# Optimal Location and Configuration of Base Stations and Frequency Assignment for Cellular Mobile Networks 

## BY

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## Dedication

> I dedicate this thesis to all my beloved family members for their always love, support and encouragement.

## ACKNOWLEDGMENT

In the name of Allah, the Most Beneficent, the Most Merciful

All praises and glory are for Almighty Allah, (the Powerful and Exalted in Might) who gave me the courage, knowledge and patience to carry out the work in this thesis. Peace and blessings of Allah be upon his prophet Mohammad (peace and blessing be upon him) and all his family and his companions (May Allah be pleased with them all).

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## THESIS ABSTRACT

Name: YASSER ADEL A. ALMOGHATHAWI<br>Title: OPTIMAL LOCATION AND CONFIGURATION OF BASE STATIONS AND FREQUENCY ASSIGNMENT FOR CELLULAR MOBILE NETWORKS

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The planning of cellular mobile networks faces several major challenges such as the rapidly growing demands for mobile communication services, the limited bandwidth available for radio networks, and the requirements of the new emerging technologies. Therefore, an efficient planning model is required to design and implement a cellular network regardless of the applied technology. The model should provide a cost-effective cellular network considering competing factors i.e. coverage, capacity, and quality requirements. The objective of this thesis is to construct a general and comprehensive framework for the problem of designing cellular mobile networks. Mathematical models are constructed using mixed integer program (MIP) to find the optimal location and antennas configuration of each base station i.e. azimuth, tilt, height, and transmitted power, and frequency allocation. The objective of the MIP models is to minimize the total network costs taking into account the constraints of area coverage, traffic capacity, and quality of service i.e. signal-to-interference-plus-noise ratio (SINR). A none-line-ofsite situation is considered while calculating the path losses of the signals using COST-231-Walfisch-Ikegami model. The constructed MIP models are solved using LINGO and the output and performance are presented.

Keywords: Cellular mobile network, optimization, mixed integer program, base stations location, base stations configuration, frequency assignment.

## خلاصرـــــة الأطــروحة

الإسم: ياسـر عـادل عـوض المغذوي

الحل الأمثل لتحديد مواقع و إعدادات المحطات اللاسلكية وتخصيص الترددات للثنبكات الخلوية للهو اتف المتنقلة
الدرجة: ماجستير في العلوم

التخصص: هندسة النظم الصناعية التاريخ: مايو 2012

التخطيط لتصميم الثبكات الخلوية للهواتف المتنقلة يو اجه عدة تحديات رئيسية منها الطلب المتز ايد بشكل كبير على خدمات الاتصالات الدتنقلة، والعدد الهحدود من التزددات المتاحة لشبكات الإتصالات، ومتطلبات النكنولوجيات الحديثة الموجودة. لذا، لا بد من وضع نموذج تخطيط فَعَال لتصميم وتنفيذ شبكات خلوية بغض النظر عن النكنولوجيا المستخدمة والذي ينبغي أن يوفر شبكة فعالة من حيث التكلفة مع النظر في عوامل المنافسة من تغطية، وققرة إستيعابية، ومتطلبات الجودة. تهدف هذه الأطروحة إلى وضع إطار عام وشامل لحل مشكلة تصميم الشبكات الخلوية للهو اتف المتنقلة. وقد تم بناء نماذج رياضية بإستخدام البرنامج الخطي الذي يحتوي على بعض المتغير ات العددية الصحيحة. تقوم هذه النماذج بتحديد أفضل الو اقع للهحطات اللاسلكية، و إعدادات هو ائيات كل محطة من زاوية القاعدة وميل وارتفاع وطاقة مرسلة، وتوزيع الترددات على كل محطة ـ والهذف من هذه النماذج هو تقليل التكاليف الإجمالية للشبكة الخلوية مع الأخذ في الاعتبار قيود تغطية المنطقة الجغر افية، والقررة الإست يعابية، وجودة الخدمة. وختمت الأطروحة بعرض بعض الأمثلة لتقييم أداء الالمماذج المقترحة وتققيم نوصيات ومقترحات للبحوث المستقبلية في هذا المجال.

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## Chapter 1

## INTRODUCTION

In this chapter, we present a brief introduction about cellular networks and the challenges faced during the planning and design of these networks. The problems of base stations location and configuration and frequency assignment are described. The problem statement of this thesis is described. Finally the organization of the thesis contents is given.

### 1.1 BACKGROUND

The cellular concept was introduced in 1970s and it led to a significant improvement towards solving the problem of spectral congestion and user capacity. It increases
networks capacity with a limited spectral allocation and without any major technological changes using the ability of frequency reuse throughout a coverage region. [1]

The coverage area of a cellular network is divided into small geographic areas called cells where each one of them is allocated a base station (BS). A set of radio channels is assigned to each BS taking into account that BSs in adjacent cells are assigned different sets of channels. The antennas of each BS are designed to achieve the desired coverage within a specific cell. By limiting the coverage area of each cell to be within its boundaries, the same set of channels can be reused to cover different cells that are far enough from each other in order to keep interference reduced to tolerable limits. [2]

The planning of cellular mobile networks faces several major challenges, one of which is the rapidly growing demands for mobile communication services. Another one is the extremely limited number of available frequencies for radio networks. In addition, existences of new emerging technologies which require more advanced cellular network design methods. Moreover, wireless operators are continuously trying to cope with the steadily increasing demands by upgrading their networks to be in line with the technology advances. Also, they are looking for minimizing the cost of hardware investments by reusing as many components as possible from the already existing networks such as sharing core network elements, base station sites, antenna, etc. which could reduce the operational expenditure on transmission power, site rental, and operation and maintenance. [3, 4, 5]

Therefore, an efficient planning and optimization method using the demand node concept is required to design and implement a cellular network regardless of the applied technology. It should be able to synthesize efficient, economic and optimal network configurations. The objective of the optimization is to provide a cost-effective cellular network taking into account competing factors i.e. area coverage, traffic capacity, and quality of service requirements.

The demand node concept, which is considered in this thesis, has been used recently as a simplified traffic load model for cellular networks which describes the traffic quantitatively. The basic step of this concept involves representing the traffic in a specific geographical area by a finite number of nodes where each of one of them represents the same portion of traffic load. Hence, densely populated areas are represented by bulks of demand nodes lying in far distant corresponding centers. [3, 4, 6]

The planning and optimization of cellular networks design consists of selecting the a set of sites for base stations from a list of potential candidate sites, finding the optimal configuration for each BS , and assigning a set of channels to each base station. The selected sites form the basis of a network which must satisfy certain the requirements of the cellular networks such as high area coverage, high traffic capacity, and low interference while the total network cost is minimized. The configuration of each selected base station includes choosing among different antenna types, e.g., various directional or
omnidirectional antennas, power control, tilt, and azimuth. None of the previous work has considered the full complexity of the problem i.e. BS location and configuration [7].

In this thesis, we consider the full complexity of the problem and also antenna heights is included for the BS configuration. Moreover, we solve the integrated problem of location and configuration of BS and frequency assignment which even harder.

Regarding the channel assignment problem, a frequency reuse scheme is required for efficient utilization of the radio spectrum. There are two channel assignment strategies: Fixed Channel Assignment (FCA) and Dynamic Channel Assignment (DCA). In FCA strategy, which is the case considered in this thesis, each cell is allocated a predetermined set of channels and if all the channels in that cell are occupied, the call is blocked and the subscriber does not receive service. On the other hand, in DCA strategy, channels are not allocated to different cells permanently and whenever there is a call request to be made, the serving BS requests a channel from the mobile switching center. [2]

Interference is the major limiting factor in the performance of cellular radio systems and it is more severe in urban areas, which is the considered environment in our work, due to the large number of BSs and mobiles. There are two types of system-generated cellular interference: co-channel interference and adjacent-channel interference. Co-channel interference, which is considered in this thesis, is the interference between signals from co-channel cells, which are the cells in which the same set of frequencies is used due to
the frequency re-use concept for cellular networks. Adjacent-channel interference is the interference resulted from signals that are adjacent in frequency to the desired signal. [2]

The performance of a cellular network is mainly characterized by the received signal strength and signal-to-noise-plus-interference ratio (SINR) for each user. The SINR for each user must be above the minimum limit, under which a certain level of service quality might not be reachable. The coverage of the network is highly affected by the environment of network, i.e. transmit power and radio propagation, and the traffic distribution. Therefore gain of better coverage and increased SINR in a cellular network are two of the key objectives of mobile networking industry. [8, 9]

### 1.2 PROBLEM STATEMENT

The design of cellular mobile networks consists of three major problems which need to be optimized. The first one is selecting the optimal sites for placing base stations. The second one is finding the optimal configuration of all used antennas for each selected base station. The last one is allocating frequencies, group of radio channels, to each selected base station.

A detailed literature review of the existing related work reveals that there is no welldesigned mathematical model, specifically using mixed integer program, for solving the problem of cellular mobile network design specifically, finding the optimal location and
configuration of base stations and frequency assignment. Therefore, an efficient planning and optimization model is required to design and implement a cellular mobile network regardless of the applied technology, which should provide a cost-effective cellular network considering competing factors namely, coverage, capacity, and quality requirements.

The objective of this thesis is to construct a general comprehensive framework for the problem of designing cellular mobile networks taking into account the constraints of area coverage, traffic capacity, and quality of service i.e. signal-to-interference-plus-noise ratio (SINR). In this thesis, we construct four mathematical models using Mixed Integer Programming (MIP) to solve this problem with the objective of minimizing the total network costs. We build the framework step by step by constructing different models for different cases precisely; base stations locations only, base station location and frequency assignment, base stations location and configuration, and finally location and configuration of base stations and frequency assignment. Also, we construct MIP models for three special cases namely, base stations configuration with the existence of base stations, base stations location with different time intervals, and finally base and relay stations location. The details of the constructed MIP model are summarized in Table 1-1.

Table 1-1 Constructed Mixed Integer Programming Models in This Thesis

|  | Model <br> No. | Model Title | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | Optimization Parameters |  |  |  |  |  |  |  | Constraints |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Base <br> Stations <br> Location | Base Stations Configuration |  |  |  |  | Relay <br> Stations <br> Location | Frequency <br> Assignment | Coverage | Capacity | Quality of Service (SINR) |
|  |  |  |  |  | no. of antenna | Azimuth | Tilt | Height | Power |  |  |  |  |  |
| 3 | I | Base Stations Location |  | $\sqrt{ }$ | - | - | - | - | - | - | - | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ |
|  | II | Base Stations <br> Location and Frequency Assignment |  | $\sqrt{ }$ | - | - | - | - | - | - | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ |
|  | III | Base Stations Location and Configuration |  | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | - | - | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ |
|  | IV | Location and Configuration of Base Stations and Frequency Assignment |  | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | - | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ |
| 4 | I | Base Stations Configuration |  | - | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | - | - | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ |
|  | II | Base Stations Location with Different Time Intervals |  | $\sqrt{ }$ | - | - | - | - | - | - | - | $\sqrt{ }$ | $\sqrt{ }$ | $\checkmark$ |
|  | III | Base and Relay Stations Location |  | $\sqrt{ }$ | - | - | - | - | - | $\sqrt{ }$ | - | $\sqrt{ }$ | $\checkmark$ | $\checkmark$ |

### 1.3 THESIS ORGANIZATION

This thesis consists of five chapters where the content of each chapter is briefly described in this section.

In Chapter 1, we introduce the concept of cellular mobile networks and considered in their design. We address the problem statement with the objective of this thesis. Finally, we provide a summary of the constructed mathematical model using MIP.

In Chapter 2, we present some backgrounds about cellular networks, mathematical programming, and propagation model used in this thesis. Also, we discuss the previous studies in which similar kind of research is done.

In Chapter 3, we present four different mathematical models using MIP for solving the cellular mobile networks design. For each MIP model, we present the model formulation, numerical experiment, and discussion about the results of the numerical experiment.

In Chapter 4, we present three further mathematical models in which MIP is used to solve some special cases related to the design problem. For each MIP model, we present the model formulation, numerical experiment, and discussion about the results of the numerical experiment.

In Chapter 5, we summarize the contributions of the work in this thesis and present some recommendations for future work and possible extensions.

## Chapter 2

## Background and Related Work

### 2.1 BACKGROUND

In this section, a brief background for the cellular network concept is given. Also, mathematical programming is briefly discussed and the general form of the mixed integer program is presented. Finally, the propagation model used in this thesis, i.e. COST-WI model, for calculating the path losses of the signals is addressed.

### 2.1.1 CELULLAR NETWORK

The cellular concept is replacing a single large cell with high power transmitter by many small cells with low power transmitters where each one is providing coverage to only a
small portion of the service area, see Figure 2-1. So, a cellular network could be defined as a radio network which consists of small land areas called cells where each cell is served by fixed-location transceivers called base stations and can provide coverage over a wide geographic area which enables a large number of portable transceivers, called mobile stations, to communicate with other transceivers anywhere in the network. These cells are often shown diagrammatically as hexagonal shapes whereas in reality they have irregular boundaries due to the terrain over which they travel such as hills, buildings and other objects which cause to that the signal is attenuated and diminish differently in each direction. A typical cellular network is shown in Figure 2-2. [1, 10]


Figure 2-1 A Single Large Cell versus Multiple Small Cells


## Figure 2-2 A Typical Cellular Network

Multiple frequencies are assigned to each cell within the cellular network which have corresponding base stations. Those frequencies can be reused in other cells as well with the condition that same frequencies are not reused in adjacent neighboring cells which would cause co-channel interference. Hence, adjacent cells must use different frequencies unless the two cells are sufficiently far enough from each other. Thus, the increased capacity in a cellular network results from the fact that the same radio frequency can be reused in a different area with a completely different transmission, see Figure 2-3. On the other hand, if there is a single plain transmitter, only one transmission can be used on any given frequency. [10]

As the demand increases, the number of base stations may be increased, thus additional radio capacity is provided with no additional increase in radio spectrum. Hence with
fixed number of channels, an arbitrarily large number of users can be served by reusing the channels throughout the coverage area [1].


Figure 2-3 Frequency Re-use

There are several techniques to increase networks capacity even more to cope with the explosive growth of mobile phone users. Cell splitting is one technique that used to increase the network capacity without new frequency spectrum allocation. Cell splitting is reducing the size of the cell by lowering antenna height and transmitter power. Also, another technique to increase the network capacity is sectoring which is dividing the cell, without changing its size, into several sectors using several directional antennas at the BS instead of a single omnidirectional antenna; see Figures 2-4 and 2-5. Using the sectoring technique will reduce the radio co-channel interference; thus the network capacity will be increased [1, 11].


Figure 2-4 Omnidirectional Antenna


Figure 2-5 Direcitional Antennae

The interference between adjacent channels in a cellular network could be minimized by assigning different frequencies to adjacent cells. Hence, cells can be grouped together to form what is called a cluster, see Figure 2-6. It is necessary to limit the interference between cells having the same frequency. The larger the number of cells in the cluster, the greater the distance between cells sharing the same frequencies. By making all the cells in a cluster smaller it is possible to increase the overall capacity of the cellular system. Hence, small low power base stations should be installed in areas where there are more users. [12]

Many advantages resulted from using the concept of cellular networks such as increased coverage and capacity by the ability to re-use frequencies, reduced the usage of transmitted power, and reduced interference from other signals.


Figure 2-6 Clusters

### 2.1.2 MATHEMATICAL PROGRAMMING

Mathematical programming is a modeling approach used for decision-making problems. Formulations of mathematical programming include a set of decision variables, which represent the decisions that need to be found and an objective function, a function of the decision variables, which assesses the quality of the solution. A mathematical program will then either minimize or maximize the value of this objective function.

The decisions of the model are subject to certain requirements and restrictions which can be included as a set of constraints in the model. Each constraint can be described as a function of the decision variables which bounds the feasible region of the solution and it
is either equal to, not less than, or not more than, a certain value. Also, another type of constraint can simply restrict the set of values to which a variable might be assigned.

Throughout this thesis, we use Mixed Integer Programming (MIP) for constructing our models which is a subset of mathematical programming. We use MIP where the constraints and objective function are all linear with the restriction that some of the variables must be integer-valued. Several applications for MIP involve decisions that are discrete, while some other decisions are continuous in nature. In this thesis, we will refer to the form of MIP as the standard form which is described as:

$$
\begin{array}{cc}
\text { Min/Max } & f(x) \\
\text { subject to } & g_{i}(x) \leq 0 \\
& h_{j}(x)=0
\end{array}
$$

Where:
$f(x)$ is the objective function to be minimized or maximized $g_{i}(x)$ are the inequality constraints to the problem for $i=1,2,3, \ldots, m$
$h_{j}(x)$ are the equality constraints to the problem for $j=1,2,3, \ldots, n$
$m$ and $n$ are the number of the constraints for the inequalities and the equalities, respectively. [13, 14, 15]

### 2.1.3 PROPAGATION MODEL

In this thesis, we use COST-Walfisch-Ikegami (COST-WI) propagation model [16] for urban city environment. This model has many features such that it can be implemented easily without expensive geographical database, captures major properties of propagation and is used widely in cellular network planning.

COST-WI model provides a high accuracy for urban environments where the propagation over the rooftops is the most dominant part by the consideration of more data to describe the character of the environment. As depicted in Figures 2-7 and 2-8, it considers buildings heights ( $h_{\text {roof }}$ ), roads widths ( $w$ ), buildings separation (b), and road orientation with respect to the direct radio path $(\varphi)$. The main parameters of the model are:

- Frequency $(f)$ which is restricted to be in the range of 800 to 2000 MHz
- Height of the transmitter $\left(h_{T X}\right)$ which is restricted to be in the range of 4 to 50 meters.
- Height of the receiver $\left(h_{R X}\right)$ which is restricted to be in the range of 1 to 3 meters.
- Distance between transmitter and receiver (d) which is restricted to be in the range of 20 to 5000 meters.


Figure 2-7 COST-WI Model Paramteres


Figure 2-8 Definition of the street orientation angle $\varphi$

The model distinguishes between two situations, line-of-sight (LOS) and none-line-ofsight (NLOS) situations. In this thesis, we consider the situation of NLOS. In the following, we describe the two cases in more details.

1- Line-Of-Sight (LOS) situation:

LOS means that there exists a direct path between the transmitter and receiver; see Figure 2-9. For this case, the path loss (PL) is determined by the following expression.

$$
P L(d B)=42.6(d B)+26 \cdot \log \left(\frac{d}{k m}\right)+20 \cdot \log \left(\frac{f}{M H z}\right) \quad \text { for } d \geq 20 m
$$

where dB indicates the decibel unit


Figure 2-9 LOS Situation

2- None-Line-Of-Sight situation:

NOLS means that the path between the transmitter and receiver is partially obstructed, usually by a physical object such as buildings, trees, hills, mountains, etc, see Figure 2-10.


Figure 2-10 NLOS Situation

For this case, the path loss calculation is more complicated where the path loss is the sum of the free space loss $\left(L_{0}\right)$, the rooftop-to-street diffraction loss $\left(L_{r t s}\right)$, and the multiple screen diffraction loss ( $L_{m s d}$ ):

$$
P L(d B)= \begin{cases}L_{0}+L_{r t s}+L_{m s d} & \text { for } L_{r t s}+L_{m s d}>0 \\ L_{0} & \text { for } L_{r t s}+L_{m s d} \leq 0\end{cases}
$$

The free space loss $\left(L_{0}\right)$ is determined by:

$$
L_{0}=32.4(d B)+20 \cdot \log \left(\frac{d}{\mathrm{~km}}\right)+20 \cdot \log \left(\frac{f}{\mathrm{MHz}}\right)
$$

and the rooftop-to-street diffraction loss $\left(L_{r t s}\right)$ determines the loss occurred on the wave coupling into the street where the receiver is located and it is calculated by:
$L_{r t s}=-16.9(d B)-10 \cdot \log \left(\frac{w}{m}\right)+10 \cdot \log \left(\frac{f}{M H z}\right)+20 \cdot \log \left(\frac{h_{\text {roof }}-h_{R X}}{M H z}\right)+L_{\text {Ori }}$
$L_{\text {Ori }}$ is the orientation loss obtained from the calibration with measurements and determined by:

$$
L_{\text {Ori }}= \begin{cases}-10+0.354 \cdot \frac{\varphi}{d e g .} & \text { for } 0^{\circ} \leq \varphi<35^{\circ} \\ 2.5+0.075 \cdot\left(\frac{\varphi}{d e g .}-35\right) & \text { for } 35^{\circ} \leq \varphi<55^{\circ} \\ 4.0-0.114 \cdot\left(\frac{\varphi}{d e g .}-55\right) & \text { for } 55^{\circ} \leq \varphi<90^{\circ}\end{cases}
$$

The multiple screen diffraction loss is determined by:

$$
L_{m s d}=L_{b s h}+k_{a}+k_{d} \cdot \log \left(\frac{d}{k m}\right)+k_{f} \cdot \log \left(\frac{f}{M H z}\right)-9 \cdot \log \left(\frac{b}{m}\right)
$$

where:

$$
L_{b s h}=\left\{\begin{array}{cl}
-18 \cdot \log \left(1+\frac{h_{T X}-h_{\text {roof }}}{m}\right) & \text { for } h_{T X}>h_{\text {roof }} \\
0 & \text { for } h_{T X} \leq h_{\text {roof }}
\end{array}\right.
$$



The factor $k_{a}$ represents the increase of the path loss for base station antennas below the rooftop of the adjacent buildings. The factors and $k_{d}$ and $k_{f}$ control the dependence of $L_{m s d}$ versus the distance and radio frequency, respectively. The relationship between the path loss using COST-WI and the distance for both LOS and NLOS situations is shown in Figure 2-11.


Figure 2-11 Path Loss versus distance (km) for COST-WI model

### 2.2 RELATED WORK

In this section a literature survey is presented of relevant previous work for the papers in which the problems of base stations (BS) location and frequency assignment for wireless networks have been studied. The reviewed papers can be classified into three main categories, namely, BSs location problem, frequency assignment problem, and combined BSs location and frequency assignment.

### 2.2.1 BASE STATIONS LOCATION PROBLEM

In the study of [7], the author developed an optimization framework based on simulated annealing to select BS sites and configurations for mobile cellular networks where the configuration of each BS involves antenna type, power, azimuth, and tilt. The optimization objective is to minimize the site cost while satisfying the coverage, capacity, handover, and interference constraints.

In the study of [17], the authors used MIP for modeling the problem of the location and configuration of BSs in a universal mobile telecommunication system (UMTS) network aiming to minimize the total cost of the network. The MIP model takes into consideration the constraints of capacity, interference, need for sufficiently strong signals, and potential gain for mobiles from being in soft hand-over.

In the studies of [18-23], the authors proposed mathematical programming models for selecting the location and configuration of BSs for UMTS network with the objective of maximizing the coverage while the total cost is reduced. They also proposed randomized greedy procedures and Tabu search algorithms to reduce the computational time required to find an approximate solution. The different models take into account installation costs, signal quality, power control mechanism and traffic coverage.

In the studies of [24, 25], the authors studied the planning of UMTS radio networks taking into consideration the coupling of coverage and capacity through interference. They provided a local search procedure and a MIP model for planning networks under quality constraints with the objective of minimizing the total cost.

In the study of [26], the authors solved the BS locations and calculation of service capacity problems using an integer programming (IP) model with the objective of netrevenue maximization satisfying the interference constraint. The model considers a set of candidate BS locations with corresponding costs, a number of customer locations with corresponding demand for traffic and the revenue potential for each unit of capacity allocated to each demand point. Also they developed some algorithms for practical solution.

In the study of [8], the authors presented a constraint satisfaction problem model using both IP and constraint programming for BSs location in 3G W-CDMA uplink environments and applied constraint satisfaction techniques such as variable ordering and value ordering to get good approximate solutions. The model objective is to minimize the total transmitted power while satisfying the coverage and quality of service constraints.

In the study of [27], the authors integrated a deterministic model for tower section and a stochastic one for revenue optimization into a stochastic integer programming model that
optimizes the BS locations under the uncertainty of the demand. They also developed algorithms using Bender's reformulation.

In the study of [28], the authors addressed the problems of planning and optimizing for the characteristics of WCDMA radio networks i.e. power and handover. They developed IP models for selecting optimal BS locations with the objective of minimizing the installation cost under while the traffic capacity and coverage are maximized. In addition, they presented four meta-heuristics that could be used for practical solutions.

In the study of [29], the authors proposed a MIP model for finding the optimal placements of BSs and Optical Network Units (ONU) in a WOBAN. The objective of model is to minimize the total cost of the networks taking into consideration coverage, capacity, and quality of service constraints. They used Lagrangian Relaxation to solve this MIP with reasonable accuracy.

In the studies of $[30,31]$, the authors studied the problem of designing Wireless Mesh Networks (WMN) considering traffic routing, interference, rate adaptation, and channel assignment. The developed MIP models for realistic size instances with the objective of minimizing the network installation cost while providing full coverage to wireless mesh clients. They also proposed a relaxation-based heuristic for large size network instances.

In the study of [32], the author solved the problem of base station placement in urban environment using 2-D convolution which searches for the best locations of base stations based on highest consumption criteria and allows simple user interface and arbitrary demand and supply patterns of power.

In the study of [33], the author introduced a novel approach for computing the number, location, and transmission powers of wireless base stations in a 2-D urban setup which utilizes 2-D convolution to extract the supply-demand correlation. This approach enables network designers to choose arbitrary antenna propagation and radio demand patterns using a simple color-coding mechanism.

In the study of [34], the authors proposed a new placement technique for WLAN APs based on 2-D convolution with the objective of covering the whole area with minimum number of access points considering the environmental features of the intended area such as radio propagation model and antenna patterns.

In the study of [35], the authors provided a mathematical model for the problem of automatic selection and configuration of BSs sites for fixed wireless access networks considering the net present value (NPV) to produce cost-effective deployments which maximize the economic performance and taking into account the constraints of coverage, interference, capacity and availability.

In the study of [36], the author studied the problem of 3 G cellular network design. He developed a series of optimization models using mathematical programming for determining the optimal locations of BSs from a set of candidate BS locations and assignment of mobiles to BSs considering capacity and quality of service constraints. He concluded that the models require special solution techniques due to its size and difficulty.

In the study of [37], the authors proposed an optimization framework for the design of CDMA-based cellular networks to determine the BS selection and BS-to-mobile user association. The objective of the optimization is to maximize the profit while satisfying the quality of service constraints.

### 2.2.2 FREQUENCY ASSIGNMENT PROBLEM

In the study of [38], the authors formulated the fixed channel assignment problem as an MIP problem with compatibility and requirement constraints. The model goal is to satisfy the maximum requirements of the BS i.e. traffic constraints. For solving the proposed MIP model, they used a special branch and bound algorithm.

In the study of [39], the authors proposed an iterative-approach heuristic for assigning fixed channels in large cellular radio networks. Their approach is to find an initial
assignment using fast heuristics then splitting the problem into smaller sub-problems which can be solved using a binary linear program.

In the study of [40], the authors developed a mathematical programming model to determine the assignment of stations to access points, signal strengths and channel assignment of both access points and stations for small scenarios of co-existing CSMA/CA-based wireless networks, such that the contention between these networks is minimized. They also proposed a genetic algorithm specifically tuned to find nearoptimal solutions for large scenarios.

In the study of [41], the authors addressed the problem of channel assignment for multichannel multi-interface WMN. They provided two MIP models to find optimal fixed channel assignment with multiple radios with the objective of maximizing the number of bidirectional links that can be activated simultaneously considering the interference constraints.

In the study of [42], the authors addressed the problem of assigning channels to communication links in multi-hop wireless mesh networks with the objective of minimizing the overall network interference using designing centralized and distributed algorithms. They developed a semi-definite program and a linear program formulation of the problem for obtaining lower bounds on overall network interference.

In the study of [43], the authors developed an MIP for obtaining the optimal frequency assignment for each access point (AP) such that the throughput is maximized and the interference between the various APs is minimized. They assumed that the location of the APs are known and considered the case where the APs are not in line of sight. They proposed also two efficient heuristic algorithms to achieve the same results.

In the study of [44], the authors studied the problem of the radio resource assignment in WMNs assuming a time division multiple access (TDMA) scheme. They provided an optimization framework using mathematical programming for routing, scheduling and channel assignment considering the signal-to-interference-and-noise ratio. They considered an alternative problem formulation where decision variables represent compatible sets of links active in the same slot and channel, called configurations and the optimization goal is to minimize the number of used slots.

### 2.2.3 BASE STATIONS LOCATION AND FREQUENCY ASSIGNMENT PROBLEMS

In the studies of [45-47], the authors studied the integrated problem of BS placement and fixed channel assignment for wireless local area network (WLAN) in an indoor environment. They proposed mathematical models using MIP with the objective of maximizing the coverage while the installation costs are reduced taking into consideration the quality of service constraints.

In the study of [6], the authors formalized the problem of BSs positions and channels assignment as a MIP model with the objective of minimizing the interferences or the number of blocked channels. Also, they developed a simulated annealing as an approximate optimization technique whenever an exact solution is out of reach.

In the study of [48], the authors solved the problem of designing cellular mobile networks using combinatorial optimization methods aiming to cover the maximum number of users considering the traffic capacity with the minimum number of sites. First, they found the BSs placement and configuration then they allocated the communication channels.

In the study of [49], the authors studied the problem of designing broadband fixed wireless access networks. They developed an optimization model for BS selection and configuration with the objective of maximizing the profits considering availability, coverage, capacity, and interference constraints. The BS configuration includes antenna type, azimuth, tilt, power, and channel assignment.

In the studies of $[50,51]$, the authors provided a MIP model for the integrated problem of the cell site selection and the frequency allocation where the objective is to cover a maximum amount of traffic at a low interference considering the capacity constraint. Also, due to the complexity of the problem, they developed a branch-and-bound algorithm to solve huge size problems.

In the study of [52], the authors studied the problem of optimizing AP placement and channel assignment in WLANs. They formulated the problem as a MIP model and the objective is to minimize the maximum of channel utilization under the constraints of coverage, capacity, and quality of service.

In the study of [53], the authors developed a MIP model for optimum locations of BSs as well as frequency channel assignment with the objective of maximizing the coverage considering the capacity constraints. They proposed also a few greedy heuristics to obtain quicker solutions. In addition, they developed a Lagrangian heuristic technique which builds on the obtained solution by the greedy heuristics in order to improve the optimality gap.

In the study of [54], the authors presented a net-revenue maximization model for cell locations and channels allocation problems using IP model. The model considers a set of candidate cell locations with corresponding costs and a number of customer locations with corresponding demand. Moreover, they developed some procedures to generate tight bounds for the model.

In the study of [55], the authors used mathematical programming for optimizing AP placement and channel assignment for WLAN. They developed a MIP model for each
problem then proposed an integrated one with the objectives of maximizing the average throughput and minimizing the overlap areas.

In the study of [2], the authors developed a mathematical programming formulation for channel allocation problem aiming to maximize the coverage taking into account the interference between adjacent-channel. They considered an interference of a channel with at most two other channels only. They developed a solution procedure for channel allocation which uses a heuristic first to locate the cell towers, then allocates the channels to the towers using a polynomial-time algorithm, and finally improves this allocation using a simulated annealing procedure.

In the study of [56], the authors provided mathematical optimization models for solving APs placement and channels assignment problems in WLANs considering the capacity and quality constraints. They presented an optimization model for AP location aimed at maximizing the average user throughput and two modeling approaches for channel assignment that use different performance metrics. They also discussed integrated models for joint optimization of AP location and channel assignment.

## Chapter 3

## Mixed Integer Programs - Main Models

### 3.1 MODEL I: BASE STATIONS LOCATION PROBLEM

In this section, we solve the problem of base stations location. The objective of this model is to minimize the total cost of the associated base stations taking into account the constraints of area coverage, capacity of base station, and quality of service requirements for each user. If the costs of base stations are equal, then problem is to find the minimum number of base stations which will satisfy all constraints.

### 3.1.1 MODEL I: MODEL FORMULATION

We assume that the demand points and candidate sites for the base stations are known. Denote the $i$ th demand point by $D P_{i}, i=1,2, \ldots, n$ and the $j$ th candidate site by $C S_{j}$, $j=1,2, \ldots, m$. A base station at candidate site $j$ can serve demand point $i$, if the power received at $D P_{i}$ exceeds its minimum power requirements, $\gamma$, so we define $S(i)$ as the set of candidate sites that can serve demand point $D P_{i}$, i.e.
$S(i)=\left\{(j) \mid j \in S\right.$, such that the power received at $\left.D P_{i} \geq \gamma\right\}$
where $S$ is the set of candidate sites.

The Mixed Integer Programming model for the base stations location problem can be described as follow:

1 - The decision variables for Model I
a. The decision variable, $Y_{j}, j=1,2, \ldots, m$ is defined as follows:

$$
Y_{j}= \begin{cases}1 & \text { if a } B S \text { is constructed at } C S_{j} \\ 0 & \text { otherwise }\end{cases}
$$

b. The decision variable, $X_{i j}, i=1,2, \ldots, n$ and $j \in S(i)$, is defined as follows:

$$
X_{i j}= \begin{cases}1 & \text { if a BS at } C S_{j} \text { has the strongest signal at } D P_{i} \\ 0 & \text { otherwise }\end{cases}
$$

## $\underline{2-T h e ~ o b j e c t i v e ~ f u n c t i o n ~ f o r ~ M o d e l ~ I ~}$

The function to be optimized is the total cost of the network. The objective function can be described as:

$$
\operatorname{Minimize} \sum_{j=1}^{m} C_{j} Y_{j}
$$

where $C_{j}$ is the cost of installing a base station at $C S_{j}$.

## 3 - The constraints for Model I

The problem has five types of constraints that bound the feasible region of the solution. These are the following:
a. Each demand point should be served by at least one base station. This set of constraints can be represented by:

$$
\sum_{j \in S(i)} Y_{j} \geq 1, \quad i=1,2, \ldots, n
$$

b. Each demand point should be assigned to exactly one base station; hence this set of constraint can be written as:

$$
\sum_{j \in S(i)} X_{i j}=1, \quad i=1,2, \ldots, n
$$

c. A candidate site $C S_{j}$ is assigned to a demand point $D P_{i}$ if it is selected to construct a base station and it can be represented by this set of constraints:

$$
Y_{j} \geq X_{i j}, \quad i=1,2, \ldots, n \text { and } j=1,2, \ldots, m
$$

d. Each base station has a capacity of $Q$ channels, so the numbers of demand points assigned to each base station must not exceed its limit of channels. This set of constraints can be written as follow:

$$
\sum_{i=1}^{n} X_{i j} \leq Q, \quad j=1,2, \ldots, m
$$

e. The quality of service constraints by which the ratio of the strongest signal received at each $D P_{i}$ to the received noise and signals from other base stations should be greater than a minimum requirement of signal-to-interference-plusnoise ratio, $S I N R$. This set of constraints can be given as follow:

$$
\frac{S P(i)}{P_{N_{i}}+T P(i)-S P(i)} \geq 10^{S I N R / 10}, \quad i=1,2, \ldots, n
$$

where:
$S P(i)$ is the strongest power at test point $D P_{i}$ and is given by:

$$
S P(i)=\sum_{j \in S(i)} X_{i j} P_{i j}
$$

where $P_{i j}$ is the received power at $D P_{i}$ from a BS at $C S_{j}$
$T P(i)$ is the total power received at $D P_{i}$ which is generated by all base stations at candidate sites that can serve $D P_{i}$ and is given by:

$$
T P(i)=\sum_{j \in S(i)} Y_{j} P_{i j}
$$

where $P_{i j}$ is the received power at $D P_{i}$ from a BS at $C S_{j}$
$P_{N_{i}} \quad$ is the noise power at $D P_{i}$
SINR is minimum signal-to-interference-plus-noise ratio

The complete MIP model for the base stations location problem can be summarized as shown in Figure 3-1.

| Minimize | $\sum_{j=1}^{m} C_{j} Y_{j}$ |
| :---: | :--- |
| Subject to |  |
|  | $\sum_{j \in S(i)} Y_{j} \geq 1$ |
|  | $\sum_{j \in S(i)} X_{i j}=1$ |
|  | $X_{i j} \leq Y_{j}$ |
|  | $\sum_{i=1}^{n} X_{i j} \leq Q$ |
|  | $\frac{S P(i)}{P_{N_{i}}+T P(i)-S P(i)} \geq 10^{S I N R / 10}$ |
|  | $X, Y \in[0,1]$ |
|  |  |

Figure 3-1 MIP Model I - Base Stations Location Problem

### 3.1.2 MODEL I: NUMERICAL EXPERIMENT

A case has been studied for an area of $11 \mathrm{~km} \times 11 \mathrm{~km}$ with a map that is converted using MATLAB into 100 demand points (DP), where each demand point represent multi users and same load of users considered for all demand points. A set of optimal location for
base stations (BS) has to be selected from 300 candidate sites (CS). The demand points and candidate sites are shown in Figure 3-2.


Figure 3-2 Demand Points and Candidate Sites Condsidered for Model I

None-line-of-sight situation (NLOS) is considered for calculating the path loss using COST-WI propagation model (Section 2.2.3) with the parameters shown in Table 3-1. The other parameters used in the numerical experiments such as transmitted power, gains, receiver sensitivity, and base stations capacity are shown in Table 3-2. The noise power is assumed to be negligible.

Table 3-1 Parameters Considered for COST-WI Propagation Model

| Parameter | Value |
| :--- | :--- |
| Frequency | 1800 MHz |
| Height of transmitter | 25 m |
| Height of receiver | 2 m |
| Height of building | 7 m |
| Buildings separation | 50 m |
| Width of streets | 25 m |
| Angle | $30^{\circ}$ |

Table 3-2 Parameters used in Numerical Experiment of Model I

| Parameter | Value |
| :--- | :--- |
| Transmitted power | 25 dBm |
| Transmitted antenna gain | 8 dBi |
| Received antenna gain | 2 dBi |
| Minimum power requirement | -95 dBm |
| Available frequencies | 1 |
| Base station capacity | 30 channels |
| SINR | 20 dB |

The MIP for base stations location problem is solved using an optimization modeling software, LINGO, LINDO Systems Inc. The optimal solution resulted into 10 base stations as shown in Figure 3-3.


Figure 3-3 Results of Model I

### 3.1.3 MODEL I: DISCUSSION

As noticed above, this MIP model recommends 10 base stations to cover all the demand points which is relatively high number considering that each base station has a capacity
up to 30 demand points. There are two reasons for having a large number of base stations. The first reason, the height of the transmitter as well as the transmitted power are low in order to satisfy the quality of service (i.e. SINR) constraint in urban areas, whereas if higher values are used, it could result in a higher coverage but the SINR constraint will not be satisfied. The second reason, the inflexibility of this model i.e. frequency and base stations configuration are not considered. However if the frequency is considered, it could result in a less number of base stations to cover all the demand points, as will be shown in Model II. In addition, if base stations configuration is considered, it could result in a less number of base stations too.

### 3.2 MODEL II: BASE STATIONS LOCATION AND FREQUENCY ASSIGNMENT PROBLEMS

In this model, we solve the integrated problem of base stations location and frequency assignment. The objective of this model is to minimize the total cost of the network taking into account the constraints of area coverage, capacity of base station, and quality of service requirements for each user. This means, if the costs of base stations are equal, then the problem is to find the minimum number of base stations with optimal frequency assignment which could achieve the objective of the model while satisfying all constraints.

### 3.2.1 MODEL II: MODEL FORMULATION

We assume that the demand points and candidate sites are known. Denote the $i$ th demand point by $D P_{i}, i=1,2, \ldots, n$ and the $j$ th candidate site by $C S_{j}, j=1,2, \ldots, m$. There is also, a set of available frequencies, $k$, to be assigned to base stations, $k=1,2, \ldots, K$. A base station at candidate site $j$ can serve demand point $i$, if the power received at $D P_{i}$ exceeds its minimum power requirements, $\gamma$, so we define $S(i)$ as the set of candidate sites whose can serve demand point $D P_{i}$, i.e.
$S(i)=\left\{(j) \mid j \in S\right.$, such that the power received at $\left.D P_{i} \geq \gamma\right\}$
where $S$ is the set of candidate sites.

The Mixed Integer Programming model for the integrated problem of base stations location and frequency assignment can be described as follows:

## 1 - The decision variables for Model II

a. The decision variable, $Y_{j k}, j=1,2, \ldots, m$ and $k=1,2, \ldots K$, is defined as follows:

$$
Y_{j k}=\left\{\begin{array}{lc}
1 & \text { if a BS is constructed at } C S_{j} \text { with a frequency } k \\
0 & \text { otherwise }
\end{array}\right.
$$

b. The decision variable, $X_{i j k}, i=1,2, \ldots, n, j \in S(i)$ and $k=1,2 \ldots K$, is defined as follows:

$$
X_{i j k}= \begin{cases}1 & \text { if a } B S \text { at } C S_{j} \text { has the strongest signal at } D P_{i} \text { with frequency } k \\ 0 & \text { otherwise }\end{cases}
$$

## $\underline{2-T h e ~ o b j e c t i v e ~ f u n c t i o n ~ f o r ~ M o d e l ~ I I ~}$

The function to be optimized is the total cost of the network. The objective function can be described as:

$$
\operatorname{Minimize} \sum_{j=1}^{m} \sum_{k=1}^{l} C_{j} Y_{j k}
$$

where $C_{j}$ is the cost of installing a base station at $C S_{j}$.

## 3 - The constraints for Model II

The problem has six types of constraints that bound the feasible region of the solution.
These are the following:
a. Each demand point should be served by at least one base station. This set of constraints can be represented by:

$$
\sum_{j \in S(i)} \sum_{k=1}^{l} Y_{j k} \geq 1, \quad i=1,2, \ldots, n
$$

b. Each base station should be allocated at most one frequency. So this set of constraints will ensure that only one frequency is assigned to each $B S$ :

$$
\sum_{k=1}^{l} Y_{j k} \leq 1, \quad j=1,2, \ldots, m
$$

c. Each demand point should be assigned to exactly one base station; hence this set of constraint can be written as:

$$
\sum_{j \in S(i)} \sum_{k=1}^{l} X_{i j k}=1, \quad i=1,2, \ldots, n
$$

d. A candidate site $C S_{j}$ is assigned to a demand point $D P_{i}$ if it is selected to construct a base station at it and it can be represented by this set of constraints:

$$
\begin{gather*}
Y_{j k} \geq X_{i j k}, \quad i=1,2, \ldots, n, j=1,2, \ldots, m \& \\
k=1,2, \ldots K
\end{gather*}
$$

e. Each frequency at each base station has a capacity of $Q$ channels, so the numbers of demand points assigned to each base station must not exceed its limit of channels. This set of constraints can be written as follow:

$$
\sum_{i=1}^{n} X_{i j k} \leq Q, \quad j=1,2, \ldots, m \& k=1,2, \ldots K
$$

f. The quality of service constraints by which the ratio of the strongest signal received at each $D P_{i}$ to the received noise and signals from other base stations
should be greater than a minimum requirement of signal-to-interference-plusnoise ratio, SINR. This set of constraints can be given as follow:

$$
\begin{array}{r}
\frac{S P(i)}{P_{N_{i}}+T P(i)-S P(i)} \geq 10^{\frac{S I N R}{10}}, \quad i=1,2, \ldots, n \& \\
\\
k=1,2, \ldots, K
\end{array}
$$

where:
$S P(i)$ is the strongest power at test point $D P_{i}$ and is given by:

$$
S P(i)=\sum_{j \in S(i)} \mathrm{X}_{i j k} P_{i j}
$$

where $P_{i j}$ is the received power at $D P_{i}$ from a BS at $C S_{j}$
$T P(i)$ is the total power received at $D P_{i}$ which is generated by all base stations at candidate sites that can serve $D P_{i}$ and is given by:

$$
T P(i)=\sum_{j \in S(i)} Y_{j k} P_{i j}
$$

where $P_{i j}$ is the received power at $D P_{i}$ from a BS at $C S_{j}$
$P_{N_{i}} \quad$ is the noise power at $D P_{i}$
SINR is minimum signal-to-interference-plus-noise ratio

The complete MIP model for the integrated problem of base stations location and frequency assignment can be summarized as shown in Figure 3-4.

| Minimize | $\sum_{j=1}^{m} \sum_{k=1}^{l} C_{j} Y_{j k}$ |
| :--- | :--- |
| Subject to | $\sum_{j \in S(i)} \sum_{k=1}^{l} Y_{j k} \geq 1$ |
|  | $\sum_{k=1}^{l} Y_{j k} \leq 1$ |
|  | $\sum_{j \in S(i)} \sum_{k=1}^{l} X_{i j k}=1$ |
|  | $Y_{j k} \geq X_{i j k}$ |
|  | $\sum_{i=1}^{n} X_{i j k} \leq Q$ |
|  | $X, Y \in[0,1]$ |
|  |  |

Figure 3-4 MIP Model II - Base Stations Location and Frequency Assignment

## Problems

### 3.2.2 MODEL II: NUMERICAL EXPERIMENT

The same case in section 3.1.2 for Model I is considered but with more demand points (DP), 265 demand points, to show the effectiveness of the frequency reuse concept,
where each demand point represent multi users and same load of users considered for all demand points. A set of optimal location for base stations (BS) has to be selected from 100 candidate sites (CS). The demand points and candidate sites are shown in Figure 3-5.


Figure 3-5 Demand Points and Candidate Sites Considered for Model II

Parameters for the COST-WI propagation model are still same as in Table 3-1. The other parameters used in the numerical experiments such as transmitted power, gains, receiver sensitivity, and base stations capacity are shown in Table 3-3. We consider a wireless operator who has 60 radio channels available and they are distributed equally into two different frequencies where each frequency has 30 completely different channels.

Table 3-3 Parameters used in Numerical Experiment of Model II

| Parameter | Value |
| :--- | :--- |
| Transmitted power | 25 dBm |
| Transmitted antenna gain | 8 dBi |
| Received antenna gain | 2 dBi |
| Minimum power requirement | -95 dBm |
| Available frequencies | 1,2 |
| Base Station Capacity | 30 channels |
| SINR | 20 dB |

The MIP for base stations location and frequency assignment problems is solved using an optimization modeling software, LINGO, LINDO Systems Inc. The optimal solution resulted into 13 base stations as shown in Figure 3-6. The selected base stations along with their allocated frequencies are shown in Table 3-4.


Figure 3-6 Results of Model II

Table 3-4 Selected BS and their Allocated Frequencies for Model II

| BS \# | Location |  | Allocated <br> Frequency |
| :---: | :---: | :---: | :---: |
|  | X | Y |  |
| 1 | 0.5 | 3.5 | 1 |
| 2 | 0.5 | 10.5 | 2 |
| 3 | 1.5 | 9.5 | 2 |
| 4 | 2.5 | 1.5 | 1 |
| 5 | 2.5 | 6.5 | 1 |
| 6 | 5.5 | 0.5 | 2 |
| 7 | 5.5 | 5.5 | 2 |
| 8 | 6.5 | 1.5 | 1 |
| 9 | 6.5 | 4.5 | 1 |
| 10 | 6.5 | 8.5 | 1 |
| 11 | 9.5 | 1.5 | 1 |
| 12 | 9.5 | 9.5 | 1 |
| 13 | 10.5 | 5.5 |  |

### 3.2.3 MODEL II: DISCUSSION

As noticed above, although the capacity of each base station is up to 30 demand points, this MIP model recommends 13 base stations to cover all the demand points. This number of recommended base stations resulted because of low height and transmitted power of the transmitter which are considered to satisfy the quality of service (i.e. SINR) constraint. Compared with Model I, frequency assignment adds more flexibility to the model which is indeed demonstrated by covering more demand points as the interference between the base stations is reduced by using different frequencies.

### 3.3 MODEL III: BASE STATIONS LOCATION AND CONFIGURATION PROBLEMS

In this model, we solve the problems of base stations location and configuration where the configuration of antennas in each base station involves azimuth, tilt, height, and transmitted power. The objective of this model is to minimize the total cost of the network taking into account the constraints of area coverage, capacity of base station, and quality of service requirements for each user.

### 3.3.1 MODEL III: MODEL FORMULATION

We assume that the demand points and candidate sites are known. Denote the $i$ th demand point by $D P_{i}, i=1,2, \ldots, n$ and the $j$ th candidate site by $C S_{j}, j=1,2, \ldots, m$. We will assume that a mast carries $l$ directional antennas where $l=1,2, \ldots, N$, i.e. $N$ is either three with $120^{\circ}$ for each sector or six with $60^{\circ}$ for each sector. In this thesis, we consider $N=3$, i.e. each base station has at most 3 directional antennas. An antenna has an azimuth angle, $A$ where $0 \leq A \leq 359$ and a tilt angle, $T \in\left[-15^{\circ}, 0\right]$. Let $P$ denote the power of an antenna, $P_{\min } \leq P \leq P_{\max }$ and $H$ denote the height of an antenna, $H_{\min } \leq H \leq H_{\max }$. Let $S(i)$ be the set of candidate sites that can serve test point $T P_{i}$ by one of its antennas at a given azimuth and tilt angles, i.e.

$$
S(i)=\left\{\begin{array}{c}
(j, l, A, T, H, P) \mid j \in S, l=1,2 \text { or } 3,0 \leq A \leq 359,-15 \leq T \leq 0, \\
H_{\min } \leq H \leq H_{\max }, P_{\min } \leq P \leq P_{\max } \\
\text { such that the power received at } D P_{i} \geq \gamma
\end{array}\right\}
$$

where S is the set of candidate sites and $\gamma$ is threshold of minimum power.

The Mixed Integer Programming model for the base stations location and configuration problems can be described as follow:

## 1 - The decision variables for Model III

a. The decision variable, $Y_{j}, j=1,2, \ldots, m$, is defined as follows:

$$
Y_{j}=\left\{\begin{array}{lc}
1 & \text { if a BS is constructed at } C S_{j} \\
0 & \text { otherwise }
\end{array}\right.
$$

b. The decision variable, $X_{i j l A T H P}, i=1,2, \ldots, n, j \in S(i), l=1,2,3,0 \leq A \leq$ 359, $-15 \leq T \leq 0, H_{\min } \leq H \leq H_{\max }$, and $P_{\min } \leq P \leq P_{\max }$ is defined as follows:

$$
X_{i j l A T H P}=\left\{\begin{array}{l}
1 \quad \text { if } B S \text { at } C S_{j} \text { with antenna l, azimuth } A, \text { tilt } T, \text { height } H, \\
0 \quad \text { and power } P, \text { has the strongest signal at } D P_{i} \\
0 \quad \text { otherwise }
\end{array}\right.
$$

c. The decision variable, $W_{\text {jlATHP }}, j \in S(i), l=1,2,3,0 \leq A \leq 359,-15 \leq T \leq$ $0, H_{\min } \leq H \leq H_{\max }$, and $P_{\min } \leq P \leq P_{\max }$, is defined as follows:

$$
W_{\text {jlATHP }}=\left\{\begin{array}{lc}
1 & \text { if at } C S_{j}, \text { antenna l has azimuth } A, \text { tilt } T, \text { height } H \\
0 & \text { otherwise }
\end{array}\right.
$$

Note that the difference of azimuth angles of the three antennas at any mast is $120^{\circ}$. Hence:

$$
W_{j, 1, A, T, H, P}=W_{j, 2, \bmod (A+120,360), T, H, P}=W_{j, 3, \bmod (A+240,360), T, H, P}
$$

d. The decision variable, $Z_{j A}, j \in S(i)$ and $0 \leq A \leq 359$, is defined as follows:

$$
Z_{j A}= \begin{cases}1 & \text { if at } C S_{j}, \text { a BS has azimuth } A \\ 0 & \text { otherwise }\end{cases}
$$

## $\underline{2-T h e ~ o b j e c t i v e ~ f u n c t i o n ~ f o r ~ M o d e l ~ I I I ~}$

The function to be optimized is the total cost of the network. The objective function can be described as:

$$
\begin{align*}
& \operatorname{Minimize} \sum_{j=1}^{n} C_{j} Y_{j} \\
& +\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l A T H P} C P(P)
\end{align*}
$$

where:
$C_{j} \quad$ is the cost of installing a base station at $C S_{j}$
$C P(P) \quad$ is the cost of having an antenna with power $P$ which might not be a linear function.

## 3 - The constraints for Model III

The problem has seven types of constraints that bound the feasible region of the solution. These are the following:
a. Each antenna, if chosen, at any base station has only one value of azimuth, tilt, height, and power. So this set of constraints can be written as:

$$
\begin{array}{r}
\sum_{A=0}^{359} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l A T H P} \leq Y_{j}, \quad j=1,2 \ldots, m \\
\text { and } l=1,2,3
\end{array}
$$

b. Each base station at any location has only one azimuth and this condition can be represented by the following two sets of constraints:

$$
\begin{gather*}
W_{j l A T H P} \leq Z_{j A}, \quad j \in S(i), l=1,2,3,0 \leq A \leq 359,-15 \leq T \\
\leq 0, H_{\min } \leq H \leq H_{\max }, \text { and } P_{\min } \leq P \leq P_{\max } \\
\sum_{A=0}^{359} Z_{j A} \leq 1, \quad j=1,2, \ldots, m
\end{gather*}
$$

c. Each demand point should be served by at least one base station. This set of constraints can be represented by:

$$
\begin{gather*}
\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l A T H P} \geq 1 \\
i=1,2, \ldots, n
\end{gather*}
$$

d. Each demand point should be assigned to exactly one base station; hence this set of constraint can be written as:

$$
\begin{gather*}
\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} X_{i j l A T H P}=1, i \\
=1,2, \ldots, n
\end{gather*}
$$

e. A candidate site $C S_{j}$ is assigned to a demand point $D P_{i}$ if it is selected to construct a base station which has an antenna $l$ with azimuth $A$, tilt $T$, height $H$, and power $P$. This set of constraints can be represented by :

$$
\begin{gather*}
W_{j l A T H P} \geq X_{i j l A T H P}, \quad i=1,2, \ldots, n, j \in S(i), l=1,2,3 \\
0 \leq A \leq 359,-15 \leq T \leq 0, H_{\min } \leq H \\
\leq H_{\max }, \text { and } P_{\min } \leq P \leq P_{\max }
\end{gather*}
$$

f. Each base station has a capacity of $Q$ channels, so the numbers of demand points assigned to each base station must not exceed its limit of channels. This set of constraints can be written as follow:

$$
\begin{align*}
& \sum_{i=1}^{n} \sum_{A=0}^{359} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} X_{i j l A T H P} \leq Q \\
& \quad j \in S(i), \text { and } l=1,2,3
\end{align*}
$$

g. The quality of service constraints by which the ratio of the strongest signal received at each $D P_{i}$ to the received noise and signals from other base stations should be greater than a minimum requirement of signal-to-interference-plusnoise ratio, $S I N R$. This set of constraints can be given as follow:

$$
\frac{S P(i)}{P_{N_{i}}+T P(i)-S P(i)} \geq 10^{\frac{S I N R}{10}}, \quad i=1,2, \ldots, n
$$

where:
$S P(i)$ is the strongest power at test point $D P_{i}$ and is given by:
$S P(i)=\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} X_{i j l A T H P} P_{i j l A T H P}$
where $P$ is the received power at $D P_{i}$.
$T P(i)$ is the total power received at $D P_{i}$ which is generated by all base stations at candidate sites that can serve $D P_{i}$ and is given by:
$T P(i)=\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l A T H P} P_{i j l A T H P}$
where $P$ is the received power at $D P_{i}$.
$P_{N_{i}} \quad$ is the noise power at $D P_{i}$
SINR is minimum signal-to-interference-plus-noise ratio

The complete MIP model for the problem of base stations location and configuration can be summarized as shown in Figure 3-7.

$$
\begin{aligned}
\text { Minimize } & \sum_{j=1}^{m} C_{j} Y_{j} \\
& +\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{A=0}^{360} \sum_{\tau=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l A T H P} C P(P)
\end{aligned}
$$

Subject to

$$
\begin{aligned}
& \sum_{A=0}^{359} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l A T H P} \leq Y_{j} \\
& W_{j l A T H P} \leq Z_{j A} \\
& \sum_{A=0}^{359} Z_{j A} \leq 1 \\
& \sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l A T H P} \geq 1 \\
& \sum_{j e s(i)} \sum_{l=1}^{3} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} X_{i j l A T H P}=1 \\
& W_{j l A T H P} \geq X_{i j l A T H P} \\
& \sum_{i=1}^{n} \sum_{A=0}^{359} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} X_{i j l A T H P} \leq Q \\
& \frac{S P(i)}{P_{N_{i}}+T P(i)-S P(i)} \geq 10^{\frac{S I N R}{10}} \\
& X, Y, W, Z \in[0,1]
\end{aligned}
$$

Figure 3-7 MIP model III - Base Stations Location and Configuration Problems

### 3.3.2 MODEL III: NUMERICAL EXPERIMENT

The same case in section 3.1.2 for Model I, is considered with the same locations of demand points and candidate sites. The parameters used for COST-WI propagation model to calculate the path loss are the same as in Table 3-1 except for the height of the transmitter as it has two values and is shown in Table 3-4. The other parameters used in the numerical experiments such as transmitted power, gains, receiver sensitivity, and base stations capacity are shown in Table 3-4. Tilt is not considered. The noise power is assumed to be negligible.

Table 3-5 Parameters used in Numerical Experiment of Model III

| Parameter | Value 1 | Value 2 |
| :---: | :---: | :---: |
| Transmitted power | 20 dBm | 25 dBm |
| Transmitted antenna gain | 8 dBi |  |
| Received antenna gain | 2 dBi |  |
| Minimum power requirement | -95 dBm |  |
| Height of transmitter | 20 m | 25 m |
| Available directional antennas | 1,2,3 |  |
| Antenna azimuth | $0^{\circ}$ | $60^{\circ}$ |
| Available frequencies | 1 |  |
| Base station capacity | 30 channels |  |
| Antenna capacity | 10 channels |  |
| SINR | 20 dB |  |

The MIP for base stations location and configuration problems is solved using an optimization modeling software, LINGO, LINDO Systems Inc. The optimal solution resulted into 9 base stations as shown in Figure 3-8. The location and configuration of each selcted base staion are shown in Table 3-5.


Figure 3-8 Results of Model III

Table 3-6 Base Stations Location and Configuration for Model III

| BS \# | LOCATION |  | AZIMUTH | ANTENNA | HEIGHT | POWER |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X | Y |  |  |  |  |
| 1 | 0.5 | 3 | 1 | 1 | 2 | 1 |
|  |  |  |  | 2 | 2 | 1 |
| 2 | 1.5 | 9 | 1 | 1 | 2 | 1 |
|  |  |  |  | 2 | 2 | 1 |
|  |  |  |  | 3 | 2 | 1 |
| 3 | 4.5 | 2 | 2 | 1 | 2 | 1 |
|  |  |  |  | 2 | 2 | 1 |
| 4 | 5 | 7.5 | 2 | 1 | 2 | 1 |
|  |  |  |  | 2 | 2 | 1 |
| 5 | 6 | 2 | 2 | 1 | 2 | 1 |
| 6 | 6 | 4.5 | 1 | 1 | 2 | 1 |
|  |  |  |  | 2 | 2 | 1 |
|  |  |  |  | 3 | 2 | 1 |
| 7 | 8 | 10 | 2 | 1 | 2 | 1 |
|  |  |  |  | 2 | 2 | 1 |
| 8 | 9 | 7.5 | 1 | 1 | 1 | 1 |
|  |  |  |  | 2 | 2 | 1 |
| 9 | 10 | 1.5 | 2 | 2 | 2 | 1 |
|  |  |  |  | 3 | 1 | 1 |

### 3.3.3 MODEL III: DISCUSSION

As observed from the results above, this MIP model recommends 9 base stations to cover all the demand points even though the capacity of each base station is 30 channels. However, we should note that most of the base stations have either one antenna or two. The average number of antennas is less than two per base station. This
number of recommended base stations resulted because of low height and transmitted power of the transmitter which are considered to satisfy the quality of service (i.e. SINR) constraint. As expected, when compared to Model I, incorporating the configuration of base stations adds more flexibility to the model resulting in all the demand points with less number of base stations. Moreover, the recommendation shows that different configuration is assigned to different base stations to reduce the interference between them and also not all sectors at each base station are working.

### 3.4 MODEL IV: LOCATION AND CONFIGURATION OF BASE STATIONS AND FREQUENCY ASSIGNMENT PROBLEMS

In this model, we solve the integrated problem of location and configuration of base stations and frequency assignment. The configuration of antennas in each base station involves azimuth, tilt, height, and transmitted power. The objective of this model is to minimize the total cost of the network taking into account the constraints of area coverage, capacity of base station, and quality of service requirements for each user. This means that the problem is to find the minimum number of base stations with optimal configuration and frequency assignment which could achieve the objective of the model while satisfying all constraints.

### 3.4.1 MODEL IV: MODEL FORMULATION

We assume that the demand points and candidate sites are known. Denote the $i$ th demand point by $D P_{i}, i=1,2, \ldots, n$ and the $j$ th candidate site by $C S_{j}, j=1,2, \ldots, m$. There is also, a set of available frequencies, $k$, to be assigned to base stations, $k=1,2, \ldots, K$. We will assume that a mast carries $l$ directional antennas where $l=1,2, \ldots, N$, i.e. $N$ is either three with $120^{\circ}$ for each sector or six with $60^{\circ}$ for each sector. In this thesis, we consider that each base station has 3 directional antennas, i.e. $N$ is three antennas. An antenna has an azimuth angle, $A$ where $0 \leq A \leq 359$ and a tilt angle, $T \in\left[-15^{\circ}, 0\right]$. Let $P$ denote the power of an antenna, $P_{\min } \leq P \leq P_{\max }$ and $H$ denote the height of an antenna, $H_{\min } \leq H \leq H_{\max }$. Let $S(i)$ be the set of candidate sites that can serve test point $T P_{i}$ by one of its antennas at a given azimuth and tilt angles, i.e.

$$
S(i)=\left\{\begin{array}{c}
(j, l, A, T, H, P) \mid j \in S, l=1,2 \text { or } 3,0 \leq A \leq 359,-15 \leq T \leq 0, \\
H_{\min } \leq H \leq H_{\max }, P_{\min } \leq P \leq P_{\max } \\
\text { such that the power received at } D P_{i} \geq \gamma
\end{array}\right\}
$$

where S is the set of candidate sites and $\gamma$ is threshold of minimum power.

The Mixed Integer Programming model for the location and configuration of base stations and frequency assignment problems can be described as follow:

## 1 - The decision variables for Model IV

a. The decision variable, $Y_{j}, j=1,2, \ldots, m$, is defined as follows:

$$
Y_{j}=\left\{\begin{array}{lc}
1 & \text { if a BS is constructed at } C S_{j} \\
0 & \text { otherwise }
\end{array}\right.
$$

b. The decision variable, $X_{i j l k a t h P}, i=1,2, \ldots, n, \quad j \in S(i), l=1,2,3, k=$ $1,2, \ldots, K, 0 \leq A \leq 359,-15 \leq T \leq 0, H_{\min } \leq H \leq H_{\max }$, and $P_{\min } \leq P \leq$ $P_{\max }$ is defined as follows:

$$
X_{i j l k A T H P}=\left\{\begin{array}{cc}
1 & \text { if } B S \text { at } C S_{j} \text { with antenna } l, \text { frequency } k, \text { azimuth } A \\
& \text { tilt } T, \text { height } H, \text { and power } P, \text { has the strongest } \\
& \text { signal at } D P_{i}
\end{array}\right.
$$

c. The decision variable, $W_{j l k a t h P}, j \in S(i), l=1,2,3, k=1,2, \ldots, K, 0 \leq A \leq$ 359, $-15 \leq T \leq 0, H_{\min } \leq H \leq H_{\max }$, and $P_{\min } \leq P \leq P_{\max }$, is defined as follows:

$$
W_{\text {jlkATHP }}=\left\{\begin{array}{lc}
1 & \text { if at } C S_{j}, \text { antenna } l \text { with frequency } k \text { has azimuth } A \\
& \text { tilt } T, \text { height } H, \text { and power } P \\
0 & \text { otherwise }
\end{array}\right.
$$

Note that the difference of azimuth angles of the three antennas at any mast is $120^{\circ}$. Hence:

$$
W_{j, 1, k, A, T, H, P}=W_{j, 2, k, \bmod (A+120,360), T, H, P}=W_{j, 3, k, \bmod (A+240,360), T, H, P}
$$

d. The decision variable, $Z_{j k A}, j \in S(i), k=1,2, \ldots, K$, and $0 \leq A \leq 359$, is defined as follows:

$$
Z_{j k A}= \begin{cases}1 & \text { if at } C S_{j}, \text { a } B S \text { has a frequency } k \text { and azimuth } A \\ 0 & \text { otherwise }\end{cases}
$$

## $\underline{2-T h e ~ o b j e c t i v e ~ f u n c t i o n ~ f o r ~ M o d e l ~ I V ~}$

The function to be optimized is the total cost of the network. The objective function can be described as:

$$
\begin{align*}
& \operatorname{Minimize} \sum_{j=1}^{n} C_{j} Y_{j} \\
& +\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{k=1}^{K} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l k A T H P} C P(P)
\end{align*}
$$

where:
$C_{j} \quad$ is the cost of installing a base station at $C S_{j}$
$C P(P) \quad$ is the cost of having an antenna with power $P$ which might not be a linear function.

## 3 - The constraints for Model IV

The problem has seven types of constraints that bound the feasible region of the solution. These are the following:
a. Each antenna, if chosen, at any base station has only one value of frequency, azimuth, tilt, height, and power. So this set of constraints can be written as:

$$
\begin{array}{r}
\sum_{k=1}^{K} \sum_{A=0}^{359} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l k A T H P} \leq Y_{j}, \quad j=1,2 \ldots, m \\
\text { and } l=1,2,3
\end{array}
$$

b. Each base station at any location is allocated with only one frequency has only one azimuth too. This condition can be represented by the following two sets of constraints:

$$
\begin{align*}
& W_{j l k A T H P} \leq Z_{j k A}, \quad j \in S(i), l=1,2,3, k=1,2, \ldots, K, 0 \leq A \\
& \quad \leq 359,-15 \leq T \leq 0, H_{\min } \leq H \leq H_{\max }, \text { and } P_{\min } \\
& \leq P \leq P_{\max } \\
& \sum_{k=1}^{K} \sum_{A=0}^{359} Z_{j A} \leq 1, \quad j=1,2, \ldots, m
\end{align*}
$$

c. Each demand point should be served by at least one base station. This set of constraints can be represented by:

$$
\begin{gather*}
\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{k=1}^{K} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l k A T H P} \geq 1 \\
i=1,2, \ldots, n
\end{gather*}
$$

d. Each demand point should be assigned to exactly one base station; hence this set of constraint can be written as:
$\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{k=1}^{K} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} X_{i j l k A T H P}=1$,

$$
i=1,2, \ldots, n
$$

e. A candidate site $C S_{j}$ is assigned to a demand point $D P_{i}$ if it is selected to construct a base station which has an antenna $l$ with frequency $k$, azimuth $A$, tilt $T$, height $H$, and power $P$. This set of constraints can be represented by :

$$
\begin{gather*}
W_{j l k A T H P} \geq X_{i j l k A T H P}, \quad i=1,2, \ldots, n, j \in S(i), l=1,2,3 \\
k=1,2, \ldots, K, \quad 0 \leq A \leq 359,-15 \leq T \leq 0, H_{\min } \leq H \\
\leq H_{\max }, \text { and } P_{\min } \leq P \leq P_{\max }
\end{gather*}
$$

f. Each antenna at each base station has a capacity of $Q$ channels, so the numbers of demand points assigned to each base station must not exceed its limit of channels. This set of constraints can be written as follow:

$$
\begin{align*}
& \sum_{i=1}^{n} \sum_{k=1}^{K} \sum_{A=0}^{359} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} X_{i j l k A T H P} \leq Q \\
& \quad j \in S(i), \text { and } l=1,2,3
\end{align*}
$$

g. The quality of service constraints by which the ratio of the strongest signal received at each $D P_{i}$ to the received noise and signals from other base stations
should be greater than a minimum requirement of signal-to-interference-plusnoise ratio, SINR. This set of constraints can be given as follow:

$$
\frac{S P(i)}{P_{N_{i}}+T P(i)-S P(i)} \geq 10^{\frac{S I N R}{10}}, \quad i=1,2, \ldots, n
$$

where:
$S P(i)$ is the strongest power at test point $D P_{i}$ and is given by:
$S P(i)=\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{k=1}^{K} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} X_{i j l k A T H P} P_{i j l k A T H P}$
where $P$ is the received power at $D P_{i}$.
$T P(i)$ is the total power received at $D P_{i}$ which is generated by all base stations at candidate sites that can serve $D P_{i}$ and is given by:
$T P(i)=\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{k=1}^{K} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l k A T H P} P_{i j l k A T H P}$
where $P$ is the received power at $D P_{i}$.
$P_{N_{i}} \quad$ is the noise power at $D P_{i}$
$\operatorname{SINR}$ is minimum signal-to-interference-plus-noise ratio

The complete MIP model for the location and configuration of base stations and frequency assignment problems can be summarized as shown in Figure 3-9.

$$
\begin{aligned}
& \begin{array}{l}
\text { Minimize } \sum_{j=1}^{m} C_{j} Y_{j} \\
+\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{k=1}^{K} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l k A T H P} C P(P)
\end{array} \\
& \text { Subject to } \\
& \sum_{k=1}^{K} \sum_{A=0}^{359} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l k A T H P} \leq Y_{j} \\
& W_{j l k A T H P} \leq Z_{j k A} \\
& \sum_{A=0}^{359} Z_{j k A} \leq 1 \\
& \sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{k=1}^{K} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l k A T H P} \geq 1 \\
& \sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{k=1}^{K} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} X_{i j l k A T H P}=1 \\
& W_{j l k A T H P} \geq X_{i j l k A T H P} \\
& \sum_{i=1}^{n} \sum_{k=1}^{K} \sum_{A=0}^{359} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} X_{i j l k A T H P} \leq Q \\
& \frac{S P(i)}{P_{N_{i}}+T P(i)-S P(i)} \geq 10^{\frac{S I N R}{10}} \\
& X, Y, W, Z \in[0,1]
\end{aligned}
$$

Figure 3-9 MIP Model IV - Location and Configuration of Base Stations and

### 3.4.2 MODEL IV: NUMERICAL EXPERIMENT

The same case in Section 3.2.2 for Model II is considered with the same locations of demand points and candidate sites. Parameters used for COST-WI propagation model to calculate the path loss are same as in Table 3-1 except the height of transmitter as it has two values and is shown in Table 3-7. The other parameters used in the numerical experiments such as transmitted power, gains, receiver sensitivity, and base stations capacity are shown in Table 3-7. Tilt is not considered. The noise power is assumed to be negligible.

Table 3-7 Parameters used in Numerical Experiment of Model IV

| Parameter | Value 1 | Value 2 |
| :--- | :---: | :---: |
| Transmitted power | 20 dBm | 25 dBm |
| Transmitted antenna gain | 8 dBi |  |
| Received antenna gain | 2 dBi |  |
| Minimum power requirement | $-95 \mathrm{dBm}$ |  |
| Height of transmitter | $1,2,3$ |  |
| Available directional antennas | 20 | 25 m |
| Antenna azimuth | 1,2 |  |
| Available frequencies | 30 channels |  |
| Base station capacity | 10 channels |  |
| Antenna capacity | 20 dB |  |
| SINR |  |  |

The MIP for location and configuration of base stations and frequency assignment problems is solved using an optimization modeling software, LINGO, LINDO Systems Inc. The optimal solution resulted into 13 base stations as shown in figure 3-10. the location and configiuration of each selected base stataions along with the allocated frequencies are shown in Table 3-7.


Figure 3-10 Results of Model IV

Table 3-8 Location and Configuration of Base Stations and their Allocated
Frequencies for Model IV

| BS \# | Location |  | Azimuth | Antenna | Height | Power | Frequency |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X | Y |  |  |  |  |  |
| 1 | 0.5 | 8.5 | 2 | 1 | 1 | 1 | 1 |
|  |  |  |  | 2 | 2 | 1 |  |
|  |  |  |  | 3 | 2 | 1 |  |
| 2 | 1.5 | 2.5 | 2 | 1 | 1 | 1 | 1 |
|  |  |  |  | 2 | 2 | 1 |  |
|  |  |  |  | 3 | 1 | 1 |  |
| 3 | 1.5 | 4.5 | 2 | 3 | 1 | 1 | 1 |
| 4 | 1.5 | 9.5 | 1 | 1 | 1 | 1 | 1 |
|  |  |  |  | 2 | 1 | 1 |  |
|  |  |  |  | 3 | 1 | 1 |  |
| 5 | 2.5 | 6.5 | 2 | 1 | 1 | 1 | 2 |
|  |  |  |  | 2 | 1 | 2 |  |
|  |  |  |  | 3 | 2 | 1 |  |
| 6 | 4.5 | 2.5 | 2 | 1 | 2 | 1 | 1 |
|  |  |  |  | 2 | 2 | 1 |  |
|  |  |  |  | 3 | 2 | 1 |  |
| 7 | 4.5 | 3.5 | 2 | 2 | 2 | 1 | 2 |
|  |  |  |  | 3 | 2 | 1 |  |
| 8 | 5.5 | 6.5 | 2 | 2 | 2 | 1 | 1 |
|  |  |  |  | 3 | 2 | 1 |  |
| 9 | 6.5 | 3.5 | 2 | 1 | 1 | 1 | 1 |
|  |  |  |  | 2 | 2 | 1 |  |
|  |  |  |  | 3 | 1 | 1 |  |
| 10 | 7.5 | 1.5 | 2 | 1 | 2 | 1 | 2 |
|  |  |  |  | 2 | 1 | 1 |  |
|  |  |  |  | 3 | 2 | 1 |  |
| 11 | 7.5 | 9.5 | 2 | 2 | 2 | 1 | 1 |
|  |  |  |  | 3 | 1 | 1 |  |
| 12 | 8.5 | 2.5 | 2 | 1 | 2 | 1 | 2 |
|  |  |  |  | 2 | 2 | 1 |  |
|  |  |  |  | 3 | 1 | 1 |  |
| 13 | 9.5 | 7.5 | 2 | 2 | 2 | 1 | 1 |
|  |  |  |  | 3 | 2 | 1 |  |

### 3.4.3 MODEL IV: DISCUSSION

As observed from the results above, this MIP recommends 13 base stations to cover all the demand points. The recommendation shows that different configuration and frequency is assigned to different base stations to reduce the interference between them and also not all sectors of each base station are working. Optimizing the configuration of each base station adds more flexibility to the model i.e. different azimuth, transmitted powers, and heights could be used. Also, number of operational sectors can be decided too. However, the limited capacity of each antenna has complicated the model and resulted in 13 base stations to cover the service area which is the same result of Model II but with different configuration of each base station. Finally, it can be conclulded that considering the antennas configuration of each base station and the frequency assignment will add more flexibility to the model and could help in reducing the interference between the base stations.

## Chapter 4

## Mixed Integer Programs - Special Cases

### 4.1 MODEL I: BASE STATIONS CONFIGURATION PROBLEM

In this model, which is a special case of Model III in Chapter 3, we solve the problem of base stations configuration using some existing base stations. The configuration of antennas in each base station involves azimuth, tilt, height, and transmitted power. The objective of this model is to minimize the total cost of the network taking into account the constraints of area coverage, capacity of base station, and quality of service requirements for each user. This means that the problem is to find the optimal configuration of the existing base stations, i.e. minimum number of antennas and transmitted power, which could achieve the objective of the model while satisfying all constraints.

### 4.1.1 MODEL I: MODEL FORMULATION

We assume that the demand points are known so we denote the $i$ th demand point by $D P_{i}$, $i=1,2, \ldots, n$. In this model, there are some exist base stations at known locations, so the $j$ th base station is located at location, $j=1,2, \ldots, m$. We will assume that a mast carries $l$ directional antennas where $l=1,2, \ldots, N$, i.e. $N$ is either three with $120^{\circ}$ for each sector or six with $60^{\circ}$ for each sector. In this thesis, we consider that each base station has 3 directional antennas, i.e. $N$ is three antennas. An antenna has an azimuth angle, $A$ where $0 \leq A \leq 359$ and a tilt angle, $T \in\left[-15^{\circ}, 0\right]$. Let $P$ denote the power of an antenna, $P_{\min } \leq P \leq P_{\max }$ and $H$ denote the height of an antenna, $H_{\min } \leq H \leq H_{\max }$. Let $S(i)$ be the set of candidate sites that can serve test point $T P_{i}$ by one of its antennas at a given azimuth and tilt angles, i.e.

$$
S(i)=\left\{\begin{array}{c}
(j, l, A, T, H, P) \mid j \in S, l=1,2 \text { or } 3,0 \leq A \leq 359,-15 \leq T \leq 0, \\
H_{\min } \leq H \leq H_{\max }, P_{\min } \leq P \leq P_{\max } \\
\text { such that the power received at } D P_{i} \geq \gamma
\end{array}\right\}
$$

where S is the set of candidate sites and $\gamma$ is threshold of minimum power.

The Mixed Integer Programming model for the base stations configuration problem can be described as follow:

## 1 - The decision variables for Model III

a. The decision variable, $Y_{j}, j=1,2, \ldots, m$, is defined as follows:

$$
Y_{j}=\left\{\begin{array}{lc}
1 & \text { if a BS is constructed at } C S_{j} \\
0 & \text { otherwise }
\end{array}\right.
$$

b. The decision variable, $X_{i j l A T H P}, i=1,2, \ldots, n, j \in S(i), l=1,2,3,0 \leq A \leq$ 359, $-15 \leq T \leq 0, H_{\min } \leq H \leq H_{\max }$, and $P_{\min } \leq P \leq P_{\max }$ is defined as follows:

$$
X_{i j l A T H P}=\left\{\begin{array}{l}
1 \quad \text { if } B S \text { at } C S_{j} \text { with antenna l, azimuth } A, \text { tilt } T, \text { height } H, \\
\text { and power } P, \text { has the strongest signal at } D P_{i} \\
0 \quad \text { otherwise }
\end{array}\right.
$$

c. The decision variable, $W_{j l A T H P}, j \in S(i), l=1,2,3,0 \leq A \leq 359,-15 \leq T \leq$ $0, H_{\min } \leq H \leq H_{\max }$, and $P_{\min } \leq P \leq P_{\max }$, is defined as follows:

$$
W_{\text {jlATHP }}=\left\{\begin{array}{lc}
1 & \text { if a BS has an antenna l with azimuth } A, \text { tilt } T, \text { height } H, \\
0 & \text { otherwise }
\end{array}\right.
$$

Note that the difference of azimuth angles of the three antennas at any mast is $120^{\circ}$. Hence:

$$
W_{j, 1, A, T, H, P}=W_{j, 2, \bmod (A+120,360), T, H, P}=W_{j, 3, \bmod (A+240,360), T, H, P}
$$

d. The decision variable, $Z_{j A}, j \in S(i)$ and $0 \leq A \leq 359$, is defined as follows:

$$
Z_{j A}=\left\{\begin{array}{lc}
1 & \text { if a BS has azimuth } A \\
0 & \text { otherwise }
\end{array}\right.
$$

## 2 - The objective function for Model III

The function to be optimized is the total cost of the network. In other words, it means, considering the base stations are already exist, the objective of this model is to minimize the transmitted power which could lead to minimizing the number of antennas used and at each existed base station. The objective function can be described as:

$$
\text { Minimize } \sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l A T H P} C P(P)
$$

where: is the cost of having an antenna with power $P$ which might not be a linear function.

## 3 - The constraints for Model III

The problem has eight types of constraints that bound the feasible region of the solution. These are the following:
a. As the locations of base stations in this model are already known, i.e. there is a constructed $B S$ at each location $j$. So this set of constraints can be written as:

$$
Y_{j}=1, \quad j=1,2 \ldots, m
$$

b. Each antenna, if chosen, at any base station has only one azimuth value, tilt, height, and power. So this set of constraints can be written as:

$$
\begin{array}{r}
\sum_{A=0}^{359} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l A T H P} \leq Y_{j}, \quad j=1,2 \ldots, m \\
\text { and } l=1,2,3
\end{array}
$$

c. Each base station at any location has only one azimuth and this condition can be represented by the following two sets of constraints:

$$
\begin{gather*}
W_{j l A T H P} \leq Z_{j A}, \quad j \in S(i), l=1,2,3,0 \leq A \leq 359,-15 \leq T \\
\leq 0, H_{\min } \leq H \leq H_{\max }, \text { and } P_{\min } \leq P \leq P_{\max } \\
\sum_{A=0}^{359} Z_{j A} \leq 1, \quad j=1,2, \ldots, m
\end{gather*}
$$

d. Each demand point should be served by at least one base station. This set of constraints can be represented by:

$$
\begin{gathered}
\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l A T H P} \geq 1 \\
i=1,2, \ldots, n
\end{gathered}
$$

e. Each demand point should be assigned to exactly one base station; hence this set of constraint can be written as:

$$
\begin{gather*}
\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} X_{i j l A T H P}=1 \\
i=1,2, \ldots, n
\end{gather*}
$$

f. A base station $B S$ is assigned to a demand point $D P_{i}$ if it has an antenna $l$ with azimuth $A$, tilt $T$, height $H$, and power $P$. This set of constraints can be represented by :

$$
\begin{gather*}
W_{j l A T H P} \geq X_{i j l A T H P}, \quad i=1,2, \ldots, n, j \in S(i), l=1,2,3 \\
0 \leq A \leq 359,-15 \leq T \leq 0, H_{\min } \leq H \\
\leq H_{\max }, \text { and } P_{\min } \leq P \leq P_{\max }
\end{gather*}
$$

g. Each base station has a capacity of $Q$ channels, so the numbers of demand points assigned to each base station must not exceed its limit of channels. This set of constraints can be written as follow:

$$
\begin{align*}
& \sum_{i=1}^{n} \sum_{A=0}^{359} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} X_{i j l A T H P} \leq Q \\
& \quad j \in S(i), \text { and } l=1,2,3
\end{align*}
$$

h. The quality of service constraints by which the ratio of the strongest signal received at each $D P_{i}$ to the received noise and signals from other base stations
should be greater than a minimum requirement of signal-to-interference-plusnoise ratio, $S I N R$. This set of constraints can be given as follow:

$$
\frac{S P(i)}{P_{N_{i}}+T P(i)-S P(i)} \geq 10^{\frac{S I N R}{10}}, \quad i=1,2, \ldots, n
$$

where:
$S P(i)$ is the strongest power at test point $D P_{i}$ and is given by:
$S P(i)=\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} X_{i j l A T H P} P_{i j l A T H P}$
where $P$ is the received power at $D P_{i}$.
$T P(i)$ is the total power received at $D P_{i}$ which is generated by all base stations at candidate sites that can serve $D P_{i}$ and is given by:
$T P(i)=\sum_{j \in S(i)} \sum_{l=1}^{3} \sum_{A=0}^{360} \sum_{T=-15}^{0} \sum_{H=H_{\min }}^{H_{\max }} \sum_{P=P_{\min }}^{P_{\max }} W_{j l A T H P} P_{i j l A T H P}$
where $P$ is the received power at $D P_{i}$.
$P_{N_{i}} \quad$ is the noise power at $D P_{i}$
SINR is minimum signal-to-interference-plus-noise ratio

The complete MIP for the problem of base stations configuration can be summarized as shown in Figure 4-1.


Figure 4-1 MIP Model I - Base Stations Configuration Problem

### 4.1.2 MODEL I: NUMERICAL EXPERIMENT

The same case of model I in Section 3.1.2 is considered here except that the locations of base stations are known as they do exist in this model where we consider 35 existing base stations. The demand points and base stations locations are shown in Figure 4-2.


Figure 4-2 Demand Points and Base Stations Location Considered in Model I

Same parameters for COST-WI propagation model are used as shown in Table 3-1 to calculate the path losses except the height of the transmitter as it has two values and is shown in Table 4-1. The other parameters used in this numerical experiment such as
transmitted power, gains, receiver sensitivity, and base stations capacity are shown in table 4-1. Also, each base station could have up to three directional antennas and the configuration of each antenna involves azimuth, height, and transmitted power. Tilt is not considered in this experiment. In this experiment, we consider two values, to choose from, for each of antennas azimuth, heights, and transmitted power which are also included in table 4-1.

Table 4-1 Parameters used in Numerical Experiment of Model I

| Parameter | Value 1 | Value 2 |
| :---: | :---: | :---: |
| Transmitted power | 20 dBm | 25 dBm |
| Transmitted antenna gain | 8 dBi |  |
| Received antenna gain | 2 dBi |  |
| Minimum power requirement | -95 dBm |  |
| Height of transmitter | 20 m | 25 m |
| Available directional antennas | 1,2,3 |  |
| Antenna Azimuth | $0^{\circ}$ | $60^{\circ}$ |
| Available frequencies | 1 |  |
| Base station capacity | 30 channels |  |
| Antenna capacity | 10 channels |  |
| SINR | 20 dB |  |

The MIP for base stations configuration problem is solved using an optimization modeling software, LINGO, LINDO Systems Inc. The optimal solution resulted into 18 base stations as shown in Figure 4-3. The configuration of each selceted base station is shown in Table 4-2.


Figure 4-3 Resuls of Model I

Table 4-2 Configuration of Selected Base Stations for Model I

| BS \# | Location |  | Azimuth | Antenna | Height | Power |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X | Y |  |  |  |  |
| 1 | 0.5 | 1.5 | 2 | 1 | 1 | 1 |
| 2 | 0.5 | 8.5 | 2 | 3 | 2 | 1 |
| 3 | 1.5 | 2.5 | 2 | 2 | 2 | 1 |
|  |  |  |  | 3 | 2 | 1 |
| 4 | 1.5 | 8.5 | 2 | 3 | 2 | 1 |
| 5 | 2.5 | 9.5 | 2 | 1 | 2 | 1 |
| 6 | 3.5 | 5.5 | 2 | 1 | 1 | 1 |
| 7 | 4.5 | 2.5 | 2 | 1 | 2 | 1 |
| 8 | 5.5 | 1.5 | 2 | 2 | 2 | 1 |
| 9 | 5.5 | 4.5 | 1 | 3 | 1 | 1 |
| 10 | 6.5 | 2.5 | 2 | 3 | 2 | 1 |
| 11 | 6.5 | 3.5 | 2 | 1 | 2 | 1 |
| 12 | 6.5 | 8.5 | 2 | 1 | 2 | 1 |
| 13 | 7.5 | 7.5 | 2 | 3 | 2 | 1 |
| 14 | 8.5 | 2.5 | 2 | 2 | 2 | 1 |
| 15 | 9.5 | 1.5 | 2 | 3 | 1 | 1 |
| 16 | 9.5 | 9.5 | 2 | 1 | 2 | 1 |
| 17 | 10.5 | 5.5 | 1 | 2 | 1 | 1 |
| 18 | 6.5 | 6.5 | 2 | 2 | 2 | 1 |

### 4.1.3 MODEL I: DISCUSSION

As noticed from the results above, even though that there are 35 existing base station, this MIP model recommends to use only 18 out of them in order to achieve the objective of the model while satisfying all the constraints. Also, even though in this we consider some
existed base stations but still same issues as raised before that this recommendation of 16 base stations is high due to the low height of the transmitter and low transmitted power used in order to satisfy the quality of service constraint. Moreover, the solution shows that different configuration is assigned to different base stations to reduce the interference between them and also not all antennas at each base station are working.

### 4.2 MODEL II: BASE STATIONS LOCATION WITH SEVERAL TIME INTERVALS

In this model, we solve the problem of base stations location with several time intervals. Each time interval has a group of demand points that may have different locations. The objective of this model is to minimize the total cost of the network taking into account the constraints of area coverage, capacity of base station, and quality of service requirements for each user at each time interval.

### 4.2.1 MODLE II: MODEL FORMULATION

We assume that the demand points and candidate sites are known. Denote the $i$ th demand point by $D P_{i}, i=1,2, \ldots, n$ and the $j$ th candidate site by $C S_{j}, j=1,2, \ldots, m$. There is also, a set of considered time intervals, $t$, to be studied in this model, $t=1,2, \ldots, T$. A base station at candidate site $j$ can serve demand point $i$, if the power received at $D P_{i}$ exceeds
its minimum power requirements, $\gamma$, so we define $S(i)$ as the set of candidate sites that can serve demand point $D P_{i}$, i.e.
$S(i)=\left\{(j) \mid j \in S\right.$, such that the power received at $\left.D P_{i} \geq \gamma\right\}$
where $S$ is the set of candidate sites.

The demand points considered in each time interval consist of two groups which are $A(t)$ and $R(t-1)$, where:
$\begin{array}{ll}A(t) & \text { is the group of additional demand points at time } t \\ R(t-1) & \text { is the group of remaining demand points from time } t-1\end{array}$
So we define $U(t)$ as the set of demand points who exist at time $t$, i.e.
$U(t)=\{(i) \mid i \in U$, such that the demand point $\in A(t) \cup R(t-1)\}$
where $S$ is the set of demand points.

The Mixed Integer Programming model for the problem of base stations location with several time intervals model can be described as follows:
$\underline{1 \text { - The decision variables for Model I }}$
a. The decision variable, $Y_{j t}, j=1,2, \ldots, m$ and $t=1,2, \ldots T$, is defined as follows:

$$
Y_{j t}= \begin{cases}1 & \text { if a BS is constructed at } C S_{j} \text { in time interval } t \\ 0 & \text { otherwise }\end{cases}
$$

b. The decision variable, $X_{i j t}, i \in U(t), j \in S(i)$ and $t=1,2 \ldots T$, is defined as follows:

$$
X_{i j t}=\left\{\begin{array}{lc}
1 & \text { if a } B S \text { at } C S_{j} \text { has the strongest signal at } D P_{i} \\
0 & \text { otherwise }
\end{array}\right.
$$

## $\underline{2-T h e ~ o b j e c t i v e ~ f u n c t i o n ~ f o r ~ M o d e l ~ I ~}$

The function to be optimized is the total cost of the network. The objective function can be described as:

$$
\text { Minimize } \sum_{j=1}^{m} \sum_{t=1}^{T} C_{j} Y_{j t}
$$

where $C_{j}$ is the cost of installing a base station at $C S_{j}$.

## 3 - The constraints for Model I

The problem has six types of constraints that bound the feasible region of the solution. These are the following:
a. Each candidate site is allowed to have only one base station to be built at it. So this set of constraint will ensure that only one BS will be built, through all time intervals, at a $\mathrm{CS}_{\mathrm{j}}$ if it is selected:

$$
\sum_{t=1}^{T} Y_{j t} \leq 1, \quad j=1,2, \ldots, m
$$

b. Each demand point should be served by at least one base station either if it is newly built or already exist from the previous period. This set of constraints can be represented by:

$$
\sum_{j \in S(i)} \sum_{t \leq T} Y_{j t} \geq 1, \quad i=1,2, \ldots, n
$$

c. Each demand point in each time interval should be assigned to exactly one base station only; hence this set of constraint can be written as:

$$
\sum_{j \in S(i)} X_{i j t}=1, \quad i=1,2, \ldots, n \& t=1,2, \ldots T
$$

d. A candidate site $C S_{j}$ is assigned to a demand point $D P_{i}$ if it is selected to construct a base station at it, regardless at which time interval is built. This set of constraints can be represented by:

$$
\begin{gather*}
Y_{j t} \geq X_{i j t}, \quad i=1,2, \ldots, n, j=1,2, \ldots, m \& \\
t=1,2, \ldots T
\end{gather*}
$$

e. Each base station, in any time interval, has a capacity of $Q$ channels, so the numbers of demand points assigned to each base station must not exceed its limit of channels. This set of constraints can be written as follow:

$$
\sum_{i \in U(t)} X_{i j t} \leq Q, \quad j=1,2, \ldots, m \& t=1,2, \ldots T
$$

f. The quality of service constraints by which the ratio of the strongest signal received at each $D P_{i}$ to the received noise and signals from other base stations should be greater than a minimum requirement of signal-to-interference-plusnoise ratio, $S I N R$. This set of constraints can be given as follow:

$$
\frac{S P(i)}{P_{N_{i t}}+T P(i)-S P(i)} \geq 10^{\frac{S I N R}{10}}, \quad i=1,2, \ldots, n \& t=1,2, \ldots, T
$$

where:
$S P(i)$ is the strongest power at test point $D P_{i}$ and is given by:

$$
S P(i)=\sum_{j \in S(i)} \mathrm{X}_{i j t} P_{i j t}
$$

where $P_{i j t}$ is the received power at $D P_{i}$ from a BS at a $C S_{j}$ in a time interval $t$.
$T P(i)$ is the total power received at $D P_{i}$ which is generated by all base stations at candidate sites that can serve $D P_{i}$ and is given by:

$$
T P(i)=\sum_{j \in S(i)} Y_{j t} P_{i j t}
$$

where $P_{i j t}$ is the received power at $D P_{i}$ from a BS at a $C S_{j}$ in a time interval $t$.
$P_{N_{i t}}$ is the noise power at $D P_{i}$ in a time interval $t$.
SINR is minimum signal-to-interference-plus-noise ratio

The complete MIP model for the problem of base stations location with several time intervals can be summarized as shown in Figure 4-4.

| Minimize | $\sum_{j=1}^{m} \sum_{t=1}^{T} C_{j} Y_{j t}$ |
| :--- | :--- |
| Subject to |  |
|  | $\sum_{t=1}^{T} Y_{j t} \leq 1$ |
|  | $\sum_{j \in S(i)} \sum_{t \leq T} Y_{j t} \geq 1$ |
|  | $\sum_{j \in S(i)} X_{i j t}=1$ |
|  | $\sum_{j t} \geq X_{i j t}$ |
|  | $X_{i \in U(t)} \leq Q$ |
|  | $X, Y \in[0,1]$ |
| $P_{N_{i t}}+T P(i)-S P(i)$ |  |
|  |  |

Figure 4-4 MIP Model II - Base Stations Location with Several Time

## Intervals Problem

### 4.2.2 MODLE II: NUMERICAL EXPERIMENT

Three different time intervals are considered in this model. So, the same case of Model I in Section 3.1.2 is considered here except that the demand points are divided into three groups to represent three different time intervals. Also, same candidate sites for base stations are considered. The demand points for the three time intervals and candidate site for base stations locations are shown in Figure 4-5.


Figure 4-5 Demand Points and Candidate Sites for Model II

Same parameters for COST-WI propagation model are used as shown in Table 3-1 to calculate the path losses. Also, other parameters used in this numerical experiment such as transmitted power, gains, receiver sensitivity, and base stations capacity are same as shown in Table 3-2.

The MIP for base stations location problem with several time intervals is solved using an optimization modeling software, LINGO, LINDO Systems Inc. The optimal solution resulted into 9 base stations as shown in Figure 4-6. Also, the usage of selected base stations is hown in Table 4-3.


Figure 4-6 Results of Model II

Table 4-3 Selected Base Stations for Model II with their Usage

| BS \# | Location |  | Used by DP in Time |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | X | Y | 1 | 2 | 3 |
| 1 | 1.5 | 3 | $\sqrt{ }$ | $\sqrt{ }$ | - |
| 2 | 4.5 | 6 | - | $\checkmark$ | - |
| 3 | 7.5 | 10 | - | $\sqrt{ }$ | $\sqrt{ }$ |
| 4 | 2 | 9.5 | $\sqrt{ }$ | $\checkmark$ | - |
| 5 | 5 | 1.5 | - | $\checkmark$ | - |
| 6 | 6 | 2.5 | - | $\sqrt{ }$ | $\sqrt{ }$ |
| 7 | 8 | 6.5 | - | $\sqrt{ }$ | $\sqrt{ }$ |
| 8 | 9 | 1.5 | - | - | $\sqrt{ }$ |
| 9 | 9 | 7.5 | - | - | $\checkmark$ |

### 4.2.3 MODLE II: DISCUSSION

As recommended by this MIP, 9 base stations need to be placed in order to cover all the demand points in different time intervals. This number is high due to the same reason mentioned above which is the consideration of low height and transmitted power of transmitters in order to satisfy the quality of service constraint. In addition, the solution of this MIP model shows that some demand points have been assigned to base stations which are already available from a previous period that reflect the concept of reuse of existed base stations.

### 4.3 MODEL III: LOCATION OF BASE STATIONS AND RELAY STATIONS

In this model, we solve the problem of location of base and relay stations with a limit on the number of base stations. Relay stations are usually used to extend the coverage of base stations. Relay stations cannot transmit power unless they are connected to base stations, see Figure 4-7. The objective of this model is to minimize the total cost of the network taking into account the constraints of area coverage, capacity of base station, and quality of service requirements for each user at each time interval. This means that the problem is to find the minimum number of base stations, without exceeding the limits for number of base stations to be installed, and relay stations which could achieve the objective of the model while satisfying all constraints.


Figure 4-7 Relay Stations Deployment

### 4.3.1 MODEL III: MODEL FORMULATION

We assume that the demand points and candidate sites, for both base stations and relay stations, are known. Denote the $i$ th demand point by $D P_{i}, i=1,2, \ldots, n$, the $j$ th candidate site for a base station by $C S_{j}, j=1,2, \ldots, m$ and the $k$ th candidate site for a relay station by $C S_{k}, k=1,2, \ldots, l$.

A base station $(B S)$ at candidate site $j$ can serve demand point $i$, if the power received at $D P_{i}$ exceeds its minimum power requirements, $\gamma$, so we define $S(i)$ as the set of candidate sites that can serve demand point $D P_{i}$, i.e.
$S(i)=\left\{(j) \mid j \in S\right.$, such that the power received at $\left.D P_{i} \geq \gamma\right\}$
where $S$ is the set of candidate sites for base stations.

A relay station $(R S)$ at candidate site $k$ can serve demand point $i$, if the power received at $D P_{i}$ exceeds its minimum power requirements, $\gamma$, so we define $U(i)$ as the set of candidate sites that can serve demand point $D P_{i}$, i.e.
$U(i)=\left\{(k) \mid k \in U\right.$, such that the power received at $\left.D P_{i} \geq \gamma\right\}$
where U is the set of candidate sites for relay stations.

The Mixed Integer Programming model for the problem of base and relay stations location can be described as follows:

1 - The decision variables for Model I
a. The decision variable, $B_{j}, j=1,2, \ldots, m$, is defined as follows:

$$
B_{j}= \begin{cases}1 & \text { if a BS is constructed at } C S_{j} \\ 0 & \text { otherwise }\end{cases}
$$

b. The decision variable, $R_{k}, k=1,2, \ldots, l$, is defined as follows:

$$
R_{k}= \begin{cases}1 & \text { if a RS is constructed at } C S_{k} \\ 0 & \text { otherwise }\end{cases}
$$

c. The decision variable, $X_{i j}, i=1,2, \ldots, n$ and $j \in S(i)$, is defined as follows:

$$
X_{i j}= \begin{cases}1 & \text { if a BS at } C S_{j} \text { has the strongest signal at } D P_{i} \\ 0 & \text { otherwise }\end{cases}
$$

d. The decision variable, $Y_{i k}, i=1,2, \ldots, n$ and $k \in U(i)$, is defined as follows:

$$
Y_{i k}= \begin{cases}1 & \text { if a } R S \text { at } C S_{k} \text { has the strongest signal at } D P_{i} \\ 0 & \text { otherwise }\end{cases}
$$

e. The decision variable, $W_{k j}, k=1,2, \ldots, l$ and $j=1,2, \ldots, m$, is defined as follows:

$$
W_{k j}= \begin{cases}1 & \text { if a RS at } C S_{k} \text { has a signal from a } B S \text { at } C S_{j} \\ 0 & \text { otherwise }\end{cases}
$$

## $\underline{2}$ - The objective function for Model I

The function to be optimized is the total cost of the network. The objective function can be described as:

$$
\text { Minimize } \sum_{j=1}^{m} C_{j} B_{j}+\sum_{k=1}^{l} C_{k} R_{k}
$$

where:
$C_{j} \quad$ is the cost of installing a base station at $C S_{j}$
$C_{k} \quad$ is the cost of installing a relay station at $C S_{k}$

## 3 - The constraints for Model I

The problem has ten types of constraints that bound the feasible region of the solution. These are the following:
a. Number of base stations to be installed must not exceed $Z$, which is the maximum number of base stations that are allowed to be installed. So this set of constraints can be written as:

$$
\sum_{j=1}^{m} B_{j} \leq Z
$$

b. Each relay station, if installed, is assigned to a single base station only. So this set of constraints will ensure that each $R S$ is assigned to at most one $B S$ :

$$
\sum_{j=1}^{m} W_{k j} \leq R_{k}, \quad k=1,2, \ldots, l
$$

c. A candidate site $C S_{j}$ is assigned to a relay station $D P_{i}$ if it is selected to construct a base station at it and it can be represented by this set of constraints:

$$
W_{k j} \leq B_{j}, \quad k=1,2, \ldots, l \text { and } j=1,2, \ldots m
$$

d. Each demand point should be served by at least one base or relay station. This set of constraints can be represented by:

$$
\sum_{j \in S(i)} B_{j}+\sum_{k \in U(i)} R_{k} \geq 1, \quad i=1,2, \ldots, n
$$

e. Each demand point should be assigned to exactly one base station or relay station; hence this set of constraint can be written as:

$$
\sum_{j \in S(i)} X_{i j}+\sum_{k \in U(i)} Y_{i k}=1, \quad i=1,2, \ldots, n
$$

f. A candidate site $C S_{j}$ is assigned to a demand point $D P_{i}$ if it is selected to construct a base station at it and it can be represented by this set of constraints:

$$
X_{i j} \leq B_{j}, \quad i=1,2, \ldots, n \text { and } j=1,2, \ldots, m
$$

g. A candidate site $C S_{k}$ is assigned to a demand point $D P_{i}$ if it is selected to construct a relay station at it and it can be represented by this set of constraints:

$$
Y_{i k} \leq R_{k}, \quad i=1,2, \ldots, n \text { and } k=1,2, \ldots, l
$$

h. Each base station has a capacity of $Q$ channels, so the numbers of demand points assigned to each base station must not exceed its limit of channels. This set of constraints can be written as follow:

$$
\sum_{i=1}^{n} X_{i j} \leq Q, \quad j=1,2, \ldots, m
$$

i. Each relay station has a capacity of $V$ channels, so the numbers of demand points assigned to each relay station must not exceed its limit of channels. This set of constraints can be written as follow:

$$
\sum_{i=1}^{n} Y_{i k} \leq V, \quad k=1,2, \ldots, l
$$

j. The quality of service constraints by which the ratio of the strongest signal received at each $D P_{i}$ to the received noise and signals from other base stations should be greater than a minimum requirement of signal-to-interference-plusnoise ratio, $S I N R$. This set of constraints can be given as follow:

$$
\frac{S P(i)}{P_{N_{i}}+T P(i)-S P(i)} \geq 10^{S I N R / 10}, \quad i=1,2, \ldots, n
$$

where:
$S P(i)$ is the strongest power at test point $D P_{i}$ and is given by:

$$
S P(i)=\sum_{j \in S(i)} X_{i j} P_{i j}+\sum_{k \in U(i)} Y_{i k} P_{i k}
$$

where:
$P_{i j}$ is the received power at $D P_{i}$ from a BS at $C S_{j}$
$P_{i k}$ is the received power at $D P_{i}$ from a RS at $C S_{k}$
$T P(i)$ is the total power received at $D P_{i}$ which is generated by all base stations at candidate sites that can serve $D P_{i}$ and is given by:

$$
T P(i)=\sum_{j \in S(i)} B_{j} P_{i j}+\sum_{k \in U(i)} R_{k} P_{i k}
$$

where:
$P_{i j}$ is the received power at $D P_{i}$ from a BS at $C S_{j}$
$P_{i k}$ is the received power at $D P_{i}$ from a RS at $C S_{k}$
$P_{N_{i}} \quad$ is the noise power at $D P_{i}$
SINR is minimum signal-to-interference-plus-noise ratio

The complete MIP model for the problem of base and relay stations location can be summarized as shown in Figure 4-8.

| Minimize | $\sum_{j=1}^{m} C_{j} B_{j}+\sum_{k=1}^{l} C_{k} R_{k}$ |
| :--- | :--- |
| Subject to |  |
|  | $\sum_{j=1}^{m} B_{j} \leq Z$ |
|  | $\sum_{j=1}^{m} W_{k j} \leq R_{k}$ |
|  | $W_{k j \leq B_{j}}$ |
|  | $\sum_{j e s(i)} B_{j}+\sum_{k e v(i)} R_{k} \geq 1$ |
|  | $\sum_{j e s(i)} X_{i j}+\sum_{k e v(i)} Y_{i k}=1$ |
|  | $X_{i j} \leq B_{j}$ |
|  | $Y_{i k} \leq R_{k}$ |
|  | $\sum_{i=1}^{n} X_{i j} \leq Q$ |
|  | $\sum_{i=1}^{n} Y_{i k} \leq V$ |
|  | $\frac{S P(i)}{P_{N_{i}}+T P(i)-S P(i)} \geq 10^{s I N R / 10}$ |
|  | $B, R, X, Y, W \in[0,1]$ |

Figure 4-8 MIP Model III - Base and Relay Stations Location Problem

### 4.3.2 MODEL III: NUMERICAL EXPERIMENT

The same case of model I in section 3.1.2 is considered here with the additional of 300 candidate sites for relay stations (RS), which are assumed to have same locations of candidate sites for base stations. Demand points and candidate sites for both base stations and relay stations are shown in Figure 4-9. The number of base stations to be placed must not exceed 8.


Figure 4-9 Demand Points and Candidate Sites for Model III

Same parameters are used for COST-WI model to calculate the path losses as shown in
Table 3-1 except the height of the transmitter as is differs from a base station to a relay station. On the other hand, line-of-sight situation (LOS) is conserved while calculating the path loss for the signals from base stations to relay stations. The other parameters used in the numerical experiments, including the height of the transmitter, such as transmitted power, gains, receiver sensitivity, and capacity are shown in Table 4-4.

Table 4-4 Parameters used in Numerical Experiment of Model III

| Parameter | Value for BS | Value for RS |
| :--- | :--- | :--- |
| Transmitted power | 25 dBm | 20 dBm |
| Transmitted antenna gain | 8 dBi | 8 dBi |
| Received antenna gain | 2 dBi | 2 dBi |
| Minimum power requirement | -95 dBm | -95 dBm |
| Height of Transmitter | 25 m | 20 m |
| Available frequencies | 1 | 1 |
| Capacity | 30 channels | 15 channels |

The MIP for base and relay stations location problem is solved using an optimization modeling software, LINGO, LINDO Systems Inc. The optimal solution resulted into 4 base stations and 8 relay stations as shown in Figure 4-10. The connections between base and realy stations are shown in Table 4-5.


Figure 4-10 Results of Model III

Table 4-5 Base and Relay Stations Connections for Model III

| BS \# | LOCATION |  | RS \# | LOCATION |  | $\begin{gathered} \text { CONNECTED } \\ \text { TO } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | X | Y |  | X | Y |  |
| 1 | 2 | 9.5 | 1 | 0.5 | 2 | BS \# 2 |
| 2 | 6 | 0.5 | 2 | 3.5 | 2 | BS \# 3 |
| 3 | 6 | 4.5 | 3 | 1 | 3.5 | BS \# 1 |
| 4 | 7 | 8.5 | 4 | 2 | 2.5 | BS \# 3 |
|  |  |  | 5 | 4 | 6.5 | BS \# 4 |
|  |  |  | 6 | 9 | 10 | BS \# 4 |
|  |  |  | 7 | 10 | 6.5 | BS \# 4 |
|  |  |  | 8 | 10 | 8.5 | BS \# 3 |

### 4.3.3 MODEL III: DISCUSSION

As shown in the results above, this MIP model recommends 4 base stations and 8 relay stations to cover all the demand points which are relatively high considering the capacity of each of them. This high number resulted due to the consideration of low height and transmitted power of the transmitters to satisfy the quality of service constraints. Also, it is clear from the results that each relay station is connected to a base station as constructed in the model as it cannot work independently.

In addition, the performance of this model is compared with the model in [57] using the same experiment considered in [57]. An area of $3 \mathrm{~km} \times 3 \mathrm{~km}$ is considered with 20
candidate sites for BS 60 candidate sites for RS, 200 demand points (DP), and 256 signal test point (STP). Also, all parameters related to the experiment and the propagation model are considered same as in [57]. The setup of the experiment is shown in Figure 411.


Figure 4-11 Numerical Experiment Setup Considered in [57]

By applying our proposed MIP model and without considering the capacity and signal-to-interference-plus-noise ratio constraints in order to match [57], The optimal solution resulted into 1 base stations and 1 relay station as shown in Figure 4-12 compared to 4 base stations and 6 relay stations as per [57].


Figure 4-12 Results of the Experiment in [57]

## Chapter 5

## Conclusion and Future Work

### 5.1 CONCLUSION

In this thesis, we provide a general comprehensive framework for the problem of designing cellular mobile networks. Mathematical models are constructed using mixed integer program (MIP) with the objective of minimizing the total network costs taking considering the constraints of area coverage, traffic capacity, and quality of service i.e. signal-to-interference-plus-noise ratio (SINR). The constructed MIP models will provide the user with the optimal base stations location, antennas configuration of each base station i.e. azimuth, tilt, height, and transmitted power, and frequency allocation to each base station which can achieve the objective while satisfying all the constraints. Moreover, we construct MIP models for some special cases i.e. base stations
configuration with the existence of base stations, base stations location with different time intervals, and finally base and relay stations location. A none-line-of-site situation is considered while calculating the path losses of the signals using COST-231-WalfischIkegami propagation model. Solved numerical experiments using LINGO are presented to show the performance and the effectiveness of the constructed MIP models. The output of the MIP models show that the models work perfectly and can be used for designing cellular mobile networks. In general, the output of the model depends on the considered location of the candidate sites, parameters used for the propagation model, and other parameters such as transmitted power, gains, capacity...etc.

### 5.2 FUTURE WORK

For future studies and extensions of the thesis, the researcher can integrate the constraints programming with the constructed mixed integer programming models in order to reduce the time for solving the problems.

As a second extension work, the researcher can modify the constructed model to consider continuous values for the configuration parameters of each base station i.e. azimuth, tilt, height, and power.

As a third extension work, the researcher can consider dynamic channel assignment strategy instead of the fixed channel assignment strategy. Also, handover could be considered too.

## REFERENCES

[1] Rappaport, T., Wireless Communications: principles and practice. Prentice Hall, 2002.
[2] Akella, M., Batta, R., Sudit, M., Rogerson, P., Blatt, A., Cellular Network Configuration with Co-Channel and Adjacent-Channel Interference Constraints. Computers and Operations Research, vol. 35, pp. 3738-3757, 2008.
[3] Tutschuka, K., Gerlich, N., Tran-Gia, P., ICEPT - An integrated cellular network planning tool. In Proceeding of: 47th IEEE Vehicular Technology Conference, pp. 765-769, 1997.
[4] Tutschku, K., Demand-based Radio Network Planning of Cellular Mobile Communication Systems. In Proceeding of: 17th Annual Joint Conference of IEEE and Communications Societies. Vol. 3, pp. 1054-1061, 1998.
[5] Al-Kanj, L., Dawy, Z., Turkiyyah, G., A Mathematical Optimization Approach for Cellular Radio Network Planning with Co-siting. Wireless Networks, pp. 1-15, 2012.
[6] Mathar, R., Niessen, T., Optimum Positioning of Base Station for Cellular Radio Networks. Wireless Networks, vol. 6, no. 6, pp. 421-428, 2000.
[7] Hurley, S., Planning Effective Cellular Mobile Radio Networks. IEEE Transactions on Vehicular Technology,vol. 51, no. 2, pp. 243-253, 2002.
[8] Sohn, S., Jo, G., Optimization of Base Stations Positioning in Mobile Networks. Computational Science and Its Applications - ICCSA, Lecture Notes in Computer Science, vol. 3981, pp. 779-787, 2006.
[9] Toros, I., Fazekas, P., An Algorithm for Automatic Base Station Placement in Cellular Network Deployment. In proceeding of: Networked Services and Applications - Engineering, Control and Management, $16^{\text {th }}$ EUNICE/IFIP WG 6.6 Workshop, Trondheim, Norway, vol. 6164, pp. 21-30, 2010.
[10] Phone Networks. The Reverse Phone. http://www.thereverse-phone.com /phonenetworks /phonenetworks/. 18 May 2012.
[11] Black, U., Mobile and wireless networks. Upper Saddle River, NJ: Prentice Hall, 1996.
[12] Cellular Concept and Basics. Radio Electronics, 2012, http://www.radio-electronics. com/info/cellulartelecomms/cellular_concepts/mobile-basicsconcepts.php.
[13] Katta M., Operations Research: Deterministic Optimization Models. Prentice Hall, 1995.
[14] Katta M., Optimization for Decision Making: Linear and Quadratic Models. Springer, 2010.
[15] Smith, J. C., Taşkin, Z. C., A tutorial guide to mixed-integer programming models and solution techniques. In: Optimization in Medicine and Biology. Lim, G. J., Lee, E. K. (Eds.), pp. 521 - 548, Taylor and Francis, Auerbach Publications, New York, 2008.
[16] Walfisch-Ikegami Propagation Model. Propagation Models, 2012, http://www.cse. hcmut.edu.vn/~1tquan/DocumentsSoftwares/CDMA/Walfisch-Ikegami.htm/.
[17] Eisenblatter, A., Fugenschuh, A., Koch, T., Koster, A., Martin, A., Pfender, T., Wegel, O., Wessaly, R., Modelling feasible network configurations for UMTS. Technical Report 02-16, Konrad-Zuse-Zentrum fur Informationstechnik (ZIB), Berlin, Germany, 2002.
[18] Amaldi, E., Capone, A., Malucelli, F., Signori, F., UMTS Radio Planning: Optimizing Base Station Configuration. IEEE $56^{\text {th }}$ Vehicular Technology Conference (VTC), vol 2, pp. 768-772, 2002.
[19] Amaldi, E., Capone, A., Malucelli, F., Planning UMTS Base Station Location: Optimization Models with Power Control and Algorithms. IEEE Transactions on Wireless Communications, vol. 2, no. 5, pp. 939-952, 2003.
[20] Amaldi, E., Capone, A., Malucelli, F., Signori, F., Optimization models and algorithms for downlink UMTS radio planning. New Orleans, LA, USA 2, pp. 827831, 2003.
[21] Amaldi, E., Capone, A., Malucelli, F., Signori, F., A Mathematical Programming Approach for W-CDMA Radio Planning with Uplink and Downlink Constraints. 58th IEEE Vehicular Technology Conference, vol. 2, pp. 806-810, 2003.
[22] Amaldi, E., Belotti, P., Capone, A., Malucelli, F., Optimizing Base Station Location and Configuration in UMTS networks, Annals of Operations Research, vol. 146, no. 1, pp. 135-151, 2006.
[23] Amaldi, E., Capone, A., Malucelli, F., Radio planning and coverage optimization of 3G cellular networks. Wireless Networks, vol. 14, no. 4, pp. 435-447, 2008.
[24] Eisenblatter, A., Fugenschuh, A., Geerdes, H., Junglas, D., Koch, T., Martin, A., Integer Programming Methods for UMTS Radio Network Planning. Proceedings of Wireless Optimization, Cambridge, UK, 2004.
[25] Eisenblatter, A., Geerdes, H., Koch, T., Martin, A., Wessaly, R., UMTS radio network evaluation and optimization beyond snapshots. Mathematical Methods of Operations Research, vol. 63, no. 1, pp. 1-29, 2006.
[26] Kalvenes, J. Kennington, J., Olinick, E., Base Station Location and Service Assignments in W-CDMA Networks. INFORMS Journal of Computing, vol. 18, no. 3, pp. 336-376, 2006.
[27] Rosenberger, J., Olinick, E., Robust Tower Location for CDMA Networks. Naval Research Logistics, vol. 54, no. 2, pp. 151-161, 2007.
[28] Zhang, J., Yang, J., Aydin, M., Wu, J., Mathematical Modelling and Comparisons of Four Heuristic Optimization Algorithms for WCDMA Radio Network Planning. IEEE International Conference on Transparent Optical Networks (ICTON), vol. 3, pp. 253-257, 2006.
[29] Sarkar, S., Yen, H., Dixit, S., Mukherjee, B., A Mixed Integer Programming Model for Optimum Placement of Base Stations and Optical Network Units in a Hybrid Wireless-Optical Broadband Access Network (WOBAN). IEEE Wireless Communications and Networking Conference, Kowloon, pp. 3907-3911, 2007.
[30] Amaldi, E., Capone, A., Cesana, M., Malucelli, F., Optimization Models for the Radio Planning of Wireless Mesh Networks, Networking. Lecture Notes in Computer Science, 4479, pp. 287-298, 2007.
[31] Amaldi, E., Capone, A., Cesana, M., Filippini, I., Malucelli, F., Optimization models and methods for planning wireless mesh networks. Computer Networks, vol. 52, no. 11, pp. 2159-2171, 2008.
[32] Aldajani, M., Efficient Placement of Wireless Base Stations in Urban Environment. International Conference on Wireless Information Networks and Systems, Barcelona, Spain, 26-29 July, 2007.
[33] Aldajani, M, Convolution-Based Placement of Wireless Base Stations in Urban Environment. IEEE Transaction on Vehicular Technology, Vol. 57, no. 6, November 2008.
[34] Al-Nasser, F., Aldajani, M., Baroudi, U., Selim, S., Placement of Access Points in Wireless Local Area Networks. IWCMC 2009 Wireless LANs and Wireless PANs Symposium, Leipzig, Germany, 21-24 June, 2009.
[35] Hurley, S., Allen, S., Ryan, D., Taplin, R., Modelling and planning fixed wireless networks. Wireless Networks, vol. 16, no. 3, pp. 577-592, 2010.
[36] Olinick, E., Mathematical Programming Models for Third GenerationWireless Network Design, Wireless Network Design: Optimization Models and Solution Preocedures, J. Kennington, E. Olinick, and D. Rajan, Springer, New York, NY 10013, pp. 101-125, 2011.
[37] Rajan, D., Olinick, E., Optimal CDMA network design with uplink and downlink rate guarantees. IEEE International Conference on Communications (ICC), Kyoto, vol. 1-6, 2011.
[38] Giortzis, A., Turner, L., Application of Mathematical Programming to The Fixed Channel Assignment Problem in Mobile Radio Networks. IEE Preceedings Communications, vol. 144, no. 4, pp. 257-264, 1997.
[39] Hellebrandt, M., Lambrecht, F., Mathar, R., Niessen, T., Starke, R., Frequency Allocation and Linear Programming. IEEE $49^{\text {th }}$ Vehicular Technology Conference, vol. 1, pp. 617-621, 1999.
[40] Zdarsky, F., Martinovic, I., Schmitt, J., On Lower Bounds for MAC Layer Contention in CSMA/CA-based Wireless Networks. DIALM-POMC '05 Proceedings of the 2005 joint workshop on Foundations of mobile computing, Cologne, Germany, 2005.
[41] Das, A., Alazemi, H., Vijayakumar, R., Roy, S., Optimization Models for Fixed Channel Assignment in Wireless Mesh Networks with Multiple Radios. proceedings IEEE SECON, pp. 463-474, 2005.
[42] Subramanian, A., Gupta, H., Das, S., Cao, J., Minimum Interference Channel Assignment in Multiradio Wireless Mesh Networks. IEEE Transactions on Mobile Computing, vol. 7, no. 12, pp. 1459-1473, 2008.
[43] El-Hajj, W., Alazemi, H., Optimal Frequency Assignment for IEEE 802.11 Wireless Networks. Wireless Communication and Mobile Computing, vol. 9, pp. 131-141, 2009.
[44] Capone, A., Carello, G., Filippini, I., Gualandi, S., Malucelli, F., Routing, scheduling and channel assignment in Wireless Mesh Networks:Optimization models and algorithms. Ad Hoc Networks, vol. 8, no. 6, pp. 545-563, 2010.
[45] Rodrigues, R., Mateus, G., Loureiro, A., Optimal Base Station Placement and Fixed Channel Assignment Applied to Wireless Local Area Network Projects. Proceedings IEEE International Conference on Networks, pp. 186-192, 1999.
[46] Rodrigues, R., Mateus, G., Loureiro, A., On the Design and Capacity Planning of a Wireless Local Area Network. Network Operations and Management Symposium, pp. 335-348, 2000.
[47] Mateus, G., Loureiro, A., Rodrigues, R., Optimal Network Design for Wireless Local Area Network. Annals of Operations Research, vol. 106, pp. 331-345, 2001.
[48] Rawnsley, R., Hurley, S., Towards Automatic Cell Planning. $11^{\text {th }}$ IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, London, vol. 2, pp. 1583-1588, 2000.
[49] Allen, S., Hurley, S., Taplin, R., Whitaker, R., Automatic Cell Planning of Broadband Fixed Wireless Networks. IEEE 53 ${ }^{\text {rd }}$ Vehicular Technology Conference (VTC), vol. 4, pp. 2808-2812, 2001.
[50] Mathar, R., Schmeink, M., Optimal BaseStation Positioning and Channel Assignment for 3G Mobile Networks by Integer Programming. Annals of Operations Research, vol. 107, no. 1-4, pp. 225-236, 2001.
[51] Mathar, R., Schmeink, M., Integrated Optimal Cell Site Selection and Frequency Allocation for Cellular Radio Networks. Telecommunication Systems, vol. 21, no. 24, pp. 339-347, 2002.
[52] Lee, Y., Kim, K., Choi, Y., Optimization of AP Placement and Channel Assignment in Wireless LANs. Proceedings 27th Annual IEEE Conference on Local Computer Networks, pp. 831-836, 2002.
[53] Akella, M., Batta, R., Delmelle, E., Rogerson, P., Blatt, A. Wilson, G., Base Station Location and Channel Allocation in A Cellular Network with Emergency Coverage Requirements. European Journal of Operational Research, vol. 164, pp. 301-323, 2005.
[54] Kalvenes, J. Kennington, J., Olinick, E., Hierarchical Cellular Network Design with Channel Allocation. European Journal of Operational Research, vol. 160, pp. 3-18, 2005.
[55] Eisenblatter, A., Geerdes, H., Siomina, I., Integrated Access Point Placement and Channel Assignment for Wireless LANs in an Indoor Office Environment. IEEE International Symposium on a World of Wireless, Mobile and Multimedia Networks, Espoo, Finland, pp. 1-10, 2007.
[56] Bosio, S., Eisenblatter, A., Geerdes, H., Siomina, I., Yuan, D., Mathematical Optimization Models for WLAN Planning, Graphs and Algorithms in Communication Networks. Texts in Theoretical Computer Science. An EATCS Series, Part 2, 283-309, 2010.
[57] Prommak, C., Wechtaison, C., WiMAX Network Design for Cost Minimization and Access Data Rate Guarantee Using Multi-hop Relay Stations. International Journal of Communications, Vol. 4, no. 2, pp. 39-46, 2010.
[58] MATLAB version 7.10.0. Natick, Massachusetts: The MathWorks Inc., 2010, http://www.mathworks.com/
[59] LINDO Systems. Solver Suite. LINDO Systems, Inc., Chicago, IL, 1996, http://www.lindo.com/

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