regarding the resettable behavior of $x_2(t)$, it clearly faster than the non resettable one.

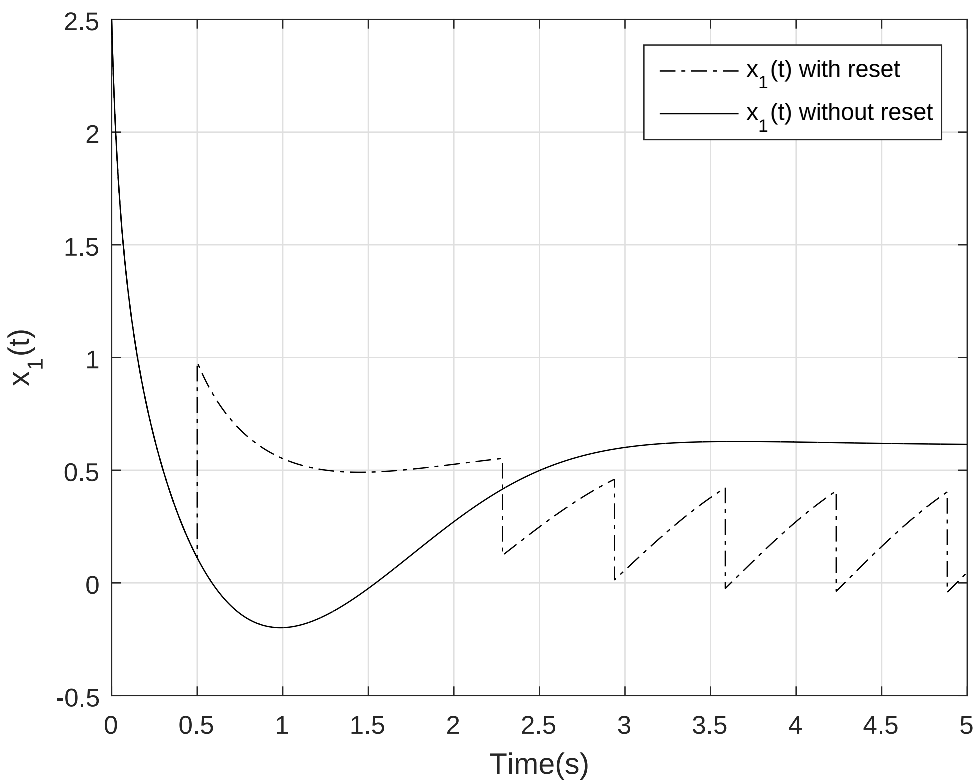

Figure 4.2: The state response $x_1(t)$ of with and without the reset mechanism.

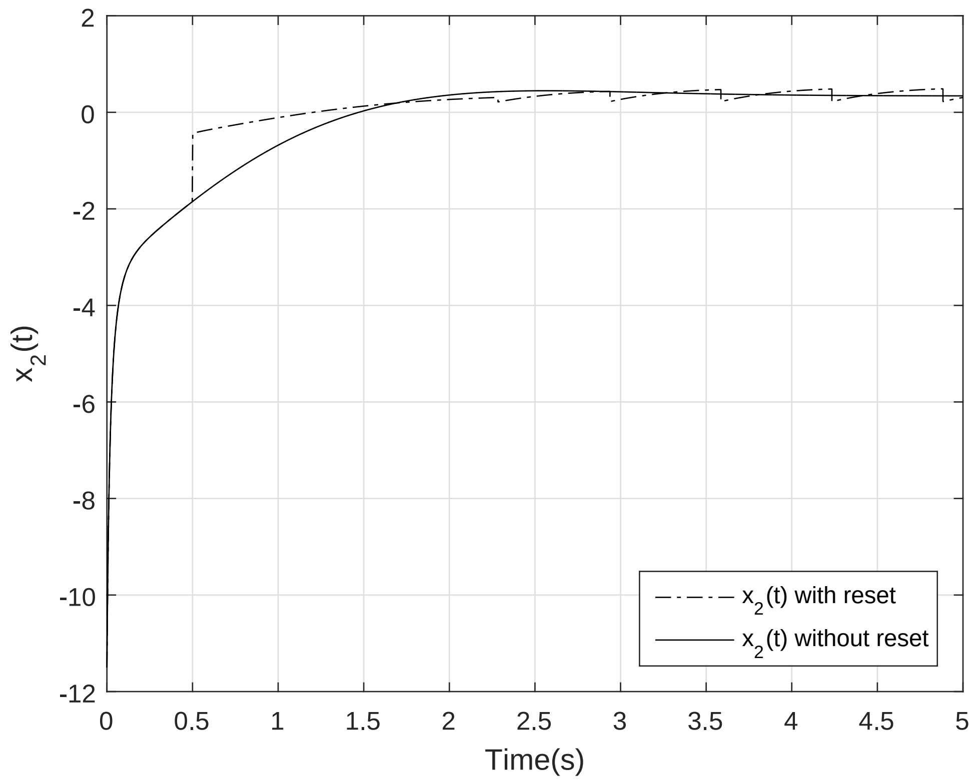

Figure 4.3: The state response $x_2(t)$ of with and without the reset mechanism.

CHAPTER 5

STABILITY ANALYSIS OF TIME-DELAY RESET SYSTEMS

5.1 Time Delay Reset Systems

Regarding time delay reset systems, consider the following reset system that is formed by a time delay continuous dynamics and a reset mechanism:

$$
\begin{cases}
\left.\begin{aligned} \dot{x}(t) &= f(t, x_t) \\ \dot{\tau}(t) &= 1, \end{aligned}\right\} & (x_t, \eta(t)) \notin \mathcal{S} \\
\\
\left.\begin{aligned} x(t^+) &= I(t, x(t)), \\ \tau(t_k^+) &= 0, \end{aligned}\right\} & (x_t, \eta(t)) \in \mathcal{S} \\
\\
\left.\begin{aligned} x(t) &= \phi(t), \\ \tau(t) &= 0, \end{aligned}\right\} & t \in [t - \tau^\star, t]
\end{cases}
\tag{5.1}
$$

where $x(t) \in \mathbb{R}^n$ is the present state of the system, the distributed state $x_t = x(t+\theta)$, $\theta \in [-\eta^\star, 0]$, $\eta^\star$ is the upper bound of the time delay $\eta(t)$, $f(t, x)$ represents the general time delay dynamics, $I(t)$ is the jump value at the reset instants, $\phi(t)$ is the initial condition, and $\tau(t)$ is the regulated variable. The variable $\tau(t)$ is used to overcome the problem of Zeno behavior. A reset system comprises Zeno solution if the trajectory encounters the resetting surface infinite times in a finite time.

Now, consider system (5.1) which includes time delay and reset mechanism. Let its reset surface $\mathcal{S}$ be defined as follows:

$$
\begin{aligned}
\mathcal{S} \;=\; & \{x(t) \in \mathbb{R}^n : \tau > \rho \text{ and} \\
& \alpha_2(\|x_t\|_W) - \alpha_1(\|x(t_k)\|) < -W(x(t_k))\}
\end{aligned}
\tag{5.2}
$$

where $W(x(t_k)) > 0$, $\alpha_1(.)$ and $\alpha_2(.)$ are class $\mathcal{K}_\infty$ functions.

Theorem 5.1 *Let there exist two absolutely continuous except at the reset instants Lyapunov-functionals $V_1(t, \phi(t))$ and $V_2(t, \phi(t))$ that satisfy the following:*

$$\alpha_1(\|\phi(0)\|) \leq V_1(t, \phi(t)) \leq \alpha_2(\|\phi\|_W) \tag{5.3}$$

$$\alpha_1(\|\phi(0)\|) \leq V_2(t, \phi(t)) \leq \alpha_2(\|\phi\|_W) \tag{5.4}$$

such that

$$\begin{aligned} \dot{V}_1(t, \phi(t)) &< -W(x(t)) \\ \Delta V_2(t_k, \phi(t_k)) &= V_2(t_k^+, \phi(t_k^+)) - V_2(t_k, \phi(t_k)) \leq 0 \end{aligned} \tag{5.5}$$

$\forall x(t) \in \mathbb{R}^n$. *Then the time delay version of reset control system in (5.1) is globally asymptotically stable.*

Proof. The proof follows parallel details to Theorem4.2. ∎

5.1.1 Linear Time delay reset systems without uncertainty

In this subsection, the stability of linear time invariant system with reset mechanism is analyzed. In this case, consider the following time delay reset system

$$
\begin{cases}
\left.\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-\eta) \\ \dot{\tau}(t) &= 1, \end{aligned}\right\} & (x_t, \eta(t)) \notin \mathcal{S} \\
\left.\begin{aligned} x(t_k^+) &= A_R x(t_k), \\ \tau(t_k^+) &= 0, \end{aligned}\right\} & (x_t, \eta(t)) \in \mathcal{S} \\
\left.\begin{aligned} x(t) &= \phi(t), \\ \tau(t) &= 0, \end{aligned}\right\} & t \in [t - \tau^\star, t]
\end{cases}
\tag{5.6}
$$

The choice of the Lyapunov-Krasoviskii functional is essential to derive non-conservative conditions for stability. Applying the same methodologies as in the field of time delay systems without reset mechanism using the powerful Theorem (5.1) leads to the following theorem.

Theorem 5.2 *Consider the dynamical reset system (5.6), with a delay* $\eta(t)$ *that satisfies* $0 \le \eta(t) \le \eta^\star$ *and* $\dot{\eta}(t) \le \eta^+$. *Given positive definite matrices* P, Q, R, Φ_1, Φ_2, Φ_3 *and any matrices with appropriate dimensions* L_1, L_2, L_3, L_4 *such that the following LMI's are satisfied*

$$
\Theta = \begin{bmatrix} -\Phi_1 & 0 & 0 & A^T L_4^T \\ \bullet & -\Phi_3 & 0 & L_4^T \\ \bullet & \bullet & -\Phi_2 & A_d^T L_4^T \\ \bullet & \bullet & \bullet & -R \end{bmatrix} \le 0 \tag{5.7}
$$

$$\Omega = \begin{bmatrix} \Omega_{11} & -L_1 + A^T L_3^T & \Omega_{13} \\ \bullet & \eta^\star(\Phi_3 - L_3^T - L_3 + \eta^\star R) & L_2^T + L_3 A_d \\ \bullet & \bullet & \Omega_{33} \end{bmatrix} \leq 0 \tag{5.8}$$

$$A_R^T P A_R^T - P \leq 0 \tag{5.9}$$

$$\Omega_{11} = A^T P + PA + L_1 A + A^T L_1^T + \eta^\star \Phi_1$$

$$\Omega_{13} = A^T L_2^T + PA_d + L_1 A_d$$

$$\Omega_{33} = \eta^\star \Phi_2 - (1 - \eta^+)Q + L_2 A_d + A_d^T L_2^T$$

then the reset system (5.6) is asymptotically stable.

Proof. Let $V_1(x_t) = V_2(x_t)$ be defined by the following functional,

$$\begin{aligned} V(x_t) &= x^T(t)Px(t) + \int_{t-\eta(t)}^{t} x^T(s)Qx(s)ds \\ &+ \int_{-\eta^\star}^{0} \int_{t+\theta}^{t} \dot{x}(s)^T R\dot{x}(s)dsd\theta \end{aligned} \tag{5.10}$$

Then the difference in the proposed Lyapunov-Krasoviskii functional (5.10) at the reset instants is given by

$$
\begin{aligned}
\Delta V &= V(t^+, x_{t^+}) - V(t, x_t) \\
&= x^T(t^+)Px(t^+) - x^T(t)Px(t) \\
&= x^T(t)\big(A_R^T P A_R^T - P\big)x(t) \qquad (5.11)
\end{aligned}
$$

Since $\eta(t) \leq \eta^\star$ the inequality

$$
[\eta^\star - \int_{t-\eta}^{t} ds] \geq 0
$$

holds for any t. Consequently, for any positive definite matrices Φ_1, Φ_2, Φ_3 the following inequality holds

$$
\begin{aligned}
[x^T(t)\Phi_1 x(t) &+ x^T(t-\eta)\Phi_2 x(t-\eta) \\
&+ \dot{x}^T(t)\Phi_3 \dot{x}(t)] \times [\eta^\star - \int_{t-\eta}^{t} ds] \geq 0
\end{aligned}
$$

On the other hand, from the base dynamics $[Ax(t) + A_d x(t-\eta) - \dot{x}(t)] = 0$. As a consequence, for any free wighting matrices with appropriate dimensions L_1, L_2, L_3, L_4, the following equation holds

$$
\begin{aligned}
2[x^T(t)L_1 &+ x^T(t-\eta)L_2 + \dot{x}^T(t)L_3 \int_{t-\eta}^{t} \dot{x}^T(s)ds L_4] \\
&\times [Ax(t) + A_d x(t-\eta) - \dot{x}(t)] = 0
\end{aligned}
$$

Now, the time derivative of $V(x)$ along the dynamics of (5.6) with some simple manipulations becomes:

$$
\begin{aligned}
\dot{V} &= x^T(t)(A^TP + PA + Q)x(t) + 2x^T(t)PA_dx(t-\eta) \\
&- x^T(t-\eta)Qx^T(t-\eta) + \dot{\eta}(t)x^T(t-\eta)Qx^T(t-\eta) \\
&+ h\dot{x}^T(t)R\dot{x}(t) - \int_{t-\eta^\star}^{t} \dot{x}^T(s)R\dot{x}(s)ds \\
&\leq x^T(t)(A^TP + PA + Q)x(t) + 2x^T(t)PA_dx(t-\eta) \\
&- (1-\eta^+)x^T(t-\eta)Qx(t-\eta) \\
&+ \eta^\star\dot{x}^T(t)R\dot{x}(t) - \int_{t-\eta}^{t} \dot{x}^T(s)R\dot{x}(s)ds \\
&+ 2[x^T(t)L_1 + x^T(t-\eta)L_2 + \dot{x}^T(t)L_3 \\
&+ \int_{t-\eta}^{t} \dot{x}^T(s)dsL_4] \times [Ax(t) + A_dx(t-\eta) - \dot{x}(t)] \\
&+ [x^T(t)\Phi_1x(t) + x^T(t-\eta)\Phi_2x(t-\eta) \\
&+ \dot{x}^T(t)\Phi_3\dot{x}(t)][\eta^\star - \int_{t-\eta}^{t} ds] \\
&= \zeta^T(t)\Omega\zeta(t) + \int_{t-\eta}^{t} \xi^T(t)\Theta\xi(t)ds
\end{aligned}
\tag{5.12}
$$

where

$$
\zeta^T(t) = [\; x^T(t) \quad \dot{x}^T(t) \quad x^T(t-\eta) \;] \tag{5.13}
$$

and

$$
\xi^T(t) = [\; x^T(t) \quad \dot{x}^T(t) \quad x^T(t-\eta) \quad \dot{x}^T(s) \;] \tag{5.14}
$$

If $\Omega < 0$ and $\Theta \leq 0$, then, Equation (5.12) is negative definite. Alongside with Equation (5.9), Theorem 5.1 ensures that the time delay reset system (5.6) is asymptotically stable.

∎

Remark 7 *The LMI's conditions of Theorem (5.2) are delay dependent.*

5.2 Uncertain Linear Reset Systems

This section concerning about the time delay stability of uncertain plants with reset system. One of the challenges that faces the control of plants is the uncertainty. The following analysis is devoted to investigate a stability criterion for uncertain dynamics using the reset policy. Consider the following uncertain base dynamical system:

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t - \eta) \tag{5.15}$$

where the uncertain part of the system is written as

$$\begin{bmatrix} \Delta A & \Delta A_d \end{bmatrix} = EF(t) \begin{bmatrix} D_0 & D1 \end{bmatrix}$$

where $F^T(t)F(t) \leq 1$ The following fact is used

Theorem 5.3 *Consider the dynamical reset system (5.15), with a delay $\eta(t)$ that satisfies $0 \leq \eta(t) \leq \eta^\star$ and $\dot{\eta}(t) \leq \eta^+$. Given positive definite matrices P, Q, R,*

Φ_1, Φ_2, Φ_3 positive scalars λ and β, and any matrices with appropriate dimensions L_1, L_2, L_3, L_4 such that the following matrices are negative semidefinite

$$\Omega_1 = \begin{bmatrix} \Omega_{11} + \lambda D_0^T D_0 & \Omega_{12} & \Omega_{13} + \lambda D_0^T D_1 & (P + L_1)E \\ \bullet & \Omega_{22} & \Omega_{23} & L_3 E \\ \bullet & \bullet & \Omega_{33} + \lambda D1^T D1 & L_2 E \\ \bullet & \bullet & \bullet & -\lambda I \end{bmatrix} \leq 0$$

$$\Theta_1 = \begin{bmatrix} \beta D_0^T D_0 - \Phi_1 & 0 & \beta D_0^T D_1 & A^T L_4^T & 0 \\ \bullet & -\Phi_3 & 0 & L_4^T & 0 \\ \bullet & \bullet & \beta D_1^T D_1 - \Phi_2 & A_d^T L_4^T & 0 \\ \bullet & \bullet & \bullet & -R & L_4 E \\ \bullet & \bullet & \bullet & \bullet & -\beta I \end{bmatrix} \leq 0$$

$$A_R^T P A_R^T - P \leq 0$$

then the reset system (5.15) is asymptotically stable.

Proof. Replacing A and A_d in the LMI's of Theorem 5.2 with $A + EF(t)D_0$ and $A_d + EF(t)D_1$, respectively, makes Equations (5.7), and (5.8) equivalent condition:

$$\Omega_1 = \Omega + \begin{bmatrix} PE + L_1 E \\ L_3 E \\ L_2 E \end{bmatrix} F(t) \begin{bmatrix} D_0^T & 0 & D_1^T \end{bmatrix} + \begin{bmatrix} D_0 \\ 0 \\ D_1 \end{bmatrix} F^T(t) \begin{bmatrix} E^T P + E^T L_1^T & E^T L_3^T & E^T L_2^T \end{bmatrix}$$

$$\Theta_1 = \Theta + \begin{bmatrix} D_0 \\ 0 \\ D_1 \\ 0 \end{bmatrix} F(t) \begin{bmatrix} E^T L_4^T & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} L_4 E \\ 0 \\ 0 \\ 0 \end{bmatrix} F^T(t) \begin{bmatrix} D_0^T & 0 & D_1^T & 0 \end{bmatrix} \quad (5.16)$$

Applying Schur complement and Fact1 completes the proof.

∎

5.3 Simulation Results

Simulation results should be presented to show the effectiveness results of reset time delay theory. The proposed method has been tested using the following example. Consider the following linear time delay reset systems:

$$
\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} -2 & 0 \\ 0 & 0.9 \end{bmatrix} x(t) + \begin{bmatrix} -2 & -1 \\ 0 & -10 \end{bmatrix} x(t-0.5) \\
\dot{\tau}(t) &= 1, \\
x(t^+) &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x(t) \\
\tau(t^+) &= 0,
\end{aligned}
\tag{5.17}
$$

with a resetting surface $\mathcal{S}$ defined by $\tau(t) \geq \rho$ means that a periodic reset mechanism is implemented. It is obvious that the reset surface is independent of the trajectory of the system. Various values of ρ is considered in simulation.

Figure 5.1 illustrates the response of the system when $\bar{\Delta} = 0.5$. The reset instant occurs every half a second. One of the states is not resettable while the other is reset to zero which increases the speed of the response. Two reset instants are enough to force the system response to settle.

Decreasing ρ to 0.1 increases the reset instants while it decrease the settling time as can be shown in Figure 5.2. The efficiency of rest mechanism can be demonstrated by comparing the response with non reset system as shown in Figure

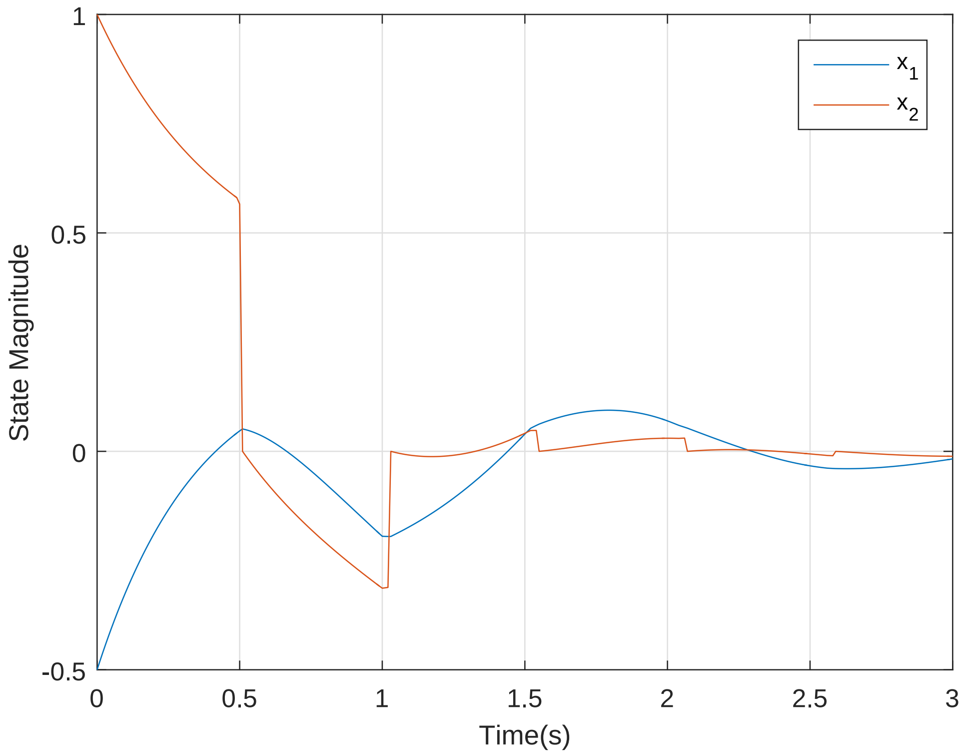

Figure 5.1: State response with reset period =0.5 second.

5.3.

5.4 Conclusion

In this thesis, a novel analysis of reset systems scheme is proposed for certain and uncertain systems. The stability conditions of the time delay reset systems are less conservative than the presented in literature since the LMI's are delay dependent inequalities and less conservative. By using the same LMI's, extending the results to obtain controller gains can be easily obtained using some modifications. More general theorems are presented in terms of multiple Lyapunov functions, which increase the flexibility of LMI's. Simulation results using numerical examples

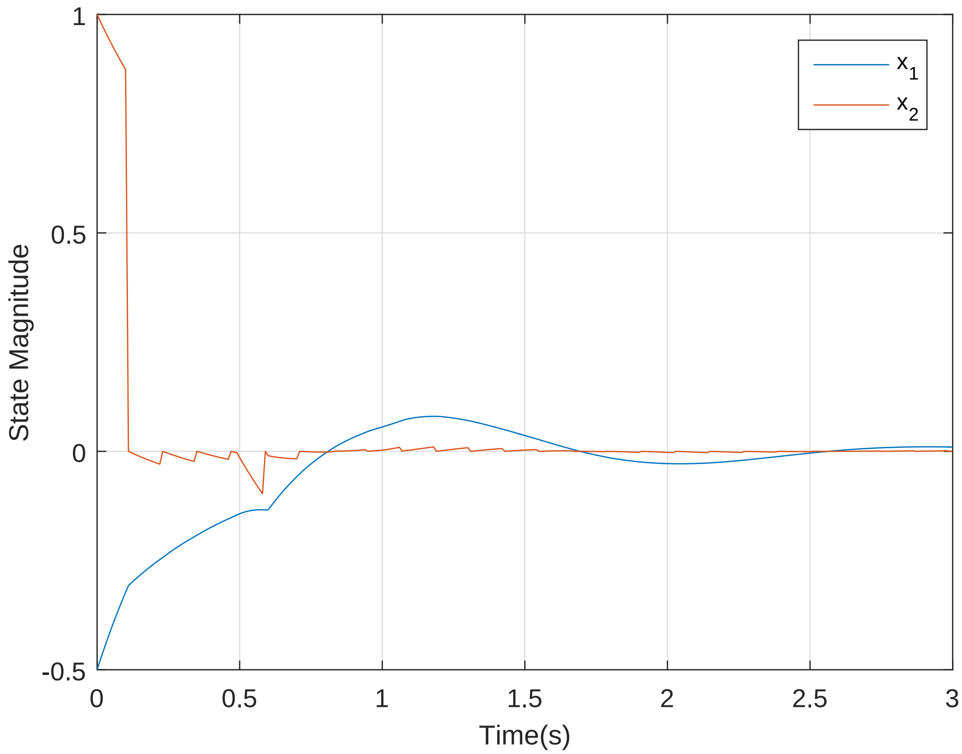

Figure 5.2: State response with reset period =0.1 second.

demonstrate the ability of the proposed method to produce a good performance for reset systems with and without delay.

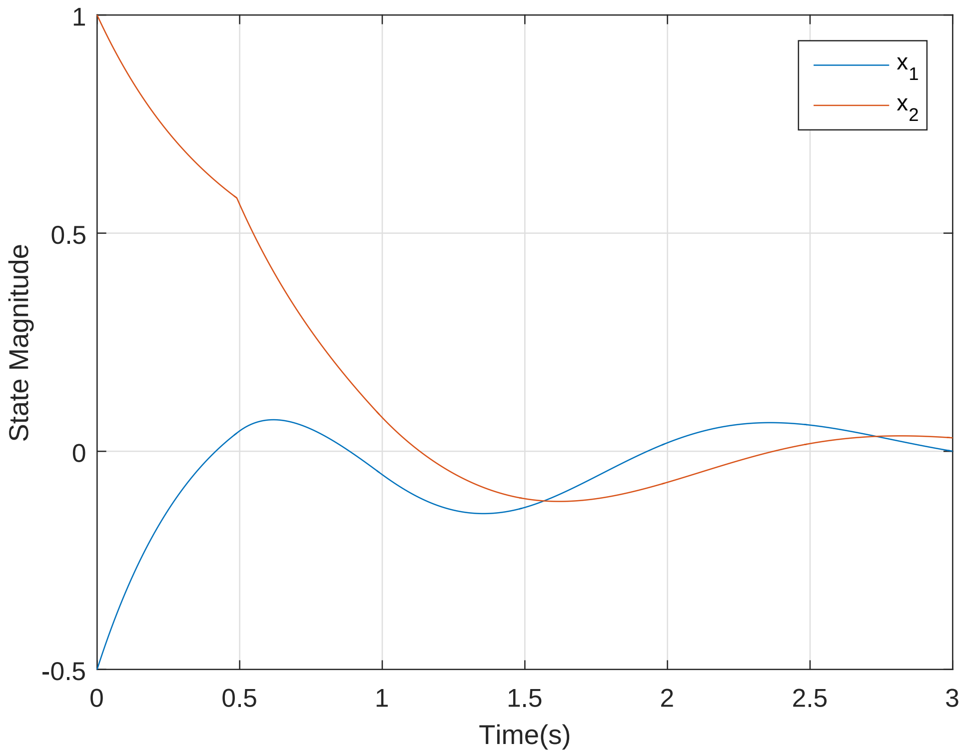

Figure 5.3: State response without reset mechanism.

CHAPTER 6

SUGGESTIONS FOR FUTURE WORK AND CONCLUSIONS

In this chapter, we give the main contributions of the thesis. Moreover, various suggestions for future work are detailed.

With the advancement in the automatic control theory, the desire to perform nonlinear operations has grown significantly. Potential benefits of reset control systems have demonstrated its applicability to complex systems. In this dissertation, we develop stability analysis and synthesis for reset dynamical systems. It has been proved that the reduced order reset control system provides a satisfactory performance. The idea of order reduction is established for linear systems with free delay. The presented methodology forces a number of states to reach the equilibrium point in a finite time. It would be interesting to extend the results to nonlinear plants with delays, and study the effect of reset mechanism on limit cycles.

Several ideas and potential line of research could be developed in future works on reset time delay systems. Since the control over networks introduces time delay, resettable observer-based control could be advantageous to be used over networks. More works need to be extended on reset control design methods rather than on reset analysis. The extended cases should also include the situation of constrained systems.

Extensions are given to study the stability of reset systems with uncertainty. The stability conditions of the time delay reset systems are delay-dependent. It is interesting to extend the aforementioned results to reset systems with multiple delays. On the other hand, any contribution to stabilizing delayed uncertain system by reset control is significant. Another creative treatment could be done by reducing the conservativeness of linear matrix inequalities that appear in time delay systems.

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  -Tawjihi Exam Hebron Industrial Secondary School, Hebron-Palestine Hebron *(89.4/100)* Excellent, 2009.
- Research Papers and Conferences

  - M. Mahmoud, and B. Karaki, New Results on Reduced-Order Reset Controllers, (Submitted).

  -M. Mahmoud, and B. Karaki, A Novel Event-triggering Scheme for Uncertain Systems. (Submitted).

- M. Mahmoud, and B. Karaki, Novel Results On the Delay Rest Control Systems, (In progress).

- B. Karaki, A new method for solving a non-homogeneous Cauchy-Euler differential equation. Third Palestinian Conference on Modern Trends in Mathematics and physics, Palestine, july 2012.

- B. Karaki, and S. Al-Takrouri, Design and Implementation of a Low Cost Control System for Bioreactors, Students Innovation Conference at Palestine Polytechnic University, 2013.

- Book Chapters

  -M. Mahmoud, and B. Karaki. Chapter 8: Rest Control Systems with Time Delays In Time-Delay Systems: Concepts, Design and Stability Analysis Book, to be published in Nova Publisher, 2018.

- Work and Experiences

  - Involved in a granted project: "New Control Methodologies of System for Systems Engineerings", DSR project number IN141003, Principal Investigator: Magdi S. Mahmoud (2016/2017).

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  - Field Training, Hebron-Palestine, Maintenance Training, Salaymeah Company (100 Hours), june, 2012.

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- Computer Skills

  - C++, and C Languages.

  - MATLAB/Simulink.

  - Maple.

  - Assembly language.

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