COOPERATIVE CONTROL OF PORT CONTROLLED HAMILTONIAN SYSTEMS

BY

AMIN-UD-DIN QURESHI

A Dissertation Presented to the

DEANSHIP OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

In

SYSTEMS ENGINEERING

APRIL 2014

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DHAHRAN 31261, SAUDI ARABIA

DEANSHIP OF GRADUATE STUDIES

This thesis, written by **AMIN-UD-DIN QURESHI** under the direction of his thesis adviser and approved by his thesis committee, has been presented to and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **DOCTOR OF PHILOSOPHY IN SYSTEMS ENGINEERING**.

Dissertation Committee

Dr. Sami Elferik (Adviser)

Dr. Fouad Al-Sunni (Co-adviser)

Dr. Moustafa El-Shafei (Member)

Dr. Abdul Wahid Abdul Aziz Al-Saif (Member)

Dr. Lahouari Cheded (Member)

Dr. Fouad Al-Sunni
Department Chairman

Dr. Salam A. Zummo
Dean of Graduate Studies

3/4/14
Date

To
my mother

ACKNOWLEDGMENTS

All praise is due to Allah, the Lord of the worlds and peace be upon Muhammad, the messenger of Allah.

Completion of a doctoral thesis requires an individual hard work spanned over several years. However, it is impossible without the support of other persons helping in several aspects. At this moment, when you have my PhD thesis in the front of your eyes, it is my pleasure to acknowledge the support of other people.

First and foremost, I would like to express my deep gratitude to my affectionate thesis advisor Dr. Sami Elferik for his continuous guidance, encouragement, appreciation and useful critiques. I would also thank my co-advisor Prof. Fouad Al-Sunnin, and other members of the dissertation committee, Prof. Moustafa El-Shafei, Dr. Abdul Wahid Abdul Aziz Al-Saif and Dr. Lahouari Cheded. Their invaluable suggestions and feedback have, no doubt, improved the outcome of this thesis. I am indebted to Prof. Frank L. Lewis (Univ. of Texas at Arlington) for his guidance and reviews.

I am also thankful to secretarial and technical staff of the systems engineering department for their help and support in many regards.

Passing a period of more than four years would be very tiresome and boring without the pleasant company of my friends, Dr. Khurram, Dr. Imran, Dr. Haris, Dr. Sabih, Raza, Ahmed Tariq, Naseer, Ahmed Adeniran, Tariq Muhammad, Bilal, Azher, Rifaqat, Alamzaib, Khwaja, Naveed, Haider and many others. Supports by Dr. Khurram, Dr. Imran and Ifthikhar Khattak in the early days of my stay at KFUPM are simply unforgettable.

I am also grateful to my parents for their support of every kind and their encouragement throughout my career. My children Sumaiyah, Maaz, Ruqaiyah, Fatima and Maryam, and, my wife have been the only source of serenity for me during my stay at the KFUPM.

I would also like to extend my thanks to the Ministry of Higher Education of Saudi Arabia and the government of Pakistan for their financial support to carry out this research.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

BCF	:	Brunovsky Canonical Form
CUUB	:	Cooperatively Uniformly Ultimately Bounded
EL	:	Euler-Lagrange
FLNN	:	Functional Link Neural Network
IDA-PBC	:	Interconnection and Damping Assignment Passivity Based Control
IP	:	Information Preserving
NLSE	:	Normalized Local Synchronization Error
NLSR	:	Normalized Local Synchronization Reference
NN	:	Neural Network
PBC	:	Passivity Based Control
PCH	:	Port Controlled Hamiltonian
RVFLNN	:	Random Vector Functional Link Neural Network
UUB	:	Uniformly Ultimately Bounded

NOTATIONS

$\lvert \cdot \rvert$	:	Absolute value of a real number
$\lVert \cdot \rVert$	:	Euclidean norm of a vector
$tr\{\cdot\}$	:	Trace of a matrix
$\lVert \cdot \rVert_F$	:	Frobenius norm of a matrix, defined as: $\lVert A \rVert_F = \sqrt{tr\{A^T A\}}$,
		where A is a square matrix
$\mathcal{C}^r$	:	A function with r continuous derivatives
$\bar{\sigma}(\cdot)$	:	Maximum singular value
$\underline{\sigma}(\cdot)$	:	Minimum singular value
$P > 0$	:	Indicates that the matrix P is positive
$(P \geq 0)$		definite (positive semi-definite)
$\mathcal{N}$	:	set $\{1, ..., N\}$
I_m	:	Identity matrix of order m
0_m	:	Square null matrix of order m
$\bar{\mathbf{1}}$	:	Unity vector $[1, \ldots, 1]^T \in \Re^n$ where n is
		the required appropriate dimension

THESIS ABSTRACT

NAME: Amin-ud-Din Qureshi

TITLE OF STUDY: Cooperative Control of Port Controlled Hamiltonian Systems

MAJOR FIELD: Systems Engineering

DATE OF DEGREE: April, 2014

Advancements in computational devices, sensors and actuators together with information technology and the need for more effective control have created a new horizon of research; the cooperative control of autonomous *multi-agent* systems. In many applications a group of autonomous systems working cooperatively can outperform a task that could be either very difficult or even impossible by the individual actions of autonomous systems. The majority of these physical systems have nonlinear dynamics and their parameters are prone to significant variations during their operations in practical scenarios. This thesis presents novel Neural Networks (NNs) based robust distributed adaptive cooperative tracking control techniques for two important classes of physical systems where the individual agents are networked through a directed graph. These systems are the *Port Con-*

trolled Hamiltonian (PCH) Systems and the higher-order nonlinear affine systems in *Brunovsky Canonical Form* (BCF). PCH systems represent a large number of physical systems including several electrical, electromechanical, chemical and particularly a major class of mechanical systems described by Euler-Lagrange (EL) models. Due to their inherent passivity property, PCH systems are best suited for passivity based energy shaping control, particularly in the applications where shaping of both the potential as well as the kinetic energy is required. These advantages associated with the controller design under PCH formalism are exploited in the design of cooperative tracking control problem in this thesis. Parametric uncertainties are dealt with using the NNs. Canonical transformation of PCH systems using NNs is also presented. A novel idea of information preserving filtering of Hamiltonian gradient is introduced to drive the NNs tuning laws by the position as well as the velocity errors. $\mathcal{L}_2$ disturbance attenuation objective is also achieved. This thesis also presents a NNs based adaptive distributed cooperative tracking control of unknown higher-order affine nonlinear systems represented in BCF. The *Cooperative Ultimate Uniform Boundedness* (CUUB) of the cooperative tracking errors is guaranteed for the proposed controllers. The proposed neuro-adaptive controllers are direct i.e. NNs are trained online without any need of the off-line training. Effectiveness of the proposed controllers are tested through simulations using several models of physical systems including planar manipulators, simple pendulums, inverted pendulums and an Autonomous Underwater Vehicle (AUV).

ملخص الرسالة

الاسم:	امين الدين قريشي

عنوان الرسالة: المتحكم التعاوني والموزع للأنظمة الهاملتونية

التخصص:	هندسةالنظم

تاريخ التحرج: ابريل 2014

نعرض في هذه الأطروحة طريقة للتحكم بالأنظمة الموزعة والمكتوبة بصيغة ال PCH(الهاملتونية). أحد الانظمة الجزئية تقوم بمقام القائد في حركة باقي الجزيئات للنظام الموزع و تنشأ المسار المقترح لحركتها. فقط مجموعة قليلة من الأنظمة الجزيئة يمكنها الحصول على المعلومات من القائد. كما أن الاتصال بين الانظمة الجزئية يقتصر فقط على الجزئيات المجاوره لها. تم استخدام الشبكات العصبية الاصطناعية ليكون المتحكم أكثر متانة في وجه المتغيرات الموجودة في النماذج المستخدمة. استخدمت الخلايا العصبية الاصطناعية بشكل جديد ليتمكن المتحكم من القيام بالتوليف الذاتي. كما يقوم المتحكم بالتغلب على العوامل المؤثرة على النظام والمؤثرة في حركته. في هذا البحث نقدم متحكم عملي وفعّال يستخدم الانظمة العصبية المعدلة، للتحكم بروبوتات ممثلة بواسطة النظم الهاملتونية مع اعتبار وجود أخطاء في تحديد القيم الدقيقة لتمثيل الروبوتات. ووجود عوامل خارجية تؤثر على استجابة الروبوتات. تم تقديم طريقة مبتكرة في عملية تصفية البيانات بالاعتماد على المتغير الهاملتوني، بالإضافة الى تعديل إعدادات المتحكم آليا بواسطة الانظمة العصبية. وقد حقق المتحكم الشروط الضرورية في تقليل آثار العوامل الخارجية التي تؤثر على حركة الروبوتات، ولبيان ذلك تم تمثيل المتحكم في روبوت ثنائي الحركة لتوضيح دقة وكفاءة المتحكم .ولقد تم التأكد من كفاءة عمل المتحكم المقترح من خلال دراسات المحاكاة لبعض الأنظمة.

CHAPTER 1

INTRODUCTION AND MOTIVATION

Evolution of feedback control theory has been fundamentally driven by the necessity of the control systems engineers to manipulate a system behavior to exhibit some desired operation dictated by the requirements of an application. Because of the vast variety of control problems, researchers have developed numerous automatic control techniques by utilizing the well-established tools of mathematics. This thesis is an attempt to step forward in the control theory by its novel contributions in the *cooperative control* of higher-order nonlinear systems. Introduction to cooperative control and motivation of the thesis are explained in the following section.

1.1 Introduction

The essence of a control system is to achieve the desired objectives with maximum accuracy, autonomy, reliability and robustness under some physical constraints. Research in control theory has been revolutionized by numerous technological advancements along the history as happened to several other disciplines of applied research. Among these advancements, the prominent ones that recently influenced the progress of control theory are the advent of ultra-fast miniaturized microprocessors, reliable means of data communication and MEMS-based smart sensors and actuators. Such advancements and emerging needs in diverse fields of applications, together with the enthusiasm of researchers have further paved the way for the unprecedented developments of a number of innovative autonomous and intelligent control systems. A substantial interest of researchers has been witnessed to develop autonomous systems with minimum human involvement in their operations. To name a few, examples of such systems are: unmanned ground, aerial and underwater vehicles, various intelligent industrial, personal and service robotic systems. They have been successfully deployed in many applications such as mineral exploration, environmental monitoring, space research, health care and several other civil and military applications. Some examples of such systems are shown in Fig. 1.1.

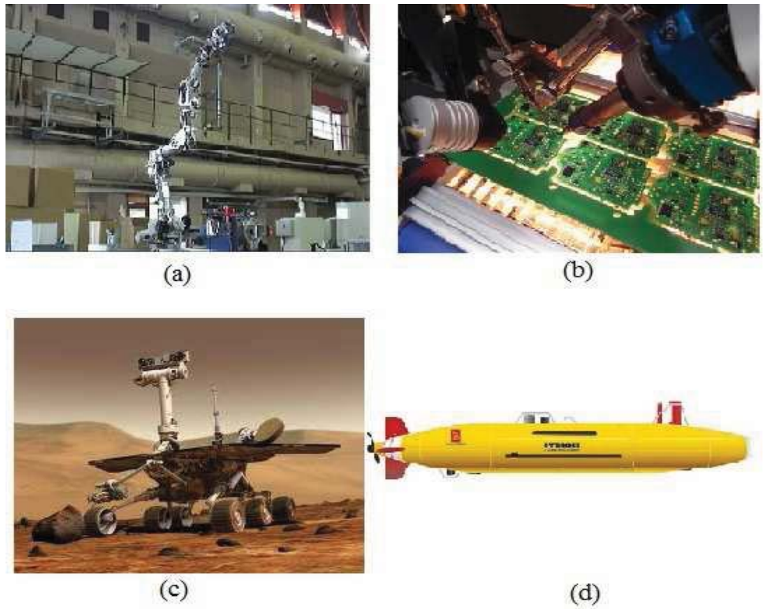

Figure 1.1: Examples of autonomous systems: (a) High access survey robot by Honda deployed at hazardous areas of abandoned Fukushima nuclear power plant. (b) Autonomous robotic solderer (Promation Inc.) (c) Mars rover (NASA Jet Propulsion Lab) and (d)Autonomous underwater vehicle Remus-6000 (Konsberg Maritime Inc.)

1.1.1 From a single system to a system of systems

When operated as single systems, the autonomous systems mentioned above have reportedly succeeded in achieving the desired objectives in their missions and tasks [1]. However, at the same time there exist numerous applications where a coordinated group or team of such autonomous systems can perform their operations with greater efficiency as compared to their individual operations. In most cases the individual systems forming a coordinated group are geographically distributed and are linked through a communication network. A group of systems operating in a coordinated way is also called a multi-agent system [2]. Some examples of applications where such systems are used are: Simultaneous Localization And Mapping (SLAM) [3] - [4], environmental monitoring and sampling [5] - [6] locomotion [7] - [8] and automated manipulation of biological cells [9]. Furthermore, there are many applications where deployment of a multi-agent system is essential rather than a matter of choice of increasing the efficiency. Examples of applications of such multi-agent systems include: Space-based interferometers [10] - [11], future combat systems [12], surveillance, reconnaissance and hazardous material handling [13], distributed reconfigurable sensor networks [14]. Some examples of multi-agent systems working cooperatively are shown in Fig. 1.2 . The feature of enhanced performance of autonomous multi-agent systems, and their inevitability in some cases, has created an active research area known as cooperative control and coordination. Compared to the control of a single autonomous system (i.e. a single-agent system) cooperative control of multi-agent systems is mainly different, firstly, in its structure; which can be centralized, decentralized or distributed and secondly, the information exchange among the agents through a communication network which can be well explained with the help of a graph. Direction of information flow, topology of the graph, limited connectivity of the communication network, complexity of the operating environment and the limitations on the embedded computational resources are the major factors that influence the multi-agent controller design in different aspects. Literature review indicates that development of any cooperative control scheme is accomplished by considering a particular class of systems. These classes mainly include systems with single or double integrator dynamics, general linear systems, nonlinear oscillators, Euler-Lagrange systems (holonomic and non-holonomic) and some other classes

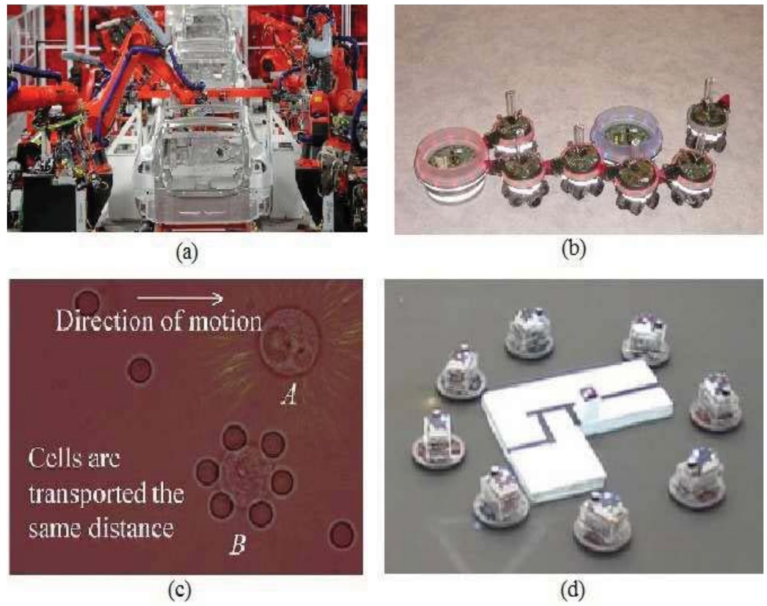

Figure 1.2: Examples of cooperative groups of autonomous systems: (a) Cooperative group of robots at a car assembly line (Tesla Cars) (b) Manipulation and locomotion by *swarm-bot* at LASA (EPFL university) (c) Automated manipulation of biological cells using gripper formations controlled by optical tweezers [9] (d) Swarm of robots cooperatively manipulate an object (University of Pennsylvania).

of nonlinear systems [15]. It is a well-established fact in control theory that unlike the case of linear systems, analysis and controller design of nonlinear systems cannot be unified in a general framework of mathematical tools. It is due to this fact that cooperative control of different classes of nonlinear systems, which represent the majority of real life physical systems, is still an area of active research around the world. The essence of this thesis is to design robust adaptive cooperative controllers which can be applied to a broader set of higher order nonlinear systems.

1.2 Cooperative Control of Higher-Order Nonlinear Multi-Agent Systems

Most of real-life practical systems are described by nonlinear dynamics, e.g. robotic manipulators, autonomous vehicles and power generation systems etc. In many cases controllers designed using the linearized models of such nonlinear systems exhibit very poor response, and it becomes necessary to base the controller design on the most realistic nonlinear model. The literature survey shows that a major research gap left in the field of cooperative control is the unavailability of results for two important classes of physical systems namely, the Port Controlled Hamiltonian (PCH) systems and the higher-order nonlinear systems described in Brunovsky Canonical Form (BCF). The major concern of this thesis is to develop distributed cooperative control schemes for these two important classes of higher order nonlinear systems, described separately in the followings.

1.2.1 Port Controlled Hamiltonian (PCH) systems

The PCH formalism represents a large number of physical systems including all of the Euler-Lagrange (EL) systems and several electrical, electromechanical, and chemical systems [16], [17]. As a distinctive feature, PCH formalism utilizes the energy-relevant properties of a physical system. Within PCH formalism a system is described by:

1. an interconnection of energy storing components to explain the phenomenon

of energy exchange among them, and

2. power ports to explain the interaction of the system with external world.

Controller design and implementation within PCH formalism becomes more evident by associating the energy-relevant properties of a system with its constituent components and the way they are interconnected to one another. Possessing the clear-cut interpretation of energy-relevant properties together with their inherent passivity, PCH systems are well suited to exploit the passivity theory, which is a strong tool of nonlinear systems analysis and control [18]. From a controller design point of view, the principal advantage of PCH formalism is that the Hamiltonian, i.e. the stored energy function acts as Lyapunov function, thus alleviating the search of candidate Lyapunov function which is an essential step in controller design, and often becomes a problem of major concern for the control of complex nonlinear systems [19]. Furthermore, in passivity-based control by energy-shaping and energy-balancing, structure preserving shaping of both the potential as well as the kinetic energies can be performed for the PCH systems, while in Euler-Lagrange such structure preserving shaping can be achieved only for potential energy. Thus PCH formalism can be successfully employed in applications which, in addition to potential energy, also require the modification of kinetic energy [16]. Some of the major theoretical contributions in the literature of PCH systems modelling, analysis and control are [16], [20], [21], [22], [23].

1.2.2 Parametric uncertainties and neural networks based adaptive control

Controllers are usually designed with the assumption of exact knowledge of the system parameters. However, in practical situations exact values of the parameters are prone to vary significantly from the nominal values provided by the manufacturers due to several factors. As an example, the effective length and mass of the free arm of a robotic manipulator significantly varies when interacting with the outside world. Ageing is also a factor of parametric uncertainty. Uncertainties in system parameters and model structure can significantly degrade the performance and stability of a controller, if they are not taken into account during the controller design stage. In this thesis, Neural Networks (NNs) are used to compensate for the parametric uncertainties, which are lumped into an unknown function. In the last two decades, closed-loop feedback applications and properties of NNs have been well studied and rigorously developed [24]. This development was achieved using the mathematical tools from a control theoretic perspective, and showed how to systematically design the *neurocontrollers* with guaranteed stability and performance. Neurocontrollers alleviate the need for several assumptions, which are very common in adaptive control theory, for example, linearity in the parameters and availability of a known regrersor matrix [24].

In the available literature of adaptive tracking control of uncertain PCH systems, the adaptive laws are driven by the velocity error only, that may result in a steady-state position error even when the velocity error converges to zero [25],

[26], [27]. Inclusion of position error in the adaptive laws is avoided for the reason of preserving the PCH structure of closed-loop dynamics [28]. In this thesis, a novel idea of *Information Preserving* filtering of Hamiltonian gradient is introduced to drive NN tuning law by both the velocity as well as the position error, while the PCH structure of the closed-loop system is preserved. In addition to compensation of parametric uncertainties, $\mathcal{L}_2$ attenuation objective of a class of external disturbance is also achieved.

It is also worth noting that neuro-adaptive controllers proposed in this thesis are *direct*. In direct adaptive controller, off-line training of NN for system identification is not needed: the parameters of the controller are tuned directly. On the other hand, in the *indirect* adaptive control an *identifier* is used to synthesize the model of the system dynamics and the information obtained from this identified model is used to tune the controller. The process of identification is performed off-line in the indirect adaptive control schemes [29], [30].

1.2.3 Nonlinear systems in Brunovsky Canonical Form (BCF)

This thesis also develops a robust neuro-adaptive distributed cooperative control methodology for a group of higher-order unknown affine nonlinear systems described in Brunovsky canonical form (BCF), also known as *companion canonical form* and *controllability canonical form*. The powerful function estimation property of NNs is utilized to estimate the unknown dynamics with guaranteed closed loop stability. Examples of nonlinear systems in BCF include the Fitzhugh-Nagumo model that describes the dynamics of the membrane potential in neuronal systems driven by electrochemical properties of sodium and potassium ion flow [31]. The Van der Pol oscillator is a special case of Fitzhugh-Nagumo models [32]. Another example of nonlinear systems in BCF is the inverted pendulum [33]. Other than the nonlinear systems directly described in BCF, many other nonlinear systems can also be transformed to BCF using feedback linearisation provided a reachability and an involutivity condition is satisfied [24]. Though practical systems usually satisfy the reachability condition, some classes of systems fail to be involutive. Nevertheless, in case of this failure, it is still possible to transform the system to BCF using partial state feedback linearisation or using input-output linearisation and taking a care of the stability of the zero-dynamics [24].

In the available literature, the actuator input function, usually denoted by $g(\cdot)$ is assumed to be known and equal to unity. Such assumption on the input function restrict the application of the available distributed controller to a relatively small class of systems. Indeed, in real-life applications, the actuators input function is different from unity, nonlinear, and depends on system's states. In this thesis the input function is assumed to be a smooth function of states, thus making the proposed controller more general as compared to the closely relevant works in [34], [35] and [36]. In this thesis, NNs are used to estimate the unknown dynamics of the individual systems with guaranteed closed loop stability.

1.3 Controller structure of Multi-Agent Systems

Theoretically, there are three possible configurations of controller structure, centralized, decentralized or distributed, as elaborated in the following sections.

1.3.1 Centralized

In a centralized control scheme, all agents send their state information to a central controller. The central controller processes this information to compute the control decision and sends the control commands to corresponding agents. Though essentially similar to the control of a single large-scale systems with relatively established theory, the centralized scheme suffers from the serious problems of increased communication load, loss of information transmitted by distant agents due to limited wireless sensor range, and increase of computational complexity with increase of number of agents and couplings among them [32]. Furthermore, any small change in the network topology, such as addition or removal of an agent from the group can require the redesign of the controller.

1.3.2 Decentralized

In decentralized control, the control command is decided at the individual agents' ends from their own state information. Decentralized control strategy has the advantages of relatively very low computational complexity and almost negligible communication load. A major shortcoming of such schemes is that some group members are not able to predict the group behavior on the basis of available information, and consequently, some of the desired cooperative objectives cannot be achieved [15].

1.3.3 Distributed

Distributed control exhibits a practical balance between the centralized and decentralized control methodologies. In such control schemes, controllers are implemented at the agents' ends with state information of itself and the neighbor nodes. The communication load and computational complexity is greater than in the decentralized case but adequately lower than in the centralized case. Distributed control approach is inspired by the group behaviors of the animals, where each group member try to cooperate as a unit [32]. Examples of such cooperative group behaviors in nature can be found in flocking of birds, swarming of insects, schooling of fish, and herding of quadrupeds. Distributed control approach is more promising for the multi-agent systems because of its robustness, flexibility, and scalability [2], [32].

1.4 Multi-Agent Systems on Graphs

Graph theory is a strong tool for the representation, analysis and controller design of networked multi-agent systems. It has been extensively used in cooperative control of multi-agent systems. An individual agent is represented by the node or vertex of a graph. Communication link between a pair of agents is represented by the edge of a graph. The topology of a graph represents the interconnection among different agents of a group. In this thesis, cooperative controllers are designed by utilizing the definitions and concepts of graph theory, explained in detail in the next chapter.

1.5 Thesis Objectives

Thesis objectives are now precisely described as follows.

1. *Consider a group of higher-order nonlinear affine systems described in Brunovsky canonical form with unknown dynamics and distributed over a communication network represented by a directed graph. Design a distributed neuro-adaptive cooperative tracking control law such that the resulting tracking error and neural network weight error dynamics are cooperatively uniformly ultimately bounded.*

2. *Consider a group of autonomous PCH systems, distributed over a communication network represented by a directed graph. Design a distributed cooperative tracking control law such that tracking error dynamics are globally asymptotically cooperatively stable.*

3. *Design a distributed neuro-adaptive cooperative tracking control law for a group of systems described in part 2, such that:*

   (a) *the resulting controller is robust to modelling and parametric uncertain-*

ties, and

(b) *the resulting tracking error and neural network weight error dynamics are cooperatively uniformly ultimately bounded, and*

(c) $\mathcal{L}_2$ *disturbance attenuation is also achieved, against the external disturbance.*

1.5.1 Thesis contributions

Development of this thesis resulted into the following contribution to the literature.

- Sami El-Ferik, Aminuddin Qureshi and Frank L. Lewis, "Neuro-adaptive cooperative tracking control of unknown higher-order affine nonlinear systems", *Automatica*, DOI: 10.1016/j.automatica.2013.12.033, Feb. 2014.

- Aminuddin Qureshi, Sami El-Ferik and Frank L. Lewis, "$\mathcal{L}_2$ Neuro-Adaptive Tracking Control of Uncertain Port Controlled Hamiltonian Systems", *submitted.*

- Sami El-Ferik, Aminuddin Qureshi and Frank L. Lewis, "Robust Neuro-Adaptive Cooperative Control of Multi-Agent Port Controlled Hamiltonian Systems', *submitted.*

- Aminuddin Qureshi, Sami El-Ferik and Frank L. Lewis, "On Neural Networks Based Canonical Transformation of Port Controlled Hamiltonian Systems", *under preparation.*

1.6 Thesis Organization

Chapter 1 introduces the subject of the thesis, i.e. cooperative control of multi-agent dynamic systems. It describes the importance of, and advantages associated with, the applications of multi-agents systems and their cooperative control. A discussion on the different structures of cooperative control is presented. Research gaps are identified and objectives of the thesis are then precisely stated.

Chapter 2 presents the detailed literature survey related to the subject of the thesis. The surveyed topics are cooperative control theory and control of BCF and PCH systems. This chapter also explains the important concepts of control theory and graph theory which are fundamental to the development of this thesis.

In chapter 3, the distributed neuro-adaptive cooperative tracking control of unknown higher-order nonlinear affine systems in BCF is presented. Chapter includes the problem formulation, underlying assumptions and controller design steps. Since the controller structure possesses discontinuity, existence and uniqueness as well as boundedness of the control action is asserted and explained in detail. Rigorous stability analysis and several simulation examples are presented as well.

In chapter 4 robust neuro-adaptive trajectory tracking control of a single uncertain PCH system is presented. Development of this controller is considered as an important step towards the cooperative control of a group of networked PCH systems. A novel idea of information preservation filtering of shaped Hamiltonian gradient is introduced which causes the neural network weight tuning law to be driven by both the position and velocity errors. This chapter also presents the neural networks based canonical transformation of PCH systems. Simulations are performed on a simple pendulum, a two-link robotic manipulator and an Autonomous Underwater Vehicle (AUV).

In chapter 5 robust neuro-adaptive distributed cooperative tracking control of a group of networked PCH systems is presented. The problem formulation of cooperative control is accomplished by taking the necessary steps to transform the PCH systems from state-space to normalized local synchronization error space. Distributed cooperative tracking control is first proposed for nominal systems, and then neural network based adaptive distributed cooperative controller is presented which is robust to parametric uncertainties. As in chapter 4, the PCH structure of the closed loop system is preserved. $\mathcal{L}_2$ disturbance attenuation objective is also achieved. Simulations are performed on a group of five PCH systems, distributed over a communication network.

The thesis is concluded in chapter 6 with a description of several interesting future works.

CHAPTER 2

LITERATURE REVIEW AND MATHEMATICAL BACKGROUND

In the literature *cooperative control* refers to a control methodology that drives a group of autonomous agents to some common state [2]. The terms consensus, synchronization, agreement and rendezvous are alternatively used for cooperative control. Research in cooperative control has attracted a great number of researchers from different disciplines of science and engineering including control theory, communication, computer science, biology, physics and economics. This chapter presents a comprehensive survey of the cooperative control literature. Important mathematical preliminaries, including some basic definitions of passivity theory, Port Controlled Hamiltonian (PCH) modelling and graph theory are also briefly described.

2.1 Review of Cooperative Control Literature

The spirit of cooperative control theory is to develop a set of mathematical rules that manipulate the individual behaviors of the members i.e the agents of a networked-group, to achieve a predefined common objective. Research in this field is believed to be first inspired by the similar research works in other disciplines of science. In 1974 DeGroot presented his work in the field of management sciences on the problem of reaching a consensus [37]. In 1986, Tsitsiklis, Bertsekas and Athans presented their work in [38] and [39] on asynchronous distributed optimization algorithms for distributed decision making problems, which is considered as a seminal work in cooperative control theory [32]. Distributed computing [40], and statistical physics [41] are also believed to have influenced the research in cooperative control [15]. In [41], the authors investigated the emergence of self-driven motion in systems of particles with biologically motivated interactions. They showed by simulations that in a group of autonomous agents moving with some constant speed, the direction of motion of an individual agent converges to the average of the directions of other agents lying within a neighborhood of radius r. These simulations showed the collective behavior of a group of autonomous agents. Control theoretic grounds for these simulations were provided by Jadbabaie, Lin and Morse in their seminal paper [42]. This paper triggered a great interest from the control theory community to further investigate the various facets of the dynamics of cooperative and coordinated multi-agent autonomous systems. Several newly-emerged cooperative control problems were then considered in another series of papers [43], [44], [45] and [46].

In [43], the authors studied the consensus problems for networks of dynamic agents with fixed and switching topologies with the analysis of three cases: (1) directed networks with fixed topology; (2) directed networks with switching topology; and (3) undirected networks with communication time-delays and fixed topology. Authors of [44] considered the problem of cooperation among a collection of vehicles performing a shared task using inter-vehicle communication to coordinate their actions. In [45] the problem of asymptotic consensus under dynamically changing interaction graph topologies and weighting factors is considered. Authors of [45] also proposed the discrete and continuous update schemes for information consensus with conditions for asymptotic consensus under dynamically changing communication environment using these update schemes.

In the following, depending on the type of system dynamics, various contributions are separately described. For each type of dynamics, a group of N agents is considered and an ith group member is denoted with a subscript i.

2.1.1 Single and double integrators

A single integrator is described as

$$\dot{x}_i = u_i, \quad i = 1, 2, \cdots N. \tag{2.1}$$

and a double integrator is described as

$$\ddot{x}_i = u_i, \quad i = 1, 2, \cdots N. \tag{2.2}$$

where $x_i \in \Re$ is the state and $u_i \in \Re$ is the control input. Cooperative control of single integrators are considered in the seminal papers [42], [43] and [45], mentioned above. Double integrator dynamics are considered in [47], [48] and [49] and references therein.

2.1.2 General linear systems

The well-known dynamics of a linear system is given by

$$\dot{x}_i = A_i x_i + B_i u_i, \quad y_i = C_i x_i \tag{2.3}$$

where $x_i \in \Re^n$ is the nth order state vector, $u_i \in \Re^m$ is the input vector and $y_i \in \Re^p$ is the output vector. A_i, B_i and C_i are real matrices of compatible dimensions.

In [50], a framework based on the matrix theory is proposed to analyze and design cooperative controls for a group of individual linear dynamic systems whose outputs are sensed by, or communicated to, others in an intermittent, dynamically changing, local manner. For arrays of identical output-coupled linear systems, [51] investigated the sufficiency of certain conditions on system matrix A_i and output matrix C_i for existence of a feedback law under which the systems synchronize for all coupling configurations with connected graphs. [52] investigated the synchronization of a network of identical linear state-space models under a possibly time-varying and directed interconnection structure. Authors of [53] studied the cooperative control of multi-agent higher-order identical MIMO linear dynamic systems where only the output information of each agents is delivered throughout the communication network. [54] addressed the consensus problem of multi-agent systems with a time-invariant communication topology consisting of general linear node dynamics and proposed a distributed observer type consensus protocol based on relative output measurements. In [55] a high-gain methodology is used to construct linear decentralized controllers for consensus, in networks with identical but general multi-input linear time-invariant (LTI) agents and general time-invariant and time-varying observation topologies. Most recently a locally optimal Riccati design approach is introduced in [56] to synthesize the distributed cooperative control protocols for both continuous and discrete-time systems networked through directed graphs. A detailed study of the fundamental definitions, mathematical tools, matrix and graph theories applied to cooperative control is presented in the the book [57].

Note that single and double integrator dynamics can be considered as special cases of general linear systems described in (2.3).

Since cooperative control and coordination is mainly concerned with the behavior of a group of dynamic systems, it is important to consider the dynamics of practical systems like robotic manipulators and vehicles during the development of a control algorithm. Research outcomes mentioned above considered the single and double integrator dynamics and general linear dynamics for the development of cooperative control theory. Since most of the practical physical systems cannot be modelled as linear systems, the trend of cooperative control research was naturally diverted to consider the models with much richer information of the system's dynamics to design practically efficient and robust controllers. A number of complex cooperative control methodologies appeared then in the literature. In the following subsections, various contributions are separately described for different types of nonlinear dynamics.

2.1.3 Coupled nonlinear oscillators

Synchronization of coupled nonlinear oscillators, often described by Kuramoto models, has been extensively studied in physics, chemistry and neuroscience [58]. Synchronization is observed when frequencies of oscillations of individual coupled oscillators converge to a common frequency despite differences in their natural frequencies. Authors of [58] studied the problem of exponential synchronization for coupled phase oscillators described by the Kuramoto model. The closed-loop dynamics of a Kuramoto oscillator is given by:

$$\dot{x}_i = \omega_i + \frac{K}{N} \sum_{j=1, j \neq i}^{N} sin(x_j - x_i) \tag{2.4}$$

where x_i is the phase and ω_i is the natural frequency of the ith oscillator, N is the number of oscillators, and K is the control gain. Oscillators can be considered as linked through a *strongly connected graph*, where each oscillator has access to any other oscillator in the group. Generally, the control gain K plays a crucial role in determining the synchronizability of the network. In [59], the problem was further studied with consideration of the effect of time-delay

2.1.4 Euler-Lagrange (EL) systems

A vast majority of physical systems can be modelled as Euler-Lagrange systems. The systems covered by EL models include mechanical, electromechanical, power electronics and a number of chemical systems. Some examples of the EL autonomous systems are robotic manipulators, unmanned aerial, ground and underwater vehicles. EL systems are generally described as

$$M_i(x_i)\ddot{x}_i + C_i(x_i, \dot{x}_i)\dot{x}_i + g_i(x_i) = \tau_i \tag{2.5}$$

where $x_i \in \Re^n$ is the vector of generalized coordinates, $M_i(x_i) \in \Re^{n\times n}$ is the symmetric positive-definite inertia matrix, $C_i(x_i, \dot{x}_i)x_i \in \Re^n$ is the vector of coriolis and centrifugal torques, $g_i(x_i) \in \Re^n$ is the vector of gravitational torques, and $\tau_i \in \Re^n$ is the vector of torques produced by the actuators associated with the ith agent. An inherent property of EL systems is that $\dot{M}_i(x_i) - 2C_i(x_i, \dot{x}_i)$ is skew-symmetric which implies that $z^T\left(\dot{M}_i(x_i) - 2C_i(x_i, \dot{x}_i)\right)z = 0, \forall z \in \Re^n$. This property plays a vital role in the Lyapunov function based controller design. In [60], the passivity-based output synchronization of EL systems was studied, for both, fixed and switching topologies. In [61], a distributed cooperative tracking control of a group of EL systems networked through an undirected graph topology is presented. In this paper the authors also considered the uncertainties in Linear-in-Parameter (LIP) form. A neural network-based distributed adaptive tracking control is presented in [62] for EL systems with completely unknown dynamics. Results of [62] are stronger than those in [61] in two aspects. Firstly, LIP assumption is not required in the controller design of [62]. Secondly, in [62] the controller is designed for a directed graph topology, as compared to the undirected graph topology considered in [61].

2.1.5 Nonholonomic mobile robots

The dynamics are described by

$$\dot{x}_i = u_i cos(\theta_i),\ \dot{y}_i = u_i sin(\theta_i),\ \dot{\theta}_i = \omega_i, \quad i \in \mathcal{N} \tag{2.6}$$

where x_i, y_i denote the location of the ith agent, and u_i and ω_i denote, respectively, its translational and rotational velocity. Note that there are three states and two control inputs. Therefore, the dynamics for nonholonomic mobile robots are underactuated. This poses substantial difficulties in designing proper consensus algorithms with corresponding stability analysis. Different facets of cooperative control of non-holonomic systems are studied in [63], [64], [65] and [66].

In [63], a decentralized feedback control strategy that drives a system of multiple nonholonomic unicycles to a rendezvous point in terms of both position and orientation is presented. The proposed nonholonomic control law is discontinuous and time-invariant. Stability of the overall system is examined using the tools from non-smooth Lyapunov theory and graph theory. In [64], the problem of output consensus for multiple non-holonomic systems in chained form is investigated for general directed graph containing a spanning tree. Authors of [65] investigated finite-time distributed tracking control problem of multiple non-holonomic mobile robots via visual servoing with unknown camera parameters. in [66], the authors studied the rendezvous problem of multiple nonholonomic unicycle-type robots interacting through unidirectional ring topology.

2.1.6 Nonlinear systems in Brunovsky canonical form (BCF)

Nonlinear systems in BCF are described as

$$\begin{aligned} \dot{x}_{i,m} &= x_{i,m+1}, \qquad m = 1, 2, ..., M-1 \\ \dot{x}_{i,M} &= f_i(x_i) + g_i(x_i)u_i \end{aligned} \tag{2.7}$$

where M is the system order. As mentioned in Section 1.2.3, several physical systems can be described in BCF either directly or through a transformation.

A series of papers, [34] - [36], have considered cooperative tracking control of BCF nonlinear systems. [34], [35] and [36] have, respectively, considered distributed cooperative tracking control of first-order, second-order and higher-order BCF nonlinear systems networked through directed graphs. In all of these three papers, the dynamics of the agents are completely unknown. The individual agents can have different dynamics of the same order. The authors used neural network (NN) structures to compensate for the unknown dynamics in the proposed controller. Furthermore, these papers assume $g_i(x_i)$ to be unity. As one of its major goals, this thesis presents a robust NN-based distributed adaptive cooperative tracking controller for general higher-order affine nonlinear systems in BCF, allowing $g_i(x_i)$ to be a smooth function of states x_i. Two separate NNs are used to estimate $f_i(x_i)$ and $g_i(x_i)$. As compared to [34] - [36], the controller proposed in this thesis is more general in the sense that the proposed controller can be applied to the systems considered in [34] - [36], but not vise versa, because $g_i(x_i)$used here is a more general function that also incorporates the unity value used in [34] - [36].

2.1.7 Port Controlled Hamiltonian systems

An autonomous PCH system is generally described as

$$\begin{aligned} \dot{x}_i &= [J_i(x_i) - R_i(x_i)]\frac{\partial H_i}{\partial x_i}(x_i,t) + g_i(x_i)u_i \\ y_i &= g_i(x_i)^T\frac{\partial H_i}{\partial x_i}(x_i,t) \end{aligned} \tag{2.8}$$

where $x_i \in \Re^n$ is the state vector. Positive semi-definite function $H_i(x_i) : \Re^n \mapsto \Re$ is the storage function and is called the Hamiltonian. The column vector $\frac{\partial H_i(x_i,t)}{\partial x_i} = [\frac{\partial H_i(x_i,t)}{\partial x_{i,1}} ... \frac{\partial H_i(x_i,t)}{\partial x_{i,n}}]^T \in \Re^n$ denotes the gradient of scalar function $H_i(x_i)$. Skew-symmetric matrix $J_i(x_i) = -J_i(x_i)^T \in \Re^{n\times n}$ and $g_i(x_i) \in \Re^{n\times m}$ collectively define the *interconnection* structure of the system. $R_i(x_i) = R_i(x_i)^T \geq 0 \in \Re^{n\times n}$ represents the *dissipation*. All these matrices may smoothly depend on x_i. The literature survey suggests that cooperative control of PCH systems has not been studied so far and will be considered for the first time in this thesis.

2.2 Passivity Theory

Passivity theory is a well-established tool for the analysis and design of nonlinear control systems. The passivity theory circles around *passive systems*, which are physical systems in which the energy consumption over any period of time is greater than or equal to the increase in the energy stored in the system over the same period [19]. A controller design scheme which exploits the *passivity* property of a dynamical systems is called the *passivity-based control* (PBC), first introduced in [67]. Since PCH systems are passive by nature, and since most of the seminal results on the control of PCH systems are based on the passivity theory, it is important to describe the basic definitions of passivity theory.

Definition 2.1 ***(Passivity of Static Nonlinearity)*** *[68].*

A static nonlinearity $y = h(u)$, where $h : \Re^p \mapsto \Re^p$, is passive if, for all $u \in \Re^p$,

$$u^T y = u^T h(u) \geq 0 \tag{2.9}$$

and strictly passive if (2.9) holds with strict inequality $\forall u \neq 0$.

Definition 2.2 ***(Passivity of Dynamical Systems)****[19].*

The dynamical system

$$\sum : \begin{cases} \dot{x} = f(x,u) \\ y = h(x,u), \quad x \in \Re^n, \; u, y \in \Re^p \end{cases} \tag{2.10}$$

is said to be passive if there exists a continuously differentiable positive semidefinite function $V(x)$ called the storage function such that

$$\dot{V} = \frac{\partial V}{\partial x}^T f(x,u) \leq u^T y, \quad \forall (x,u) \in \Re^n \times \Re^p \tag{2.11}$$

Moreover, it is said to be

- *lossless if $u^T y = \dot{V}$.*
- *input-feedforward passive if $u^T y \geq \dot{V} + u^T \varphi(u)$ for some function φ.*
- *input strictly passive if $u^T y \geq \dot{V} + u^T \varphi(u)$ and $u^T \varphi(u) > 0, \quad \forall u \neq 0$.*
- *output-feedback passive if $u^T y \geq \dot{V} + u^T \rho(y)$ for some function ρ.*
- *output strictly passive if $u^T y \geq \dot{V} + u^T \rho(y)$ and $u^T \rho(y) > 0, \quad \forall y \neq 0$.*
- *strictly passive if $u^T y \geq \dot{V} + u^T \psi(x)$ for some positive definite function ψ.*

In all cases, the inequality should hold for all (x, u).

2.2.1 Passivity and stability

A very attractive feature of the passivity theory is its appealing link with the Lyapunov stability, i.e. *a system is at least Lyapunov stable if it is passive.* The following two important properties follow from the fact that the storage function is non positive along the state trajectories.

1. If $u = 0$ in (2.11) then

$$\dot{V} \leq 0 \tag{2.12}$$

   Thus $x = 0$ is a Lyapunov stable equilibrium point.

2. If $y = 0$, then Eq. (2.12) still holds. Thus the *zero dynamics* of the system is Lyapunov stable.

Moreover, asymptotic stability of passive systems is followed by the following definition and lemma.

Definition 2.3 ***Zero State Observability***[19] *The system (2.10) is said to be zero-state observable if no solution of* $\dot{x} = f(x, 0)$ *can stay identically in* $S = \{x \in \Re^n | h(x, 0) = 0\}$*, other than the trivial solution* $x(t) = 0$*.*

Lemma 2.1 ***Asymptotic stability of passive systems [19]***

Consider the system (2.10). The origin of $\dot{x} = f(x, 0)$ *is asymptotically stable if the system is*

- *strictly passive*
- *output strictly passive and zero-state observable.*

Furthermore, if the storage function is radially unbounded, then the origin will be globally asymptotically stable.

2.2.2 Output feedback stabilization of passive systems

In Sec. 2.2.1, the stability of passive systems was discussed. Another attractive feature of passive systems, i.e. the asymptotic stabilization by output feedback, is considered in this section which has been well established in the literature. The global stabilization of passive systems is illustrated in the following theorem.

Theorem 2.1 *[19]. If the system (2.10) is*

1. *passive with a radially unbounded positive definite storage function and*

2. *zero-state observable,*

then the origin $x = 0$ *can be globally stabilized by* $u = -\phi(y)$*, where* ϕ *is any locally Lipschitz function such that* $\phi(0) = 0$ *and* $y^T\phi(y) > 0$ *for all* $y \neq 0$.

Remark 2.1: *Theorem 2.1 gives the basic idea of well-known Passivity based control. A simple choice of* $\phi(y)$ *is* $(-ky)$*, for some* $k > 0$*, which is also called the damping injection.*

2.3 Modeling Examples and Passivity of PCH Systems

A single PCH system is described as

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\frac{\partial H}{\partial x}(x,t) + g(x)u \\ y &= g(x)^T\frac{\partial H}{\partial x}(x,t) \end{aligned} \tag{2.13}$$

with all the variables and parameter matrices as defined in section 2.1.7. The following examples illustrate the fundamental concepts of modeling a system in PCH form.

2.3.1 Example 1: mass spring damper systems

Figure 2.1 shows a typical mass-spring-damper system with mass m, spring constant k and damping coefficient b. Let q be the distance moved by mass. Equation of motion of this system can be obtained by simple force balancing and is given by

$$m\ddot{q} + b\dot{q} + kq = F(t) \tag{2.14}$$

The momentum p of mass m is given by

$$p = m\dot{q} \tag{2.15}$$

In mechanics, q is called the vector of generalized coordinates and p is the momentum vector. In this example q and p are scalers. The Hamiltonian, $H(q, p)$, for this system is defined as:

$$\begin{aligned} H(q,p) &= \text{K.E of the mass} + \text{P.E. of the spring} \\ H(q,p) &= \frac{1}{2}m\dot{q}^2 + \frac{1}{2}kq^2 \\ H(q,p) &= \frac{1}{2m}p^2 + \frac{1}{2}kq^2 \end{aligned} \tag{2.16}$$

Where K.E = Kinetic energy and P.E = Potential energy. The gradient of Hamiltonian $H(q,p)$ is given as:

$$\frac{\partial H(q,p)}{\partial(q,p)} = \begin{bmatrix} \frac{\partial H(q,p)}{\partial q} \\ \frac{\partial H(q,p)}{\partial p} \end{bmatrix} = \begin{bmatrix} kq \\ \frac{p}{m} \end{bmatrix} \tag{2.17}$$

A simple re-arrangement of (2.14)-(2.15) and (2.17) leads to

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \left(\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \right) \begin{bmatrix} \frac{\partial H(q,p)}{\partial q} \\ \frac{\partial H(q,p)}{\partial p} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{2.18}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial H(q,p)}{\partial q} \\ \frac{\partial H(q,p)}{\partial p} \end{bmatrix} \tag{2.19}$$

Note that the port variables i.e. input and out put are, $u = F(t) =$ external force and $y = \dot{q} =$ velocity of mass. Equations (2.18)-(2.19) represent the mass-spring-damper system in PCH form described by (2.13) with $x = \begin{bmatrix} q & p \end{bmatrix}^T$,
$J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $R = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix}$ and $\frac{\partial H(x)}{\partial x} = \frac{\partial H(q,p)}{\partial(q,p)}$
Note that $J = -J^T$ and $R = R^T$.

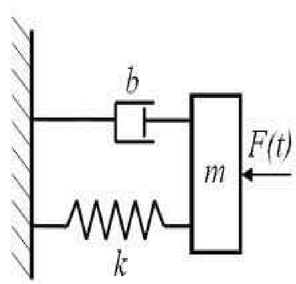

Figure 2.1: Mass-Spring-Damper System

2.3.2 Example 2: DC motor

The schematic diagram of a loaded DC motor is shown in Fig. 2.2. The inductor with inductance L and mass of load with inertia m are the energy storing elements of the system, with flux ϕ on the inductor and momentum p on the load as the energy variables. The Hamiltonian of the system is given by

$$\begin{aligned} H &= \text{Magnetic energy stored by the inductor} + \text{K.E. of the coupled system} \\ &= \frac{\phi^2}{2L} + \frac{p^2}{2m} \qquad (2.20) \end{aligned}$$

Resistance R_m of the circuit and damping b on the load are energy dissipat-

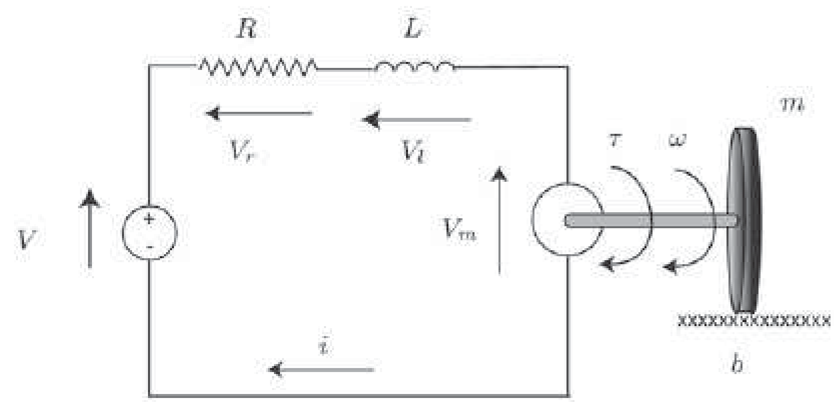

Figure 2.2: Equivalent circuit of a DC motor [18]

ing elements of the system. Let k be the electro-mechanical coupling constant. Application of Kirchoff's Voltage Law (KVL) yields

$$\dot{\phi} = -\frac{kp}{m} - \frac{R\phi}{L} + V \qquad (2.21)$$

The torque balance equation is given by

$$\dot{p} = \frac{k\phi}{L} - \frac{bp}{m} \tag{2.22}$$

The gradient of Hamiltonian $H(\phi, p)$ is given as:

$$\frac{\partial H(\phi,p)}{\partial(\phi,p)} = \begin{bmatrix} \frac{\partial H(\phi,p)}{\partial \phi} \\ \frac{\partial H(\phi,p)}{\partial p} \end{bmatrix} = \begin{bmatrix} \frac{\phi}{L} \\ \frac{p}{m} \end{bmatrix} \tag{2.23}$$

A simple re-arrangement of (2.21)-(2.23) leads to

$$\begin{bmatrix} \dot{\phi} \\ \dot{p} \end{bmatrix} = \left(\begin{bmatrix} 0 & k \\ -k & 0 \end{bmatrix} - \begin{bmatrix} R_m & 0 \\ 0 & b \end{bmatrix} \right) \begin{bmatrix} \frac{\partial H(\phi,p)}{\partial \phi} \\ \frac{\partial H(\phi,p)}{\partial p} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \tag{2.24}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H(\phi,p)}{\partial \phi} \\ \frac{\partial H(\phi,p)}{\partial p} \end{bmatrix} \tag{2.25}$$

The port variables are: $u = V =$ supply voltage and $y = i =$ armature current. The dynamics of the DC motor is represented in PCH form by (2.24) and (2.25) with $x = \begin{bmatrix} \phi & p \end{bmatrix}^T$, $J = \begin{bmatrix} 0 & k \\ -k & 0 \end{bmatrix}$ and $R = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix}$.
Note again that that $J = -J^T$ and $R = R^T$.

2.3.3 Passivity of PCH systems

The passivity property has a strong connection with the nonlinear version of Kalman-Yakubovitch-Popov (KYP) lemma, which is very useful to prove the passivity of PCH systems. At this point, the following nonlinear affine system is considered.

$$\sum_a : \begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x), \quad x \in \Re^n, \ u, y \in \Re^p \end{cases} \tag{2.26}$$

Definition 2.4 ***(KYP property)****[18] A non linear system described by (2.26) enjoys the KYP property if there exists a non negative $\mathcal{C}^1$ function $V : \Re^n \mapsto \Re$ with $V(0) = 0$ such that, for each $x \in \Re^n$*

$$L_f V(x) \leq 0 \tag{2.27}$$

$$L_g V(x) = h^T(x) \tag{2.28}$$

where the Lie derivatives, $L_f V(x)$ and $L_g V(x)$ are given by:

$L_f V(x) = \frac{\partial V(x)}{\partial x} f(x)$ *and*

$L_g V(x) = \frac{\partial V(x)}{\partial x} g(x)$

Note that the notation $\mathcal{C}^r$ denotes a function with r continuous derivatives, and therefore, the $\mathcal{C}^1$ storage function V has at least one continuous derivative. The following proposition relates the KYP property with the passivity of a system.

Proposition 2.1 *[69] If a system enjoys KYP property then it is passive. Conversely, if a system is passive with $\mathcal{C}^1$ storage function then it enjoys the KYP property.* $\diamond$

Passivity of the PCH systems is shown in a very obvious way by utilizing the link between the KYP lemma and passivity, using the following proposition.

Proposition 2.2 *[18] A port-controlled Hamiltonian system is a passive system with hamiltonian as a storage function.*

Proof. Comparison of a PCH system (2.13) with the affine system (2.26), yields

$$
\begin{aligned}
f(x) &= [J(x) - R(x)]\frac{\partial H}{\partial x}(x,t) && (2.29)\\
g(x) &= g(x) && (2.30)\\
h(x) &= g^T(x)\frac{\partial H}{\partial x}(x,t) && (2.31)
\end{aligned}
$$

Exploiting the skew symmetry of $J(x)$ and the positive semidefiniteness of $R(x)$, it is straightforward to show that

$$
L_{[J(x)-R(x)]\frac{\partial H}{\partial x}} H(x) = \frac{\partial^T H}{\partial x}\left[\begin{array}{c} J(x) - R(x) \end{array}\right]\frac{\partial H}{\partial x} = -\frac{\partial^T H}{\partial x} R(x)\frac{\partial H}{\partial x} \leq 0 \quad (2.32)
$$

Furthermore,

$$
L_g H(x) = \frac{\partial^T H}{\partial x} g(x) = \left(g^T(x)\frac{\partial^T H}{\partial x}\right)^T \quad (2.33)
$$

Eqs. (2.32)-(2.33) together with KYP lemma imply the proof of proposition. □

In what follows, most prominent seminal results on stabilization and trajectory tracking control of PCH systems are briefly described.

2.3.4 Interconnection and damping assignment passivity-based control (IDA-PBC) of PCH systems

The objective of IDA-PBC of PCH system (2.13) is to find a state-feedback control $u = \beta(x)$ such that the closed-loop dynamics is again a PCH systems with dissipation of the form

$$\dot{x} = [J_d(x) - R_d(x)]\frac{\partial H_d(x)}{\partial x} \tag{2.34}$$

where the new energy function $H_d(x)$ has a strict local minimum at some desired equilibrium x_*, and, $J_d(x) = -J_d(x)^T$ and $R_d(x) = R_d(x)^T \geq 0$ are some *desired* interconnection and damping matrices, respectively. The following proposition presents the IDA-PBC for the PCH system (2.13).

Proposition 2.3 *[16] Given $J(x,u)$, $R(x)$, $H(x)$, $g(x,u)$ and the desired equilibrium point $x_* \in \Re^n$. Assume we can find functions $\beta(x)$, $J_a(x)$, $R_a(x)$ and a vector function $K(x)$ satisfying*

$$[J(x,\beta(x)) + J_a(x) - \{R(x) + R_a(x)\}]K(x) = -[J_a(x) - R_a(x)]\frac{\partial H(x)}{\partial x} + g(x)\beta(x) \tag{2.35}$$

and such that

(i)(Structure preservation)

$$
\begin{aligned}
J_d(x) &:= J(x,\beta(x)) + J_a(x) = -[J(x,\beta(x)) + J_a(x)]^T \\
R_d(x) &:= R(x) + R_a(x) = [R(x) + R_a(x)]^T \geq 0
\end{aligned}
$$

(ii)(Integrability) $K(x)$ is the gradient of a scalar function. That is,

$$\frac{\partial K}{\partial x}(x) = \left[\frac{\partial K}{\partial x}(x)\right]^T \tag{2.36}$$

(iii)(Equilibrium assignment)$K(x)$ at x_* *verifies*

$$K(x) = -\frac{\partial H}{\partial x}(x_*) \tag{2.37}$$

(iv)(Lyapunov stability) The Jacobian $K(x)$, at x_*, satisfies the bound

$$\frac{\partial K}{\partial x}(x_*) > -\frac{\partial^2 H}{\partial x^2}(x_*) \tag{2.38}$$

Under these conditions, the closed-loop system with $u = \beta(x)$ will be a PCH system with dissipation of the form (2.34), where

$$H_d(x) := H(x) + H_a(x) \tag{2.39}$$

and

$$\frac{\partial H_a}{\partial x}(x) = K(x) \tag{2.40}$$

Furthermore, x_* will be a (locally) stable equilibrium of the closed loop. It will be asymptotically stable if, in addition, the largest invariant set under the closed-loop dynamics contained in

$$\left\{ x \in \Re^n | \frac{\partial H_d^T}{\partial x}(x) R_d \frac{\partial H_d}{\partial x}(x) = 0 \right\} \tag{2.41}$$

equals $\{x_*\}$. An estimate of its domain of attraction is given by the largest bounded level set $\{x \in \Re^n | H_d(x) \leq c\}$. $\diamond$

In literature, (2.35) is also known as *matching condition* for stability [70]. A controller designed on the basis of finding $H_a(x)$ so as to obtain the desired energy function $H_d(x)$, as in (2.39), is also called the *energy shaping* based controller. Additions of $J_a(x)$ and $R_a(x)$ in (2.35) are called, respectively, the interconnection and damping assignment. Addition of $R_a(x)$ in (2.35) is also called *damping injection*. Proposition 2.3 is a standard result for IDA-PBC of PCH systems. Moreover, as mentioned in [16], this IDA-PBC becomes the so called *energy balancing stabilizer* without damping injection i.e. $R_a = 0$ and assuming that the natural damping $R(x)$ verifies

$$R(x)\frac{\partial H_a(x)}{\partial x} = 0 \tag{2.42}$$

2.3.5 Static output-feedback stabilization control of PCH systems

As mentioned earlier, the IDA-PBC technique is based on state-feedback. In what follows, a static-output-feedback stabilization control of PCH systems is presented.

Theorem 2.2 *[23]. (i) Consider the system (2.13). Suppose that the Hamiltonian* $H(x,t)$ *satisfies* $H(x,t) \geq H(0,t) = 0$ *and that* $\frac{\partial H(x,t)}{\partial t} \leq 0$ *holds for all* x *and* t*. Then the input-output mapping* $u \mapsto y$ *of the system is passive with respect to the storage function* $H(x,t)$*, and the feedback*

$$u = -C(x,t)y \tag{2.43}$$

with a matrix $C(x;t) \geq \eta I > 0$ *renders* $(u,y) \to 0$*.*

(ii) Suppose moreover that $H(x,t)$ *is positive definite, and that the system is zero-state detectable. Then the feedback (2.43) renders the system asymptotically stable.*

(iii) Suppose moreover that $H(x,t)$ *is decrescent and that the system is periodic. Then the feedback (2.43) renders the system uniformly asymptotically stable.* $\diamond$

2.3.6 Tracking control of PCH systems

In order to construct an error system of a given port-controlled Hamiltonian system (2.13) and to stabilize it, generalized canonical transformations are introduced in [23]. A generalized canonical transformation is a set of transformations

$$
\begin{aligned}
\bar{x} &= \Phi(x), \\
\bar{H} &= H(x,t) + U(x,t) \\
\bar{y} &= y + \alpha(x,t) \\
\bar{u} &= u + \beta(x,t)
\end{aligned}
\tag{2.44}
$$

which preserves the structure of port-controlled Hamiltonian systems with dissipation as described in (2.13). Here $\bar{x}$, $\bar{H}$, $\bar{y}$ and $\bar{u}$ denote the new state, the new Hamiltonian, the new output and the new input, respectively. The generalized canonical transformation is a natural generalization of a canonical transformation which is widely used for the analysis of conventional Hamiltonian systems in classical mechanics. The resultant transformed systems is given by

$$
\dot{\bar{x}} = \left[\bar{J}(\bar{x},t) - \bar{R}(\bar{x},t)\right] \frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x},t) + \bar{g}(\bar{x})\bar{u} \tag{2.45}
$$

$$
\bar{y} = \bar{g}(\bar{x})^T \frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x},t) \tag{2.46}
$$

Definition 2.5 ***:*** *A system described by*

$$
\dot{\bar{x}} = f(\bar{x},u,t), \qquad \bar{x}(t_0) = \psi(x(t_0), x_d(t_0)) \tag{2.47}
$$

with a smooth function $\psi : \Re^n \times \Re^n$, is said to be an error system of (2.13) with *respect to the desired trajectory* $x_d(t)$ *if the following holds for each* $t > t_0$

$$\bar{x}(t) = 0 \Longleftrightarrow x(t) = x_d(t) \tag{2.48}$$

It should be noted at this point that the PCH formalism allows the shaping of both the kinetic as well as the potential energy and at the same time preserves the PCH structure, thus exploiting all the advantages of PBC. This fact will become more evident when the tracking controller is designed for robotic manipulators discussed in Ch. 4. On the other hand, as stated in [16], PBC has been successfully applied to physical systems described by EL equations of motion. However, though still defining a passive operator, the closed-loop is no longer an EL system for applications that require the modification of the kinetic energy, and the storage function of the passive map (which is typically quadratic in the errors) does not have the interpretation of total energy [16].

The following theorem proposes the generalized canonical transformation for construction of structure preserving or structure-invariant error dynamics.

Theorem 2.3 *[23]. (i) Consider the system (2.13). For any functions* $U(x,t) \in \Re$ *and* $\beta(x,t) \in \Re^m$*, there exists a pair of functions* $\Phi(x,t) \in \Re^n$ *and* $\alpha(x,t) \in \Re^m$ *such that the set (2.44) yields a generalized canonical transformation. A function* Φ *yields a generalized canonical transformation if and only if a partial differential* *equation (PDE)*

$$\frac{\partial \Phi}{\partial(x,t)} \begin{pmatrix} [J-R]\frac{\partial H(x)}{\partial x} + [K-S](\frac{\partial H(x)}{\partial x} + \frac{\partial U(x)}{\partial x}) + g\beta \\ -1 \end{pmatrix} = 0 \tag{2.49}$$

holds with a skew-symmetric matrix $K(x,t) \in \Re^n$ *and a symmetric matrix* $S(x,t) \in \Re^n$ *satisfying* $R + S > 0$. *Further the change of output* α *and the* $\bar{g}$ *are given by*

$$\alpha = g^T \frac{\partial U(x)}{\partial x} \tag{2.50}$$

$$\bar{g} = \frac{\partial \Phi}{\partial x} \tag{2.51}$$

$$\bar{J} = \frac{\partial \Phi}{\partial(x,t)}(J+K)\frac{\partial \Phi^T}{\partial(x,t)} \tag{2.52}$$

$$\bar{R} = \frac{\partial \Phi}{\partial(x,t)}(R-S)\frac{\partial \Phi^T}{\partial(x,t)} \tag{2.53}$$

(ii) If the system (2.13) is transformed by the generalized canonical transformation with U, β *and* S *such that* $H + U \geq 0$, *then the new input-output mapping* $\bar{u} \mapsto \bar{y}$ *is passive with the storage function* $\bar{H}$ *if and only if*

$$\frac{\partial(H+U)}{\partial(x,t)} \begin{pmatrix} [J-R]\frac{\partial U(x)}{\partial x} - S(\frac{\partial H(x)}{\partial x} + \frac{\partial U(x)}{\partial x}) + g\beta \\ -1 \end{pmatrix} \geq 0 \tag{2.54}$$

(iii) Suppose moreover that (2.54) holds, that $H + U$ *is positive definite, and that the system is zero-state detectable. Then the feedback* $u = \beta - C(x,t)(y+\alpha)$ *with* $C(x,t) \geq \eta I > 0 \in \Re^{m \times m}$ *renders the system asymptotically stable. Suppose*

moreover that $H + U$ is decrescent and the transformed system is periodic. Then the feedback renders the system uniformly asymptotically stable. $\diamond$

2.4 Preliminary Graph Theory

The structure of the information sharing among the members of a group of agents interlinked through a communication network can be well analyzed with the help of graph theory. With the recent development of multi-agent control, graph theory has accordingly evolved with a control theoretic basis. In this section preliminary graph theory is briefly described as necessary in the development of this thesis.

A graph is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with a nonempty finite set of nodes (or vertices) $\mathcal{V} = \{\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_n\}$, and a set of edges (or arcs) $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. $(\mathcal{V}_i, \mathcal{V}_j) \in \mathcal{E}$ if there is an edge from node i to node j. The topology of a weighted graph is often described by the adjacency matrix $A = [a_{ij}] \in \Re^{N \times N}$ with weights $a_{ij} > 0$ if $(\mathcal{V}_j, \mathcal{V}_i) \in \mathcal{E}$: otherwise $a_{ij} = 0$. Throughout this thesis, the topology is fixed, i.e. A is time-invariant, and the self-connectivity element $a_{ii} = 0$. A graph can be directed or undirected. A directed graph is called diagraph. The weighted in-degree of a node i is defined as the sum of i-th row of A, i.e., $d_i = \sum_{j=1}^{N} a_{ij}$. Define the diagonal in-degree matrix $D = diag(d_1, ...d_N) \in \Re^{N \times N}$ and the graph Laplacian matrix $L = D - A$. The set of neighbors of a node i is $N_i = \{j | (\mathcal{V}_j \times \mathcal{V}_i) \in \mathcal{E}\}$. If node j is a neighbor of node i, then node i can get information from node j , but not necessarily vice versa. For undirected graph, neighborhood is a mutual relation. A direct path from node i to node j is a sequence of successive edges in the form $\{(\mathcal{V}_i, \mathcal{V}_k), (\mathcal{V}_k, \mathcal{V}_l), ..., (\mathcal{V}_m, \mathcal{V}_j\}$. A diagraph has a spanning tree, if there is a node (called the root), such that there is a directed path from the root to every other node in the graph. A diagraph is strongly connected, if for any ordered pair of nodes $[\mathcal{V}_i, \mathcal{V}_j]$ with $i \neq j$, there is a directed path from node i to node j [2], [35].

CHAPTER 3

NEURO-ADAPTIVE COOPERATIVE TRACKING CONTROL OF UNKNOWN HIGHER-ORDER AFFINE NONLINEAR SYSTEMS

In this chapter a practical design method for distributed cooperative tracking control of a class of higher-order nonlinear heterogeneous multi-agent systems is proposed. Dynamics of the agents, (also called the nodes,) are assumed to be unknown to the controller and are estimated using Neural Networks. Linearisation-based robust neuro-adaptive controller driving the follower nodes to track the trajectory of the leader node is proposed. The nodes are connected through a weighted directed graph with a time-invariant topology. In addition to the fact that only few nodes have access to the leader, communication among the follower nodes is limited with some nodes having access to the information of their neighbor nodes only. Command generated by the leader node is ultimately followed by the followers with bounded synchronization error. The proposed controller is well-defined in the sense that the control effort is restrained to practical limits. The closed-loop system dynamics are proved to be stable and simulations results demonstrate the effectiveness of the proposed control scheme.

3.1 Introduction

In the available studies, much attention has been paid to the cooperative tracking control of single and double integrator and general linear systems [15] and references therein. On the other hand, due to the vast variety of the dynamics of nonlinear systems, research in the cooperative control of nonlinear system is still worthy of the attention of researchers in the control community. This gap in the knowledge has been highlighted in many papers (see for example: [71], [72] and [73]).

Recently a series of papers [34] - [36], addressed the cooperative tracking of nonlinear systems with unknown dynamics using neuro-adaptive feedback linearisation technique. The first two papers considered first and second order systems, respectively. On the other hand, [36] considered higher-order systems described in Brunovsky form and connected through a directed graph, in which access to leader is limited to a few follower nodes. The *input function*, $g_i(\cdot)$ for each agent i, $i = 1, \ldots, N$, is assumed to be known and equal to unity. Such assumptions on the input function restrict the application of the developed distributed controller to a relatively small class of systems. Indeed, in real-life applications, the actuator input function is different from unity, nonlinear, and depends on the system's states. To avoid the complexity introduced by the actuator dynamics, many published studies ignore the actuator and assume that the input to the system is directly the thrust generated by the different propulsion units. In addition to the fact that the manipulated variables are usually the supplied power, fuel flow rate, or voltage, the thrust generated by the power system depends on different factors such as altitude, forward and relative airspeed, engine temperature, etc. Assuming $g_i(\cdot) = 1$ introduces a discrepancy between both theoretical and in-field performances.

The goal of this chapter is to propose a new controller that allows the input function to be an unknown function of the system states. Thus, the proposed results cover a broader class of systems and are closer to real-life applications than any published results.

It is argued that the contribution of this chapter is nontrivial in two major aspects. Firstly, due to an additional estimation of the input nonlinearities, the controller structure is much different from the one proposed in [36] and the stability analysis is performed in different regions of the controller's operating space as a function of the magnitude of the $g_i(\cdot)$ estimate. Secondly, the centralized controller for a single independent system presented in [74] cannot be directly used in the present case because in this chapter a networked multi-agent system is considered and the associated controller is distributed in nature, i.e. every node has access to the state information of its neighbor nodes only. Furthermore it is important to emphasize that the controller design methodology presented in this chapter is more generic and equally applicable to the systems considered by [36] and therefore to [34] and [35], while the converse is not true.

In the feedback linearisation based adaptive control of single nonlinear systems, the estimate of $g(x)$ (denoted by $\hat{g}(x)$) appears as a denominator term in the control law. Therefore special attention has to be paid to avoid very high control magnitudes in case of a very small estimate of $g(x)$, or even the control singularity in the case of zero estimate of $g(x)$. This problem has been investigated by several researchers and they proposed a number of solutions [33], [75], [74] and [29]. For a similar problem, recently, an adaptive neuro-fuzzy approach for a single system is proposed in [76], in which the approximation accuracy of the unknown functions depends upon an initial estimate of the centers of the output membership functions. These centers are obtained from experts or by off-line techniques based on gathered data [76]. The approach presented in this chapter is inspired from [74] to cope with additional unknown input nonlinearities. This approach does not require any kind of off-line data gathering. However, as will be seen in the sequel, few design parameters need to be evaluated. In addition, adaptive control assumes the unknown part of a system to be linear-in-parameter which is not needed in our proposed approach.Moreover, as mentioned in Sec. 1.2.2, the proposed neuro-adaptive controller is direct, i.e. no off-line training of the NN is required.

Chapter Organization: Sec. 3.1 introduces the cooperative control problem discussed in this chapter. Problem statement is given in Sec. 3.2. The proposed controller design steps are detailed in Sec. 3.3. Simulation examples are presented in Sec. 3.4. The chapter is concluded in Sec. 3.5.

3.2 Problem Statement

Consider a multi-agent system composed of ($N \geq 2$) agents, having distinct Mth order dynamics. The dynamics of the i^{th} agent, ($i = 1, 2, ..., N$), is described in Brunovsky canonical form as

$$\begin{aligned} \dot{x}_{i,m} &= x_{i,m+1}, \qquad m = 1, 2, ..., M-1 \\ \dot{x}_{i,M} &= f_i(x_i) + g_i(x_i)u_i + \xi_i \end{aligned} \tag{3.1}$$

where $x_i = [x_{i,1}, x_{i,2}, ..., x_{i,M}]^T \in \Re^M$ is the state vector of i^{th} agent and $f_i(x_i),\ g_i(x_i)$ are unknown nonlinear functions $\Re^M \rightarrow \Re$, locally Lipschitz in $\Re^M$, with $f_i(0) = 0$. $u_i \in \Re$ is the control input to agent i and $\xi_i \in \Re$ is an unknown bounded external disturbance. Furthermore, we assume that

$$|g_i(x_i)| \geq \underline{g_i} \neq 0, \quad i = 1, ..., N \tag{3.2}$$

where $\underline{g} = min(\underline{g}_1, ..., \underline{g}_N)$ and $\underline{g}_i$ is the assumed known lower bound of $g_i(\cdot)$. The smooth function $g_i(x_i)$ is strictly either positive or negative for all $x_i(t)$. The sign of $g_i(x_i)$ is assumed to be known . Without loss of generality, it is assumed that $g_i(x_i) > 0$. It should be noted that this assumption is required for the current study and as revealed from the literature survey, there is no general approach that analyzes this class of systems without any knowledge of the sign of $g_i(x_i)$. In addition, a simple test in practice can determine if the system is direct-acting ($g_i(x_i) > 0$) or reverse-acting ($g_i(x_i) < 0$).

System (3.1) can be collectively described as

$$\begin{aligned} \dot{x}^m &= x^{m+1}, \quad m = 1, 2, ..., M-1 \\ \dot{x}^M &= f(x) + g(x)u + \xi \end{aligned} \tag{3.3}$$

where , for $m = 1, ..., M$, $x^m = [x_{1,m}, ..., x_{N,m}]^T$,

$f(x) = [f_1(x_1), ..., f_N(x_N)]^T$,

$g(x) = diag\{g_i(x_i), ..., g_N(x_N)\}$, $u = [u_1, ..., u_N]^T$ and $\xi = [\xi_1, ..., \xi_N]^T$. For instance if $M = 3$, then x^1, x^2, x^3 can represent global position, global velocity and global acceleration vectors respectively. On the other hand, the leader node, indexed as node 0, has the following dynamic model:

$$\begin{aligned} \dot{x}_{0,m} &= x_{0,m+1}, \quad m = 1, 2, ..., M-1 \\ \dot{x}_{0,M} &= f_0(t, x_0) \end{aligned} \tag{3.4}$$

where $x_0 = [x_{0,1}, ..., x_{0,m}, ..., x_{0,M}]^T$ is the state vector of the leader such that $x_{0,m} \in \Re$ and $x_0 \in \Re^M$. $f_0(t, x_0(t)) : [0, \infty) \times \Re^M \rightarrow \Re$ is piecewise continuous in t and locally Lipschitz in x_0 with $f_0(t, 0) = 0$ for all $t \geq 0$. It is assumed that system (3.4) is forward complete, which means that for every initial condition, the solution $x_0(t)$ exists for all $t \geq 0$ and there is no finite escape time.

Remark 3.1: *It should be noted at this point that, compared to [36], where g_i is restricted to unity, in this study the dynamics of a node i is further generalized by allowing the function g_i to be a smooth unknown function of state x_i . Thus, the proposed approach to design a distributed cooperative tracking controller for a more generalized multi-agent system.*

The m^{th} order tracking error for node i is defined as $\delta_{i,m} = x_{i,m} - x_{0,m}$. Let $\delta^m = [\delta_{1,m}, ..., \delta_{N,m}]$; then $\delta^m = x^m - \underline{x}_0^m$, where $\underline{x}_0^m = [x_{0,m}, ..., x_{0,m}]^T = \bar{\mathbf{1}}.x_{0,m} \in \Re^N$. The objective is to develop a well-defined distributed control law capable of driving the tracking error δ^m to a small neighborhood of the origin. At this stage, one needs to define *Cooperative Uniform Ultimate Boundedness (CUUB)*, which represents an extension of the standard concept of uniform ultimate boundedness (UUB) to cooperative control systems.

Definition 3.1 *For any m $(m = 1, ..., M)$, the tracking error δ^m is said to be Cooperatively Uniformly Ultimately Bounded (CUUB) if there exists a compact set $\Omega^m \subset \Re^N$ containing the origin, so that $\forall \delta^m(t_0) \in \Omega^m$, there exist a bound B^m and a finite time $T_m(B^m, \delta_1(t_0), ..., \delta_M(t_0))$, such that $\|\delta^m(t)\| \leq B^m, \forall t \geq t_0 + T_m$.*

Thus, if δ^m is CUUB then $x_{i,m}$ is bounded within the neighborhood of $x_{0,m}$, for all $i \in \mathcal{N}$ and $t \geq t_0 + T_m$, thereby rendering all the follower nodes synchronized with the leader. As such, the practical notion of "close enough" synchronization is guaranteed. In this thesis, the scenario in which the individual nodes are connected through a digraph is considered . A particular node can access the state information of its neighbor nodes only. Following [77], the neighborhood synchronization error is defined as:

$$e_{i,m} = \sum_{j \in N_i} a_{ij}(x_{j,m} - x_{i,m}) + b_i(x_{0,m} - x_{i,m}), \tag{3.5}$$

for $m = 1, ..., M$ and where $b_i \geq 0$ is the weight of the edge from the leader node 0 to node i, $(i \in N)$. Note that $b_i > 0$ if and only if there is an edge from the node 0 to node i. Let $e^m = [e_{1,m}, ..., e_{N,m}]^T$, $\underline{f}_0 = \bar{\mathbf{1}}.f_0(t, x_0) \in \Re^N$, and $B = diag(b_1, ..., b_N) \in \Re^{N \times N}$. Algebraic manipulations of (3.1) and (3.5) lead to

$$\begin{aligned} \dot{e}^m &= e^{m+1}, \quad m = 1, ..., M-1 \\ \dot{e}^m &= -(L + B)(f(x) + g(x)u + \xi - \underline{f}_0), \quad m = M \end{aligned} \tag{3.6}$$

where $L = D - A$ is the Laplacian matrix. Define the augmented graph as $\bar{\mathcal{G}} = \{\bar{\mathcal{V}}, \bar{E}\}$, where $\bar{\mathcal{V}} = \{\nu_0, \nu_1, ..., \nu_N\}$ and $\bar{E} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. The following assumption on the graph topology is necessary.

Assumption 3.1: *There exists a spanning tree in the augmented graph $\bar{\mathcal{G}}$ with the leader node 0 acting as a root node.*

Assumption 3.1 implies that all the follower nodes have access to the leader node either directly or indirectly through directed paths. Without such assumption, there would be at least one follower node that is isolated or acting as a leader, making the synchronization among all of the nodes impossible. The paper [36] enumerates a number of topologies that can verify Assumption 3.1.

The following two lemmas are important in the subsequent stability analysis of the system under consideration. Their proofs can be found in [36].

Lemma 3.1 ***Graph Lyapunov Equation*** *[36].*

Define

$$\begin{aligned} q &= [q_1, ..., q_N]^T = (L+B)^{-1}\bar{\mathbf{1}}, \\ P &= diag(p_i) = diag(1/q_i), \\ Q &= P(L+B) + (L+B)^T P \end{aligned} \tag{3.7}$$

Then $P > 0$ *and* $Q > 0$. ◊

Note that the graph Lyapunov equation (3.7) captures the structure of the underlying graph topology.

Lemma 3.2 *[36].*

$\|\delta^m\| \le \|e^m\|/\underline{\sigma}(L+B), \quad m = 1, ..., M.$ ◊

Lemma 3.4 relates the convergence of δ^m to that of e^m. The distributed cooperative controller is designed to guarantee $e^m \to 0$, which will imply that $\delta^m \to 0$ and thus $x_{i,m} \to x_{0,m}$.

3.3 Neuro-Adaptive Distributed Cooperative Controller design

In this section, the design of the distributed adaptive neural network controllers for the follower nodes is presented. The stability analysis of the proposed control law and tuning rules for the NN weights are explained. The development of these tasks is carried out as follows:

- The sliding mode error is formulated for each node as well as for the whole network.
- Then it is explained how the two sets of neural networks are used to estimate the unknown nonlinear functions $f_i(x_i)$ and $g_i(x_i)$ for each follower node i.
- The distributed control design and the tuning rules for the NN weights are then presented.
- Existence of solution and boundedness of the discontinuous control input is shown. In addition, it will be shown that under the switching conditions, the control input remains smooth and well-defined.
- Stability of the closed-loop dynamics is proved.

3.3.1 The sliding mode error

Define the sliding mode error r_i for node i ($i \in \mathcal{N}$) as

$$r_i = \lambda_1 e_{i,1} + \lambda_2 e_{i,2} + ... + \lambda_{M-1} e_{i,M-1} + e_{i,M} \tag{3.8}$$

where $\lambda_1, ..., \lambda_{M-1}$ are design constants chosen such that the resulting sliding manifold at $r_i = 0$ is stable, and $e_i = [e_{i,1},\ e_{i,2},\ \ldots, e_{i,M}]^T \ \rightarrow 0$ exponentially as $t \rightarrow \infty$. The proposed controller aims at keeping the individual sliding mode error r_i within a close neighborhood of the sliding manifold. The collective sliding mode error for the group is defined as $r = [r_1, ..., r_N]^T$;

$r = \lambda_1 e^1 + \lambda_2 e^2 + ... + \lambda_{M-1} e^{M-1} + e^M$.

Define $E_1 = [e^1, ..., e^{M-1}]^T \in \Re^{N \times (M-1)}$,

$E_2 = \dot{E}_1 = [e^2, ..., e^M]^T,\ l = [0, ..., 0, 1]^T \in \Re^{M-1}$, and

$\Lambda = \left[\begin{array}{c|c} 0 & I \\ \hline -\lambda_1 & -\lambda_2 ... -\lambda_{M-1} \end{array}\right] \in \Re^{(M-1)\times(M-1)}$.

which leads to

$$E_2 = E_1 \Lambda^T + r l^T \tag{3.9}$$

Since Λ is Hurwitz, then given any positive number β, there exists a matrix $P_1 > 0$, such that the following Lyapunov equation holds.

$$\Lambda^T P_1 + P_1 \Lambda = -\beta I \tag{3.10}$$

The dynamic equation of the sliding mode error r takes the form

$$\dot{r} = \rho - (L + B)(f(x) + g(x)u + \xi - \underline{f}_0) \tag{3.11}$$

where

$$\rho = \lambda_1 e^2 + \lambda_2 e^3 + ... + \lambda_{M-1} e^M = E_2 \bar{\lambda} \tag{3.12}$$

with $\bar{\lambda} = [\lambda_1, ..., \lambda_{M-1}]^T$. The goal of the stability analysis is to prove that the sliding mode error r_i is ultimately bounded. The following lemma shows that (ultimate) boundedness of r_i implies (ultimate) boundedness of $e_i, \forall i \in \mathcal{N}$.

Lemma 3.3 *[36]. For all $i = 1, ..., N$, suppose*

$|r_i(t)| \le \psi_i, \quad \forall t \ge t_0$

$|r_i(t)| \le \zeta_i, \quad \forall t \ge T_i$

for some bounds $\psi_i > 0$, $\zeta_i > 0$, and time $T_i > t_0$. Then there exist bounds $\Psi_i > 0$, $\Xi_i > 0$ and time $\Delta_i > t_0$, such that

$|e_i(t)| \le \Psi_i, \quad \forall t \ge t_0$

$|e_i(t)| \le \Xi_i, \quad \forall t \ge \Delta_i$. $\diamondsuit$

3.3.2 Approximation of $f_i(x_i)$ and $g_i(x_i)$ using NNs

Two neural networks are used to approximate $f_i(x_i)$ and $g_i(x_i)$. Assume that on a compact set $\Omega \subset \Re^M$ there exist ideal weights so that

$$
\begin{aligned}
f_i(x_i) &= W_{f_i}^T \phi_{f_i}(x_i) + \epsilon_{f_i} \\
g_i(x_i) &= W_{g_i}^T \phi_{g_i}(x_i) + \epsilon_{g_i}
\end{aligned}
\tag{3.13}
$$

where for $\ell \in \{f_i,\ g_i\}$, $W_\ell \in \Re^{v_i}$ is the ideal neural network weight vector, $\phi_\ell \in \Re^{v_i}$, is a suitable set of basis functions, and ϵ_j represents a functional approximation error such that

$$
\begin{aligned}
\|\epsilon_{f_i}\| &\leq \epsilon_{f_i N} \\
\|\epsilon_{g_i}\| &\leq \epsilon_{g_i N}
\end{aligned}
\tag{3.14}
$$

where $\epsilon_{f_i N}$ and $\epsilon_{g_i N}$ are known bounds.

Functional Link NN

The NNs used for the approximation of f_i and g_i belongs to a special class of NNs called the *Functional Link NN (FLNN)*, which is a simplification of two layer NN with the input layer replaced by a matrix of fixed weights, i.e. $\phi_j(x_i) = \sigma_j V_j^T(x_i)$. V is a matrix of fixed weights of the input (first) layer. These NNs are *Linear In Parameter (LIP)* and can be easily trained compared to the classical two-layer NNs. As shown in [78], for randomly selected V_j, the function $\phi_j(x_i)$ is a basis, and the resulting *Random Vector Functional Link (RVFL)* NN achieves the universal approximation property. In these NNs, $\sigma(\cdot)$ can be the standard sigmoid function.

Practically one cannot achieve neither the ideal NNs weights nor the zero functional approximation errors. Therefore estimates of f_i's and g_i's are defined as

$$\begin{aligned}
\hat{f}_i(x_i) &= \hat{W}_{f_i}^T(t)\phi_{f_i}(x_i) \\
\hat{g}_i(x_i) &= \hat{W}_{g_i}^T(t)\phi_{g_i}(x_i)
\end{aligned} \tag{3.15}$$

where $\{\hat{W}_{f_i}(t), \hat{W}_{g_i}(t)\} \in \Re^{v_i}$ are the current weights of the corresponding two NNs at node i. These weights are tuned with local state information only, as will be described subsequently.

Define $W_f = diag(W_{f_1}, ..., W_{f_N}), \hat{W}_f = diag(\hat{W}_{f_1}, ..., \hat{W}_{f_N})$,
$\epsilon_f = [\epsilon_{f_1}, ..., \epsilon_{f_N}]^T, \phi_f = [\phi_{f_1}^T, ..., \phi_{f_N}^T]^T$,
$W_g = diag(W_{g_1}, ..., W_{g_N}), \hat{W}_g = diag(\hat{W}_{g_1}, ..., \hat{W}_{g_N})$,
$\epsilon_g = diag(\epsilon_{g_1}, ..., \epsilon_{g_N}), \phi_g = diag(\phi_{g_1}^T, ..., \phi_{g_N}^T)$. Note that due to the diagonal structure of $g(x)$, the structure of ϕ_g and ϵ_g is also diagonal, which is not the case for ϕ_f and ϵ_f. The global nonlinearities $f(x)$ and $g(x)$ and their approximations can be written as

$$f(x) = W_f^T\phi_f(x) + \epsilon_f \tag{3.16}$$

$$g(x) = W_g^T\phi_g(x) + \epsilon_g \tag{3.17}$$

$$\hat{f}(x) = \hat{W}_f^T \phi_f(x) \tag{3.18}$$

$$\hat{g}(x) = \hat{W}_g^T \phi_g(x) \tag{3.19}$$

Consequently, the associated NNs weights estimation errors are $\tilde{W}_f = W_f - \hat{W}_f$ and $\tilde{W}_g = W_g - \hat{W}_g$.

Remark 3.2: *Let* $\phi_{f_iM} = \max_{x_i \in \Omega} \|\phi_{f_i}(x_i)\|$, $W_{f_iM} = \|W_{f_i}\|$ *and* $\phi_{g_iM} = \max_{x_i \in \Omega} \|\phi_{g_i}(x_i)\|$, $W_{g_iM} = \|W_{g_i}\|$. *Then following the definitions of* ϕ_j, W_j *and* ϵ_j *for* $j \in \{f, g\}$, *there exist positive numbers* ϕ_{jM}, W_{jM} *and* ϵ_{jM}, *such that* $\|\phi_j\| \leq \phi_{jM}, \|W_j\| \leq W_{jM}$ *and* $\|\epsilon_j\| \leq \epsilon_{jM}$.

3.3.3 Controller design

In this section, a new approach to design a neuro-adaptive distributed controller based on the estimation of the system's nonlinear dynamics using NNs is presented. The tuning rules of the NN weights are also addressed. Before proceeding to the controller design, the following assumptions are made.

Assumption 3.2:

1. *There exists a positive number* $X_M > 0$ *such that* $\|x_0(t)\| \leq X_M, \forall\ t \geq t_0$.
2. *There exists a continuous function* $\varphi(\cdot) : \Re^M \to \Re$, *such that* $|f_0(t, x_0)| \leq |\varphi(x_0)|, \forall\ x_0 \in \Re^M, \forall\ t \geq t_0$.
3. *For each node* i, *the disturbance* ξ_i *is unknown but bounded. Or, equivalently, the overall disturbance vector* ξ *is bounded by* $\|\xi\| \leq \xi_M$ *where* ξ_M *can be unknown.*

4. *There exist positive numbers* $g_M > 0$ *and* $g_m > 0$ *such that* $\|g(x)\| \leq g_M, \forall x \in \Omega_g$ *and* $\|\hat{g}(x)\| \leq g_m, \forall x \in \Omega_g$, *respectively, where* $\Omega_g = \{x \in \Re^M | \|x\| \leq B_g\}$ *and* B_g *is a finite positive number.*

Remark 3.3: *The nonlinearity of the leader node,* $f_0(x_0)$ *is known a priori in most of the practical scenarios making Assumption 3.2(1) reasonable. One can infer from Assumption 3.2(2) , that there exists a positive number* F_M *such that* $|f_0(t, x_0)| \leq F_M, \forall x_0 \in \Omega_0$ *and* $\forall t \geq t_0$, *where* $\Omega_0 = \{x_0 \in \Re^M \|x_0\| \leq X_M\}$. *Assumption 3.2(4) can be deduced from equation (3.17) and Remark 3.2.*

Assumption 3.2(1, 3, 4) and Remarks 3.2 and 3.3 have bounded several quantities by arbitrary constants $X_M, \xi_M, F_M, W_{fM}, W_{gM}, \epsilon_{fM}, \epsilon_{gM}, g_M$ and g_m. However, the bounds are not needed in the controller design and therefore their computation is not necessary. These bounds are only required for the stability analysis. The bounds Φ_{fM} and Φ_{gM} can be explicitly expressed, because we can choose some squashing functions, such as sigmoids, Gaussians, and hyperbolic tangents, as the basis set.

3.3.4 Distributed control law

The control law u_i at node i is composed of two components, namely u_{ci} and u_{di}, such that

$$u_i = u_{ci} + u_{di} \tag{3.20}$$

u_{ci} mainly accounts for the cancellation of the nonlinearities f_i and g_i. Its structure can be adequately selected as:

$$u_{ci} = \frac{1}{\hat{g}_i}(-\hat{f}_i + \nu_i) \tag{3.21}$$

where

$$\nu_i = cr_i + \frac{1}{d_i + b_i}(\lambda_i e_{i,2}, ..., \lambda_{M-1} e_{i,M}) \tag{3.22}$$

and

$$c > \frac{2}{\underline{\sigma}(Q)} \left(\frac{\gamma_f^2 + \gamma_g^2}{\kappa} + \frac{2}{\beta}\varrho^2 + h \right) \tag{3.23}$$

with

$\gamma_f = -\frac{1}{2}\Phi_{fM}\bar{\sigma}(P)\bar{\sigma}(A)\|$, $\gamma_g = -\frac{1}{2}s\Phi_{gM}\|\bar{\sigma}(P)\bar{\sigma}(A)\|$, $h = \frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|\bar{\lambda}\|$ and $\varrho = -\frac{1}{2}\left(\frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|\Lambda\|_F\|\bar{\lambda}\| + \bar{\sigma}(P_1)\right)$

The structure of u_{ci} shows that its magnitude can become undefined or extremely high if $\hat{g}_i$ becomes zero or very small. This problem invokes the notion of *well-defined controller* which restrains the magnitude of the control action to a feasible limit during these conditions while insuring the stability of the closed loop system.

In the literature, various approaches have been proposed to ensure the boundedness of the control signal. A simple solution can be proposed for a very restricted class of systems by assuming that the estimate $\hat{g}_i$ is constant.

Authors of [79] proposed an adaptive scheme which assumes that the initial estimates are close to the actual values of $g_i(x_i)$ and that they do not leave a feasible invariant set in which $\hat{g}_i \neq 0$. [80] considers these initial estimates within a region of attraction of a stable equilibrium point that forms a feasible set. However, the selection of initial NN weights is difficult, even with a very good knowledge of the system.

Parameter projection is a popular approach to keep $\hat{g}_i$ away from zero by projecting the weights $\hat{W}_{g_i}$ inside an estimated convex subset of the parameter space (i.e. the W_{g_i} space) through the weight tuning law [81]. (A brief discussion on the projection operator can be found in [82] and appendix E of [83]). A possible candidate of such subset was shown to be the set of NN weight vectors $\hat{W}_{g_i}$ with all elements positive. A drawback of this approach is that the actual W_{g_i} do not necessarily belong to this subset.

In [24], an additional smoothly switching control term (u_{di}) is employed to ensure a well-defined control, even when $\hat{g}_i \to 0$. In this study, the approach of [24], which was originally proposed for a single system, is modified for the distributed cooperative tracking control problem at hand. This approach does not require: (i) any assumption on the initial estimate of W_{g_i} and (ii) estimation of a feasible subset of W_{g_i} to project NN weights therein. More recently, [76] employed the so called hopping approach, which is similar to the one proposed in [24].

The control component u_{di} is defined as

$$u_{di} = \begin{cases} \frac{1}{2}(u_{ri} - u_{ci}) \exp^{\gamma(|u_{ci}| - s)}, & \text{If } \mathcal{I}_i = 1 \\ (u_{ri} - u_{ci})(1 - \frac{1}{2}\exp^{-\gamma(|u_{ci}| - s)}), & \text{If } \mathcal{I}_i = 0 \end{cases} \tag{3.24}$$

where $s > 0$ and $\gamma > 0$ are design parameters. The robustifying term u_{ri} is given by

$$u_{ri} = \mu \frac{|\hat{g}_i|}{\underline{g}} |u_{ci}| sgn(r_i), \quad \mu > 0 \tag{3.25}$$

$\mathcal{I}_i$ is the *indicator function*

$$\mathcal{I}_i = \begin{cases} 1, & \text{If } |\hat{g}_i| \geq \underline{g} \text{ and } |u_{ci}| \leq s \\ 0, & \text{Otherwise} \end{cases} \tag{3.26}$$

Using (3.24) in (3.20) gives

$$u_i = \begin{cases} u_{ci} + \frac{u_{ri} - u_{ci}}{2} \exp^{\gamma(|u_{ci}| - s)}, & \text{If } \mathcal{I}_i = 1 \\ u_{ri} - \frac{u_{ri} - u_{ci}}{2} \exp^{-\gamma(|u_{ci}| - s)}, & \text{If } \mathcal{I}_i = 0 \end{cases} \tag{3.27}$$

For simplicity, define

$\hat{g} = diag(\hat{g}_1, ..., \hat{g}_N)$

such that

$\hat{g}^{-1} = diag(\frac{1}{\hat{g}_1}, ..., \frac{1}{\hat{g}_N})$, and

$\nu = [\nu_1, ..., \nu_N]^T = cr + (D + B)^{-1}\rho.$

Collectively

$u_c = [u_{c1}, ...u_{cN}]^T = \hat{g}^{-1}(-\hat{f} + \nu)$

$= \hat{g}^{-1}(-\hat{f} + cr + (D + B)^{-1}\rho)$

such that

$$\nu = \hat{g}u_c + \hat{f} \tag{3.28}$$

Also

$$u_d = \begin{cases} \frac{1}{2}\mathcal{E}_1(u_r - u_c), & \mathcal{I} = 1 \\ (I_N - \frac{1}{2}\mathcal{E}_0)(u_r - u_c) & \mathcal{I} = 0 \end{cases} \tag{3.29}$$

and

$$\begin{aligned} u &= u_c + u_d \\ &= \begin{cases} u_c + \mathcal{E}_1 \frac{u_r - u_c}{2}, & \text{If } \mathcal{I} = 1 \\ u_r - \mathcal{E}_0 \frac{u_c - u_r}{2}, & \text{If } \mathcal{I} = 0 \end{cases} \end{aligned} \tag{3.30}$$

with $u_c = [u_{c1}, ..., u_{cN}]^T$, $u_d = [u_{d1}, ..., u_{dN}]^T$,

$u_r = \mu \frac{1}{\underline{g}} \hat{g} \mathcal{S} |u_c|$,

$\mathcal{S} = diag(sgn(r_1), ..., sgn(r_N))$

$\mathcal{I} = diag(\mathcal{I}_i, ..., \mathcal{I}_N)$

$\mathcal{E}_1 = diag(\exp^{\gamma(|u_{c1}|-s)}), ..., \exp^{\gamma(|u_{cN}|-s)}),$

and

$\mathcal{E}_0 = diag(\exp^{-\gamma(|u_{c1}|-s)}), ..., \exp^{-\gamma(|u_{cN}|-s)}).$

Boundedness of control and existence of solution

The control proposed law switches its structure on the basis of the value of binary indicator $\mathcal{I}_i$. Addition of switching action in the control law has been a common practice in adaptive control theory, to avoid control singularity or very high magnitude (see for instance: [81] , [84], [85]). Literature survey reveals that the existence of solution in the closed-loop feedback systems with switching and discontinuous terms in the control law can be explained using two prominent approaches: (i) Filippov's theory of differential equations with discontinuous right hand side, for example, [84], (ii) modifications to smooth out the switching term in the control or parameter tuning laws. For example, modifications to smoothen the parameter projection can be found in [81] and [85].

The switching of the proposed controller is smoothed out by adding an exponential term in Eq. (3.27), originally proposed in [24]. One should note that the second term on the right hand side of equation (3.27) has been introduced to smooth out the transition from u_{ci} to u_{ri}. The controller operation strategy is that when $\hat{g}_i \geq \underline{g}$ and $|u_{ci}| < s$, the total control action is set to u_{ci}; otherwise control is switched to the auxiliary input u_{ri}. Following (Lewis *et al* 1998), the boundedness of control input and smooth transition between u_{ci} and u_{ri} and vice-versa are guaranteed by the addition of the exponential term $+\exp^{\gamma(|u_{ci}|-s)}$, when switching from u_{ri} to u_{ci} and $-\exp^{-\gamma(|u_{ci}|-s)}$ when switching form u_{ci} to u_{ri}. These exponential terms are necessary when the switching is due to having $\hat{g}_i < \underline{g}$. In the case where the switching is due to the control input $|u_{ci}| \geq s$, the switching occurs at $|u_{ci}| = s$. Therefore both exponential terms on the right hand side of (3.27) are equal to one and $u_i = \frac{1}{2}(u_{ci} + u_{ri})$ before and after the switching. This creates the *hopping* feature of the smoothing scheme as it hops over the expected singularity.

From the above discussion, it can be concluded that the transition of u_i from

u_{ci} to u_{ri} and vice versa is smooth and therefore the existence of the solution is guaranteed. Furthermore, the smoothly switching control (3.27) keeps the control action u_i well defined when $\hat{g}_i \rightarrow 0$. Further details of the existence of solution and boundedness of the control can be found in [24].

3.3.5 NNs tuning rule

In the following, the tuning rules for W_{f_i} and W_{g_i} are proposed.

$$\dot{\hat{W}}_{f_i} = -F_i \phi_{f_i} r_i p_i (d_i + b_i) - \kappa F_i \hat{W}_{f_i} \tag{3.31}$$

$$\dot{\hat{W}}_{g_i} = -\mathcal{I}_i \left[G_i \phi_{g_i} u_{ci} r_i p_i (d_i + b_i) + \kappa G_i \hat{W}_{g_i} \right] \tag{3.32}$$

or collectively,

$$\dot{\hat{W}}_f = -F \phi_f r_d P(D + B) - \kappa F \hat{W}_f \tag{3.33}$$

$$\dot{\hat{W}}_g = -\mathcal{I} \left[G \phi_g u_c r_d P(D + B) + \kappa G \hat{W}_g \right] \tag{3.34}$$

where the design parameters $F_i = F_i^T \in \Re^{(v_i \times v_i)}$ and $G_i = G_i^T \in \Re^{(v_i \times v_i)}$ are arbitrary positive definite matrices and $F = diag(F_1, ..., F_N)$, $G = diag(G_1, \ldots, G_N)$; the tuning gain κ is a positive scalar, P is as defined in Lemma 1, and $u_c = diag(u_{c1}, \ldots, u_{cN})$ and $r_d = diag(r_1, ..., r_N)$. At this stage, it is emphasized that the control law and NNs tuning rules do not require global state information and are implemented using the local state information of node i and its $\mathcal{N}_i$ neighbors, making this cooperative tracking scheme a distributed one. Fig. 3.1 shows the

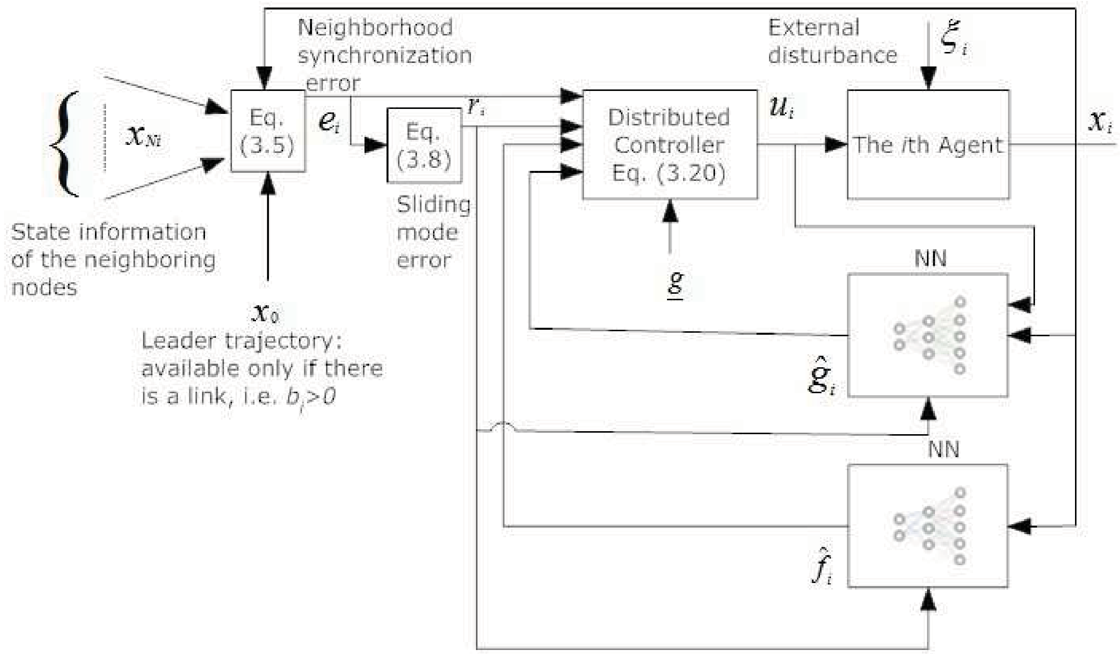

Figure 3.1: The proposed neuro-apative distributed cooperative control scheme

block-diagram representation of the proposed controller.

Remark 3.4: *The proposed approach for controller design is different from [24], in two major aspects. First and most important, the proposed controller is distributed and hence the control law in [24] cannot be applied directly. Second, in the same work, the authors assume a prior knowledge of some design constants, e.g.* c_3 *and* c_4 *relating* r *with some unknown nonlinearity, say* $f(x)$ *as,* $f(x) \leq c_3 + c_4|r|$*. On the other hand, the current approach does not require such constraints.*

Remark 3.5: *The proposed controller at hand can be applied to the type of systems considered in [36], [34], and [35], but not vice versa.*

Before proceeding to the stability analysis, the controller operating space is divided the into four different regions, according to the magnitude of $\hat{g}_i$ and u_{ci} (Table 3.1). However, it should be noted that the ith indicator function $\mathcal{I}_i$ has the

Table 3.1: Regions of controller operation

Region no.	Condition on $\lvert\hat{g}_i\rvert$	Condition on $\lvert u_{ci}\rvert$	Indicator $\mathcal{I}_i$
1	$\lvert u_{ci}\rvert \leq s$	$\lvert\hat{g}_i\rvert \geq \underline{g}$	1
2	$\lvert u_{ci}\rvert \leq s$	$\lvert\hat{g}_i\rvert < \underline{g}$	0
3	$\lvert u_{ci}\rvert > s$	$\lvert\hat{g}_i\rvert \geq \underline{g}$	0
4	$\lvert u_{ci}\rvert > s$	$\lvert\hat{g}_i\rvert < \underline{g}$	0

same value in regions 2-4. Consequently, there is no switching within regions 2-4. Among the four regions in Table 3.1, region 1 is the one in which the controller is expected to operate. However, it is extremely important that the magnitude of the control signal remains within practical limits and preserves closed-loop stability when the controller operates in the other regions (2-4).

Remark 3.6: *It can be easily inferred from the control law structure (3.20) - (3.26) and the tuning rules (3.31) - (3.32) that the magnitude of the control effort is always bounded by some finite limit. At this stage, let's assume that in regions 2-4, $\|u_c\| \leq S_m < \infty$, where $S_m \in \Re$ is unknown but $S_m > s$. Its knowledge for controller design is not required and will be only used in the subsequent stability analysis.*

In the following, stability analysis is performed under the proposed control law, firstly in the region 1, and then in the other regions.

3.3.6 Stability analysis

After adding $[-(L+B)(\nu - \nu)]$ to the sliding mode error dynamics (3.11), and considering (3.28), one has

$$
\begin{aligned}
\dot{r} &= \rho - (L+B)\Big(f(x) + g(x)u + \xi - \underline{f}_0 + \nu - \nu\Big) \\
&= \rho - (L+B)\Big(f(x) + g(x)u + \xi - \underline{f}_0 - \hat{g}u_c - \hat{f} + \nu\Big) \\
&= \rho - (L+B)\Big(f - \hat{f} + (g-\hat{g})u_c + gu_d + \xi - \underline{f}_0 + \nu\Big) \\
&= \rho - (L+B)\Big(f - \hat{f} + (g-\hat{g})u_c + \xi - \underline{f}_0 + cr + (D+B)^{-1}\rho + gu_d\Big)
\end{aligned}
\tag{3.35}
$$

The stability of the cooperative system (3.1) to synchronize with the leader (3.4) under the proposed controller is established in the following theorem.

Theorem 3.1 *Consider the leader-follower system described by (3.1) and (3.4). Suppose that Assumptions 3.1 and 3.2 hold. Then the use of distributed control law described by (3.20)- (3.24), with the NNs tuning rules, (3.33)-(3.34), results into the following:*

1. *The tracking errors $\delta^1, ..., \delta^M$ are cooperatively uniformly ultimately bounded, implying that all nodes in graph $\mathcal{G}$ synchronize to the leader node 0 with bounded residual errors.*

2. *The states $x_i(t)$, $(i = 1, ..., N)$ are bounded $\forall\ t \geq t_0$*

proof: To prove part *1*, one can select the following Lyapunov function candidate

$$V = V_1 + V_2 + V_3 + V_4 \tag{3.36}$$

where $V_1 = \frac{1}{2} r^T P r$, $V_2 = \frac{1}{2} tr\{\tilde{W_f}^T F^{-1} \tilde{W}_f\}$,

$V_3 = \frac{1}{2} tr\{\tilde{W_g}^T G^{-1} \tilde{W}_g\}$ and $V_4 = \frac{1}{2} tr\{E_1 P_1 E_1^T\}$

The stability analysis can be performed region-by-region, as follows:

Region 1: $(|\hat{g}_i| \geq \underline{g}$ and $|u_c| \leq s)$

$$\begin{aligned}
\dot{V}_1 &= r^T P \dot{r} \\
&= r^T P \Big[\rho - (L+B)(f - \hat{f} + (g - \hat{g}_d)u_c + \xi - \underline{f}_0 + cr \\
&\quad +(D+B)^{-1}\rho + g u_d)\Big]
\end{aligned} \tag{3.37}$$

Using, Eqs. (3.16)-(3.19) and definitions of $\tilde{W}_f$ and $\tilde{W}_g$,

$$\begin{aligned}
\dot{V}_1 &= -r^T P(L+B)(\epsilon_f + \epsilon_g u_c + g u_d + \xi - \underline{f}_0) \\
&\quad -c r^T P(L+B) r + r^T P \rho - r^T P(L+B)(D+B)^{-1}\rho \\
&\quad -r^T P(L+B)\tilde{W}_f^T \phi_f - r^T P(L+B)\tilde{W}_g^T \phi_g u_c \\
&= -c r^T P(L+B) r \\
&\quad -r^T P(L+B)(\epsilon_f + \epsilon_g u_c + g u_d + \xi - \underline{f}_0) \\
&\quad -r^T P(D+B)\tilde{W}_f^T \phi_f + r^T P A \tilde{W}_f^T \phi_f \\
&\quad +r^T P A \tilde{W}_g^T \phi_g u_c \\
&\quad -r^T P(D+B)\tilde{W}_g^T \phi_g u_c + r^T P A (D+B)^{-1}\rho
\end{aligned} \tag{3.38}$$

Using the identity, $x^T y = tr\{yx^T\} \forall \{x, y\} \in \Re^N$ and considering (3.7) yields

$$\begin{aligned}
\dot{V}_1 &= -\frac{1}{2} c r^T Q r - r^T P(L+B)(\epsilon_f + \epsilon_g u_c + g u_d + \xi - \underline{f}_0) - tr\{\tilde{W}_f^T \phi_f r^T P(D+B)\} \\
&\quad + tr\{\tilde{W}_f^T \phi_f r^T P A\} - tr\{\tilde{W}_g^T \phi_g u_c r^T P(D+B)\} \\
&\quad + tr\{\tilde{W}_g^T \phi_g u_c r^T P A\} + r^T P A (D+B)^{-1} \rho \qquad (3.39)
\end{aligned}$$

One should note that $\dot{\tilde{W}}_j = \dot{W}_j - \dot{\hat{W}}_j = -\dot{\hat{W}}_j,\ j \in \{f, g\}$, and consider

$$\begin{aligned}
\dot{V}_2 \ &+\ \dot{V}_3 = -tr\{\tilde{W}_f^T F^{-1} \dot{\hat{W}}_f\} - tr\{\tilde{W}_g^T G^{-1} \dot{\hat{W}}_g\} \\
&=\ tr\{\tilde{W}_f^T \phi_f r_d P(D+B) + \kappa \tilde{W}_f^T \hat{W}_f\} \\
&+\ tr\{\tilde{W}_g^T \phi_g u_c r_d P(D+B) + \kappa \tilde{W}_g^T \hat{W}_g\} \qquad (3.40)
\end{aligned}$$

Since in this region $|u_{ci}| \le s$, therefore, $\|u_c\| \le \sqrt{N} s$. Also, define $T_M = \epsilon_f + \epsilon_g s + \xi_M + F_M$ and $\varsigma = g_m / \underline{g}$. Combining (3.39), (3.40) and considering (3.9) and (3.12), one has

$$\begin{aligned}
&\dot{V}_1 + \dot{V}_2 + \dot{V}_3 = -\frac{1}{2} c r^T Q r \\
&- r^T P(L+B)(\epsilon_f + \epsilon_g u_c + \xi - \underline{f}_0) \\
&- r^T P(L+B) g \frac{1}{2} \mathcal{E}_1 (\frac{\mu}{\underline{g}} \mathcal{S} |\hat{g}||u_c| - u_c) \\
&+ \kappa tr\{\tilde{W}_f \hat{W}_f\} + tr\{\tilde{W}_f^T \phi_f r^T P A\} \\
&+ \kappa tr\{\tilde{W}_g \hat{W}_g\} + tr\{\tilde{W}_g^T \phi_g u_c r^T P A\}
\end{aligned}$$

$$
\begin{aligned}
& +r^T PA(D+B)^{-1}E_1\Lambda^T\bar{\lambda} + r^T PA(D+B)^{-1}rl^T\bar{\lambda} \\
\le\ & -\frac{1}{2}c\underline{\sigma}(Q)\|r\|^2 + \bar{\sigma}(P)\bar{\sigma}(L+B)T_M\|r\| \\
& +\frac{1}{2}\bar{\sigma}(P)\bar{\sigma}(L+B)g_M(\frac{\mu}{\underline{g}}\|\hat{g}\|\|u_c\| + \|u_c\|)\|r\| \\
& -\kappa(\|\tilde{W}_f\|_F^2 + \|\tilde{W}_g\|_F^2) \\
& +\bar{\sigma}(P)\bar{\sigma}(A)(\Phi_{fM}\|\tilde{W}_f\|_F + s\Phi_{gM}\|\tilde{W}_g\|_F)\|r\| \\
& +\frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|r\|^2\|l\|\|\bar{\lambda}\| \\
& +\|r\|\frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|E_1\|_F\|\Lambda\|_F\|\bar{\lambda}\| \\
& +\kappa W_{fM}\|\tilde{W}_{fM}\|_F + \kappa W_{gM}\|\tilde{W}_{gM}\|_F \\
\le\ & -\frac{1}{2}c\underline{\sigma}(Q)\|r\|^2 + \bar{\sigma}(P)\bar{\sigma}(L+B)T_M\|r\| \\
& +\frac{1}{2}\sqrt{N}sg_M\bar{\sigma}(P)\bar{\sigma}(L+B)(\mu\varsigma+1)\|r\| \\
& -\kappa(\|\tilde{W}_f\|_F^2 + \|\tilde{W}_g\|_F^2) \\
& +\bar{\sigma}(P)\bar{\sigma}(A)(\Phi_{fM}\|\tilde{W}_f\|_F + s\Phi_{gM}\|\tilde{W}_g\|_F)\|r\| \\
& +\frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|r\|^2\|l\|\|\bar{\lambda}\| \\
& +\|r\|\frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|E_1\|_F\|\Lambda\|_F\|\bar{\lambda}\| \\
& +\kappa W_{fM}\|\tilde{W}_{fM}\|_F + \kappa W_{gM}\|\tilde{W}_{gM}\|_F \\
\le\ & -\frac{1}{2}c\underline{\sigma}(Q)\|r\|^2 + \bar{\sigma}(P)\bar{\sigma}(L+B)T_M\|r\| \\
& +\frac{1}{2}\sqrt{N}sg_M\bar{\sigma}(P)\bar{\sigma}(L+B)(\mu\varsigma+1)\|r\| \\
& -\kappa(\|\tilde{W}_f\|_F^2 + \|\tilde{W}_g\|_F^2) \\
& +\bar{\sigma}(P)\bar{\sigma}(A)(\Phi_{fM}\|\tilde{W}_f\|_F + s\Phi_{gM}\|\tilde{W}_g\|_F)\|r\| \\
& +\frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|r\|^2\|l\|\|\bar{\lambda}\|
\end{aligned}
$$

$$+\|r\|\frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|E_1\|_F\|\Lambda\|_F\|\bar{\lambda}\| \qquad (3.41)$$
$$+\kappa W_{fM}\|\tilde{W}_{fM}\|_F+\kappa\|W_{gM}\|\tilde{W}_{gM}\|_F$$

Now consider,

$$\dot{V}_4 = tr\{E_2 P_1 E_1^T\} \qquad (3.42)$$

Substituting (3.9) into (3.42) and applying (3.10)

$$\begin{aligned}\dot{V}_4 &= -\frac{\beta}{2}tr\{E_1E_1^T\}+tr\{rl^TP_1E_1^T\}\\ &\le -\frac{\beta}{2}\|E_1\|_F^2+\bar{\sigma}(P_1)\|r\|\|E_1\|_F \qquad (3.43)\end{aligned}$$

Thus

$$\dot{V} \le -\left(\frac{1}{2}c\underline{\sigma}(Q)-\frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|\bar{\lambda}\|\right)\|r\|^2-\frac{\beta}{2}\|E_1\|_F^2$$

$$\begin{aligned}&+\left(\bar{\sigma}(P)\bar{\sigma}(A)(\Phi_{fM}\|\tilde{W}_f\|_F+s\Phi_{gM}\|\tilde{W}_g\|_F\right)\|r\|\\ &+\left(\frac{1}{2}\sqrt{N}sg_M\bar{\sigma}(P)\bar{\sigma}(L+B)(\mu\varsigma+1+2\frac{T_M}{\sqrt{N}sg_M})\right)\|r\|\\ &-\kappa(\|\tilde{W}_f\|_F^2+\|\tilde{W}_g\|_F^2)\\ &+\left(\frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|\Lambda\|_F\|\bar{\lambda}\|+\bar{\sigma}(P_1)\right)\|r\|\|E_1\|_F\\ &+\kappa W_{fM}\|\tilde{W}_{fM}\|_F+\kappa W_{gM}\|\tilde{W}_{gM}\|_F \qquad (3.44)\end{aligned}$$

Let $\gamma_f = -\frac{1}{2}\Phi_{fM}\bar{\sigma}(P)\bar{\sigma}(A)\|$,

$\gamma_g = -\frac{1}{2}s\Phi_{gM}\|\bar{\sigma}(P)\bar{\sigma}(A)\|$, $h = \frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|\bar{\lambda}\|$

and $\varrho = -\frac{1}{2}\left(\frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|\Lambda\|_F\|\bar{\lambda}\| + \bar{\sigma}(P_1)\right)$. Rearranging (3.44) leads to

$$\dot{V} \leq -z^T K z + \omega_1^T z = -V_z(z), \tag{3.45}$$

where

$$z = \left[\|E_1\|_F, \|\tilde{W}_f\|_F, \|\tilde{W}_g\|_F, \|r\|\right]^T \tag{3.46}$$

$$\omega_1 = [0, \kappa W_{fM}, \kappa W_{gM}, \Upsilon_1]^T \tag{3.47}$$

with $\Upsilon_1 = \frac{1}{2}\sqrt{N}sg_M\bar{\sigma}(P)\bar{\sigma}(L+B)(\mu\varsigma + 1 + 2\frac{T_M}{\sqrt{N}sg_M})$,

$K = \begin{bmatrix} \frac{\beta}{2} & 0 & 0 & \varrho \\ 0 & \kappa & 0 & \gamma_f \\ 0 & 0 & \kappa & \gamma_g \\ \varrho & \gamma_f & \gamma_g & \theta \end{bmatrix}$ and $\theta = -\frac{1}{2}c\underline{\sigma}(Q) - h$.

Eq. (45) implies that $V_z(z)$ is positive definite if the following two conditions are satisfied:

C1. K is positive definite

C2. $\|z\| > \frac{\|\omega_1\|}{\underline{\sigma}(K)}$.

According to Sylvester's criterion, K is positive definite if

$\beta > 0$

$\beta\kappa > 0$

$\beta\kappa^2 > 0$

$\kappa(\beta\theta - 2\varrho^2) - \beta(\gamma_f^2 + \gamma_g^2) > 0.$

The solution of the above inequalities gives the condition on c described in (3.23). At this stage of the proof, the discussion of condition (C2) is postponed after the remaining regions have been investigated and the corresponding ω_i vectors obtained.

Region 2: $|u_c| < s$ and $|\hat{g}| < \underline{g}$

In the remaining three regions $\dot{\hat{W}}_g = 0$, and therefore

$$\begin{aligned} \dot{V} &= -\frac{1}{2}cr^TQr \\ &\quad -r^TP(L+B)(\epsilon_f + (g-\hat{g})u_c + gu_d + \xi - \underline{f}_0) \\ &\quad +\kappa tr\{\tilde{W}_f\hat{W}_f\} + tr\{\tilde{W}_f^T\phi_f r^TPA\} \\ &\quad +r^TPA(D+B)^{-1}E_1\Lambda^T\bar{\lambda} + r^TPA(D+B)^{-1}rl^T\bar{\lambda} \\ &\quad -\frac{\beta}{2}tr\{E_1E_1^T\} + tr\{rl^TP_1E_1^T\} \end{aligned} \tag{3.48}$$

One can see that in this region as well as in regions 3 and 4, apart from the terms involving g and $\tilde{g}$, the stability analysis essentially remains the same as in region 1. To simplify the analysis, define

$$\begin{aligned} \dot{V}_g &= -r^TP(L+B)(\tilde{g}u_c + gu_d) \\ &= -r^TP(L+B)((g-\hat{g})u_c + gu_d) \\ &= -r^TP(L+B)(-\hat{g}u_c + gu) \end{aligned} \tag{3.49}$$

Substituting the controller corresponding to this region, one has

$$
\begin{aligned}
\dot{V}_g &= r^T P(L+B)\hat{g}u_c - r^T P(L+B)g(u_c+u_d) \\
&= r^T P(L+B)\hat{g}u_c \\
&\quad -r^T P(L+B)g\left(u_r - \frac{1}{2}\mathcal{E}_0(u_r-u_c)\right) \\
&= r^T P(L+B)\hat{g}u_c - \frac{\mu}{\underline{g}} r^T PBg\left(I_N - \frac{1}{2}\mathcal{E}_0\right)|\hat{g}|\mathcal{S}
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{\mu}{\underline{g}} r^T PLg\left(I_N - \frac{1}{2}\mathcal{E}_0\right)|\hat{g}|\mathcal{S} \\
&-\frac{1}{2} r^T P(L+B)g\mathcal{E}_0 u_c
\end{aligned} \tag{3.50}
$$

Since $P>0$, $B\geq 0$, $g>0$, $|\hat{g}|>0$, $\left(I_N - \frac{1}{2}\mathcal{E}_0\right)>0$ and $r^T\mathcal{S}>0$, the term $\frac{\mu}{\underline{g}} r^T PBg\left(I_N - \frac{1}{2}\mathcal{E}_0\right)|\hat{g}|\mathcal{S}$ in (3.50) is always non-negative. Furthermore, the K matrix in regions 2-4 is slightly affected, i.e. $\gamma_g = 0$ maintaining the condition (3.23) on c essentially the same. The variations in $\omega_i, i=1,...,4$, which have no influence on the overall controller structure, represent the only changes.

Define

$\Upsilon_2 = \frac{1}{2}\bar{\sigma}(P)\left((2s\underline{g} + sg_m)\bar{\sigma}(L+B) + \mu\varsigma g_M\bar{\sigma}(L)\right)$,

then

$$
\dot{V}_g \leq \Upsilon_2\|r\| \tag{3.51}
$$

Note that in this case $\gamma_g = 0$

Region 3: $|\hat{g}| > \underline{g}$ and $|u_c| > s$

With straightforward manipulations, it can be shown that, in this region,

$$\dot{V}_g \leq \Upsilon_3 \|r\| \tag{3.52}$$

where

$\Upsilon_3 = \frac{1}{2}\bar{\sigma}(P)\left((2S_m \underline{g} + S_m g_m)\bar{\sigma}(L+B) + \mu\varsigma g_M \bar{\sigma}(L)\right)$,

<u>Region 4:</u>$|\hat{g}| < \underline{g}$ and $|u_c| > s$

Similarly it can be shown that in this region

$$\dot{V}_g \leq \Upsilon_4 \|r\| \tag{3.53}$$

where

$\Upsilon_4 = \frac{1}{2}\bar{\sigma}(P)\left((2S_m \underline{g} + S_m g_m)\bar{\sigma}(L+B) + \mu g_M \bar{\sigma}(L)\right)$,

Define $\Upsilon = max(\Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4)$ and

$\omega = [0, \kappa W_{fM}, \kappa s W_{gM}, \Upsilon]^T$.

Since $\|\omega\|_1 > \|\omega\|$, condition (C2) holds if $\|z\| \geq B_d$ with

$$B_d = \frac{\kappa W_{fM} + \kappa s W_{gM} + \Upsilon}{\underline{\sigma}(K)} \tag{3.54}$$

Therefore, under condition (3.23), we obtain

$\dot{V} \leq -V_z(z), \quad \forall \|z\| \geq B_d$.

with $V_z(z)$ being a positive definite function.

To complete the proof, the same steps are followed as in the proof of Theorem 1 in [36]. Eqs. (3.36) and (4.24) imply that

$$\underline{\sigma}(\Gamma)\|z\|^2 \leq V \leq \bar{\sigma}(T)\|z\|^2, \tag{3.55}$$

where

$\Gamma = diag\left(\frac{\underline{\sigma}(P_1)}{2}, \frac{1}{\bar{\sigma}(F)}, \frac{1}{\bar{\sigma}(G)}, \frac{\underline{\sigma}(P)}{2}\right) \in \Re^{4\times 4}$ and $T = diag\left(\frac{\bar{\sigma}(P_1)}{2}, \frac{1}{\underline{\sigma}(F)}, \frac{1}{\underline{\sigma}(G)}, \frac{\bar{\sigma}(P)}{2}\right) \in \Re^{4\times 4}$. Then following Theorem 4.18 in [19], it can be concluded that for any initial condition $z(t_0)$ (or equivalently $V(t_0)$), there exists a time T_0 such that

$$\|z(t)\| \leq \sqrt{\frac{\bar{\sigma}(T)}{\underline{\sigma}(\Gamma)}}B_d, \quad \forall t \geq t_0 + T_0. \tag{3.56}$$

Define $k = min_{\|z\|\geq B_d} V_z(z)$. Then proceeding essentially same as in [19], it can be shown that

$$T_0 = \frac{V(t_0) - \bar{\sigma}(t)B_d^2}{k} \tag{3.57}$$

The definition of z, (4.24) and (3.56) imply that $r(t)$ is ultimately bounded. Then $r_i(t)$ is ultimately bounded. By Lemma 3.3, $e_i(t)$ is ultimately bounded ($\forall i \in \mathcal{N}$), which implies that $e^m(t)$ ($\forall m = 1, ..., M$) is ultimately bounded. Then, following Lemma 3.2, the tracking errors $\delta^1, ..., \delta^M$ are CUUB and all nodes in graph $\mathcal{G}$ synchronize, in the sense of Definition 3.1, to the trajectory $x_0(t)$ of the leader node.

To prove part *2* of the Theorem, it can be shown that the state $x_i(t)$ is bounded

$\forall i \in \mathcal{N}$ and $\forall t \geq t_0$. Equation (3.45) implies that

$$\dot{V} \leq -\underline{\sigma}(K)\|z\|^2 + \|\omega\|\|z\| \tag{3.58}$$

The combination of (3.55) and (3.58) leads to

$$\frac{d}{dt}(\sqrt{V}) \leq -\frac{\underline{\sigma}(K)}{2\bar{\sigma}(T)}\sqrt{V} + \frac{\|\omega\|}{2\sqrt{\underline{\sigma}(\Gamma)}}$$

Thus $V(t)$ is bounded for all $t \geq t_0$ by Corollary 1.1 in [86]. Since (3.36) implies that $\|r\|^2 \leq \frac{2V(t)}{\underline{\sigma}(P)}$, boundedness of $r(t)$ is guaranteed for all $t \geq t_0$. By Lemmas 3.2 and 3.3, $\delta^m(t)$ is also bounded. Since by definition $\delta^m = x^m - \underline{x}_{0,m}$ and considering Assumption 3.2, $x_m(t), \forall m = 1, ..., M$, is bounded for all t, i.e., $x_i(t)$ is bounded. Equivalently, $x_i(t)$ is contained in a compact set $\Omega_i, \forall i \in \mathcal{N}$ and $t \geq t_0$. This completes the proof. □

Remark 3.7: *Since the existence of ideal weights is assumed for f_i and g_i on a compact set $\Omega \subset \mathcal{R}^M$, our results are semi-global. However, the size of Ω has no impact on the controller, and it can be arbitrarily very large [24]. The results will be global if the ideal weights exist for all $x_i \in \Re^M$.*

3.4 Simulation Examples

In the following, the proposed approach is applied to two different examples. In both examples, five followers nodes have to be synchronized with the leader node, connected through the digraph $\mathcal{G}$ as shown in Fig. 3.2.

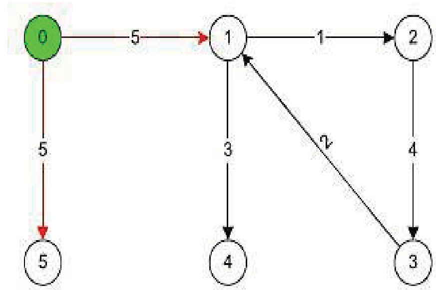

Figure 3.2: Topology of augmented graph $\bar{\mathcal{G}}$

3.4.1 Example 1:

In this example, the dynamics of the leader and all of the followers are similar to those in the example of [36], with the only difference that in this thesis, all $g_i(x_i)$'s are non-unity but unknown functions of the states x_is. The example has been selected to illustrate the performance of the proposed approach. The dynamic model of the leader is

$$\begin{aligned}
\dot{x}_{0,1} &= x_{0,2} \\
\dot{x}_{0,2} &= x_{0,3} \\
\dot{x}_{0,3} &= -x_{0,2} - 2x_{0,3} + 1 + 3sin(2t) + 6cos(2t) \\
&\quad \frac{1}{3}(x_{0,1} + x_{0,2} - 1)(x_{0,1} + 4x_{0,2} + 3x_{0,3} - 1)
\end{aligned}$$

The dynamics of the follower nodes are expressed as in (3.1).

$$\begin{aligned}
\dot{x}_{1,3} &= x_{1,2}sin(x_{1,1}) + cos^2(x_{1,3}) + (0.1 + x_{1,2}^2)u_1 + \xi_1 \\
\dot{x}_{2,3} &= -x_{2,1}x_{2,2} + 0.01x_{2,1} - 0.01x_{2,1}^2 \\
&\quad +(1 + sin^2(x_{2,1}))u_2 + \xi_2
\end{aligned}$$

$$
\begin{aligned}
\dot{x}_{3,3} &= x_{3,2} + sin(x_{3,3}) + (1 + cos^2(x_{3,2}))u_3 + \xi_3 \\
\dot{x}_{4,3} &= -3(x_{4,1} + x_{4,2} - 1)^2(x_{4,1} + x_{4,2} + x_{4,3} - 1) - x_{4,2} \\
&\quad -x_{4,3} + 0.5sin2t + cos(2t) + (1 + 0.5x_{4,2}^2)u_4 + \xi_4 \\
\dot{x}_{5,3} &= cos(x_{5,1}) - x_{5,2} + (1 + x_{5,1}^2)u_5 + \xi_5
\end{aligned}
$$

When initially at rest, the unit step responses of the first three nodes are unstable. The fourth node is stable and similar to the leader node with different parameters. The fifth node has a ramp-like response to unit step input. The unknown disturbances ξ_i's are random but bounded ($|\xi_i| < 1$). In the simulation, each neural network is composed of six neurons. The NN weights are initialized as $W_{f_i}(0) = [0, 0, 0, 0, 0, 0]^T \forall i$ and $W_{g_i}(0) = [1, 1, 1, 1, 1, 1]^T \forall i$ to avoid $\hat{g}_i = 0, \forall i$, initially. Also, the simulations are performed with $\lambda_1 = 1, \lambda_2 = 2, \gamma = 0.05, \underline{g} = 0.01, c = 1000, \mu = 1, \kappa = 0.001, p_i > 0, F_i = G_i = I$. Profiles of the tracking errors (Figs.3.4–3.6) show that they converge to a close vicinity of zero within a small period of time. Fig.3.7 shows the magnitudes of control efforts. Controller switching is shown in Fig.3.8 for the first 100 ms. This switching is indicated by the magnitude of indicator function $\mathcal{I}_i$.

3.4.2 Example 2:

In this example the synchronization of five inverted pendulums (such as the one shown in Fig.3.3) is considered. They are networked through the same graph as in example 1.

Table 3.2: Parameters in Example 2

Node No.	M_i [Kg]	m_i [Kg]	L_i [m]
1	0.8	0.08	0.4
2	0.9	0.09	0.45
3	1	0.1	0.5
4	1.1	0.11	0.55
5	1.2	0.12	0.6

The virtual leader's dynamics are:

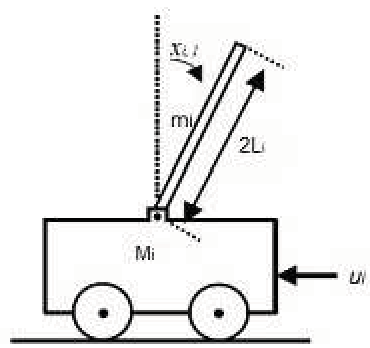

Figure 3.3: Inverted pendulum in example 2.

$\dot{x}_{0,1} = x_{0,2}$

$\dot{x}_{0,2} = -\frac{3\pi^3}{40}cos(2\pi f t)$

where $f = 3/4$ Hz. The follower nodes are described by (3.1) with $f_i(x_i)$ and $g_i(x_i)$, for $i = 1, ..., 5$, defined in [33] and [86]:

$f_i(x_i) = \frac{gsin(x_{i,1}) - 0.5m_iL_ix_{i,2}^2 sin(2x_{i,1})(M_i+m_i)^{-1}}{L_i(\frac{4}{3} - (M_i+m_i)^{-1}mcos^2(x_{i,1}))}$

$g_i(x_i) = \frac{(M_i+m_i)^{-1}cos(x_{i,1})u_i}{L_i(\frac{4}{3} - (M_i+m_i)^{-1}m_icos^2(x_{i,1}))}$

where $x_{i,1}$ and $x_{i,2}$ are the angular position and speed of the pole of the i^{th} pendulum, respectively. Furthermore, M_i and m_i are masses of the cart and pole, respectively, and L_i is half-length of the pole of the i^{th} pendulum. the values of the simulation parameters are given in Table 3.2. It is assumed that the initial angular positions, i.e. $x_{i,1}$ of all the pendulums' poles are much less then $\pi/2$ radians, to ensure that $g_i(x_i) > 0$. The controller and NNs tuning parameters as well as the external disturbances remain the same as in example 1. Figs. 3.9–3.10 show the evolution of the leader and followers' states and the corresponding tracking errors, respectively.

In both of the above examples, though with small residual errors, all of the followers are synchronized with the leader in a short period of time. The system is stable and the control signal is within practical limits.

3.5 Conclusion

The distributed cooperative tracking control of higher order nonlinear multi-agent systems is presented in this chapter. The nonlinearities in the systems' dynamics are completely unknown Lipschitz functions. The proposed controller at a certain node does not require the global state knowledge and works with the state information of neighboring nodes only. The control signal is well defined and ensures system's stability. Two neural networks are used to estimate the unknown system nonlinearities. The leader acts as a command generator and the followers synchronize with the leader in a finite time with a small residual error.

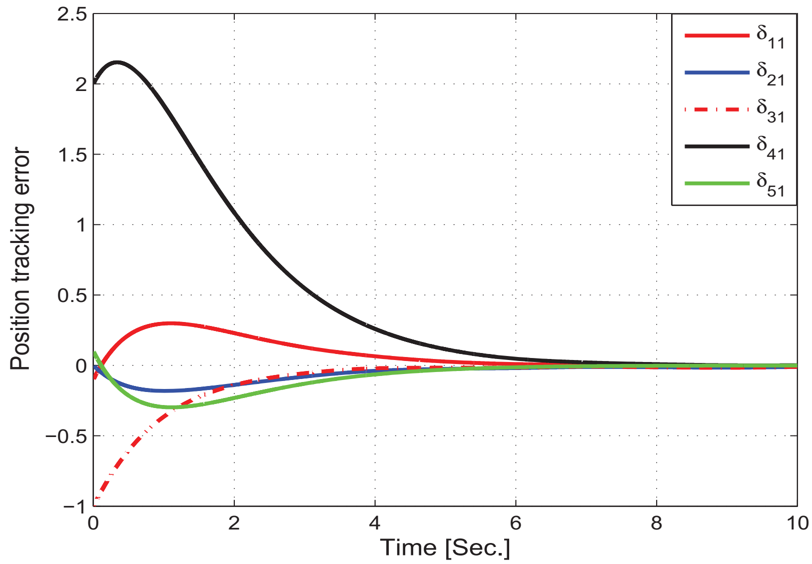

Figure 3.4: Evolution of position error, $\delta_{i,1}$

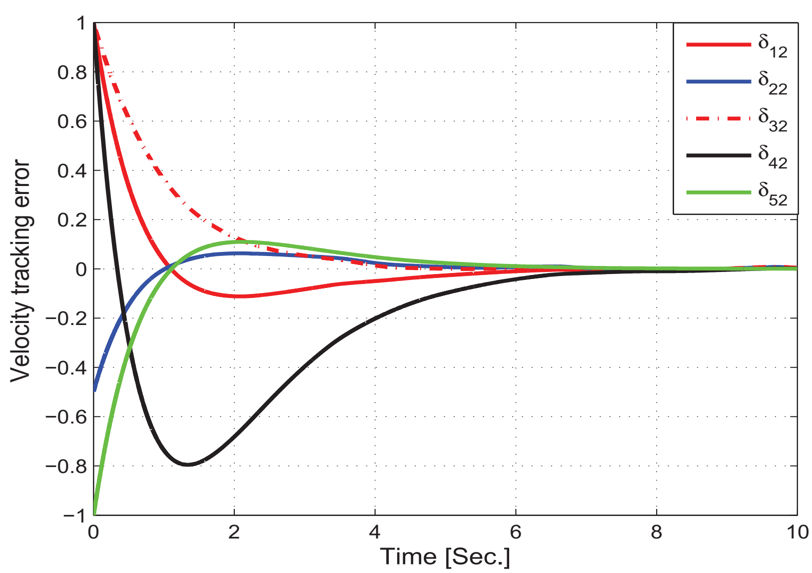

Figure 3.5: Evolution of velocity error, $\delta_{i,2}$

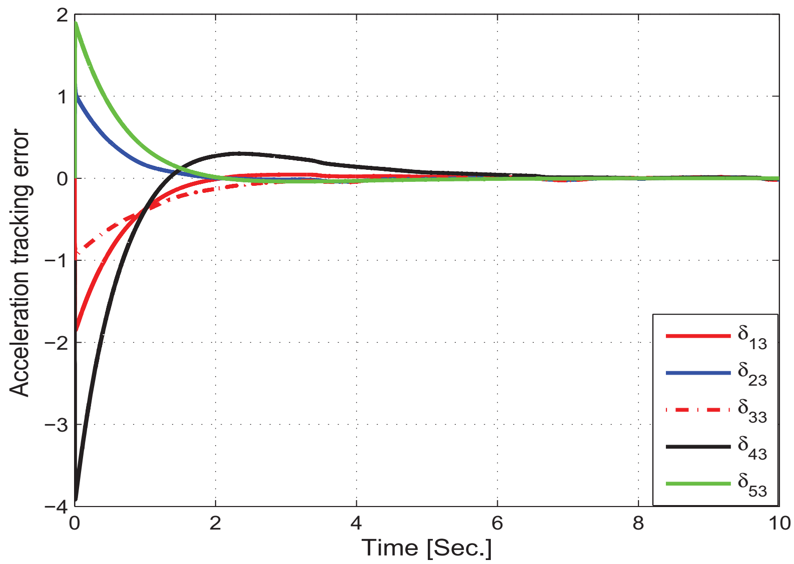

Figure 3.6: Evolution of acceleration error, $\delta_{i,3}$

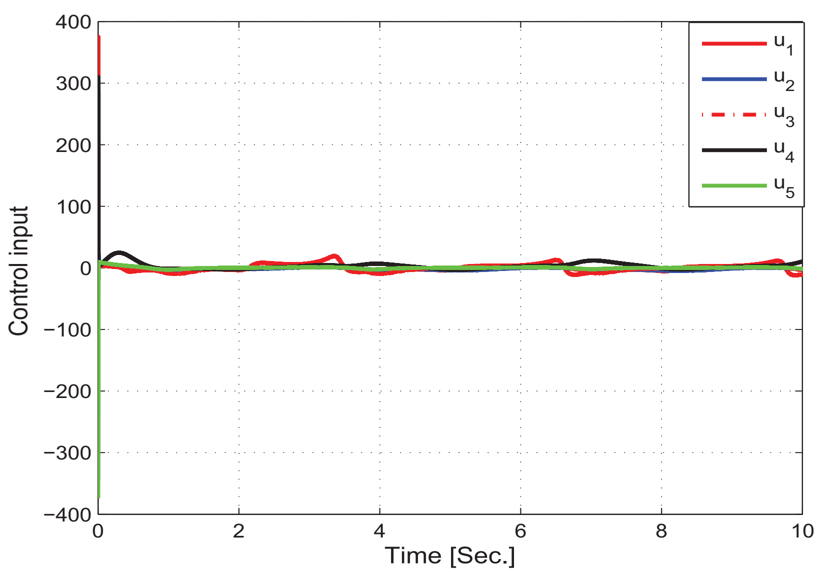

Figure 3.7: Control inputs, u_i

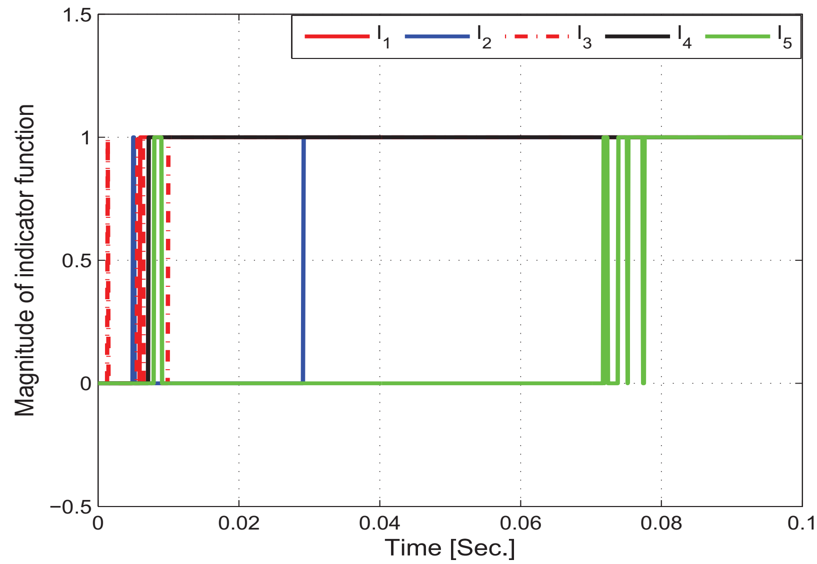

Figure 3.8: Indicator $\mathcal{I}_i$

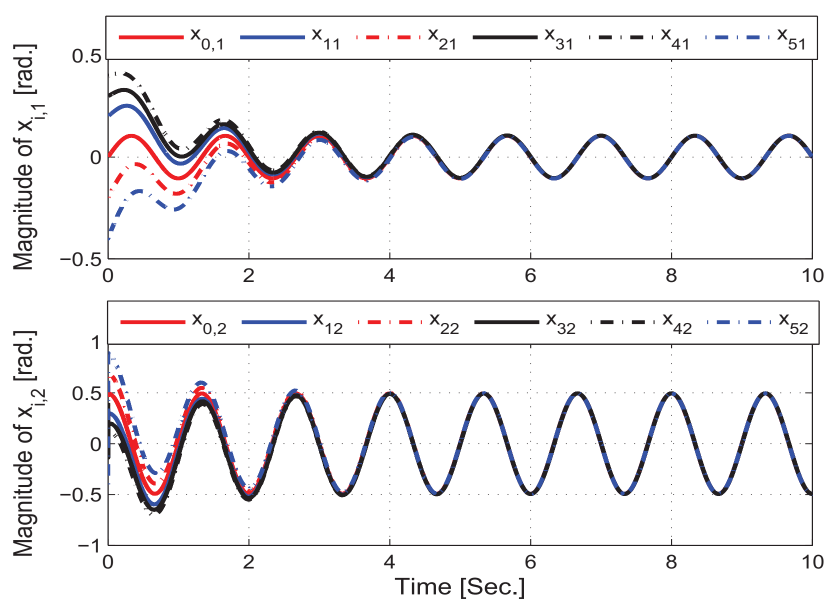

Figure 3.9: Evolution of inverted pendulums states, $x_{i,1}$ and $x_{i,2}$

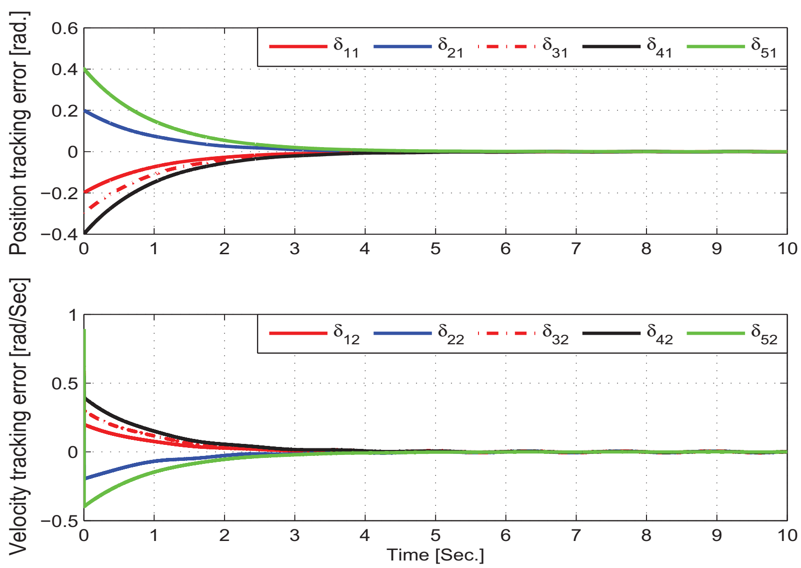

Figure 3.10: Evolution of tracking errors of inverted pendulums, $\delta_{i,1}$ and $\delta_{i,2}$

CHAPTER 4

$\mathcal{L}_2$ NEURO-ADAPTIVE TRACKING CONTROL OF UNCERTAIN PORT CONTROLLED HAMILTONIAN SYSTEMS

In this chapter a practical method of neural network adaptive tracking control of uncertain Port Controlled Hamiltonian (PCH) systems is presented. The developments of this chapter are important as a significant step prior to the design of robust cooperative controller for a networked group of PCH systems discussed in the next chapter. A neural network is used to compensate for the parametric uncertainty. The dynamics of the the proposed neural network tuning law are driven by both the position as well as the velocity errors by introducing the novel idea of information preserving filtering of Hamiltonian gradient. PCH structure of the closed-loop system is preserved. Moreover, the controller also achieves the $\mathcal{L}_2$ disturbance attenuation objective. Simulation examples show the efficacy of the proposed controller.

Chapter Organization: Sec. 4.1 introduces the problem discussed in this chapter and the relevant literature. Sec. 4.2 briefly describes the PCH systems and the generalized canonical transformation theory which is important for the development of tracking controllers. Sec. 4.3 explains in detail the main problem treated in the chapter. The main results of this chapter are described in Sec. 4.4. Sec. 4.5 explains the application of the proposed controller to standard mechanical systems. A couple of simulation examples is detailed in Sec. 4.6. The chapter ends with a conclusion in Sec. 4.7.

4.1 Introduction

The major concern of this chapter is to design a robust controller for single PCH systems. If uncertainties in the system parameters and model structure are not taken into account during the controller design stage, they can significantly degrade the performance and stability of the closed-loop system. Few attempts on the robust control of uncertain PCH systems have been reported in the literature. A variable structure controller is proposed in [87] and [18] to cope with parametric uncertainties. However, in most practical situations, chattering caused by discontinuous control actions hinder the implementation of the proposed variable structure controller. The authors of [88] used an adaptive controller to achieve robust simultaneous stabilization of multiple PCH systems. However a restriction on the form of Hamiltonian limits the application of such approach to a narrow class of systems. An adaptive control approach is proposed in [27] for stabilizing uncertain PCH systems. In [26] a similar approach is presented for time-varying uncertain PCH systems, by employing the canonical transformation proposed in [23], to address both, stabilization and tracking problems. A potential drawback of [27] and [26] is that the adaptive laws proposed in these papers are driven by the passive output, i.e. the velocity error, and therefore significant position error can occur in the steady state. The inclusion of the position error in the adaptive laws is avoided to preserve the PCH structure of the closed loop system. Recognizing such drawback, authors of [28] proposed the power-based Brayton-Moser (BM) formalism as an alternative to energy-shaping based PCH formalism with the motive of incorporating position error in the adaptive law. This has been achieved at the cost of losing the freedom of tuning certain design parameters, a feature well exploited in the canonical transformation of PCH models.

In addition, the adaptive control schemes mentioned above, require the parametric uncertainty to be in *Linear-In-Parameter* (LIP) form, which asserts that uncertainty is expressed as a product of a known regressor matrix and a vector of unknown parameters. In many practical situations, finding the regressor matrix is either very difficult or, in some cases impossible, and therefore LIP property may not be exactly fulfilled.

Motivated by the limitations of the above works(i.e. the adaptive laws driven by the velocity errors only, and the LIP requirement), in this thesis a novel approach based on neural network (NN) adaptive control of PCH systems with $\mathcal{L}_2$ disturbance attenuation is presented. The novel idea of *Information Preserving* (IP) filtering of Hamiltonian gradient is introduced. With the incorporation of this IP filtering, NN tuning law is driven by, both, the position as well as velocity errors. PCH structure of the closed loop dynamics is preserved. The intuition behind the proposed NN tuning law stems from the well-known filtered-error approach of adaptive and NN control theory, where the adaptive laws are driven by filtered error, and thus by position as well as velocity error [89], [24]. The concept of IP is introduced to reflect that in the proposed NN tuning law with IP filtering of Hamiltonian gradient, the information contained in the position error is preserved and utilized, in contrast to available results, for example, [27] and [26], where this information is lost and only the velocity error information is utilized. This idea has been recognized in [26], but their approach proposes to further transform the system dynamics into filtered error space and drive the adaptive law by position error as well as velocity error. However, such transformation complicates the controller design. The NN tuning law proposed in this chapter does not require any extra transformation. Moreover the proposed neuro-adaptive approach is direct i.e. off-line training of the NN is not required. Another advantage of the NN based control is that, both the LIP assumption on the uncertainty, and therefore, the evaluation of a regressor matrix are no longer required, making the proposed approach much simpler. Furthermore, the so called *Nerendra e-modification* [90] is employed in the NN tuning law to avoid the need for *persistence of excitation condition* and therefore making the NN tuning law more robust.

In summary, the contribution of this chapter to the literature is twofold:

1. $\mathcal{L}_2$ neuro-adaptive control of PCH systems is proposed and,
2. a novel NN tuning law is proposed which is driven by both the position as well as velocity errors, by introducing the concept of IP filtering.

4.2 Preliminaries

A PCH system is generally described as:

$$\begin{aligned} \dot{x} &= [J(x,t) - R(x,t)] \frac{\partial H}{\partial x}(x,t) + g(x,t)u \\ y &= g(x,t)^T \frac{\partial H}{\partial x}(x,t) \end{aligned} \tag{4.1}$$

where $x \in \Re^n$ is the state vector. The Hamiltonian function $H(x,t) : \Re^n \mapsto \Re$ is a Positive semi-definite function and represents the energy stored in the system. The column vector $\frac{\partial H}{\partial x}(x,t) = [\frac{\partial H(x,t)}{\partial x_1} ... \frac{\partial H(x,t)}{\partial x_n}]^T$ denotes the gradient of the scalar function $H(x,t)$. Matrices $J(x,t) = -J(x,t)^T \in \Re^{n \times n}$ and $g(x,t) \in \Re^{m \times n}$ collectively define the *interconnection* structure of the system. $R(x,t) = R^T(x,t) \geq 0 \in \Re^{n \times n}$ represents the *dissipation*. All these matrices, may, smoothly depend on x.

In this chapter the tracking problem is directly addressed, however, these theoretical developments are equally applicable to stabilization and regulation problems as well. Depending upon the problem at hand, (stabilization, regulation or tracking,) an appropriate canonical transformation is performed before the particular controller design is carried out. In control theory, such canonical transformations are very common to transform a dynamic system to an equivalent one, to facilitate, both, the analysis and synthesis of the controller. The theory of canonical transformation of PCH systems is briefly described in Sec. 2.3.6. and further details can be found in [22] and [23]. The main concern of this chapter is now stated in the following section.

4.3 Problem Statement

The Generalized canonical transformation and trajectory tracking problems described in Sec. 2.3.6 are based on the exact knowledge of system parameters and the assumption that external disturbances are zero. However, in many real world applications, parameters deviate from their nominal values caused by several factors. For instance, certain parameters of robot manipulators can suffer from significant variations due to unknown loads during the operations. An *autonomous under-water vehicle* (AUV) experiences parameter variations and external disturbances due to waves and ocean currents. As another example, a helicopter transporting slung load may experience significant variations in its dynamics due to cable length flexibility and aerodynamic drags and wind gusts due to varying weather conditions. Ignoring such uncertainties and disturbances will ultimately degrade the controller performance and sometimes may affect the stability of the closed-loop system. Thus, a scheme for the estimation of the collective impact of the parameter uncertainties and robustness against external disturbances can effectively enhance the controller performance. Controller design with estimation of such parametric uncertainties using NN and $\mathcal{L}_2$ attenuation of external disturbances is the main subject of this chapter.

In this chapter the following uncertain autonomous PCH system is considered.

$$\begin{aligned} \dot{x} &= [J(x,\varepsilon_J) - R(x,\varepsilon_R)]\frac{\partial H}{\partial x}(x,\varepsilon_H) + g(x)(u+\xi) && (4.2) \\ y &= g(x)^T \frac{\partial H}{\partial x}(x,\varepsilon_H) && (4.3) \end{aligned}$$

where $\varepsilon_\jmath, \jmath \in \{J, R, H\}$ denotes the uncertainties in the parameters and $\xi \in \Re^m$ is the unknown but bounded disturbance.

To facilitate the analysis, let the uncertain structure matrices in (4.2) be represented as:

$$\begin{aligned} J(x,\varepsilon_J) &= J_0(x) + \Delta_J(x,\varepsilon_J) \\ R(x,\varepsilon_R) &= R_0(x) + \Delta_R(x,\varepsilon_R) \\ \frac{\partial H}{\partial x}(x,\varepsilon_H) &= \frac{\partial H_0}{\partial x}(x) + \Delta_H(x,\varepsilon_H) \end{aligned} \qquad (4.4)$$

where $J_0(x)$, $R_0(x)$ and $H_0(x)$ denote the nominal values. Define $\varepsilon = [\varepsilon_R \;\; \varepsilon_J \;\; \varepsilon_H]^T$ and let $\beta(x,\varepsilon_\beta)$ as obtained from the solution of PDE (2.49) for the uncertain system (4.2) and expressed as

$$\beta(x, \varepsilon_\beta) = \beta_0(x) + \Delta_\beta(x, \varepsilon_\beta) \tag{4.5}$$

where $\beta_0(x)$ corresponds to the solution for the nominal system. $\Delta_\beta(x, \varepsilon_\beta)$ is the corresponding uncertainty.

Application of $\beta_0(x)$ transforms the uncertain system(4.2) from state space to the following system in tracking-error space.

$$\begin{aligned} \dot{\bar{x}} &= \left[\bar{J}(\bar{x}, \varepsilon_J) - \bar{R}(\bar{x}, \varepsilon_R)\right] \frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x}, \varepsilon_H) \\ &\quad + \bar{g}(\bar{x})(\Delta_\beta(x, \varepsilon_\beta) + \bar{u} + \xi) && (4.6) \\ \bar{y} &= \bar{g}(\bar{x})^T \frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x}, \varepsilon_H) && (4.7) \end{aligned}$$

Define $\bar{\mathcal{F}}(\bar{x}, \varepsilon) \in \Re^n$ as

$$\begin{aligned} \bar{\mathcal{F}}(\bar{x}, \varepsilon) &= [\bar{J}(\bar{x}, \varepsilon_J) - \bar{R}(\bar{x}, \varepsilon_R)] \frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x}, \varepsilon_H) \\ &\quad - [\bar{J}_0(\bar{x}) - \bar{R}_0(\bar{x})] \frac{\partial \bar{H}_0}{\partial \bar{x}}(\bar{x}) + \bar{g}(\bar{x}, t)\Delta_\beta(x, \varepsilon_\beta) && (4.8) \end{aligned}$$

Using (4.8), the uncertain error system (4.6) can be expressed as:

$$\begin{aligned} \dot{\bar{x}} &= [\bar{J}_0(\bar{x}) - \bar{R}_0(\bar{x})] \frac{\partial \bar{H}_0}{\partial \bar{x}}(\bar{x}) \\ &\quad + \bar{\mathcal{F}}(\bar{x}, \varepsilon) + \bar{g}(\bar{x})(\bar{u} + \xi) && (4.9) \end{aligned}$$

Note that $\bar{\mathcal{F}}$ collectively represents the parametric uncertainty of transformed system (4.6).

Assumption 3.1: *The collective parametric uncertainty $\bar{\mathcal{F}}$ in (4.8) can be expressed as:*

$$\bar{\mathcal{F}}(\bar{x}, \varepsilon) = \mathcal{Z}\bar{g}(x)\mathcal{F}(\bar{x}, \varepsilon) \tag{4.10}$$

where the so called information preserver $\mathcal{Z} \in \Re^{n\times n}$ is a symmetric design matrix and the unknown function $\mathcal{F}(\bar{x}, \varepsilon) \in \Re^{m}$ is locally Lipschitz.

Further explanation of $\mathcal{Z}$ is given in the Sec. 4.3.3. The unknown function $\mathcal{F}(\bar{x}, \varepsilon)$ will be approximated by a neural network.

Let

$$\bar{u} = -C(\bar{x}, t)\bar{y} + \vartheta \tag{4.11}$$

where $-C(\bar{x}, t)\bar{y}$ is the passive output feedback part of the control input according to *Theorem 2.2* (Ch. 02), and ϑ is the $\mathcal{L}_2$ neuro-adaptive component of the control to account for all parametric uncertainties and disturbance attenuation. Applying $\bar{u}$ of (4.11) to the uncertain system (4.6) yields

$$\dot{\bar{x}} = [\bar{J}_0(\bar{x}) - \bar{R}_c(\bar{x})]\frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x}) + \mathcal{Z}\bar{g}\mathcal{F}(\bar{x}, \varepsilon) + \bar{g}(\bar{x})(\vartheta + \xi) \tag{4.12}$$

where

$$\bar{R}_c(\bar{x}) = \bar{R}_0(\bar{x}) + \bar{g}(\bar{x}, t)C(\bar{x}, t)\bar{g}^T(\bar{x}, t) \tag{4.13}$$

Technically, the objective of this chapter is threefold;

1. *find a NN tuning law for estimation of $\mathcal{F}(\bar{x},\varepsilon)$. The NN tuning law should be driven by both, the position as well as the velocity error.*

2. *design the feedback controller ϑ to compensate for the uncertainties in PCH error system (4.2) using the NN approximation of $\mathcal{F}(\bar{x},\varepsilon)$. The controller should achieve the $\mathcal{L}_2$ disturbance attenuation objective and preserve the PCH structure of the closed-loop system as well.*

3. *ensure the Uniform Ultimate Boundedness(UUB) of the error vector $\bar{x}$ and $\tilde{W}$(defined in eq. (4.32)) under the proposed controller.*

4.3.1 Approximation of uncertainties using neural networks

Let $\Omega \subset \Re^n$ be a compact, simply connected set and $\mathcal{F}(\cdot) : \Omega \rightarrow \Re^m$. Define $\mathcal{C}^m(\Omega)$ as the space of continuous function $\mathcal{F}(\cdot)$. Then, for all $\mathcal{F}(\cdot) \in \mathcal{C}^m(\Omega)$, there exists a weight vector W such that $\mathcal{F}(\bar{x},\varepsilon)$ can be expressed as

$$\mathcal{F}(\bar{x},\varepsilon) = \phi(\bar{x}_N)^T W + \epsilon \tag{4.14}$$

The weights $W = \left[W_1^T ... W_m^T\right] \in \Re^{m\nu}$ with $W_i \in \Re^{\nu},\ i = 1, ..., m$ are the ideal neural network weights, $\phi = diag(\phi_1, ...\phi_m^T)\Re^{m\nu \times m}$ with $\phi_i \in \Re^{\nu}, i = 1, ...m$ are some basis functions. $\bar{x}_N$ is the NN input given by:

$$\bar{x}_N = \left[\bar{x}^T\ \dot{\bar{x}}^T\ x_d^T\ \dot{x}_d^T\ \ddot{x}_d^T\right]^T \tag{4.15}$$

$\epsilon \in \Re^m$ is the NN approximation error.

In practical applications the ideal weights W and error ϵ can not be evaluated, rather, the function $\mathcal{F}(\cdot)$ is approximated as:

$$\hat{\mathcal{F}}(\bar{x}, \varepsilon) = \phi(\bar{x}_N)^T \hat{W}(t) \tag{4.16}$$

where $\hat{W}(t) \in \Re^{m\nu}$ are the actual time-varying weights of the NNs.

Remark 4.1: *Similar to the NNs used in Ch. 3, the FLNN is also used here for the approximation of $\mathcal{F}(\cdot)$.*

In addition to guarantying the closed-loop system's stability, the objective here is to propose a NN tuning law that

1. preserves the PCH structure of the closed-loop dynamics augmented with the NNs' dynamics, and
2. its dynamics are driven by both position as well as velocity errors, unlike in [27] and [26], where only velocity error is used and may result in a steady state position error.

4.3.2 NN tuning law

The following NN tuning law is proposed for the approximation of $\mathcal{F}(\cdot)$.

$$\dot{\hat{W}} = \Gamma \phi(x) \bar{g}^T(\bar{x}) \mathcal{Z} \frac{\partial \bar{H}}{\partial \bar{x}} - \kappa \Gamma \| \bar{g}^T(\bar{x}) \mathcal{Z} \frac{\partial \bar{H}}{\partial \bar{x}} \| \hat{W} \tag{4.17}$$

where $\Gamma = diag(\Gamma_1, ..., \Gamma_m)$ and $\Gamma_i \in \Re^{\nu \times \nu}$, for $i = 1, ..., m$, is a user defined symmetric positive definite design matrix. $\kappa > 0$ is a scaler design parameter.

Define the manifold Ξ as:

$$\Xi = \{\bar{x} \mid \bar{g}^T(\bar{x})\mathcal{Z}\frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x}) = 0\} \tag{4.18}$$

The following subsection explains the advantages associated with the introduction of $\mathcal{Z}$ in the NN tuning law (4.17).

4.3.3 Information-preserving gradient filtering

Incorporation of $\mathcal{Z}$ causes the NN tuning law (4.17) to be driven by a combination of position and velocity errors. The manifold Ξ can be judiciously termed as an information-preserving *filtered Hamiltonian-gradient*. In some cases, e.g. simple pendulum and linear spring-mass-damper systems, it can be shown that Ξ is equivalent to the *filtered error* or *sliding manifold* in error space. The proposed approach of driving the NN tuning law by IP filtered Hamiltonian-gradient is inspired by filtered error approach, which is very popular in the literature of adaptive and NN control theory, [24] and [89]. The advantage of the proposed NN tuning law is that information contained in $\frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x})$ is fully utilized. The concept of information preservation is well explained by considering the example of a standard mechanical system (explained in Sec. 4.5) for which

$$\bar{H}(\bar{q}, \bar{p}) = \frac{1}{2}\bar{p}^T M^{-1}(q)\bar{p} + \frac{1}{2}\bar{q}^T K_p \bar{q} \tag{4.19}$$

where $[\bar{q}^T \quad \bar{p}^T]^T = \bar{x}$ and

$$\frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x}) = \frac{\partial \bar{H}}{\partial(\bar{q},\bar{p})}(\bar{q},\bar{p}) = \begin{bmatrix} K_p\bar{q} + f_{\bar{H}}(\bar{q},\bar{p}) \\ \\ \dot{\bar{q}} \end{bmatrix} \tag{4.20}$$

where $\bar{q}$ and $\bar{p}$ are the *error* vectors of the generalized configuration coordinates and generalized momenta, respectively. $f_{\bar{H}}(\bar{q},\bar{p}) = \frac{\partial\left(\bar{p}^T M(q)\bar{p}\right)}{\partial \bar{q}}$ and K_p is a positive definite gain matrix. $M(q) \in \Re^{n\times n}$ is the inertial matrix. Note that $\bar{q}$ and $\dot{\bar{q}}$ represent the *position* and *velocity* errors, respectively. Let $[\bar{q}^T \quad \bar{p}^T]^T \in \Re^4$, then for a fully actuated system $m = 2$ and

$$\bar{g} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \\ 0 & 0 & 0 & 1 \end{bmatrix}^T$$

Now, for $\bar{q} = [\bar{q}_1 \;\; \bar{q}_2]^T$, consider

$$\bar{g}^T \frac{\partial \bar{H}}{\partial(\bar{q},\bar{p})} = \begin{bmatrix} \dot{\bar{q}}_1 \\ \\ \dot{\bar{q}}_2 \end{bmatrix} = \dot{\bar{q}} \tag{4.21}$$

In contrast to (4.21), it is possible to choose $\mathcal{Z}$ and U such that for a positive definite matrix $\Lambda = diag(\lambda_1, \; \lambda_2) \in \Re^{2\times 2}$,

$$\begin{aligned} \bar{g}^T \mathcal{Z} \frac{\partial \bar{H}}{\partial(\bar{q},\bar{p})} &= \begin{bmatrix} \lambda_1\bar{q}_1 + \dot{\bar{q}}_1 \\ \\ \lambda_2\bar{q}_2 + \dot{\bar{q}}_2 \end{bmatrix} + \begin{bmatrix} f_1(\bar{q},\bar{p}) \\ \\ f_2(\bar{q},\bar{p}) \end{bmatrix} \\ &= \Lambda\bar{q} + \dot{\bar{q}} + f(\bar{q},\bar{p}) \end{aligned} \tag{4.22}$$

where $f(\cdot) = [f_1(\cdot) \quad f_2(\cdot)]^T$. A comparison of (4.21) and (4.22) explains the fact that all of the information contained in $\frac{\partial H}{\partial \bar{x}}(\bar{x})$ is fully preserved and utilized by the proposed NN tuning law (4.17) driven by $\Lambda\bar{q} + \dot{\bar{q}} + f(\bar{q}, \bar{p})$, while in contrast, the adaptive laws in [27] and [26] are driven by only $\dot{\bar{q}}$. With this additional feature of information preservation, the proposed law is applicable to reference tracking as well, and hence is more general in its application than in [27] and [26].

With the above explanation, it is justified to name the RHS of (4.22) as the filtered Hamiltonian-gradient which is similar to the filtered error in [24] and [89].

Remark 4.2: *Another possibility of driving the NN tuning law by both, the position and velocity errors, as suggested in [26], is to perform another coordinate transformation resulting into an equivalent PCH system with a passive output which is function of both position and velocity errors. However, analysis and design of controller becomes more complex with such transformation.*

Selection of $\mathcal{Z}$

In the filtered error based adaptive control, the coefficients of the filter are selected such that the resulting dynamics of the filter are stable [24], [89]. Likewise, the filter $\mathcal{Z}$ is selected such that the the dynamics of the resulting filtered Hamiltonian gradient represent a stable auxiliary dynamic system.

The $\mathcal{L}_2$ disturbance attenuation objective, in the context of this work is now described in the following (see [27]):

Given the desired trajectory $x_d \in \Re^n$, a penalty signal $z = q(\bar{x})$ such that $q(\bar{x}_0) = 0$ and a disturbance attenuation level $\gamma > 0$. Let ω be the combination of external *disturbance ξ and some multiple of the NN approximation error ϵ. Find a feedback control law $\vartheta = f(x, \hat{W})$, NN approximation of $\mathcal{F}(\cdot)$ and a modified positive definite, (with respect to $\bar{x}_0$), storage function $V(\bar{x})$ such that for the closed loop system consisting of (4.12) with the feedback law, the γ-dissipation inequality*

$$\dot{V} + Q(\bar{x}) \leq \frac{1}{2}\left\{\gamma^2\|\omega\|^2 - \|z\|^2\right\}, \forall\omega \tag{4.23}$$

holds along all trajectories remained in $\mathcal{X} \subset \Re^n$, where $Q(\bar{x})$ is a given non-negative definite function, with $Q(\bar{x}_0) = 0$. This problem is called the $\mathcal{L}_2$ neuro-adaptive disturbance attenuation. *The γ-dissipation inequality (4.23) ensures that $\mathcal{L}_2$-gain from the disturbance ω to the penalty signal z is less than the given level γ.*

4.4 $\mathcal{L}_2$ Neuro-Adaptive Control

Let the penalty signal z be defined as:

$$z(\bar{x}) = h(\bar{x})\bar{g}^T(\bar{x})\frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x}) \tag{4.24}$$

where $h(\bar{x})$ ($h(0) = 0$) is a weighting matrix.

The following assumption is made on $\bar{g}(\bar{x})$.

Assumption 4.2: *The Moore-Penrose pseudoinverse of $\bar{g}$ denoted by $\bar{g}^\dagger(\bar{x})$ exists for all $\bar{x}$.*

Assumption 4.2 can appear as a restriction on the application of the proposed controller. Fortunately most of the models of physical systems have $\bar{g}$ independent of $\bar{x}$, e.g. n-link robot manipulator, magnetic levitation systems and power generation systems etc.

The proposed $\mathcal{L}_2$ neuro-adaptive control law is given as:

$$\vartheta(x, W) \;\; = \;\; -\mathcal{K}\frac{\partial \bar{H}}{\partial \bar{x}} - \bar{g}^{\dagger}\mathcal{Z}\bar{g}\hat{\mathcal{F}}(\bar{x}, \epsilon) \tag{4.25}$$

where

$$\mathcal{K} = \frac{1}{2}\left\{\frac{1}{\gamma^2}I + h^T(\bar{x})h(\bar{x})\right\}\bar{g}^T \tag{4.26}$$

Note that the first term on the RHS of (4.25) represents the $\mathcal{L}_2$ disturbance attenuation component, while the second term represents the neuro-adaptive component of the control law (4.25).

Application of (4.25) to (4.12) yields

$$\begin{aligned}\dot{\bar{x}} \;\; = \;\; & [\bar{J}_0(\bar{x}) - \bar{R}_c(\bar{x}) - \bar{g}(\bar{x})\mathcal{K}]\frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x}) \\ & + \mathcal{Z}\bar{g}(\bar{x})\phi^T(W - \hat{W}(t)) + \bar{g}(\bar{x})(\xi + \bar{g}^{\dagger}\mathcal{Z}\bar{g}\epsilon)\end{aligned} \tag{4.27}$$

To augment the NNs dynamics with the closed-loop system's dynamics and to obtain the augmented dynamics in PCH form, define the shaped Hamiltonian as:

$$\bar{H}_S(\bar{x}, \hat{W}) = \bar{H}(\bar{x}) + \bar{H}_{N}(\hat{W}) \tag{4.28}$$

where

$$\bar{H}_N(\hat{W}) = \frac{1}{2}(W - \hat{W}(t))^T \Gamma^{-1} (W - \hat{W}(t)) \tag{4.29}$$

To facilitate the analysis, the transformed state vector $\bar{x}$ is augmented with $\hat{W}$ to form the augmented state vector $\mathcal{X}$ as:

$$\mathcal{X} = \begin{bmatrix} \bar{x} \\ \hat{W} \end{bmatrix} \tag{4.30}$$

Gradient of $\bar{H}_S(\bar{x}, \hat{W})$ w.r.t. $\mathcal{X}$ is given by:

$$\frac{\partial \bar{H}_S}{\partial(\bar{x}, \hat{W})}(\bar{x}, \hat{W}) = \frac{\partial \bar{H}_S}{\partial \mathcal{X}}(\mathcal{X}) = \begin{bmatrix} \frac{\partial \bar{H}(\bar{x})}{\partial \bar{x}} \\ \frac{\partial \bar{H}_N(\hat{W})}{\partial \hat{W}} \end{bmatrix} = \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x}) \\ -\Gamma^{-1}\tilde{W} \end{bmatrix} \tag{4.31}$$

where

$$\tilde{W} = W - \hat{W}(t) \tag{4.32}$$

Applying controller (4.25) to system (4.12), and augmenting the resulting dynamics to NN dynamics (4.17) yields the following PCH system

$$\dot{\mathcal{X}} = \begin{bmatrix} \dot{\bar{x}} \\ \dot{\hat{W}} \end{bmatrix} = \Bigg[\begin{pmatrix} \bar{J}_0 & -\mathcal{Z}\bar{g}\phi^T\Gamma \\ \left(\mathcal{Z}\bar{g}\phi^T\Gamma\right)^T & 0 \end{pmatrix}$$

$$
- \left(\begin{array}{cc} \bar{R}_c + \bar{g}\mathcal{K} & 0 \\ 0 & \kappa\Gamma \| \bar{g}^T \mathcal{Z} \frac{\partial \bar{H}}{\partial \bar{x}} \| \Gamma \end{array} \right) \Bigg] \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{x}} \\ \frac{\partial \bar{H}_N}{\partial \hat{W}} \end{bmatrix}
$$

$$
+ \begin{bmatrix} \bar{g} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \omega \\ -\kappa\Gamma \| \bar{g}^T \mathcal{Z} \frac{\partial \bar{H}}{\partial \bar{x}} \| W \end{bmatrix} \tag{4.33}
$$

where $\omega = \xi + \bar{g}^{\dagger}\mathcal{Z}\bar{g}\epsilon$.

The following theorem establishes the main results of this chapter.

Theorem 4.1 *Consider the uncertain PCH system (4.2) and its corresponding error system (4.6). Suppose Assumptions 4.1–4.2 hold, then*

1. *for any $\gamma > 0$, the $\mathcal{L}_2$ disturbance attenuation objective is achieved by application of control law (4.25).*

2. *$\bar{x}$ and $\tilde{W}$ are UUB with practical bounds given by (4.47) and (4.48), respectively .*

Proof:1). Let the shaped Hamiltonian (4.28) be the Lyapunov function candidate. Taking its time derivative along the trajectory of augmented system (4.33) yields

$$
\begin{aligned}
\dot{\bar{H}}_S(\bar{x}, \hat{W}) &= \frac{\partial \bar{H}}{\partial \bar{x}}^T \dot{\bar{x}} - (W - \hat{W})^T \Gamma^{-1} \dot{\hat{W}} \\
&= \begin{bmatrix} \frac{\partial \bar{H}^T}{\partial \bar{x}} & \frac{\partial \bar{H}_N^T}{\partial \hat{W}} \end{bmatrix} \begin{bmatrix} \dot{\bar{x}} \\ \dot{\hat{W}} \end{bmatrix} \\
&= \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{x}}^T & \frac{\partial \bar{H}_N}{\partial \hat{W}}^T \end{bmatrix} \Bigg[\left(\begin{array}{cc} \bar{J}_0 & -\mathcal{Z}\bar{g}\phi^T\Gamma \\ \left(\mathcal{Z}\bar{g}\phi^T\Gamma\right)^T & 0 \end{array} \right)
\end{aligned}
$$

$$
\begin{aligned}
& \left. - \begin{pmatrix} \bar{R}_c + \bar{g}K & 0 \\ 0 & \kappa\Gamma\|\bar{g}^T\mathcal{Z}\frac{\partial \bar{H}}{\partial \bar{x}}\|\Gamma \end{pmatrix} \right] \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{x}} \\ \frac{\partial \bar{H}_N}{\partial \hat{W}} \end{bmatrix} \\
& + \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{x}}^T & \frac{\partial \bar{H}_N}{\partial \hat{W}}^T \end{bmatrix} \begin{bmatrix} \bar{g} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \omega \\ -\kappa\Gamma\|\bar{g}^T\mathcal{Z}\frac{\partial \bar{H}}{\partial \bar{x}}\|W \end{bmatrix} \\
= & -\frac{\partial \bar{H}}{\partial \bar{x}}^T R_c \frac{\partial \bar{H}}{\partial \bar{x}} - \kappa \frac{\partial \bar{H}_N}{\partial \hat{W}}^T \Gamma\|\bar{g}^T\mathcal{Z}\frac{\partial \bar{H}}{\partial \bar{x}}\|\Gamma\frac{\partial \bar{H}_N}{\partial \hat{W}} \\
& -\frac{\partial \bar{H}}{\partial \bar{x}}^T \bar{g}K\frac{\partial \bar{H}}{\partial \bar{x}} + \frac{\partial \bar{H}}{\partial \bar{x}}^T \bar{g}\omega \\
& -\kappa\frac{\partial \bar{H}_N}{\partial \hat{W}}^T \Gamma\|\bar{g}^T\mathcal{Z}\frac{\partial \bar{H}}{\partial \bar{x}}\|W
\end{aligned}
\tag{4.34}
$$

Using the definitions $r = \bar{g}^T\mathcal{Z}\frac{\partial \bar{H}}{\partial \bar{x}}$, $\mathcal{R}_c = (\mathcal{Z}^T\bar{g})^\dagger R_c(\bar{g}^T\mathcal{Z})^\dagger$, $W_B = \|W\|$, substituting $\mathcal{K}$ from (4.26) and noting that $\frac{\partial \bar{H}_N}{\partial \hat{W}} = -\Gamma^{-1}\tilde{W}$, Eq. (4.34) becomes

$$
\begin{aligned}
\dot{\bar{H}}_S = & -r^T\mathcal{R}_c r - \kappa\tilde{W}^T\|r\|\tilde{W} + \kappa\tilde{W}\|r\|W \\
& -\frac{1}{2}\frac{\partial \bar{H}}{\partial \bar{x}}^T \bar{g}\left\{\frac{1}{\gamma^2}I + h^T(\bar{x})h(\bar{x})\right\}\bar{g}^T\frac{\partial \bar{H}}{\partial \bar{x}} \\
& +\frac{\partial \bar{H}}{\partial \bar{x}}^T \bar{g}\omega \\
\leq & -\underline{\sigma}(\mathcal{R}_c)\|r\|^2 - \kappa\|r\|\|\tilde{W}\|^2 + \kappa\|\tilde{W}\|\|r\|\|W\| \\
& -\frac{1}{2}\frac{\partial \bar{H}}{\partial \bar{x}}^T \bar{g}\left\{\frac{1}{\gamma^2}I + h^T(\bar{x})h(\bar{x})\right\}\bar{g}^T\frac{\partial \bar{H}}{\partial \bar{x}} \\
& +\frac{1}{2\gamma^2}\frac{\partial \bar{H}}{\partial \bar{x}}^T \bar{g}\bar{g}^T\frac{\partial \bar{H}}{\partial \bar{x}} + \frac{1}{2}\gamma^2\omega^T\omega \\
\leq & -\underline{\sigma}(\mathcal{R}_c)\|r\|^2 - \kappa\|r\|\|\tilde{W}\|^2 + \kappa\|\tilde{W}\|\|r\|W_B \\
& +\frac{1}{2}\left\{\gamma^2\|\omega\|^2 - \|z\|^2\right\}
\end{aligned}
\tag{4.35}
$$

Let

$$\rho = \begin{bmatrix} \|r\| & \|\tilde{W}\| \end{bmatrix}^T \tag{4.36}$$

$$\mathcal{M} = \begin{bmatrix} \underline{\sigma}(\mathcal{R}_c) & 0 \\ 0 & \kappa\|r\| \end{bmatrix} \tag{4.37}$$

$$\theta = \begin{bmatrix} 0 & \kappa\|r\|W_B \end{bmatrix}^T \tag{4.38}$$

Inequality (4.35) can be written as

$$\begin{aligned} \dot{\bar{H}}_S &\leq -\rho^T\mathcal{M}\rho + \theta^T\rho + \frac{1}{2}\left\{\gamma^2\|\omega\|^2 - \|z\|^2\right\} \\ &\leq -Q(\rho) + \frac{1}{2}\left\{\gamma^2\|\omega\|^2 - \|z\|^2\right\} \end{aligned} \tag{4.39}$$

where $Q(\rho) = \rho^T\mathcal{M}\rho - \theta^T\rho$

$Q(\rho)$ is non-negative definite if the following conditions, C1 and C2, hold:

(C1) $\mathcal{M}$ is non-negative definite.

(C2) $\|\rho\| > \frac{\|\theta\|}{\underline{\sigma}(\mathcal{M})}$

Condition (C1) is obviously satisfied. Since $\|\theta\|_1 > \|\theta\|$, condition (C2) holds if $\|\rho\| \geq B$ with

$$B = \frac{\|\theta\|_1}{\underline{\sigma}(\mathcal{M})} = \frac{\kappa\|\bar{g}^T r\|W_B}{\underline{\sigma}(\mathcal{M})} \tag{4.40}$$

Thus,

$$Q(\rho) \geq 0, \quad \forall\|\rho\| \geq B \tag{4.41}$$

Since ρ explicitly depends on $\bar{x}$, (4.39) is re-written as:

$$\dot{H}_S + Q(\bar{x}) \leq \frac{1}{2}\left\{\gamma^2\|\omega\|^2 - \|z\|^2\right\} \tag{4.42}$$

Equation (4.41) with B defined in (4.40) implies that the function $Q(\bar{x})$ is non-negative definite. This completes the the proof of the first part of this theorem.

2). **UUB of $\bar{x}$ and $\tilde{W}$**

Let

$$V_r(r) = -\underline{\sigma}(\mathcal{R}_c)\|r\|^2 - \kappa\|r\|\|\tilde{W}\|^2 + \kappa\|\tilde{W}\|\|r\|W_B \tag{4.43}$$

Using definition (4.43), inequality (4.35) can be written as

$$\dot{H}_S \leq V_r(r) + \frac{1}{2}\left\{\gamma^2\|\omega\|^2 - \|z\|^2\right\} \tag{4.44}$$

UUB of $\bar{x}$ and $\tilde{W}$ is proven by showing the negative definiteness of V_r, as follows. Re-write $V_r(r)$ as

$$V_r(r) = -\|r\|\left[k_{c_{min}}\|r\| + \kappa\|\tilde{W}\|\left(\|\tilde{W}\| - W_B\right)\right] \tag{4.45}$$

where $k_{c_{min}} = \underline{\sigma}(\mathcal{R}_c)$. $V_r(r)$ is negative as long as the term in brackets is strictly positive. Completing the square yields

$$\begin{aligned} & k_{c_{min}}\|r\| + \kappa\|\tilde{W}\|\left(\|\tilde{W}\| - W_B\right) \\ = \; & \kappa\left(\|\tilde{W}\| - \frac{1}{2}W_B\right)^2 - \kappa W_B^2/4 + k_{c_{min}}\|r\| \end{aligned} \tag{4.46}$$

which is positive as long as

$$\|r\| > \frac{\kappa W_B^2/4}{k_{c_{min}}} \equiv b_r \tag{4.47}$$

or

$$\|\tilde{W}\| > W_B/2 + \sqrt{\kappa W_B^2/4} \equiv b_W \tag{4.48}$$

Thus $V_r(r)$ is negative outside the compact set $\{S_r | \|r\| < b_r\}$. Eq. (4.47) implies that sufficiently large value of $k_{c_{min}}$ obtained by selecting a large C in eq. (4.11), significantly reduce the magnitude of the ball b_r, Thus ensuring the UUB of both $\bar{x}$ and $\tilde{W}$. This completes the proof □

Remark 4.3: *The arbitrary bounds W_B, b_W and b_r are just needed in the stability analysis. Their knowledge is not needed in the controller design, though in most practical problems, it is possible to find the numerical values of these bounds. Similar arguments can be found in [24].*

Remark 4.4: *The concept of filtered Hamiltonian-gradient introduced in this chapter can also be applied to the adaptive laws proposed in [27] and [26] without affecting the PCH structure and stability of the closed-loop system, further strengthening their results.*

Remark 4.5: *Similar to the NN tuning laws in Ch.3, Persistence of excitation (PE) in this chapter is also not needed, thus making the NN tuning law (4.17) robust against unmodeled dynamics and unforeseen situations.*

4.5 Application to Standard Mechanical Systems

Consider the following standard mechanical system in PCH form.

$$\begin{aligned}\begin{bmatrix}\dot{q}\\ \dot{p}\end{bmatrix} &= \begin{bmatrix}0 & I\\ -I & -D(q,p)\end{bmatrix}\begin{bmatrix}\frac{\partial H}{\partial q}\\ \frac{\partial H}{\partial p}\end{bmatrix}+\begin{bmatrix}0\\ G\end{bmatrix}u\\ y &= \begin{bmatrix}0 & G^T\end{bmatrix}\begin{bmatrix}\frac{\partial H}{\partial q}\\ \frac{\partial H}{\partial p}\end{bmatrix}=G^T\frac{\partial H}{\partial p}\end{aligned}\tag{4.49}$$

where $q \in \Re^n$ and $p \in \Re^n$ are vectors of generalized configuration coordinates and generalized momenta, respectively. I is the nth-order identity matrix and $D(q,p) \in \Re^{n\times n}$ is a positive definite damping matrix. Note that the system order is $2n$. $u \in \Re^m$ and $y \in \Re^m$ is the pair of passive inputs and outputs, respectively. Compared to the PCH system defined in (4.1), the interconnection matrix J, the dissipation matrix R and input matrix g for a standard mechanical system (4.49) are expressed as

$$J = \begin{bmatrix}0 & I\\ -I & 0\end{bmatrix}\tag{4.50}$$

$$R = \begin{bmatrix}0 & 0\\ 0 & D(q,p)\end{bmatrix}\tag{4.51}$$

and

$$g = \begin{bmatrix} 0 \\ G \end{bmatrix} \tag{4.52}$$

The Hamiltonian of this system is equal to the sum of kinetic and potential energies, and is given by

$$H(q,p) = \frac{1}{2}p^T M^{-1}(q)p + V(q) \tag{4.53}$$

where $M(q) = M(q)^T > 0$ is the system's mass matrix. Let q_d, be the twice differentiable desired trajectory to be tracked by the system (4.49). In [23], and [91], system (4.49) is transformed to a passive error system in PCH form using the following canonical transformation:

$$\Phi(q,p,t) = \begin{bmatrix} \bar{q}^T & \bar{p}^T \end{bmatrix}^T \tag{4.54}$$

$$\begin{bmatrix} \bar{q} \\ \bar{p} \end{bmatrix} = \begin{bmatrix} q - q_d(t) \\ p - M(q)\dot{q}_d(t) \end{bmatrix} \tag{4.55}$$

$$\bar{y} = M^{-1}(q)p - \dot{q}_d(t) \tag{4.56}$$

The positive definite Hamiltonian function of the transformed system is

$$\begin{aligned} \bar{H} &= H(q,p) + \frac{1}{2}\dot{q}_d^T(t)M(q)\dot{q}_d(t) \\ &\quad -p^T q_d(t) + \bar{U}(q - q_d(t)) \\ &= \frac{1}{2}(\dot{q} - \dot{q}_d(t))^T M(q)(\dot{q} - \dot{q}_d(t)) + \bar{U}(q - q_d(t)) \end{aligned} \tag{4.57}$$

For nominal systems, the transformed input $\bar{u}$ defined in (2.44) is given by

$$\begin{aligned}\bar{u} &= u + \beta_0 \\ &= u - M\ddot{q}_d + \frac{\partial \bar{U}}{\partial q} + \frac{\partial V}{\partial q} + \left(\frac{1}{2}\frac{\partial (M(q)\dot{q}_d)^T}{\partial q}\right. \\ &\quad \left. -\frac{1}{2}\frac{\partial M(q)\dot{q}_d}{\partial q} + \frac{1}{2}M(q)\frac{(\partial (M^{-1}(q)p)}{\partial q} - D\right)\dot{q}_d \end{aligned} \tag{4.58}$$

The resultant error PCH system is

$$\begin{aligned}\begin{bmatrix}\dot{\bar{q}} \\ \dot{\bar{p}}\end{bmatrix} &= \begin{bmatrix} 0 & I \\ -I & \bar{J}_2 - \bar{D}\end{bmatrix}\begin{bmatrix}\frac{\partial \bar{H}}{\partial \bar{q}} \\ \frac{\partial \bar{H}}{\partial \bar{p}}\end{bmatrix} + \begin{bmatrix} 0 \\ \bar{G}\end{bmatrix}\bar{u} \\ \bar{y} &= \bar{G}^T\frac{\partial \bar{H}}{\partial \bar{p}}\end{aligned} \tag{4.59}$$

with $\bar{J}_2$ a skew symmetric matrix. Using (2.49) - (2.53), one has $\bar{D} = D$, $\bar{G} = G$, and as shown in [25],

$\bar{J}_2 = \frac{\partial (M(q)\dot{q}_d)^T}{\partial q} - \frac{\partial M(q)\dot{q}_d}{\partial q}$

For a positive definite matrix K_p, let

$$\bar{U} = \frac{1}{2}\bar{q}^T K_p \bar{q} \tag{4.60}$$

For the transformed system, the Hamiltonian (4.57) can be expressed as

$$\bar{H}(\bar{q}, \bar{p}) = \frac{1}{2}\bar{p}^T M^{-1}(q)\bar{p} + \frac{1}{2}\bar{q}^T K_p \bar{q} \tag{4.61}$$

Note the fact that, since $\bar{q} = q - q_d$, therefore, $\frac{\partial\left(\bar{p}^T M(q)\bar{p}\right)}{\partial q} = \frac{\partial\left(\bar{p}^T M(q)\bar{p}\right)}{\partial \bar{q}}$. The gradient of $\bar{H}$ is given by

$$\frac{\partial \bar{H}}{\partial(\bar{q},\bar{p})}(\bar{q},\bar{p}) = \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{q}}(\bar{q},\bar{p}) \\ \frac{\partial \bar{H}}{\partial \bar{p}}(\bar{q},\bar{p}) \end{bmatrix} = \begin{bmatrix} K_p\bar{q} + \frac{\partial\left(\bar{p}^T M(q)\bar{p}\right)}{\partial q} \\ \dot{\bar{q}} \end{bmatrix} \tag{4.62}$$

A proper selection of $\mathcal{Z}$ yields

$$\Xi = \bar{g}^T \mathcal{Z} \frac{\partial \bar{H}}{\partial(\bar{q},\bar{p})}(\bar{q},\bar{p}) = \dot{\bar{q}} + \Lambda\bar{q} + f(\bar{q}) \tag{4.63}$$

where $\Lambda > 0 \in \Re^{n\times n}$. Note that by the virtue of IP filter $\mathcal{Z}$, The proposed NN tuning law (4.17) is driven by the RHS of (4.63), which is a combination of velocity as well as position error.

4.6 Simulation Examples

In the following subsections, the proposed $\mathcal{L}_2$ neuro-adaptive controller is applied to some benchmark dynamic systems.

4.6.1 Simple pendulum

A simple pendulum is shown in Fig. 4.1. A ball of mass m is attached to the lower end of the pendulum rod of length l and negligible mass. The upper end of the pendulum rod is attached to a motor which provides the input torque u to the system. In this example, θ is the generalized configuration coordinate,

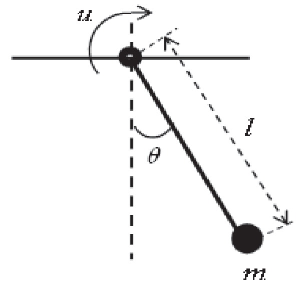

Figure 4.1: Simple pendulum

Table 4.1: Pendulum Parameters & Uncertainties

Parameter	Nominal magnitude	Uncertainty (%)
m	1Kg	100
l	1m	20

i.e. $q = \theta$ and $G = 1$. Assume that there is no friction, i.e. $D = 0$. Further description of the system is given as

$$M = ml^2 \tag{4.64}$$

The Hamiltonian of the system is given by:

$$\begin{aligned} H(\theta, p) &= K.E + P.E \\ &= \frac{1}{2}\frac{p^2}{ml^2} + mgl(1 - cos\theta) \end{aligned} \tag{4.65}$$

where $p = ml^2\dot{\theta}$. The nominal magnitudes of pendulum parameters and uncertainties are given in Table 4.1. It is desired to stabilize the system at $\theta_d = \frac{\pi}{2}$ rad. As a straightforward choice,

$\Phi(\theta, t) = \bar{\theta} = \theta - \theta_d$.

With such θ_d, $p_d = ml^2\dot{\theta}_d^2 = 0$ and therefore $\bar{p} = p$. Hamiltonian of the transformed system is given by

$$\bar{H}(\bar{\theta}, \bar{p}) = \frac{1}{2ml^2}p^2 + mgl\left(1 - cos\bar{\theta}\right) + \frac{1}{2}k_p\bar{\theta}^2 \tag{4.66}$$

such that, from (2.44),

$$\begin{aligned} U &= mgl\left(1 - cos\bar{\theta}\right) + \frac{1}{2}k_p\bar{\theta}^2 - mgl\left(1 - cos\theta\right) \\ &= mgl\{cos\theta - cos\bar{\theta}\} + \frac{1}{2}k_p\bar{\theta}^2 \end{aligned} \tag{4.67}$$

NN and controller parameters: To estimate the uncertainty $\mathcal{F}(\cdot)$, a NN with six neurons ($\nu = 6$) is used. $\Gamma = 1000I_6$ and $\kappa = 0.001$. A Hyperbolic tangent is used as an activation function. Note that there is no need to find a regressor, as required in other adaptive control schemes. Furthermore, to compute the penalty signal $z(\cdot)$ in Eq. (4.24), $h(\cdot)$ is chosen as

$h(\bar{x}) = h(\bar{\theta}) = \theta - \theta_d$,

$C = K_p = 10$

The initial condition is $[\theta_0 \quad \dot{\theta}_0]^T = [0 \quad 0]^T$. The NN weights are initialized to zero. The disturbance attenuation gain γ is set to 0.1. The external disturbance ξ is the unit variance Gaussian random noise. The filter $\mathcal{Z}$ is chosen as:

$$\mathcal{Z} = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

which yields $\Lambda = 5$, and

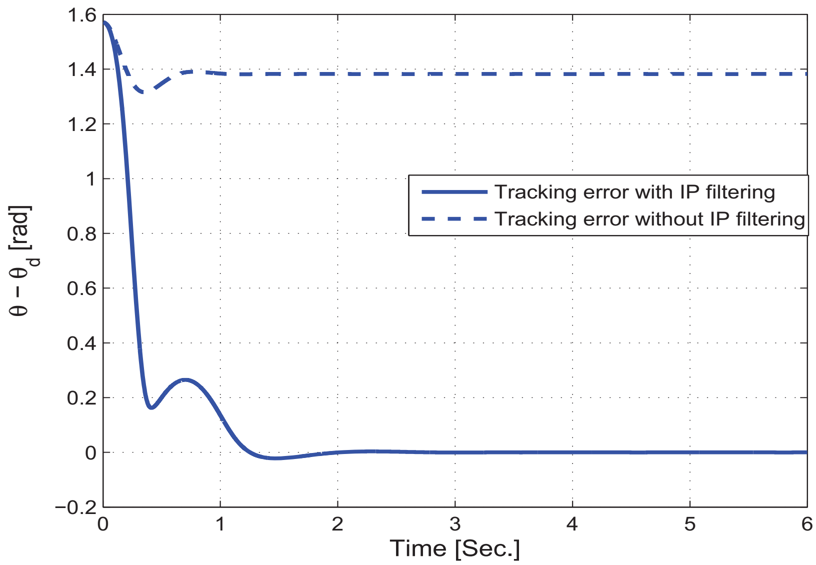

Figure 4.2: Tracking error

$$\bar{g}^T \mathcal{Z} \frac{\partial(\bar{H})}{\partial(\bar{\theta})} = \dot{\bar{\theta}} + 5\bar{\theta} + f(\bar{\theta}) \tag{4.68}$$

where $f(\bar{\theta}) = 0.5mglsin(\bar{\theta})$.

Fig. 4.2 shows that the application of the proposed controller drives the difference between the actual angle and the desired angle to zero when the IP filtering is employed in the NN tuning law. A significant error is observed when IP filtering is not employed in the NN tuning law. This controller performance is achieved within the PCH formalism only by incorporating the IP filtering and unlike [28] no extra transformation is needed. Furthermore, the steady-state error is minimized without any integral action in the control law.

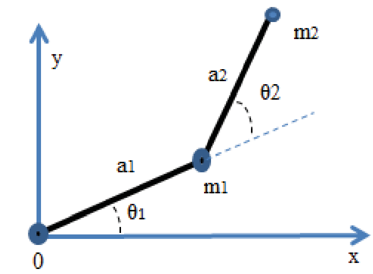

Figure 4.3: 2-Link manipulator

4.6.2 2-link planar manipulator

A fully actuated 2-link, 2 degrees of freedom planar manipulator is a benchmark mechanical system . This robot arm has all the nonlinear effects common to general robot system. Consider a manipulator, shown in Fig. 4.3, with masses m_i and link lengths a_i for $i = 1, 2$. The generalized configuration coordinates are the link angles θ_1 and θ_2, and therefore $q = \theta = [\theta_1 \ \theta_2]^T$. The system is described by (4.49) with
$u = \begin{bmatrix} \tau_1 & \tau_2 \end{bmatrix}^T$, where τ_1 and τ_2 are the torques applied to joint 1 and 2, respectively. $G = I_2$, $D = diag(d_1, d_2)$,
$M(\theta) = \begin{bmatrix} M_{11}(\theta) & M_{12}(\theta) \\ M_{21}(\theta) & M_{22}(\theta) \end{bmatrix}$ where
$M_{11}(\theta) = (m_1 + m_2)a_1^2 + m_2 a_2^2 + 2m_2 a_1 a_2 cos\theta_2$,

$M_{12}(\theta) = M_{21}(\theta) = m_2 a_2^2 + m_2 a_1 a_2 cos\theta_2$,

$M_{22}(\theta) = m_2 a_2^2$.

Note that $\bar{\theta} = \theta - \theta_d$. The manipulator works in the horizontal plane, therefore, the gravity effects are negligible and $V(\theta) = 0$. The nominal magnitudes of the system parameters and uncertainties are given in Table 4.2. The desired

Table 4.2: Manipulator Parameters & Uncertainties

Parameter	Nominal magnitude	Uncertainty (%)
$m_1,\ m_2$	1Kg	100
$a_1,\ a_2$	1m	10
$d_1,\ d_2$	1N-s/m	150

trajectories are:

$\theta_{1d} = sin(t)$, $\theta_{2d} = cos(t)$. **NN and controller parameters:** Since there are two inputs, two separate NNs are used, each with six neurons ($\nu = 6$). All parameters of the two NNs are the same as those used in Example 1. Furthermore,

$h(\bar{x}) = h(\bar{\theta}) = \theta - \theta_d$,

$C = K_p = diag(20, 20)$

The initial condition is $[\theta_0^T \quad \dot{\theta}_0^T] = [1 \quad 0 \quad 0 \quad 0]$rad. The disturbance attenuation gain γ is set to 0.1. The external disturbance ξ is the unit-variance random Gaussian noise. The filter $\mathcal{Z}$ is chosen as:

$$\mathcal{Z} = \begin{bmatrix} 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.25 \\ 0.25 & 0 & 1 & 0 \\ 0 & 0.25 & 0 & 1 \end{bmatrix}$$

which yields $\Lambda = diag(5,\ 5)$, and

$$\bar{g}^T \mathcal{Z} \frac{\partial(\bar{H})}{\partial(\bar{\theta})} = \dot{\bar{\theta}} + \Lambda\bar{\theta} + f(\bar{\theta}) \tag{4.69}$$

with $f(\bar{\theta}) = [0 \quad -0.125\{m_2 a_1 a_2 \dot{\bar{\theta}}_1 (\dot{\bar{\theta}}_1 + \dot{\bar{\theta}}_2) sin\theta_2\}]^T$. Fig. 4.4 shows that the links follow the desired trajectories with negligible errors. Fig. 4.5 shows the tracking errors.

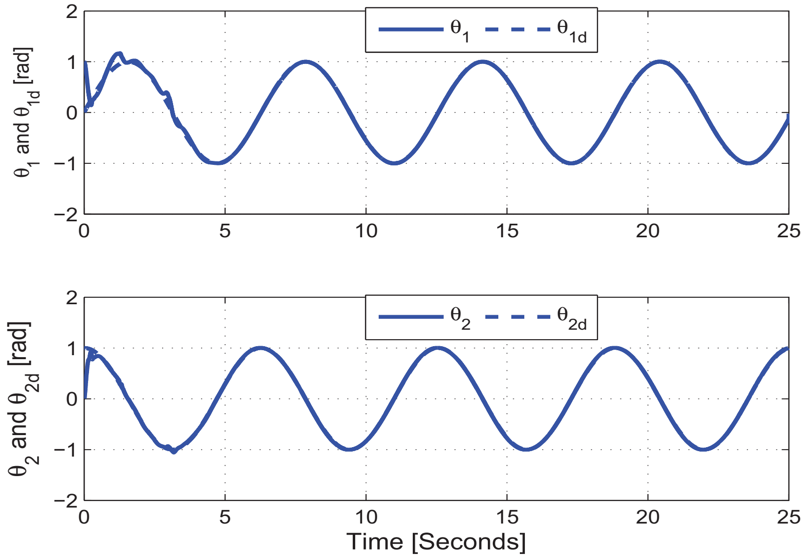

Figure 4.4: State evolution

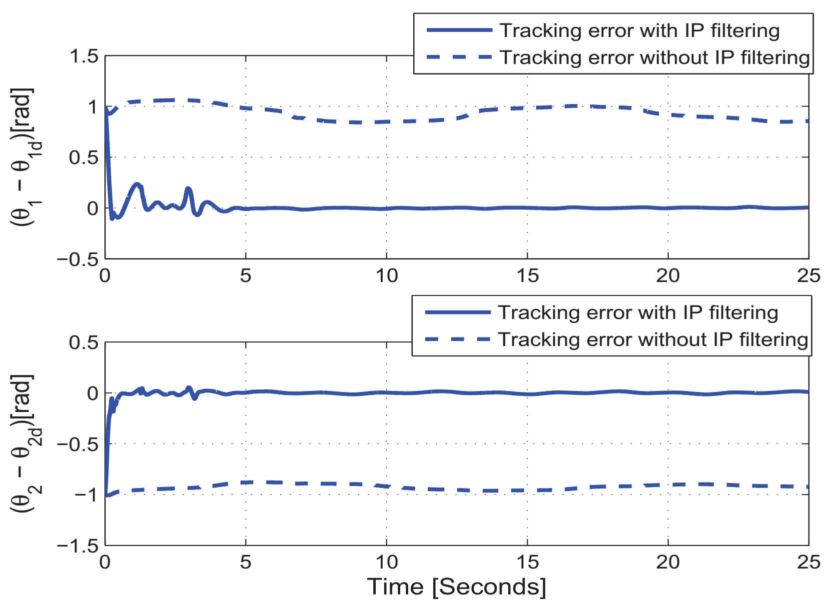

Figure 4.5: Tracking error

4.7 Canonical Transformation of PCH Systems Using Neural Networks

For the trajectory tracking control, a PCH system is first transformed from state-space to error-space using the state feed-back $\beta(x)$, which is obtained from the solution the PDE (2.49). Solution of a similar PDE is also required in the IDA-PBC, described in Sec. 2.3.4, to obtain a state feedback component of the control law. As a matter of fact, as mentioned in [16], it is well known that solving PDEs is, in general, not easy.

On the other hand, literature survey reveals that NNs have been successfully applied to the problem of feedback-linearisation of nonlinear systems, when there are uncertainties in the systems's dynamics, or, when a particular system fails to fulfill the conditions of feedback-linearisation [74], [29], [92], [93]. Indeed, the so-called feedback-linearisation is a process of transforming a nonlinear system to a linear one by some state-feedback control. Such successful application of NNs to the problem of feedback-linearisation (or transformation) gives a strong motivation to use the NNs in a likewise manner for the canonical transformation of PCH systems, since, solving the PDEs to obtain $\beta(x)$ can become very complicated in some practical cases. Estimation of the stat-feedback $\beta(x)$ for the canonical transformation of PCH systems using NNs is explained in the following.

Recall the lumped uncertainty, represented by the unknown function $\bar{\mathcal{F}}$ and defined in Eq. (4.8). This unknown function is redefined to include the state-feedback component $\beta(x)$ as well. The new unknown function, denoted by $\bar{\mathcal{F}}_\beta$, is obtained by a slight modification of (4.8), as follows:

$$\begin{aligned} \bar{\mathcal{F}}_{\beta}(\bar{x},\varepsilon) &= [\bar{J}(\bar{x},\varepsilon_J) - \bar{R}(\bar{x},\varepsilon_R)]\frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x},\varepsilon_H) \\ &\quad -[\bar{J}_0(\bar{x}) - \bar{R}_0(\bar{x})]\frac{\partial \bar{H}_0}{\partial \bar{x}}(\bar{x}) + \bar{g}(\bar{x},t)\beta(x) \end{aligned} \tag{4.70}$$

Note that the last term, $\bar{g}(\bar{x},t)\Delta_{\beta}(x,\varepsilon_{\beta})$, in Eq. (4.8) is replaced by $\bar{g}(\bar{x},t)\beta(x)$ in Eq. (4.70), which simply means that, this time, the modified unknown function $\bar{\mathcal{F}}_{\beta}(\bar{x},\varepsilon)$ also includes the state-feedback component $\beta(x)$. In this case, the NN will be used to estimate the $\bar{\mathcal{F}}_{\beta}(\bar{x},\varepsilon)$ to compensate for the uncertainties as well as to perform the canonical transformation. It is straightforward to observe that the NN tuning law (4.17) as well as the control law (4.25) do not require any alteration and essentially remain unchanged, and thus, the closed-loop stability is guaranteed.

4.7.1 Trajectory tracking control of an Autonomous Underwater Vehicle (AUV)

The dynamics of an AUV in PCH form is given by:

$$\begin{aligned} \begin{bmatrix} \dot{\eta} \\ \dot{p} \end{bmatrix} &= \begin{bmatrix} 0 & J_a \\ -J_a^T & -\bar{D}(\nu) \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial \eta} \\ \frac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} [\tau - g(\eta) + \xi] \\ y &= \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} = \frac{\partial H}{\partial p} \end{aligned} \tag{4.71}$$

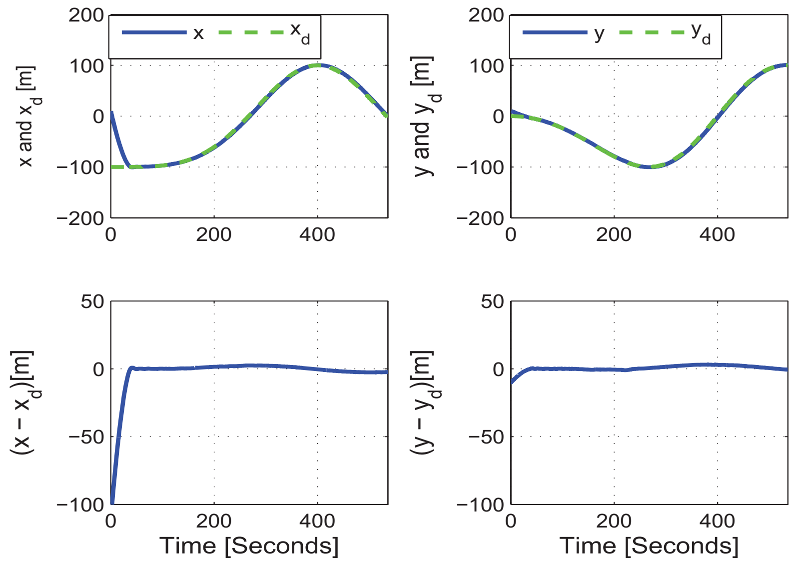

Figure 4.6: MARES trajectories

where $\eta \in \Re^6$ denotes position and attitude vector in the earth fixed frame, $p \in \Re^6$ denotes the vector of generalized momenta. The Hamiltonian, $H = \frac{1}{2}\nu^T M \nu$, where $\nu \in \Re^6$ denotes the body fixed linear and angular velocity vector. $M \in \Re^{6\times6}$ is the inertia matrix. $J_a \in \Re^6$ is a rotation transformation matrix. $\bar{D} \in \Re^{6\times6}$ is the drag matrix. $g(\eta) \in \Re^6$ is the vector of gravitational forces and moments. Simulations are performed on MARES AUV [94], which is a highly nonlinear system due to the structures of drag matrix $D(\nu)$ and inertia M. Details of the MARES model can be found in [94], [95] and [96]. Solving the PDE (2.49) in this case becomes very difficult to obtain $\beta(\cdot)$ for the canonical transformation.

The AUV is desired to move in the xy-plan with $x_d = -100cos(\frac{\pi}{400}t)$ and $y_d = -100sin(\frac{\pi}{400}t)$.

NN and controller parameters: A total of six NNs are used, each with six neurons. $\Gamma = 0.1I_6$ and $\kappa = 0.001$.

Information preserver $\mathcal{Z} = \begin{bmatrix} I_6 & I_6 \\ I_6 & I_6 \end{bmatrix}$.

Hamiltonian in the transformed space is: $\bar{H} = \frac{1}{2}\bar{\nu}^T M \bar{\nu} + \frac{1}{2}\bar{\eta}^T \bar{\eta}$. Disturbance attenuation gain, $\gamma = 0.1$. Fig. 4.6 shows that the AUV follows the desired trajectories, with bounded error.

4.8 Conclusion

In this chapter, a novel structure preserving neuro-adaptive control of uncertain PCH system is presented. The proposed controller is applicable to stabilization as well as regulation and tracking problems. The NN tuning law is driven by both the position and velocity errors by introducing the so-called information preserving filtered Hamiltonian-gradient. The proposed controller achieves the $\mathcal{L}_2$ disturbance attenuation objective. The UUB of the tracking errors and NN weight errors has been shown. The proposed controller is successfully applied to some standard mechanical systems. Simulations are performed on a simple pendulum and a 2-link planar manipulator to illustrate the proposed control scheme. In addition to neuro-adaptive control of PCH systems, this chapter also presents the NN-based canonical transformation of PCH systems. Simulations on an AUV illustrates the idea of NN-based canonical transformation.

CHAPTER 5

ROBUST NEURO-ADAPTIVE COOPERATIVE CONTROL OF MULTI-AGENT PORT CONTROLLED HAMILTONIAN SYSTEMS

This chapter presents the distributed cooperative tracking control of multi-agent Port Controlled Hamiltonian (PCH) systems networked through a directed graph. A leader node generates the desired trajectory to be tracked by all the individual PCH nodes in the group. Only few nodes can access the information from the leader node. Communication among the follower nodes is limited to the information of their neighbor nodes only. The controller is made robust against parametric uncertainties using Neural Networks. The robust neuro-adaptive control strategy developed in the previous chapter is modified for application to the cooperative control problem in this chapter with guaranteed stability of the closed loop dynamics. The PCH structure of the closed-loop system is preserved. The dynamics of the individual PCH agents are transformed to a *normalized local synchronization error (NLSE)* space using the canonical transformation theory described in the previous chapter. The controller also achieves the $\mathcal{L}_2$ disturbance attenuation objective. Simulations are performed on a group of robotic manipulators to demonstrate the efficacy of the proposed controller.

Chapter Organization: Sec. 5.1 presents the introduction to the chapter. In Sec. 5.2, some mathematical preliminaries are detailed. Problem formulation is described in Sec. 5.3. Distributed controller design for nominal PCH system is presented in Sec. 5.4, and then robustified in Sec. 5.5. Simulations are presented in Sect. 5.6. Finally Sec. 5.7 concludes the chapter.

5.1 Introduction

It has been asserted in Sec. 2.1 and Sec. 3.1 that most real-life systems possess nonlinear dynamics and researchers have developed a number of cooperative control schemes on the basis of a particular class of such nonlinear systems. The present work steps forward in the same direction by considering an important class of dynamics systems, namely the Port Controlled Hamiltonian (PCH) Systems, networked through a communication link. With the study of the available literature, it can be argued that the cooperative control of a group of PCH systems is addressed for the first time in this thesis. The only exception is [97], where passivity theory is utilized as a tool for group coordination.

In this chapter, a group of PCH systems interlinked through a communication network is considered. Exchange of information among the individual PCH systems can be well described by a graph with vertices representing the individual nodes i.e. the PCH systems and edges representing communication links among the nodes. The leader node generates the trajectory or command to be followed by other members of the group. Only a subset of the follower nodes have access to the trajectory information of the leader node. Information-sharing among the follower nodes is also limited to the neighbor nodes only. All of the follower nodes have to track (or follow) the leader node in spite of such constrained communication. In other words, all of the group nodes have to synchronize with the leader node as well as with one another. The strategy to achieve this goal is composed of the following steps.

1. a *normalized local synchronization error* (NLSE) at each node is defined, and then the dynamics of the individual PCH nodes is transformed from state-space to the NLSE space by employing the canonical transformation theory developed in [22] - [22] and briefly described in Ch. 2.

2. In the second step a distributed cooperative controller is designed using passive output feedback of the individual transformed systems with nominal dynamics and no external disturbance.

3. In the third step the real world scenario is considered by taking into account the parametric uncertainties and external disturbance. At this step use of Neural Network (NN) is made for the compensation of parametric uncertainties. A robust neuro-adaptive distributed cooperative controller is proposed, which also achieves the $\mathcal{L}_2$ disturbance attenuation.

Note that the neuro-adaptive controller is *direct*, i.e. off-line training of NN is not required.

In short, the contribution of this chapter is threefold, as described in the following.

- A mathematical framework for cooperative control of PCH systems is presented for the first time in the literature.
- To robustify the proposed approach against parametric uncertainties, neural network based adaptive controller with information-preserving filtered-gradient based novel tuning laws is proposed.
- Due to the generality of PCH systems, application of the proposed framework is not limited to standard mechanical systems, rather it can be applied to several other engineering systems, such as power systems and process control to name a few.

5.2 Preliminaries

Consider a group of $N(N \geq 2)$ PCH systems networked through a communication link. The $i^{th}(i = 1, 2, ..., N)$ PCH system is described as:

$$
\begin{aligned}
\dot{x}_i &= [J_i(x_i, t) - R_i(x_i, t)] \frac{\partial H_i}{\partial x_i}(x_i, t) + g_i(x_i, t)u_i \\
y_i &= g_i(x_i, t)^T \frac{\partial H_i}{\partial x_i}(x_i, t)
\end{aligned}
\tag{5.1}
$$

where, for the i^{th} agent, $x_i \in \Re^n$ is the state vector. Positive semi-definite function $H_i(x_i, t) : \Re^n \mapsto \Re$ is the Hamiltonian function representing the energy stored in the ith system. The column vector $\frac{\partial H_i}{\partial x_i}(x_i, t) = [\frac{\partial H(x_i,t)}{\partial x_{i_1}} ... \frac{\partial H(x_i,t)}{\partial x_{i_n}}]^T$ denotes the gradient of scalar function $H(x, t)$. Matrices $J_i(x_i, t) = -J_i(x_i, t)^T \in \Re^{n\times n}$ and $g_i(x_i, t) \in \Re^{m\times n}$ collectively define the *interconnection* structure of the system. $R_i(x_i, t) = R_i^T(x, t) \geq 0 \in \Re^{n\times n}$ represents the *dissipation*. All these matrices, may, smoothly depend on x_i. Note that all these definitions are the same as those used in the previous chapter, and are repeated here for the sake of clarity.

In this chapter, the cooperative tracking problem is directly addressed, however, the theoretical developments are equally applicable to the consensus and (in the case of Euler-Lagrange systems,) to the attitude alignment problems as well. Depending upon the problem at hand, (tracking, alignment or consensus) appropriate canonical transformation is performed before the required controller is designed. The theory of canonical transformation of PCH systems has been described in the Ch. 2. The canonical transformation theory is applicable to a PCH system which is operated either independently or as part of multi-agent group. The subscript i is retained for notational uniformity throughout the thesis.

5.3 Problem Formulation

In this section, the cooperative tracking control problem of a networked group of N PCH systems is formulated. Define

$x = [x_1^T \;\; x_2^T \;\; ... \;\; x_N^T]^T \in \Re^{nN}$

Then the group of N PCH systems can be collectively described as

$$\begin{aligned} \dot{x} &= [J(x,t) - R(x,t)]\,\frac{\partial H}{\partial x}(x,t) + g_i(x,t)u \\ y &= G(x,t)^T \frac{\partial H}{\partial x}(x,t) \end{aligned} \tag{5.2}$$

where $J = -J^T = diag(J_1, J_2, ..., J_N) \in \Re^{nN \times nN}$

$R = R^T = diag(R_1, R_2, ..., R_N) \geq 0 \in \Re^{nN \times nN}$

$H = H_1 + H_2 + ... + H_N \in \Re$

$\frac{\partial H(x,t)}{\partial x} = [\frac{\partial H_1^T(x_1,t)}{\partial x_1} \;\; \frac{\partial H_2^T(x_2,t)}{\partial x_2} \;\; ... \;\; \frac{\partial H_N^T(x_N,t)}{\partial x_N}]^T \in \Re^{nN}$

$g = diag(g_1, g_2, ..., g_N) \in \Re^{nN \times mN}$

$u = [u_1^T \;\; u_2^T \;\; ... \;\; u_N^T] \in \Re^{mN}$

$y = [y_1^T \;\; y_2^T \;\; ... \;\; y_N^T] \in \Re^{mN}$

Let the virtual leader generate the desired trajectory $x_0(t) = [x_{0_1} \;\; x_{0_2} \; ... \; x_{0_n}]^T \in \Re^n$ to be tracked by all the agents and let $x_i = [x_{i_1} x_{i_2} \;\; ...x_{i_n}]^T$. For $k = 1, 2, ..., n$, the k^{th} order tracking error for node $i (i \in \mathcal{N})$ is defined as $\delta_{i_k} = x_{i_k} - x_{0_k}$. Let

$\delta_i = [\delta_{i_1} \quad \delta_{i_2} \cdots \delta_{i_n}]^T \in \Re^n$,

$\delta^k = [\delta_{1_k}, \cdots, \delta_{N_k}]^T \in \Re^N$,

$x^k = [x_{1_k} \quad x_{2_k}, \cdots, x_{N_k}]$ and

$\underline{x}_{0_k} = [x_{0_k}, \cdots, x_{0_k}] \in \Re^N$ then $\delta^k = x^k - \underline{x}_{0_k}$.

Note the difference between $\delta_i \in \Re^n$ and $\delta^k \in \Re^N$. Further developments of this chapter are accomplished in the following two stages:

1. Design of distributed controllers with global asymptotic stability of tracking errors $\delta_i^k(t)$.

2. In the second stage, the parametric uncertainty and the presence of external disturbance in the dynamics of all the followers is considered and is followed by the design of the distributed controllers such that the global tracking error $\delta_i^k(t)$ for $k = 1, 2, ..., n$ converges to a close neighborhood of zero.

Convergence of global tracking error $\delta(t)$ can be well explained by extending the well known notion of cooperative uniform ultimate boundedness (CUUB) defined in Ch. 3. Since the structure of controller to be designed in this chapter is distributed, only relative state information is used in the controller design. Equivalently, following [77], the neighborhood synchronization error, $e_i \in \Re^n$, at node i is given by

$$e_i = \sum_{j \in N_i} a_{ij}(x_i - x_j) + b_i(x_i - x_0), \qquad i = 1, ..., N \tag{5.3}$$

where $b_i \geq 0$ is the weight of the edge from the leader node 0 to node i, $(i \in N)$. Note that $b_i > 0$ if and only if there is an edge from the node 0 to node i.

Let $e = [e_1^T \quad e_2^T ... e_N^T]^T \in \Re^{nN}$ be the global synchronization error vector, then straightforward algebra yields

$$\begin{aligned} e &= (L+B) \otimes I_n(x - \underline{x}_0) \\ e &= (L+B) \otimes I_n \delta \end{aligned} \tag{5.4}$$

where $\otimes$ represents the Kronecker product operator, and

$$\delta = [\delta_1^T \quad \delta_2^T \quad \cdots \quad \delta_N^T]^T \in \Re^{nN} \tag{5.5}$$

Let $e^k = [e_{1_k} \quad ... \quad e_{N_k}]^T \in \Re^N$, for $k = 1, 2, ..., n$, then from (5.4), one has

$$e^k = (L+B)\delta^k \tag{5.6}$$

Define

$x_A = [x^T \quad x_0^T]^T \in \Re^{n(N+1)}$

$B_c = [b_1 \quad b_2 \quad ... \quad b_N]^T \in \Re^N$

$A_B = [A \quad B_c] \in \Re^{N \times (N+1)}$,

Let A_{B_i} be the i^{th} row of A_B, then (5.3) can be rewritten as

$$e_i = (d_i + b_i)x_i - A_{B_i} \otimes I_n x_A \tag{5.7}$$

Define the *normalized local synchronization reference* (NLSR) as

$$x_{d_i} = \frac{1}{d_i + b_i} A_{B_i} \otimes I_n x_A \tag{5.8}$$

The *normalized local synchronization error* (NLSE) $\bar{x}_i = \in \Re^n$ is defined as

$$\bar{x}_i \quad = \quad \frac{1}{d_i + b_i} e_i = x_i - x_{d_i} \tag{5.9}$$

Let $\bar{x} = [\bar{x}_1 \;\; \bar{x}_2 \;\; ... \;\; \bar{x}_N]^T \in \Re^{nN}$, then (5.7) and (5.9) imply that

$$\bar{x} = (D + B)^{-1} \otimes I_n e = x - (D + B)^{-1} A_B \otimes I_n x_A \tag{5.10}$$

Now the canonical transformation is performed at each node, to transform the system from state space x_i to the NLSE space $\bar{x}_i$. Defining $\Phi_i = \bar{x}_i$, it can be argued that the transformed system represents the error system, because

$$\bar{x}_i = 0 \Longrightarrow x_i = x_{d_i} \tag{5.11}$$

The following Lemma relates the convergence of $\bar{x}$ with the convergence of δ, and will be required later in the stability analysis.

Lemma 5.1

$$\bar{x} = 0 \Longleftrightarrow \delta = 0 \tag{5.12}$$

Proof: If $\bar{x} = 0$ then Eq. (5.10) implies that $e = 0$. Eq. (5.4) further implies that if $e = 0$ then $\delta = 0$. $\square$

The ith PCH system in transformed space is given by

$$\dot{\bar{x}}_i = \left[\bar{J}_i(\bar{x}_i, t) - \bar{R}_i(\bar{x}_i, t)\right] \frac{\partial \bar{H}_i}{\partial \bar{x}_i}(\bar{x}_i, t) + \bar{g}_i(\bar{x}_i)\bar{u}_i \tag{5.13}$$

$$\bar{y}_i = \bar{g}_i(\bar{x}_i)^T \frac{\partial \bar{H}_i}{\partial \bar{x}_i}(\bar{x}_i, t) \tag{5.14}$$

The combination of N tracking error systems described by (5.13) operating cooperatively through a communication network can be termed as *cooperative tracking error system.*

The augmented graph $\bar{\mathcal{G}}$ is defined as, $\bar{\mathcal{G}} = \{\bar{\mathcal{V}}, \bar{E}\}$, where $\bar{\mathcal{V}} = \{\nu_0\ \nu_1, ..., \nu_N\}$ and $\bar{E} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. As in Ch. 3, the following assumption is made on the graph topology for the distributed cooperative control of this paper.

Assumption 5.1 *There exists a spanning tree in the augmented graph $\bar{\mathcal{G}}$ having the leader node* 0 *as the root node.*

5.4 Distributed Cooperative Controller Design

Design of the distributed cooperative tracking controller is accomplished in two steps. First, the individual PCH systems (5.1) are transformed to the NLSE space using both Theorem 2.3 and the following transformation

$$\begin{aligned}\Phi_i(x_i, x_{d_i}) &= x_i - x_{d_i} \\ &= \bar{x}_i \end{aligned} \tag{5.15}$$

In the second step, the transformed system is stabilized with the static output feedback controls, (using Theorem 2.2). The β_i for Φ_i defined in (5.15) is computed as follows. Eq. (5.15) implies that

$$\begin{aligned}\frac{d\bar{x}}{dt} &= \frac{\partial \Phi_i}{\partial x_i}\frac{dx}{dt} + \frac{\partial \Phi_i}{\partial t} \\ \Longrightarrow \frac{\partial \Phi_i}{\partial t} &= -\dot{x}_{di}\end{aligned} \tag{5.16}$$

. Using Eq. (5.15) and (5.16), one has

$$\frac{\partial \Phi_i}{\partial (x_i, t)} = \begin{bmatrix} 1 & -\dot{x}_{di} \end{bmatrix} \tag{5.17}$$

Putting (5.17) in (2.49) and noting that $\frac{\partial \Phi_i}{\partial x} = \frac{\partial \Phi_i}{\partial \bar{x}}$, one obtains

$$g_i\beta_i = -[J_i - R_i]\frac{\partial H_i}{\partial x_i} - [K_i - S_i]\frac{\partial (H_i + U_i)}{\partial x_i} - \dot{x}_{di} \tag{5.18}$$

Applying $u_i = \bar{u} - \beta_i$, transforms the PCH system (5.1) to the equivalent PCH system (5.13). Applying the output feedback control,

$$\bar{u}_i = -C\bar{y}_i = -C\bar{g}_i^T\frac{\partial \bar{H}}{\partial \bar{x}} \tag{5.19}$$

to system (5.13) yields the following closed-loop dynamics

$$\dot{\bar{x}}_i = \left[\bar{J}_i(\bar{x}_i, t) - \bar{R}_{ci}(\bar{x}_i, t)\right] \frac{\partial \bar{H}_i}{\partial \bar{x}_i}(\bar{x}_i, t) \tag{5.20}$$

where $\bar{H}_i = H_i + U_i$ and

$\bar{R}_{ci} = \bar{R}_i + g_i C g_i^T$. For $i = 1, 2, ..., N$ the networked closed loop PCH systems (5.20) can be combined as

$$\dot{\bar{x}} = \left[\bar{J}(\bar{x}, t) - \bar{R}_c(\bar{x}, t)\right] \frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x}, t) \tag{5.21}$$

where $\bar{x} = [\bar{x}_i^T \quad \bar{x}_2^T \cdots \bar{x}_N^T]^T \in \Re^{nN}$,

$\bar{J} = diag\left(\bar{J}_1, \bar{J}_2, \cdots \bar{J}_N\right) \in \Re^{nN \times nN}$,

$\bar{R}_c = diag\left(\bar{R}_{c1i}, \bar{R}_{c2i}, \cdots \bar{R}_{cN}\right) \in \Re^{nN \times nN}$ and

$\frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x}) = \left[\frac{\partial \bar{H}_1^T}{\partial \bar{x}_1}(\bar{x}_1) \quad \frac{\partial \bar{H}_2^T}{\partial \bar{x}_2}(\bar{x}_2) \cdots \frac{\partial \bar{H}_N^T}{\partial \bar{x}_N}^T(\bar{x}_N)\right] \in \Re^{nN}$.

The following corollary of Theorem 2.2 establishes the fact that the group of N PCH follower systems cooperatively track the leader with asymptotic stability using local information only.

Corollary 5.1 *Suppose that Assumption 5.1 holds. The group of N PCH systems (5.1) networked through a directed communication graph $\mathcal{G}$, cooperatively track the leader trajectory x_0 using the distributed cooperative tracking controller defined in (5.18) and (5.19), rendering the local synchronization error system (5.13) asymptotically sta ble.* $\diamondsuit$

Corollary 5.1 presents the fundamental result of distributed cooperative tracking control of PCH systems. The controller performance is further improved by considering the practical issues of parametric uncertainty and external disturbance, in the next section.

5.5 $\mathcal{L}_2$ Neuro-Adaptive Distributed Cooperative Tracking Control

The performance of the distributed cooperative controller (5.18) and (5.19) depends upon the exact knowledge of system parameters (J, R) and on the accurate computation of Hamiltonian H. However, in practice, parametric uncertainty may exist and, in some cases, such uncertainties may become so significant as to adversely affect the controller performance. Furthermore, external disturbances and sensor noise may also significantly degrade the controller performance. In this section, the controller given by (5.18) and (5.19), is modified such that the resulting controller is robust to parametric uncertainties and external disturbances.

The strategy is to employ Neural Networks (NNs) for the estimation of the collective impact of the parameter uncertainties. This estimate will be incorporated in the controller structure to significantly reduce the effect of parametric uncertainties. In addition, the controller design will also consider the $\mathcal{L}_2$ disturbance attenuation of a class of external disturbances.

The analysis given in the subsequent sections of this chapter is a straightforward application of the theory developed in Ch. 4. However, in most places the analysis is repeated for the purpose of clarity. Similar to Ch. 4, consider the following uncertain autonomous PCH system.

$$\begin{aligned} \dot{x} &= [J_i(x_i, \varepsilon_{J_i}) - R_i(x_i, \varepsilon_{R_i})] \frac{\partial H_i}{\partial x_i}(x_i, \varepsilon_{H_i}) \\ &\quad + g_i(x_i)(u_i + \xi_i) \qquad (5.22) \\ y_i &= g_i(x_i)^T \frac{\partial H_i}{\partial x_i}(x_i, \varepsilon_{H_i}) \qquad (5.23) \end{aligned}$$

where $\xi_i \in \Re^m$ is the unknown but bounded disturbance.

To facilitate the analysis, let the uncertain structure matrices in (5.22) be represented as:

$$\begin{aligned} J_i(x_i, \varepsilon_{J_i}) &= J_{0i}(x_i) + \Delta_{J_i}(x_i, \varepsilon_{J_i}) \\ R_i(x_i, \varepsilon_{R_i}) &= R_{0i}(x_i) + \Delta_{R_i}(x_i, \varepsilon_{R_i}) \\ \frac{\partial H_i}{\partial x_i}(x_i, \varepsilon_{H_i}) &= \frac{\partial H_{0i}}{\partial x_i}(x_i) + \Delta_{H_i}(x_i, \varepsilon_{H_i}) \end{aligned} \qquad (5.24)$$

where $\varepsilon_j, j \in \{J_i, R_i, H_i\}$ denotes the uncertainties in the parameters. Define $\varepsilon = [\varepsilon_{R_i} \quad \varepsilon_{J_i} \quad \varepsilon_{H_i}]^T$. Let $\beta_i(x_i, \varepsilon_{\beta_i})$ as obtained from the solution of the PDE (2.49) for the system (5.22) with $\varepsilon = 0$ and expressed as:

$$\beta(x_i, \varepsilon_{\beta_i}) = \beta_{0i}(x_i) + \Delta_{\beta_i}(x_i, \varepsilon_{\beta_i}) \qquad (5.25)$$

where $\beta_{0i}(x_i)$ corresponds to the solution for the nominal system. $\Delta_{\beta_i}(x_i, \varepsilon_{\beta_i})$ is the corresponding uncertainty.

Application of $\beta_i(x_i)$ transforms the uncertain system(5.22) from state space to the following system in neighborhood synchronization error space.

$$\begin{aligned} \dot{\bar{x}}_i &= \left[\bar{J}_i(\bar{x}_i, \varepsilon_{J_i}) - \bar{R}_i(\bar{x}_i, \varepsilon_{R_i})\right] \frac{\partial \bar{H}_i}{\partial \bar{x}_i}(\bar{x}_i, \varepsilon_{H_i}) \\ &\quad + \bar{g}_i(\bar{x}_i)(\Delta_{\beta_i}(x_i, \varepsilon_{\beta_i}) + \bar{u}_i + \xi_i) \qquad (5.26) \\ \bar{y}_i &= \bar{g}_i(\bar{x}_i)^T \frac{\partial \bar{H}_i}{\partial \bar{x}_i}(\bar{x}_i, \varepsilon_{H_i}) \qquad (5.27) \end{aligned}$$

Define $\bar{\mathcal{F}}_i(\bar{x}_i, \varepsilon_i) \in \Re^n$ as

$$\begin{aligned} \bar{\mathcal{F}}_i(\bar{x}_i, \varepsilon_i) \quad &= [\bar{J}_i(\bar{x}_i, \varepsilon_{J_i}) - \bar{R}_i(\bar{x}_i, \varepsilon_{R_i})] \frac{\partial \bar{H}_i}{\partial \bar{x}_i}(\bar{x}_i, \varepsilon_{H_i}) \\ &\quad - [\bar{J}_{0i}(\bar{x}_i) - \bar{R}_{i0}(\bar{x}_i)] \frac{\partial \bar{H}_{0i}}{\partial \bar{x}_i}(\bar{x}_i) \\ &\quad \bar{g}_i(\bar{x}_i, t)\Delta_{\beta_i}(x_i, \varepsilon_{\beta_i}) \qquad (5.28) \end{aligned}$$

Using (5.28), the uncertain error system (5.26) can be expressed as:

$$\begin{aligned} \dot{\bar{x}}_i &= [\bar{J}_{0i}(\bar{x}_i) - \bar{R}_{0i}(\bar{x}_i)] \frac{\partial \bar{H}_{0i}}{\partial \bar{x}_i}(\bar{x}_i) \\ &\quad + \bar{\mathcal{F}}_i(\bar{x}_i, \varepsilon_i) + \bar{g}_i(\bar{x}_i)(\bar{u}_i + \xi_i) \qquad (5.29) \end{aligned}$$

Note that $\bar{\mathcal{F}}$ collectively represents the parametric uncertainty of the transformed system (5.26).

Assumption 5.2 *The collective parametric uncertainty $\bar{\mathcal{F}}$ in (5.28) can be ex-*

pressed as:

$$\bar{\mathcal{F}}_i(\bar{x}_i, \varepsilon_i) = \mathcal{Z}_i \bar{g}_i(x_i) \mathcal{F}_i(\bar{x}_i, \varepsilon_i) \tag{5.30}$$

where the so called information preserver $\mathcal{Z}_i \in \Re^{n \times n}$ is a symmetric design matrix and the unknown function $\mathcal{F}_i(\bar{x}_i, \varepsilon_i) \in \Re^m$ is locally Lipschitz.

Let

$$\bar{u}_i = -C_i(\bar{x}_i, t)\bar{y}_i + \vartheta_i \tag{5.31}$$

where $[-C_i(\bar{x}_i, t)\bar{y}_i]$ is the passive output feedback part of the control input according to Theorem 2.2 and ϑ_i is the $\mathcal{L}_2$ neuro-adaptive component of the control to account for all parametric uncertainties and disturbance attenuation. Applying $\bar{u}_i$ in (5.31) to the uncertain system (5.26) leads to

$$\begin{aligned} \dot{\bar{x}}_i &= [\bar{J}_{0i}(\bar{x}) - \bar{R}_{ci}(\bar{x})] \frac{\partial \bar{H}_{0i}}{\partial \bar{x}_i}(\bar{x}_i) + \mathcal{Z}_i \bar{g}_i \mathcal{F}_i(\bar{x}_i, \varepsilon_i) \\ &\quad + \bar{g}_i(\bar{x}_i)(\vartheta_i + \xi_i) \end{aligned} \tag{5.32}$$

where

$$\bar{R}_{ci}(\bar{x}_i) = \bar{R}_{0i}(\bar{x}_i) + \bar{g}_i(\bar{x}_i, t) C_i(\bar{x}_i, t) \bar{g}_i^T(\bar{x}_i, t) \tag{5.33}$$

The focus of this chapter can now be stated as:

1. *Find a NN tuning law for the estimation of $\mathcal{F}_i(\bar{x}_i, \varepsilon_i)$. The NN tuning law should be driven by both, the position as well as the velocity error.*

2. *Design the feedback controller ϑ_i to compensate for the uncertainties in the*

PCH error system (5.22) using the NN approximation of $\mathcal{F}_i(\bar{x}_i, \varepsilon_i)$. The controller should achieve the $\mathcal{L}_2$ disturbance attenuation objective and preserve the PCH structure of the closed loop system as well.

3. *The controller should ensure the Cooperative Uniform Ultimate Boundedness(CUUB) of the local synchronization error $\bar{x}$ and $\tilde{W}$ (as defined later in eq. (5.49)).*

5.5.1 Approximation of uncertainties using neural network

Let $\Omega_i \subset \Re^n$ be a compact, simply connected set and $\mathcal{F}_i(\cdot) : \Omega_i \to \Re^m$. Define $\mathcal{C}_i^m(\Omega_i)$ as the space of continuous function $\mathcal{F}_i(\cdot)$. Then, for all $\mathcal{F}_i(\cdot) \in \mathcal{C}_i^m(\Omega_i)$, there exist weights W_i such that

$$\mathcal{F}_i(\bar{x}_i, \varepsilon_i) = \phi_i(\bar{x}_{nni})^T W_i + \epsilon_i \tag{5.34}$$

The weights $W_i = \left[W_{i1}^T ... W_{im}^T\right] \in \Re^{m\nu}$ with $W_{ij} \in \Re^{\nu},\ i = 1, ..., N, j = 1, ..., m$ are the ideal neural network weights, $\phi_i = diag(\phi_{i1}, ...\phi_{im}^T)\Re^{m\nu \times m}$ with $\phi_{ij} \in \Re^{\nu},\ i = 1, ..., N, j = 1, ..., m$ are some basis functions. $\bar{x}_{nni}$ is the input to the NN at node i, given by:

$$\bar{x}_{nni} = \left[\bar{x}_i^T\ \dot{\bar{x}}_i^T\ x_d^T\ \dot{x}_{id}^T\ \ddot{x}_{di}^T\right]^T \tag{5.35}$$

$\epsilon_i \in \Re^m$ is the NN approximation error.

In practical applications ideal weights W_i and error ϵ_i can not be evaluated, rather, the function $\mathcal{F}_i(\cdot)$ is approximated instead as:

$$\hat{\mathcal{F}}_i(\bar{x}_i) = \phi_i(\bar{x}_{nni})^T \hat{W}_i(t) \tag{5.36}$$

where $\hat{W}_i(t) \in \Re^{m\nu}$ are the actual time-varying weight of the NNs.

Remark 5.1 *Similar to Ch. 3 & 4, FLNN is used in this chapter for the approximation of* $\mathcal{F}_i(\cdot)$

In addition to the closed-loop system's stability, the particular motivations behind our proposed NN tuning law are:

1. PCH structure of the closed-loop dynamics augmented with NN's dynamics is preserved.

2. NN dynamics are driven by position as well as velocity errors.

The concept of IP filtering introduced in Ch. 4 is utilized here to drive the NN tuning law by position as well as velocity errors.

5.5.2 NN tuning law

The following NN tuning law is proposed for the approximation of $\mathcal{F}_i(\cdot)$.

$$\dot{\hat{W}}_i = \Gamma_i \phi_i(x) \bar{g}_i^T(\bar{x}_i) \mathcal{Z}_i \frac{\partial \bar{H}_{0i}}{\partial \bar{x}_i} - \kappa \Gamma_i \| \bar{g}_i^T(\bar{x}_i) \mathcal{Z}_i \frac{\partial \bar{H}_i}{\partial \bar{x}_i} \| \hat{W}_i \tag{5.37}$$

where $\Gamma_i = diag(\Gamma_{i1}, ..., \Gamma_{im})$ and $\Gamma_{ij} \in \Re^{\nu \times \nu}$, for $i = 1, ..., N, \quad j = 1, ..., m$, is a user-defined symmetric positive-definite design matrix. $\kappa > 0$ is a scalar design parameter.

NN dynamics for the whole group of N agents can be combined as:

$$\dot{\hat{W}} = \Gamma\phi(x)\bar{g}^T(\bar{x})\mathcal{Z}\frac{\partial\bar{H}_0}{\partial\bar{x}} - \kappa\Gamma\|\bar{g}^T(\bar{x})\mathcal{Z}\frac{\partial\bar{H}}{\partial\bar{x}}\|\hat{W} \tag{5.38}$$

where $\hat{W} = [\hat{W}_1^T \quad \hat{W}_2^T \cdots \hat{W}_N^T]^T \in \Re^{m\nu N}$, $\Gamma = diag\,(\Gamma_1, \Gamma_2, \cdots\Gamma_N)$,

$\phi = diag\,(\phi_1, \phi_2, \cdots\phi_N) \in \Re^{m\nu N\times mN}$, $\bar{g} = diag\,(\bar{g}_1, \bar{g}_2, \cdots\bar{g}_N) \in \Re^{nN\times mN}$ and

$\mathcal{Z} = diag\,(\mathcal{Z}_1, \mathcal{Z}_2, \cdots\mathcal{Z}_N) \in \Re^{nN\times nN}$.

Define the manifold Ξ_i as:

$$\{\Xi_i \mid \bar{g}_i^T(\bar{x}_i)\mathcal{Z}_i\frac{\partial\bar{H}_i}{\partial\bar{x}_i}(\bar{x}_i) = 0\} \tag{5.39}$$

The advantages associated with the introduction of $\mathcal{Z}_i$ have already been explained in detail in Ch. 4.

5.5.3 Controller design

Let the penalty signal z_i for $\mathcal{L}_2$ disturbance attenuation be defined as:

$$z_i(\bar{x}_i) = h_i(\bar{x}_i)\bar{g}_i^T(\bar{x}_i)\frac{\partial\bar{H}_i}{\partial\bar{x}_i}(\bar{x}_i) \tag{5.40}$$

where $h_i(\bar{x}_i)$ $(h_i(0) = 0)$ is a weighting matrix. The following assumption is made on $\bar{g}_i(\bar{x}_i)$.

Assumption 5.3 *The Moore-Penrose pseudoinverse of $\bar{g}_i$ denoted by $\bar{g}_i^\dagger(\bar{x}_i)$ exists* *for all* $\bar{x}_i$.

As explained in Ch. 4, most of the models of physical systems have $\bar{g}_i$ independent of $\bar{x}_i$, therefore, Assumption 5.3 does not prevent applicability of the proposed controller to the practical systems.

The proposed $\mathcal{L}_2$ neuro-adaptive control is given as:

$$\vartheta(\bar{x}_i, \hat{W}_i) = -\mathcal{K}_i \frac{\partial \bar{H}_i}{\partial \bar{x}_i} - \bar{g}_i^{\dagger} \mathcal{Z}_i \bar{g}_i \hat{\mathcal{F}}_i(\bar{x}_i) \tag{5.41}$$

where

$$\mathcal{K}_i = \frac{1}{2}\left\{\frac{1}{\gamma^2} I_n + h_i^T(\bar{x}_i) h_i(\bar{x}_i)\right\} \bar{g}_i^T \tag{5.42}$$

Note that the first term in the RHS of (5.41) represents the $\mathcal{L}_2$ disturbance attenuation component, while the second term represents the neuro-adaptive component of the control law (5.41).

Application of (5.41) to (5.32) yields

$$\begin{aligned}
\dot{\bar{x}}_i &= [\bar{J}_i(\bar{x}_i) - \bar{R}_{ci}(\bar{x}_i) - \bar{g}_i(\bar{x}_i)\mathcal{K}_i] \frac{\partial \bar{H}_i}{\partial \bar{x}_i}(\bar{x}_i) \\
&\quad + \mathcal{Z}_i \bar{g}_i(\bar{x}_i) \phi_i^T (W_i - \hat{W}_i(t)) \\
&\quad + \bar{g}_i(\bar{x}_i)(\xi_i + \bar{g}_i^{\dagger} \mathcal{Z}_i \bar{g}_i \epsilon_i)
\end{aligned} \tag{5.43}$$

To augment the NNs dynamics with the closed-loop system's dynamics and to obtain the augmented dynamics in PCH form, define the shaped Hamiltonian as:

$$\bar{H}_{S_i}(\bar{x}_i, \hat{W}_i) = \bar{H}_i(\bar{x}_i) + \bar{H}_{N_i}(\hat{W}_i) \tag{5.44}$$

where

$$\bar{H}_{N_i}(\hat{W}_i) = \frac{1}{2}(W_i - \hat{W}_i(t))^T \Gamma_i^{-1} (W_i - \hat{W}_i(t)) \tag{5.45}$$

The shaped Hamiltonian of the whole group of N agents is given by:

$$\begin{aligned} \bar{H}_S(\bar{x}, \hat{W}) &= \sum_{i=1}^{N} \bar{H}_{S_i}(\bar{x}_i, \hat{W}_i) \\ &= \sum_{i=1}^{N} \bar{H}_i(\bar{x}_i) + \sum_{i=1}^{N} \bar{H}_{N_i}(\hat{W}_i) \\ &= \bar{H}(\bar{x}) + \bar{H}_N(\hat{W}) \end{aligned} \tag{5.46}$$

where

$$\bar{H}(\bar{x}) = \sum_{i=1}^{N} \bar{H}_i(\bar{x}_i) \tag{5.47}$$

and

$$\begin{aligned} \bar{H}_N(\hat{W}) &= \frac{1}{2}(W - \hat{W}(t))^T \Gamma^{-1} (W - \hat{W}(t)) \\ &= \frac{1}{2}\tilde{W}(t)^T \Gamma^{-1} \tilde{W}(t) \end{aligned} \tag{5.48}$$

where $W = [W_1^T \quad W_2^T \cdots W_N^T]^T \in \Re^{m\nu N}$ and

$$\tilde{W} = W - \hat{W} \in \Re^{m\nu N} \tag{5.49}$$

To facilitate the analysis, the error vector $\bar{x}$ defined earlier is augmented with $\hat{W}$ to form the augmented state vector $\mathcal{X}$ as:

$$\mathcal{X} = \begin{bmatrix} \bar{x} \\ \hat{W} \end{bmatrix} \tag{5.50}$$

Gradient of $\bar{H}_S(\bar{x}, \hat{W})$ w.r.t. $\mathcal{X}$ is given by:

$$\begin{aligned} \frac{\partial \bar{H}_S}{\partial \mathcal{X}}(\mathcal{X}) &= \frac{\partial \bar{H}_S}{\partial(\bar{x}, \hat{W})}(\bar{x}, \hat{W}) \\ &= \begin{bmatrix} \frac{\partial \bar{H}(\bar{x})}{\partial \bar{x}} \\ \frac{\partial \bar{H}_N(\hat{W})}{\partial \hat{W}} \end{bmatrix} = \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{x}}(\bar{x}) \\ -\Gamma^{-1}\tilde{W} \end{bmatrix} \end{aligned} \tag{5.51}$$

Applying the controller (5.41) to system (5.32), and augmenting the resulting dynamics to NN dynamics (5.37) yields the following PCH system

$$\begin{aligned} \dot{\mathcal{X}} &= \begin{bmatrix} \dot{\bar{x}} \\ \dot{\hat{W}} \end{bmatrix} = \left[\begin{pmatrix} \bar{J} & -\mathcal{Z}\bar{g}\phi^T\Gamma \\ \left(\mathcal{Z}\bar{g}\phi^T\Gamma\right)^T & 0 \end{pmatrix} \right. \\ &\quad \left. - \begin{pmatrix} \bar{R}_c + \bar{g}\mathcal{K} & 0 \\ 0 & \kappa\Gamma\|\bar{g}^T\mathcal{Z}\frac{\partial \bar{H}}{\partial \bar{x}}\|\Gamma \end{pmatrix} \right] \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{x}} \\ \frac{\partial \bar{H}_N}{\partial \hat{W}} \end{bmatrix} \\ &\quad + \begin{bmatrix} \bar{g} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \omega \\ -\kappa\Gamma\|\bar{g}^T\mathcal{Z}\frac{\partial \bar{H}}{\partial \bar{x}}\|W \end{bmatrix} \end{aligned} \tag{5.52}$$

where $\mathcal{K} = diag\left(\mathcal{K}_1, \mathcal{K}_2, \cdots \mathcal{K}_N\right) \in \Re^{Nn \times Nn}$, $\omega = [\omega_1^T \quad \omega_2^T \cdots \omega_N^T]^T \in \Re^{Nm}$ and $\omega_i = \xi_i + \bar{g}_i^{\dagger}\mathcal{Z}_i\bar{g}_i\epsilon_i \in \Re^m$.

The following theorem establishes the main results of this chapter.

Theorem 5.1 *Consider the uncertain PCH system (5.22) and its corresponding error system (5.26).*

1). Suppose Assumptions 5.1-5.3 hold. Then for any $\gamma > 0$, the $\mathcal{L}_2$ disturbance attenuation objective is achieved by application of control law (5.41).

2). $\bar{x}$ and $\tilde{W}$ are CUUB with practical bounds given by (5.66) and (5.67), respectively.

Proof: 1). Let the shaped Hamiltonian (5.44) be the candidate Lyapunov function. Taking its time derivative along the trajectory of augmented system (5.52) yields

$$
\begin{aligned}
\dot{\bar{H}}_S &= \frac{\partial \bar{H}}{\partial \bar{x}}^T \dot{\bar{x}} - (W - \hat{W})^T \Gamma^{-1} \dot{\hat{W}} \\
&= \begin{bmatrix} \frac{\partial \bar{H}^T}{\partial \bar{x}} & \frac{\partial \bar{H}_N^T}{\partial \hat{W}} \end{bmatrix} \begin{bmatrix} \dot{\bar{x}} \\ \dot{\hat{W}} \end{bmatrix} \\
&= \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{x}}^T & \frac{\partial \bar{H}_N}{\partial \hat{W}}^T \end{bmatrix} \left[\begin{pmatrix} \bar{J} & -\mathcal{Z}\bar{g}\phi^T\Gamma \\ \left(\mathcal{Z}\bar{g}\phi^T\Gamma\right)^T & 0 \end{pmatrix} \right. \\
&\quad \left. - \begin{pmatrix} \bar{R}_c + \bar{g}K & 0 \\ 0 & \kappa\Gamma \|\bar{g}^T \mathcal{Z} \frac{\partial \bar{H}}{\partial \bar{x}}\| \Gamma \end{pmatrix} \right] \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{x}} \\ \frac{\partial \bar{H}_N}{\partial \hat{W}} \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
& + \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{x}}^T & \frac{\partial \bar{H}_N}{\partial \hat{W}}^T \end{bmatrix} \begin{bmatrix} \bar{g} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \omega \\ -\kappa \Gamma \|\bar{g}^T \mathcal{Z} \frac{\partial \bar{H}}{\partial \bar{x}}\| W \end{bmatrix} \\
= \; & -\frac{\partial \bar{H}}{\partial \bar{x}}^T R_c \frac{\partial \bar{H}}{\partial \bar{x}} - \kappa \frac{\partial \bar{H}_N}{\partial \hat{W}}^T \Gamma \|\bar{g}^T \mathcal{Z} \frac{\partial \bar{H}}{\partial \bar{x}}\| \Gamma \frac{\partial \bar{H}_N}{\partial \hat{W}} \\
& -\frac{\partial \bar{H}}{\partial \bar{x}}^T \bar{g} K \frac{\partial \bar{H}}{\partial \bar{x}} + \frac{\partial \bar{H}}{\partial \bar{x}}^T \bar{g}\omega \\
& -\kappa \frac{\partial \bar{H}_N}{\partial \hat{W}}^T \Gamma \|\bar{g}^T \mathcal{Z} \frac{\partial \bar{H}}{\partial \bar{x}}\| W
\end{aligned} \tag{5.53}
$$

Using the definitions $r = \bar{g}^T \mathcal{Z} \frac{\partial \bar{H}}{\partial \bar{x}}$, $\mathcal{R}_c = (\mathcal{Z}^T \bar{g})^\dagger R_c (\bar{g}^T \mathcal{Z})^\dagger$, $W_B = \|W\|$, substituting $\mathcal{K}$ from (5.42) and noting that $\frac{\partial \bar{H}_N}{\partial \hat{W}} = -\Gamma^{-1}\tilde{W}$, eq. (5.53) becomes

$$
\begin{aligned}
\dot{\bar{H}}_S \; = \; & -r^T \mathcal{R}_c r - \kappa \tilde{W}^T \|r\| \tilde{W} + \kappa \tilde{W} \|r\| W \\
& -\frac{1}{2} \frac{\partial \bar{H}}{\partial \bar{x}}^T \bar{g} \left\{ \frac{1}{\gamma^2} I + h^T(\bar{x}) h(\bar{x}) \right\} \bar{g}^T \frac{\partial \bar{H}}{\partial \bar{x}} \\
& + \frac{\partial \bar{H}}{\partial \bar{x}}^T \bar{g}\omega \\
\leq \; & -\underline{\sigma}(\mathcal{R}_c) \|r\|^2 - \kappa \|r\| \|\tilde{W}\|^2 + \kappa \|\tilde{W}\| \|r\| \|W\| \\
& -\frac{1}{2} \frac{\partial \bar{H}}{\partial \bar{x}}^T \bar{g} \left\{ \frac{1}{\gamma^2} I + h^T(\bar{x}) h(\bar{x}) \right\} \bar{g}^T \frac{\partial \bar{H}}{\partial \bar{x}} \\
& + \frac{1}{2\gamma^2} \frac{\partial \bar{H}}{\partial \bar{x}}^T \bar{g}\bar{g}^T \frac{\partial \bar{H}}{\partial \bar{x}} + \frac{1}{2}\gamma^2 \omega^T \omega \\
\leq \; & -\underline{\sigma}(\mathcal{R}_c) \|r\|^2 - \kappa \|r\| \|\tilde{W}\|^2 + \kappa \|\tilde{W}\| \|r\| W_B \\
& + \frac{1}{2} \left\{ \gamma^2 \|\omega\|^2 - \|z\|^2 \right\}
\end{aligned} \tag{5.54}
$$

Let

$$
\rho = \begin{bmatrix} \|r\| & \|\tilde{W}\| \end{bmatrix}^T \tag{5.55}
$$

$$\mathcal{M} = \begin{bmatrix} \underline{\sigma}(\mathcal{R}_c) & 0 \\ 0 & \kappa\|r\| \end{bmatrix} \tag{5.56}$$

$$\theta = \begin{bmatrix} 0 & \kappa\|r\|W_B \end{bmatrix}^T \tag{5.57}$$

Inequality (5.54) can be written as

$$\begin{aligned} \dot{\bar{H}}_S &\leq -\rho^T\mathcal{M}\rho + \theta^T\rho + \frac{1}{2}\left\{\gamma^2\|\omega\|^2 - \|z\|^2\right\} \\ &\leq -Q(\rho) + \frac{1}{2}\left\{\gamma^2\|\omega\|^2 - \|z\|^2\right\} \end{aligned} \tag{5.58}$$

where $Q(\rho) = \rho^T\mathcal{M}\rho - \theta^T\rho$

$Q(\rho)$ is non-negative definite if the following conditions, C1 and C2, hold:

(C1) $\mathcal{M}$ is non-negative definite.

(C2) $\|\rho\| > \frac{\|\theta\|}{\underline{\sigma}(\mathcal{M})}$

Condition (C1) is obviously satisfied. Since $\|\theta\|_1 > \|\theta\|$, condition (C2) holds if $\|\rho\| \geq \mathcal{B}$ with

$$\mathcal{B} = \frac{\|\theta\|_1}{\underline{\sigma}(\mathcal{M})} = \frac{\kappa\|\bar{g}^T r\|W_B}{\underline{\sigma}(\mathcal{M})} \tag{5.59}$$

Thus,

$$Q(\rho) \geq 0, \qquad \forall\|\rho\| \geq \mathcal{B} \tag{5.60}$$

Since ρ explicitly depends on $\bar{x}$, (5.58) is re-written as:

$$\dot{\bar{H}}_s + Q(\bar{x}) \leq \frac{1}{2}\left\{\gamma^2\|\omega\|^2 - \|z\|^2\right\} \tag{5.61}$$

Equation (5.60) with $\mathcal{B}$ defined in (5.59) implies that the function $Q(\bar{x})$ is non-negative definite. This completes the the proof of the first part of the theorem.

2). **CUUB of $\bar{x}$ and $\tilde{W}$**

Let

$$V_r(r) = -\underline{\sigma}(\mathcal{R}_c)\|r\|^2 - \kappa\|r\|\|\tilde{W}\|^2 + \kappa\|\tilde{W}\|\|r\|W_B \tag{5.62}$$

Using definition (5.62), inequality (5.54) can be written as

$$\dot{\bar{H}}_S \leq V_r(r) + \frac{1}{2}\left\{\gamma^2\|\omega\|^2 - \|z\|^2\right\} \tag{5.63}$$

CUUB of $\bar{x}$ and $\tilde{W}$ is proven by showing the negative definiteness of V_r, as follows. Re-write $V_r(r)$ as

$$V_r(r) = -\|r\|\left[k_{c_{min}}\|r\| + \kappa\|\tilde{W}\|\left(\|\tilde{W}\| - W_B\right)\right] \tag{5.64}$$

where $k_{c_{min}} = \underline{\sigma}(\mathcal{R}_c)$. $V_r(r)$ is negative as long as the term in the brackets is strictly positive. Completing the square yields

$$\begin{aligned} & k_{c_{min}}\|r\| + \kappa\|\tilde{W}\|\left(\|\tilde{W}\| - W_B\right) \\ = \;& \kappa\left(\|\tilde{W}\| - \frac{1}{2}W_B\right)^2 - \kappa W_B^2/4 + k_{c_{min}}\|r\| \end{aligned} \tag{5.65}$$

which is positive as long as

$$\|r\| > \frac{\kappa W_B^2/4}{k_{c_{min}}} \equiv b_r \tag{5.66}$$

or

$$\|\tilde{W}\| > W_B/2 + \sqrt{\kappa W_B^2/4} \equiv b_W \tag{5.67}$$

Thus $V_r(r)$ is negative outside the compact set $\{S_r | \|r\| < b_r\}$. Eq. (5.66) implies that a sufficiently large value of $k_{c_{min}}$ obtained by selecting a large C_i in eq. (5.31), will significantly reduce the magnitude of the ball b_r, thus ensuring the CUUB of both $\bar{x}$ and $\tilde{W}$. According to Lemma 5.1, CUUB of $\bar{x}$ implies the CUUB of both the synchronization error e and tracking error δ. The proof is complete. □

Remark 5.2 *As mentioned in the previous chapter, the arbitrary bounds W_B, b_W and b_r are just needed for the stability analysis. Their knowledge is not needed in the controller design, though in most practical problems, it is possible to find the numerical values of these bounds. Similar arguments can be found in [24].*

5.6 Simulation on a networked group of 2-link planar manipulators

The proposed controller is applied to a group of five 2-link planar manipulators, shown in Fig. 5.1, networked through the graph shown in Fig. 5.2. Application of canonical transformation from state space $[q_i^T \quad p_i^T]^T$ to NLSE space $[\bar{q}_i^T \quad \bar{p}_i^T]^T$ can be explained exactly as in Sec. 4.5. Consider an ith manipulator with masses m_{i1}, m_{i2} and link lengths a_{i1}, a_{i1}. The generalized configuration coordinates are link angles θ_{i1} and θ_{i2}, and therefore $q_i = \theta_i = [\theta_{i1} \ \theta_{i2}]^T$. Similar to the case of a single system described by (4.49), the ith agent in this example can also be

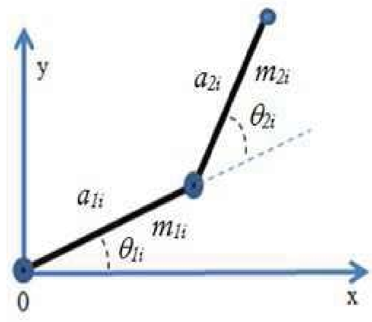

Figure 5.1: i^{th} 2-Link manipulator

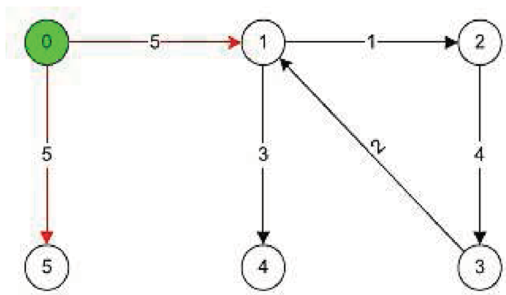

Figure 5.2: Communication graph

described by

$u_i = \begin{bmatrix} \tau_{i1} & \tau_{i2} \end{bmatrix}^T$, $G_i = I_2$, $D_i = diag(d_{i1}, d_{i2})$,

$M_i(\theta) = \begin{bmatrix} M_{i11}(q_i) & M_{i12}(q_i) \\ M_{i21}(q_i) & M_{i22}(q_i) \end{bmatrix}$ where

$M_{i11}(q_i) = (m_{i1} + m_{i2})a_{i1}^2 + m_{i2}a_{i2}^2 + 2m_{i2}a_{i1}a_{i2}cos(q_{i2})$,

$M(q_i)_{12} = M(q_i)_{21} = m_{i2}a_{i2}^2 + m_{i2}a_{i1}a_{i2}cos(q_{i2})$,

$M_{i22}(q_i) = m_{i2}a_{i2}^2$.

The manipulators work on the horizontal plane, therefore, gravity effects are negligible and $V_i(\theta_i) = 0$. The nominal magnitudes of the system parameters and uncertainties are given in Table 5.1. The virtual leader trajectories are:

$q_{01} = sin(t)$, $q_{02} = cos(t)$.

Table 5.1: Manipulator Parameters & Uncertainties

Parameter	Nominal magnitude	Uncertainty (%)
$m_{i1},\ m_{i2}$	1Kg	100
$a_{i1},\ a_{i2}$	1m	10
$d_{i1},\ d_{i2}$	1N-s/m	150

NN and controller parameters: Controller parameters are similar to those in the example of Sec. 4.6.2. Since there are two inputs for each manipulator, two separate NNs are used at every node, each with six neurons ($\nu = 6$). $\Gamma_i = 10I_6$ and $\kappa = 0.001$. Hyperbolic tangent is used as activation function. Note that there is no need to find a regressor here, as is required in other adaptive control schemes. Furthermore, to compute the penalty signal $z_i(\cdot)$ in Eq. (5.40), $h_i(\cdot)$ is chosen as

$h_i(\bar{q}_i) = q_i - q_{di},$

$C_i = K_{pi} = 20$

The initial condition is $[q_{i_0} \quad \dot{q}_{i_0}]^T = [0 \quad 0]^T$. NN weights are initialized to zero. The disturbance attenuation gain γ_i is set to 0.1. The external disturbance ξ_i is the unit variance Gaussian random noise. The filter $\mathcal{Z}_i$ is chosen as:

$$\mathcal{Z}_i = \begin{bmatrix} 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.25 \\ 0.25 & 0 & 1 & 0 \\ 0 & 0.25 & 0 & 1 \end{bmatrix}$$

which yields $\Lambda_i = diag(5,\ 5)$.

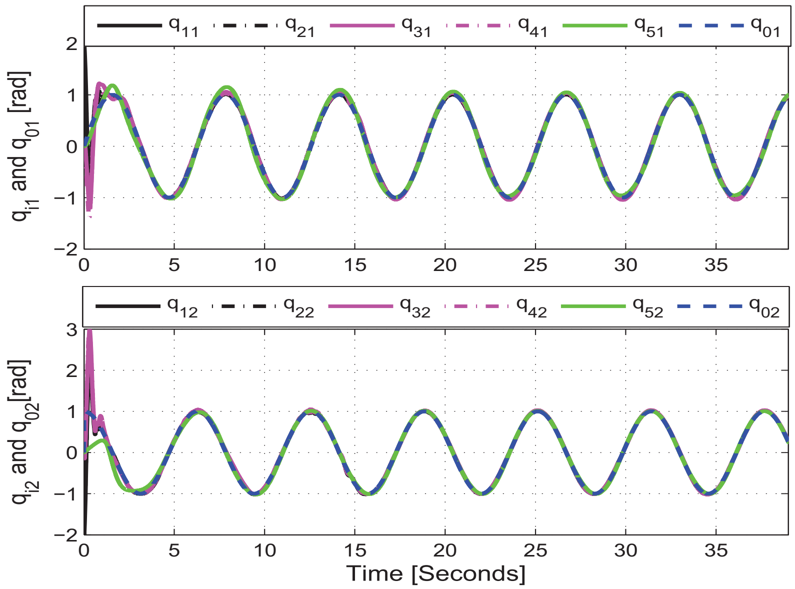

Figure 5.3: State evolution

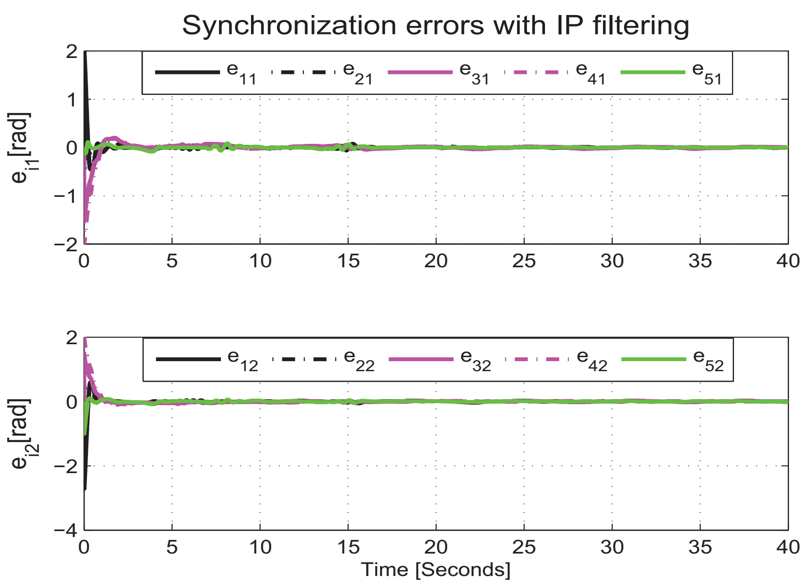

Figure 5.4: Synchronization errors with IP filtering

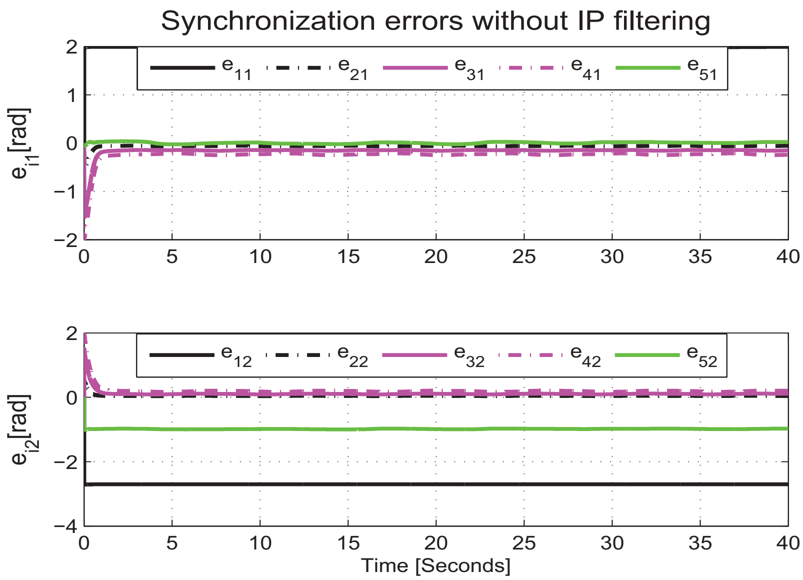

Figure 5.5: Synchronization errors without IP filtering

Fig. 5.3 shows that links follow the desired trajectories with negligible errors. Figs. 5.4–5.5 show the synchronization errors, with and without IP filtering, respectively. Significant synchronization errors are observed when IP filtering is not employed in the NN tuning law (i.e. $\mathcal{Z}_i = I_4$). Flatness of these errors indicates that the corresponding velocity errors are zero.

5.7 Conclusion

The distributed cooperative tracking control of PCH systems has been addressed in this chapter. Canonical transformation theory of PCH systems is employed to formulate the tracking of the leader node by all the follower nodes, and the synchronization among the follower nodes themselves as well. Neural networks (NN) are employed to deal with the parametric uncertainties. The NN tuning laws are driven by both position as well as velocity errors. The proposed controller also achieves the $\mathcal{L}_2$ disturbance attenuation objective. The CUUB of the tracking errors and NN weight errors has been shown. The proposed controller is successfully applied to standard mechanical systems. Simulations are performed on a group of five 2-link planar manipulators to verify the proposed scheme.

CHAPTER 6

CONCLUSION AND FUTURE WORK

This chapter summarizes the contributions of this thesis and recommends some important future extensions to them. Due to several attractive features as well as the growing needs in several civil and military applications, cooperative control of multi-agent autonomous systems has become an area of active research. On the other hand, it has been asserted in the literature survey presented in Chap. 2 that due to the vast variety of the nonlinear systems dynamics representing the majority of the real-life systems, research in the cooperative control of nonlinear multi-agents systems needs greater attention from researchers in the control community. This thesis considers cooperative control of two important classes of nonlinear systems, the Port Controlled Hamiltonian (PCH) systems and higher-order nonlinear affine systems in the Brunovsky Canonical Form (BCF).

6.1 Neuro-Adaptive Cooperative Tracking Control of Unknown Higher-Order Affine Nonlinear Systems

In Ch.3, a neural network-based distributed adaptive cooperative tracking control of nonlinear BCF systems is presented. Dynamics of the individual agents are unknown to the controller and can be different from one another i.e. the group can be heterogenous. Similar works in the available literature assume the input function of individual agent to be known and equal to unity. The results presented in this thesis generalize the available results by allowing the input function to be a smooth function of the states. The synchronization error and the NNs weight error are proved to be CUUB. Efficacy of the proposed controller is demonstrated through simulations on a group of five heterogeneous nonlinear systems with leader dynamics described by a Fitzhugh-Nagumo model and, also on another group of five inverted pendulums networked through a directed graph. It should be noted that all of the neuro-adaptive controller proposed in this thesis are direct i.e. off-line system identification is not needed.

6.2 Cooperative Control of PCH Systems

PCH systems represent a wide range of real life physical systems like robotic manipulators and several types of unmanned vehicles. PCH systems are inherently passive and are therefore, best suited for Passivity-Based Control (PBC). A noticeable flexibility offered by the PCH formalism is its ability to accept the shaping of kinetic energy in addition to potential energy as compared to the only allowable shaping of potential energy in another class of passive systems described by the Euler-Lagrange (EL) formalism. This feature of PCH systems is particularly useful in the design of trajectory tracking controllers. As another attractive feature, controller design under PCH formalism offers a straightforward interpretation of controller implementation in terms of energy storing and energy dissipating components. This feature is demonstrated in Interconnection and Damping Assignment Passivity Based Control (IDA-PBC) methodology. This thesis exploits these attractive features of PCH systems in the design of cooperative control applicable to numerous real life multi-agents systems.

6.2.1 Neuro-Adaptive Trajectory Tracking Control of Single Uncertain PCH Systems

Parametric uncertainty has always been an issue of great concern in control engineering and several other engineering fields and, if ignored, can result in serious performance and stability problems. In Chap. 4, a robust neuro-adaptive controller design is presented for PCH systems. Unlike in other adaptive control schemes, in this thesis the parametric uncertainties are not restricted to be in the so called *Linear-in-Parameter* form, thus making the proposed scheme applicable to much complicated uncertainty forms. The proposal of this controller is considered as an important step prior to the design of cooperative controller of multi-agent PCH system.

Introduction of novel information-preserving filtering

A novel contribution of the proposed neuro-adaptive controller is that the neural networks tuning law is driven by velocity as well as position errors, when applied to mechanical systems. In the available literature, the adaptive laws are driven by only the velocity error, which can result in significant steady state position error, even when velocity error is zero. Inclusion of position error in these adaptive laws is avoided to preserve the PCH structure of the closed loop system. In this thesis, a novel idea of Information-preserving (IP) filtering of the Hamiltonian gradient is introduced to drive the NN tuning law by both the velocity and position error, while preserving the PCH structure of the closed-loop systems as well.

$\mathcal{L}_2$ attenuation of bounded external disturbance

In addition to the parametric uncertainties, the proposed controller also achieves the $\mathcal{L}_2$ disturbance attenuation objective against a class of external disturbance with known bounds appearing through input channels.

Canonical transformation of PCH systems using neural networks

Canonical transformation of PCH systems, which is required in the trajectory tracking control, often involves the solution of PDEs, which is not easy, in general. In the literature, NNs have been successfully applied to the state-feedback linearisation (i.e. the transformation) of nonlinear systems. In addition to neuro-adaptive control of PCH systems, this thesis also presents the NN-based canonical transformation of PCH systems.

6.2.2 Distributed Cooperative Control of Multi-Agent PCH Systems

As a major goal of this thesis, the cooperative control of PCH systems is presented in Chap.5. The individual systems are networked through a directed graph. The generalized canonical transformation theory of PCH systems is utilized as a tool in the formulation of the cooperative control problem. Dynamics of each group member are transformed from state space to a local synchronization error space, and then a stabilizing controller is designed for the resultant *synchronization error* system. Stability of the transformed global error system is guaranteed by a straightforward application of the stability theory of the trajectory tracking control of a single PCH system.

6.2.3 $\mathcal{L}_2$ Neuro-Adaptive Trajectory Tracking Control of Uncertain PCH Systems

Cooperative control of multi-agent PCH system is robustified against parametric uncertainties and bounded external disturbances using NNs. The novel idea of information-preserving filtering introduced in the Ch.4 is utilized to drive the NNs tuning law by velocity as well as position errors. The synchronization error and the NNs weight error are proved to be CUUB. Simulations are performed on a group of five networked 2-link robotic manipulators to demonstrate the efficacy of the proposed controller.

6.3 Future Recommendations

The developments in this thesis can be further extended in several aspects. These extensions will expectedly result in several contributions to further enhance the cooperative control theory literature towards the goal of developing robust and practical autonomous multi-agent systems. The following extension are recommended to be considered in the future.

6.3.1 Formation control of PCH systems

In the formation control problem, a group of autonomous agents is coordinated to achieve some desired formation i.e. a desired geometrical shape, and then accomplish some tasks through the collaboration of the agents. Applications of formation control include formation flying, cooperative transportation, sensor networks, as well as combat intelligence, surveillance, and reconnaissance [15]. Formation control of PCH systems can be a natural extension of this thesis. Advantages associated with the passivity based control (PBC) together with the PCH formalism can expectedly result in more attractive formation control algorithms. In particular, the generalized canonical transformation theory of PCH systems can be well utilized to study the formation tracking control problem. Moreover, the experience of direct neuro-adaptive control can be further exploited to develop robust formation control algorithms.

6.3.2 Cooperative control of underactuated systems

This thesis considers the cooperative control of fully actuated PCH systems. There are several mechanical systems with number of actuators less than the degree of freedom. These are called the underactuated systems. Some researchers also categorize the non-holonomic systems as underactuated systems [15]. Failure of an actuator of a fully actuated system can also make it an underactuated system. Extension of the present work to the case of underactuated systems can be an interesting research work. Stabilization control of underactuated systems under PCH formalism has already been studied in [98] which can be a good starting point for this future extension.

6.3.3 Cooperative control under communication constraints

In multi-agent systems, information among the constituent group members is shared through a communication network which is often a wireless one. Problems associated with the data communication cause spurious effects on the cooperative control performance and can even destabilize the cooperative system in some cases. In the following, some future extensions are suggested for consideration of significant communication constraints.

Time delay and packet drops

Network latency or time delay is an essential phenomenon in data communication. A major reason for the occurrence of time delay is the limited communication speed or the limited channel bandwidth. In addition to time delay, packet drops can also affect the performance. Packet drops are modeled as a special case of time delay when dropped data packets are re-transmitted and then received. Extension of the results of this thesis by considering the time delay will further enhance the applicability of the proposed controller.

Intermittent connectivity, switching and reconfigurable topology

In many practical situations, the topology of the communication graph does not remain fixed, and rather varies due to several factors, like distance variation among the agents, loss of communication link, mobility of the agents, or even sometimes dictated by the requirements of an application e.g. in simultaneous localization and mapping (SLAM). A cooperative control scheme must be *pliable* enough to maintain the group objective in the cases of intermittent connectivity, switching and reconfigurable topology. This has been a problem of great concern for the researchers and as a result numerous schemes have been appeared in the literature for particular classes of dynamics systems [15]. In future, such pliability of cooperative control of the systems considered in this thesis should also be targeted.

6.4 Concluding Remarks

Cooperative control of multi-agents systems has emerged as an active area of research in control community due to its numerous valuable benefits to society. Research in the cooperative control of nonlinear systems has several problems to be solved yet. Much has already been done but still much more is yet to be done. This thesis contributes to the literature by presenting robust cooperative control of two important classes of higher order nonlinear systems: higher order nonlinear systems in BCF and the PCH systems. As such, it represents a small step in the drive to expand and exploit this powerful control tool.

REFERENCES

[1] G. Bekey, R. Ambrose, K. Kumar, Vijay, D. Lavery, A. Sanderson, B. Wilcox, J. Yuh, and Y. Zheng, *ROBOTICS: State of the Art and Future Challenges*. World Scientific, 2008.

[2] W. Ren and C. Yongcan, *Distributed Coordination of Multi-agent Networks: Emergent Problems, Models, and Issues*. Springer-Verlag London, 2011.

[3] A. Howard, G. S. Sukhatme, and M. J. Mataric, "Multirobot Simultaneous Localization and Mapping Using Manifold Representations ," *Proceedings of the IEEE*, vol. 94, no. 7, pp. 1360–1369, 2006.

[4] P. M., S. B., and V. A., "A scalable hybrid multi-robot SLAM method for highly detailed maps," *Proceedings of the International Symposium, LNAI, Atlanta, GA.*, 2007.

[5] E. Fiorelli, N. E. Leonard, P. Bhatta, D. A. Paley, R. Bachmayer, and D. M. Fratantoni, "Multi-AUV Control and Adaptive Sampling in Monterey Bay," *IEEE JOURNAL OF OCEANIC ENGINEERING*, 2006.

[6] M. F. Mysorewala, L. Cheded, and D. Popa, "A distributed multi-robot adaptive sampling scheme for the estimation of the spatial distribution in widespread fields," *EURASIP Journal on Wireless Communications and Networking*, no. 2012:213, 2012. [Online]. Available: http://jwcn.eurasipjournals.com/content/2012/1/223

[7] M. J. Mataric, M. Nilsson, and K. T. Simsarian, "Cooperative Multi-Robot Box-Pushing," *Proceedings of 1995 IEEE/RSJ International Conference on Intelligent Robots and Systems, 'Human Robot Interaction and Cooperative Robots',*, vol. 3, no. 12, pp. 556–561, 1995.

[8] T. Fukuda, Y. Hasegawa, K. Sekiyama, and T. Aoyama, *Multi-Locomotion Robotic Systems: New Concepts of Bio-inspired Robotics* . Springer-Verlag Berlin Heidelberg, 2012.

[9] S. Chowdhury, A. Thakur, P. Svec, C. Wang, W. Losert, and S. K. Gupta, "Automated Manipulation of Biological Cells Using Gripper Formations Controlled By Optical Tweezers," *IEEE Transections on Automation Sciences and Engineering*, 2013.

[10] R. Smith and F. Hadaegh, "Cooperative Multi-Robot Box-Pushing," *Journal of Guidance, Control of Dynamics*, vol. 28, no. 1, pp. 106–114, 2005.

[11] A. Labeyrie, S. G. Lipson, and P. Nisenson, *An Introduction to Optical Stellar Interferometry.* Cambridge University Press, 2006.

[12] T. Potok, L. Phillips, R. Pollock, and L. A., "Suitability of Agent Technology for Military Command and Control in the Future Combat System Environment," *8th International Command and Control Research Technology Symposium, Washington*, 2003.

[13] D. Casbeer, D. Kingston, R. Beard, S. Li, T. McLain, and R. Mehra, "Cooperative forest fire surveillance using a team of small unmanned air vehicles," *International Journal of Systems Sciences*, vol. 37, no. 6, pp. 351–360, 2006.

[14] P. Ogren, E. Fiorelli, and N. E. Leonard, "Cooperative Control of Mobile Sensor Networks: Adaptive Gradient Climbing in a Distributed Environment ," *IEEE Transections on Automatic Control*, vol. 49, no. 8, pp. 1292–1302, 2004.

[15] Y. Cao, W. Member, W. Ren, and G. Chen, "An Overview of Recent Progress in the Study of Distributed Multi-Agent Coordination," *IEEE Transections on Industrial Informatics*, vol. 9, no. 1, pp. 427–438, 2012.

[16] R. Ortega, A. Schaft, B. Maschke, and G. Escobar, "Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian Systems," *Automatica*, vol. 38, pp. 585–596, 2002.

[17] A. v. Schaft and B. Maschke, *A Port-Hamiltonian Formulation of Open Chemical Reaction Networks in Advances in the Theory of Control, Signals and Systems with Physical Modeling Levine J. and Mullhaupt P. (Ed.)*. Springer-Verlag, Berlin Heidelberg, 2010.

[18] C. Secchi, S. Stramigioli, and C. Fantuzzi, *Control of Interactive Robotic Interfaces: A Port-Hamiltonian Approach*. Springer-Verlag Berlin Heidelberg, 2007.

[19] H. K. Khalil, *Nonlinear systems*. Prentice Hall, UpperSaddle River, NJ, 2002.

[20] B. M. Maschke, R. Ortega, and A. J. Van der Schaft, "Lyapunov functions for forced Hamiltonian systems with dissipation," *IEEE Transactions on Automatic Control*, vol. 45, no. 8, pp. 1498–1502, 2000.

[21] V. A. Schaft, $\mathcal{L}_2$ *Gain and passivity techniques in nonlinear control*. Springer, Berlin, 1999.

[22] K. Fujimoto and T. Sugie, "Canonical transformation and stabilization of generalized Hamiltonian systems," *Systems and Control Letters*, vol. 42, no. 3, pp. 217–227, 2002.

[23] K. Fujimoto, K. Sakurama, and T. Sugie, "Trajectory tracking control of port-controlled Hamiltonian systems via generalized canonical transformations," *Automatica*, vol. 39, no. 12, pp. 2059–2069, 2003.

[24] F. L. Lewis, S. Jagannathan, and A. Yesilderek, *Neural Network Control of Robot Manipulator and Nonlinear Systems*. London: Taylor and Francis, 1998.

[25] D. A. Dirksz and J. M. A. Sherpen, "Adaptive tracking control of fully actuated port-Hamiltonian mechanical systems," *Proc. 2010 IEEE International*

Conference on Control Applications Part of 2010 IEEE Multi-Conference on Systems and Control, pp. 1678–1683, 2010.

[26] ——, "Structure preserving adaptive control of port-Hamiltonian system," *IEEE Trans. Automatic Control*, vol. 57, no. 11, pp. 2880–2884, 2012.

[27] T. Shen, R. Ortega, Q. Lu, S. Mei, and K. Tamura, "Adaptive $\mathcal{L}_2$ distubance attenuation of Hamiltonian systems with parametric purturbation and application to power systems," *Asian Journal of Control*, vol. 5, no. 1, pp. 143–152, 2003.

[28] J. M. A. Dirksz, D. A. & Sherpen, "Power-based control: Canonical coordinate transformations, integral and adaptive control," *Automatics*, vol. 48, no. 3, pp. 1045–1056, 2012.

[29] J. T. Spooner and K. M. Passino, "Stable Adaptive Control Using Fuzzy Systems and Neural Networks," *IEEE Transection on Fuzzy Systems*, vol. 4, no. 3, pp. 339–259, 1996.

[30] F. L. Lewis and S. S. Ge, *"Neural Networks in Feedback Control Systems" in 2nd volume of "Mechanical Engineers' Handbook" Myer Kutz (editor).* John Wiley, New York, 2005.

[31] J. Rinzel, *A formal classification of bursting mechanisms in excitable systems, in Mathematical Topics in Population Biology, Morphogenesis and Neurosciences, E. Teramoto and M. Yamaguti, Eds.* Springer-Verlag, New York, 1987.

[32] H. Zhang, L. L. Frank, and Z. Qu, "Lyapunov, Adaptive, and Optimal Design Techniques for Cooperative Systems on Directed Communication Graphs ," *IEEE Transections on Industrial Electronics,*, vol. 59, no. 7, pp. 3026–3041, 2012.

[33] T. Zhang, S. S. Ge, and C. C. Hang, "Design and performance analysis of a direct adaptive controller for nonlinear systems," *Automatica*, vol. 35, no. 11, pp. 1809–1817, 1999.

[34] A. Das and F. L. Lewis, "Distributed adaptive control for synchronization of unknown nonlinear networked systems," *Automatica*, vol. 46, no. 12, pp. 2014–2021, 2010.

[35] ——, "Cooperative adaptive control for synchronization of second-order systems with unknown nonlinearities," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 13, pp. 1509–1524, 2011.

[36] H. Zhang and F. L. Lewis, "Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics," *Automatica*, vol. 48, no. 7, pp. 1432–1439, 2012.

[37] M. H. DeGroot, "Reaching a consensus," *Journal of American Statistical Association*, vol. 69, no. 345, pp. 118–121, 1974.

[38] J. N. Tsitsiklis, *Problems in decentralized decision making and computation.* Ph.D. dissertation, MIT, 1984.

[39] J. N. Tsitsiklis, D. P. Bertsekas, and M. Athans, "Distributed asynchronous deterministic and stochastic gradient optimization algorithms," *IEEE Transection on Automatic Control*, vol. 31, no. 9, pp. 803–812, 1986.

[40] N. A. Lynch, *Distributed Algorithms*. Morgan Kaufmann Publishers, Inc., San Francisco, California, 1996.

[41] T. Vicsek, A. Czirok, E. B. Jacob, I. Cohen, and O. Schochet, "Novel type of phase transitions in a system of self-driven particles," *Physical Review Letters*, vol. 75, no. 6, pp. 1226–1229, 1995.

[42] A. Jadbabaie, J. Lin, and S. m. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," vol. 48, no. 6, pp. 988–1001, 2003.

[43] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.

[44] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1465–1476, 2004.

[45] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.

[46] L. Moreau, "Stability of multi-agent systems with time-dependent communication links," *IEEE Transection on Automatic Control*, vol. 50, no. 2, pp. 169–182, 2005.

[47] G. Lafferriere, A. Williams, J. Caughman, and J. J. P. Veerman, "Decentralized control of vehicle formations," *Systems and Control Letters*, vol. 54, no. 9, pp. 899–910, 2005.

[48] Y. Hong, L. Gao, D. Cheng, and J. Hu, "Lyapunov-based approach to multiagent systems with switching jointly connected interconnection," *IEEE Transactions on Automatic Control*, vol. 52, no. 5, pp. 943–948, 2007.

[49] W. Yu, G. Chen, M. Cao, and J. Kurths, "Second-order consensus for multiagent systems with directed topologies and nonlinear dynamics," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 40, no. 3, pp. 881–891, 2010.

[50] Z. Qu, J. Wang, and R. A. Hull, "Cooperative control of dynamical systems with application to autonomous vehicles," *IEEE Transactions on Automatic Control*, vol. 53, no. 4, pp. 894–911, 2008.

[51] S. E. Tuna, "Conditions for synchronizability in arrays of coupled linear systems," *IEEE Transection on Automatic Control*, vol. 54, no. 10, pp. 2416–2420, 2009.

[52] S. L. and S. R., "Synchronization in networks of identical linear systems," *Automatica*, vol. 45, no. 11, pp. 2557–2562, 2009.

[53] J. H. Seo, H. Shim, and B. J., "Consensus of high-order linear systems using dynamic output feedback compensator: Low gain approach," *Automatica*, vol. 45, no. 11, pp. 2659–2664, 2009.

[54] Z. Li, Z. Duan, G. Chen, and H. L., "Consensus of multiagent systems and synchronization of complex networks: A unified viewpoint," *IEEE Transactions on Circuits And SystemsI*, vol. 57, no. 1, pp. 213–224, 2010.

[55] T. Yang, S. Roy, Y. Wan, and S. A., "Constructing consensus controllers for networks with identical general linear agents," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 11, pp. 1237–1256, 2011.

[56] F. L. Lewis, K. Zhang, Hongwei Hengster-Movric, and A. Das, *Cooperative Control of Multi-Agent Systems Optimal and Adaptive Design Approaches*. Springer-verlag, London, 2014.

[57] Z. Qu, *Cooperative Control of Dynamical Systems: Applications to Autonomous Vehicles*. Springer-Verlag, London, 2009.

[58] N. Chopra and M. W. Spong, "On exponential synchronization of kuramoto oscillators," *IEEE Transactions on Automatic Control*, vol. 54, no. 2, p. 2009, 353–357.

[59] A. Papachristodoulou, A. Jadbabaie, and U. Munz, "Effects of Delay in Multi-Agent Consensus and Oscillator Synchronization," *IEEE Transactions on Automatic Control*, vol. 55, no. 6, pp. 1471–1478, 2010.

[60] N. Chopra and M. W. Spong, *Passivity-Based Control of Multi-Agent Systems in Advances in Robot Control, Ed.* . Springer-Verlag, 2007.

[61] J. Mei, W. Ren, and G. Ma, "Distributed Coordinated Tracking With a Dynamic Leader for Multiple Euler-Lagrange Systems," *IEEE Transactions on Automatic Control*, vol. 56, no. 6, pp. 1415–1421, 2011.

[62] G. Chen and F. L. Lewis, "Distributed adaptive tracking control for synchronization of unknown networked lagrangian systems," *IEEE Transactions on Systems, Man, and Cybernetics*.

[63] D. V. Dimarogonas and K. J. Kyriakopoulos, "On the rendezvous problem for multiple nonholonomic agents," *IEEE Transactions on Automatic Control*, vol. 55, no. 5, pp. 916–922, 2007.

[64] Y. Xua, Y.-P. Tiana, and Y. Q. Chen, "Output consensus for non-holonomic systems under directed communication topology," *International Journal of Systems Science*, 2013.

[65] M. Ouab, S. Lib, and C. Wangc, "Finite-time tracking control for multiple non-holonomic mobile Journal of Control, vol. 86, no. 12, pp. 2175–2188, 2013.

[66] R. Zheng and D. Sun, "Rendezvous of unicycles: A bearings-only and perimeter shortening approach," *Systems and Control Letters*, vol. 62, pp. 401–407, 2013.

[67] R. Ortega and S. M. W., "Adaptive Motion Control of Rigid Robots: a Tutorial," *Automatica*, vol. 25, no. 6, pp. 877–888, 1989.

[68] H. Bai, M. Arcak, and W. J., *Cooperative Control Design: A Systematic, Passivity-Based Approach*. Springer-Verlag, New York, 2011.

[69] C. I. Byrnes, A. Isidori, and J. C. Willems, "Passivity, feedback equivalence and the global stabilization of minimum phase nonlinear systems," *IEEE transections on Automatic Control*, vol. 36, no. 11, pp. 1228–1240, 1991.

[70] Z. Wang and P. Goldsmith, "Modified energy-balancing-based control for the tracking problem ," *IET Control Theory Applications*, vol. 2, no. 4, pp. 310–322, 2008.

[71] W. Ren, K. L. Moore, and Y. Chen, "High-order and model reference consensus algorithms in cooperative control of multivehicle systems," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 129, no. 5, pp. 678–688, 2007.

[72] J. Wang and C. D., "Consensus of multi-agent systems with higher order dynamics," *Proceedings of the Chinese control conference*, pp. 761–765, 2007.

[73] F. Jiang, L. Wang, and Y. Jia, "Consensus in leaderless networks of highorder-integrator agents," *Proceedings of the American control conference*, pp. 4458–4463, 2009.

[74] A. Yesilderek and F. L. Lewis, "Feedback Linearisation using neural Networks," *Automatica*, vol. 31, no. 11, pp. 1659–1664, 1995.

[75] F.-C. Chen and C.-C. Liu, "Adaptively Controlling Nonlinear Continuous-Time Systems Using Multilayer Neural Networks," *IEEE Transactions on Automatic Control*, vol. 39, no. 6, pp. 1306–1310, 1994.

[76] D. C. Theodoridis, Y. S. Boutalis, and M. A. Christodoulou, "Direct Adaptive Neuro-Fuzzy Trajectory Tracking of Uncertain Nonlinear Systems," *International Journal of Adaptive Control and Signal Processing*, vol. 26, pp. 660–688, 2012.

[77] S. Khoo, X. L., and Z. Man, "Robust finite-time consensus tracking algorithm for multirobot systems," *IEEE/ASME Transactions on Mechatronics*, vol. 14, no. 2, pp. 219–228, 2009.

[78] B. Igelink and Y. H. Pao, "Stochastic choice of basis function in adaptive function approximation and functional-link net," *IEEE Transection on Neural Networks*, vol. 6, no. 6, pp. 1320–1329, 1995.

[79] C. C. Liu and F. C. Chen, "Adaptive control of non-linear continuous-time systems using neural networksgeneral relative degree and MIMO cases," *International Journal of Control*, vol. 58, no. 2, pp. 317–335, 1993.

[80] I. Kanellakopoulos, P. Kokotovic, and A. Morse, "Systematic design of adaptive controllers for feedback linearizable systems," *IEEE Transactions on Automatic Control*, vol. 36, no. 11, pp. 1241–1253, 1991.

[81] M. Polykarpou and P. Ioannou, “On the existence and uniqueness of solutions in adaptive control systems,” *IEEE Transactions on Automatic Control*, vol. 38, no. 3, pp. 474–479, 1993.

[82] B. Egardt, *Stability of adaptive controllers.* Springer-Verlag, Berlin, Heidelberg, 199=79.

[83] M. Kristic, I. Kanellakopoulos, and P. Kokotovic, *Nonlinear and adaptive control design.* Wiley-Interscience Publications, New York, 1995.

[84] L. T. Hsu and . A. A. D. Lizarralde, L., “New results on output feedback variable structure model reference adaptive control: design and stability analysis,” *IEEE Transections on Automatic Control*, vol. 42, no. 3, pp. 386–393, 1997.

[85] J. Baptiste and Praly, “Adaptive nonlinear regulation: estimation from the Lyapunov equation,” *IEEE Transactions on Automatic Control*, vol. 31, no. 6, pp. 729–740, 1992.

[86] S. S. Ge and J. Zhang, “Neural network control of nonaffine nonlinear systems with zero dynamics by state and output feedback,” *IEEE Transactions on Neural Networks*, vol. 14, no. 4, pp. 900–918, 2003.

[87] A. Macchelli, C. Melchiorri, C. Secchiy, Fantuzziy, and C., “A variable structure approach to energy shaping,” *Proceeding of European Control Conference*, 2003.

[88] Y. Wang, G. Feng, and D. Cheng, "Simultaneous stabilization of a set of nonlinear port-controlled Hamiltonian systems," *Automatica*, vol. 43, no. 3, pp. 403–415, 2007.

[89] J. J. E. Slotine and W. Li, *Applied Nonlinear Control.* Printice-Hall, Englewood Cliffs, NJ., 1991.

[90] K. S. Nerendra and A. M. Annaswami, "A new adaptive law for robust adaptation without persistent excitation," *IEEE Transection on Automatic Control*, vol. AC-32, no. 2.

[91] K. Sakurama, *Trajectory Tracking Control of Hamiltonian and Hybrid Control Systems.* PhD Dissertation, Kyoto University, Japan, 2004.

[92] H. A. A. Fattah, A. F. Sakr, and A. Bahgat, "Nonlinear System Control Via Feedback Linearization Using Neural Networks," *Proceedings of IEEE International Conference on Neural Networks*, vol. 4, pp. 2238–2243, 1996.

[93] S. He, K. Reif, and U. R., "A Neural Approach for Control of Nonlinear Systems with Feedback Linearization," *IEEE Transection on Neural Networks*, vol. 9, no. 6, pp. 1409–1421, 1998.

[94] B. Ferreira, A. Matos, N. Cruz, and M. Pinto, "Modeling and Control of the MARES Autonomous Underwater Vehicle," *Marine Technology Society Journal*, vol. 44, no. 2, pp. 19–36, 2010.

[95] B. Ferreira, M. Pinto, A. Matos, and N. Cruz, "Control of the MARES Autonomous Underwater Vehicle ," *Proeedings OCEANS 2009, MTS/IEEE*

Biloxi - Marine Technology for Our Future: Global and Local Challenges, pp. 1–10, 2009.

[96] ——, “Hydrodynamic modeling and motion limits of AUV MARES,” *Proceeding 35th IEEE Annual Conference on Industrial Electronics*, pp. 2241–2246, 2009.

[97] M. Arcak, “Passivity As a Design Tool for Group Coordination,” *IEEE Transactions on Automatic Control*, vol. 52, no. 8, pp. 1380–888, 2007.

[98] R. Ortega, M. W. Spong, F. G. Estern, and G. Blankenstein, “Stabilization of a Class of Underactuated Mechanical Systems Via Interconnection and Damping Assignment,” *IEEE Transection on Automatic Control*, vol. 47, no. 8, pp. 1218–1233, 2002.

Vitae

Name:	Amin-ud-din Qureshi
Nationality:	Pakistani
Date of Birth:	3rd January, 1979
Email Address:	*qureshiamin@{kfupm.edu.sa, yahoo.com}*
Permanent Address:	Street of the flower sellers, Qissa Khwani, Peshawar, Khyber Pakhtunkhwa, Pakistan

Education

BSc Electrical Engineering (2001):	NWFP University of Engineering and Technology, Peshawar, Pakistan
MSc Systems Engineering (2003):	Pakistan Institute of Engineering and Applied Sciences, Islamabad, Pakistan
PhD Systems Engineering (2014):	King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia

Experience: From 2003 to 2010 he worked in the Informatics Complex, Pakistan, on several R&D projects in the area of digital signal processing, and during the same period, he has been a junior adjunct faculty member in the electrical engineering department of Pakistan Institute of Engineering and Applied Sciences-Islamabad. From Feb. 2010 to April 2014, he worked in the systems engineering department of King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia as Lecturer-B, and pursued his PhD studies during the same period. His research interest includes sliding mode control, neural networks based adaptive control and signal processing applied to surveillance systems.