

**INTEGRATION OF MULTIVARIATE STATISTICAL PROCESS  
CONTROL AND ENGINEERING PROCESS CONTROL**

BY

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*Dedicated to my parents, my wife and my sisters*

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## THESIS ABSTRACT (ENGLISH)

<b>Name</b>	Yasir Abdullah Siddiqui
<b>Title</b>	Integration of Multivariate Statistical Process Control and Engineering Process Control
<b>Degree</b>	Master in Science
<b>Major Field</b>	Systems Engineering
<b>Date of Degree</b>	April 2014

Statistical Process Control is being used along with classical feedback control systems (also termed as Engineering Process Control) for the purposes of detecting faults and avoiding over adjustment of the processes. This thesis evaluates the effectiveness of integrating SPC with EPC for both fault detection and control. A novel framework for fault detection using Multivariate Statistical Process Control (MSPC) has been demonstrated. The simultaneous application of MSPC control charts to process inputs and outputs or in other words “Joint Monitoring” of process inputs and outputs is shown here to provide efficient fault detection capabilities.

The proposed method was simulated for different levels of shifts in noise as assignable cause/fault along with two different EPC schemes using a numerical example. The results indicated that the “Joint Monitoring” provides earliest detection as compared to monitoring of either inputs or outputs alone.

An example of Heating Ventilation and Air Conditioning (HVAC) systems is simulated here and used as a case study to demonstrate the detection capabilities of

the proposed framework. Three different kinds of faults namely sensor malfunction, stuck damper and leakage in cooling coil were simulated and hence successfully detected by the proposed mechanism. Moreover, a corrective action scheme was briefly discussed as well to illustrate a complete control system with fault detection and correction.

## THESIS ABSTRACT (ARABIC)

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الدرجة العلمية : ماجستير العلوم

مجال التخصص الرئيسي: الهندسة النظم

تاريخ الحصول علي الدرجة: أبريل 2014

لقد تم استخدام طريقة التحكم الاحصائية (SPC) مع انظمة تحكم التغذية الراجعة التقليدية (EPC) و التي تعرف ايضا بطرق التحكم الهندسية ن اجل تحسس الاخطاء و تجنب التقدير المبالغ فيه للعمليات المختلفة. تقييم هذه الرسالة فاعلية الموائمة بين طريقتي SPC و EPC في التحكم و تحسس الاخطاء. تم توضيح نموذج فريد لتحسس الاخطاء بالاعتماد على طرق التحكم الاحصائية متعددة المتغيرات. MSPC التطبيق المتزامن لجداول التحكم الخاصة ب MSPC لمدخلات و مخرجات النظام, او ان شئت قل المراقبة المتزامنة لكليهما, اظهر كفاءة هذه الطريقة في تحسس الاخطاء.

لقد تمت عملية المحاكاة للطريقة المقترحة لمستويات مختلفة من التشويش على شكل سبب/خطأ لنموذجين مختلفين من EPC باستخدام مثال عددي. اظهرت النتائج ان المراقبة المتزامنة اقدر على التحسس المبكر للاخطاء إذا ما قورنت بنتائج مراقبة المدخلات و المخرجات بشكل منفرد.

تم التعامل مع نظام للتكييف و التبريد HVAC كحالة للدراسة حيث تم عمل محاكاة له لتوضيح كفاءة الطريقة المقترحة لتحسس الاخطاء. تمثلت هذه الاخطاء بثلاثة انواع مختلفة حيث تم تحسسها جميعا بنجاح باستخدام الطريقة المقترحة. زيادة على هذا, فقد تم مناقشة طريقة لمعالجة الاخطاء بشكل مختصر لبيان نظام تحكم كامل يشمل تحسس الاخطاء و معالجتها.

# CHAPTER 1 INTRODUCTION

## 1.1 BACKGROUND

For products, unstable process can lead to poor quality, which significantly affects customers' satisfaction and companies' goodwill. A good process control has a final aim for incorporations to achieve stable quality. Statistical Process Control (SPC) and Engineering Process Control (EPC), which have been used in quality improvement for decades, are the most used tools for process control. These two strategies focus on different quality aspects. EPC gives sequential adjustment in the process without finding the assignable causes[1]. The main goal of EPC is to compensate for the effect of inertia in the process and to keep the process on target. EPC regulates the process input in order to minimize the deviation of output from target while ignoring the root cause behind this deviation. EPC techniques are extensively applied in the chemical and other processes, where variations in process outputs are often largely correlated[2]. The benefits of using EPC can be concluded as follows[3]:

- EPC prevents injury to factory personnel, emission and waste to environment, and damage to equipment.
- EPC keeps product quality in customers' demand at minimum cost.

- EPC enhances plant production rate at minimum cost.
- EPC makes efficiency of process operation maximal by adjustment of a controller.

EPC focuses on process regulation which assumes that there is a set of manipulatable variables that can be adjusted to compensate for the drift in process outputs and keep the process outputs close to the desired targets[1]. It makes no attempt to identify and remove assignable causes that impact the processes. However, there are still some unknown assignable causes, which can disturb the process. When the disturbance to the process is beyond a certain range, EPC alone is not able to keep the system output close to the target. Therefore, it is necessary to apply SPC to detect non-random patterns which cause the abnormal disturbance to the process. As soon as the types of non-random patterns are identified, the corresponding root causes should be removed to bring the process back to normal conditions. On the other hand, SPC is used to detect an assignable cause which makes the process out of control. The main goal of SPC is to achieve product quality by monitoring whether certain variables of the process in specific range[2]. SPC tools, such as control charts, are used to determine the stability of process mean and variation by measuring output characteristics. SPC have had popularity for a long time worldwide in the industries because of the following benefits[1]:

- SPC reduces scrap and rework for improving productivity.
- SPC prevents defects to appear in the product/process.



- SPC prevents unnecessary process adjustment for process stability.
- SPC provides diagnostic information for current decision-making.
- SPC provides information about process capability.

SPC only can help to detect the evident assignable cause that drives a process “out of control” from “in-control” state especially in the processes that tends to stay on target for relatively long period of time without continual ongoing adjustment. However, practical manufacturing processes still have a tendency, or called “inertia”, to drift away from the target. This inertia primarily results from material, machine, tools, machine settings, human factors, etc. If the process is drifted away from the target by inertia and the extent of drifting doesn’t exceed the control limit, SPC still presume that the process is still “in-control” and there is no need to change the process. This implies that an “in-control” process is not related to whether the units it produces are acceptable or not. It is the main blind spot of using SPC, where EPC can conquer. Based on previous description, SPC and EPC, which respectively contain different control disciplines, can supplement each other. Therefore, integration of SPC and EPC offers an attractive trend and option for process control study. The need for an integrated approach to process control increases when the processes designs are adopting more hybrid framework, especially in multi-input multi-output (MIMO) system. Moreover, controls from one discipline may not be effective enough to achieve higher demands in certain processes[2]. The need of combining SPC and EPC arise because of following reasons:

EPC, in the presence of a range of disturbances, regularly adjusts the manipulatable process variables to keep the process outputs on target while ignoring the causes behind

the disturbances; however, its capability is limited when the strong disturbances appear in the system which cannot be avoided until the underlying causes behind them are rectified. SPC is instrumental in detecting such assignable causes, removal of which can relieve the adjustment procedure of EPC. Moreover, some disturbances that have a certain cause behind them can be conquered using EPC yet at the expense of energy and recourses. Detection of these assignable causes using SPC minimizes the energy losses. Furthermore, life-time of a plant, equipment or a machine reduces when it is over-adjusted using EPC; hence, timely detection and removal of assignable cause can increase the life-time as well. Therefore, the integrated scheme containing SPC and EPC should be essentially studied and applied in practical process control domains.

Integration of Statistical Process Control and Engineering Process Control acquired first attention in 1988 when [4], [5] proposed this concept of integration and convinced the SPC research community that control charts can be used to monitor a “controlled” system. The two schemes, their similarities, overlap, contradictions, reasons behind their isolation and the need to integrate them were reviewed. [6] formulated the model for integration using Shewhart and CUSUM control charts as monitoring tools and added the minimum mean-squared error (MMSE) EPC rule in their further work and, as such, they were among initiators in the development of this integration technique.

All of the above researches suggested that combined application of both EPC and SPC can outperform the application of either of them alone in most of the cases. The fundamental work of the above mentioned researchers was followed by many others that can be broadly classified into two categories based on the integration approach.

### *SPC triggered EPC*

One of the popular schemes of SPC/EPC integration involves triggering of EPC controller only in case when SPC signals presence of assignable cause or out-of-control signal; [7] were the earliest of many in this horizon who have advocated that EPC based process adjustments should only be triggered if SPC detects the out-of-control state of the system. [8] provided a concept similar to that of [7] by suggesting a cost based model in which the EPC adjustments were only supposed to be triggered through an out-of-control signal provided by SPC based monitoring. They only considered the out-of-control and in-control costs and made a handful of assumptions to simplify the problem. A contrary approach is to continuously use EPC for controlling and process adjustment while using SPC for detection of assignable cause by monitoring output or input variables of the process. Applying EPC continuously implies loss of resources, whereas EPC only triggered by SPC in out-of-control condition amounts for loss of quality. Therefore, [9] proposed a scheme that takes short comings of both the approaches into consideration and proposed an integrated scheme comprising Taguchi's Quality Engineering. In the mentioned approach SPC plays dual role; apart from being used to search for assignable cause, it also provides required quantities to a Taguchi quality loss function that estimates the cost of associated quality loss. Meanwhile, the cost of EPC implementation for the same instant is also calculated. Finally, EPC is only allowed process adjustments when cost of adjustment is less than the cost of quality loss.

### ***Integration for assignable cause detection:***

The most powerful approach of SPC and EPC integration involves continuous adjustments using EPC and detection of assignable cause using SPC monitoring. Several researchers have explored different EPC techniques along with different control charts for this purpose. Shewhart, Exponentially Weighted Moving Average (EWMA) and Cumulative Sum (CUSUM) control charts were used in integrated models before [10] introduced Cumulative Score (CUSCORE) control charts as SPC tools in the arena of EPC/SPC integration. Furthermore, [11] formulated a graphical aid technique meant to recognize the type of disturbance or assignable cause (either shift in mean or a drift). Later on [12] demonstrated an adaptive controller technique that is triggered by SPC based assignable cause detection and aims at identifying the changing parameters of the disturbance and consequently adjusting the process until the underlying cause is completely eliminated. Process subjected to a slowly changing trend was considered by [13]. It is a special case of SPC and EPC integration in which it is insufficient to only monitor the process that changes with time using SPC. Accordingly a model had been developed that makes adjustment to the process after regular intervals of time and the process output itself is monitored with changing control limits instead of its variation from the target value. SPC/EPC integration for univariate case was comprehensively discussed by [14] and the associated issues had been addressed. In the mentioned study, effects of Shewhart and CUSUM control charts on an MMSE regulated system with shifting and drifting mean disturbances had been taken into account. [14] noted that Shewhart control charts are more effective than CUSUM control charts in detecting the

shifts. In case of drifting disturbance with smaller slope, CUSUM proves to be more effective whereas for larger slopes Shewhart is more efficient. Moreover, it was noted that an EPC feedback compensation mechanism affects SPC out-of-control detection and disturbs the output when suddenly assignable cause is removed. To account for this so called overcompensation issue, a joint monitoring scheme of Shewhart and CUSUM charts had been used to recognize the disturbance type and a cost based decision rule is provided to decide whether the assignable cause removal will be cost efficient owing to the fact that the overcompensation phenomenon is irresolvable and in some cases renders the system unstable.

## **1.2 MOTIVATION**

Although a substantial amount of research has been done in the area of integration of SPC and EPC during last two decades yet it still seems very much insufficient due to following reasons. Most of the work done by early researchers was based on many unrealistic assumptions that were inevitable for convincing the then researchers in a simpler way that the integration of these techniques can prove beneficial. Later on, a handful of researchers got attracted towards the approach and started to take into account some realistic considerations as well. Some considered different types of costs associated with different operations while others focused on the time delays during various steps. Some investigated different kinds of disturbances in the systems that were meant to be detected whereas others explored the detecting powers of different control charts in

integrated systems. Apart from this, there were some aspects that acquired very little attention due to complexity and lack of foundation.

A few of the researchers in this area[15]–[17] have tried to form a foundation for integration of SPC/EPC in multiple input multiple output systems; however, there is a drastic need for further research in this aspect owing to the fact that most of the industrial processes are MIMO with a strong coupling in between that can hardly be neglected.

[18] and [19] have shown that SPC, when applied at input of the process, proves to be quiet useful. Despite the exceptional performance of this method, especially in cases of small shifts and slowly varying drifts, this approach has gained very little attention. There remains a need for further exploration in this procedure starting from evaluation of effects of different EPC controllers on SPC at input to generalization of this concept to MIMO systems.

In addition to this, designing of optimal time for removing assignable causes, quality characteristics considerations, over compensation phenomenon, effect of intelligent controllers in integrated scheme are some of the aspects that need to be addressed.

Furthermore, there are very few case studies carried out in this area while most of the researchers have stuck to numerical examples with ideal assumptions for illustrating their findings. On the contrary, case studies provide a path way for ideas to get adopted into practices. In particular, any case study involving MIMO systems has not been carried out in this area; however, a good amount of work has been done in the area of Fault

Detection and Diagnosis[20]–[24] which can be used as a reference to conduct a good case study in integration of multivariate SPC and EPC for detecting assignable causes.

### **1.3 OBJECTIVES**

Reasons mentioned in previous section have motivated the author of this research thesis to define following objectives meant to be carried out as result of this work:

- Formulation of a framework for integration of SPC and EPC for MIMO systems by adding joint monitoring of inputs and outputs to the work of [16]
- Evaluation of different EPC controllers, such as Output Feedback and fuzzy controllers etc., in the model of [16].
- A case study on assignable cause detection by integrating SPC and EPC in MIMO systems using the model provided by [23], [24].

### **1.4 THESIS ORGANIZATION**

Organization of the remainder of the thesis will be as follows:

Chapter 2 provides detailed literature review outlining the gap in recent researches and the areas needed to be addressed related to the topic. It is followed by a chapter on a model formulation for integration of SPC and EPC in MIMO systems. Chapter 4

illustrates a case study on fault detection using integrated use of EPC/SPC in MIMO systems. Finally chapter 5 concludes the findings and achievements of this research.



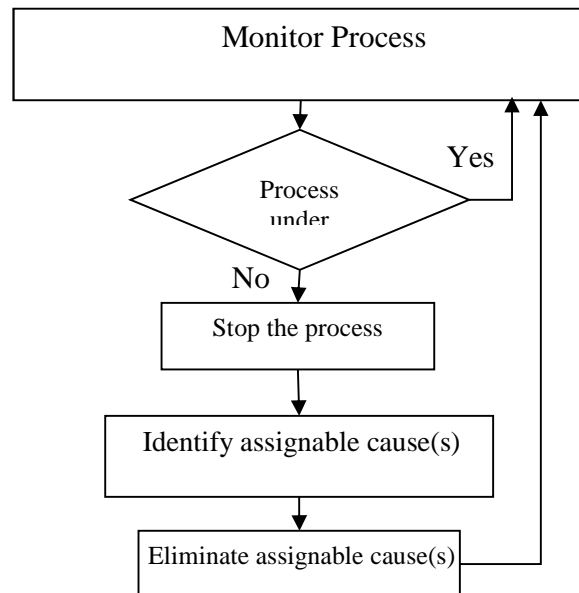
## **CHAPTER 2 LITERATURE REVIEW**

Integrated control approach utilizing both Engineering Process Control and Statistical Process Control can be beneficial for the processes; however, integration of the two isolately developed techniques requires acquaintance with both the fields along with in-depth knowledge of recent advancements in the integrated models. This chapter outlines basics of SPC/ EPC and provides a comprehensive review on integration of both the complementary techniques.

### **2.1 STATISTICAL PROCESS CONTROL**

The earliest statistical process control procedure can be traced back to the work of Shewhart [25] which began in the 1920's and resulted in the publication of his seminal book in 1931. Since its inception over 80 years ago, SPC techniques have been used to obtain significant reductions in product/process variability in the discrete manufacturing environment. However, these SPC techniques did not address real-time, automatic correction of the output. As a result, control engineers adopted EPC to monitor and adjust system variability in the continuous process industry. SPC techniques have been developed for monitoring processes where the output deviations (errors) are independent and also the cases where they are correlated. The objective of SPC techniques is to identify assignable or special causes of variability and hence aid in elimination of these special causes of variability that result in driving the process out of control. The SPC

methodology is basically a graphical test of statistical hypothesis. Figure shows how SPC keeps a process under control.



**Figure 1: Typical Statistical Process Control Procedure**

The output observations  $Y_t$  are monitored and collected for a pre-determined time period. These observations are sequentially plotted and traditionally compared against  $3\sigma$  control limits. Any point falling outside these control limits is considered out of control and is used to identify assignable causes. These assignable causes are eliminated from the process in order to return the process to a state of statistical control. In addition to serving as a historical visual aid, control charts help operation and control engineering personnel to make objective decisions and reduce the tendency to over-control the process. When the output deviations are assumed to independent and normally distributed, the Shewhart, CUSUM and EWMA control charts can easily be applied for process monitoring.

### 2.1.1 Shewhart Control Chart

Dr. Walter A. Shewhart developed these control charts in the 1920's and they are extensively used in many industries for process monitoring. He developed the idea of using past and present observations as a tool to make future predictions about the process being monitored. However, it is expected that the process be in a state of statistical control before making future predictions. The model for the process is

$$Y_t = \eta + \varepsilon_t \quad (2.1)$$

where  $Y_t$  is the observation at time  $t$ ,  $\eta$  is the process mean, which is assumed to be constant, and  $\varepsilon_t$  is the error at time  $t$ , which is assumed to be  $[NID(0, \sigma^2)]$ . Typically the limits of the Shewhart control chart are set at  $\pm 3\sigma$ .

### 2.1.2 CUSUM Control Charts

The CUSUM control chart first proposed by Page [26] is an alternative to the Shewhart control chart. The CUSUM control chart incorporates all the information in a sequence of observations over time by plotting the cumulative sums of deviation of the observations from target. The CUSUM control chart uses the statistics  $S_N$  defined as

$$S_N = \sum_{t=1}^n \frac{Y_t - \tau}{\sigma_y} \quad (2.2)$$

where  $Y_t$  is the observation at time  $t$ ,  $\tau$  is the target value,  $N$  is the number of observations on the process and  $\sigma_y$  is the standard deviation of the process. In order to apply the CUSUM, the observations are usually assumed to be independent and normally distributed with fixed mean  $\eta$  and constant variance  $\sigma$ . CUSUM procedures for other

types of process data are available, but not discussed in this work. The CUSUM defined in equation 2.2 fluctuates statistically around 0 when the process remains under statistical control. If the mean of  $Y_t$  shifts upward, then the sum increases and if the mean of  $Y_t$  shifts downward, then the sum decreases. Therefore an increasing or a decreasing trend is a sign that the process mean has shifted and a search for the assignable cause should be carried out. The CUSUM technique is based on Sequential Probability Ratio Test (SPRT). There are two methods for designing and displaying the CUSUM. The first method is the V-mask procedure and the second method is called the tabular (h and K) CUSUM, which is gaining popularity due to its ease of use and the increase in computer implementations. Let  $S_H(t)$  be the upper one-sided tabular CUSUM for period t (for increasing mean for averages) and  $S_L(t)$  be the lower one-sided tabular CUSUM for period t (for decreasing mean for averages). Accordingly,  $S_H(t)$  and  $S_L(t)$  are computed as follows:

$$S_H(t) = \max[0, (\bar{Y}_t - \tau) - K\hat{\sigma}_{\bar{y}} + S_H(t - 1)]$$

$$S_L(t) = \max[0, (\tau - \bar{Y}_t) - K\hat{\sigma}_{\bar{y}} + S_L(t - 1)]$$

where K is the reference value, which is usually chosen about halfway between target  $\tau$  and the out-of-control value of the mean  $Y_t$  that is of interest. The CUSUM control limits are set at  $h\sigma_{\bar{y}}$  where h is the decision interval.

[27] developed a combined Shewhart and CUSUM scheme that will work for both large and small shifts in the mean. [28] have proposed a modification to the CUSUM called the Fast Initial response (FIR) to improve the sensitivity at process set-up in order

to accommodate any delay in resetting the mean to the target value after a corrective action is applied.

### **2.1.3 EWMA Control Charts**

The Exponentially Weighted Moving Average (EWMA) control chart introduced by Roberts [29] is a good alternative to the Shewhart control chart for detecting small shifts. The EWMA is a smoothing technique, and is given by:

$$z_t = \lambda x_t + (1 - \lambda)z_{t-1}$$

where  $Z_t$ = smoothed value at time t.  $Y_t$  = observed value at time t,  $\lambda$ = constant,  $0 < \lambda < 1$ .  $\lambda$  is the weight given to the most recent observation and  $(1 - \lambda)$  is the weight given to the most recent prediction. The original use of the EWMA was in time series analysis, because it is often a good predictor of the next value of the variable of interest x.

Unlike Shewhart and CUSUM control charts, the EWMA control chart can also be effectively used for autocorrelated data. [30] also suggested using a combination of Shewhart and EWMA control charts in order to identify large as well as small shifts in the mean. [1] provides comprehensive coverage of all these control schemes.

### **2.1.4 Average Run Length (ARL) as a control chart performance measure**

In order to compare the performance of control charts, the Average Run Length (ARL) is widely used. It is the average number of observations that are taken before the control chart indicates an out-of-control condition. The optimal control strategy will aim for a large ARL when the process is in statistical control and a very small ARL when the

process goes out of control due to the presence of an assignable cause. This approach will ensure that the number of false alarms is minimal when the process is under control. Many researchers prefer an in-control ARL of 370.4, because this is the theoretical value achieved by a Shewhart control chart with  $3\sigma$  limits.

ARL is associated with the probability of TYPE-I and TYPE-II errors. Let  $\alpha$  be the probability of alarm when process is in control:

$$\alpha = P[\text{Type-I Error}] = P[\text{point falls outside control limits} | \text{process is in control}]$$

Let  $\beta$  be the probability of alarm when process is out of control:

$$\beta = P[\text{Type-II Error}] = P[\text{point falls inside control limits} | \text{process is out of control}]$$

Therefore the in control ( $ARL_0$ ) and out of control ( $ARL_1$ ) average run lengths can be defined as:

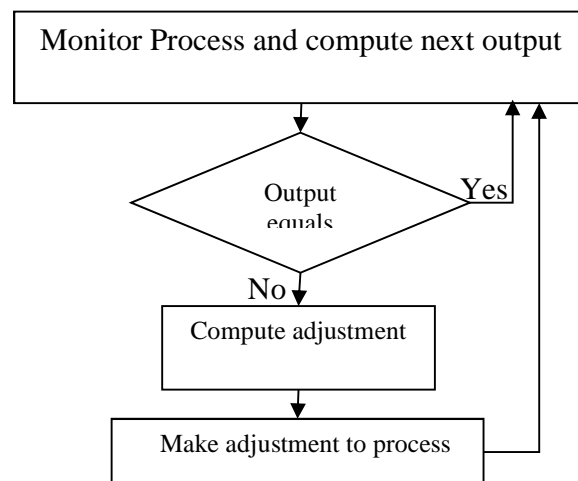
$$ARL_0 = 1 / \alpha \tag{2.3}$$

$$ARL_1 = 1 / (1 - \beta) \tag{2.4}$$

## 2.2 ENGINEERING PROCESS CONTROL

Although SPC techniques are extensively used for reduction of process variability, it is not necessarily the best method for all cases and this is particularly true in the case of a

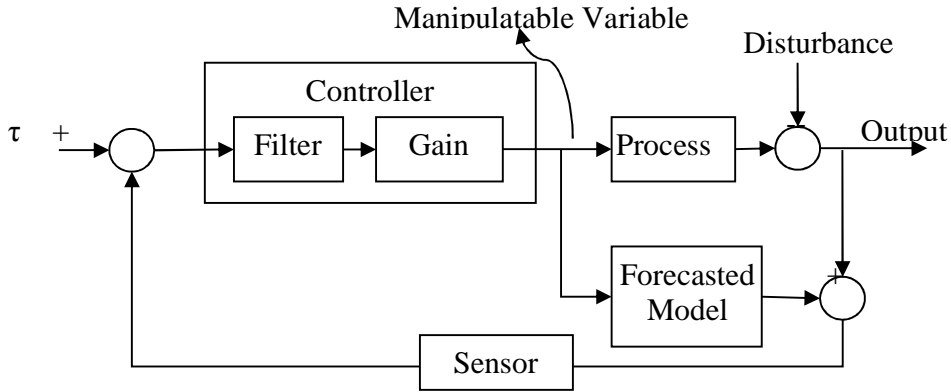
drifting mean process as illustrated by [31]. EPC has been effectively used for these cases, which are common in chemical and process industries. EPC is based on control theory, which operates in the following manner: (1) Predict the next observation on the process, (2) Identify some other variable, which can be manipulated in order to affect the process output and, (3) Understand the effect of this manipulated variable in order to determine how much control action to apply so as to make an adjustment in the manipulated variable at time  $t$  that is most likely to produce an on-target value of the process output at time  $t+1$



**Figure 2: Typical Engineering Process Control Procedure**

A clear understanding of the process dynamics and the relationship between manipulated variable and output variable is necessary to accomplish this task. Control theory accomplishes this task through the use of deterministic models, stochastic models (for disturbance) and transfer function models. The model equations are proportional,

integral, derivative or a combination of each other. The compensation is applied in the form of feedback, feedforward or combination of both.



**Figure 3: Typical Feedback Control Loop**

Authors of [32] have explained various types of deterministic control schemes. Proportional control refers to the correction, which is proportional to the error (the difference between actual response and target), that is,

$$X_t = K_p \varepsilon(t) \tag{2.5}$$

where  $K_p$  is the proportional gain. Integral control refers to the correction, which is proportional to the time integral of the error, that is,

$$X_t = K_i \int_0^t \varepsilon(u) du \tag{2.6}$$

where  $K_i$  is the integral controller gain. Derivative control refers to the correction, which is a measure of the rate of change of error, that is,



$$X_t = K_D(d\varepsilon(t)/dt) \quad (2.7)$$

where  $K_D$  is the derivative controller gain. Proportional-Integral (PI) and Proportional-Integral-Derivative control (PID) schemes are also widely used. These controls can be applied in a feedback, feedforward, cascade and combination of feedback and feedforward control schemes. PID controllers base the control action empirically on a mixture of proportional, integral and derivative control. A PID controller is given by

$$X_t = K_p\varepsilon(t) + K_i \int_0^t \varepsilon(u)du + K_D(d\varepsilon(t)/dt) \quad (2.8)$$

Controllers of this kind are usually operated automatically and employ continuous rather than discrete measurement and adjustment. Here,  $X_t$  is the deviation of the input from some equilibrium to compensate for the continuous deviation  $e_t$  of the output from target.

Spectral analysis by [33] is a method for dealing with autocorrelated, sensor-based data from machines, instruments or metrology systems. These methods are appropriate when the sampling interval is short that the data points are not independent as required by Shewhart charting. They also described a minimal variance controller (MVC), which is designed to keep the output variance to a minimum. Any automatic controller can be tested this way to assure that gain, reset, and proportional band are adjusted correctly for the lag time of the system. MacGregor [34] indicated techniques to model discrete dynamic stochastic models using ARIMA time series models or by a state-variable model. The state-variable model was developed by Kalman [35], which, uses state variable models to characterize the system and solves the optimal control problems using dynamic programming and Kalman filtering techniques.

## 2.3 SPC VS EPC

As explained in the previous sections, both SPC and EPC have the goals of reducing the process variability from target while keeping the process stable and under control. However, they take different routes for accomplishing the similar goals. Classically there existed a large gulf of mistrust between statisticians and the process control professionals. This was mainly due to insufficient knowledge possessed by statisticians about control systems and vice versa.

Since SPC and EPC represent two different approaches to reducing variability, it had been a challenge to integrate or use both techniques for process monitoring and control. However, lately there has been much more interest in this area since an effective integration of SPC and EPC is likely to result in improved quality through further reduction of variability.

McGregor [36] emphasized the importance of SPC/EPC integration and indicated that a typical control engineer is inadequately trained in the statistical methods and data analysis. Deshpande [37] reinforced this thought by proposing statistical analysis classes in addition to the traditional control engineering classes in the undergraduate control-engineering curriculum. Statisticians, who are experts in the discrete realm, have very little knowledge of the process dynamics and classical continuous control. Box [38] stressed the need for control engineering knowledge to the traditional statistical quality practitioner in order to reap the benefit of combined schemes. Lack of communication is not limited to the control engineers and statisticians. Hoerl and Palm [39] indicated that at

any given time, 25% to 35% of the world's most advanced automatic control systems are in manual mode. One compelling reason for this is the lack of operators' confidence in a "black box" that makes decisions beyond their grasp. They seem to be comfortable with the hands-on SPC techniques. However, with advances in technology, this is changing and more use of automation is occurring resulting in near lights out factories. In addition to providing research strategies for the integration of SPC and EPC via simulation, Messina [31] also presented the differences between SPC and EPC in a tabular form shown in Table 1

**Table 1. Comparison of SPC and EPC by Messina (1992) [31]**

<b>Philosophy</b>	<b>Minimize variability by detection of and removal of process upsets</b>	<b>Minimize variability by adjustment of process to counter-act process upsets</b>
Application Deployment	Expectation of process stationarity	Expectation of continuous process drift
Level	Strategic	Tactical
Target	Quality characteristics	Process parameters
Function	Detecting disturbances	Monitoring setpoints
Cost	Large	Negligible
Focus	People and Methods	Equipment
Correlation	None	Low to High
Results	Process improvement	Process optimization

Comparison of these two methodologies based on Table 1 indicates that SPC and EPC have little in common; however, the later developments proved this assumption wrong.

## **2.4 INTEGRATION OF SPC AND EPC**

MacGregor [4] was the first who convinced the SPC community that control charts can be used to monitor a “controlled” system. His work became the cornerstone of the

integration of the isolated strategies. His review of the schemes, their similarities, overlap, contradictions and reasons behind their isolation was followed by couple of more [5], [40] reviews highlighting the need to integrate these methods. MacGregor [4] suggested that stochastic control theory connects these two fields and the application of on-line quality control demands the integration of these two schemes. Palm [40] also emphasized that SPC and APC (automatic process control) actually form a complementary nature in process improvement and demonstrates with an example of a continuous baking operation. Box and Kramer [5] also discussed the benefits of using SPC methods in conjunction with a system under APC.

Vander Weil *et al.* [41] used the term algorithmic statistical process control (ASPC) for an integrated approach devised in order to improve quality. This approach focuses on quality gains through appropriate process adjustment and also through identification and elimination of root causes of variability detected by SPC techniques.

Vander Weil and Tucker [42] discussed the scenarios in which the integration becomes highly useful. They expressed the approach with the help of a case study on a batch polymerization example.

Montgomery *et. al.* [6] used the famous model of funnel experiment to explain SPC and EPC integration and showed the potential effectiveness of this new approach especially when assignable causes take place. It was testified by them that SPC is capable of detecting assignable causes rapidly by monitoring the outputs while EPC effectively keeps the process on target. In their study they investigated how the system operates

when additional assignable causes occur. They used the average squared deviation from the target as performance measure and pointed out that the model is robust to the misspecification of the disturbance model. They concluded that integrating SPC and EPC by applying SPC to the output deviation from target results in reducing overall variability if the system experiences certain assignable causes.

The so far research involving the integration of SPC and EPC can be categorized in two basic classifications based on the roles of SPC and EPC respectively in the integrated scheme.

The first classification involves inherently stable systems that are continuously being monitored using SPC. EPC starts to play its role in process adjustment whenever SPC detects an out-of-control signal. Advocates of this approach argue that there is a cost associated with continuous adjustment and it should not be done until needed.

The second classification involves continuous regulation/adjustment of the process by EPC whereas SPC is meant to monitor the system for assignable cause. Most of the researchers have focussed on this approach owing to the fact that most of the processes are inherently unstable and they need continuous adjustment/regulations for stability.

Following subsections review the research done using the above mentioned two approaches:

#### **2.4.1 SPC triggered EPC and cost based approaches:**

English and Case [43] were the earliest of many in this horizon who have advocated that EPC based process adjustments should only be triggered if SPC detects the out-of-control state of the system. They used the SPC to monitor the process while APC (as called by the authors) was used as a feedback filter, taking control action whenever an out-of-control signal was given by SPC. The drawback in this approach was that only compensatory control action was taken each time an alarm was raised by SPC without recognizing and eliminating the underlying cause.

Nembhard and Mastrangelo [7] employed the same approach while using the term Integrated Process Control (IPC). They stated that EPC can refer to many forms of feedback and feedforward regulation, while SPC can refer to many forms of monitoring tools such as Shewhart charts and EWMA charts. They utilized Proportional Integral (PI) controller as an EPC tool in their IPC scheme where as a Moving Center-line Exponentially Weighted Moving Average (MCEWMA) chart was used as an SPC monitoring tool. They concluded that IPC design develops adjustment policies to reduce the length of the transient period, decrease the out-of-control points and lower the variation.

Jiang and Tsui [8] developed an economic model for SPC monitoring of EPC controlled processes. They also developed an economic loss-based criterion, the Average Quality Cost (AQC), to evaluate the performance of SPC charting methods. The AQC and the traditional average run length of three common SPC charts were investigated and compared. They stated that when the feedback control is a MMSE control scheme and the

underlying process can be perfectly estimated, the outputs of the control system are independent, and identically distributed. When a constant (step) mean shift of magnitude  $\mu$  occurs, the control action can compensate the mean shift and result into an independent process output with a dynamic mean value. When the MMSE control scheme is applied to AR(1) process, the means of the process output before and after the shift occurrence are:

$$\mu_t = \begin{cases} \mathbf{0} & \text{at } t < \mathbf{0} \\ \mu & \text{at } t = \mathbf{0} \\ (\mathbf{1} - \phi)\mu & \text{at } t > \mathbf{0} \end{cases} \quad (2.9)$$

It follows that the total cost of a production cycle (denoted as the total quality cost) consists of two parts: the in-control cost and the out-of control cost as:

$$L_T = L_{in} + L_{out} \quad (2.10)$$

where  $L_{in}$  is the in-control cost,  $L_{out}$  is the out of control cost, and  $L_T$  is the total quality cost. By assuming the adjustment cost to be negligible, and averaging the total quality cost over the entire production cycle, the AQC was obtained from:

$$L_A = \frac{L_T}{(1/p) + ARL_1} \quad (2.11)$$

where  $ARL_1$  is the average run length when the process is out-of-control, and  $L_A$  is the average quality cost  $p$  is equal to  $1-\beta$ . They applied the AQC criterion to compare three common SPC charts: the Individual Shewhart Chart (IS chart), the EWMA chart. and the combined EWMA-Shewhart charts (ES chart), under AR(1) and ARMA(1,1) processes, They found that the AQC criterion was generally consistent with the ARL criterion



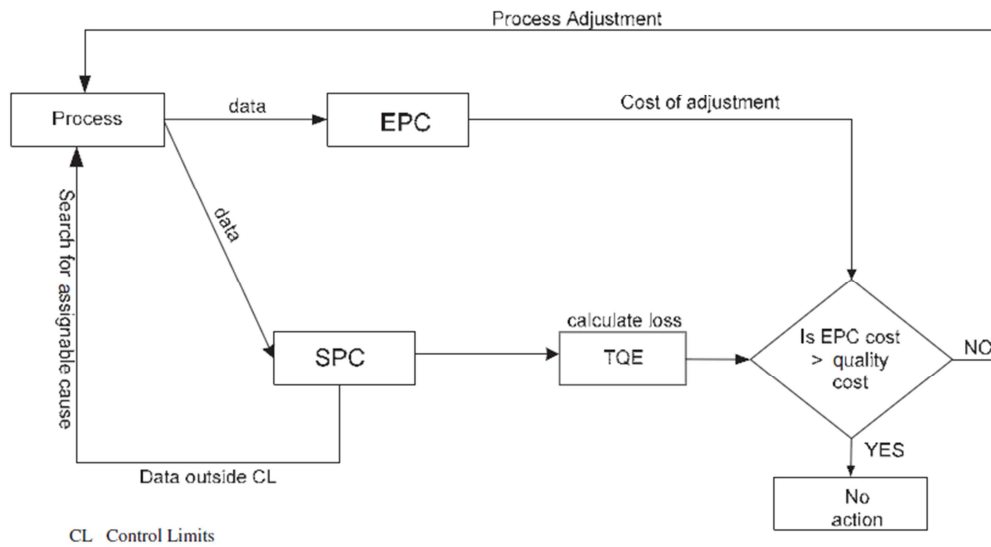
except when the APC control action significantly compensates the process shift. When this happens, the performance of the control chart will depend critically on the size of the diagnosis cost. They concluded that the AQC criterion is generally consistent with the ARL criterion and gives more economic information than the ARL by providing an integrated measure to evaluate the performance of an SPC chart.

A contrary approach is to continuously use EPC for controlling or process adjustment while using SPC for detection of assignable cause by monitoring output or input variables of the process. Applying EPC continuously implies to loss of resources, whereas EPC only triggered by SPC in out-of-control condition amounts for loss of quality. Therefore, Duffua et. al. [9] proposed a scheme that takes short comings of both the approaches into consideration and propose an integrated scheme comprising Taguchi's Quality Engineering.

In the mentioned approach SPC plays dual role; apart from being used to search for assignable cause, it also provides required quantities to a TQL function that estimates the cost of associated quality loss.

Meanwhile, the cost of EPC implementation for the same instant is also calculated. Finally, EPC is only allowed process adjustments hen cost of adjustment is less than the cost of quality loss.

This approach is illustrated by a flow chart in following Figure 4:



**Figure 4: Model of Duffua et. al. (2004)**

Park et. al. [44] proposed a frame work for selection of EPC and SPC tools based on economic cost models with more reasonable considerations as compared to earlier researches. The authors considered disturbance cost, diagnosis cost, false alarm cost and reworking/scrapping cost while employing quadratic loss function for overall calculations. Furthermore, in this study, the authors evaluated performances of both EPC and SPC tools based on cost models. In addition to formulation of economic cost model, a new performance measure parameter called ‘Long Run Expected Cost’ (LREC) was proposed which provides more realistic performance measure as compared to classically used ‘Average Run Length’ especially in a situation having infinite horizon.

LRECs of SPC/EPC integrated systems with Proportional, Proportional-Integral, Minimum Mean Square Error controllers and EWMA monitoring were investigated in different scenarios and following conclusions were drawn:

- Performance of MMSE controller is higher than that of Proportional controller.
- Changes in re-working cost have the most significant impact on the LREC whereas disturbance cost, autoregressive and moving average co-efficients seldom affect the LREC.
- Variation in proportional gain and mean-shift magnitude alter the LREC considerably.

Apart from the above findings a comparative study was provided on the consideration of cost in SPC and EPC integration. This comparison is summarized in the following table:

**Table 2. Comparison of cost based analysis in SPC/EPC integration.**

<b>Reference</b>	<b>Costs Considered</b>	<b>Measurement and Tools</b>
Elsyed and Chen (1994)	Measurement cost, False alarm cost, Cost of finding and fixing	Quadratic Loss Function, Expected cost per unit sample size, Sampling interval and control limits
Jiang and Tsui (2000)	Diagnosis cost, Loss cost/ unit, Adjustment cost, Off target cost	Quadratic Loss Function, MMSE controller, Average quality cost
Duffua et. al. (2004)	Diagnosis cost, Adjustment cost, Loss cost per unit	Quadratic Loss Function
Yang and Sheu (2007)	Non-conformity cost, Diagnosis cost, Adjustment cost, Sampling and testing cost, False alarm cost, Repairing cost	Multivariate EPC/EWMA, Average quality cost and Euclidean distance as performance measure.
Kandanand (2007)	False alarm cost, loss cost per unit	Quadratic Loss function, Expected net savings
Park and Reynolds (2008)	Monitoring cost, Adjustment cost, Off target cost, False alarm cost	Expected Cost per unit, Repeated adjustment, Feedback adjustment, EWMA
Park et. al.	disturbance cost, diagnosis cost, false alarm cost and reworking/scrapping cost	Quadratic Loss function, PI and MMSE controllers

#### **2.4.2 Integration for assignable cause detection:**

The most powerful approach of SPC and EPC integration involves continuous adjustments using EPC and detection of assignable cause using SPC monitoring. Several researchers have explored different EPC techniques along with different control charts for this purpose.

Shao [10] introduced Cumulative Score (CUSCORE) control charts in the arena of EPC/SPC integration as an assignable cause detection tool. The author evaluated effectiveness of using CUSCORE charts along with MMSE control technique of EPC. CUSCORE chart monitoring had been formulated for use with MMSE regulated process subjected to a linearly varying disturbance (drift). The efficiency of CUSCORE charts had been discussed and compared with that of Shewhart and Cumulative Sum (CUSUM) charts. It was thus shown that CUSCORE control charts outperformed Shewhart and CUSUM charts in detecting the drifting disturbance with different values of slope.

Shao et. al. [11] focussed on the eradication of one of the assumptions that had always been taken into account in all earlier SPC/EPC integration research. Earlier researchers used to assume that the SPC detects the disturbance caused by an assignable cause and the cause is removed as soon as it is detected. However, there should be recognition of disturbance associated with different assignable causes for the sake of correct identification of the culprit underlying cause. Therefore, a graphical aid technique was proposed that is capable of distinguishing between shift (step change) and drift (linear change) disturbances by examining the output patterns. Furthermore, a neural network

based methodology was introduced to automate this recognition process and to find out underlying cause linked with the type of disturbance.

Later on, Shao et. al. [12] build on their previous work in order to eliminate one of the unrealistic assumptions that had continuously been taken into account during earlier researches involving SPC and EPC integration. Considering the roles of EPC and SPC in integrated systems, that is, to automate adjustment of process input to keep process output on target and to monitor process output in for the sake of detecting the assignable causes, the earlier scholars had assumed that the assignable cause is eliminated as soon as it's detected. Shao et. al. in the mentioned study formulated and demonstrated an adaptive controller technique that is triggered by SPC based assignable cause detection. This scheme aims at identifying the changing parameters of the disturbance and consequently adjusting the process until the underlying cause is completely eliminated.

Jiang and Tsui [45] also researched the application of control charts on MMSE and PI regulated systems having continuous adjustments. This technique, similar to other works, was meant to detect assignable causes of variation.

Xie et. al. [13] noted that SPC deals with the type of problems in which a process is assumed to be under control initially and the focus of SPC techniques is to detect the out-of-control state of the process monitoring the quality characteristics using control charts. Process subjected to a slowly changing trend was considered by Xie et. al. [13] which is a special case of SPC and EPC integration where it is insufficient to only monitor the process that changes with time using SPC. Accordingly a model had been developed that

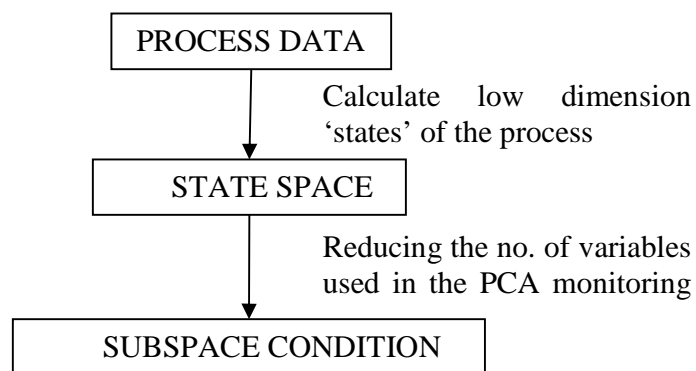
makes adjustment to the process after regular intervals of time and the process output itself is monitored with changing control limits instead of its variation from the target value. This model is only valid for slowly changing univariate processes in which process adjustments are done after regular yet larger intervals of time. On the contrary the classical engineering processes need regulations rather rapidly as their rate of change with time is very high.

SPC/EPC integration for univariate case was comprehensively discussed by Huang and Lin [14] and the associated issues had been addressed. In the mentioned study, effects of Shewhart and CUSUM control charts on an MMSE regulated system with shifting and drifting mean disturbances had been taken into account.

Huang and Lin [14] noted that Shewhart control charts are more effective than CUSUM control charts in detecting the shifts. In case of drifting disturbance with smaller slope, CUSUM proves to be more effective whereas for larger slopes Shewhart is more efficient.

Moreover, it was noted that an EPC feedback compensation mechanism affects SPC out-of-control detection and disturbs the output when suddenly assignable cause is removed. To account for this so called overcompensation issue, a joint monitoring scheme of Shewhart and CUSUM charts had been used to recognize the disturbance type and a cost based decision rule is provided to decide whether the assignable cause removal will be cost efficient owing to the fact that the overcompensation phenomenon is irresolvable and in some cases renders the system unstable.

Treasure et. al. [46] elaborated a rather advanced integration of EPC and SPC by utilizing Principal Component Analysis(PCA) and Subspace Model Identification(SMI). They noted that dynamic extension to classical MSPC (multivariate statistical process control) procedures such as PCA and PLS, can lead to addition of numerous variables to condition monitor. To prevent this issue, they presented a scheme that, as a first step, uses the popular subspace identification technique to identify the process parameters that may or may not be changing with time. Moreover, a monitoring technique was introduced that integrates principal component analysis (PCA) into subspace model identification (SMI) in order to give rise to error in variable (EIV) approach. This allows significant variation to be extracted in order to identify the state-space matrices and to establish  $T^2$  and  $Q$  statistics for the sake of addressing the deficiencies of the earlier SMI applications. Furthermore, it results in the reduction of number of process variables to be identified that considerably account for a deteriorating or faulty event. Treasure et. al. [46] also offered step-wise procedure for designing of contribution charts meant to diagnose anomalous behavior of the system.



**Figure 5: Model of Treasure et. al. (2004)**



Sun and Wang [47] added to the research of SPC/EPC integration by considering quality characteristics while designing parameters and control limits of EWMA chart. The process they considered for their study was assumed to possess first order dynamic model with white noise (i.i.d.) disturbance. An MMSE controller was used to regulate the system for variation due to disturbance and inherent system dynamics. A linearly varying co-related assignable cause was induced into the system. Consequently, an EWMA controller was optimally designed considering costs associated with monitoring, false alarm and process adjustment for the sake of eliminating assignable cause.

Tsung and Shi [18] were the first who hinted that SPC monitoring can be done on the process input or control actions. They devised a scheme for univariate processes in which they jointly monitored input and output of the process by augmenting both of them in a matrix.

Joint monitoring of outputs and manipulated inputs using SPC had been considered by Huang and Lin [14] rather in detail for the first time after Tsung and Shi [18].

Effectiveness of applying SPC control charts on either process output or the control actions (process inputs) was also investigated by Tsung and Kwok-leung [19] who discussed the integration of SPC and APC by considering MMSE regulated processes with ARMA(1,1) disturbances. It was indicated that the detection of out-of-control state of the process is dependent on the mean shift pattern of the disturbance represented by the different ARMA(1,1) parameters. Moreover, effectiveness of using SPC control charts on either process output or the control actions (process inputs) was investigated

using different magnitudes and patterns of mean-shifts of the disturbance induced into the system. It was showed, in general, that it is more effective to monitor control actions using SPC for smaller shifts while output monitoring is efficient for larger shifts. Furthermore, it was argued that that the mean-shift pattern is the key dominating factor upon which ARL performance depends whereas it does also depend upon the autocorrelation structure of the process itself.

## **2.5 MULTIPLE INPUT MULTIPLE OUTPUT SYSTEMS**

Most of the real life processes comprise multiple inputs and multiple outputs. For instance, all chemical processes, boilers, pulp and paper manufacturing, stock exchange etc. all have more than one inputs and outputs. Therefore, there had always been a need to control systems involving more than one variables especially in the presence of internal coupling among them.

### **2.5.1 Multivariable Engineering Process Control**

Macfarlane[48] states that the beginning of research on Multivariable Control Systems dates back to 1930s when Bode, Nyquist and other scholars provided corner stone for the establishment of a huge and beautiful building of control theory. In the initial decades most of the work had been focused on the development of representation of multivariable systems. Later on, numerous techniques were developed with an aim to keep multiple-input-multiple-output systems stable. Some of the commonly used control strategies for multivariable systems are as follows:

- State Feedback Control
- Output Feedback Control
- Feed forward Control
- Linear Quadratic Control
- Dynamic De-coupling Control etc.

Statistical process control community and researchers from stochastic control turned their attention to multivariate systems later on in order to develop adjustment schemes for keeping the multiple-input-multiple-output processes on target. Tseng et al. [49] suggested a multivariate EWMA controller for a linear multi-input and output model which is described by the below equation:

$$\hat{\alpha}_i = \hat{\alpha}_{i-1} + \omega(y_i - \tau) \quad (2.12)$$

where  $\omega$  is a discount factor,  $y_i$  is the output and  $\tau$  is the target value. In addition, Del Castillo and Rajagopal [50] proposed a MIMO double EWMA feedback controller for drifting processes.

### 2.5.2 Multivariate Statistical Process Control

Hotelling was first to propose a multivariate control chart in the middle of twentieth century. Hotelling's  $\chi^2$  and  $T^2$  Charts find numerous applications owing to the fact that they are easily implementable in multivariate problems. Hotelling's  $T^2$  control chart is a direct analogue of univariate Shewhart  $\bar{X}$  chart and it is used to monitor the whole process mean vector. An out-of-control condition is signaled by Hotelling's  $T^2$  control chart as soon as the statistic  $T_i^2$  given by the following equation exceeds the upper control limit.

$$T_i^2 = \mathbf{y}_i' \Sigma_{y_i}^{-1} \mathbf{y}_i \quad (2.13)$$

where,  $y_i$  is the process output and  $\Sigma_{y_i}$  is the covariance matrix. The upper control limit is selected so as to achieve desired average run length.

Due to the fact that Hotelling's  $T^2$  was prone to smaller shifts in the process outputs, other univariate control charts were ultimately transformed into their multivariate counter parts.

Healy [51] formulated the CUSUM control chart for multivariate processes using the fact that Multivariate CUSUM can be viewed as a series of sequential probability ratio tests. The MCUSUM for detecting a change in the variance –covariance matrix, may be written as

$$S_i = \max[(S_{i-1} + a^t(x_i - \mu_0) - 0.5\lambda(\mu_1)), 0] \quad (2.14)$$

where  $\lambda(\mu_1)$  is the square root of the non-centrality parameter and  $a^t = [(\mu_1 - \mu_0)^t \Sigma_0^{-1}] / \lambda(\mu_1)$ .

Further vigorous research on multivariate CUSUM charts was carried out by Hawkins[52], Crosier [53] and Pignatiello and Runger [54] etc.

Lowry et. al. [55] devised Multivariate EWMA charts for the first time due to their efficiency in univariate cases. The MEWMA chart proposed by Lowry et. al. [55] can be described by the below equations:

$$\mathbf{z}_i = \mathbf{R}\mathbf{x}_i + (\mathbf{I} - \mathbf{R})\mathbf{z}_{i-1} \quad (2.15)$$

$$T_i^2 = \mathbf{z}_i' \Sigma_{z_i}^{-1} \mathbf{z}_i \quad (2.16)$$

where  $I$  is the identity matrix,  $R = \text{diag}(r_1, r_2, r_3, \dots, r_p)$  and  $0 \leq r_k \leq 1$  for  $k = 1, 2, 3, \dots, p$  and  $\Sigma_{z_i}^{-1}$  is the covariance matrix. Later on, robustness properties of MEWMA charts were discussed by Testik et. al. [56] for the cases when the data follow multivariate  $t$  and multivariate gamma distributions. Research on MEWMA charts has continuously gained the attention of many researchers [44], [57]–[59] since its formulation by Lowry et. al [55].

One of the major concerns about applying multivariate control charts is its capability to recognize the source of assignable cause of variation. For instance, a multivariate control chart that detects the occurrence of assignable cause is practically useless until it points out the variable or variables that have been victims of this assignable cause. Owing to this reason, some scholars have advocated the idea of applying univariate control charts on each of the variable in a multivariate process. Woodal and Ncube [60] presented this idea for CUSUM charts stating that  $p$  univariate CUSUM charts can be used for a process involving  $p$  variables. Another approach is to use graphical methods in order to diagnose the type and source of assignable cause. Subramanyam and Houshmand [61], Fuchs and Benjamin [62], Nottingham et. al.[63] and Francisco et. al.[64] are some of the authors who have introduced the graphical methods in the arena of multivariate statistical process control; however, primary drawback of this technique is that it requires interpretation of the results by an expert. For catering this problem many decomposition schemes have also been proposed in the literature. Mason et. al. [65] illustrated a method of

decomposition of  $T^2$  statistics in order to diagnose the source of a shift in the process mean. This was accomplished by a series of orthogonal decomposition. Chen et. al.[66] used the eigenfactor decomposition for the purpose of identification of primary source of assignable cause. Recently, Tan and Shi [67] have used Bayesian approach in solving this kind of problems. Finally, some of the researchers [68]–[70] have successfully applied artificial neural networks in the above mentioned scenarios in order to diagnose the reasons behind drifting and shifting means of multivariate processes.

### **2.5.3 Integration of Multivariate Statistical Process Control and Engineering Process Control**

A lot of research had been done in the fields of Statistical Process Control and Engineering Process Control on multiple-input-multiple-output or multivariate systems. Inspired by the research on integrating the SPC and EPC in univariate cases, Ling Yang et. al. [15], [16] paved the way for integration of Multivariate SPC and Multivariable EPC. There had been control practices prevailing in multivariable systems for quite a long time whereas, there had been tools of SPC developed for multivariate cases in complete isolation with MEPC.

Ling Yang et. al. [16] used MEWMA controller as an MEPC tool in order to observe its integrator with three multivariate SPC control charts, namely, Hotelling  $T^2$ , MEWMA and MGWMA charts. These charts were used to detect a mean shift in disturbance with different magnitudes.

Among the above mentioned monitoring schemes hailing from multivariate SPC, the MGWMA charts proved to be the most efficient both in terms of ARL and performance measure for all magnitudes of shifts. The GWMA chart's equations are detailed as below:

$$\mathbf{T}_i^2 = \mathbf{g}_i' (\mathbf{Q}_i \Sigma_{y_i})^{-1} \mathbf{g}_i \quad (2.17)$$

where,

$$\mathbf{Q}_i = (\mathbf{q}^{0\alpha} - \mathbf{q}^{1\alpha})^2 + (\mathbf{q}^{1\alpha} - \mathbf{q}^{2\alpha})^2 + \dots + (\mathbf{q}^{(i-1)\alpha} - \mathbf{q}^{i\alpha})^2 \quad (2.18)$$

And 
$$\mathbf{g}_i = \sum_{t=1}^i [\mathbf{q}^{(i-t)\alpha} - \mathbf{q}^{(i-t+1)\alpha}] \mathbf{y}_t$$

The idea of integrating MIMO systems was illustrated using an example and simulation study was performed to confirm consistency of the findings in different scenarios.

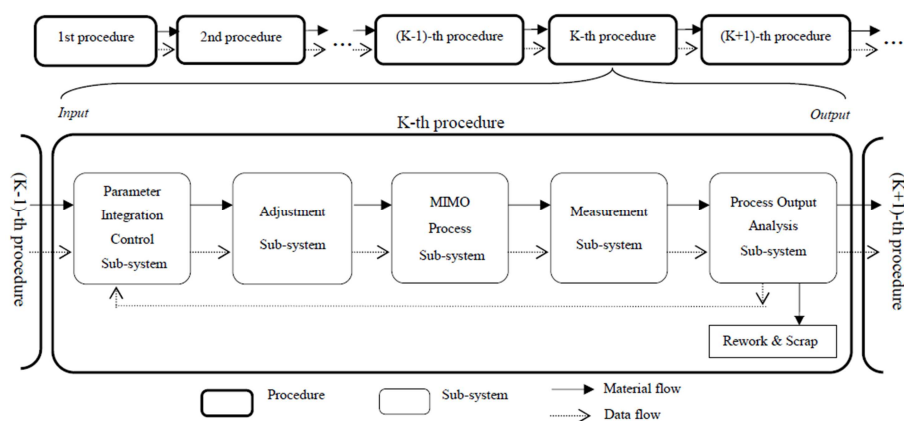
Jiang et. al. [71] proposed a MIMO process control model that comprises mathematical model of the process, disturbance characteristics, SPC, EPC and an adaptive controller for parameters and set-point (target) adjustments.

According to Jiang et. al.[71], the overall system can be divided into following sub-systems:

- MIMO Process Sub-system: This consists of process itself that takes inputs (feeds), processes them and renders the output that is measured by the next sub-system.

- **Measurement Sub-system:** Primary purpose of this sub-system is to measure and record the needed output quantities (also known as quality characteristics). Disturbance is also supposed to enter this sub-system.
- **Process Output Analysis:** This sub-system applies SPC techniques on the measured data acquired from the previous sub-system in order to detect and-where possible-eliminate the assignable cause.
- **Parameter Integration Control Sub-system:** This system basically performs two tasks. First is to adjust the process “mathematical model” owing to variation in inputs and second is to change the target/set-point provided to EPC. These tasks are accomplished using adaptive controllers.
- **Adjustment Sub-system:** EPC procedures are applied herein to adjust the process inputs for the sake of keeping the outputs (quality characteristics) on target.

This model is summarized in the following flow diagram:



**Figure 6: Flow Diagram of Model of Jiang et. al. (2008)**



## 2.6 CASE STUDIES ON INTEGRATION OF SPC AND EPC

There are very few case studies carried out in this area while most of the researchers have stuck to numerical examples with ideal assumptions for illustrating their findings. On the contrary, case studies provide a path way for ideas to get adopted into practices.

Capilla et. al.[72] carried a case study on integrating SPC and EPC techniques in a continuous polymerization process. The case study testifies that the integrated application of EPC and SPC together out classes the application of either of them alone.

The system taken under consideration by Capilla et. al [72] is a commercial scale polymerization that produces large amount of polymers. The key quality characteristic was polymer viscosity measured by Melt Index (MI), whose variation with time is dependent on temperature and was given by

$$\nabla MI_t = \omega_1 \nabla I_{t-1} + \omega_2 \nabla I_{t-2} + a_t - \theta a_{t-1} \quad (2.19)$$

where  $a_t$  was assumed to be an independent variable having normal distribution with zero mean and  $\delta^2$  variance.

Parameter estimation was performed before applying three EPC techniques, namely, Clarke's Constrained Controller, Minimum Mean Square Error Controller and two step ahead forecasting controller. Performance and stability robustness of the mentioned EPC schemes were evaluated under different circumstances i.e. without disturbance or assignable cause, with assignable cause but without SPC and with assignable cause along

with SPC. EWMA and Shewhart charts were used as SPC tools to detect assignable cause.

Matos et. al. [73] provide the literature with a very comprehensive case study based on a real system. They integrated SPC and EPC in Pulp and Paper production industry to come up with a multivariate applications.

In the paper and pulp production industry, Bleached Eucalyptus Kraft Pulp is produced in the plant using the Elemental Chlorine Free (ECF) process. The Kraft Pulping process comprises of two different phases, either of which influences the final pulp quality. These phases are the cooking process of wood chips (eucalyptus globules) and the pulp bleaching. Former of the two phases contributes more to the final quality of the paper which is mainly measured by viscosity of the bleached pulp along with other parameters. The process inputs are temperature and concentration measures of various components of a digester.

The methodology acquired by the authors is as follows:

**System Identification:** System identification techniques were implemented on real time data taken from the system on three different occasions in order to find the best fitted model.

The Multiple-Input-Single-Output (MISO) model was ultimately found out to be:

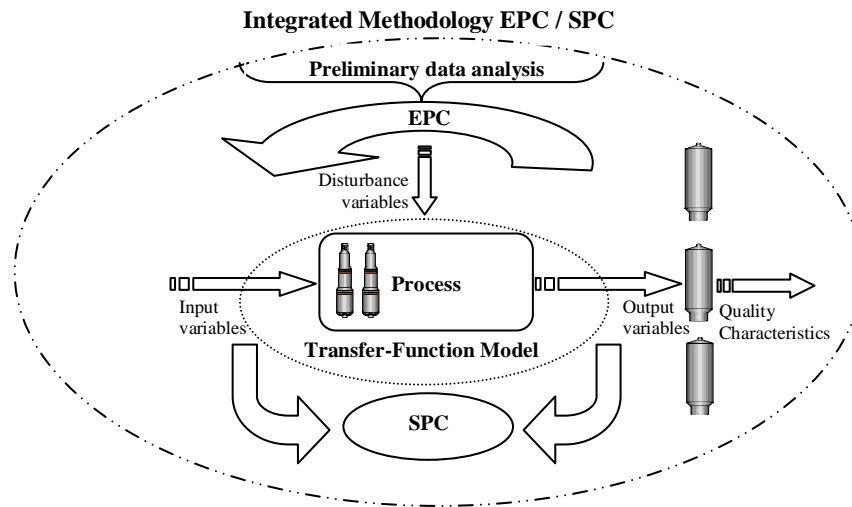
$$y_t = \frac{(-2,86 - 7,46\mathcal{B})}{(1 + 0,99\mathcal{B})} WLC8_{t-2}^{D1} + 5,90SI_{t-1}^{D2} - 4,01WLC4_{t-2}^{D2} - 5,23TemC4_{t-1}^{D2} - 2,90TemC5_{t-1}^{D2} + 1,66AA_{t-2}^{D2} + \frac{1}{(1 - 0,56\mathcal{B})} \varepsilon_t \quad \mathbf{2.20)}$$

Where  $y_t$  is the deviation of viscosity from target at time  $t$ ,  $\varepsilon_t$  is the white noise sequence and  $\mathcal{B}$  defines the backshift operator. WLC8, SI, WLC4, TemC4, TemC5 and AA represent the variables of digester 1 (D1) and digester 2 (D2)

EPC Scheme: The Ridge Controller (del Castillo, 2002) based on a minimum variance criterion was adopted in order to keep the process output on target.

SPC Schemes: EWMA, CUSCORE and EWMAST charts were applied on the output quality characteristics (viscosity of the bleached pulp) and inputs (digester temperatures and concentrations of various components) in order to detect the assignable cause. These charts were based on dynamic principle component analysis owing to the large amount and auto-correlated structure of the data.

The above mentioned methodology is summarized in the figure below:



**Figure 7: Methodology of case study by Matos et. al. (2008)**

Asymptotic Mean Square deviation AMSD and ARL were evaluated for different scenarios in order to testify that SPC/EPC integrated control outperforms the use of either of them alone.

## **CHAPTER 3 A NOVEL FRAMEWORK FOR INTEGRATION OF MULTIVARIATE SPC AND EPC**

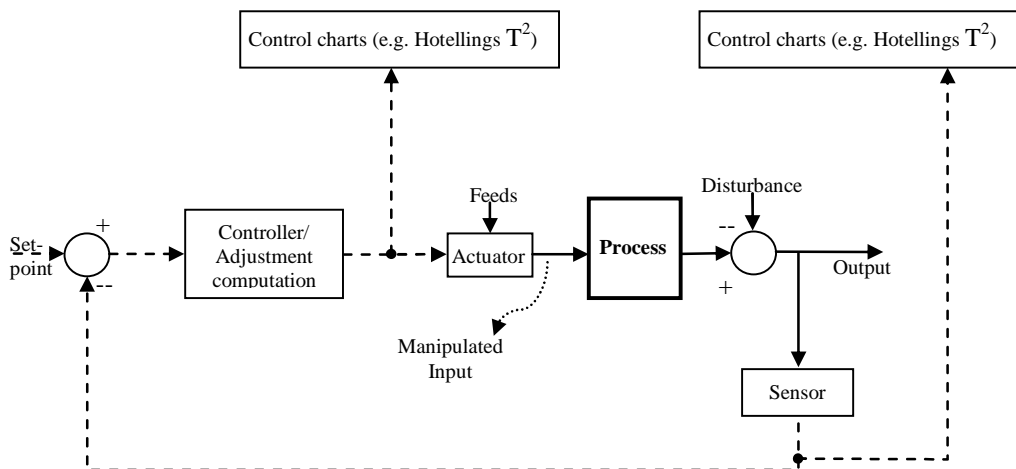
This chapter elaborates the usefulness of integrating SPC and EPC for the multivariate cases, proposes a novel scheme of integration and discusses its effectiveness using a numerical example. The subsequent sub-section of this chapter discusses the novel scheme developed herein for integration of multivariate statistical process control and engineering process control followed by a numerical example elaborating the idea. Lastly, sensitivity analysis comprising various shift magnitudes and EPC schemes proves the effectiveness of the said scheme in general.

### **3.1 AN INTEGRATED SPC/EPC MIMO CONTROL SYSTEM**

In light of the above mentioned literature survey, an integrated control system model has been proposed here. It is well established in the literature that applying control charts on process output yield to detection of assignable causes. Hotelling's  $T^2$  Chart is the simplest and the most fundamental control chart meant to monitor a system already being regulated by EPC scheme. A short coming in applying Hotelling's  $T^2$  Chart to process output is its inability to detect assignable causes that appear small in magnitude on output; for instance, a mean shift in noise culminating the output. The popular solution to this problem is the use of rather complex control charts such as EWMA, CUSCORE or

GWMA control charts. [19] investigated a rather unique approach i.e. to apply control charts on process inputs instead of output.

Inspired by [19] and other researchers, we adopt a “Joint Monitoring” of process inputs and outputs in order to detect the assignable cause in MIMO systems. As it is argued in the subsequent sections, the proposed method is the best way to detect assignable causes of broader ranges. This eliminates the need for using complex control charts relieving engineers, who have seldom in-depth knowledge of statistical techniques, from designing control charts. The proposed MIMO control system is illustrated by the following block diagram:



**Figure 8: Block diagram of the proposed control system**

In the Figure 8, the process is illustrated by a bold block that is fed through actuator(s); data flow and material flow are represented by dashed and solid lines

respectively. As it is clear from the figure, the output is measured by sensor(s) and the measured value(s) are subtracted from the set-point or target values in order to generate the error. This error is used in the controller block that performs mathematical calculations based on the amount of error and the plant mathematical model in order to adjust or manipulate the process input through actuator(s). In the proposed scheme, the multivariate SPC control charts (generally Hotelling's  $T^2$  will be enough) are employed both at process inputs and outputs for detecting assignable causes of variation. Following sections discuss application of the proposed scheme using a simple system and shed light on its effectiveness under different situations.

### **3.2 SYSTEM DESCRIPTION AND MEPC SCHEME**

[74] targeted the process control problem and discussed the conditions for the stability of a process using a single EWMA (exponentially weighted moving average) controller having taken into account a first-order process/system. [75] introduced a double EWMA controller and found it useful in eradicating the deterministic drift within the process. Furthermore, Tseng et al. suggested a multivariate EWMA controller for a linear multi-input and output model. [50] proposed an MIMO double EWMA feedback controller for drifting processes.

For MEPC scheme, [16] consider a linear MIMO system with  $m$  inputs and  $p$  outputs after [76], described by the below equation:

$$\mathbf{y}_i = \boldsymbol{\alpha} + \boldsymbol{\beta}\mathbf{c}_{i-1} + \boldsymbol{\varepsilon}_i \quad (3.1)$$

where,  $\mathbf{y}_i$  is a vector of dimensions  $(p \times 1)$  comprising the outputs,  $\boldsymbol{\alpha}$  is a  $(p \times 1)$  vector containing the offset parameters of each output,  $\boldsymbol{\beta}$  is a process gain matrix having  $p$  rows and  $m$  columns,  $\mathbf{c}_{i-1}$  is an  $(m \times 1)$  vector comprising the values of manipulatable inputs, and  $\boldsymbol{\varepsilon}_i$  is a  $(p \times 1)$  vector denoting the noise or process disturbance.  $\boldsymbol{\varepsilon}_i$  is assumed to be contributing in the dynamics of the system.

The offset in the output or the intercept will be updated online after each iteration. For simplicity we shall assume that the estimate of  $\boldsymbol{\beta}$  denoted by  $\mathbf{B}$  is known. Let  $\hat{\boldsymbol{\alpha}}_0$  denote the estimate of  $\boldsymbol{\alpha}$  at  $i = 0$ , then the predicted model will be:

$$\hat{\mathbf{y}}_i = \hat{\boldsymbol{\alpha}}_0 + \mathbf{B}\mathbf{c}_{i-1} \quad (3.2)$$

Prior to implementation of the feedback control scheme, the process (manipulatable) input will look like:

$$\mathbf{c} = \mathbf{B}^{-1}(\boldsymbol{\tau} - \hat{\boldsymbol{\alpha}}_0) \quad (3.3)$$

where  $\boldsymbol{\tau}$  is the target vector. Multivariate EWMA controller proposed by [16] is described by the following equation:

$$\hat{\boldsymbol{\alpha}}_i = \hat{\boldsymbol{\alpha}}_{i-1} + \omega(\mathbf{y}_i - \boldsymbol{\tau}) \quad (3.4)$$

where  $\omega$  is a discount factor.

Let  $\boldsymbol{\alpha}_0 = 0$  and  $\boldsymbol{\tau} = 0$ ; then, the off-target amount at iteration  $i$  can be described as :



$$\mathbf{y}_i - \boldsymbol{\tau} = \mathbf{y}_i \quad (3.5)$$

$$\mathbf{y}_i = (\mathbf{1} - \boldsymbol{\omega})^{i-1} \boldsymbol{\gamma}_0 + \sum_{t=0}^{i-1} (\mathbf{1} - \boldsymbol{\omega})^t (\boldsymbol{\varepsilon}_{i-t} - \boldsymbol{\varepsilon}_{i-t-1}) \quad (3.6)$$

When the  $\boldsymbol{\varepsilon}_i$  is a white noise with mean vector  $\boldsymbol{\mu}$  and variance  $\boldsymbol{\Sigma}$ , the covariance of  $\mathbf{y}_i$  will be:

$$\boldsymbol{\Sigma}_{\mathbf{y}_i} = \left( \mathbf{1} + \frac{\boldsymbol{\omega}}{2-\boldsymbol{\omega}} (\mathbf{1} - (\mathbf{1} - \boldsymbol{\omega})^{2(i-1)}) \right) \boldsymbol{\Sigma} \quad (3.7)$$

It is considered in control action of equation 3.5, taken by EWMA controller, that assignable cause doesn't exist. Therefore, the only source of common cause of disturbance is a white noise series  $\boldsymbol{\varepsilon}_i$  that is described by equation 3.1. Now, the performance of this system is investigated under additional assignable causes. Let us consider that that this MIMO process model is generally controlled by MEPC and that the MSPC monitoring scheme will only report assignable causes i.e. external changes. Assignable causes can be rapidly detected by application of MSPC control charts to the deviation of output from target; it's considered that the assignable cause takes the form of a sustained shift in the process mean vector. The output deviation will obviously reduce upon successful detection and eradication of external changes or assignable causes. Firstly, in this paper, using MEPC scheme alone is compared with using MSPC together with MEPC.

Another MEPC scheme considered in the subsequent section of sensitivity analysis is a direct analogue of famous Proportional Integral Controller having following mathematical form (in discrete case):

$$c(k) = Kp * e(k) + Ki * T * (c(k) - c(k-1)) \quad (3.8)$$

where  $c(k)$  is manipulatable input to the process,  $e(k)$  is the error between output and the target value whereas  $Kp$  and  $Ki$  are proportional and integral gains.

### 3.3 HOTELLING'S $T^2$ CHART

A counter part of univariate Shewhart  $\bar{X}$  is Hotelling's  $T^2$  control chart; it's used to monitor the process mean vector. As per the multi input multi output system established in Sect. III, let us consider a white noise series  $\epsilon_i$  (used in Equation 3.1) be independent multivariate normal random vectors with mean vectors  $\mu_i$  and a common covariance matrix  $\Sigma$ , which is non-singular. The covariance matrix of  $y_i$  (denoted by  $\Sigma_{y_i}$ ), that is given by the Equation 3.8, is calculated by measuring the deviations of  $y_i$  from the target vector ( $\tau = 0$ ). Moreover, an out-of-control condition is signalled by hotelling's  $T^2$  control chart as soon as the statistic  $T_i^2$ ,

$$T_i^2 = y_i' \Sigma_{y_i}^{-1} y_i \quad (3.9)$$

exceed the UCL at iteration  $i$ , where UCL (h1) is selected so as to achieve desired ARL. For detailed discussion on Hotelling's charts [77], [78] can be referred.

### 3.4 A NUMERICAL EXAMPLE

In this example a simple case of 2 variables has been taken into account to elaborate the idea of integration of MEPC and MSPC. Two integration schemes have been demonstrated using this example. The first scheme i.e. application of MSPC on the output has been elaborated using four control charts whereas the second scheme i.e. application of MSPC on the process input has been briefly illustrated using one control chart. However, subsequent section considers sensitivity analysis for establishing that both of the techniques are effective in contrasting scenarios.

For simplicity the example considered by Yang and Sheu[19] is taken into account here. Let the number of production runs  $n = 100$ . The mean vector of  $\epsilon_i$  is assumed to be on target at  $[0 \ 0]'$  for the first 20 observations where the white noise series  $\epsilon_i$  in Equation 1 follows the bivariate normal distribution. A disturbance of the form of shift having mean vector  $[0.875 \ 0]'$  is introduced into the process at time  $i = 21$  i.e.

$$\mu_0 = [0 \ 0]', \mu_1 = [0.875 \ 0]'$$

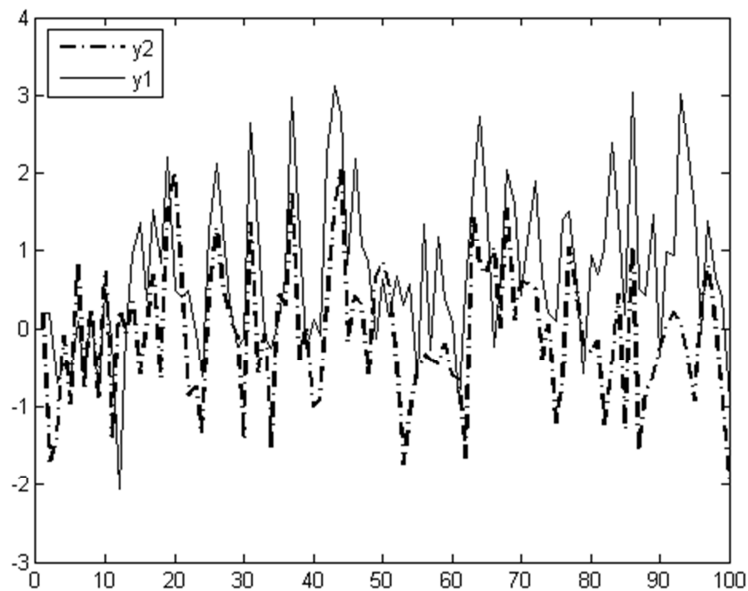
Let  $\omega = 0.1$  and

$$\hat{\alpha}_0 = [1 \ 1]', B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \Sigma = \begin{bmatrix} 1.0 & 0.5 \\ 0.5 & 1.0 \end{bmatrix}, \gamma_0 = [0.2 \ 0.2]'$$

From Equation 3.2, we get  $c_0 = [-1 \ -1]'$

For the simulation of this example, Mathwork's Matlab has been used. The white noise vector has been generated by a built-in command of Matlab.

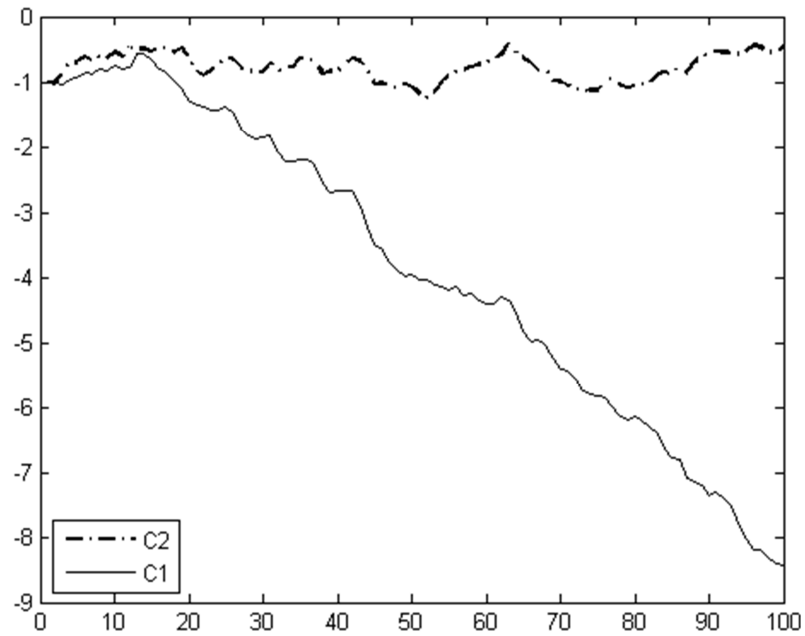
Output observations of the MIMO model described by Equation 3.1 during 100 iterations are illustrated by Figure 9 where only MEWMA controller (given by Equation 3.4 and 3.5) is applied. A disturbance of the form of shift having mean vector  $[0.875 \ 0]'$  is introduced into the process at time  $i = 21$ . Figure 11 illustrates the control actions of Equation 3.5. In the absence of MSPC control charts meant to detect the shift, the control action produced by MEWMA controller ( $c_1$ ) increases to a very large extent in order to compensate for this sustained shift.



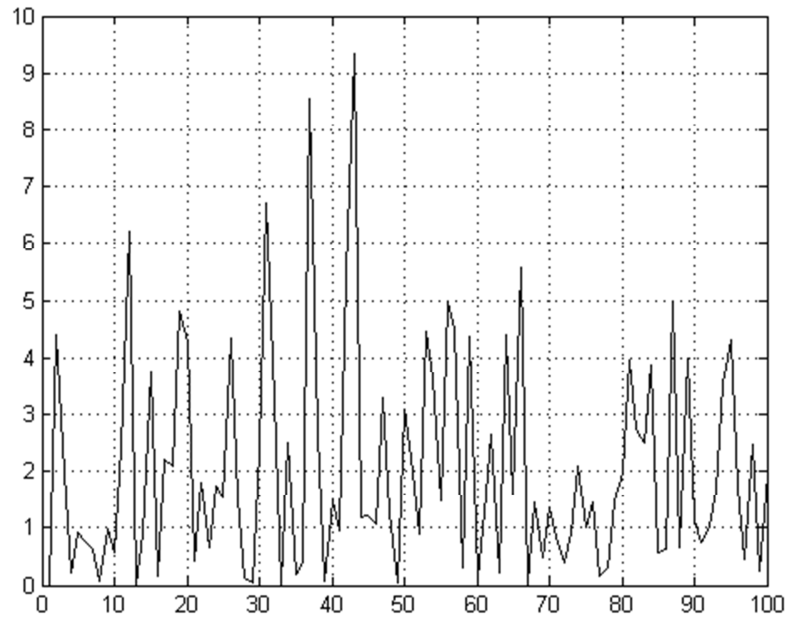
**Figure 9: Output fluctuation of the process employing only MEPC. A shift of mean vector  $[0.875 \ 0]'$  is introduced at  $i = 21$ .**

The statistics of  $T^2_i$  have been illustrated in Figure 11 for the case in which a Hotelling's  $T^2$  chart is applied to the deviation of outputs from the target in addition to the MEPC rule. Corresponding values of  $T^2_i$  for Hotelling's  $T^2$  chart have been calculated

using Equation 6. The calculations of [16] have been adapted in order to insure ARL0's of 200; therefore, the control limit is  $h1 = 9.2$ .



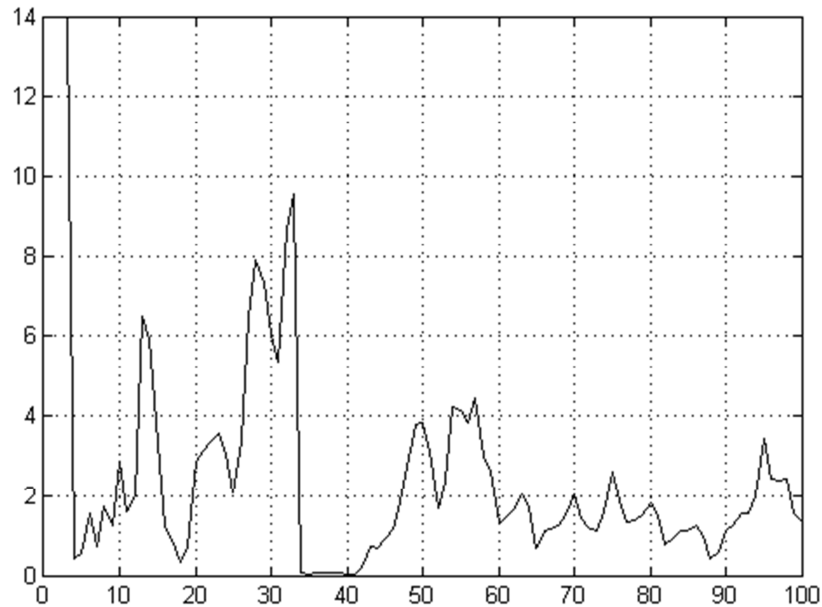
**Figure 10: Control actions for the process when only MEPC scheme has been applied. A shift of mean vector  $[0.875, 0]'$  is introduced at 21st iteration.**



**Figure 11:  $T_1^2$  statistics after joint application of MEPC and a Hotelling's  $T^2$  chart. A shift of mean vector  $[0.875, 0]'$  is introduced at 21st iteration.**

Now let us consider the case of applying MSPC on the process input along with MEPC. Hotelling's  $T^2$  control chart is applied on input in the same example. Since the mean and covariance vectors of input are unknown, the Hotelling's  $T^2$  chart is implemented after at least 5 iterations have taken place under MEPC only. Once enough samples are available, the mean and covariance vectors are determined using Matlab built-in commands. The UCL of the chart ( $h_4 = 10.0$ ) is adjusted for  $ARL_0=200$  using as many as 500 simulations. Using MSPC at input, it was observed that the shift was detected on 38<sup>th</sup> iteration and the performance measure was 1.2292 in contrast with the detection on 41<sup>st</sup> iteration and performance measure of 1.2590. The graph of input

variations has been shown in Figure 12. It is evident from the figure that this method is more prone to false detection when compared with Figure 11.



**Figure 12: Application of MSPC on input recipes**

Findings, using above example, suggest that applying MSPC at input has an edge over applying MSPC on output; however, it needs to be verified for different magnitudes of shifts. The following section discusses the idea in more detail.

### **3.5 SENSITIVITY ANALYSIS**

For comparison of the two schemes developed in previous section, a detailed and general case analysis is considered here. An assignable cause of the form of sustained shift is considered. The shift magnitudes of 0.25, 0.5, 1, 2 and 5 are investigated. Hotelling's  $T^2$  control chart is applied on both output and input of the process one by one.

An in-control (zero shift) ARL ( $ARL_0$ ) is maintained at approximately 200 by changing the width of the control limits. For each case, 500 simulations were run, whereas, 200 iterations were done in each simulation. A sustained shift is introduced on 21<sup>st</sup> iteration in each simulation run and it's assumed that the assignable cause (sustained shift) is removed as soon as it is detected. The out-of-control ARLs ( $ARL_1$ s) and performance measures (Euclidean average) are compared for both the schemes. Both  $ARL_1$ s and PMs are averaged for 500 simulation runs where each simulation run comprises 200 iterations. Summary of these simulations performed using Matlab is illustrated in Table 3.

**Table 3: Comparison of ARLs and PMs when MSPC is applied at Output and Input**

Shift Magnitude	Hotelling's $T^2$ chart at Output $h1 = 9.2$ for $ARL_0 = 200$		Hotelling's $T^2$ chart at Input $h1 = 10.1$ for $ARL_0 = 200$	
	$ARL_1$	PM	$ARL_1$	PM
<b>5</b>	1.02	1.2699	17.10	1.2904
<b>2</b>	3.92	1.2771	18.83	1.2786
<b>1</b>	23.02	1.2937	36.07	1.2778
<b>0.75</b>	60.58	1.2843	46.34	1.2621
<b>0.5</b>	128.90	1.2731	59.38	1.2528
<b>0.25</b>	183.30	1.2521	92.30	1.2513



In Table 3, first column shows the shift magnitude that was introduced in the output of the process. The subsequent columns display the  $ARL_{1s}$  and PMs when Hotelling's  $T^2$  control chart is applied on output/input of the process. When applied on output, Hotelling's  $T^2$  chart detects shift faster than its application at input when the shift magnitude is higher. However, the trend inverses down the Table 3 where shift magnitude is reduced. Furthermore, PMs of first scheme under larger shifts is better while the PMs of second are better for smaller shifts.

Therefore, application of Hotelling's  $T^2$  control chart on output is more effective than its application on input when the shift magnitude is higher; whereas, for smaller shift magnitudes, application of Hotelling's  $T^2$  control chart on input shows better results than its application at output. This finding is evident from both  $ARL_{1s}$  and performance measures. Hence, simultaneous application of both the schemes is recommended for general cases.

Furthermore, detection capabilities of Hotelling's  $T^2$  charts (applied at input) were investigated by using two different EPC schemes. Moreover, different EPC controller parameters were also used in order to identify the factor that affects detection capabilities of control charts in an integrated system.

**Table 4: Comparisons of ARL<sub>1</sub>s while using two different MEPC controllers with different gains.**

Shift Magnitude	ARL <sub>1</sub> s of Hotelling's T <sup>2</sup> at input along with MEWMA controller		ARL <sub>1</sub> s of Hotelling's T <sup>2</sup> at input along with Output Feedback controller	
	$\omega = 0.1$	$\omega = 0.5$	K <sub>p</sub> =0.02, K <sub>I</sub> =0.1	K <sub>p</sub> =0.02, K <sub>I</sub> =1
<b>5</b>	17.10	9.45	5.61	3.927
<b>2</b>	18.83	15.20	8.36	4.174
<b>1</b>	36.07	19.17	12.14	4.471
<b>0.75</b>	46.34	24.20	14.61	4.667
<b>0.5</b>	59.38	25.85	18.42	5.174
<b>0.25</b>	92.30	29.30	22.59	6.125

It is evident from the Table 4 that detection capability of Hotelling's Chart at input is directly affected by the weight of integrator term in EPC. As the weight  $\omega$  was changed from 0.1 to 0.5, the ARLs have reduced significantly. Similarly, in case of Output feedback controller, the ARLs have reduced when integrator's gain was increased slightly. This finding is logically consistent as well. On the contrary, in real life examples, the integrators' gains are kept as low as possible in order to avoid over adjustment that often leads to failure of actuators; however, there are cases where high integrators' gain can be bearable. Although the finding is important in some special cases

but our conclusion that “Joint Monitoring” of inputs and outputs is the best assignable cause detection scheme holds in practice.

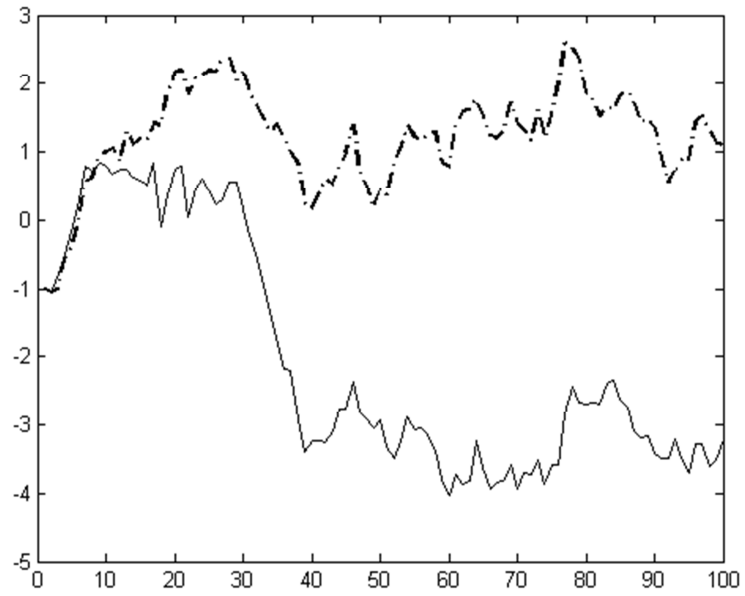
### **3.6 CORRECTIVE ACTION FOLLOWING ASSIGNABLE CAUSE DETECTION**

The logical subsequent step that should be followed by assignable cause detection is the corrective action. Corrective actions heavily depend upon the nature of underlying assignable causes of variations. Most of the assignable causes of variations incur due to some physical fault either in the process or at input/output. For instance, actuators (that control the input feed to the process according to EPC controller’s signal) can start to malfunction. Another common example of reason behind an assignable can be change in process parameters with the passage of time due to wear and tear. The most critical of all is malfunctioning of output sensors; this affects the whole control loop and can lead the system to undesirable conditions such as instability.

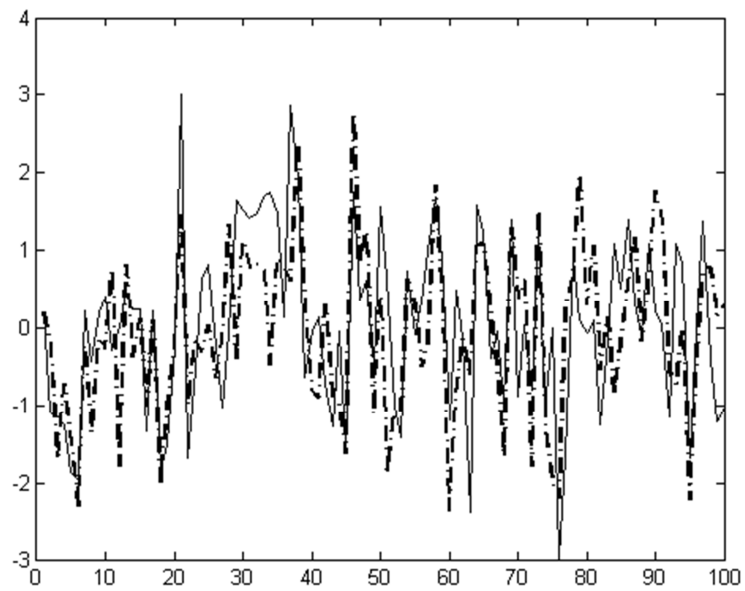
In this section we present a corrective action scheme for such an assignable cause i.e. a significant change in sensor’s measurement error at the process output. Let us assume in our previous example that the noise being added at the output represents measurement error. A mean shift in this noise vector can be a best estimate of induction of an offset into the sensor. Therefore, considering the same example we can investigate the effects of the said corrective actions.

The idea being proposed here is the adjustment of setpoint or target value following the detection of sensor offset assignable cause. The mean shift in noise vector implies culmination of sensed output value by an amount equal to magnitude of the shift. It follows that the EPC scheme will try to bring the wrongly measured value of output closer to the target. To get rid of this situation, the target value or set point can be adjusted by an equal amount to that of shift in the measurement noise vector, assuming that the magnitude of shift is measurable. The idea is illustrated in the following figure.

Simulations were performed using the same example of previous sections in order to probe into the effects of the proposed corrective action. As illustrated in figure, the process input ceases to deviate as soon as this correction is applied; consequently, the wastage of energy at the process input in leaving the assignable cause uncorrected can be avoided while keeping the process output on target.



**Figure 13: Control actions stop deviating after corrective action**



**Figure 14: Output deviation with corrective scheme in place**

## **CHAPTER 4 A CASE STUDY OF HEATING VENTILATION AND AIR CONDITIONING SYSTEM**

A One of the biggest energy consuming systems these days are buildings. The energy consumption of buildings accounts for more than one third of global energy consumptions. Specifically, in the domain of buildings, Heating Ventilation and Air Conditioning (HVAC) systems are the most energy consuming ones along with being top ranked in terms of client complaints[79]. Concerns behind the bulk of energy utilized in building/construction sector have prompted the idea of green buildings that are aimed at least energy acquiring designs. On the other hand, one of the major reasons behind losses of energy is persistence of faults in the system. Therefore, fault detection and isolation is equally important in reducing energy losses. In the systems with feedback control loop, the controller tries to hide or compensate for the faults; however, they continue to dissipate energy and cause reduction in the life time of the equipment.

Numerous researchers in the area of Fault Detection and Diagnosis (FDD) have applied variety of techniques, most of which require modelling of the system[80]. Many have come up with data driven techniques such as Neural Networks and Principal Components Analysis for fault detection[80]. There are few of them who have explored fault detection capabilities of SPC control charts in HVAC; however, their work is limited to application of univariate control charts at process outputs.

[81] were one of the earliest to apply control charts for fault detection in the field of HVAC. They successfully applied CUSUM control charts on Variable Air Volume Terminal units in order to detect four kinds of faults, namely, stuck damper, stuck cooling/heating coil, failed flow sensor and unstable flow. The work of [81] is very motivational for HVAC solution providers; however, it accompanies a major issue as well i.e. the CUSUM control limits were selected by manual observation of trending data instead of some automatic procedure.

[79] developed a rather comprehensive FDD approach to HVAC problem and came up with rule based FDD techniques incorporating CUSUM and EWMA control charts. Their approach improved the diagnosis capabilities as compared to previous works.

[82] have integrated SPC and Kalman Filter to detect faults in the system whereby considering a simple SPC rule that 3 consecutive points falling outside 2 sigma limit indicate fault.

[83] built on the work of [81] by eliminating two associated issues. They incorporated rule based classifier for fault diagnosis and added estimation of CUSUM parameters instead of manual selection.

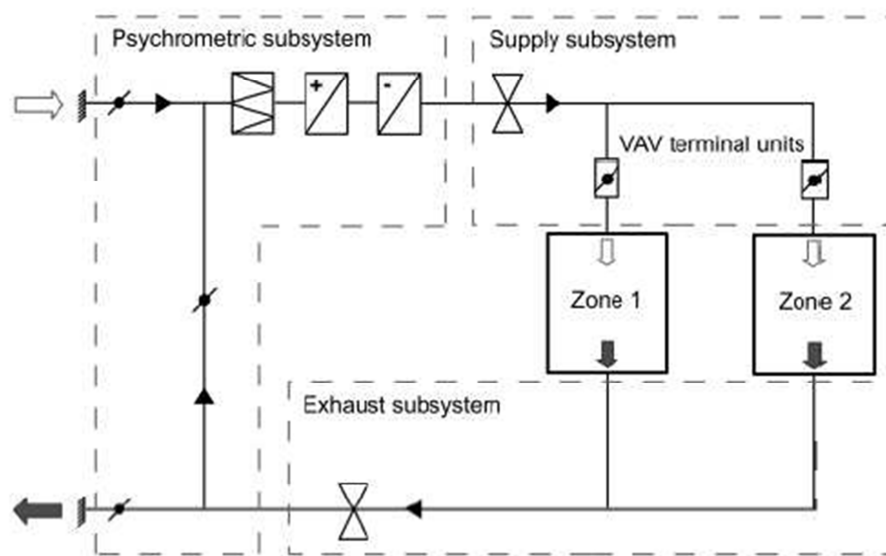
[84] have also come up with an extension to the work of [81] by introducing fault counter method as a fault diagnosis procedure.

This section of the paper demonstrates the effectiveness of “Joint Monitoring” of process inputs and outputs using multivariate control charts such as Hotelling’s  $T^2$  charts.

Moreover, it is argued that they are instrumental in detecting all sorts of faults that occur at sensors, actuators or system level.

## 4.1 SYSTEM MODELLING AND CONTROL

A single-duct VAV system with two thermal zones is shown in Figure 15



**Figure 15: Schematic Diagram of VAVAC system**

The figure shows that there are four primary subsystems

### I. Psychrometric Subsystem

- It consists of chilled water coil, air filter, heating coil, recirculation and exhaust air dampers, etc.



- The key component in this system is the cooling/heating coil, where the heat exchange between air and water takes place

## II. Supply Subsystem

- It consists of variable-duty supply fan and a network of air distribution ducts and VAV terminal units.
- The main purpose of this system is to regulate the inlet flow of conditioned air entering in the thermal zones.

## III. Exhaust Subsystem

- It consists of return ducts, return air diffusers, exhaust fans etc.
- This subsystem takes a part of return air and recirculates it through the supply subsystem.

## IV. Thermal Zones

- Apart from the above subsystems, there are 2 individual control zones/thermal zones.
- Conditioned air is supplied to these zones through supply subsystem and the return air coming out is extracted by exhaust subsystem
- These zones consist of internal loads (occupants, lights, electronic devices etc.) and external loads (heat from windows, walls etc.)

According to the modelling done by [24] the non-linear model of Variable Air Volume Air Conditioning System can be expressed as follows:

$$\dot{x}_1 = a_1 u_1 (x_3 - x_1) + a_2 u_4 + a_3 (u_6 - x_1) \quad (4.1)$$

$$\dot{x}_2 = b_1 u_2 (x_3 - x_2) + b_2 u_5 + b_3 (u_6 - x_1) \quad (4.2)$$

$$\dot{x}_3 = \left[ \frac{C_{pw}}{C_{pa}} \frac{u_3}{(u_1+u_2)} (T_{wi} - x_4) + \left( r \frac{x_1 u_1 + x_2 u_2}{u_1 + u_2} + (1-r)u_6 - x_3 \right) \right] \frac{(UA)_c}{M_c C_{pc}} \quad (4.3)$$

$$\dot{x}_4 = \left[ C_{pw} u_3 (T_{wi} - x_4) + C_{pa} (u_1 + u_2) \left( r \frac{x_1 u_1 + x_2 u_2}{u_1 + u_2} + (1-r)u_6 - x_3 \right) \right] \cdot \frac{1}{M_c C_{pc}} \quad (4.4)$$

where

$$a_1 = \frac{1}{V_{z1} \rho_a}, \quad a_2 = \frac{1}{V_{z1} \rho_a C_{pa}}, \quad a_3 = \frac{U_{z1} A_{z1}}{V_{z1} \rho_a C_{pa}}$$

$$b_1 = \frac{1}{V_{z2} \rho_a}, \quad b_2 = \frac{1}{V_{z2} \rho_a C_{pa}}, \quad b_3 = \frac{U_{z2} A_{z2}}{V_{z2} \rho_a C_{pa}}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} T_{z1} \\ T_{z2} \\ T_{sa} \\ T_{wo} \end{bmatrix} = \begin{bmatrix} \text{Temperature of zone 1} \\ \text{Temperature of zone 2} \\ \text{Temperature of supply air} \\ \text{Temperature of chilled water} \end{bmatrix},$$

$$U = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} = \begin{bmatrix} \dot{m}_{a1} \\ \dot{m}_{a2} \\ \dot{m}_w \\ \dot{Q}_1 \\ \dot{Q}_2 \\ T_{ext} \end{bmatrix} = \begin{bmatrix} \text{flow rate of air in zone 1} \\ \text{flow rate of air in zone 2} \\ \text{flow rate of chilled water} \\ \text{Internal heat load of zone 1} \\ \text{Internal heat load of zone 2} \\ \text{External Temperature} \end{bmatrix}$$

The simulation test case consists of a single duct VAVAC system with two zones. Volumes of each zones are  $V_{z1} = 36\text{m}^3$  and  $V_{z2} = 90\text{m}^3$ , respectively. The external walls areas are  $A_{z1} = 12\text{m}^2$  and  $A_{z2} = 18\text{m}^2$ . The portion of exhausted air equals 25% ( $r = 0.25$ ). The external air temperature is equal to  $27^\circ\text{C}$ , if not otherwise specified. The initial inlet chilled water temperature  $T_{wi}$  is set to  $7^\circ\text{C}$ . It is assumed that the initial supply air temperature,  $T_{sa}$ , is equal to  $15^\circ\text{C}$ . The initial indoor air temperatures  $T_{z1}$  and  $T_{z2}$  are equal to  $22^\circ\text{C}$ .

The disturbances related to internal heat gains (including people occupancy, electric devices, etc.) were assumed to follow a normal distribution with mean value of 500W and variance of 200W. Moreover, External air temperature was simulated to vary from  $27^\circ\text{C}$  to  $33^\circ\text{C}$  sinusoidally over the course of 12 hours.

The feedback controller for the system was designed by using dynamic feedback linearization method in order to accommodate various operating conditions. It was supposed in the design that each zone's temperature is controlled by its respective air flow through VAV terminal box whereas, the overall supply air temperature was governed by the opening of chilled water valve. Consequently, three new inputs were defined:

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} := \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix}$$

And a decoupled, linear system is obtained as follows:

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

The control inputs are given by

$$u_1 = \frac{v_1 - a_2 u_4 - a_3 (u_6 - x_1)}{a_1 (x_3 - x_1)} \quad (4.5)$$

$$u_2 = \frac{v_2 - b_2 u_5 - b_3 (u_6 - x_1)}{b_1 (x_3 - x_2)} \quad (4.6)$$

$$u_3 = \left[ \frac{M_c C_{pc}}{(UA)_c} v_3 - (T_{ai} - x_3) \right] \frac{C_{pa} (u_1 + u_2)}{C_{pw} (T_{wi} - x_4)} \quad (4.7)$$

It is worth mentioning that the values of disturbances  $u_4$  and  $u_5$  used in the calculation of above control inputs can only be estimates (assumed in the simulation to be 500 W) whereas external air temperature disturbance is measurable.

A conventional closed loop proportional control scheme, described below, was adopted and with the appropriate choice of  $k_i = (1, 2, 3)$ , the closed-loop poles of the linearized system can be placed arbitrarily. For this study, values of  $k_i$  were used as 1 for all three loops.

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} k_1(y_1 - y_{1,set}) \\ k_2(y_2 - y_{2,set}) \\ k_3(y_3 - y_{3,set}) \end{bmatrix}$$

The set points for zone's temperature and supply air temperature were set out to be 21°C, 25°C and 15°C. The controller was given ample time for transients before faults were introduced into the system. Furthermore, under steady state conditions, dead-bands of 0.1°C were applied around set-points in order to avoid over-adjustments of process inputs.

## 4.2 FAULT DETECTION USING HOTELLING'S T<sup>2</sup> CHARTS

The fault detection scheme developed in chapter 2 i.e. "Joint Monitoring" of process inputs and outputs was simulated on VAVAC system using following three faults:

- Temperature Sensor Offset (Sensor/Output Fault)
- Cooling Coil leakage (Actuator/Input Fault)
- Stuck mixing damper (System Level Fault)

In order to make the fault detection scheme more convenient for the end users, the input and output measurements were normalized before the application of Hotelling's Control Charts. Therefore, the input observations were normalized using the following formula:

$$u_i' = (u_i - \mu_i)/\sigma_i$$

where  $u_i'$  is the normalized input observation and  $u_i$  the measured input observation whereas  $\mu_i$  and  $\sigma_i$  are the mean and standard deviation of the input observations in the data window used for calculating Hotelling's  $T^2$ . The output observations were normalized by tracking their deviation from the set-point, using the following formula:

$$y_i' = (y_i - y_{sp})/R(y_i)$$

where  $y_i'$  is the normalized output observation and  $y_i$  the measured output observation whereas  $y_{sp}$  and  $R(y_i)$  are the set-point and maximum absolute deviation of output from set-point. The maximum absolute deviation,  $R(y_i)$  is calculated based on the data window of Hotelling's  $T^2$  and is given by:

$$R(y_i) = \max |y_m - y_{sp}|_{m=i-n \text{ to } i}$$

Two Hotelling's Control chart parameters, namely data window size and control limit were to be selected for efficient detection capabilities. The former was selected heuristically by observing the trending data under no fault condition, following the approach of [81]. whereas the latter was selected based on the basis of popular  $ARL_0$  criterion. The system was simulated for a long period of time (approximately 55 hours) without fault and the  $T^2$  statistics were recorded. Based on false alarm probability of 1%, the control limits of outputs and inputs were calculated to be 1.0 and 0.2.

Furthermore, simulations were then run for 100 minutes for simulating each fault and the fault was introduced into the system at 3000<sup>th</sup> second (50<sup>th</sup> minute).

#### 4.2.1 Temperature Sensor Offset Fault:

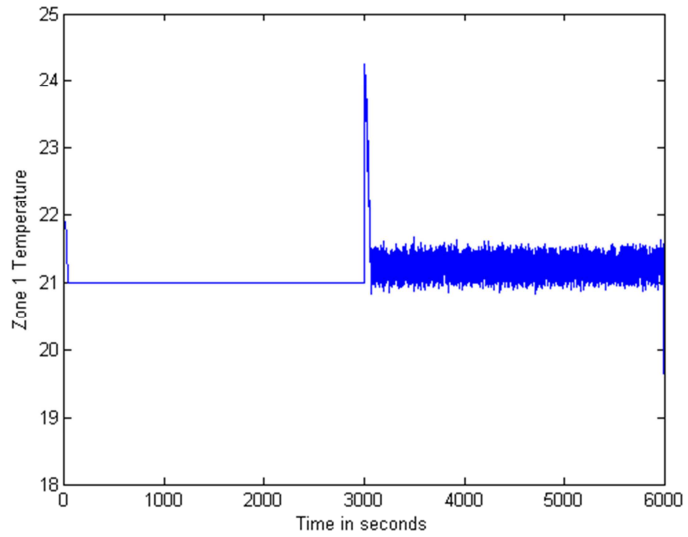
The most important and critical faults in any feedback control systems are related to sensor faults of process variables. Wrong or culminated values of process variable, if progressed through the control loop, can lead the system to instability. Zone Temperature is of such importance in our case. Therefore, sensor faults in zone temperature are the most critical faults in VAVAC systems.

Let us consider that the temperature sensor's readings were culminated with a random noise as shown below:

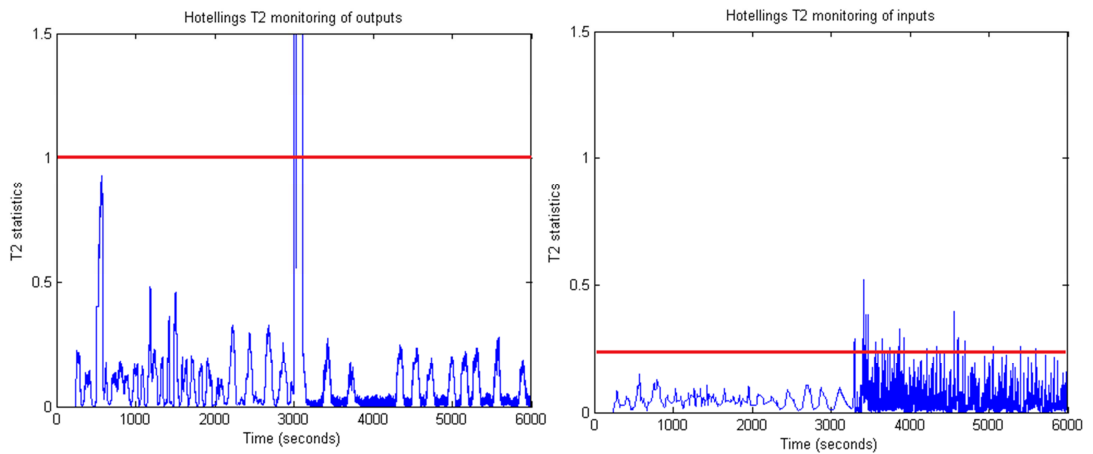
$$T_{z,m} = T_z + \varepsilon_m$$

where  $T_{z,m}$  represents measured value,  $T_z$  represents actual value and  $\varepsilon_m$  is a random noise with mean 0 and variance of  $0^\circ\text{C}$  during the simulation before fault. Two levels of temperature sensor faults were introduced in the system in order to evaluate the detection capabilities of the proposed method.

Firstly, a mean shift of  $3^\circ\text{C}$  and a variance of  $0.25^\circ\text{C}$  are introduced in the measurement noise  $\varepsilon_m$ . The variation of the output in this case is shown in figure 3 whereas figure 4 compares the Hotelling's  $T^2$  statistics observations at output and input. It's apparent from the figure 4 that the fault is readily detected by the both as the plotted  $T^2$  statistics go well beyond the control limits (0.2 and 1.0 for inputs and outputs respectively) after the fault is introduced.



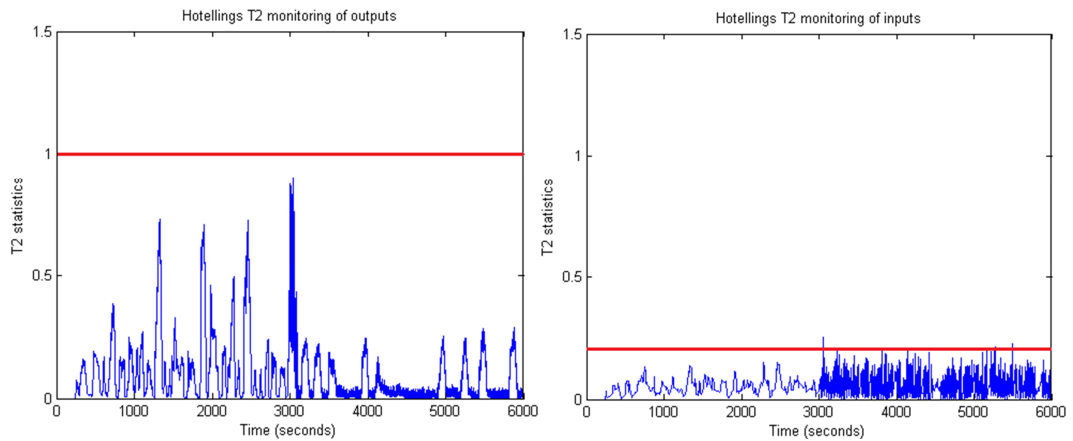
**Figure 16: Output variations with faulty temperature sensor**



**Figure 17: Comparison of  $T^2$  statistics at inputs and outputs with faulty sensor  
(3°C offset)**



Secondly, a mean shift of 0.1°C and a variance of 0.05°C are introduced in the measurement noise  $\varepsilon_m$ . The Hotelling's  $T^2$  statistics observations at output and input are shown in figure 5. Close observation of figure 5 reveals that such a small level of offset is detected by the both as the plotted  $T^2$  statistics exceed the control limits (illustrated as horizontal red line) after the fault is introduced.



**Figure 18 Comparison of  $T^2$  statistics at inputs and outputs with faulty sensor (0.1°C offset)**

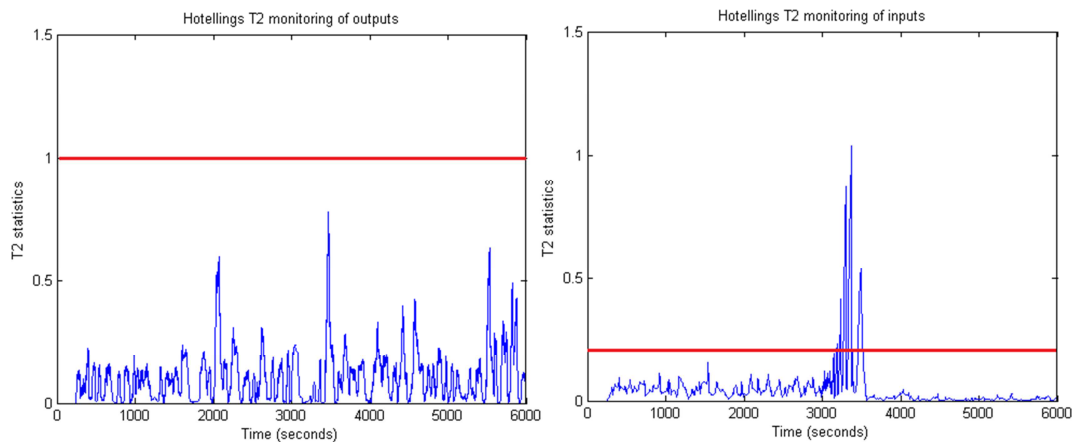
Comparison of Hotelling's charts at input and output vectors reveal that two different levels of faults were quickly and significantly detected by both control charts.

#### 4.2.2 Cooling Coil leakage:

Another important fault in VAVAC systems is the cooling coil fouling and leakage. When the cooling coil valve is stuck, suppose the cooling coil valve position is fixed at  $k$  ( $0 < k < 1$ ), then the actual chilled water flow rate is

$$\dot{m}_r = k \times \dot{m}_r \tag{4.8}$$

A 20% leakage of coolant or chilled water from cooling coil was simulated by selecting  $k = 0.80$ . This is another common yet critical fault in the system that the feedback controller tries to hide by over adjusting the amount of coolant or chilled water flow. The fault was successfully detected by the proposed approach of Joint Monitoring of inputs and outputs as illustrated below:

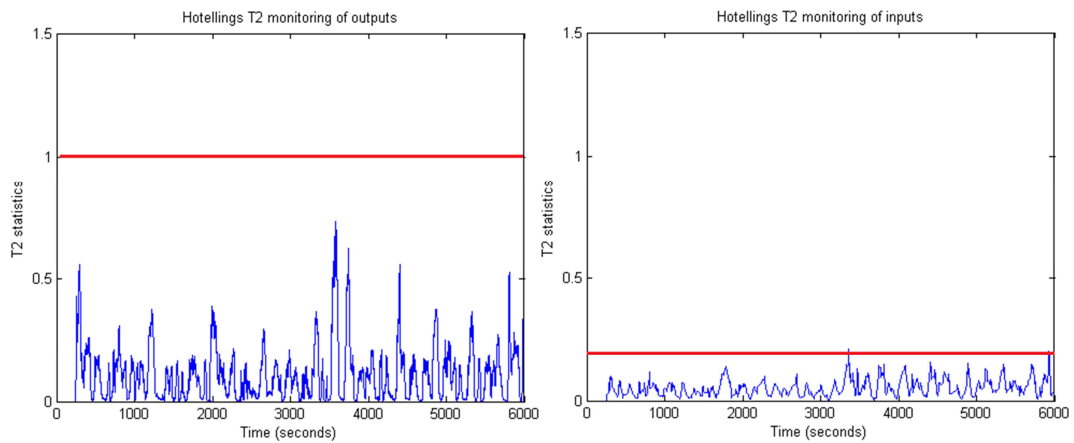


**Figure 19: Comparison of T2 statistics at inputs and outputs with faulty valve (actuator)**

Figure 19 depicts that the  $T^2$  statistics' variation at process input exceeds the control limits as soon as the fault is introduced; However,  $T^2$  statistics at process output remain under control limit and thus cannot detect the fault.

#### 4.2.3 Stuck Mixing Damper:

Stuck mixing damper is one of the most common VAVAC faults encountered by the practicing engineers. Early detection can avoid turbulent flow that disturbs the entire system's pressure. The fault was simulated by shifting the value of 'r' from 0.5 to 0 in state equations.



**Figure 20:  $T^2$  statistics at inputs with faulty damper**

In this kind of faults i.e. fault in the system (neither at output nor at input), detection capability of Hotelling's chart at input is better than its capacity at output in the presence of a good controller. Hotelling's control chart at output doesn't detect this fault as the  $T^2$  statistics don't cross the control limit; however, the control chart at input ultimately detects the fault after approximately 500 seconds as the  $T^2$  exceeds the control limit.

### 4.3 CORRECTIVE ACTION FOLLOWING ASSIGNABLE CAUSE DETECTION

The logical subsequent step that should be followed by fault detection and diagnosis is the corrective action. Corrective actions heavily depend upon the nature of underlying faults. Most of the faults incur due to some physical fault either in the process or at input/output. For instance, actuators (that control the input feed to the process according to EPC controller's signal) can start to malfunction. Another common example of reason behind a fault can be change in process parameters with the passage of time due to wear and tear. The most critical of all is malfunctioning of output sensors; this affects the whole control loop and can lead the system to undesirable conditions such as instability.

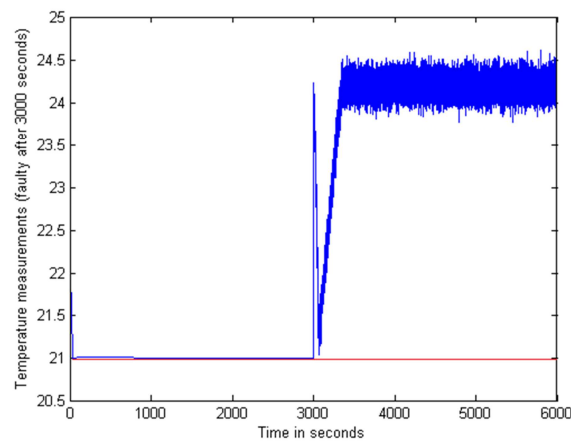
In this section we present a corrective action scheme for such a fault i.e. a significant shift/offset in sensor's measurement error at the process output. Let us consider the first fault as discussed in section 4.2. The output temperature measurement was considered to have following form:

$$T_{z,m} = T_z + \varepsilon_m \quad (4.9)$$

Let us consider that the shift in mean value of  $\varepsilon_m$  is measurable for simplicity and investigate the effects of the said corrective actions in our example.

The idea being proposed here is the adjustment of setpoint or target value following the detection of sensor offset assignable cause. The mean shift in noise vector implies culmination of sensed output value by an amount equal to magnitude of the shift. It

follows that the EPC or feedback control scheme will try to bring the wrongly measured value of output closer to the target. To get rid of this situation, the target value or set point can be adjusted by an equal amount to that of shift in the measurement noise vector, assuming that the magnitude of shift is measurable. Therefore, the faulty measurement will jump to shifted setpoint whereas keeping the original temperature on target as illustrated below where red line represents original temperature and blue represents measurements.



**Figure 21: Effect of corrective action to faulty measurements**

A detailed case study of HVAC systems has been presented in order to explain the idea behind integration of the two complementary schemes along with its practicality and usefulness. Furthermore, it has been established in this work that joint monitoring of an EPC regulated process' outputs and inputs using SPC leads to detection of assignable causes in all cases. Different types of faults have been simulated to ensure that the findings hold in different scenarios of faults.

## **CHAPTER 5 CONCLUSION AND FUTURE WORK**

### **5.1 CONCLUSION**

Multivariate Statistical Process Control and Engineering Process Control are two complementary techniques used in the area of process control. EPC tries to minimize the deviation of process from target, or in other words, prevents the effect of disturbance (common causes of variation) by manipulating process input. On the contrary, SPC aims at monitoring the process for assignable causes of variation, detecting them and ultimately eliminating them as soon as possible. Various schemes of integration between SPC and EPC had been proposed in the literature with a view to complement each other's shortcomings while benefitting from their advantages.

This thesis evaluates the effectiveness of using SPC and EPC together for fault detection and control. A novel scheme of integration has been proposed and evaluated in this thesis considering Multiple-Input-Multiple-Output (MIMO) systems. Simultaneous application of MSPC control charts to process inputs and outputs or in other words "Joint Monitoring" of process inputs and outputs renders very efficient fault detection capabilities.

A numerical example has been presented in order to explain the idea behind integration of the two complementary schemes. Furthermore, it has been established in this work that joint monitoring of an EPC regulated process' outputs and inputs using

SPC leads to the earliest detection of assignable causes. Sensitivity analysis has been performed to ensure that the findings hold in different scenarios of assignable causes and in the presence of different EPC controllers.

A detailed case study of HVAC systems has been presented in order to explain the idea behind integration of the two complementary schemes along with its practicality and usefulness. Furthermore, it has been established in this work that joint monitoring of an EPC regulated process' outputs and inputs using SPC leads to detection of assignable causes in all cases. Different types of faults have been simulated to ensure that the findings hold in different scenarios of faults.

## **5.2 FUTURE WORK**

Research and innovation have no bounds. There can be many directions in which this work of integrating SPC and EPC can be extended. The first and foremost is the consideration of multivariate statistics in economical design of SPC/EPC integrated models. In the economic design configuration, the SPC plays dual role of determining whether or not EPC adjustment is needed as well as the role to detect assignable causes of variation. Model of [9] is very useful addition to the literature but needs extension to multivariate systems and application to real life examples.

One of the major issues in multivariate statistics is that they can detect assignable cause or fault in the system but they are not capable of diagnosing it. For instance, multivariate Hotelling's chart can detect fault due to any single culprit output/input but it is always

needed to know which parameter has caused the fault or assignable cause to occur. Joint Monitoring of inputs and outputs can be very useful in this regard. It can provide some data training mechanism such as neural network or regression to learn the situation faster.

Furthermore, a complete scheme should involve corrective actions, following assignable cause detection and diagnosis. In other words, fault detection and diagnosis should lead to fault isolation as well. An interesting corrective action example is provided in this work; however, it needs to be formulated mathematically in order to acquire a more general form. Process Targeting can be integrated within the current framework for this purpose.

Moreover, there are numerous examples in the arena of feedback control systems where the proposed method can be evaluated. Real-time implementation of these techniques should be considered as they seem more applicable and easy to use than many other data driven fault detection techniques.



## **ACRONYMS AND ABBREVIATIONS**

EWMA	Exponentially Weighted Moving Average
GWMA	Generally Weighted Moving Average
CUSUM	Cumulative Sum
CUSCORE	Cumulative Score
ARIMA	Auto Regressive Integrated Moving Average
FDD	Fault Detection and Diagnosis
HVAC	Heating, Ventilation and Air Conditioning System
VAVAC	Variable Air Volume Air Conditioning System

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