ECONOMIC PRODUCTION QUANTITY MODEL WITH IMPERFECT QUALITY DURING A PROCESS ADJUSTMENT PERIOD - SOME CONSIDERATIONS

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This thesis, written by Ismail Ibrahim Al-Me'raj under the direction of his thesis advisor and approved by his thesis committee, has been presented and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN SYSTEMS ENGINEERING.**

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To my parents, Ibrahim and Madinah,

And my nephews, Ahmed, Mustafa and Abdull Allah

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LIST OF ABBREVIATIONS

EPQ	:	Economic Production Quantity
Q	:	Order quantity size
P	:	Production rate
P_s	:	The defective items processing rate.
P_C	:	Items processing rate
D	:	Demand rate
A	:	Setup cost
A_d	:	Machine adjustment cost
d	:	Proportion of defective items produced during the machine adjustment period
C	:	The purchasing cost of unit product
C_P	:	Production cost
r	:	The screening cost per item
t	:	The process adjustment period
T_s	:	The defective items processing time
T	:	Cycle time
T_P	:	Production time
T_C	:	Items processing time
h	:	Holding cost
h_s	:	Holding cost of defective items

h_C : Holding cost of processed items

$f(t)$: Probability density function of the random variable t

USL : Upper specification limit

LSL : Lower specification limit

K : Taguchi loss parameter

X : Actual value of the quality characteristic

$g(x)$: Normal probability density function of the quality characteristic X

$L(x)$: Loss of poor quality per unit product

μ : Target quality characteristic

σ : Standard deviation of quality characteristic

Δ : Deviation from the target value

S_{max} : Maximum shortage allowed

$\hat{\pi}$: Shortage cost per unit short per year

$\breve{\pi}$: Shortage cost per unit short, independent of the duration of the shortage

e_1 : Probability of type I error

e_2 : Probability of type II error

R_L : Reworking cost for an item produced below the specification limit

R_U : Reworking cost for an item produced above the specification limit

ABSTRACT

Full Name : [Ismail Ibrahim Abass Al-Me'raj]

Thesis Title : [Economic Production Quantity Model with Imperfect Quality During a Process Adjustment Period-Some Considerations]

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Economic Production Quantity EPQ in its classical version does not include a scenario of imperfect quality items. However, many researchers have considered this situation. We consider a manufacturing process that generates non-conforming items until proper adjustment of the process is reached. The time to perform the adjustment is treated as a fixed value and a random variable. Items produced after the adjustment period is always conforming and the demand rate for the produced items is constant. The process stops when the production of conforming items is sufficient to cover the demand, and then the cycle is repeated uninterrupted. Other considerations such as maximum allowable shortage, inspection errors, defective items processing and Taguchi's quality loss function approach are incorporated as extensions. We determined the optimal production quantity and maximum allowable shortage that result in minimum expected total cost. Data example and sensitivity analysis are provided for illustration and is expected to provide more insights in managing this important problem.

Keywords: Economic Production Quantity EPQ, adjustment period, non-conforming items, Taguchi's quality loss function, inspection errors.

ملخص الرسالة

الاسم الكامل: إسماعيل إبراهيم عباس المعراج

عنوان الرسالة: نموذج الكمية الاقتصادية للإنتاج المتضمن للإنتاج المعيوب خلال ضبط عملية التصنيع – بعض الاعتبارات.

التخصص: هندسة نظم صناعية

تاريخ الدرجة العلمية: مايو 2013

لا تتضمن الكمية الاقتصادية للإنتاج في صيغتها التقليدية لسيناريو الإنتاج المعيوب. بيد أن الكثير من الباحثين قد أدخلوا هذا الاعتبار في أبحاثهم مؤخراً. أطروحتنا تدور حول عملية التصنيع التي تشتمل على إنتاج معيوب، وبعد أن يتم ضبط عملية التصنيع يصبح الإنتاج بالجودة المطلوبة. الوقت المطلوب لضبط عملية التصنيع عومل على أنه إما قيمة ثابتة أو متغير عشوائي، كما أن معدل الطلب يكون ثابتاً. وعندما يكون الإنتاج ذات الجودة المطلوبة كافياً لتغطية الطلب، يتوقف الإنتاج. وبذلك تنتهي دورة إنتاجية كاملة وبعدها تتكرر دورات إنتاجية أخرى بنفس الأسلوب. ونناقش في أطروحتنا كذلك بعض الاعتبارات الأخرى ككمية الإنتاج العليا غير المستوفاة للطلب، أخطاء الفحص، عملية التحسين للإنتاج المعيوب و دالة تاجوشي للجودة. ومن خلال النماذج الرياضية التي قمنا بتطويرها، أوجدنا الكمية المثالية للإنتاج وكمية الإنتاج العليا غير المستوفاة للطلب وذلك بأقل التكاليف المتوقعة. كما ناقشنا في أطروحتنا أمثلة وتحليلات رياضية لتوضيح فكرتنا والتي نأمل أن تفتح آفاقاً جديدةً للباحثين في المستقبل.

CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

1.1 Literature Review

We have divided this review into three areas: economic production quantity model with imperfect quality items and inspection errors, optimum economic production quantity model and shortage, and optimum process parameters and economic production quantity model with quality loss function.

1.1.1 Economic production quantity model with imperfect quality items and inspection errors:

The economic order quantity EOQ model of Harris [1] is the foundation of modern-day inventory models. Many recent research papers have integrated areas of quality control, production systems, and process targeting together. This is because, today, a successful business needs to produce the proper quantity with high quality at a reasonable cost.

Economic Production Quantity EPQ and EOQ models have been extended by introducing the assumption of imperfect quality items and inspection errors. Two types of errors are committed in the inspection process. A Type I error is committed when a conforming item is classified as non-conforming and a Type II error is committed when a non-conforming item is classified as conforming. Inspection errors are traced back to 1952 when Bennett and Jacobson studied their influences on quality control systems.

Study of EOQ for items with imperfect quality was conducted by Salameh and Jaber [2]. Papachristos and Konstantaras [3] extended their work by assuming that imperfect items are withdrawn from the end of the planning horizon. Liao [4] considered imperfect production processes that require production correction and maintenance in his research. The production process is performed both under a Type I state (out-of-control state) and a Type II state (in-control state). Gary and Gong [5] studied the impact of random machine breakdowns on the EPQ model for a product subject to exponential decay and under a non-resumption inventory control policy. Darwish [6] generalized the EPQ model by considering a relationship between the setup cost and the production run length and their relationship to process deterioration, and learning and forgetting effects. Chakraborty and Chaudhuri [7] proposed production lot sizing with process deterioration and machine breakdown. They presented a generalized economic manufacturing quantity model for an unreliable production system in which the production facility may shift from an 'in-control' state to an 'out-of-control' state at any random time and may ultimately break down. Seung et al. [8] extended the EPQ model by considering imperfect-quality items, two-way imperfect inspection, and sales return. Chung et al. [9] presented a two-warehouse inventory model with imperfect quality production processes. Leopoldo et al. [10] focused on the EPQ model with the rework process at a single-stage manufacturing system with planned backorders. Wen and Hog [11] investigated an EOQ inventory model for imperfect items under a one-time-only discount, where the defectives can be screened out by a 100% screening process and then sold in a single batch by the end of screening.

Liao et al. [12] integrated maintenance and production programs with the EPQ model for an imperfect process involving a deteriorating production system with an increasing hazard rate, and imperfect repair and rework upon failure (out of control state). Yuan et al. [13] worked on optimization of the finite production rate model with processing the defective items, rework and stochastic machine breakdown. Lin et al. [14] studied the impact of inspection errors, imperfect maintenance, and minimal repairs on an imperfect production system. Khan et al. [15] extended the work of Salameh and Jaber [2] by incorporating inspection errors. Sana [16] developed a production-inventory model of imperfect quality products in a three-layer supply chain. Wang et al. [17] investigated integrating the acquisition of input materials, material inspection, and production planning, where Type I and Type II inspection errors are allowed, and the unit acquisition cost is dependent on the average quality level. Widyadana and Wee [18] considered a production-inventory model with deteriorating items with random machine breakdown and stochastic repair time. In the same area, Gwo [19] studied an optimum policy for a production system with major repair and preventive maintenance. Liao and Sheu [20] presented an EPQ model for a randomly failing production process with minimal repair and imperfect maintenance. Liu and Zheng [21] introduced fuzzy economic order quantity model with imperfect items, shortages, and inspection errors. Tolgari et al. [22] studied an inventory model with imperfect items and inspection error under inflationary conditions. Seung et al. [23] investigated both internal and external effects of defective production and delivery from imperfect production and inspection processes in a stable production and inventory system, and subsequent defective returns and disposing.

1.1.2 Optimum economic production quantity model and shortage:

The assumption of shortage allowance and backordering make EPQ models more realistic and hence more practical. Wee et al. [24] developed an inventory model for items with imperfect quality and shortage backordering. Later Cheng and Chih [25] investigated an imperfect production system with allowable shortages for products sold with free minimal repair warranty. They sought to minimize the total cost per item through optimal determination of the production run length and the time length when backorder is replenished. Eroglu and Ozdemir [26] proposed an EOQ model with defective items and shortages. To deal with the uncertainties and randomness of defective percentage and shortages occurring in real-life situations, the order inventory model with a mixture of shortages and imperfect items was presented and investigated in the fuzzy environment by Li and Zhang [27]. Jaber and El Saadany [28] discussed the production, remanufacture, and waste disposal model with lost sales.

1.1.3 Optimum process parameters and economic production quantity model with quality loss function:

Economic selection of process parameters has been an important topic in modern statistical process control. The optimal mean setting has a significant impact on production, the expected total profit/cost, defective fraction, and inspection/reprocessing cost. However, few papers have discussed the incorporation of the Taguchi's quality loss function [29] into the EPQ model. Tsou [30] proposed the use of the EOQ model with Taguchi's cost of poor quality. His model considered a situation where the process parameters are known. Chen and Lai [31] discussed EMQ, optimum process mean, and economic specification limits setting under a rectifying inspection plan model. Later Chen and Khoo [32] extended the work of Chen and Lai by considering a serial production system. Wang and Yeh [33] studied utilizing an approximate solution to obtain the optimal solution for a production and inspection model. Jeang [34] developed the simultaneous determination of production lot size and process parameters under process deterioration and process breakdown.

1.2 The General View of the Model

1.2.1 Machine adjustment period:

One essential step that must be taken before or while running a production process is to ensure that all machines, equipment and tools used are properly adjusted. As it is common that laborers and technicians do not know, in advance, how much time the machine adjustment takes, the machine adjustment period is a random variable. In some chemical processes, the chemical composition is properly adjusted during the machine adjustment period. . In practice there are examples where machines, equipment, and tools can be adjusted while the production is running such as:

- Manufacturing or repairing parts and items of equipment by using a variety of machine tools and performing the following processes; the initial planning of the work, selecting the material, laying out the work to be machined, determining the machines to be used and proper machining sequences, setting up the work on the machine, performing necessary machining operations, and performing precision handwork to fit, finish, and assemble machined parts and equipment, Fig. 1.1 and 1.2.

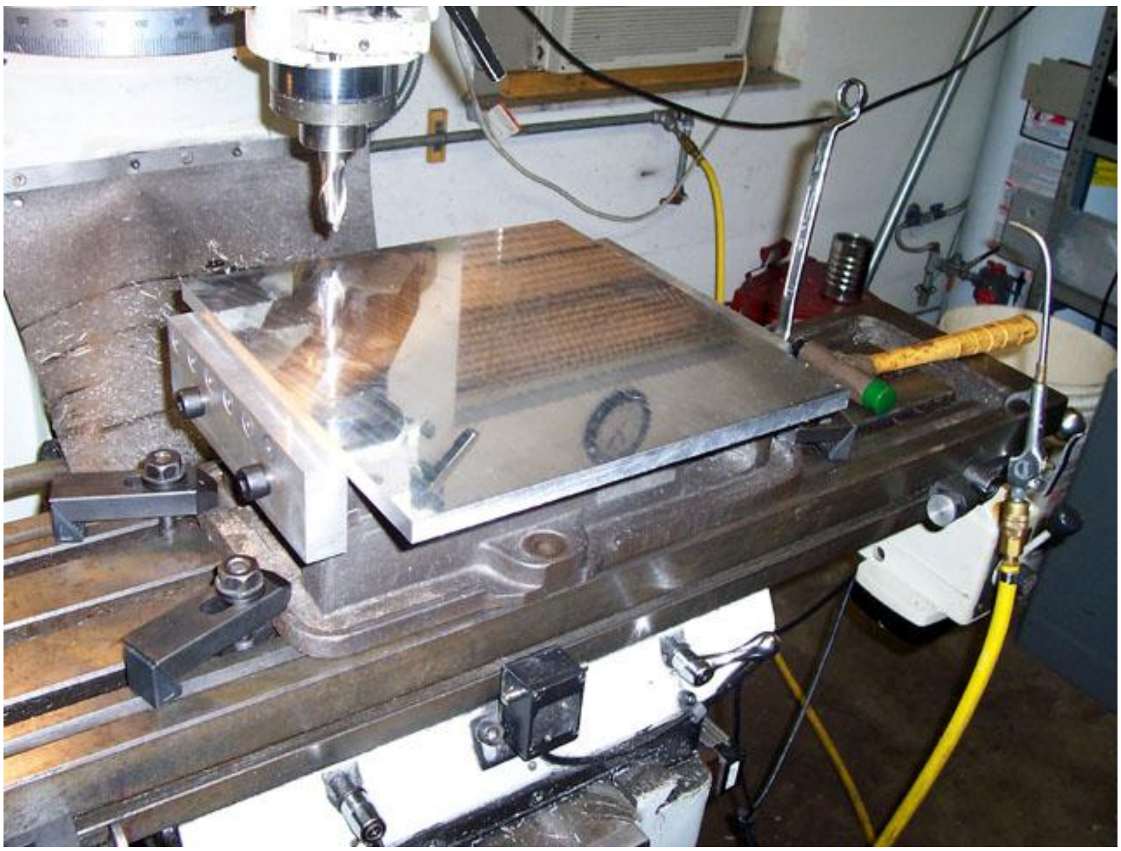

Figure 1.1: A Kurt Vise is used as an adjustment tool in Milling Techniques

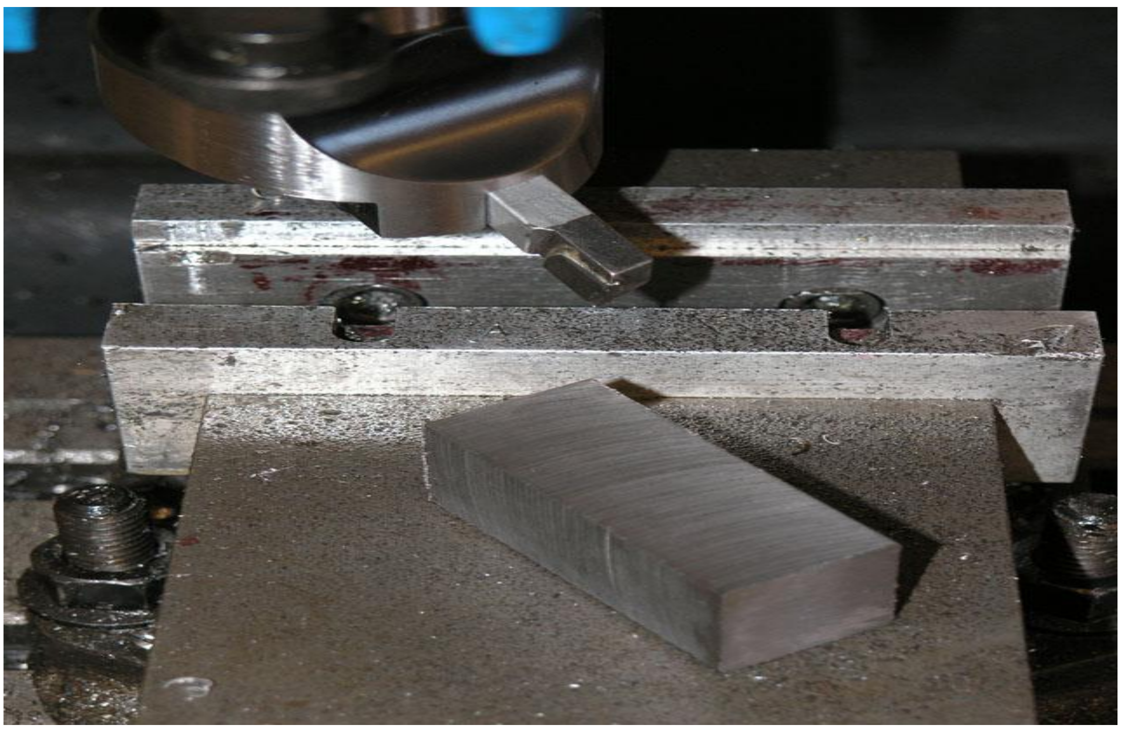

Figure 1.2: A fly cutter and an off-the-shelf brazed carbide lathe tool which is used in the surface finish

Figure 1.3: An electric arc furnace and the machines used in the steel making process.

- Steel production provides an example of adjusting the chemical composition by adjusting furnace and machine parameters. In steel production iron is smelted from its ore which contains some carbon. To produce the desirable type of steel, iron must be melted and reprocessed by adjusting the furnace and machine parameters to change the carbon content to the desired amount, at which point other elements can be added, Fig. 1.3.

1.2.2 General model description:

In this thesis we consider an inventory system with a constant demand rate D. In traditional EPQ models it is assumed that items produced are of perfect quality. However, product quality in this model is not always perfect and is usually a function of the production process. The demand of the produced item is continuous and constant and all demands must be met (production rate > demand rate). Also, the production rate is finite at a fixed rate.

The behavior of the inventory level in our model is illustrated in Fig 1.4. When production starts, the inventory level will increase at a rate $P(1-d)-D$. Within the screening/adjustment time t, the lot is inspected and the machines are adjusted. A product which is outside the specification limits will be detected and processing the defective items will be stopped. Then, production continues at a rate $P-D$. After the production period ends, the inventory level decreases at a rate D and this is one complete cycle of length T.

Several extensions to the existing models have been proposed in this thesis. They are classified into different models in which each model takes some considerations into account (Table 1.1).

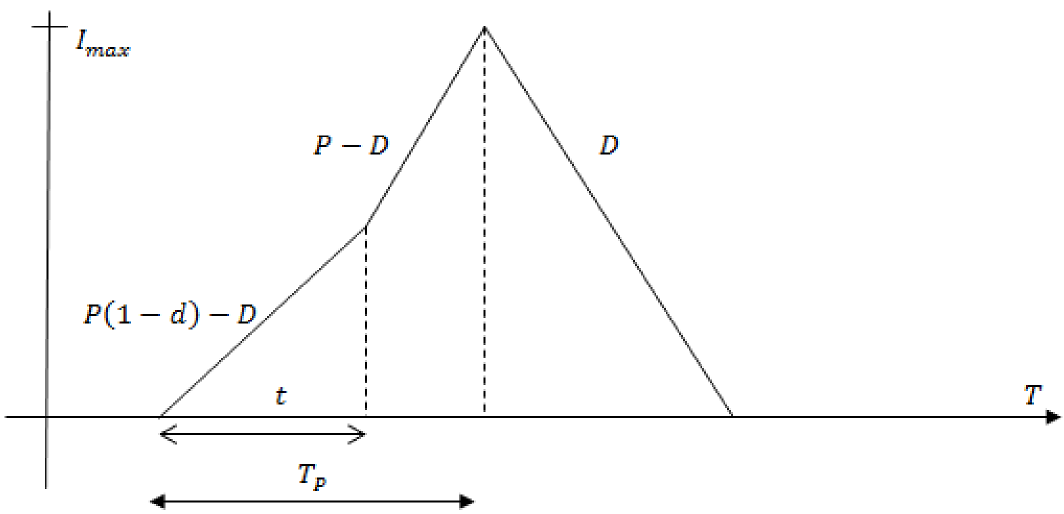

Figure 1.4: The inventory level during a production cycle.

Table 1.1: The models discussed in this thesis.

Chapter #	The Adjustment Machine Period	Decision Variable(s)	Cases
2	Deterministic	Q	Two cases of t are treated : $t < T_P$ and $t > T_P$
	Stochastic		t has a probability density function.
3	Deterministic	$Q \ and \ S_{max}$	Three cases of t are treated : $t < T_0, T_0 < t < T_P$ $and \ t > T_P$
	Stochastic		t has a probability density function.
4	Deterministic	$Q \ and \ S_{max}$	Taguchi's quality loss function is introduced. The product's quality distribution is normal.
			Two types of errors are committed in the inspection process. Reworked items are perfect.
5	Deterministic	$Q \ and \ S_{max}$	The two types of errors are random. Further processed items are sold at a reduced price at a secondary market.
	Stochastic		

CHAPTER 2

THE OPTIMAL LOT SIZE UNDER MACHINE ADJUSTMENT PERIOD

In this chapter we treat the adjusting machine period, t, as a deterministic value and a random variable and the order quantity size, Q, as the decision variable. Accordingly , two cases of t are treated in this model. They are cases where the adjusting machine period ends before the completion of the production period, $t < T_P$, (Fig. 2.1), and it takes longer than the production period, $t > T_P$, (Fig. 2.2).

Fig. 2.1 shows the situation where the production is running and machines are being adjusted, and some defective items are produced. When the adjustment reaches the desired level, the production becomes always perfect and that leads to the increase of the slope from $P(1-d)-D$ to $P-D$. Unlike Fig. 2.1, Fig. 2.2 shows no change in the slope during the production period and that is because machine adjustment takes place during the entire production period.

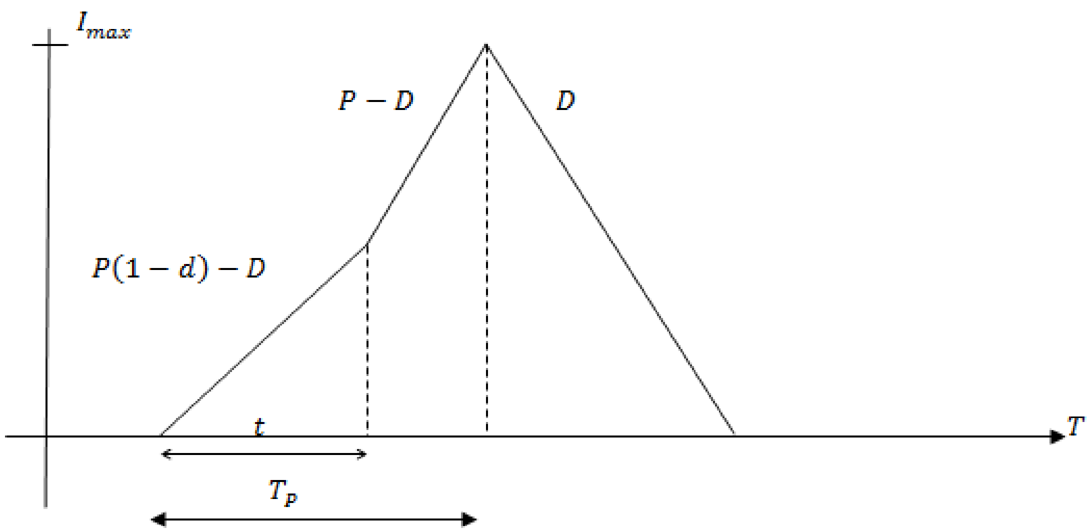

Figure 2.1: The relationship between the inventory level and time when $< T_P$.

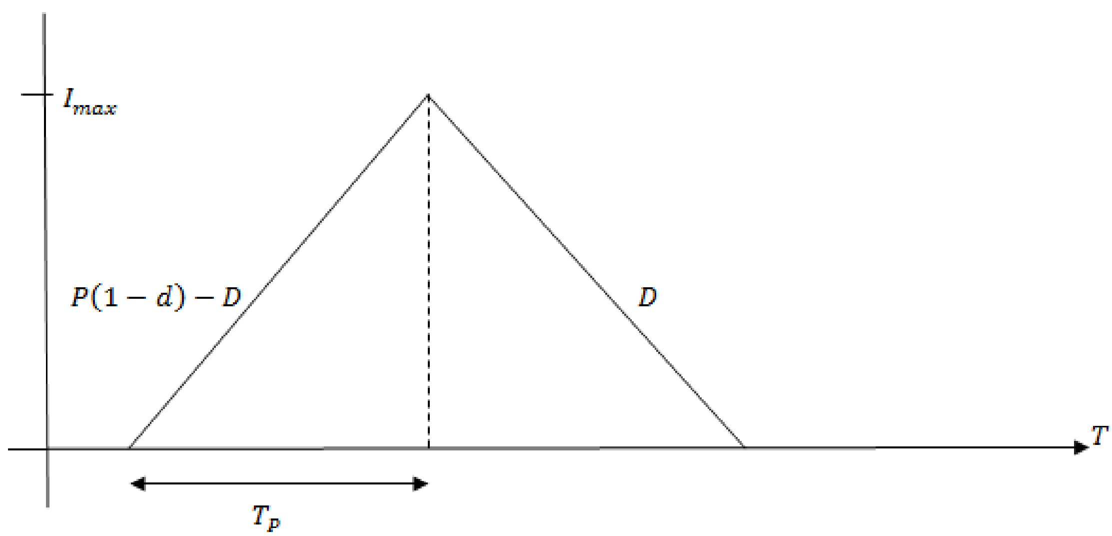

Figure 2.2: The relationship between the inventory level and time when $t \geq T_P$.

2.1 The case where *t* is deterministic:

When t is a fixed value, the total costs are separately set for the two cases, namely, $t < T_P$, and $t \geq T_P$.

2.1.1 The case of $t < T_P$

In this case, the total inventory, I, during one cycle can be computed as the area under the graph in (Fig. 2.1), and is given by

$$I = t.Z/2 + T_1.(Z + I_{max})/2 + T_D.I_{max}/2 = [T_p.Z + (T - t)I_{max}]/2$$

where, the cycle length $T = T_P + T_D$, the production period $T_p = \frac{Q}{P}$, the demand during a production cycle, $T.D = Q - t.P.d$, $Q=P.T_P$, Z is the inventory level at the end of the adjustment period and is given by; $Z = t.(P \times (1 - d) - D)$ and I_{max} is the maximum inventory during a cycle and is given by;

$$I_{max} = Q - t.P.d - D * T_p$$

And thus:

$$\bar{I}(Q|t < T_P) = \frac{P(Q-dPt)^2+D\left(-Q^2+dP^2t^2\right)}{2P(Q-dPt)} \quad (2.1)$$

The cost per cycle is the sum of setup cost, A, the manufacturing cost, CQ, the process adjustment cost per cycle and the screening cost $r.t.P.d+A_d.t$, and the holding cost, *h:*

$$A + CQ + r\ t\ P + A_d.t + h\ T\ \bar{I}(Q/t) \quad (2.2)$$

The total annual cost, for a given adjustment time, is obtained by multiplying Eq. 2.2 by the number of cycles per year 1/ $T=D/(Q-t.P.d)$,

$TCY(Q|t < T_P) =$

$$\frac{AD}{Q-t\,.P\,.d} + \frac{C\,Q\,D}{Q-t.P.d} + \frac{r\,t\,P\,D\,d}{Q-t.P.d} + \frac{A_d\,t\,D}{Q-t.P\,.d} + h(\frac{P(Q-dPt)^2 + D\,(-Q^2+dP^2t^2)}{2P(Q-dPt)}) \qquad (2.3)$$

where, t is the process adjustment period and is assumed to be constant and $P(1-d) > D$. The convexity of the total cost per year can be demonstrated by finding the second derivative of $TCY(Q)$:

$$\frac{d^2TCY(Q)}{dQ^2} = \frac{D(2\,A + t\,(2\,F + d\,P(2\,r + 2\,C + h\,t - d\,h\,t)))}{(Q - d\,P\,t)^3} > 0$$

Then the production quantity, Q is obtained by setting $TCY(Q)$ to zero and solving for Q. Thus,

$$Q^* = d.P.t + \sqrt{-DP(-2A + t(-2A_d - d\,P(2\,r + d(2C + h\,t - d\,h\,t))))} \qquad (2.4)$$

If $t = 0$, the formula for Q^* is reduced to

$$Q^* = \sqrt{\frac{2\,A\,D}{h(1-\frac{D}{P})}} \qquad (2.5)$$

which is the traditional *EPQ* formula with a finite production rate.

2.1.2 The case of $t \geq T_P$

If we follow the same procedure used in the first case, we can compute the average inventory level as the area under the graph in Fig. 2.2 and it is given by:

$$\bar{I}(Q|t \geq T_P) = \frac{(-D+(1-d)P)Q}{2\,P} + \frac{(-D+(1-d)P)^2Q^2}{2\,D\,P^2} \qquad (2.6)$$

The total annual cost, for a given adjustment time, is obtained by multiplying Eq. 2.2 by the number of cycles per year $\frac{\mathbf{1}}{\boldsymbol{T}} = \frac{\boldsymbol{D}}{\boldsymbol{Q}(\mathbf{1}-\boldsymbol{d})}$,

$TCY(Q|t \geq T_P) =$

$$\frac{AD}{Q(1-d)} + \frac{C\ D}{1-d} + \frac{r\ D\ d}{1-d} + \frac{A_d\ D}{P(1-d)} + h\left(\frac{(-D+(1-d)P)Q}{2\ P} + \frac{(-D+(1-d)P)^2 Q^2}{2\ D\ P^2}\right) \qquad (2.7)$$

Eq. 2.7 is a convex function, but it has no closed form solution for the optimal Q. It is useful to know in advance which case applies, which can be achieved by dividing both sides of Eq. 2.4 by P;

$$T_P = \frac{d.P.t + \sqrt{-DP(-2A+t(-2A_d - d\,P(2\,r+d(2C+h\,t-d\,h\,t))))}}{P} \qquad (2.8)$$

For feasibility purposes, $\boldsymbol{Q} > \boldsymbol{t}\ .\boldsymbol{P}\ .\boldsymbol{d}$.

2.2 The case where *t* is a random variable:

It may happen that the machine adjustment period is a random variable where it finishes before the end of production period or takes a longer time than usual. In this model, *t* is treated as a random variable and has some probability density function $f(t)$.

In such a situation the total costs must be calculated over the total cycles based on renewal theory;

$$\frac{E[TC(Q)]}{E[T(Q)]} = \frac{TC_1(Q)+TC_2(Q)+\ ...+TC_n(Q)}{T_1(Q)+T_2(Q)+\cdots+T_n(Q)} \qquad (2.9)$$

According to renewal theory the formulation of this model is obtained as follows:

$$\frac{E[TC(Q|t)]}{E[T(Q|t)]} = \left(\int_0^{\frac{Q}{P}} \left[A + C\,Q + r\,t\,P\,\;d\; + A_d\,t + h\frac{P(Q-dPt)^2+D\left(-Q^2+dP^2t^2\right)}{2D\,P}\right] f(t)dt + \right.$$

$$\int_{\frac{Q}{P}}^{\infty} \left[A + C\,Q + r\,Q\;\,d\; + A_d\,\frac{Q}{P} + \right.$$

$$\left. h\left(-\frac{(-1+d)Q^2((-1+d)P+D)(P((-1+d)Q-D)+QD)}{2P^2D^2}\right)\right] f(t)dt\;\Bigg) \div \left(\int_0^{\frac{Q}{P}} \left[\frac{Q-P\,t\,d}{D}\right] f(t)dt + \right.$$

$$\left. \frac{Q(1-d)}{D}\int_{\frac{Q}{P}}^{\infty} f(t)dt\right) \qquad (2.10)$$

where the limits of the first and second integrals indicate the scenarios of, $t < T_P$, and ,$t \geq T_P$, respectively.

2.3 Numerical Examples:

2.3.1 Example I:

Consider the following data:

P	=	25,000 units per year
r	=	\$1 per unit
D	=	20,000 units per year
h	=	\$ 4 per unit/year
C	=	\$ 5 per unit
t	=	1 hour
A_d	=	\$ 50 per hour
A	=	\$ 100 per order
d	=	0.0455

By using Eq. 2.8 to determine which case is applicable, we get:

$$T_P = 0.41 < t$$

Therefore the case, $t \geq T_P$ applies.

Then the total annual cost is given by Eq. 2.7 as follows:

$$TCY(Q) = 105{,}762.18 + \frac{2{,}095{,}337.87}{Q} + 4(0.08Q + 5.96 \times 10^{-7}Q^2)$$

This function is convex as Fig. 2.3 shows and its minimum is attained at $Q^* = 2{,}554.13$ units and $TCY(2{,}554.13) = \$107{,}387$. Note that $T_P = Q/P = 0.102 < 1 = t$. Hence the assumption is valid.

Table 2.1 shows the effect of t on the optimal lot size. As the adjustment time increases, more defectives are generated and hence a larger lot size is needed. Once t exceeds a certain value, 0.1779, the lot size becomes constant.

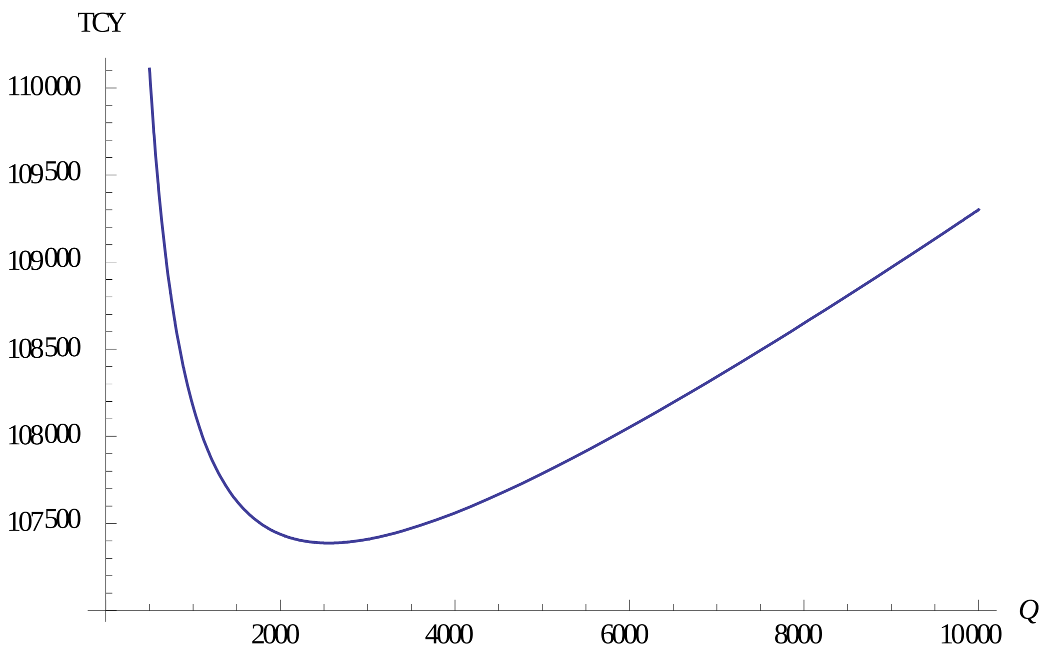

Figure 2.3: The convexity of $t \geq T_P$ for the data given in Example I.

Table 2.1: The effect of t on the optimal lot size.

Adjustment Period t	Lot Size Q^*
0.000	2236.1
0.025	2638.2
0.050	2994.2
0.075	3318.2
0.100	3618.2
0.125	3899.4
0.150	4165.4
0.175	4418.8
$\geq$ 0.1779	4447.5

2.3.2 Example II:

Suppose that the adjustment period is a uniform random variable where $f(t) = 1/8$, $0 \leq t \leq 8$. Substituting the data of Example I into Eq. 2.10 gives the following function shown also in Fig. 2.4:

$$\frac{E[TC(Q)]}{E[T(Q)]} = \frac{100 + 5.05Q + 1.47 \times 10^{-5}Q^2 + 1.24 \times 10^{-10}Q^3 - 5.70 \times 10^{-16}Q^4}{4.77 \times 10^{-5}Q + 5.69 \times 10^{-12}Q^2}$$

The optimal lot size is given by $Q^* = 2{,}612.37$ and the corresponding cost is $\$107{,}349$. Table 2.2 shows the effect of r and A_d on Q^*.

As r is fixed and A_d goes up, there is a slight increase in $Q*$, indicated by the results across the rows. Likewise, as the results in the columns indicate, when A_d is fixed and r increases, Q^* increases at a constant rate as well.

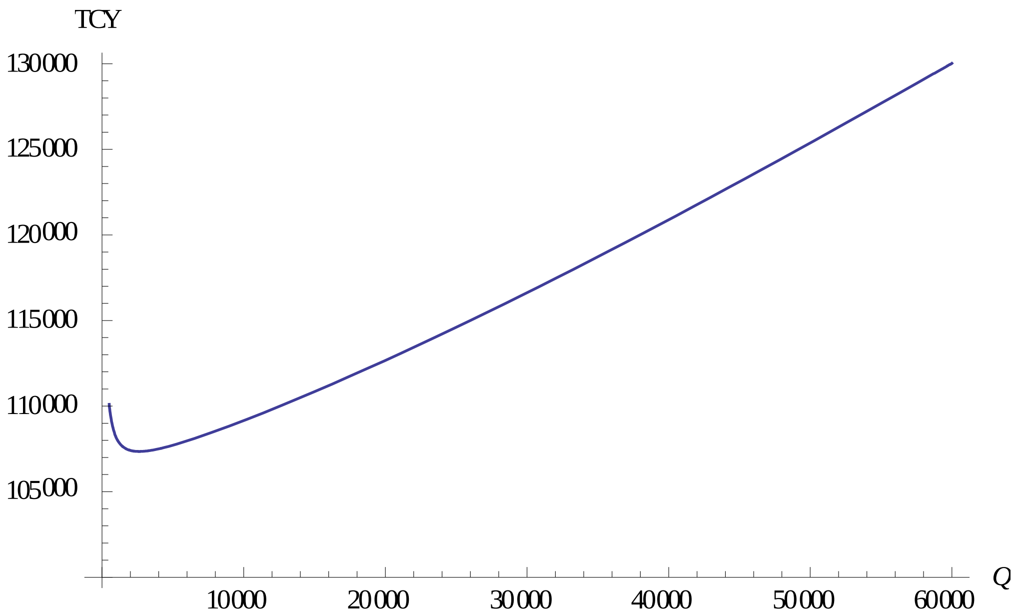

Figure 2.4: The plot of calculations of Example II where t is a random variable.

Table 2.2: The effect of r and A_d on Q^*

r	A_d 30.00	40.00	50.00	60.00	70.00	80.00
0.50	2,607.00	2,607.09	2,607.19	2,607.28	2,607.37	2,607.46
1.00	2,612.19	2,612.28	2,612.37	2,612.46	2,612.55	2,612.64
1.50	2,617.40	2,617.49	2,617.58	2,617.68	2,617.77	2,617.86
2.00	2,622.64	2,622.74	2,622.83	2,622.92	2,623.01	2,623.11
2.50	2,627.92	2,628.01	2,628.11	2,628.20	2,628.29	2,628.38
3.00	2,633.23	2,633.32	2,633.41	2,633.51	2,633.60	2,633.69

CHAPTER 3

THE OPTIMAL LOT SIZE UNDER MAXIMUM SHORTAGE ALLOWANCE

In this chapter we extend the models in Chapter 2 by allowing for a shortage. A shortage cost is incurred if units of inventory are not available on demand. It includes the cost of lost sales, loss of goodwill, overtime payments, customer dissatisfaction, and special administrative efforts resulting from inability to meet the demand. There are two types of shortage costs: (1) one- time shortage cost per unit short, independent of the duration of the shortage, $\breve{\pi}$, and (2) shortage cost per unit short per unit time, $\hat{\pi}$.

There are three cases to be considered. The adjustment period can take place during the shortage period, can take place within the production period, or can exceed the production period. We consider t as a fixed value (Section 3.1) and as a random variable (Section 3.2).

3.1 The case where t is deterministic:

In Section 3.1.1 we discuss the case of $t < T_0$ where T_0 is the period that starts with the resumption of the production cycle and ends when the shortage is zero as shown in Figure 3.1. In Section 3.1.2 we discuss the case of $T_0 \leq t < T_p$ where T_p is the production period. Finally, in Section 3.1.3, we disuses the case of $t \geq T_p$.

3.1.1 The case of $t < T_0$

In this case we assume that the adjustment and/or screening is completed before satisfying the shortage. So, during the interval T_0 the shortage is initially satisfied at a rate $P(1-d)-D$, and after t it is satisfied at the rate $P - D$, as shown in Fig. 3.1.

The quantity produced in a production cycle, Q, is used to replenish the shortage, S_{max}, build up inventory up to level I_{max}, and satisfy the demand during the production interval $D\,T_p$. In addition, some items are discarded, which is equal to $t.P.d$. Hence:

$$Q = S_{max} + I_{max} + D\,T_P + t.P.d \tag{3.1}$$

The total inventory is the area above the horizontal axis in Figure 3.1 which is given by:

$$I = I_{max}(T_1 + T_2)/2 \tag{3.2}$$

Note that $T_1 = I_{max}/(P-D)$, $T_2 = I_{max}/D$ and $T_p = Q/P$ Substituting T_1 and T_2 into (3.2) and I_{max} from (3.1) into (3.2) we get

$$\bar{I}\big((Q, S_{\max})|t \leq T_0\big) = \frac{(P(-Q+S_{max})+d\,P^2t+QD)^2}{2P(Q-d\,P\,t)(P-D)} \tag{3.3}$$

$$T_0 = T_P - T_1 = \frac{Q}{P} - \frac{I_{max}}{P-D} = \frac{S_{max}+d\,P\,t}{P-D} \tag{3.4}$$

$$T_3 = \frac{S_{max}}{D}$$

The average shortage $\bar{S}$ over the cycle T is the sum of the two areas, T_0 and T_3, divided by T:

$$\bar{S} = \frac{P(S_{max}^2 + 2\ d\ D\ S_{max}\ t + d\ D(D + (-1+d)P)\ t^2)}{2(P-D)(Q-d\ P\ t)} \tag{3.5}$$

The average shortage cost during the cycle T is:

$$\hat{\pi}\ T\ \bar{S} + \ \breve{\pi}\ S_{max}$$

Therefore, the total annual cost is given by:

$$TCY(Q, S_{max}|t \leq T_0) = \frac{AD}{Q-t\ .P\ .d} + \frac{C\ Q\ D}{Q-t.P.d} + \frac{r\ t\ P\ D\ d}{Q-t.P.d} + \frac{A_d\ t\ D}{Q-t.P\ .d} +$$

$$h\left(\frac{(P(-Q+S)+dP^2t+QY)^2}{2P(Q-dPt)(P-Y)}\right) + \hat{\pi}\ \left(\frac{P\left(S_{max}^2+2\ d\ D\ S_{max}\ t+d\ D(D+(-1+d)P)\ t^2\right)}{2(P-D)(Q-d\ P\ t)}\right) + \breve{\pi}\frac{S_{max}\ D}{Q-t.P.d}$$

(3.6)

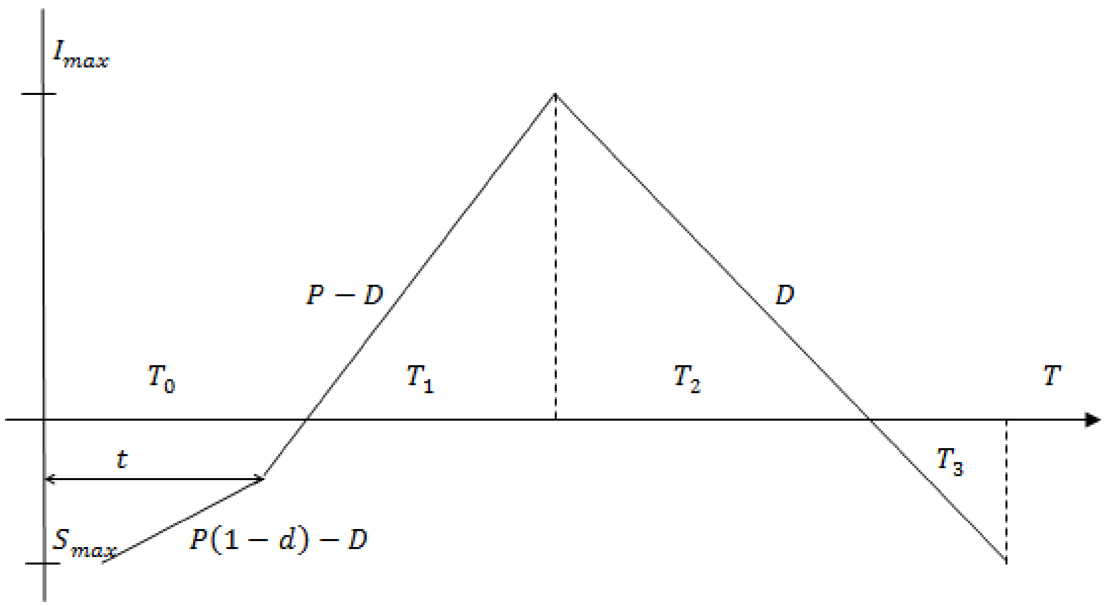

Figure 3.1: The relationship between the inventory level and time when $t < T_0$.

3.1.2 Numerical Example:

Let us consider the following data:

t	=	0.15 hour
P	=	25,000 units per year
r	=	$1 per unit
D	=	23,000 units per year
h	=	$ 4 per unit/year
C	=	$ 5 per unit
t	=	0.15 hour
A_d	=	$ 50 per hour
A	=	$ 100 per order
$\hat{\pi}$	=	$ 5/unit/year
$\breve{\pi}$	=	$ 0.3
d	=	0.0455

The optimal values of the order quantity and the maximum shortage permitted are Q^*= 16,367.6 units and S_{max}^*= 357.585 units with minimum cost of $118,124.8. The plots of *TCY* versus Q at $S_{max} = S_{max}^*$ and *TCY* versus S_{max} at $Q = Q^*$ are shown in Fig. 3.2 and 3.3, respectively.

Note that $0.15 = t < T_0$ = 0.264, therefore this case applies. Table 3.1 and Fig. 3.4 show the effect of increasing t on Q, S_{max} and TCY.

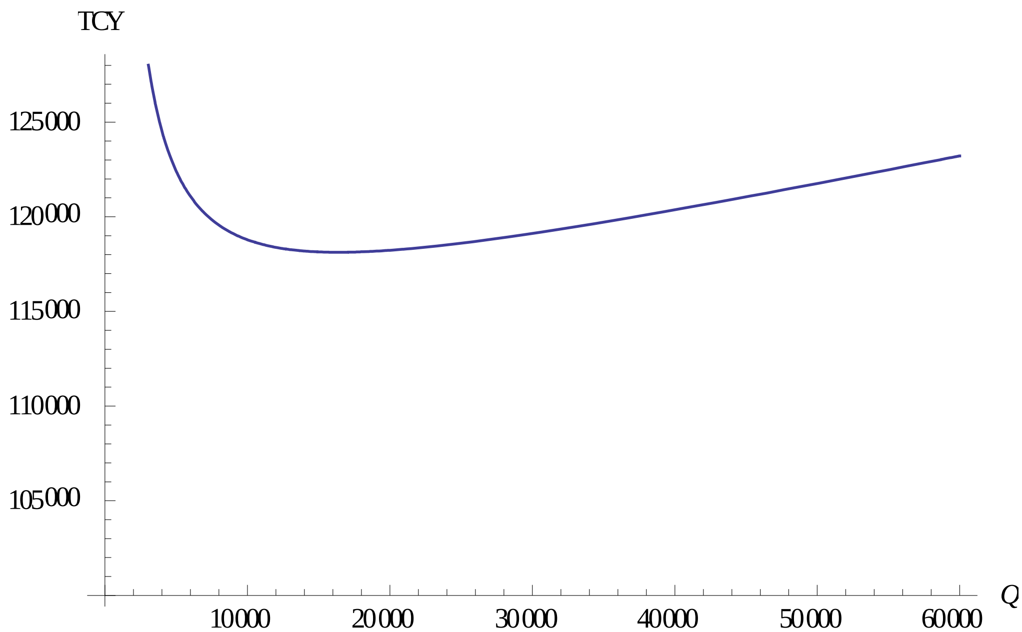

Figure 3.2: The behavior of Q as S^{*}_{max} is fixed for $t < T_0$.

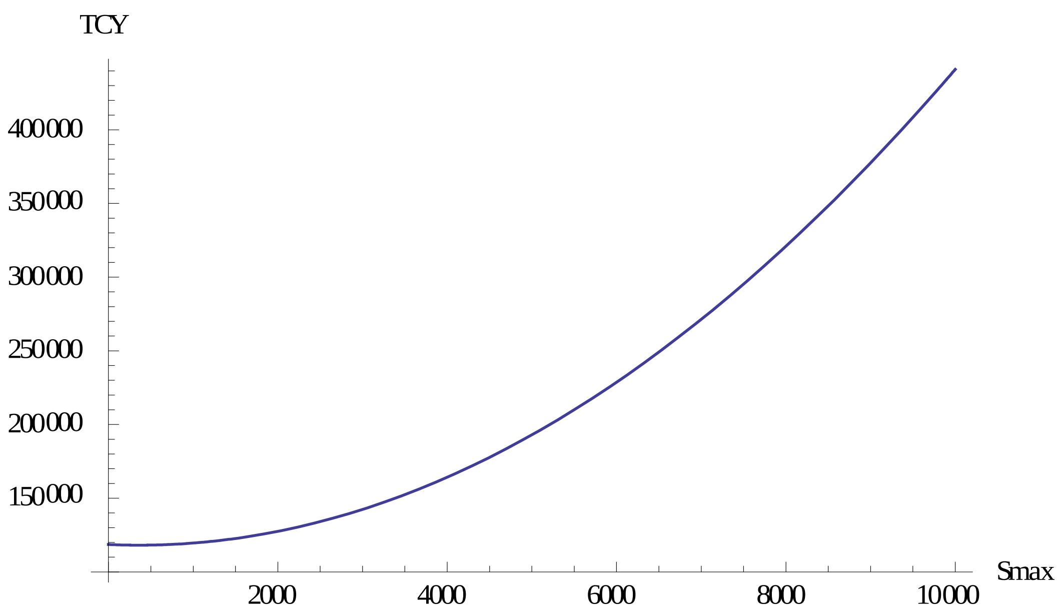

Figure 3.3: The behavior of S_{max} as Q^* is fixed for $t < T_0$.

Table 3.1: The effect of increasing t on Q, S_{max} and TCY for $t < T_0$.

t	Q	S_{max}	TCY
0	4,847.11	111.01	116,107.42
0.05	10,382.7	253.48	117,081.03
0.1	13,760.7	319.24	117,671.45
0.15	16,367.62	357.58	118,124.8
0.2	18,528.74	380.08	118,499
0.25	20,384.53	391.71	118,818.69
0.3	22,011.17	395.20	119,097.76
0.4	24,748.8	383.846	119,344.42

It is obvious from Table 3.1 and Fig. 3.4 that as t increases both Q and S_{max} increase as well, and as a result, TCY goes up. Indeed as t increases more units are discarded and hence Q should increase to compensate for these units. However, when t > 0.3, S_{max} decreases. Fig. 3.5 shows the interaction of Q, S_{max} and TCY.

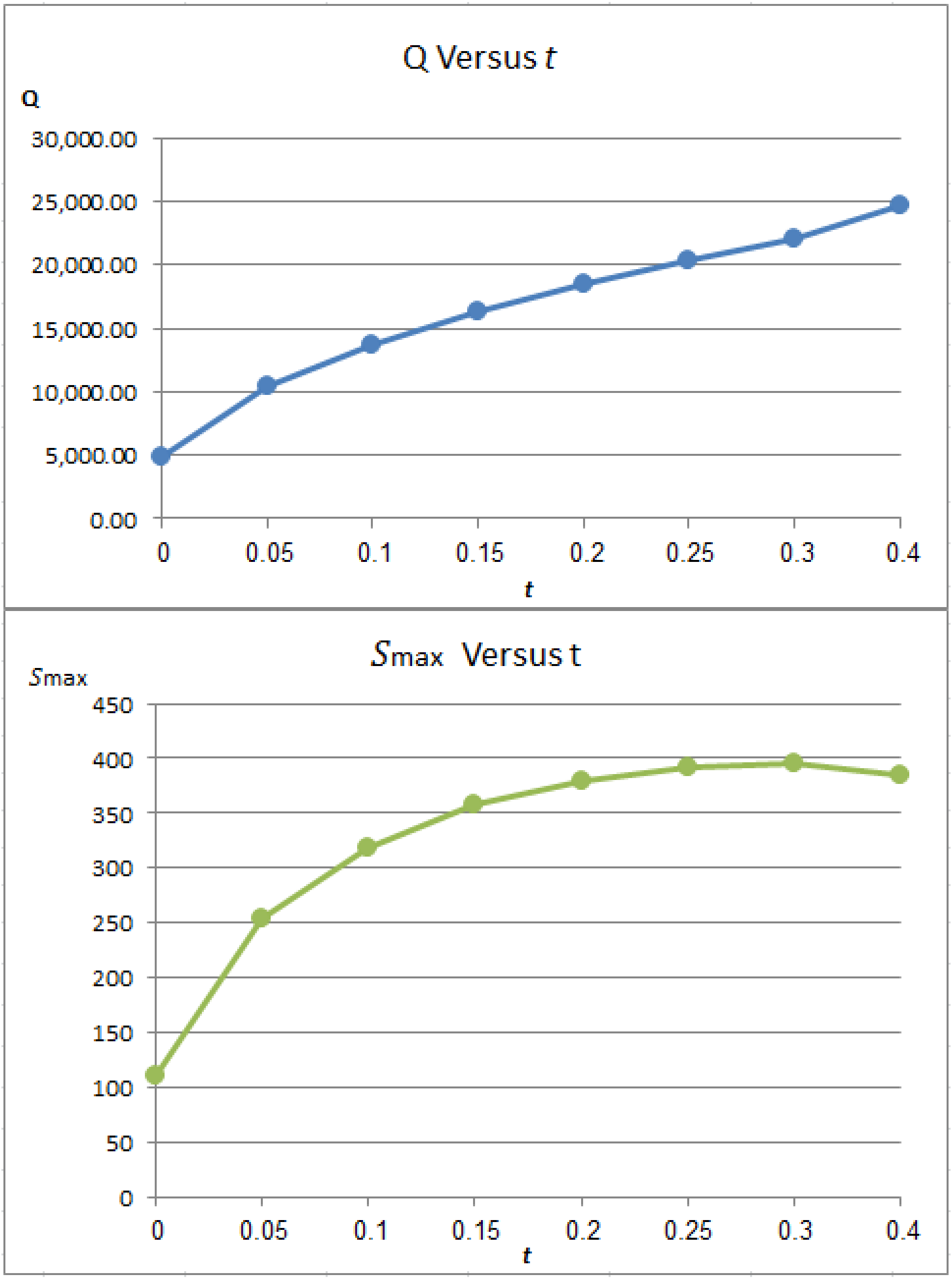

Figure 3.4: The behavior of Q and S_{max} **as** t **increases for case of** $t < T_0$

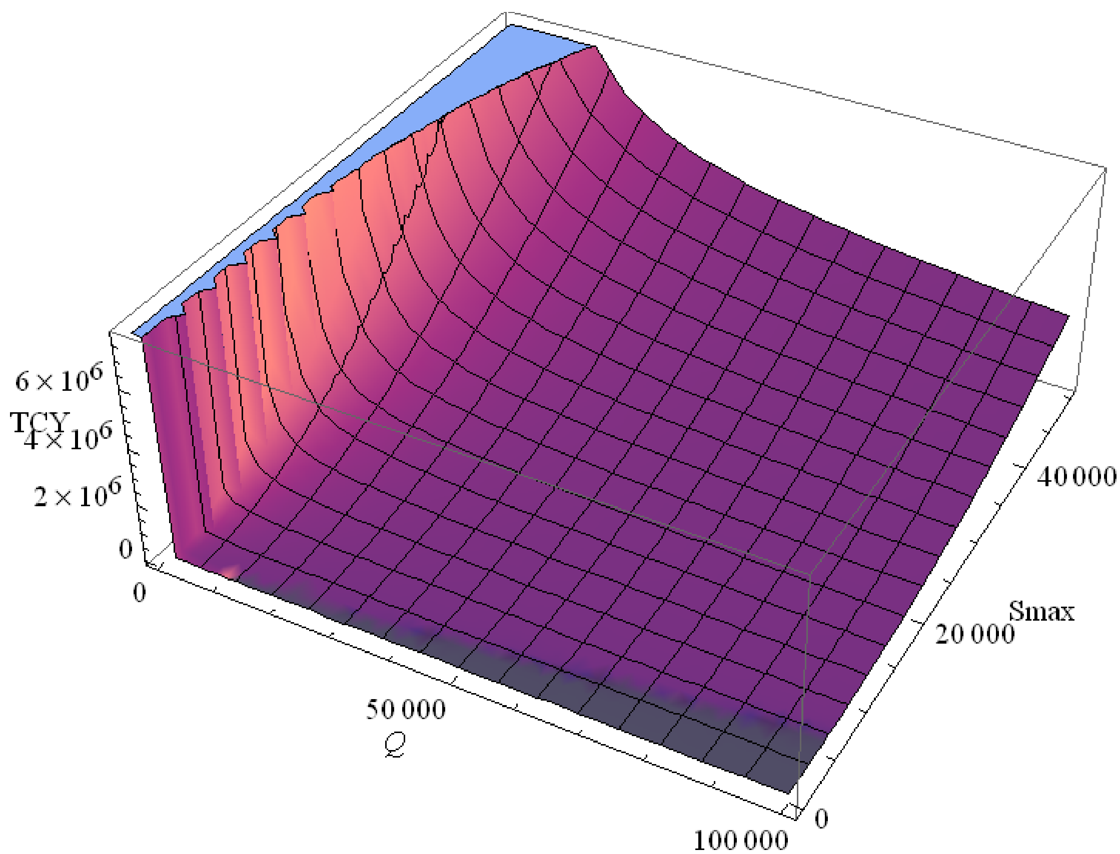

Figure 3.5: Three dimensional plot of Q, S_{max} and TCY for $t < T_P$.

3.1.3 The case of $T_0 \leq t < T_P$

In this case t extends beyond the shortage period but ends before end of production. Therefore t exceeds $T_0 = \frac{S_{max}}{P(1-d)-D}$,and the inventory increases at slope of $P(1-d) - D$, then the slope changes to $P - D$ until the end of production period, Fig. 3.6.

The total inventory, I, during one cycle is given by the area under the graph above the horizontal axis and is given by

$$I = T_1.\frac{Z}{2} + (T_P - t).(Z + I_{max})/2 + T_D.I_{max}/2$$

Thus, the average inventory level is obtained as follows:

$$\bar{I}\left((Q, S_{\max})|T_0 \leq t < T_P\right) =$$

$$\frac{D\left(-\frac{(S_{max}+t((-1+d)P+D))^2}{(-1+d)P+D}+\frac{(S_{max}+d\,Pt+Q(-1+\frac{D}{P}))^2}{D}+\frac{(-Q+P\,t)((-1+2\,d)P^2t+Q\,D+P(-Q+2\,S_{max}+t\,D))}{P^2}\right)}{2(Q-d\,P\,t)} \quad (3.7)$$

The average shortage $\bar{S}$ over the cycle T is the sum of two areas of T_0 and T_2 divided by T:

$$\bar{S} = \frac{1}{T}\,\frac{S_{max}^2\,P\,(1-d)}{2(P(1-d)-D)D} = \frac{S_{max}^2\,P\,(1-d)}{2\,(Q-t.P.d)\,(P(1-d)-D)} \quad (3.8)$$

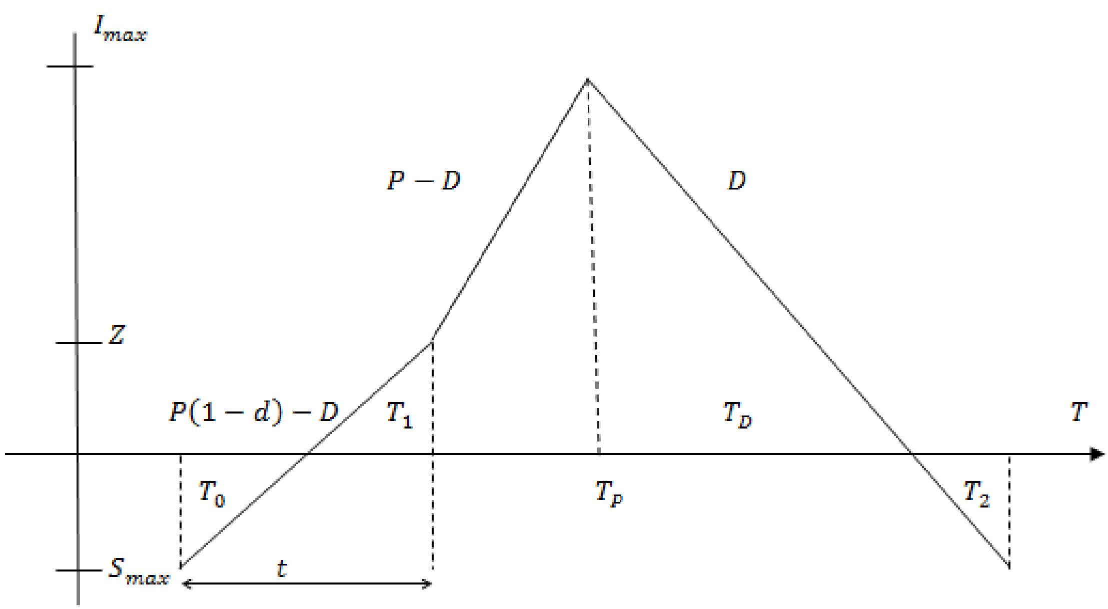

Figure 3.6: The relationship between the inventory level and time when $T_0 \leq t < T_P$.

Therefore, the total annual cost per cycle can be obtained as follows:

$$TCY(Q, S_{max}|T_0 \leq\ t\ < T_P) = \frac{AD}{Q-t\,.P\,.d} + \frac{C\,Q\,D}{Q-t.P.d} + \frac{r\,t\,P\,D\,d}{Q-t.P.d} + \frac{A_d\,t\,D}{Q-t.P\,.d} +$$

$$h\,\frac{D\left(-\frac{\left(S_{max}+t\left((-1+d)P+D\right)\right)^2}{(-1+d)P+D}+\frac{\left(S_{max}+d\,Pt+Q\left(-1+\frac{D}{P}\right)\right)^2}{D}+\frac{(-Q+P\,t)\left((-1+2\,d)P^2t+Q\,D+P(-Q+2\,S_{max}+t\,D)\right)}{P^2}\right)}{2(Q-d\,P\,t)} +$$

$$\hat{\pi}\,\frac{S_{max}^2\,P\,(1-d)}{2\,(Q-t.P.d)\,(P(1-d)-D)} + \breve{\pi}\,\frac{S_{max}\,D}{Q-t.P.d} \tag{3.9}$$

3.1.4 Numerical Example

Considering the data given in section 3.1.2 where $t = 3.5$ hours, we find that the optimal values of order quantity and maximum shortage permitted are obtained $at\ \ Q^*$= 99,531.95 units and S^*_{max}= 1,507.24 units with a minimum cost of \$ 121,295.57 .The plots of TCY versus Q at $S_{max} = S^*_{max}$ and TCY versus S_{max} at $Q = Q^*$ are shown in Fig. 3.7 and 3.8, respectively. Also, we see that the condition $T_0 <\ t\ < T_P$ is satisfied such that $1.75 < 3.5\ < 3.98$, and hence this case applies. Table 3.2 and Fig. 3.9 show the effect of increasing t on Q, S_{max} and TCY.

The second case shows that as t increases, S_{max} , Q and TCY. This can be justified from the first case. Fig. 3.10 shows the interaction of Q, S_{max} and TCY.

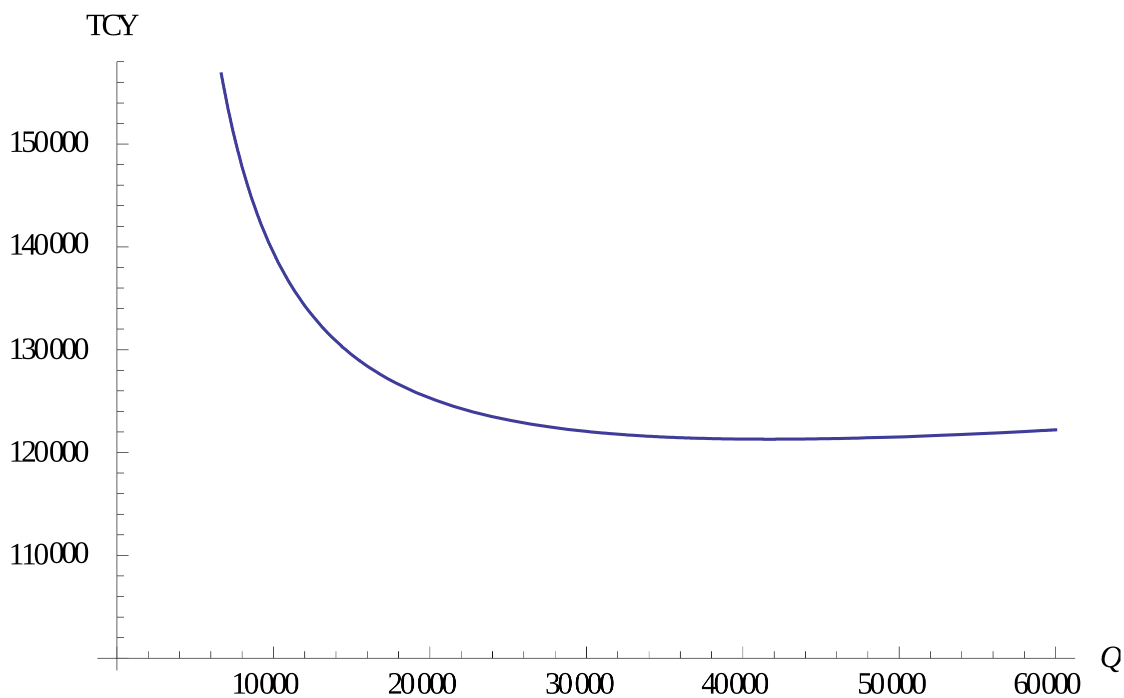

Figure 3.7: The behavior of Q as S^{*}_{max} is fixed for $T_0 \leq t < T_P$.

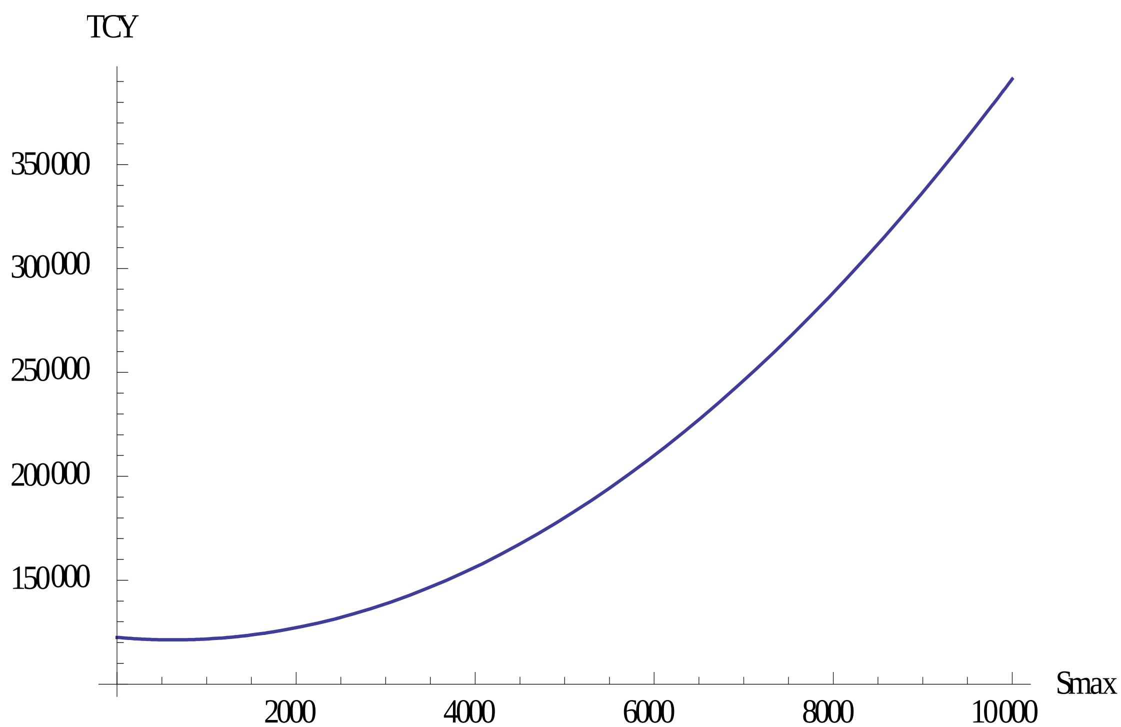

Figure 3.8: The behavior of S_{max} as Q^* is fixed for $T_0 \leq t < T_P$.

Table 3.2: Shows the effect of increasing t on Q, S_{max} and TCY for $T_0 \leq t < T_P$.

t	Q	S_{max}	TCY
0.5	27,646.1	407.27	119,942.68
1.25	48,040.15	721.18	121,800.64
2	65,936.22	994.96	123,019.75
2.75	82,950.68	1254.58	124,013.40
3.5	99,531.95	1507.24	124,896.26
4.25	115,864.26	1755.9	125,715.46
5	132,039.14	2002.03	126,494.39
5.75	148,107.73	2246.46	125,007.48

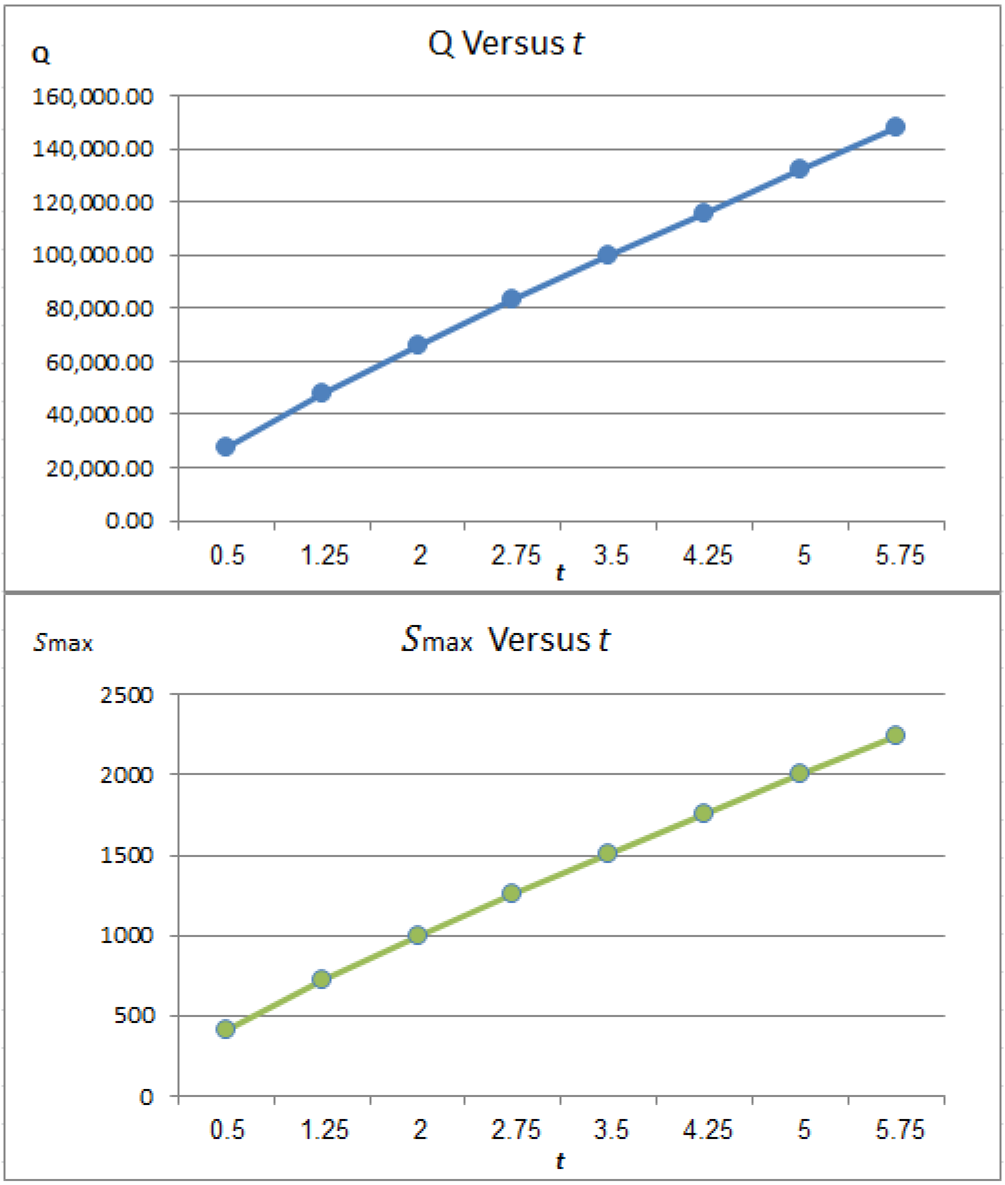

Figure 3.9: The behavior of Q and S_{max} as t increases for case of $T_0 \leq t < T_P$.

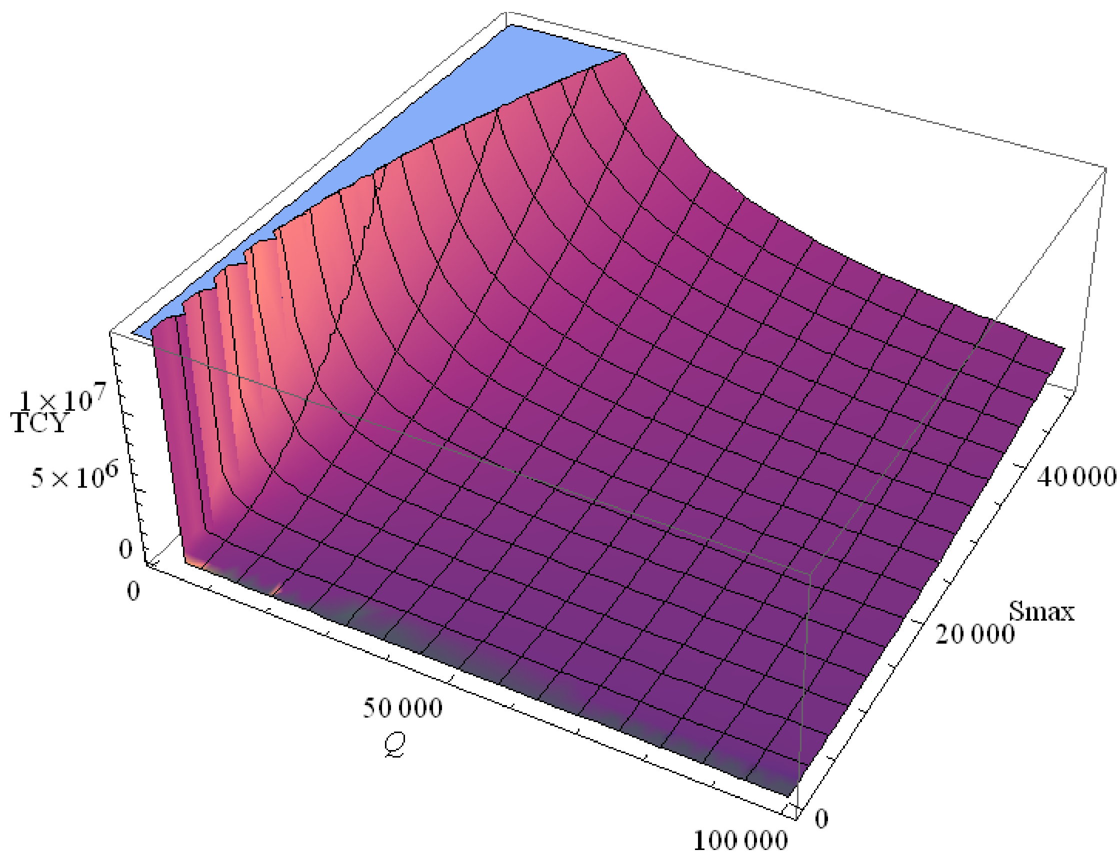

Figure 3.10: Three dimensional plot of Q, S_{max} and TCY for $T_0 \leq t < T_P$.

3.1.5 The case of $t \geq T_P$

In this section we consider the case where t is longer than the production period. Therefore, the inventory level keeps increasing at a rate of $P(1-d)-D$ during the production period, as shown in Fig. 3.11.

The total inventory, I, during one cycle can be computed as the area under the graph in Fig. 3.11 and is given by:

$$\bar{I}\big((Q, S_{\max})|t \geq T_P\big) = -\frac{(P((-1+d)Q+S_{max})+Q\ D)^2}{2\ P\ Q\ ((-1+d)P+D)} \tag{3.10}$$

The shortage cost occurs over the time intervals, T_0, and, T_1, during the cycle T.

$T_0 = \frac{S_{max}}{P(1-d)-D}$ (Time to eliminate a backorder)

$T_1 = \frac{S_{max}}{D}$ (Time to build a backorder of S_{max})

The average shortage, $\bar{S}$, over the cycle T is the sum of two areas divided by T:

$$\bar{S} = \frac{1}{T}\ \frac{S_{max}^2\ P\ (1-d)}{2(P(1-d)-D)D} = \frac{S_{max}^2\ P}{2\ Q(P(1-d)-D)} \tag{3.11}$$

Therefore, the total annual cost per cycle can be obtained as follows:

$$TCY(Q, S_{max}|t \geq T_P) =$$

$$\frac{AD}{Q(1-d)} + \frac{C\ D}{1-d} + \frac{r\ D\ d}{1-d} + \frac{A_d\ D}{P(1-d)} + h\left(-\frac{(P((-1+d)Q+S_{max})+Q\ D)^2}{2\ P\ Q\ ((-1+d)P+D)}\right) + \hat{\pi}\ \frac{S_{max}^2\ P}{2\ Q(P(1-d)-D)} + \breve{\pi}\frac{S_{max}\ D}{Q\ (1-d)} \tag{3.12}$$

The plot in Fig. 3.11 is similar to the graph of the classical version of EPQ where the slope is $(P - D)$, and hence TCY is a convex function, (A. Elsayed and O.Boucher, 1985). Thus, if we treat this case similar to the classical version of EPQ except for a slope of $P(1 - d) - D$, then we can prove its convexity easily.

As the production time is shorter than t, then the activity of adjusting the machine terminates with the end of production and one may substitute T_p for t. For a feasible solution the condition $Q > 0$ must be met.

3.1.6 Numerical Example:

We use the same data given in examples of previous two cases. We find that at $t = 6.9$ hours, $t = T_P$. Therefore at any value of t which meets the condition $t \geq 6.9$, the lot size Q and maximum shortage allowed S_{max} become constant. Using Eq. 3.12, we find that the optimal values of order quantity and maximum shortage permitted are obtained at Q^*= 7,761.91 unit and S^*_{max}= 91.3051 unit with a minimum cost of \$ 122,332. Fig. 3.12 shows the behavior of maximum shortage allowed S_{max} for the three cases of t. Fig. 3.13 shows the interaction of Q, S_{max} and TCY.

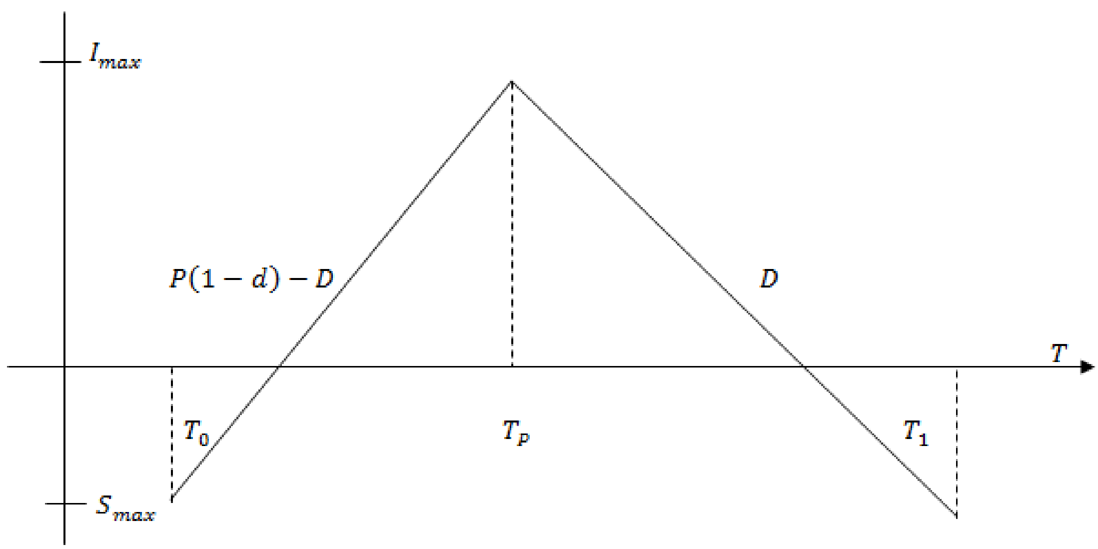

Figure 3.11: The relationship between the inventory level and time when $t \geq T_P$.

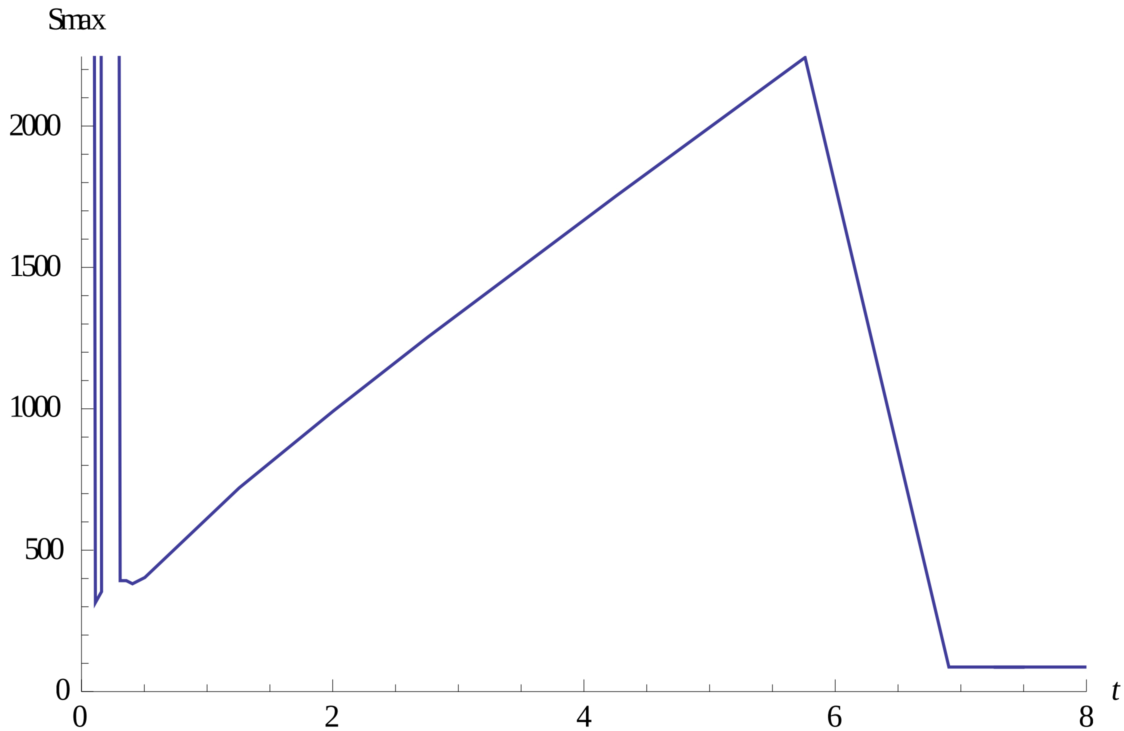

Figure 3.12: The behavior of maximum shortage allowed S_{max} for the three cases of t.

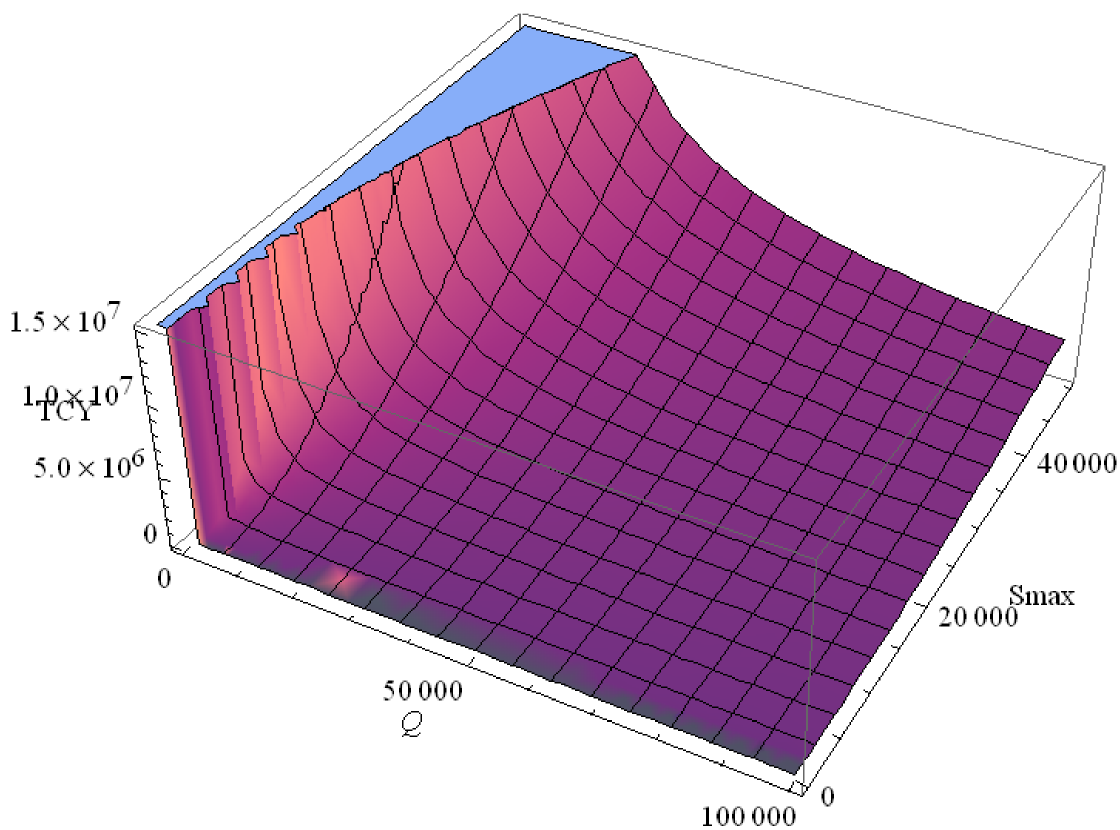

Figure 3.13: Three dimensional plot of Q, S_{max} and TCY for $t \geq T_P$.

3.2 The case where t is random variable:

In Chapter 2 we treated t as a random variable where the order quantity Q is the only decision variable. In this section the optimal shortage S_{max}^* allowed is a decision variable too and t has some probability density distribution $f(t)$.

3.2.1 Model Formulation:

As t is a random variable, the three cases considered separately earlier are combined based on the renewal theory. The limits of integrals refer to the period at which each case may occur. Note that $t \leq T_0$, implies that $t \leq \frac{S_{max} + t\,P\,d}{P-D}$ which in turn simplifies to $t \leq \frac{S_{max}}{P(1-d)-D}$.

The total cost can be expressed as the sum of the following: ordering cost, A, the manufacturing cost, CQ, the process adjustment cost per cycle and the screening cost $r.t.P.d + A_d.t$, the average shortage cost, $\hat{\pi}\,T\,\bar{S} + \breve{\pi}\,S_{max}$, and the holding cost, $T\bar{I}h$.

Therefore, the expected total cost is obtained as follows:

$E_i[TC(Q, S_{max})] =$

$$\int_{a_i}^{b_i} (A + C\,Q + r\,t\,P\,d + A_d\,t + +T\bar{I}h + \hat{\pi}T\,\bar{S} + \breve{\pi}\,S_{max}) f(t)\,dt \qquad (3.13)$$

The expected cycle length is determined as follows:

$$E_i[T(Q, S_{max})] = \int_{a_i}^{b_i} \left(\frac{Q - t\,P\,d}{D}\right) f(t)\,dt \qquad (3.14)$$

where a, and b, are the lower and upper bounds of each integral, the index i refers to the case number.

Thus the expected total cost per cycle is obtained as follows:

$$\frac{\sum_{j=1}^{3} E_j[TC(Q,S_{max})]}{\sum_{j=1}^{3} E_j[T(Q,S_{max})]} \tag{3.15}$$

The objective of Eq. 3.15 is to find the optimal values of Q and S_{max} such that the total cost per year is minimized. This model cannot be valid for any value of $S_{max} > 0$. One condition that must be satisfied is $T_0 < T_P$ or $\frac{S_{max}}{P(1-d)-D} < \frac{Q}{P}$, thus:

$$S_{max} < \frac{Q\,[P(1-d)-D]}{P} \tag{3.16}$$

3.2.2 Numerical Example:

Suppose that the adjustment period, t, is a uniform random variable where $f(t) = 1/8$, $0 \leq t \leq 8$. We found that the optimal values of order quantity and maximum shortage permitted are obtained at Q^*= 9,822.8 units and S^*_{max}= 123.69 units with a minimum cost of $ 122,193.01. Let us consider that t is exponentially distributed, with $f(t) = 1.25\, e^{-1.25\,t}$ where the expected value of t is $\frac{1}{1.25}$. We found that the optimal values of order quantity and maximum shortage permitted are obtained at Q^*= 24,349.5 units and S^*_{max}= 407.96 units with a minimum cost of $ 120,520.35.

The plots of TC versus Q at $S_{max} = S^*_{max}$ and TC versus S_{max} at $Q = Q^*$ for both distributions are shown in Fig. 3.14 and Fig. 3.15, respectively.

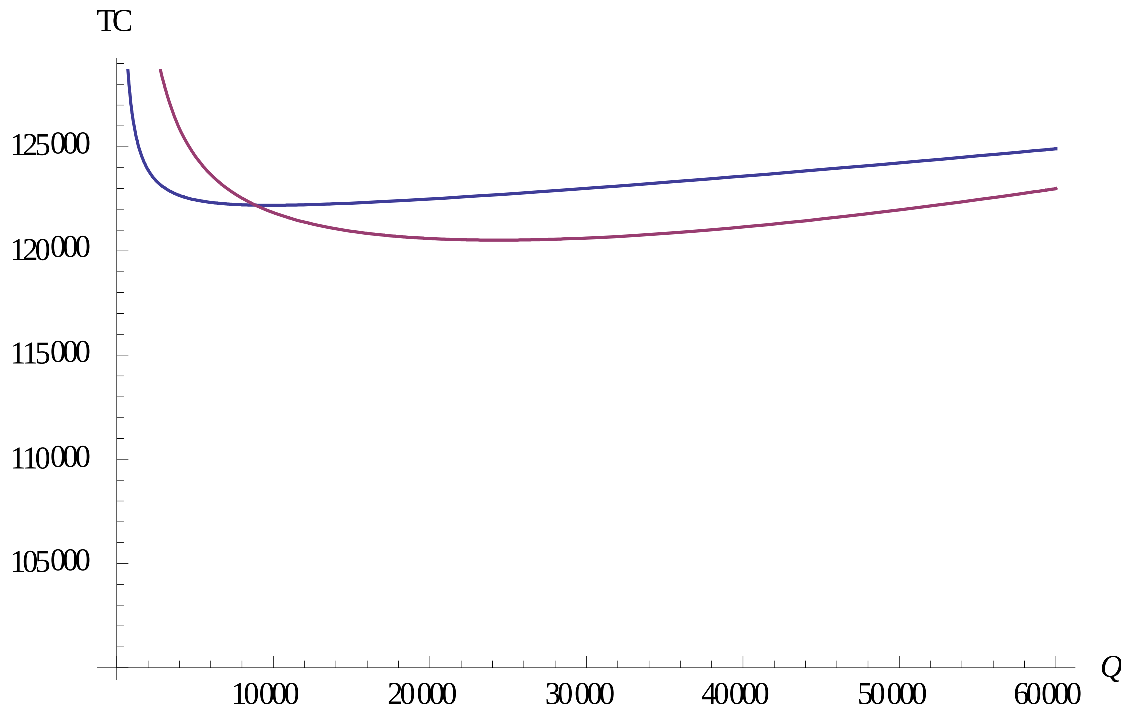

Figure 3.14: The behavior of Q at $S_{max} = S^*_{max}$ is fixed. Blue is uniform and Red is exponential.

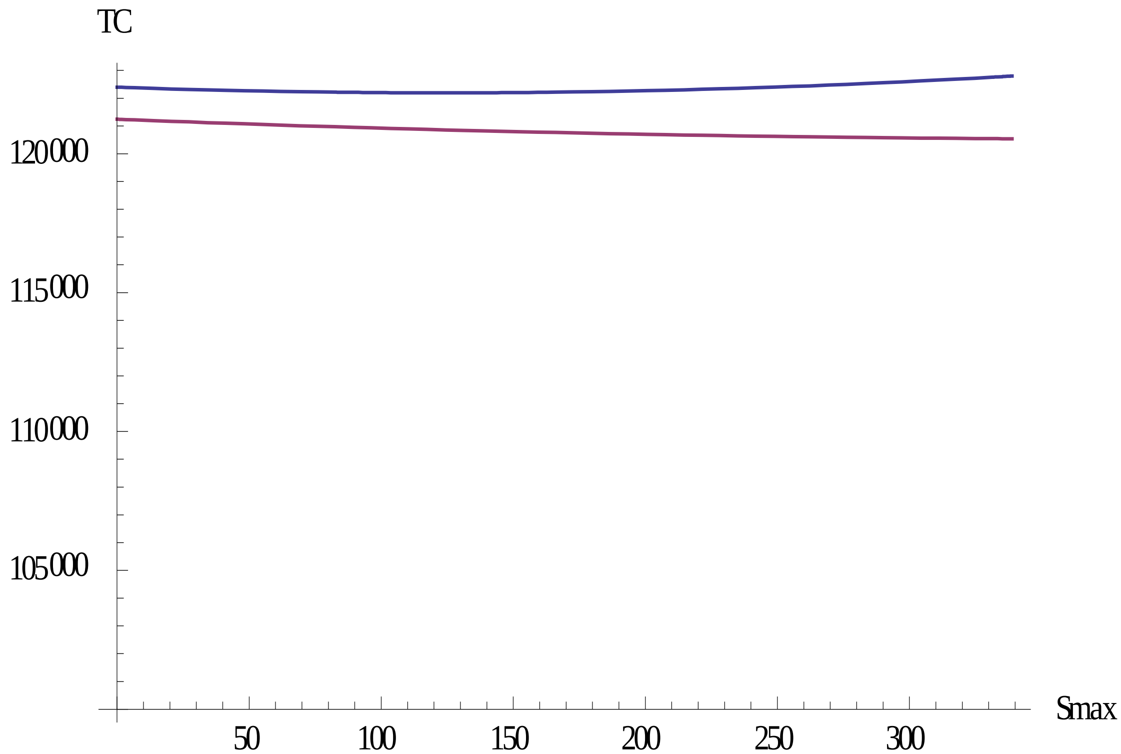

Figure 3.15: The behavior of S_{max} at $Q = Q^*$ is fixed. Blue is uniform and Red is exponential.

CHAPTER 4

THE OPTIMAL LOT SIZE AND SHORTAGE ALLOWANCE UNDER PROCESSING THE DEFECTIVE ITEMS WITH TAGUCHI'S QUALITY LOSS FUNCTION AND INSPECTION ERRORS

In this chapter we describe the results of incorporating the Taguchi's quality loss function and inspection errors into our model. Also, we will address the processing the defective items situation and study how these aspects will affect the model's decision variables. The machine adjustment period t is treated as a fixed value.

4.1 Taguchi's Quality Loss Function Approach:

Since 1960, Taguchi methods have been used for improving the quality of Japanese products with great success. During the 1980s, many companies finally realized that the old methods for ensuring quality were not competitive with the Japanese methods. The old methods for quality assurance relied heavily upon inspecting products as they rolled off the production line and rejecting those products that did not fall within a certain acceptance range. To measure quality, Taguchi defines a Quality Loss Function. The quality loss function is a continuous function that is defined in terms of the deviation of a design parameter from an ideal or a target value.

This function penalizes the deviation of a parameter from the specification value that contributes to deteriorating the performance of a product, resulting in a loss to the customer. The product's quality distribution is assumed to follow a normal distribution function and the loss function given in our model is referred to as "nominal is best". But there are also expressions for cases when higher or lower values of parameters are better.

4.1.1 Model Formulation:

Taguchi's quality loss function approach can be used to determine the economic impact of the quality characteristic, X, as it deviates from the target value, μ:

$$g(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(X-\mu)^2}{2\,\sigma^2}}$$

Taguchi's quality loss function $L(X)$ is defined as follows:

$$L(x) = K(X-\mu)^2 \qquad\qquad LSL \leq X \leq USL$$

where K is Taguchi loss parameter

$$K = \frac{V}{\Delta^2}$$

$$\Delta = (USL - \mu) = (\mu - LSL)$$

Thus, the amount of loss is expressed as follows:

$$(Q - t.P.d)\int_{LSL}^{USL} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(X-\mu)^2}{2\sigma^2}} K(X-\mu)^2 dx$$

The defective items will be sent to the storehouse for the reworking process. We assumed that there is a negligible inventory holding cost associated with the storehouse. Once the non-conforming items are reworked, they become as good as new (Fig. 4.1). We assume that there are two different rework costs: one is for items produced below the specification limits, R_L, and the other one is for items produced above the upper specification limits, R_U.

$$\text{Rework Process Cost} = (t.P.d)\left(R_L \int_0^{LSL} g(x;\mu)dx \; + R_U \int_{USL}^{\infty} g(x;\mu)dx\right)$$

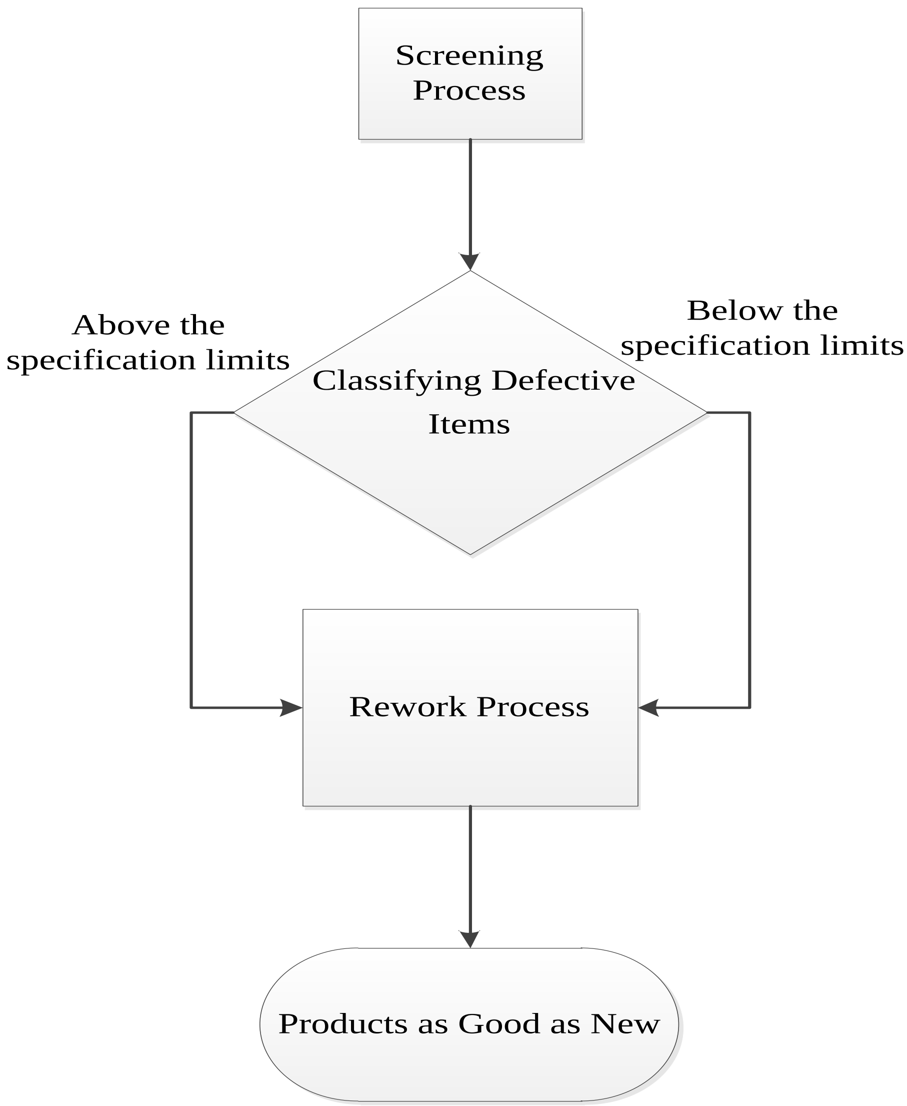

Figure 4.1: Flow chart of the rework process.

4.2 Committing Inspection Errors with Processing the defective items Processing:

Two types of errors are committed in the inspection process. Type I error, e_1, is committed when a conforming item is classified as non-conforming and Type II error, e_2, is committed when a non-conforming item is classified as conforming. Both types of errors are assumed to be known. The apparent conforming items fraction can be determined as follows:

$$(1-d)(1-e_1)+d\,e_2 = 1-e_1-d(1-e_1-e_2) = 1-\grave{d} \tag{4.1}$$

where,

$$\grave{d} = e_1 + d(1-e_1-e_2) \tag{4.2}$$

Apparent defective items are sent to the processing the defective items plant which has an inventory holding cost of, $\hat{h}$. Once defective items are processed, they become as good as new and are sent back to the original plant to meet the demand.

This situation yields three different cases of the process adjustment time, t. The three cases and their sub-cases are shown in Table 4.1.

Each case will be discussed in Sections 4.2.1, 4.2.2 and 4.2.3, which are followed by formulation and numerical examples in Sections 4.3.1 and 4.3.2. We assume t to be provided.

Table 4.1: The three cases and their subcases with respect to t

Case Number	Case Status	Sub-cases
1	$t \leq T_0$	1. Shortage is not met while processing the defective items 2. Shortage is met before Processing the defective items is completed 3. Time for processing the defective items exceeds T_P 4. Processing the defective items reaches the cycle end zone
2	$T_0 \leq t \leq T_P$	1. Shortage is met before processing the defective items is completed 2. Time for processing the defective items exceeds T_P 3. Processing the defective items reaches the cycle end zone
3	$t \geq T_P$	1. Time for processing the defective items exceeds T_P 2. Processing the defective items reaches the cycle end zone

4.2.1 The Case of $t < T_0$

The sub-cases of P_s:

In all the four sub-cases of this case, t lapses before meeting the shortage. We address the four subcases as follows:

1. *Shortage is not met while processing the defective items:*

The production starts at a rate of $P(1 - \grave{d}) - D$, and an inventory of apparent defective items is built up at a rate of $P\grave{d}$. During, T_s, processing the defective items are processed at a rate of P_s ,and then the processed items are used to meet the shortage at a rate of $P_s + P - D$. Subsequently, the rest of the production period continues with a slope of $P - D$. Once the production stops, the inventory level decreases until the end of cycle with a slope of D (Fig. 4.2).

From Fig 4.2, the total shortage S, is computed as follows:

$$S = \frac{t.(S_{max}+\gamma_2+\gamma_3)}{2} + \frac{T_s\,.(2\gamma_2+\gamma_3)}{2} + \frac{\gamma_1^2(P-D)}{2} + \frac{S_{max}^2}{2D} \quad (4.3)$$

where,

$$\gamma_1 = T_P - (t + T_s), \gamma_2 = \gamma_1(P - D), \gamma_3 = T_s(P_s + P - D) \text{ and } T_s = \frac{t\,P\,\grave{d}}{P_s}$$

Therefore Eq. 4.3 can be rewritten as follows:

$$S = \frac{\grave{d}^2P^4t^2D + \grave{d}P^3P_s\,t^2\,D - P_s\,Q^2D^2 + P\,P_s\,Q\,D(Q + t\,D) + P^2P_s({S^2}_{max} - Q\,t\,D + S_{max}t\,D)}{2P^2P_s\,D} \quad (4.4)$$

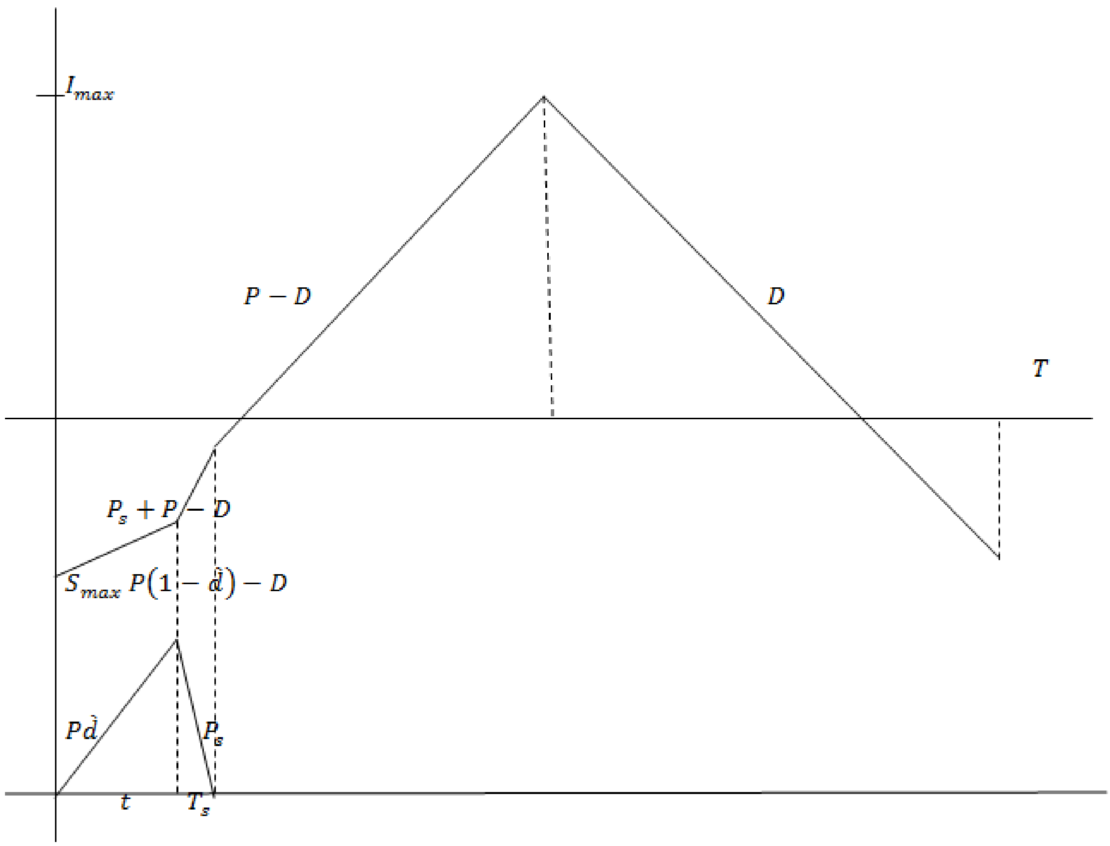

Figure 4.2: The relationship between the inventory level of conforming and defective items and the time when the shortage is not met while processing the defective items.

Also, the total inventory level is obtained as follows:

$$I = \frac{I_{max}^2}{2\,(P-D)} + \frac{I_{max}^2}{2\,D} \tag{4.5}$$

where,

$$I_{max} = \; Q - D.T_p - S_{max}$$

Therefore Eq. 4.5 can be rewritten as follows:

$$I = \frac{(Q-S_{max}-\frac{Q\,D}{P})^2}{2(P-D)} + \frac{(Q-S_{max}-\frac{Q\,D}{P})^2}{2D} \tag{4.6}$$

Also, we compute the total inventory level of defective items as follows:

$$I_s = \frac{\grave{d}\;P\;t^2 + P_s\,T_s^2}{2} \tag{4.7}$$

We may come up with a relationship that governs the value of t. from Fig. 4.2:

$$t \leq T_0 \qquad \text{and} \qquad t + T_s \leq T_0$$

$$\text{Thus, } t \leq \frac{S_{max}}{(P-D)} \qquad \text{and} \qquad t \leq \frac{S_{max}}{(P-D)(1+\frac{P.\grave{d}}{P_s})} \tag{4.8}$$

2. *Shortage is met before Processing the defective items is completed:*

Once the apparent defective items are sent for processing, shortage is met while defective items are being processing (Fig. 4.3).

From Fig. 4.3, the total shortage S, is computed as follows:

$$S = \frac{t.(2S_{max}-\beta_1)}{2} + \frac{(S_{max}-\beta_1)^2}{2(P+P_s-D)} + \frac{S_{max}^2}{2D} \tag{4.9}$$

where, $\beta_1 = t.[P(1-\grave{d}) - D]$

Thus,

$$S = \frac{(-1+\grave{d})\grave{d}\,P^2t^2D + P_s(S_{max}+t\,D)^2 + P(S_{max}^2 + 2\,\grave{d}\,S_{max}\,t\,D + t^2D((-1+\grave{d})P_s + \grave{d}\,D))}{2(P+P_s-D)D} \quad (4.10)$$

We compute the total inventory level as follows:

$$I = \frac{\beta_2^2(P+P_s-D)}{2} + \frac{[T_P-(t+T_S)].[\beta_2(P+P_s-D)+I_{max}]}{2} + \frac{I_{max}^2}{2D} \quad (4.11)$$

where,

$$\beta_2 = T_s - \frac{S_{max} - \beta_1}{P + P_s - D}$$

Thus,

$$I = \frac{1}{2}\left(\frac{\left(S_{max}+Q(-1+\frac{D}{P})\right)^2}{D} + \frac{(-\grave{d}\,P^2t + P_s(Q-P\,t))((-1+\grave{d})P^2t - Q\,D + P(Q+t\,D))}{P^2P_s} + \frac{\left(\grave{d}\,P\,t(-P+D) + P_s(S_{max}+t\,(-P+D))\right)^2}{{P_s}^2(P+P_s-D)}\right) \quad (4.12)$$

We compute the total inventory level of defective items as follows:

$$I_s = \frac{\grave{d}\,P\,t^2 + P_s\,T_S^2}{2} \quad (4.13)$$

We may come up with a relationship that governs the value of t. from Fig. 4.3:

$$t \le T_0 \qquad \text{and} \qquad T_0 \le t + T_s < T_P$$

Thus,

$$t \le \frac{S_{max}}{P(1-\grave{d})-D} \qquad \text{and} \qquad \frac{S_{max}}{(P-D)(1+\frac{P.\grave{d}}{P_S})} \le t < \frac{Q}{P[1+\frac{P\,\grave{d}}{P_S}]} \quad (4.14)$$

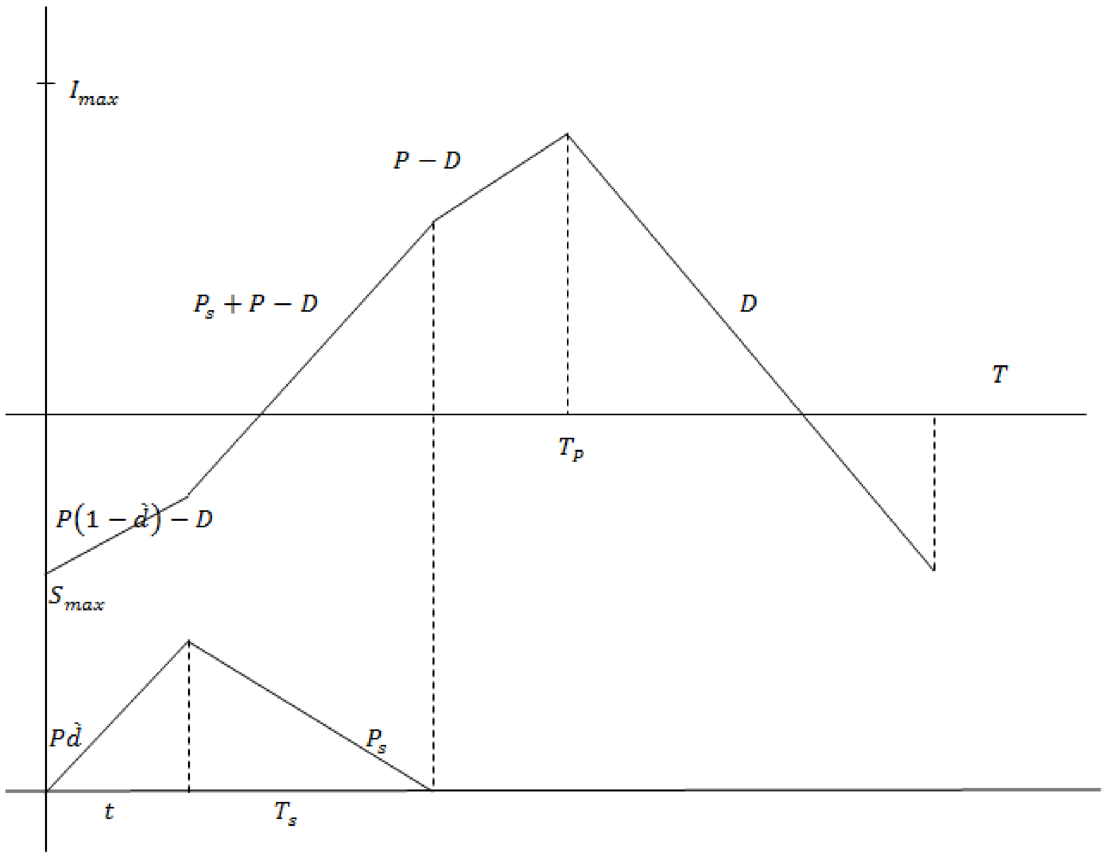

Figure 4.3: The relationship between the inventory level of conforming and defective items and the time when shortage is met before processing the defective items is completed.

3. *Processing the defective items exceeds* T_P*:*

This sub-case assumes that processing the defective items takes a longer time such that the production stops before processing the defective items is completed (Fig. 4.4).

It is obvious from Fig. 4.4 that the total shortage, S, is the same as the second case which is given in Eq. 4.10.

We compute the total inventory level as follows:

$$I = \frac{\alpha^2.(P+P_S-D)}{2} + \frac{(T_S+t-T_P).[2I_{max} - (T_S+t-T_P).(P_S-D)]}{2} + \frac{I_{max}^2}{2D} \quad (4.15)$$

where,

$$\alpha = T_P - \ [t + \frac{S_{max}-\beta_1}{P+P_S-D}] \quad \text{and}$$

$$I_{max} = t.\left[P\left(1-\grave{d}\right) - D\right] + (T_P - t).(P + P_s - D) + (t + T_s - T_P).(P_s - D) - S_{max}$$

Thus,

$$I = \frac{1}{2}\left(\frac{Q}{P} - t - \frac{S_{max} - t(1-\grave{d}-D)}{P+P_S-D}\right)^2 (P + P_s - D) + \frac{\left(Q - \frac{P_S\, S_{max} + \grave{d}\, Pt\, D + P_S\, t\, D}{P_S}\right)^2}{2D} + \frac{1}{2}\left(-\frac{Q}{P} + t + \frac{\grave{d}\, P\, t}{P_S}\right)\left(-\left(-\frac{Q}{P} + t + \frac{\grave{d}\, P\, t}{P_S}\right)(P_s - D) + 2\left(Q - \frac{P_S\, S_{max} + \grave{d}\, P\, t\, D + P_S\, t\, D}{P_S}\right)\right) \quad (4.16)$$

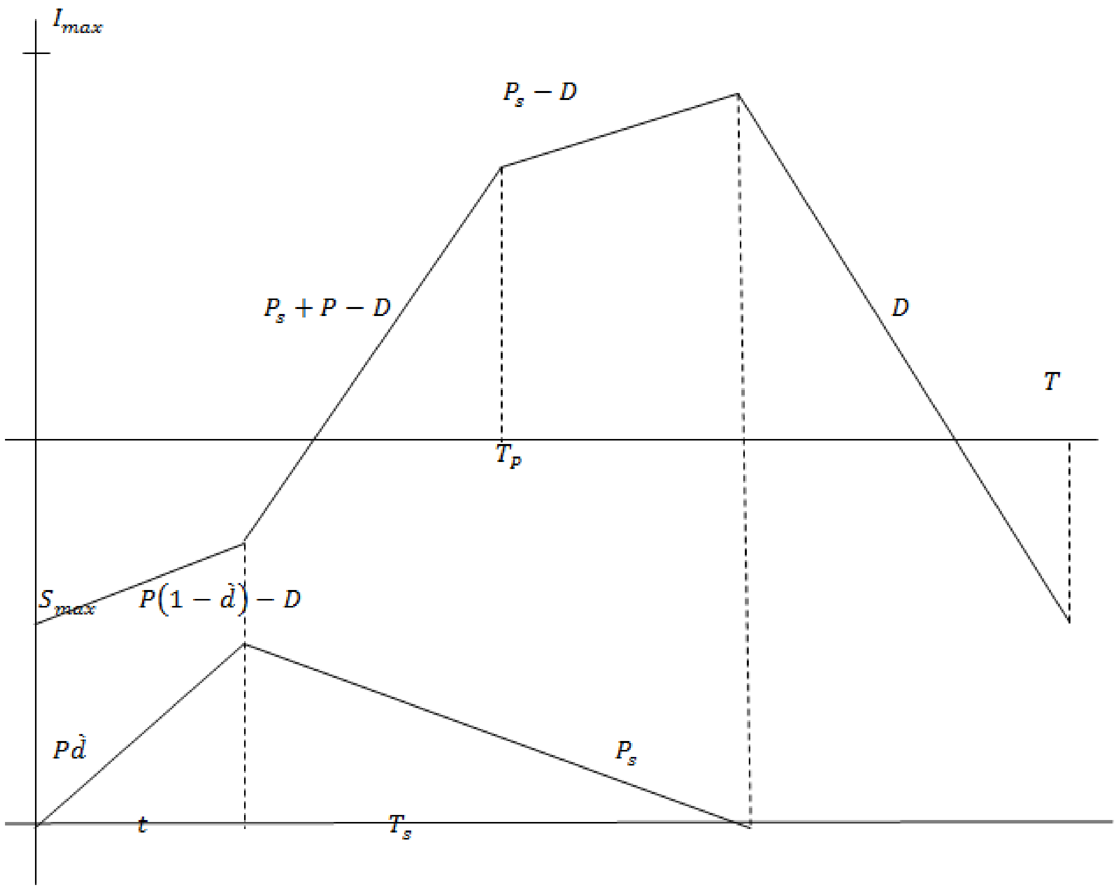

Figure 4.4: The relationship between the inventory level of conforming and defective items and the time when processing the defective items exceeds T_P and when t is small.

The formula for the total inventory level of defective items is the same as in the previous two sub-cases.

Also we notice that:

$$t \le T_0 \qquad \text{and} \qquad T_P \le t + T_s < T - \frac{S_{max}}{D}$$

Thus,

$$t \le \frac{S_{max}}{P(1-\grave{d})-D} \qquad \text{and} \qquad \frac{Q}{P[1+\frac{P\,\grave{d}}{P_S}]} \le t < \frac{Q-S_{max}}{D[1+\frac{P\,\grave{d}}{P_S}]} \tag{4.17}$$

4. *Processing the defective items reaches the cycle end zone:*

This sub-case is valid only for $P_s - D < 0$, because when $P_s - D \ge 0$, the previous case is valid. Here we assume that processing the defective items reaches the end of the cycle where the shortage zone has not been met (Fig. 4.5).

From Fig 4.5, the total shortage, S, is computed as follows:

$$S = \frac{t.(2S_{max}-\beta_1)}{2} + \frac{(S_{max}-\beta_1)^2}{2(P+P_S-D)} + \frac{1}{2}\left[(t+T_S-T_P) - \frac{I_{max}}{P_S-D}\right]^2 (P_s - D) + \frac{1}{2}[T - (t+T_s)][2S_{max} - \big(T-(t+T_s)\big)D] \tag{4.18}$$

Thus,

$$S = \frac{1}{2}((-\frac{(\grave{d}\,P+P_S)t}{P_S} + \frac{Q}{D})(2\,S_{max} - \frac{Q}{D} + \frac{(\grave{d}\,P+P_S)t\,D}{P_S}) + \frac{(S_{max}+t((-1+\grave{d})P+D))^2}{P+P_S-D} + t(2S_{max} + t((-1+\grave{d})P+D)) - \frac{(\grave{d}\,P\,t\,D+\mathrm{Ps}(-Q+S_{max}+t\,D))^2}{P_S^2(P_S-D)}) \tag{4.19}$$

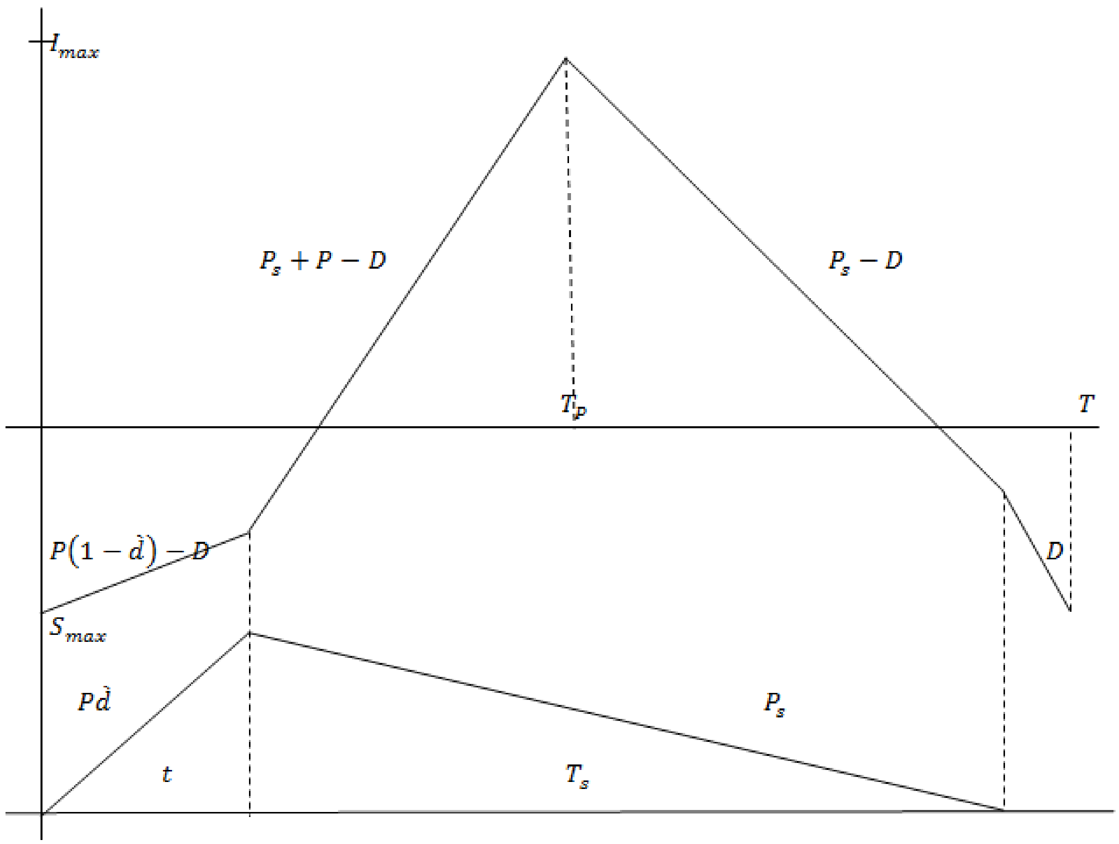

Figure 4.5: The relationship between the inventory level of conforming and defective items and the time when processing the defective items reaches the cycle end zone and when *t* is small.

We compute the total inventory level as follows:

$$I = \frac{1}{2}\frac{I_{max}^2}{P_s+P-D} + \frac{1}{2}\frac{I_{max}^2}{D-P_s} \tag{4.20}$$

where,

$$I_{max} = t.\left[P\left(1-\grave{d}\right)-D\right] + (T_P - t).(P + P_s - D) - S_{max}$$

Thus,

$$I = \frac{1}{2}(S_{max} + (d\ P + P_s)t - \frac{Q(P+P_s-D)}{P})^2(\frac{1}{P+P_s-D} + \frac{1}{-P_s+D}) \tag{4.21}$$

The total inventory level of defective items can be obtained from Eq. 4.13. We may come up with a relationship that governs the value of t. from Fig. 4.5:

$$t \le T_0 \qquad \text{and} \qquad T - \frac{S_{max}}{D} \le t + T_s < T$$

$$\text{Thus, } t \le \frac{S_{max}}{P\left(1-\grave{d}\right)-D} \qquad \text{and} \qquad \frac{Q-S_{max}}{D[1+\frac{P\,\grave{d}}{P_s}]} \le t < \frac{Q}{D[1+\frac{P\,\grave{d}}{P_s}]} \tag{4.22}$$

Formulation and numerical example:

The total cost can be stated as the sum of the following costs divided by T : ordering cost, A, the manufacturing cost, CQ, the process adjustment cost per cycle and the screening cost, $r.t.P.d + A_d.t$, the production cost, $Q\,\mu\,C_P$, Taguchi's quality loss,$Q\int_{LSL}^{USL}\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(X-\mu)^2}{2\sigma^2}}K(X-\mu)^2dx$, rework process cost, and the defective items inventory holding cost of $T\bar{I}_s\hat{h}$.

Therefore, the total cost per cycle is obtained as follows:

$$TCY_{i,j}(Q, S_{max}) = \frac{A\,D}{Q} + C\,D + \frac{r\,t\,P\,d\,D}{Q} + \frac{A_d\,t\,D}{Q} + D\,\mu\,C_P + D\int_{LSL}^{USL} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(X-\mu)^2}{2\sigma^2}}\, K(X - \mu)^2 dx + \frac{(t.P.d)D}{Q}\left(R_L \int_0^{LSL} g(x;\mu)dx \; + R_U \int_{USL}^{\infty} g(x;\mu)dx\right) + \bar{I}h + \bar{I}_s\hat{h} + \hat{\pi}\,\bar{S} + \breve{\pi}\,S_{max} \tag{4.23}$$

where the index i refers to the case number and j refers to sub-case number.

The objective of Eq. 4.23 is to find the optimal values of $Q \; and \; S_{max}$ such that the total cost per cycle is minimized.

To illustrate how to apply the four sub-cases, let us consider the following numerical example. Consider the following data:

P = 25,000 units per year
D = 23,000 units per year
P_s= 1,500 units per year
r = \$1 per unit
h = \$ 4 per unit/year
h_s = \$ 2 per unit/year
C = \$ 5 per unit
C_P= \$ 3 per unit
A_d = \$ 50 per hour
A = \$ 100 per order
$\hat{\pi}$= \$ 5 per unit/year
$\breve{\pi}$= \$ 0.3
R_L = \$ 4 per unit
R_U = \$ 6 per unit
$\mu = 5$
σ = 0.05
USL = 5.2

LSL = 4.8

K= $ 120

$e_1 = e_2 = 0.02$

$d = 0.0455$

Table 4.2 shows the optimal values of Q and S_{max} as well as $TCY_{i,j}$ at two different values of t for the four sub-cases and their subjected constraints. For our example, the decision maker in terms of costs would choose the second sub-case where the shortage is met before processing the defective items is completed because it yields the minimum cost.

Table 4.2: The optimal values of Q and S_{max} as well as $TCY_{i,j}$ at two different values of t for the four sub-cases.

$TCY_{1,j}$	$t = 0.01$	$t = 0.1$
$TCY_{1,1}$ Subject to $t \leq \frac{S_{max}}{(P-D)(1+\frac{P.\grave{d}}{P_S})}$	$Q = 3{,}325.9\ units$ $S_{max} = 105.97 units$ $TCY_{1,1} = \$\ 468{,}718$	$Q = 7{,}354.93\ units$ $S_{max} = 412.27\ units$ $TCY_{1,1} = \$\ 469{,}851$
$TCY_{1,2}$ Subject to $t \leq \frac{S_{max}}{P(1-\grave{d})-D}$ $\frac{S_{max}}{(P-D)(1+\frac{P.\grave{d}}{P_S})} \leq t < \frac{Q}{P(1+\frac{P.\grave{d}}{P_S})}$	$Q = 4136.65\ units$ $S_{max} = 4.08\ units$ $TCY_{1,2} = \$\ 468{,}139$	$Q = 7{,}728.43\ units$ $S_{max} = 40.8\ units$ $TCY_{1,2} = \$\ 468{,}526$
$TCY_{1,3}$ Subject to $t \leq \frac{S_{max}}{P(1-\grave{d})-D}$ $\frac{Q}{P(1+\frac{P.\grave{d}}{P_S})} \leq t < \frac{Q-S_{max}}{D(1+\frac{P.\grave{d}}{P_S})}$	$Q = 515.333\ units$ $S_{max} = 4.08\ units$ $TCY_{1,3} = \$\ 473{,}164$	$Q = 5{,}153.33\ units$ $S_{max} = 40.8\ units$ $TCY_{1,3} = \$\ 478{,}270$
$TCY_{1,4}$ Subject to $t \leq \frac{S_{max}}{P(1-\grave{d})-D}$ $\frac{Q-S_{max}}{D(1+\frac{P.\grave{d}}{P_S})} \leq t < \frac{Q}{D(1+\frac{P.\grave{d}}{P_S})}$	$Q = 478.187\ units$ $S_{max} = 4.08\ units$ $TCY_{1,4} = \$\ 472{,}671$	$Q = 4{,}741.07\ units$ $S_{max} = 87.5289\ units$ $TCY_{1,4} = \$\ 469{,}217$

4.2.2 The Case of $T_0 \leq t \leq T_P$

We address three subcases under this case, where t is always performed after meeting the shortage, which takes place at the beginning of the cycle, i.e., $t > T_0$. However, we assume that t ends before the end of the production or, $t \leq T_P$.

The three sub-cases of P_s:

1. *Shortage is met before processing the defective items is completed:*

In this scenario the production starts with a rate of $P\left(1 - \grave{d}\right) - D$, while inventory level of defective items is being built at a rate of $P\grave{d}$ till the end of t. Then the processed items will be sent to the inventory which is given by the slope $P_s + P - D$. Once all the defective items have been processed and consumed, the inventory level continues at a rate of $P - D$. Finally, the production stops and the inventory level starts decreasing with a slope of D (Fig. 4.6).

From Fig. 4.6, the total shortage S, is computed as follows:

$$S = \frac{(\grave{d}-1)PS_{max}^2}{2D((\grave{d}-1)P+D)} \quad (4.24)$$

We compute the total inventory level as follows:

$$I = \frac{1}{2}\theta^2.\left(P\left(1 - \grave{d}\right) - D\right) + \frac{T_S.[2\theta\left(P\left(1-\grave{d}\right)-D\right)+T_S.(P_S+P-D)]}{2} + \frac{(T_P-(t+T_S))}{2}\left[\left(\theta.\left(P\left(1 - \grave{d}\right) - D\right) + T_s(P_s + P - D)\right) + I_{max}\right] + \frac{I_{max}^2}{2D} \quad (4.25)$$

where,

$$\theta = t - \frac{S_{max}}{P(1-\grave{d}) - D} \text{, and}$$

$$I_{max} = Q - D.T_p - S_{max}$$

Thus,

$$I = \frac{\grave{d}\ P\ t(2(t - \frac{S_{max}}{(1-\grave{d})P-D})((1-\grave{d})P-D) + \frac{\grave{d}\ P\ t(P+P_S-D)}{Ps})}{2P_S} + \frac{1}{2}(t - \frac{S_{max}}{(1-\grave{d})P-D})^2((1-\grave{d})P-D) + \frac{(Q-S_{max}-\frac{QD}{P})^2}{2\ D} + \frac{1}{2}(\frac{Q}{P} - t - \frac{\grave{d}\ P\ t}{P_S})(Q - S_{max} + (t - \frac{S_{max}}{(1-\grave{d})P-D})((1-\grave{d})P-D) + \frac{\grave{d}\ P\ t(P+P_S-D)}{P_S} - \frac{QD}{P}) \quad (4.26)$$

The total inventory level of defective items can be obtained from Eq. 4.13.

The lower and upper bounds of t are:

$$T_0 \le t < T_P \qquad \text{and} \qquad T_0 \le t + T_s < T_P$$

Thus,

$$\frac{S_{max}}{P(1-\grave{d})-D} \le t < \frac{Q}{P} \qquad \text{and} \qquad \frac{S_{max}}{[P(1-\grave{d})-D](1+\frac{P\,\grave{d}}{P_S})} < t < \frac{Q}{P[1+\frac{P\,\grave{d}}{P_S}]} \quad (4.27)$$

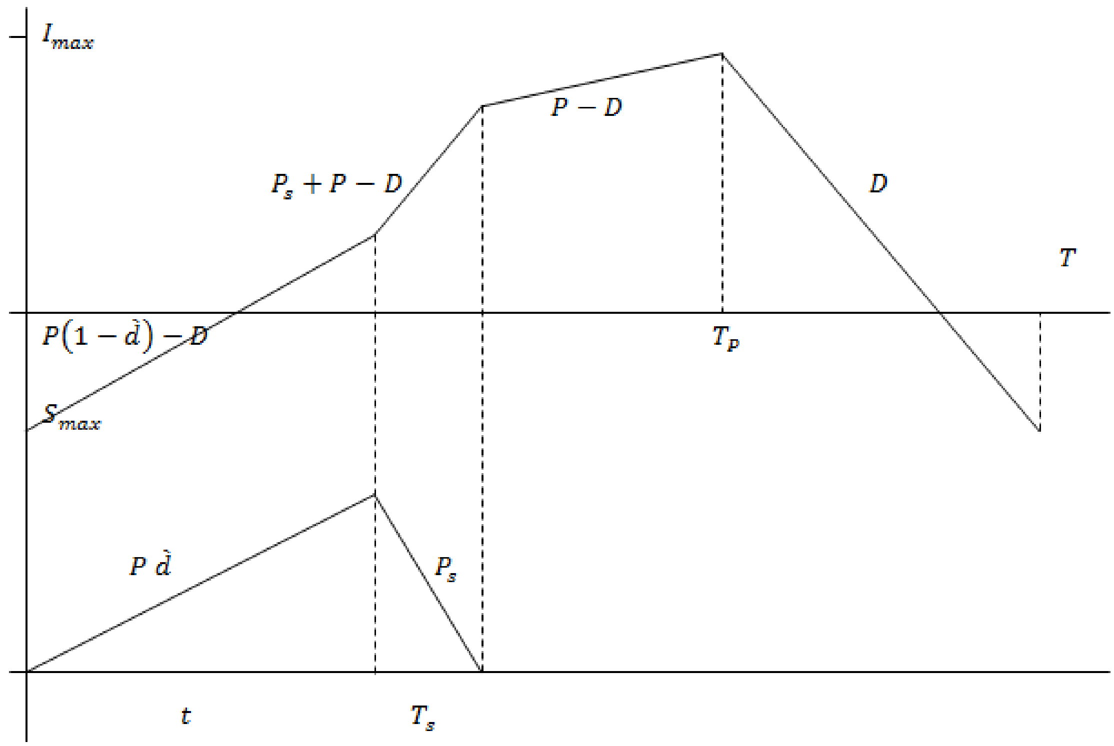

Figure 4.6: The relationship between the inventory level of conforming and defective items and the time when shortage is met before processing the defective items is completed.

2. *Processing the defective items period exceeds T_P:*

This sub-case assumes that processing the defective items continues after the production has been completed (Fig 4.7).

The total shortage, S, is obtained from Eq. 4.18. We compute the total inventory level as follows:

$$I = \frac{1}{2}\theta^2.\left(P\left(1-\grave{d}\right)-D\right) + \frac{1}{2}\left(T_P - t\right)\left[2\theta.\left(P\left(1-\grave{d}\right)-D\right) + \left(T_P - t\right).\left(P_s + P - D\right)\right] + \frac{1}{2}\left(T_s - \left(T_P - t\right)\right).\left[2I_{max} - \left(T_s - \left(T_P - t\right)\right).\left(P_s - D\right)\right] + \frac{I_{max}^2}{2D} \quad (4.28)$$

where,

$$\theta = t - \frac{S_{max}}{P\left(1-\grave{d}\right)-D}, \text{ and}$$

$$I_{max} = t.\left[P\left(1-\grave{d}\right)-D\right] + \left(T_P - t\right).\left(P + P_s - D\right) + \left(t + T_s - T_P\right).\left(P_s - D\right) - S_{max}$$

Thus,

$$I = \frac{1}{2}\left(\frac{Q}{P} - t\right)\left(2\left(t - \frac{S_{max}}{(1-\grave{d})P-D}\right) + \left(\frac{Q}{P} - t\right)\left(P + P_s - D\right)\right) + \frac{1}{2}\left(t - \frac{S}{(1-\grave{d})P-D}\right)^2\left((1-\grave{d})P - D\right) + \frac{\left(Q - \frac{P_s\, S_{max} + \grave{d}\, P t\, D + P_s\, t\, D}{P_s}\right)^2}{2\, D} + \frac{1}{2}\left(-\frac{Q}{P} + t + \frac{\grave{d}\, P\, t}{P_s}\right)\left(-\left(-\frac{Q}{P} + t + \frac{\grave{d}\, P\, t}{P_s}\right)\left(P_s - D\right) + 2\left(Q - \frac{P_s\, S_{max} + \grave{d}\, P\, t\, D + P_s\, t\, D}{P_s}\right)\right) \quad (4.29$$

The total inventory level of defective items can be obtained from Eq. 4.13.

The lower and upper bounds of t are:

$$T_0 \le t < T_P \qquad \text{and} \qquad T_P \le t + T_s < T - \frac{S_{max}}{D}$$

Thus,

$$\frac{S_{max}}{P\left(1-\grave{d}\right)-D} \le t < \frac{Q}{P} \qquad \text{and} \qquad \frac{Q}{P\left(1+\frac{P.\grave{d}}{P_s}\right)} \le t < \frac{Q - S_{max}}{D\left(1+\frac{P.\grave{d}}{P_s}\right)} \quad (4.30)$$

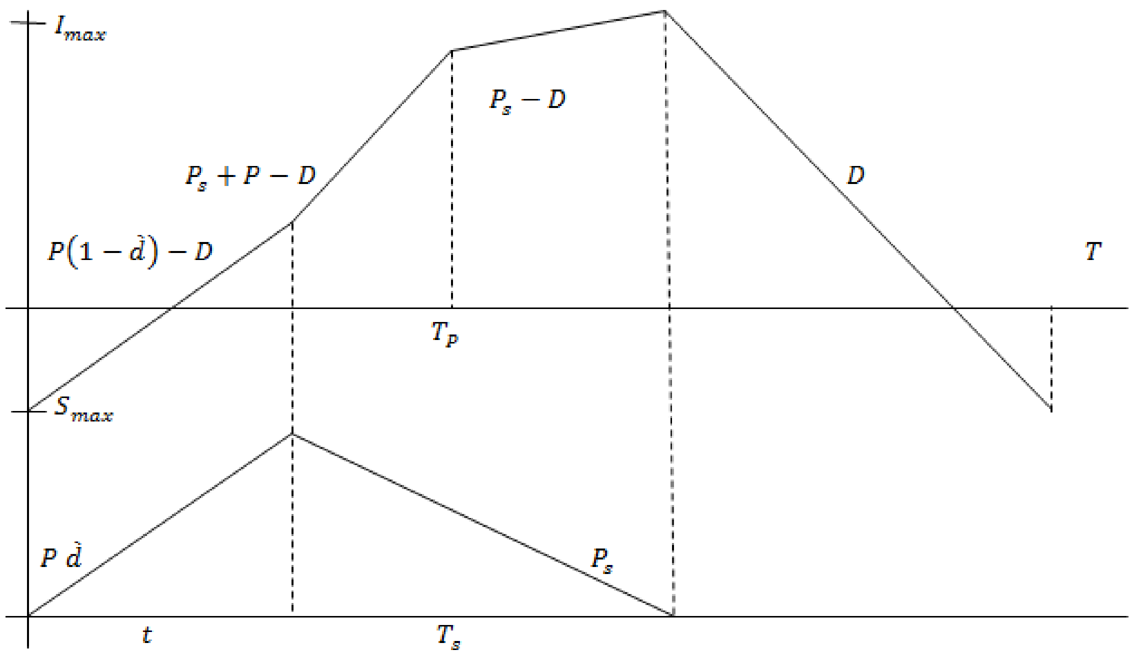

Figure 4.7: The relationship between the inventory level of conforming and defective items and the time when processing the defective items exceeds T_P and t is between T_0 and T_P.

3. Processing the defective items reaches the cycle end zone:

This sub-case is valid for $P_s - D < 0$, because when $P_s - D \geq 0$, the previous case is valid. Here we suggest that processing the defective items reaches the end of the cycle where the shortage zone has not been met (Fig 4.8).

From Fig 4.8, the total shortage, S, is computed as follows:

$$S = \frac{1}{2}\frac{S_{max}^2}{P(1-\grave{d})-D} + \frac{1}{2}\left[(t+T_s-T_P) - \frac{I_{max}}{D-P_s}\right]^2 (D-P_s) + \frac{1}{2}[T-(t+T_s)][2S_{max} - \big(T-(t+T_s)\big)D] \quad (4.31)$$

where,

$$I_{max} = t.\left[P\left(1-\grave{d}\right) - D\right] + (T_P - t).(P + P_s - D) - S_{max}$$

Thus,

$$S = \frac{1}{2}\left(-\frac{S_{max}^2}{(-1+\grave{d})P+D} + \left(-t - \frac{\grave{d}\,P\,t}{P_s} + \frac{Q}{D}\right)\left(2\,S_{max}^2 - \frac{Q}{D} + \frac{(\grave{d}\,P+P_s)\,t\,D}{P_s}\right) - \frac{(\grave{d}\,P\,t\,D + P_s(-Q+S_{max}^2+t\,D))^2}{P_s^2(P_s-D)}\right) \quad (4.32)$$

We compute the total inventory level as follows:

$$I = \frac{1}{2}\theta^2.\left(P\left(1-\grave{d}\right) - D\right) + \frac{1}{2}(T_P - t)[2I_{max} - (T_P - t)(P_s + P - D)] + \frac{1}{2}\frac{I_{max}^2}{P_s - D} \quad (4.33)$$

Thus, $I = \quad \frac{1}{2}[-\frac{\left(S_{max}+t\left((-1+\grave{d})P+D\right)\right)^2}{(-1+\grave{d})P+D}+\frac{\left(S_{max}+\grave{d}\,P\,t+Q\left(-1+\frac{D}{P}\right)\right)^2}{P_S-D}+$

$$\frac{(-Q+P\,t)\left((-1+2d)P^2t+Q(P_S+D)-P(Q-2S_{max}+P_S\,t-t\,D)\right)}{P^2}] \quad (4.34)$$

The total inventory level of defective items can be obtained from Eq. 4.13. The lower and upper bounds of t are:

$$T_0 \leq t < T_P \qquad \text{and} \qquad T-\frac{S_{max}}{D} \leq t+T_s < T$$

Thus,

$$\frac{S_{max}}{P(1-\grave{d})-D} \leq t < \frac{Q}{P} \qquad \text{and} \qquad \frac{Q-S_{max}}{D(1+\frac{P.\grave{d}}{P_S})} \leq t < \frac{Q}{D(1+\frac{P.\grave{d}}{P_S})} \quad (4.35)$$

Formulation and numerical example:

By referring to Eq. 4.23 we can obtain a model for each sub-case. To illustrate how to apply the three sub-cases, let us consider the data given in Subsection 4.2.1.

Table 4.3 shows the optimal values of Q and S_{max} as well as $TCY_{i,j}$ at two different values of t for the four sub-cases and their subjected constraints. For our example, the decision maker in terms of costs would choose the second sub-case where processing the defective items exceeds, T_P because it yields the minimum cost.

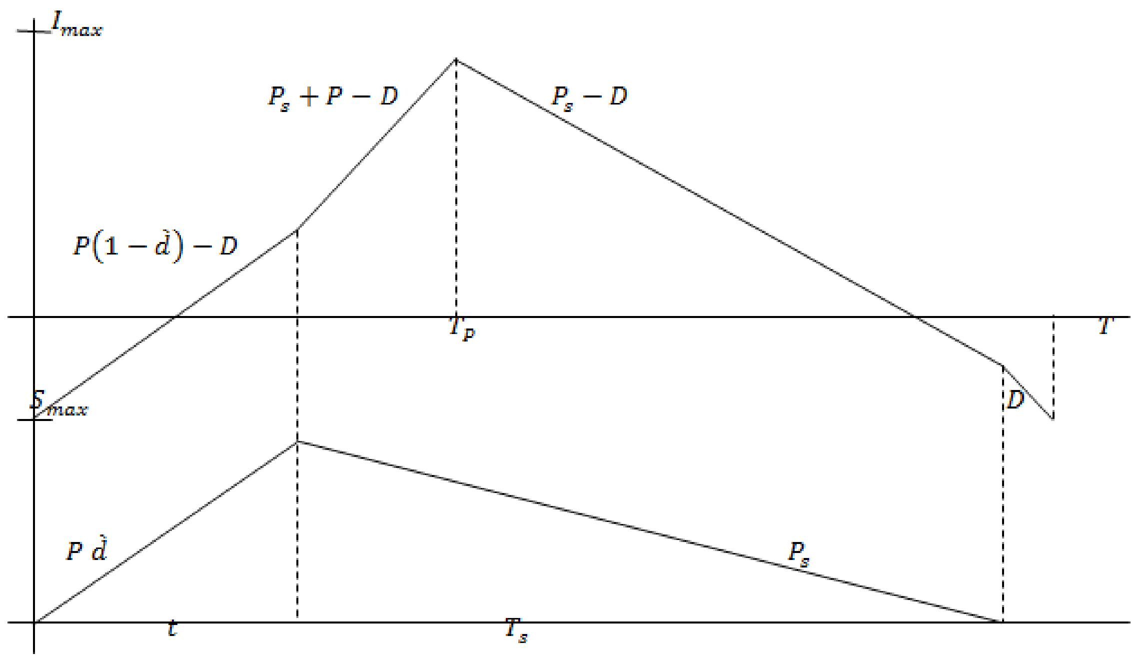

Figure 4.8: The relationship between the inventory level of conforming and defective items and the time when processing the defective items reaches the cycle end zone and t is between T_0 and T_P.

Table 4.3: The optimal values of Q and S_{max} as well as $TCY_{i,j}$ at two different values of t for the three sub-cases.

$TCY_{2,j}$	$t = 0.5$	$t = 1$
$TCY_{2,1}$ Subject to $\frac{S_{max}}{P(1-\grave{d})-D} \leq t < \frac{Q}{P}$ $\frac{S_{max}}{[P(1-\grave{d})-D](1+\frac{P\grave{d}}{P_S})} < t < \frac{\frac{Q}{P}}{1+\frac{P\grave{d}}{P_S}}$	$S_{max} = 186.242\ units$ $Q = 25{,}766.7\ units$ $TCY_{2,1} = \$470{,}765$	$S_{max} = 385.848\ units$ $Q = 51{,}533.3\ units$ $TCY_{2,1} = \$473{,}712$
$TCY_{2,2}$ Subject to $\frac{S_{max}}{P(1-\grave{d})-D} \leq t < \frac{Q}{P}$ $\frac{Q}{P(1+\frac{P.\grave{d}}{P_S})} \leq t < \frac{Q-S_{max}}{D(1+\frac{P.\grave{d}}{P_S})}$	$S_{max} = 84.8247\ units$ $Q = 23{,}790.2\ units$ $TCY_{2,2} = \$470{,}720$	$S_{max} = 179.661\ units$ $Q = 47{,}590.3\ units$ $TCY_{2,2} = \$472{,}855$
$TCY_{2,3}$ Subject to $\frac{S_{max}}{P(1-\grave{d})-D} \leq t < \frac{Q}{P}$ $\frac{Q-S_{max}}{D(1+\frac{P.\grave{d}}{P_S})} \leq t < \frac{Q}{D(1+\frac{P.\grave{d}}{P_S})}$	$S_{max} = 170.067\ units$ $Q = 23{,}705.3\ units$ $TCY_{2,3} = \$470{,}475$	$S_{max} = 353.481\ units$ $Q = 47{,}410.7\ units$ $TCY_{2,3} = \$\ 473{,}057$

4.2.3 The Case of $t \geq T_P$

This case discusses the situation where t is large, $t \geq T_P$ such that during the production period the inventory is built at a rate of $\left(1-\grave{d}\right)-D$, before its slope changes to $P_s - D$. Finally, the inventory level decreases at a rate of D.

The two sub-cases of P_s:

1. *Processing the defective items exceeds T_P:*

The total shortage,S, is obtained from Eq. 4.18. We compute the total inventory level from Fig. 4.9 as follows:

$$I = \frac{1}{2}\tilde{\theta}^2.\left(P\left(1-\grave{d}\right)-D\right) + \frac{1}{2}T_s\,.\left[2I_{max} - T_s\,.\left(D-P_s\right)\right] + \frac{I_{max}^2}{2D} \tag{4.36}$$

where,

$$I_{max} = T_P.\left[P\left(1-\grave{d}\right)-D\right] + \frac{Q\,\grave{d}}{P_s}\left(D-P_s\right) - S_{max} \text{ and } \tilde{\theta} = T_P - \frac{S_{max}}{P\left(1-\grave{d}\right)-D}\text{, thus:}$$

$$I = \frac{1}{2}\left(\frac{Q}{P} - \frac{S_{max}}{(1-\grave{d})P-D}\right)^2\left((1-\grave{d})P-D\right) + \frac{\left(Q-\grave{d}\,Q-S_{max}-\frac{Q\,D}{P}\right)^2}{2\,D} + \frac{\grave{d}\,Q\left(-\frac{\grave{d}\,Q(-P_S+D)}{P_S}+2\left(-S_{max}+\frac{Q((1-\grave{d})P-D)}{P}+\frac{d\,Q(-P_S+D)}{P_S}\right)\right)}{2\,P_S} \tag{4.37}$$

Eq. 4.13 becomes:

$$I_s = \frac{\grave{d}\,\left(\frac{Q}{P}\right)^2 + P_S\,T_S^2}{2} \tag{4.38}$$

where, $T_s = \frac{Q\,\grave{d}}{P_S}$.

One condition must be satisfied which is:

$$t = T_P = \frac{Q}{P} \qquad \text{and} \qquad T_P \leq T_P + T_s < T - \frac{S_{max}}{D}$$

Also, it may be noticed that as,

$$T_P + T_s < T - \frac{S_{max}}{D} \Rightarrow Q(\frac{1}{P} + \frac{\grave{d}}{P_s} - \frac{1}{D}) \leq \frac{-S_{max}}{D}$$

And therefore,

$$\frac{1}{P} + \frac{\grave{d}}{P_s} - \frac{1}{D} < 0 \tag{4.39}$$

In other words condition 4.39 is satisfied only if P and P_c are very large quantities, and $\grave{d}$ and D are small enough. The data given in Subsection 4.2.1 do not satisfy condition 4.39.

2. *Processing the defective items reaches the cycle end zone:*

This sub-case is valid for $P_s - D < 0$. Here we consider a case where processing the defective items continues while the shortage zone has not been met (Fig. 4.10).

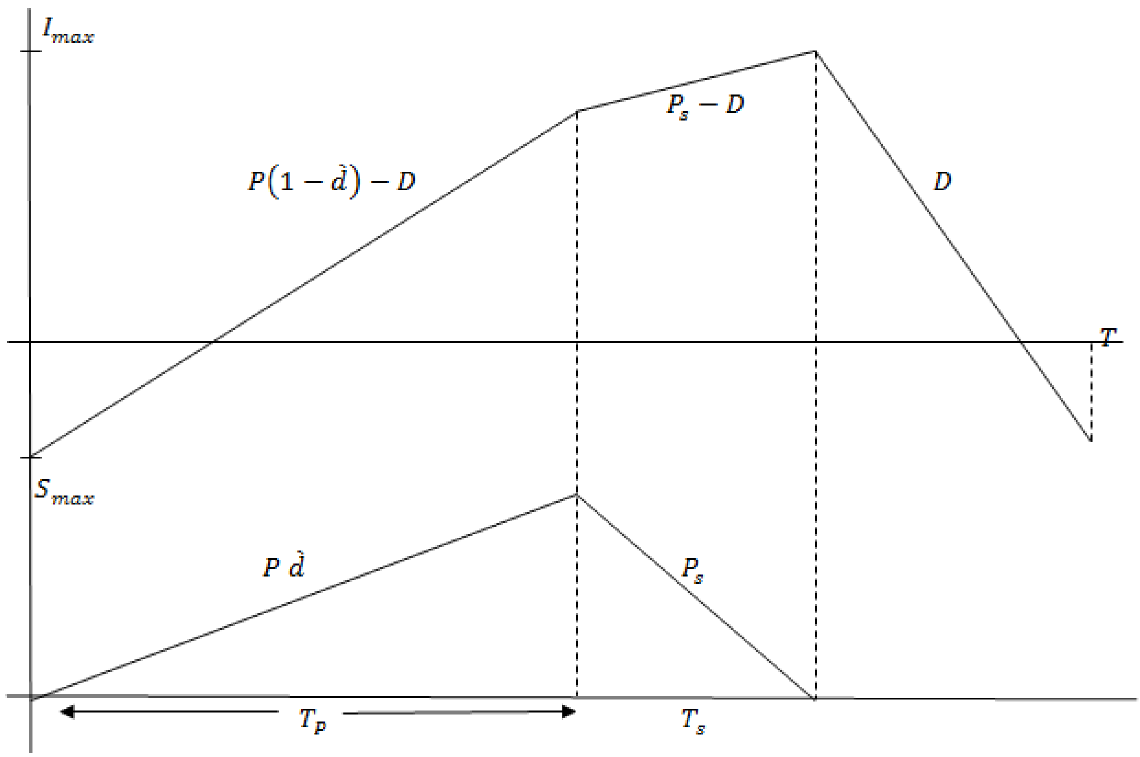

Figure 4.9: The relationship between the inventory level of conforming and defective items and the time when $t \geq T_P$.

The total shortage, S, is obtained from Eq. 4.32. We compute the total inventory level as follows:

$$I = \frac{1}{2}\frac{I_{max}^2}{P(1-\grave{d})-D} + \frac{1}{2}\frac{I_{max}^2}{P_s-D} \tag{4.40}$$

where,

$$I_{max} = T_P.\left[P\left(1-\grave{d}\right)-D\right]$$

Thus,

$$I = \frac{-((-1+\grave{d})P+\mathrm{P_s})Q^2((-1+\grave{d})P+D)}{2P^2(\mathrm{P_s}-D)} \tag{4.41}$$

Also, it may be noticed as,

$$t = T_P = \frac{Q}{P} \qquad \text{and} \qquad T - \frac{S_{max}}{D} \leq T_P + T_s < T$$

the condition 4.39 might not be satisfied.

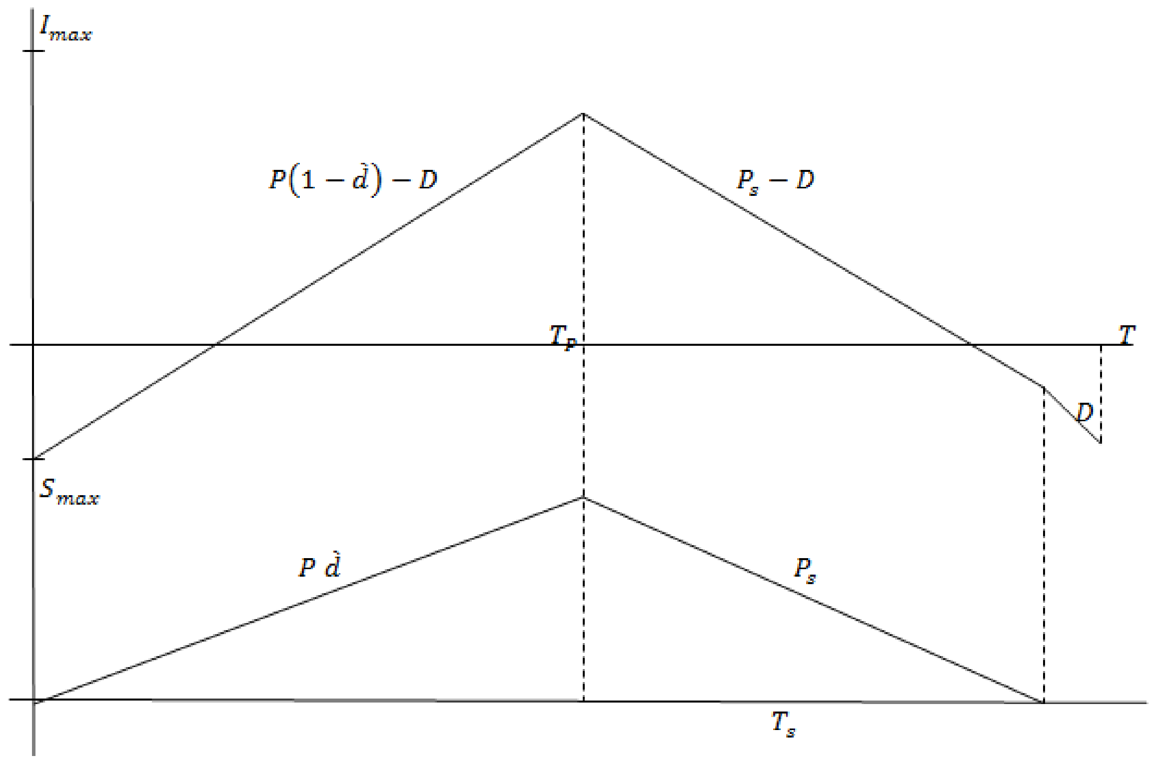

Figure 4.10: The relationship between the inventory level of conforming and defective items and the time when processing the defective items reaches the cycle end zone and $t \geq T_P$.

The lower and upper bounds of t for each case and sub-case are summarized as follows:

The case of $t \leq T_0$

1. Shortage is not met while processing the defective items:

$$t \leq T_0 \qquad \text{and} \qquad t + T_s \leq T_0$$

$$\text{Thus: } t \leq \frac{S_{max}}{(P-D)} \qquad \text{and} \qquad t \leq \frac{S_{max}}{(P-D)(1+\frac{P.\grave{d}}{P_S})}$$

2. Shortage is met before processing the defective items is completed:

$$t \leq T_0 \qquad \text{and} \qquad T_0 \leq t + T_s < T_P$$

Thus,

$$t \leq \frac{S_{max}}{P(1-\grave{d})-D} \qquad \text{and} \qquad \frac{S_{max}}{(P-D)(1+\frac{P.\grave{d}}{P_S})} \leq t < \frac{Q}{P[1+\frac{P\,\grave{d}}{P_S}]}$$

3. Processing the defective items exceeds T_P:

$$t \leq T_0 \qquad \text{and} \qquad T_P \leq t + T_s < T - \frac{S_{max}}{D}$$

Thus,

$$t \leq \frac{S_{max}}{P(1-\grave{d})-D} \qquad \text{and} \qquad \frac{Q}{P[1+\frac{P\,\grave{d}}{P_S}]} \leq t < \frac{Q-S_{max}}{D[1+\frac{P\,\grave{d}}{P_S}]}$$

4. Processing the defective items reaches the cycle end zone:

$$t \leq T_0 \qquad \text{and} \qquad T - \frac{S_{max}}{D} \leq t + T_s < T$$

$$\text{Thus: } t \leq \frac{S_{max}}{P(1-\grave{d})-D} \qquad \text{and} \qquad \frac{Q-S_{max}}{D[1+\frac{P\,\grave{d}}{P_S}]} \leq t < \frac{Q}{D[1+\frac{P\,\grave{d}}{P_S}]}$$

The Case of $T_0 \leq t < T_P$

1. Shortage is met before processing the defective items is completed:

$$T_0 \leq t < T_P \qquad \text{and} \qquad T_0 \leq t + T_s < T_P$$

Thus, $\frac{S_{max}}{P(1-\grave{d})-D} \leq t < \frac{Q}{P}$ and $\frac{S_{max}}{[P(1-\grave{d})-D](1+\frac{P\,\grave{d}}{P_S})} < t < \frac{Q}{P[1+\frac{P\,\grave{d}}{P_S}]}$

2. Processing the defective items exceeds T_P:

$$T_0 \leq t < T_P \qquad \text{and} \qquad T_P \leq t + T_s < T - \frac{S_{max}}{D}$$

Thus, $\frac{S_{max}}{P(1-\grave{d})-D} \leq t < \frac{Q}{P}$ and $\frac{Q}{P[1+\frac{P\,\grave{d}}{P_S}]} \leq t < \frac{Q-S_{max}}{D[1+\frac{P\,\grave{d}}{P_S}]}$

3. Processing the defective items reaches the cycle end zone:

$$T_0 \leq t < T_P \qquad \text{and} \qquad T - \frac{S_{max}}{D} \leq t + T_s < T$$

Thus: $\frac{S_{max}}{P(1-\grave{d})-D} \leq t < \frac{Q}{P}$ and $\frac{Q-S_{max}}{D[1+\frac{P\,\grave{d}}{P_S}]} \leq t < \frac{Q}{D[1+\frac{P\,\grave{d}}{P_S}]}$

The case of $t \geq T_P$

1. Processing the defective items exceeds T_P:

$$t = T_P = \frac{Q}{P} \qquad \text{and} \qquad T_P \leq T_P + T_s < T - \frac{S_{max}}{D}$$

2. Processing the defective items reaches the cycle end zone:

$$t = T_P = \frac{Q}{P} \qquad \text{and} \qquad T - \frac{S_{max}}{D} \leq T_P + T_s < T$$

CHAPTER 5

THE OPTIMAL LOT SIZE WITH RANDOM INSPECTION ERRORS AND PROCESSED ITEMS SOLD AT A SECONDARY MARKET

In this chapter we treat models discussed in Chapter 4 as if the inspection errors are random variables. Moreover, we assume that defective items that are further processed will be sold at a reduced cost at a secondary market.

5.1 Random Inspection Errors and Processed Items Sold at a Secondary Market:

5.1.1 Random Inspection Errors:

Assuming that inspection errors are fixed or constant is not always a valid assumption. Random inspection errors are more likely to take place in practice such that the fractions of Type I and II errors are calculated according to some probability distribution. That is, e_1 (a probability of conforming item is classified as non-conforming) and e_2 (a probability of non-conforming item is classified as conforming) are randomly distributed. The expected value of defective items is taken as follows:

$$\grave{d} = e_1 + d(1 - e_1 - e_2)$$

Thus,

$$E(\grave{d}) = E(e_1) + d[1 - E(e_1) - E(e_2)] \tag{5.1}$$

and

$$V(\grave{d}) = V(e_1) + 0 - d^2\, V(e_1) - d^2\, V(e_2) \tag{5.2}$$

We assume both types of inspection errors are normally distributed then,

$e_1 \sim N(\mu_1, \sigma_1)$ and $e_2 \sim N(\mu_2, \sigma_2)$. Hence, Eq. 5.1 and 5.2 become,

$$E(\grave{d}) = \mu_1 + d(1 - \mu_1 - \mu_2) \tag{5.3}$$

$$V(\grave{d}) = {\sigma_1}^2 - d^2\, {\sigma_1}^2 - d^2\, {\sigma_2}^2 \tag{5.4}$$

Therefore, the expected value of $E(\grave{d})$ is given by Eq. 5.3 and its variance is given by Eq. 5.4.

In this case $e_1 + e_2 = e$, is also a normal random variable with the probability density function given as:

$$g(e) = \frac{1}{V(\grave{d})\sqrt{2\pi}} e^{\frac{-(e - E(\grave{d}))^2}{2\,\sigma^2}} \tag{5.5}$$

5.1.2 Further Processed Items Sold at a Secondary Market:

When apparently defective items are detected, they are sent for further processing. In this model, the processed items are not perfect as new and therefore will be sold at a reduced cost at a secondary market. Also, we assume that the items processing starts after t ends. This makes the behavior of further processed items inventory level completely independent of the behavior of apparently perfect items inventory level. Thus, the behavior of further processed items inventory level always follows the graph in Fig. 5.1.

The average inventory level of processed items is obtained from Fig. 5.1 as follows:

$$\bar{I}_c = \frac{\frac{1}{2}t^2.P\,.E(\grave{d})+\frac{1}{2}T_C{}^2.P_C}{T} \tag{5.6}$$

One condition must be satisfied to ensure that the adjustment machine period and items processing period will not exceed the cycle length. That is,

$$t + T_C \leq T \tag{5.7}$$

Thus,

$$t + \frac{t\ P\ E(\grave{d})}{P_c} \leq \frac{Q - t\ P\ E(\grave{d})}{D} \tag{5.8}$$

We also observe that the behavior of the apparently perfect items inventory level as t varies, is exactly what we obtained in Chapter 3, except that the inspection errors are random variables.

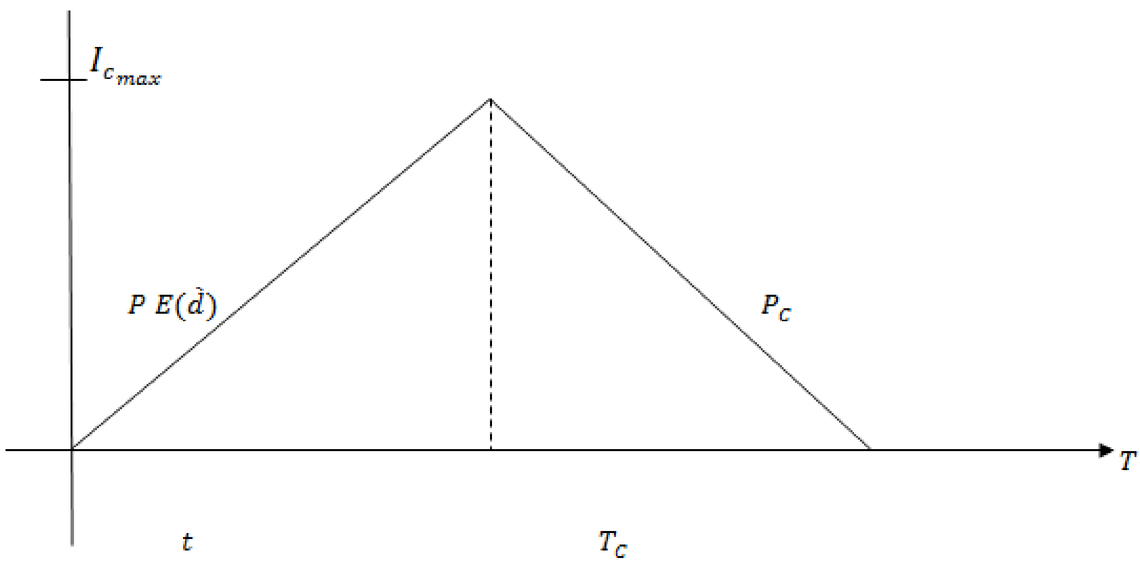

Figure 5.1: The behavior of the further processed items and the inventory level.

5.2 The case when t is deterministic:

There are three cases to be considered. In Section 5.2.1 we discuss the case of $t < T_0$, where T_0 is the period that starts with the resumption of the production cycle and ends when shortage is zero, as shown in Fig. 3.1. In Section 5.2.3 we discuss the case of $T_0 \leq t < T_p$ where T_p is the production period. Finally, in Section 5.2.5, we consider the case where $t \geq T_p$.

Total cost per cycle is obtained as follows:

$$TCY(Q, S_{max}) =$$

$$\frac{A\ D}{Q-t\ P\ E(\grave{d})} + C\ D + \frac{r\ t\ P\ E(\grave{d})D}{Q-t\ P\ E(\grave{d})} + \frac{A_d\ t\ D}{Q-t\ P\ E(\grave{d})} + D\ \mu\ C_P + D\int_{LSL}^{USL} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(X-\mu)^2}{2\sigma^2}} K(X-\mu)^2 dx$$

$$+ \frac{(t.P.E(\grave{d}))D}{Q-t\ P\ E(\grave{d})}\left(R_L \int_0^{LSL} g(x;\mu)dx \ + R_U \int_{USL}^{\infty} g(x;\mu)dx\right) + \bar{I}h + \bar{I}_c\ \hat{h} + \hat{\pi}\ \bar{S} + \breve{\pi}\ S_{max} \qquad (5.9)$$

5.2.1 The case of $t < T_0$

By referring to Fig. 3.1 where we replace d by $E(\grave{d})$, we calculate the average inventory as follows:

$$\bar{I} = \frac{(P(-Q+S_{max})+E(\grave{d})\ P^2 t+QD)^2}{2P(Q-E(\grave{d})\ P\ t)(P-D)} \qquad (5.10)$$

Also, the average shortage is calculated as follows:

$$\bar{S} = \frac{-P(S_{max}^2+2\ E(\grave{d})\ D\ S_{max}\ t+E(\grave{d})\ D(D+(-1+E(\grave{d}))P)\ t^2)}{2(D-P)(Q-E(\grave{d})\ P\ t)} \qquad (5.11)$$

5.2.2 Numerical Example:

Let us consider the following data:

P = 25,000 units per year
D = 23,000 units per year
P_C= 1,500 units per year
r = $1 per unit
h = $ 4 per unit/year
h_C = $ 2 per unit/year
C = $ 5 per unit
C_P= $ 3 per unit
A_d = $ 50 per hour
A = $ 100 per order
$\hat{\pi}$= $ 5 per unit/year
$\breve{\pi}$= $ 0.3
R_L = $ 4 per unit
R_U = $ 6 per unit
$\mu = 5$
σ = 0.05
USL = 5.2
LSL = 4.8
K= $ 120
$\mu_1 = 0.02$
$\mu_2 = 0.03$
$d = 0.0455$

The optimal values of the order quantity and the maximum shortage permitted when $t = 2\ hours$ are Q^*= 131,730 units and S^*_{max}= 1,601.64 units with a minimum cost of $ 490,002. The plots of TCY versus Q at $S_{max} = S^*_{max}$ and TCY versus S_{max} at $Q = Q^*$ are shown in Fig. 5.2 and 5.3 respectively.

Note that $2 = t < T_0$ = 2.38 which means that this case applies. Table 5.1 and Fig. 5.4 show the effect of increasing t on Q, S_{max} and TCY.

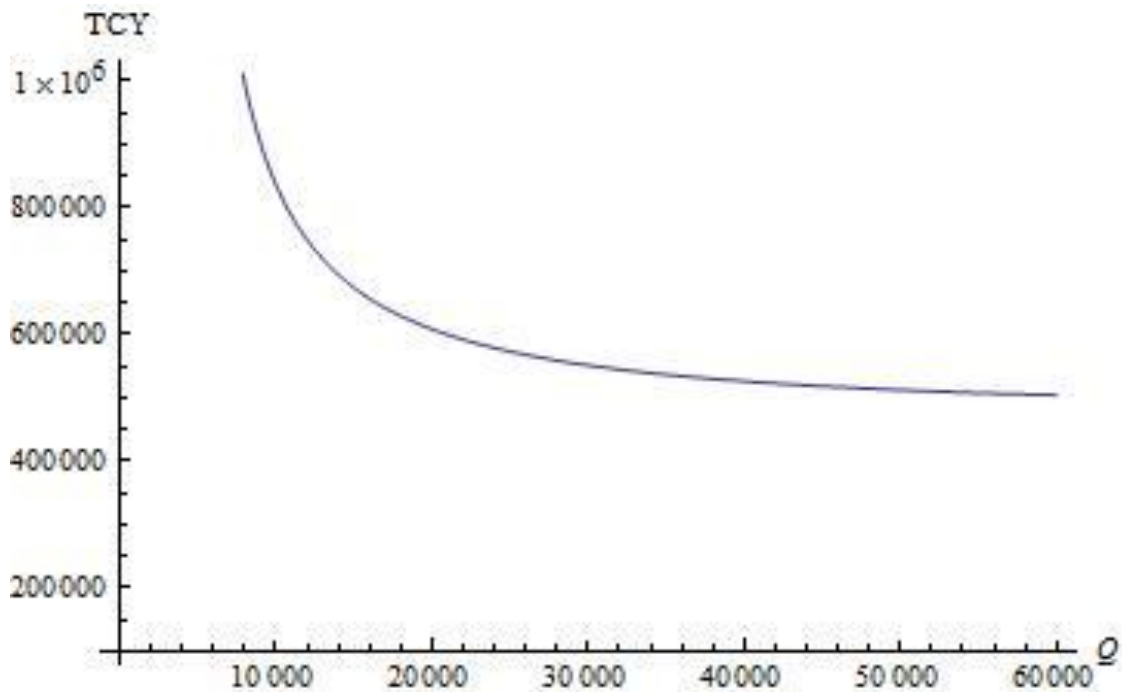

Figure 5.2: The behavior of Q as S^*_{max} is fixed.

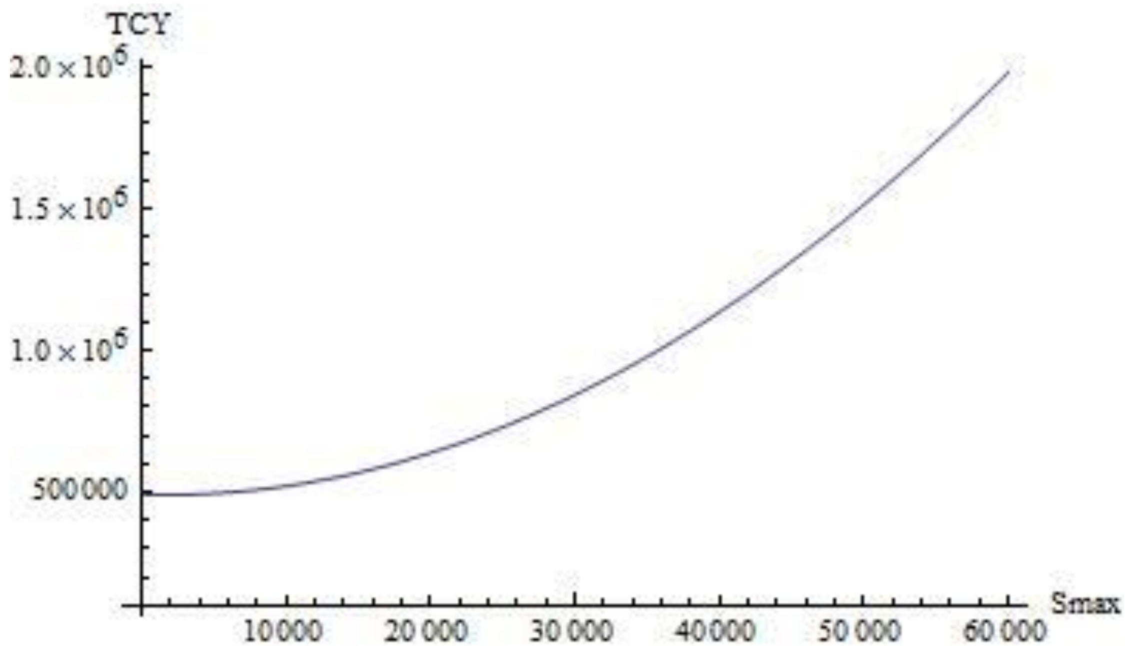

Figure 5.3: The behavior of S_{max} as Q^* is fixed.

Table 5.1: The effect of increasing t on Q, S_{max} and TCY

t	Q	S_{max}	TCY
0	4,847.11	111.01	468,007
0.4	58,943.48	1,430.28	477,511
0.8	83,357.05	1,694.17	481,739
1.2	102,099	1,756.4	484,959
1.6	117,872	1,713.07	487,650
2	131,730	1,601.64	490,002
2.4	144,214	1,441.38	492,109
2.8	155,649	1,243.79	494,029

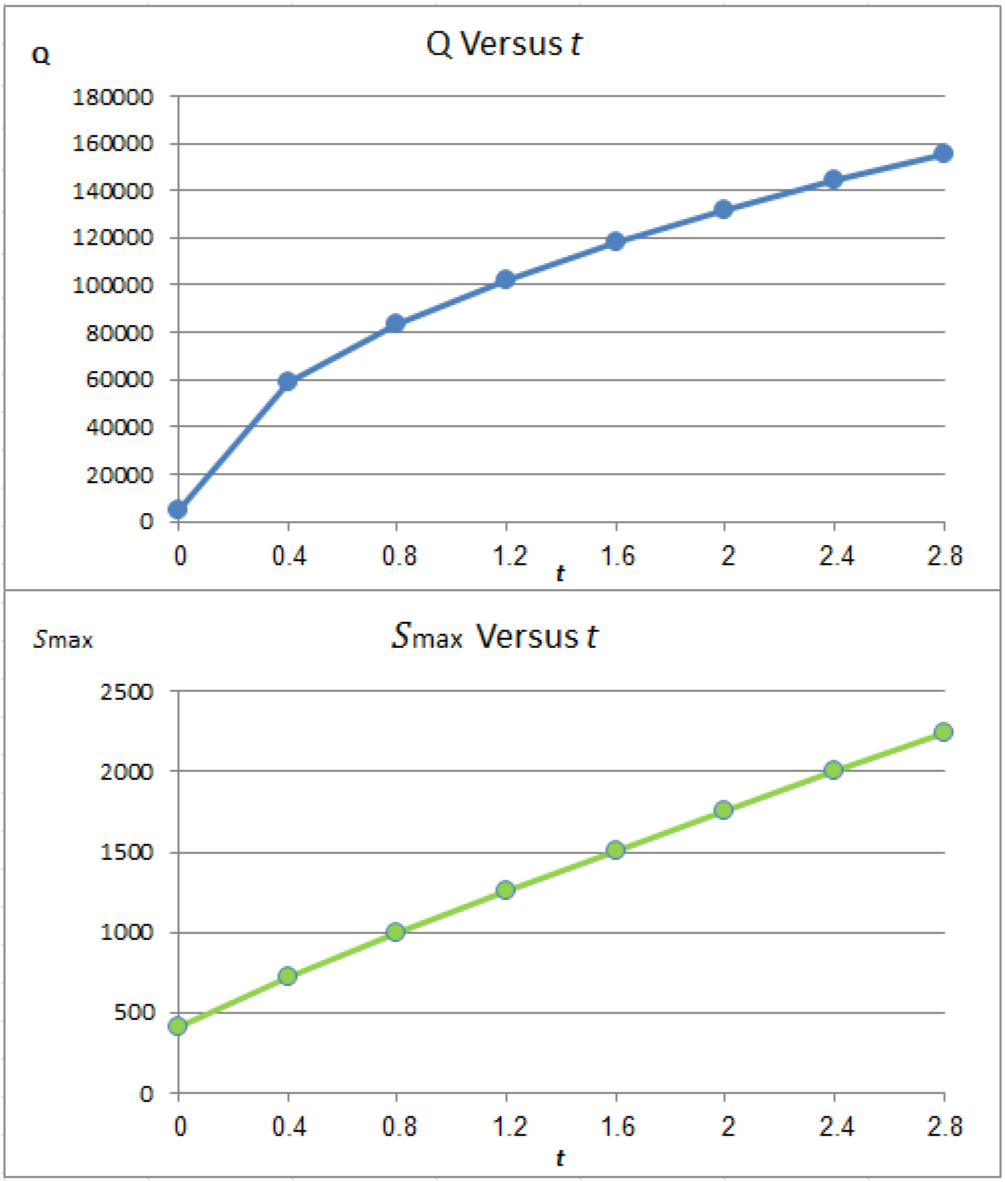

Figure 5.4: The behavior of Q and S_{max} as t increases for case: $t < T_0$

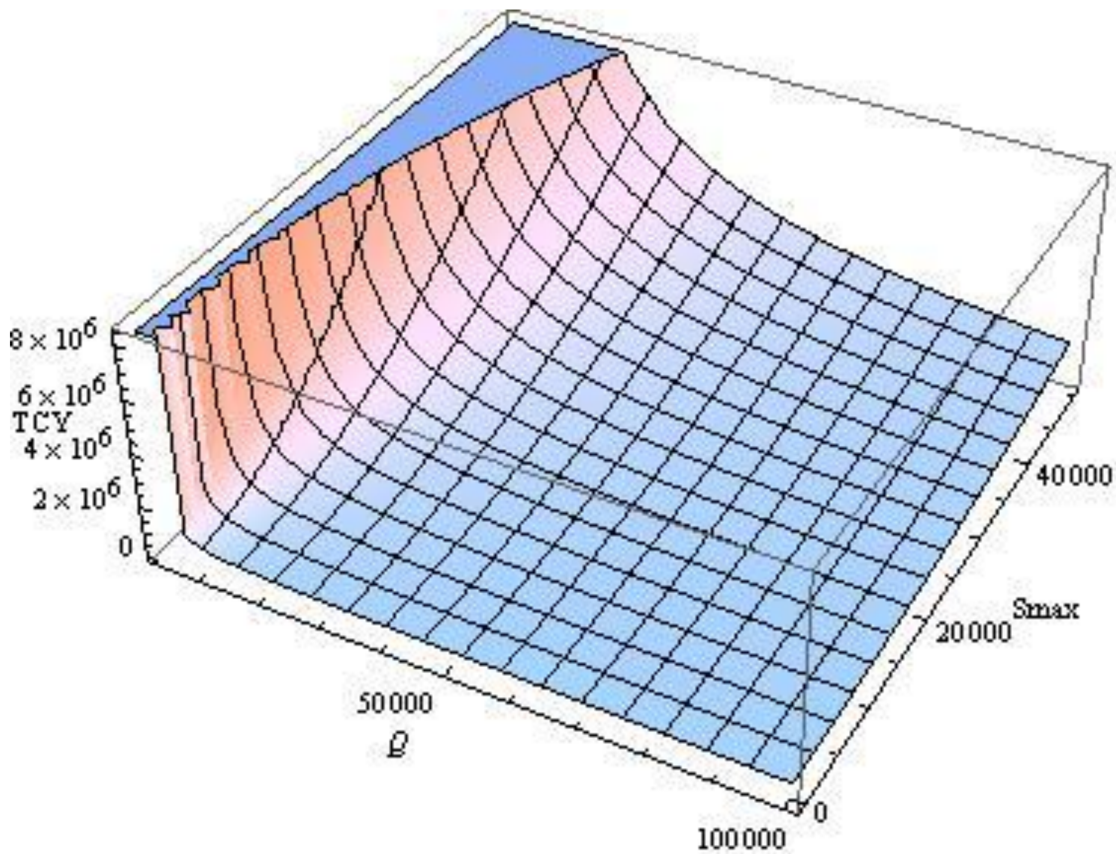

Figure 5.5: Three dimensional plot of Q, S_{max} and TCY

It is obvious from Table 5.1 and Fig. 5.4 that as t increases both Q and S_{max} increase as well, and as a result, TCY goes up. Indeed as t increases more units are discarded and hence Q should increase to compensate for these units. However, at $t > 2$, S_{max} goes down. Fig. 5.5 shows the interaction of Q, S_{max} and TCY.

5.2.3 The case of $T_0 \leq t < T_P$

Referring to Fig. 3.5, the average inventory level is obtained as follows:

$$\bar{I} = \frac{D\left(-\frac{(S_{max}+t((-1+E(\grave{d}))P+D))^2}{(-1+E(\grave{d}))P+D}+\frac{(S_{max}+E(\grave{d})Pt+Q(-1+\frac{D}{P}))^2}{D}+\frac{(-Q+P\,t)((-1+2\,E(\grave{d}))P^2 t+Q\,D+P(-Q+2\,S_{max}+t\,D))}{P^2}\right)}{2(Q-E(\grave{d})\,P\,t)} \quad (5.12)$$

Also, the average shortage is calculated as follows:

$$\bar{S} = \frac{1}{T}\,\frac{S_{max}^2\,P\,(1-E(\grave{d}))}{2\big(P\big(1-E(\grave{d})\big)-D\big)D} = \frac{S_{max}^2\,P\,(1-E(\grave{d}))}{2\,\big(Q-t.P.E(\grave{d})\big)\,\big(P\big(1-E(\grave{d})\big)-D\big)} \quad (5.13)$$

Therefore, total cost per year is obtained from Eq. 5.9.

5.2.4 Numerical Example:

Let's consider the data given in section 5.2.2. This case applies for any value of $t > 2.8$ hours, and this aspect can be derived from Eq. 5.8 as follows:

$$t + \frac{t\,P\,E(\grave{d})}{P_c} \leq \frac{Q - t\,P\,E(\grave{d})}{D}$$

Thus:

$$Q > t\,.D\,\left(1 + \frac{P\,E(\grave{d})}{P_c} + \frac{P\,E(\grave{d})}{D}\right) \text{ and this yields that } \frac{Q}{P} > t.$$

Therefore, any given value of $t > 2.8$ hours is applicable on this model. Now let's consider $\mathrm{t} = 5$ hours. The optimal values of the order quantity and the maximum shortage permitted when are Q^*= 244,084 units and S^*_{max}= 1,865.98 units with minimum cost of $ 503,210. The plots of TCY versus Q at $S_{max} = S^*_{max}$ and TCY versus S_{max} at $Q = Q^*$ are shown in Fig. 5.6 and Fig. 5.7, respectively.

Table 5.2 and Fig. 5.8 show the effect of increasing t on Q, S_{max} and TCY.

It is obvious from Table 5.2 and Fig. 5.8 that as t increases both Q and S_{max} increase as well, and as a result, TCY goes up. Fig. 5.9 shows the interaction of Q, S_{max} and TCY.

5.2.5 The case of $t \geq T_P$

Referring to Fig. 3.9, the average inventory level is obtained as follows:

$$\bar{I} = -\frac{(P((-1+E(\grave{d}))Q+S_{max})+Q\,D)^2}{2\,P\,Q\,((-1+E(\grave{d}))P+D)} \tag{5.14}$$

Also, the average shortage is calculated as follows:

$$\bar{S} = \frac{1}{T}\,\frac{S^2_{max}\,P\,(1-E(\grave{d}))}{2\big(P\big(1-E(\grave{d})\big)-D\big)D} = \frac{S^2_{max}\,P}{2\,Q\big(P\big(1-E(\grave{d})\big)-D\big)} \tag{5.15}$$

Therefore, total cost per cycle is obtained from Eq. 5.9.

The following conditions must be satisfied:

$$T_P + T_C \leq T$$

$$\frac{Q}{P} + \frac{Q\,E(\grave{d})}{P_c} \leq \frac{Q(1-\,E\big(\grave{d}\big))}{D}$$

$$\text{Thus: } \frac{1}{P} + \frac{E(\grave{d})}{P_c} \leq \frac{1-E(\grave{d})}{D} \tag{5.16}$$

In other words condition 5.16 is satisfied only if P and P_c are very large quantities, and $E(\grave{d})$ and D are small enough. So, when we refer to the data given in Section 5.2.2, we find that this case does not apply.

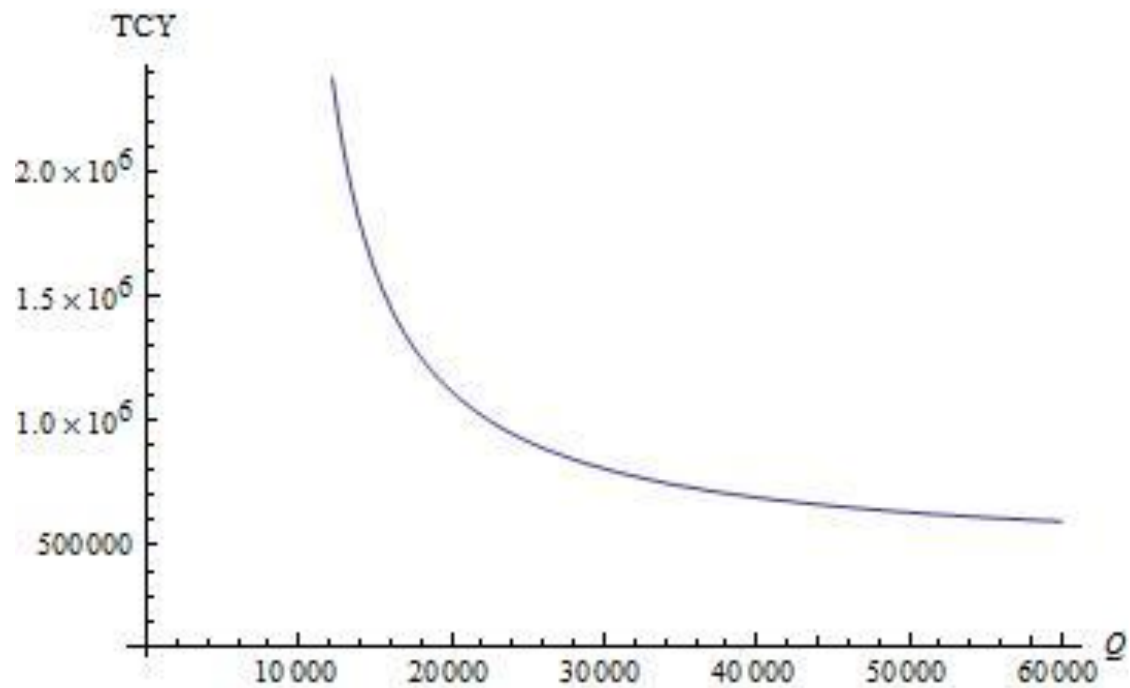

Figure 5.6: The behavior of Q as S^{*}_{max} is fixed.

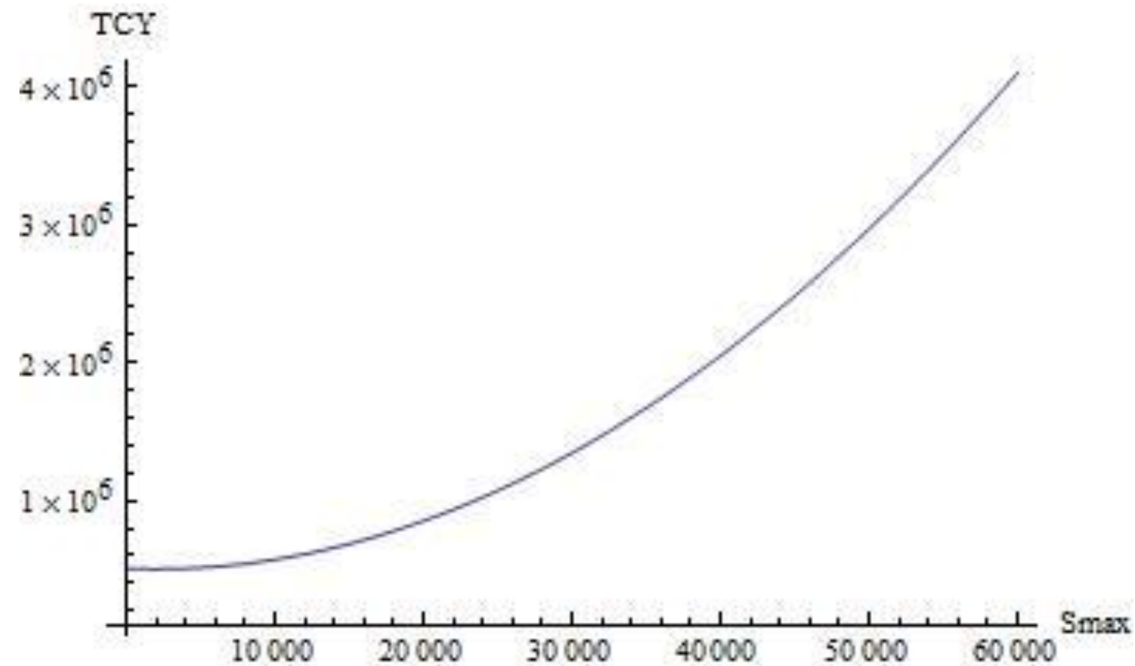

Figure 5.7: The behavior of S_{max} as Q^{*} is fixed.

Table 5.2: The effect of increasing t on Q, S_{max} and TCY

t	Q	S_{max}	TCY
3	162,394	1,240.98	494,934
4	198,929	1,519.18	499,190
5	244,084	1,865.98	503,210
6	292,901	2,241.92	507,220
7	341,718	2,617.86	511,231
8	390,535	2,993.8	515,241
9	439,352	3,369.74	519,253
10	488,169	3,745.68	523,264

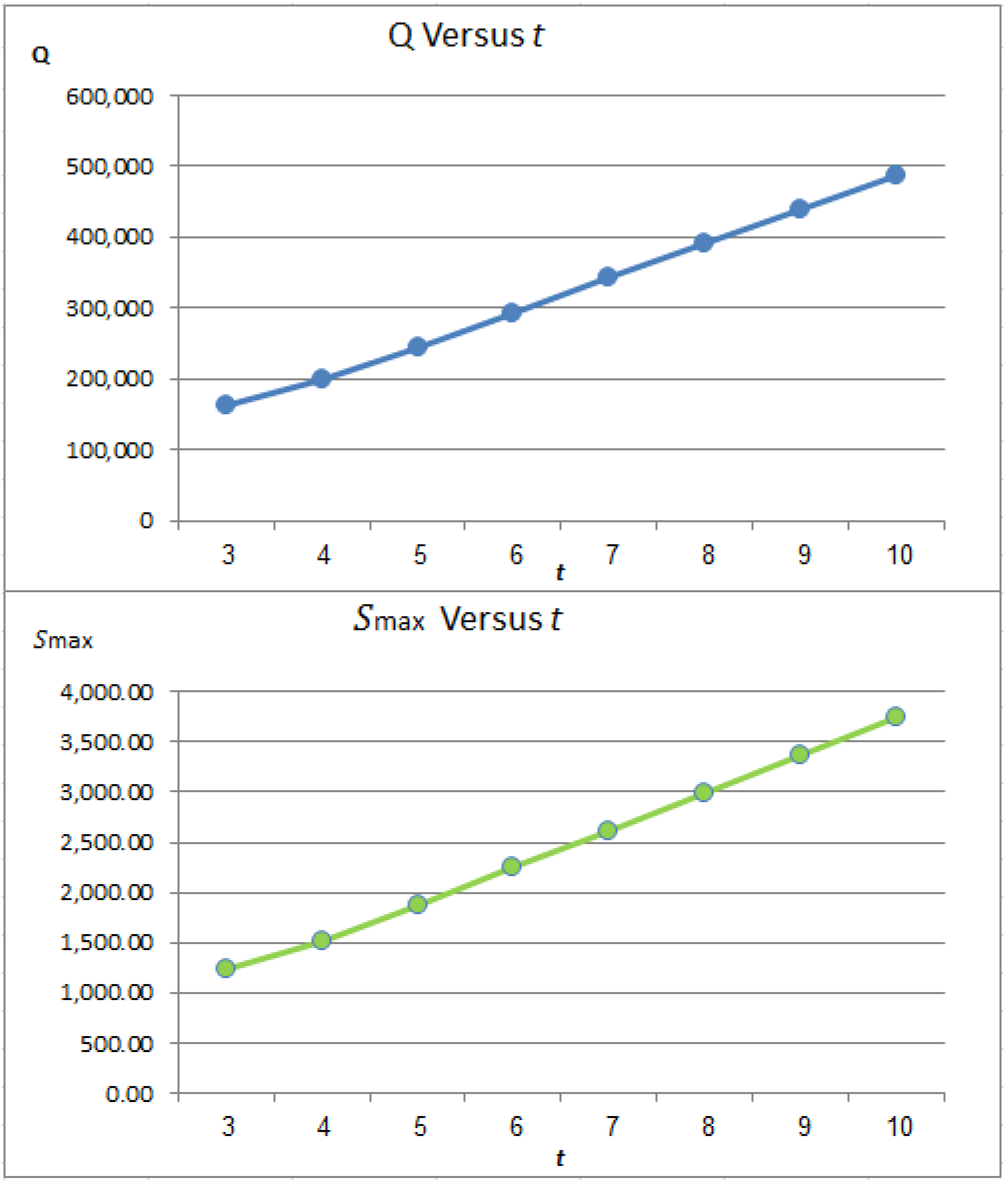

Figure 5.8: The behavior of Q and S_{max} **as** t **increases for the case of** $T_0 \leq t < T_P$

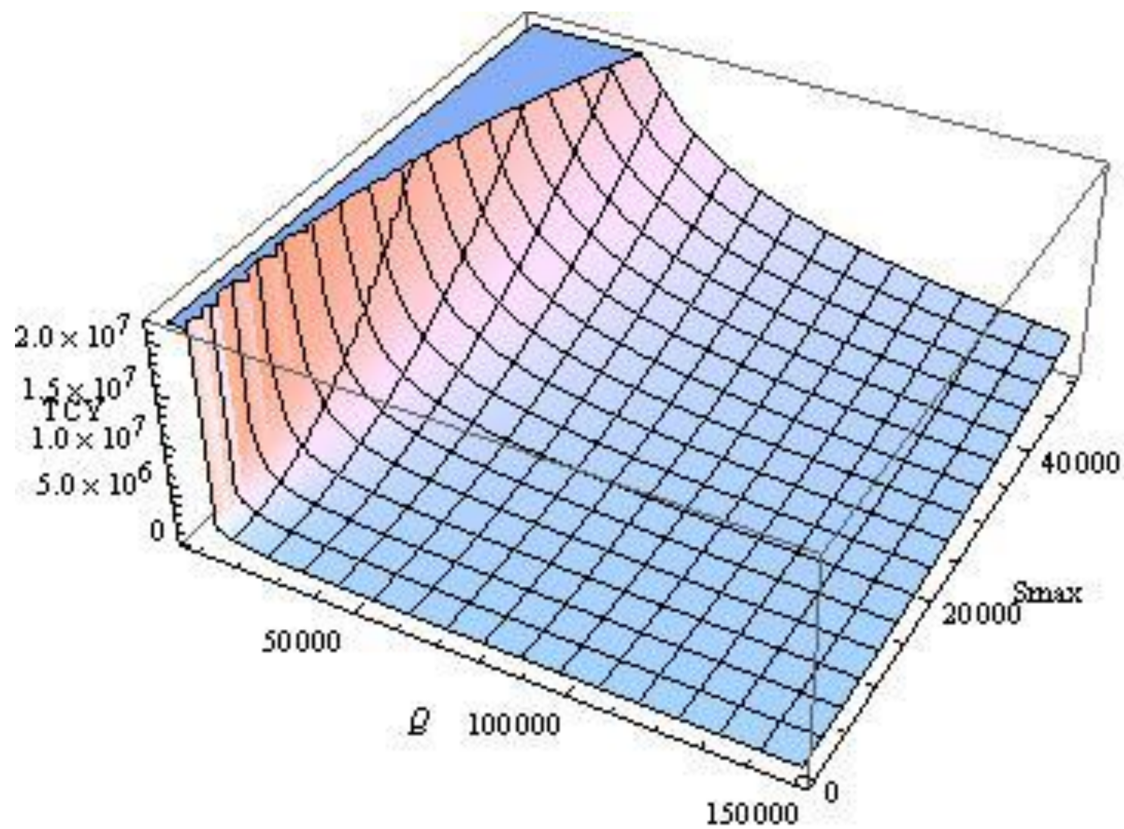

Figure 5.9: Three dimensional plot of Q, S_{max} and TCY

5.3 The Case when t is a random variable:

In this section we treat t as a random variable and hence some probability density distribution $f(t)$ must be defined. Section 5.3.1 considers model formulation and Section 5.3.2 provides a numerical example.

5.3.1 Model Formulation:

As t ,is a random variable, the three cases are combined based on the renewal theory, according to Eq. 2.1. The limits of integrals refer to the period during which each case may occur according to Table 5.3.

Thus:

$$E_i[TC(Q,S_{max})] = \int_{a_i}^{b_i} (A + C\,Q + r\,t\,P\,E(\grave{d}) + A_d\,t + Q\,\mu\,C_P + Q\int_{LSL}^{USL} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(X-\mu)^2}{2\sigma^2}} K(X - \mu)^2 dx + \left(t.P.E(\grave{d})\right)\left(R_L \int_0^{LSL} g(x;\mu)dx\ + R_U \int_{USL}^{\infty} g(x;\mu)dx\right) + T\bar{I}h + T\bar{I}_c h_C + \hat{\pi} T\,\bar{S} + \check{\pi}\,S_{max}) f(t)\,dt \tag{5.17}$$

Moreover, the expected cycle length is determined as follows:

$$E_i[T(Q,S_{max})] = \int_{a_i}^{b_i} (\frac{Q - t\,P\,E(\grave{d})}{D}) f(t)\,dt \tag{5.18}$$

where, a and b are the lower and upper bounds of each integral, the index i refers to the case number. Thus the expected total cost per cycle is obtained as follows:

$$\frac{\sum_{i=1}^{3} E_i[TC(Q,S_{max})]}{\sum_{i=1}^{3} E_i[T(Q,S_{max})]} \tag{5.19}$$

The objective of Eq. 5.19 is to find the optimal values of $Q \; and \; S_{max}$ such that the total cost per cycle is minimized.

This model cannot be valid for any value of $S_{max} > 0$. The condition $T_0 < T_P$ or $\frac{S_{max}}{P\left(1-E(\grave{d})\right)-D} < \frac{Q}{P}$ must be satisfied, thus,

$$S_{max} < \frac{Q\left[P(1-E(\grave{d}))-D\right]}{P} \tag{5.20}$$

Table 5.3: The limits of integrals for the three cases.

i	a_i	b_i
1	0	$\frac{S_{max}}{P\left(1-E(\dot{d})\right)-D}$
2	$\frac{S_{max}}{P\left(1-E(\dot{d})\right)-D}$	$\frac{Q}{P}$
3	$\frac{Q}{P}$	∞

5.3.2 Numerical Example:

Let us consider the following data:

P = 25,000 units per year
D = 23,000 units per year
P_C= 1,500 units per year
r = \$1 per unit
h = \$ 4 per unit/year
h_C = \$ 2 per unit/year
C = \$ 5 per unit
C_P= \$ 3 per unit
A_d = \$ 50 per hour
A = \$ 100 per order
$\hat{\pi}$= \$ 5 per unit/year
$\breve{\pi}$= \$ 0.3
R_L = \$ 4 per unit
R_U = \$ 6 per unit
$\mu = 5$
σ = 0.05
USL = 5.2
LSL = 4.8
K= \$ 120
$\mu_1 = 0.02$
$\mu_2 = 0.03$
$d = 0.0455$

When we use the above data, we discard the third case because the condition 5.16 is not satisfied. Suppose that the adjustment period t, is a uniform random variable where $f(t) = 1/8, \ 0 \le t \le 8$. We find that the optimal values of order quantity and maximum shortage permitted are obtained at Q^*=219,250 units and S^*_{max}=2,284.71 units with a minimum cost of \$ 502,631. The plots of TC versus Q at $S_{max} = S^*_{max}$ and TCY versus S_{max} at $Q = Q^*$ are shown in Fig. 5.9 and 5.10, respectively.

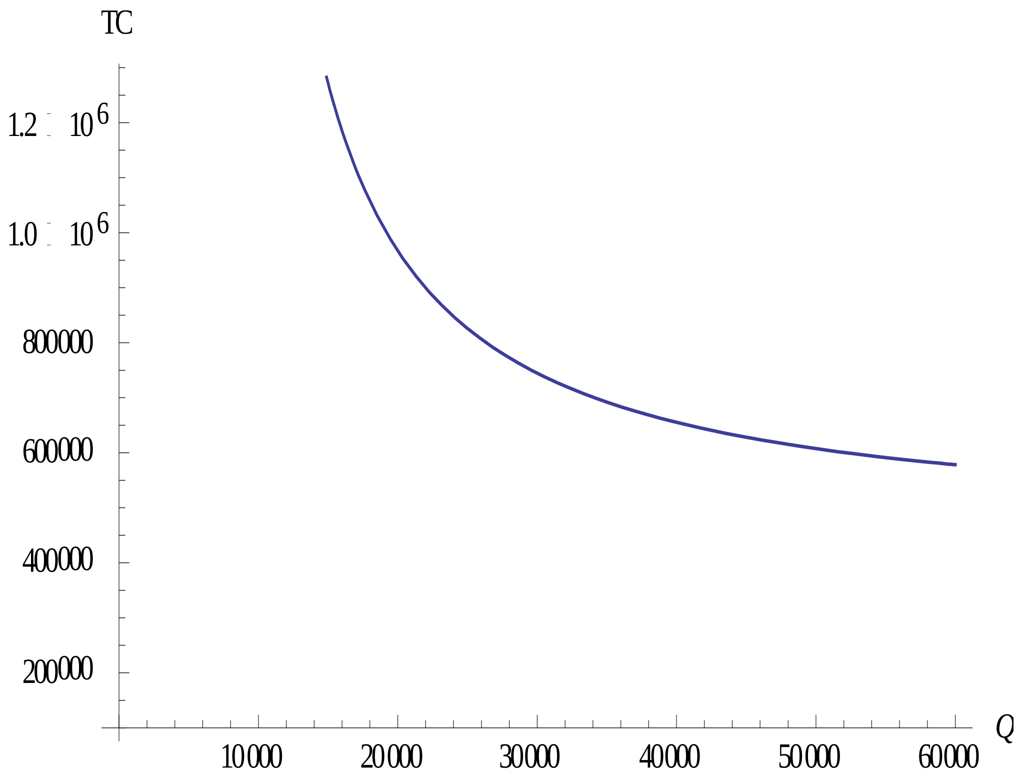

Figure 5.10: The behavior of Q as S^*_{max} is fixed.

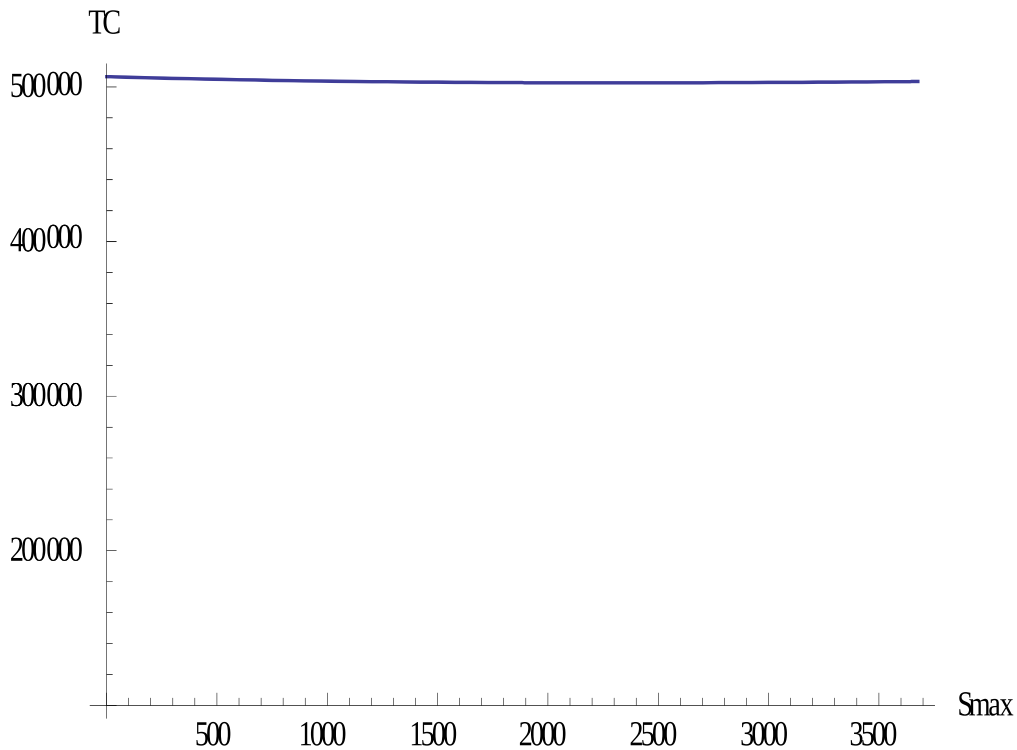

Figure 5.11: The behavior of S_{max} as Q^* is fixed.

CHAPTER 6

CONCLUSIONS AND FUTURE WORK

In this chapter we summarize the models proposed in the previous chapters and briefly describe the results obtained from these models. Studies which can be undertaken in the future and possible extensions to the proposed models are also discussed. This chapter is divided into two sections; in Section 6.1 we summarize our work, and in Section 6.2 we propose several ideas which could be considered as future extensions.

6.1 Summary:

The research discussed in this thesis investigated economic production quantity models where items with imperfect quality are produced during the process adjustment period. The main objective of these models is to find the allowed lot size quantity and 1shortage that will result in a minimum inventory and production cost.

In Chapter 1 we presented a literature review of the researches that are relevant to our work. The issue of the process adjustment period has not been addressed in the way we did in our research. Moreover, we found that there are not many papers which address the concept of the process adjustment period. We introduced the concept of the process adjustment period, during which all machines, equipment and tools associated with the production process are properly adjusted while the production is taking place. In some chemical processes, the machine adjustment period is the time taken for the attainmentofthe proper chemical composition. Some practical real life examples such as milling techniques, metal surface finishing and the steel making process are provided in Chapter 1.

- In Chapter 2 “The Optimal Lot Size under Machine Adjustment Period” model is discussed. The two cases considered are when the machine adjustment takes place during the production period, $t < T_P$, and when the machine adjustment takes a longer time such that it exceeds the production period ,$t \geq T_P$. We treat the machine adjustment period, t, as a deterministic value and a random variable and the order quantity size as the decision variable, Q. We developed a mathematical model representing each case. When t is a random variable the two cases are combined mathematically, based on the renewal theory. Numerical examples for t as a deterministic value and a random variable are provided for illustration purposes. We also showed that the optimal production size increases as the adjustment period increases, then at a certain value, it becomes constant.

- In Chapter 3 we discussed “The Optimal Lot Size under Maximum Shortage Allowance”. In this model three cases of the machine adjustment and screening process, time for which is t, are considered. It may take place during the shortage period, $t < T_0$, or could take place within production period, $T_0 \leq t < T_p$, or can exceed the production period, $t \geq T_p$. Moreover, t is treated as a fixed value and as a random variable. A mathematical model was developed and a numerical example is provided for each of the three cases of t. We showed that as t

increases both Q and S_{max} increase for $t < T_0$ and $T_0 \leq t < T_p$, and as a result, TCY goes up. For $t \geq T_p$, Q and S_{max} become constant.

- Chapter 4 titled "The Optimal Lot Size and Shortage allowance under Processing the defective items with Taguchi's Quality Loss Function and Inspection Errors", incorporates three contributions; processing the defective items , Taguchi's quality loss function and inspection errors. In processing the defective items , the apparent defective items are sent to the recycling process plant which has an inventory holding cost of $\hat{h}$. Once defective items have been recycled, they become as good as new and are sent back to the original plant to cover the demand. In Taguchi's quality loss function, the product's quality distribution is assumed to follow a normal distribution function and the loss function in our model is referred to as "nominal is best". Two types of errors are committed in the inspection process. Type I error (e_1) is committed when a conforming item is classified as non-conforming and Type II error (e_2) is committed when a non-conforming item is classified as conforming. Both types of errors are assumed to be known. Processing the defective items yields four different cases of t which is treated as a fixed value. The four cases and their sub-cases are shown in table 4.1. We developed a mathematical model and provided a numerical example for each case where the decision variables are Q and S_{max}.

- In Chapter 5 another extension titled "The Optimal Lot Size with Random Inspection Errors and Processed Items Sold at Secondary Market" is discussed. In this extension the fractions of Type I and II errors are randomly distributed and the processed items are not defect-free and therefore will be sold at a reduced price on a secondary market. Also, we assume that the processing of the items starts after t has lapsed and t is treated as a fixed value and as a random variable. In this model also three cases of the machine adjusting and screening process, time for which is t, are considered. It may take place during the shortage period, $t < T_0$, or could take place within the production period, $T_0 \leq t < T_p$, or can exceed the production period, $t \geq T_p$. We developed a mathematical model and provide a numerical example for each case where the decision variables are Q and S_{max}.

6.2 Future Work:

The models that we have developed in this work can be extended in several ways as shown below:

- Integrating this model with design of control charts.
- Treating the adjusting machine period as a fuzzy.
- Incorporating the lead time and the stochastic demand.
- Incorporating more than one adjusting machine period into a cycle.
- Integrating this model with vendor selection problem in a supply chain.

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