AN INTEGRATED MODEL FOR THE ECONOMIC MANUFACTURING QUANTITY AND PROCESS TARGET UNDER IMPERFECT QUALITY AND INSPECTION SYSTEM

BY

AHMED ADNAN ZAID

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In the name of Allah, the Most Gracious and the Most Merciful

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This thesis, written by **<u>AHMED ADNAN ZAID</u>** under the direction of his thesis advisors and approved by his thesis committee, has been presented to and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN <u>INDUSTRIAL AND SYSTEM ENGINEERING</u>.**

<u>Thesis Committee</u>

20/2/2012
Prof. Salih Duffuaa (Advisor)

20/2/2012
Dr. Mazen Al-Otaibi (Co-Advisor)

20/2/2012
Prof. Abdul-Basit Andijani (Member)

Dr. Umar Al-Turki (Member)

20/02/2012
Dr. Malick M.Ndiaye (Member)

Feb 21, 12
Prof. Fouad Al-Sunni
Department Chairman

Dr. Salam A. Zummo
Dean of Graduate Studies

25/2/12
Date

Dedicated to

My Family Members

Father (May Allah have mercy on his soul),

Mother, Iyad, Nizar, Abdul-Alraheem, Mohammad,

Amal, Leena & My Wife Salsabeel

Whose Prayers and Perseverance led to this accomplishment

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THESIS ABSTRACT (ENGLISH)

Name:	AHMED ADNAN ZAID
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Recently, there has been much interest among researchers in the economic manufacturing quantity and quality control, hence it is natural to integrate the determination of economic manufacturing quantity, process targeting and the approach used for maintain good quality product. The objective of the thesis was to develop a modified economic manufacturing quantity (EMQ) and process target model under imperfect product quality and inspection system. Three models are developed. The first model was developed under100% error-free inspection system. In the second model the inspection error free assumption is relaxed using cut-off point for inspection instead of the original specification limits. The results showed that the total profit in the first model is greater than the second model. The third model introduced Chen (2006) model where the inspection system is error prone. The results showed that the total cost increases. The utility of the modified economic manufacturing quantity models has been demonstrated using numerical examples. Sensitivity analysis has been performed to study the effect of inspection error and other parameters like production, inspection costs…etc. The sensitivity analysis has shown that as the correlation coefficient gets closer to one, the expected total cost decrease. The thesis concluded by suggesting a number of extensions to be considered in future research.

MASTER OF SCIENCE DEGREE

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THESIS ABSTRACT (ARABIC)

خلاصة الاطروحة

الاسم: أحمد عدنان زيد

عنوان الرسالة: نموذج متكامل لانتاج كمية تصنيع اقتصادية وتحديد القيم المثلى للعمليات الصناعية في اطار الجودة ونظام التفتيش

التخصص: هندسة الأنظمة الصناعية

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ازداد مؤخرا بين الباحثين الكثير من الاهتمام في موضوع ضبط الجودة لما لها من أهمية في خفض التكاليف وزيادة الأرباح للمؤسسات الصناعية. معظم هذه الأبحاث تستهدف التأثير على جودة المنتج من خلال نوعية نهج التحكم المستخدمة لمراقبة الجودة ، وبالتالي فمن الطبيعي أن تدمج ما بين تحديد كمية التصنيع الاقتصادية ، و تحديد القيم المثلى للعملية الصناعية تحت ظروف غير جيدة للمحافظة على جودة المنتج .

يهدف هذا البحث الى تطوير عدة نماذج تجمع ما بين انتاج كمية تصنيع اقتصادية مع الأخذ بعين الاعتبار تحديد القيم المثلى لبرامترات العملية الصناعية تحت ظروف غير جيدة في العملية التصنيعية. في النموذج الأول تم بناء النموذج الأول تحت فرضية أن كل عناصر المنتج تفحص بلا أخطاء في عملية الفحص .طوّرالنموذج الثاني حيث تفحص كل عناصر المنتج ولكن بفرض وجود أخطاء في نظام الفحص .النتائج أوضحت نقصان ملحوظ في الربح عندما تكون الأخطاء موجودة في خط الانتاج.النموذج الثالث تم فيه تطوير نموذج (2006، البروفيسور شن) لكن بفرض وجود أخطاء في نظام الفحص.النتائج أظهرت زيادة في التكاليف عندما تكون الأخطاء موجودة. تم حل امثلة عملية على هذا االنماذج مع دراسة واختبار حساسية هذه النموذج للتغيير في البارمترات المختلفة وللاخطاء في الفحص. دلت نتائج تحليل الحساسية على أنه كلما كانت قيمة معامل الارتباط أقرب الى واحد صحيح،أدى ذلك الى انخفاض التكلفة الانتاجية المتوقعة. ختمت الأطروحة بتوصيات ومقترحات للبحوث المسقبلية في هذا المجال

درجة الماجستير العلوم

جامعة الملك فهد للبترول والمعادن

الظهران ، المملكة العربية السعودية

CHAPTER 1

INTRODUCTION

1.1 PREFACE

The objective of this chapter is to provide an overview of the economic manufacturing quantity quality control and quality assurance approaches. The overview includes the basic definitions of economic manufacturing quantity, quality models and thesis organization.

1.2 ECONOMIC MANUFACTURING QUANTITY

Traditional economic manufacturing quantity (EMQ) model assume implicitly that items produced are perfect in its quality. However, product quality is not always perfect and is usually a function of the production process. The total inventory cost of EMQ model includes the set-up cost and the holding cost.

Eqs.(1.1) is presented by **Makis and Fung (1998)** :

$$TC = S_t \,.\, \frac{D}{Q} + 0.5\,.\,Q.\left(1 - \frac{O}{I}\right).\,h \qquad (1.1)$$

where TC is the total inventory cost per unit time; D is the demand quantity in units per unit time; Q is the economic manufacturing quantity; St is the set-up cost for each production run; O is the demand rate in units per day; I is the production rate in units per day, where ($I > O$) and h is the holding cost per unit item per unit time. By differentiating TC with respect to Q, we get:

$$\frac{dTC}{dQ} = -S_t \cdot \frac{D}{Q^2} + 0.5 \cdot \left(1 - \frac{O}{I}\right) \cdot h \qquad (1.2)$$

The second derivative of *TC* with respective to *Q* is

$$\frac{d^2TC}{dQ^2} = 2S_t \cdot \frac{D}{Q^3} \qquad (1.3)$$

Eq. (1.3) is positive ($\frac{d^2TC}{dQ^2} \geq 0$), so the total cost function is convex and have a unique minimum solution. Hence, one sets the first derivative of Eq.(1.2) equal to zero, and solves for *Q*. We have the optimum economic manufacturing quantity is:

$$Q^* = \sqrt{\frac{I\ D\ S_t}{0.5\ h\ (I - O)}} \qquad (1.4)$$

1.3 DEFINITIONS OF QUALITY

In any production process, the product passes through a number of operations before it takes its final form. During these operations, a certain amount of variability will exist due to the presence of variation of raw material, environment etc. Therefore, quality control is considered as an essential method to minimize this variability and improve the final product quality.

Quality itself is difficult to define, it is an abstract term. The definition has evolved over time. The following are the classical definitions of quality as reported by **Montgomery (2005)**

- Definition 1: Quality is fitness for use.

- Definition 2: Meeting specifications.

- Definition 3: inversely proportional to variability.

Based on the second definition, the Quality Control (QC) can be defined as a procedure or set of procedures intended to ensure that a manufactured product or performed service adheres to a defined set of quality criteria or meets the requirements of the client or customer. In the next subsection established areas of quality will be presented.

1.3.1 STATISTICAL PROCESS CONTROL

Statistical process control (SPC) is the application of statistical methods to monitor and control a process to ensure that it operates at its full potential to produce conforming products. Under SPC, a process behaves predictably to produce as much conforming product as possible with the least possible waste. While SPC has been applied most frequently to control manufacturing lines, it applies equally well to any process with a measurable output. Key tool in SPC are control charts, a focus on continuous improvement and designed experiments. **Montgomery (2005)**

1.3.2 QUALITY ASSURANCE

It is a planned and systemic set of activities to ensure that variances in processes are clearly identified, assessed and improving defined processes for fulfilling the requirements of customers and product or service makers. This is usually done through standards such as ISO and quality auditing.

1.3.3 QUALITY ENGINEERING

Quality engineering (QE) is the set of operational, managerial, and engineering activities that a company uses to ensure that the quality characteristics of a product are at the nominal or required levels. The main idea behind quality engineering is to involve the quality concept and quality cost through all phases of a product's life cycle rather than involving it at the final product phase. The life cycle begins with product planning and continues through the phases of product design, production process design, online production process control, market development, and packaging, as well as maintenance and product service. QE can be divided into on-line QE and off-line QE. On-line QE is in contrast with traditional statistical process control (SPC), and off-line QE is in contrast with classical design and analysis of experiments. The difference between on-line and off-line QE is that activities of the former are done along mass production lines while those of the latter are done prior to mass production. There are three steps in off-line QE, i.e. system design, parameter design and tolerance design.

Taguchi defined quality of a product/process as the loss to society. Losses are incurred because performances of products are deviated from their targets (ideal function), as shown in figure (1.1). This deviation is caused by noises which are uncontrollable variables such as environmental factors (temperature, humidity, dust, magnetism, etc ...), variation within components and deterioration (wear-out). Therefore, noises are always there and we can never eliminate them. So are losses. The best we can do is to make our products/processes "strong" enough to be less sensitive to the effects of noises. That is, through minimization of loss, we obtain a robust product/ process. Taguchi presented a quadratic penalty for this deviation known as "Taguchi quadratic loss function". This loss function can be used to evaluate the effect of quality improvement, determine the economic impact of tightening the tolerance to improve product quality, to justify improvements of the process, and to determine whether 100% inspection can be justified or not.

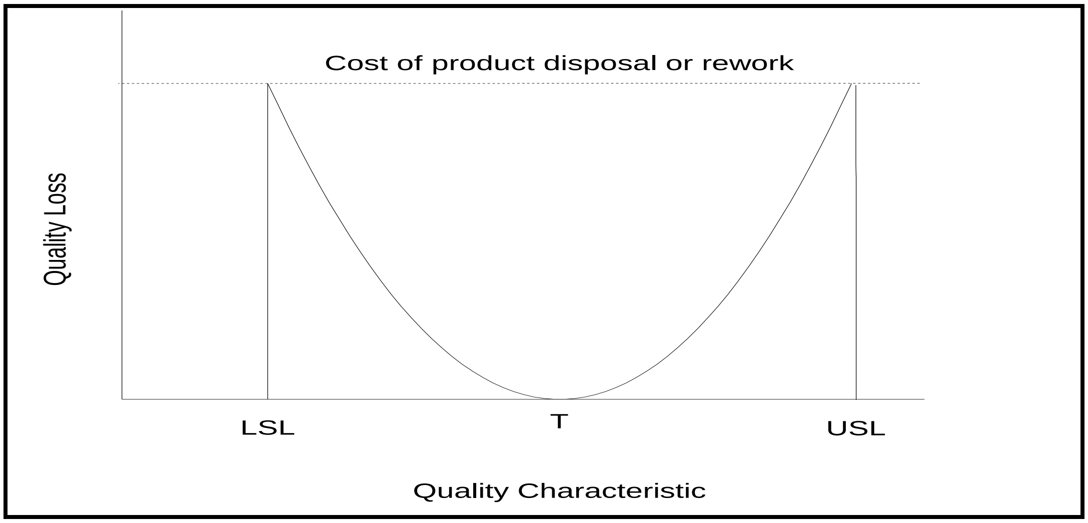

Figure 1.1 Relationship between quality loss and deviation from target

1.4 PROCESS TARGETING

Studies estimate that 6 to 40 percent of the total cost of sales can be attributed to the cost of quality in a typical company. For this reason, many companies have turned to improving the processes of achieving quality in order to reduce costs.

The new perspective has led companies to reexamine the traditional assumptions and approaches used to achieve quality improvement. The classical approach of SQC, which focused on screening and correction of defects, is giving way to new methodologies that emphasize prevention. Unlike the classical approach, which assumed that the process settings (mean and variance) were given, the new approaches view the process settings as variables that can be controlled through investments in improved raw materials, worker training, and process capabilities. To effectively carry out the new approach, companies need methods to evaluate investments that are aimed at changing the process settings. From here the concept of process targeting has been raised.

In general, process targeting problem can be described as follows: consider a production process where items are produced continuously. Specification limits are specified for the quality characteristic of interest, and an item is defective if its value of the quality characteristic is not within the specification limits. The process mean may be set higher to reduce the chance of producing defective items. However, doing so may result in a higher production cost when production cost is an increasing function of the quality characteristic. Defective items can be identified by inspection, in which they may be scraped, reworked or sold at reduced prices. Consequently, the decision of setting a process mean should be based on the tradeoff among production cost, payoff of nondefective items and the costs incurred by the disposition of the defective items. Many papers related to this topic have been published. Each paper considers the same problem, in general, with different assumptions. As a result, different models and solution methods exist, see the literature review.

1.5 MEASURMENT ERROR

Since inspection is used to determine whether or not a product meets specifications, the manufacture of quality products demands measurements that are both high precision and high accuracy. The inspection results are commonly used to influence the operation in making the current part or the production of the next part, there by correcting a potential quality problem before a product is completed. Hence, the accuracy and effectiveness of the inspection procedures and equipments are essential for precision manufacturing.

Unfortunately, there are always sources of errors in manufacturing equipments and measurement systems. The sources of errors that come from the measuring equipment include imperfect mechanical structure, errors in control systems, and environmental disturbances. As measurement error is defined as the discrepancy between actual and measured dimensions, it will be affected not only by the error resulted from the measuring equipment and the repeatability of the measurement, but also by the error resulted from the compound effect of machine errors and the geometric characteristic of the measured surfaces. A variety of techniques have been developed to deal with machine error modeling and compensation as well as uncertainty in inspection.

In this thesis, an attempt is made to study the effect of measurement errors on process targeting and economic manufacturing quantity models in this area by incorporating Taguchi concept in these models.

1.6 THESIS ORGANIZATION

The problem of joint modified economic manufacturing quantity (EMQ) model under the imperfect product quality with process targeting is the focus of this thesis. The problem is formulated to obtain both the optimum combination of EMQ and process mean in order to have the maximum expected total profit per unit of time.

The rest of the thesis is organized as follows: chapter 2 presents the literature review. Chapter 3 presents a modified economic manufacturing quantity model with 100% error-free inspection system. Chapter 4 presents a modified economic manufacturing quantity model with100% error-prone inspection system**.** Chapter 5 presents **Chen (2006)** model with100% error-prone inspection system. Finally, conclusions and further research are outlined in chapter 6.

CHAPTER 2

LITERATURE REVIEW

2.1 PREFACE

The purpose of this chapter is to present the literature review on the joint modified economic manufacturing quantity (EMQ) with process targeting area. Next, the objectives of this thesis are presented. The problem definition and formulation are presented at the end of the chapter.

2.2 LITRETURE REVEIEW

This section includes a brief literature review in the areas of “Process Targeting”, “Economic Manufacturing Quantity (EMQ)”and “Integrated Process Targeting with economic manufacturing quantity (EMQ)”. It covers most important papers that concerned in the field’s that mentions above in chronological order.

2.2.1 PROCESS TARGETTING

The filling process problem has received considerable attention from researchers in recent years. **Springer (1951)** developed a method for determining the most economic position of a process mean. He considered a manufacturing process in which both upper and lower specification limits were of interest and in which the financial loss due to producing a product above the upper specification limit was not necessarily equal to the loss when producing one below the lower specification limit. He suggested a simple method for determining the optimum target mean to minimize the optimum cost. The distribution of the product quality characteristic was assumed to be normal. **Bettes (1962)** studied a similar problem with a lower specification limit; however he assumed any arbitrary upper specification limit. Furthermore he assumed under sized and over sized items are reprocessed at a fixed cost. **Hunter and Kartha (1977)** investigated the optimization of a target mean when a lower specification limit is employed. Their study provides a simple procedure for obtaining the optimal process mean. **Nelson (1979)** provided a similar solution to this problem. **Bisgaard et al. (1984)** extended **Hunter and Kartha (1977)** work to include the selection of the most favorable quality characteristic distribution of the product. **Carlsson (1984)** modified the work of **Hunter and Kartha (1977)** to include both fixed and variable costs. Both **Hunter and Kartha (1977)** and **Carlsson (1984)** assumed that rejected products are sold in a secondary market.**Arcelus and Banerjee (1985)** extended the work of **Bisgaard et al. (1984)** assuming a linear drift. **Golhar (1987)** addressed the problem of finding economic setting of process mean. He modeled a situation where over filled cans could only be sold at a fixed price where under filled cans would be emptied and refilled with a penalty of extra cost .**Golhar and Pollock (1988)** extended this work to include an upper specification limit and provided solutions for determining both the process mean and the upper specification limit. **Rahim and Banerjee (1988)** are the first to consider a process with linear drift. They have proposed a search algorithm and graphical method to find the optimum production run length. **Schmidt and Pfeiffer (1991)** used Golhar's model to evaluate the economic effects of process variance reduction. **Boucher and Jafari (1991)** examined the problem of choosing the optimal set for an automatic filling operation with a lower specification limit. **Arcelus and Rahim (1994)** examined a quality selection problem in which the target means for both a variable and an attribute quality characteristic are simultaneously determined. **Chen and Chung (1996)** determined the most profitable target value measuring precision value for a production process.

Liu and Raghavachari (1997) studied the economic selection of the process target mean and the upper specification limit of filling process under capacity constraints. The filling amount assumed to follow an arbitrary continuous distribution, and the upper specification limit can be presented by a very simple formulation regardless of the shape of distribution. **Al-Sultan and Al-Fawzan (1997)** developed a model to determine the optimal initial process mean and production run which minimizes the total cost. They studied a multistage production system where the processing at each stage was performed by a process that deteriorated randomly with time. **Wen and Mergen (1999)** proposed a model that helps minimize the quality costs when the process is not capable of meeting specification limits. The proposed method, which is a special case of the one proposed by **Springer (1951)**, is a short-term measure to deal with the loss due to incapability of the process. The process is assumed to be in statistical control but not 100% capable of meeting the specification limits. **Hong et al. (1999)** studied the effect of measurement error on the optimal target mean for the case of two-class screening process. **Rahim and Shaibu (2000)** proposed a model similar to the model in **Springer (1951)** but in term of profit instead of cost. A product within the specifications incurs a profit *p*. a product below the lower specification limit or above the upper specification limit incurs cost *Cl* or *Cu*, respectively. The model determines the optimum process target mean which maximizes the expected total profit.

Lee et al. (2001) proposed a model to determine the optimum process target mean and specification limits under single and two-stage screening. **Duffuaa and Siddiqui (2002)** proposed two process targeting models for three-class screening. Product uniformity considered in the models using Taguchi quadratic loss function. **Teeravaraprug and Cho (2002)** extended Taguchi univariate loss function to a multivariate quality loss function. The model included the same three cost elements. Their model could also be used for the case where co-variances among the quality characteristics exist. **Chen and Chou (2003)** proposed another modification in **Wen and Mergen (1999)** model. They have studied the effect of multiple quality characteristics in the original model. The bivariate quality characteristic and asymmetric quadratic loss function are taking into account in the development of the cost model. **Duffuaa and Siddiqui (2003)** proposed a process targeting model for three-class screening. The case of measurement error present in inspection system is considered in this model. **Lee et al. (2004)** used a similar concept as **Golhar (1987)**, with upper and lower specification limits. Over and under filled cans are empted and refill again, with the assumption that the reprocessing cost is proportional of the amount of ingredient in a container can that is not changed after reprocessing. The proposed economic model consists of the selling price and the cost of production, inspection, reprocessing and quality, the later cost evaluated using Taguchi quadratic loss function. The objective of the model is to determine the optimum process target mean where the process standard deviation is known. **Li (2005)** stated that, using a quadratic loss function when the actual loss function is non quadratic may yield incorrect input parameter levels. In certain situations, a linear loss function is more appropriate in industrial applications. Hence, the optimum process target mean is determined under a truncated asymmetrical linear loss function to describe unbalanced tolerance design, which minimizes the total expected cost.

Chen and Chou (2005) proposed a modified **Pulak and Al-Sultan (1997)** model, by considering both the lot tolerance percentage defective (LTPD) and the average outgoing quality limit (AOQL). In this model the optimum process target mean which maximizes the expected total profit is obtained **Lee et al. (2005)** considered the problem of determining the optimum process target mean and screening limits under single-screening procedure. Two surrogate variables correlated to the quality characteristic of interest are observed simultaneously in the single-screening procedure. **Jordan (2006)** proposed a profit model with fixed selling price, a linear cost to produce and fixed reprocessing cost under the uniform distribution. The objective of this model is to find the optimum process target mean and upper specification limit. **Chen (2006)** proposed a modified **Wen and Mergen (1999)** cost model with mixed quality loss function to determine the optimum process target mean. The mixed quality loss function includes a quadratic loss function for products within the specifications and a piecewise linear loss function for products out of specifications. **Fareeduddin (2008)** developed four process targeting models with different inspection policies for two stage production process in series for a product with two quality characteristics.

2.2.2 ECNOMIC MANUFACTURING QUANTITY

In the classical Economic Manufacturing Quantity (EMQ) model, it is assumed that all items produced are of perfect quality and the production facility never breaks down. However, in real production, the product quality is usually a function of the state of the production process which may deteriorate over time and the production facility may fail randomly. However, in real-life manufacturing settings, generation of imperfect quality items is almost inevitable.

When a machine breakdown takes place in the production phase, however, the basic (deterministic) EMQ model loses its usefulness since the cyclic behavior of the production system changes by interrupted failures. In addition, from a practical perspective, the manufacturer should design the production lot from the standpoint of safety, and then effects of machine breakdown in economic manufacturing quantity decisions should be examined in uncertain environment with unreliable manufacturing facilities.

Numerous research efforts have been undertaken to extend the manufacturing model subject to stochastic machine breakdowns. **Rosenblatt and Lee (1986)** focused on the imperfections in the production process and equipment, and determined the optimal EMQ policy and/or inspection schedule when machine breakdowns. They analyzed a deteriorating production process. Different approaches to maintain the EMQ model with stochastic machine breakdown were tried by **Groenevelt et al. (1992)** and others **Ibrahim and Kee (1994)**, **Dohi and Osaki (1996)** and **Dohi et al. (1998)**.**Liu and Cao (1999; Makis and Fung (1998)**. Studied effects of machine failures on the optimal lot size as well as on optimal number of inspections. Formulas for the long-run expected average cost per unit time was obtained. Then the optimal production/inspection policy that minimizes the expected average costs was derived. **Chung (2003)** has studied manufacturing systems with machine breakdowns as stated earlier, they described the EMQ model subject to stochastic machine breakdowns, by proposing an asymptotic approximation formula of the expected cost function to examine the impact of machine breakdowns and repairs on the production lot sizing.

Chiu et al. (2007) concerned in determination of optimal run time for an economic production quantity (EPQ) model with scrap, rework, and stochastic machine breakdowns, he supposed that a portion of the defective items is considered to be scrap, while the other is assumed to be repairable. Total production-inventory cost functions are derived respectively for both EPQ models with breakdown (no-resumption policy is adopted) and without breakdown taking place. These cost functions are integrated and the renewal reward theorem is used to cope with the variable cycle length.

The fundamental assumption of an economic manufacturing quantity (EMQ) model is that 100% of items that are produced are perfect. This assumption is not always pertinent for production processes because of process deterioration or other factors. In many real-life conditions, stockout is unavoidable because of various uncertainties in the related system. Therefore, the occurrence of shortages in inventory could be considered as a natural phenomenon. In the classical models while shortage is considered, the issue of quality was ignored , a few of them have considered shortage problem. In this direction we have several works, for example **Salameh and Jaber (2000)** extends the traditional EPQ/EOQ model by accounting for imperfect quality items when using the backorder EPQ/EOQ formulae. They studied the effect of imperfect quality products and rework of them on the finite economic production quantity model where shortages are allowed and backordered and considered that the percentage of defective products follows a known probability density function. Related to this work is the paper by **Cárdenas-Barrón**

(2000) where an error appearing on **Salameh and Jaber (2000)** model is corrected. **Huang (2004)** extended the **Salameh and Jaber (2000)** model to incorporate the view of the integrated single-vendor and single-buyer relationship, considering the presence of imperfect products into the lot size. **Chiu (2006)** developed a mathematical modeling for production system with backlogging and failure in repair, they assumed a random portion of reworked items fails the repairing process and becomes scrap items; hence the renewal reward thermo is employed to cope with variable cycle length. Disposal cost per scrap item and repairing and holding cost per reworked items are included. Furthermore, the optimal lot size and allowable backlogging level that minimizes the overall production-inventory costs is derived.

Eroglu and Ozdemir (2007) extended **Salameh and Jaber (2000)** model by allowing shortages backordered. Also, they had been studied effects of different levels of defectives fractions on lot size and expected total profit **Chiu et al. (2008)** provides a complete solution procedure for determining optimal run time for EMQ model with backordering of excess demand, failure-in rework, and breakdown happening in stock-piling time. This procedure includes the mathematical modeling, the use of renewal reward theorem to cope with variable cycle length and derivation of the long-run average production-inventory cost function. **Cárdenas-Barrón (2009)** developed an EOQ model for that each ordered lot contains some defective items and shortages backordered. They assumed that 100% of each lot are screened to separate good and defective items which are collection of imperfect quality and scrap items. The effect of percentage defective on optimal solution was studied.

2.2.3 INTEGRATED ECONOMIC MANUFACTURING QUANTITY WITH PROCESS TARGETTING

Since **F. Harris (1913)** proposed the famous EOQ model to the world, it has been broadly applied in many places. However, there are some drawbacks in the assumption of the original EOQ model and many researchers have tried to improve it with different viewpoints, and the absence of the inventory quality is one of these shortcomings.

In a traditional EOQ model, there is no defect on the quality of inventory or production line. However, this assumption does not exist in the real world.

The relationship between quality and EOQ model has been diversely studied over the last decade and the work by **Porteus (1986)** was believed to be the starting point **.** In **Porteus (1986)** paper, the concept of quality control has been brought into a production system. Following his work, a stream of research has examined the relationship between the economics of inventory and quality of products **Rosenblatt and Lee (1986)** concluded that the presence of defective products motivates smaller lot sizes. In a subsequent paper. **Taguchi (1986)** redefined the product quality as the loss of society and proposed the quadratic quality loss function for measuring the quality cost. His quality loss function has been successfully applied in the on-line and off-line quality control problem. **Lee and Rosenblatt (1987)** considered using process inspection during the production run so that the shift to out-of-control state can be detected and restored earlier. Furthermore,**Tapiero (1987)** links optimal quality inspection policies and the resulting improvements in the manufacturing costs. **Fine (1988)** uses a stochastic dynamic programming model to characterize optimal inspection policies. Fine refine the original work of **Porteus (1986)** to allow smaller investments over time with potential process improvement of random magnitude. **Chand (1989)** brought the learning effect into the model. In a series of papers, **Cheng (1989)** has involved the production process reliability into a classic economic order quantity model. **Lee and Park (1991)** introduced some inspection and maintenance mechanisms in order to monitor the production process. They assumed that the shift of the production process follows an exponential distribution and extended it to type I inspection error. **Hong et al. (1993)** have established the relationship between process quality and investment. **Liou et al. (1994)** extended **Lee and Rosenblatt (1987)** work. They considered the shift of the production process following a general distribution, the inspection interval being arbitrary, and type I and type II inspection errors existing in the EMQ model. **Pulak and Al-Sultan (1996)** extended the application of rectifying inspection plan in determining the optimum process mean setting. For the rectifying inspection plan, the 100% inspection will be executed when the lot is rejected. All the non-conforming products during the inspection stage are usually replaced by conforming ones. **Makis and Fung (1998)** further incorporated a preventive maintenance policy into **Lee and Rosenblatt (1987)**deteriorating production system. **Salameh and Jaber (2000)** considered a special inventory situation where items, received or produced, are not of perfect quality. **Roan, et al. (2000)** incorporated the issues associated with production setup and raw material procurement into the classical process targeting problem. The product is assumed to have a lower specification limit, and the non-conforming items are scrapped with no salvage value. The production cost of an item is a linear function of the amount of the raw material used in producing the item. The proposed model aims to determine the optimum process target mean, production run size and material order quantity which minimize the expected total cost. **Shao, et al. (2000)** proposed a model where several grades of consumer specifications may be sold within the same market. In such situations, manufacturers may hold goods that have been rejected by one customer to sell the same goods to another consumer in the same market later. The expected profit function for such firms must consider the holding costs as well as the profits associated with this sales strategy. The model objective is to determine the optimum process target mean that maximizes the expected total profit.

Siddiqui (2001) developed a multi class targeting model under error and error free measurement system. The effect of measurement error eliminate by set optimal cut off points. The product uniformity also considered using Taguchi quadratic loss function. **Chen (2006)** proposes a modified EMQ model with the producer's loss and the customer's loss. The total inventory cost of his model includes the set-up cost, the holding cost, and the product cost. The 100% inspection, perfect rework, and imperfect rework of product are considered. By solving the modified model, he obtains both the optimum combination of EMQ and process mean in order to have the minimum total loss of society. However, his model does not consider the problem of economic specification limits selection for screening the non-conforming product. **Chen and Lai (2007)** presented a modified EMQ model by applying the modified Al-Sultan's model with **Taguchi (1986)** symmetric quadratic quality loss function. However, the asymmetric quadratic quality loss function maybe occurs in the industrial application. In this paper, the author presents a modified EMQ model based on the modified **Al-Sultan's (1994)** model with **Taguchi (1986)** asymmetric quadratic quality loss function for obtaining the maximum expected total profit per unit of time. The EMQ, maximum inventory level, and optimum process mean will be determined simultaneously. The advantage of this integrated model is to obtain a joint control of manufacturing quantity, inventory level, and production process.

The literature review revealed that the integrated economic manufacturing quantity with process targeting problem under imperfect production process quality and inspection process has not been modeled before. Hence, a need for research in this area exists. In this thesis work extends the work done by Chen (2006), their model is presented in (Chapter 5).This model is used as basis for the extension made in this thesis, though assuming different scenario (the objective function is maximization total profit and different assumptions) is presented in this chapter.

2.3 THESIS OBJECTIVES

The following objectives are planned to be accomplished during the course of the thesis

1. Develop a profit maximization model for determining the optimal manufacturing quantity and process targetting under perfect inspection system.
2. Develop a profit maximization model for determining the optimal manufacturing quantity and process targeting under imperfect inspection system.
3. Modify **Chen (2006)** Model for the case of imperfect inspection system.
4. Study the impact of measurement error on the models 2& 3.

2.4 A MODIFIED EMQ WITH PROCESS TARGETING MODEL

The problem formulated in this section will be used in different settings in this thesis. It will be the basis for the research work in all of the coming chapters.

2.4.1 DESCRIBTION OF THE PRODUCTION PROCESS

Consider a production process producing a product with a quality characteristic y that is normally distributed with unknown mean μ and known variance σ^2. Let L_2 denote the lower specification limit (LSL) for the quality characteristic and L_1 denote the upper specification limit (USL) for the quality characteristic. A product whose quality falls between the two limits ($L_2 < y < L_1$) is accepted and sold in a primary market at a regular price \$$a$, a product with quality characteristic below lower specification limit ($y < L_2$), is sold in a secondary market at reduced price \$$r$ where$(a > r)$. Finally the product whose quality characteristic fall above the upper specification limit ($y > L_1$) need to be reworked as shown in figure 3.

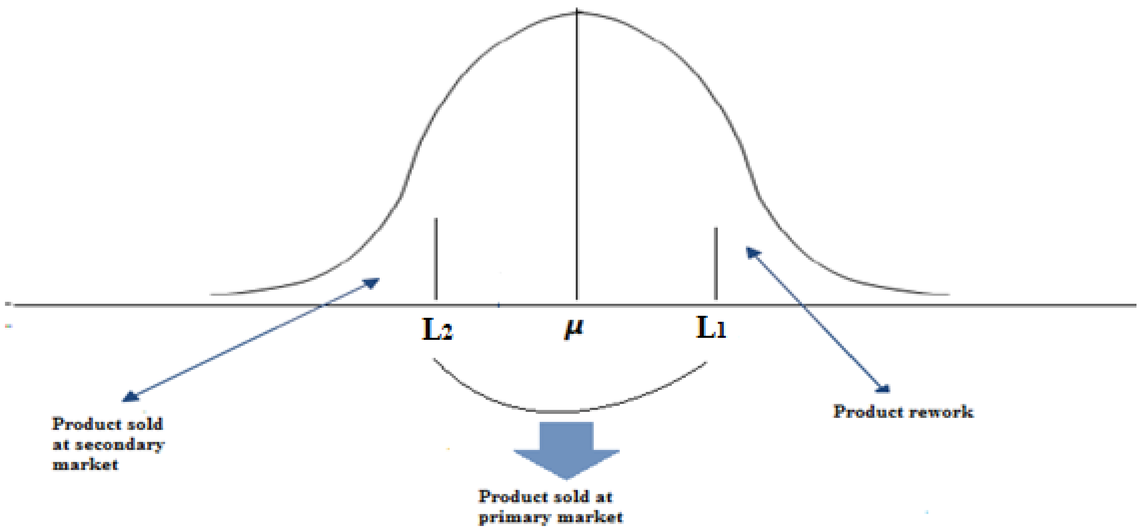

Figure 2.1 The classifications of the production process

A schematic flowchart for the production process described above is given in (Figure 2-2).

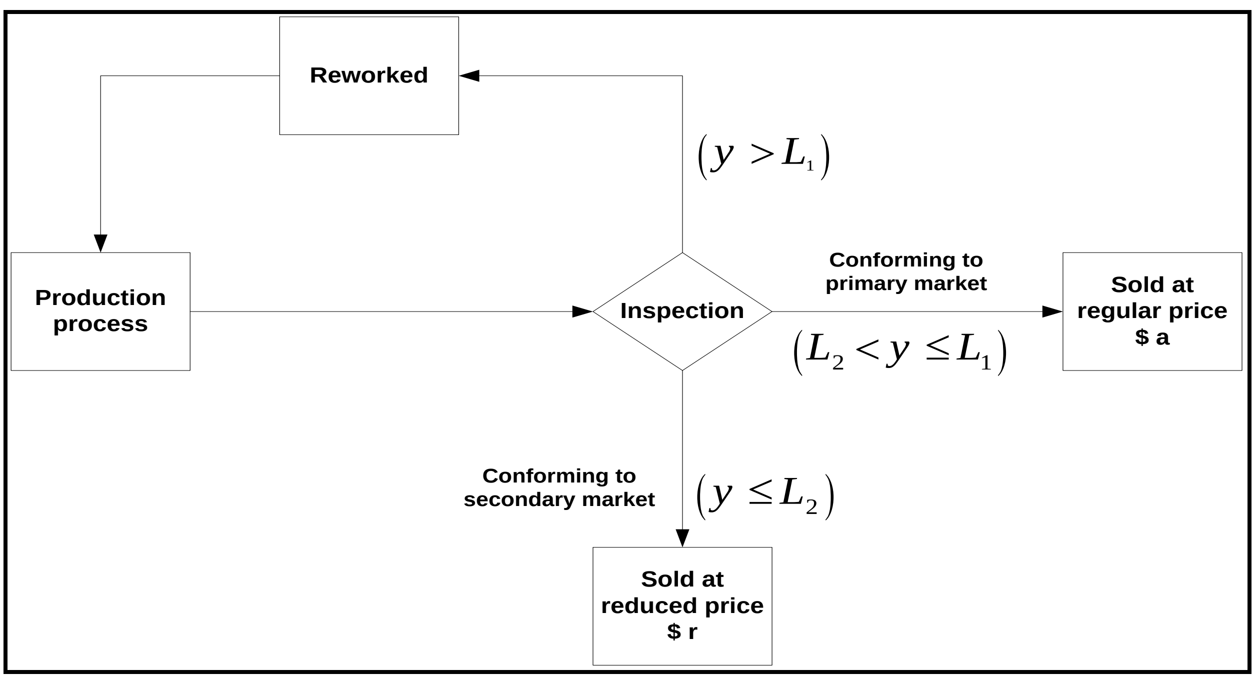

Figure 2.2The basic production process

The producer will ship the conforming units (primary and secondary units) to the customers. For the rework process. The product is reworked only once and the product of the rework will be conformance. Two cases are considered; in the first case the non-uniformity of the product is addressed. While, in the second case the uniformity of product is considered. The quality loss of conformance will be measured by Taguchi quadratic quality loss function. The problem is to find both the optimum combination of EMQ and process mean in order to have the maximum expected total profit per unit time. In real life, this problem can be applied in industrial area, e.g: a packing plant of cement factory, the plant consists of two processes which are a filling process and inspection process. Each cement bag processed by filling machine is moved to the loading and dispatching phase on conveyor belt. Inspection is performed by automatic weighing system. The quality characteristic which interested is the weight of cement bag.

2.4.2 MODEL ASSUMPTION

The following assumptions are made to develop the EMQ model.

1. The manufacturing system consists of a single process or machine engaged in the production of a single product.
2. There is no shortage cost.
3. Demand of the produced item is continuous and constant and the process has capacity to meet all demands (production rate > demand rate).
4. The price of per unit material in production is at a fixed cost.
5. The quality characteristic of product y is normally distributed with unknown mean μ and known standard deviation σ.

2.5 CONCLUSION

In this chapter, the literature in the area of economic manufacturing quantity and process targeting is reviewed, followed by the objectives of the thesis and a clear statement of the problem and the modeling framework for the problem. Next, Two model are given using 100% error-free inspection system.

CHAPTER 3

A MODIFIED (EMQ) MODEL WITH 100% ERROR-FREE INSPECTION SYSTEM

3.1 PREFACE

The purpose of this chapter is to develop a modified economic manufacturing quantity model for the problem stated in chapter 2, and will be described in section 3.2 of this chapter. The model developed in this chapter assumes an error-free 100% inspection policy for product quality control. The uniformity penalty similar to that of Taguchi quadratic loss function is introduced in section 3.3.2. The utility of the two models is demonstrated using an example from the literature. Sensitivity analysis is conducted for the model's parameters to assess the sensitivity of the results in section 3.4.

3.2 STATEMENT OF PROBLEM

Consider the production process that is mentioned in chapter two (figure 2-2). The quality characteristic of product is normally distributed with unknown μ and the known standard deviation σ. In the rework, the product is reworked only once and the product of the rework will have two chances, either be sold in a primary market at regular price $\$a$ or sold in a secondary market at reduced price $\$r$,$(a > r)$.The production cost is assumed to be known and constant $\$c$. After the items are being produced they are 100% inspected using an error-free measurement system. Two cases are considered. In the first case the non-uniformity of the product is addressed. While, in the second case the uniformity of product is addressed. The quality loss of conformance will be measured by Taguchi quadratic quality loss function. The problem is to find both the optimum combination of EMQ and process mean in order to have the maximum expected total profit per unit of time.

3.3 MODEL DEVELOPMENT

In this section, the modified of EMQ model with perfect measurement system will be presented. Two cases will be handled, the first case is assuming non-uniformity penalty will be conducted in the model (section 3.3.1), while the second case is assuming a uniformity penalty will be conducted (section 3.3.2).

3.3.1 EMQ MODEL WITH NON-UNIFORMITY PENALTY

The cost function of product is:

$$C = \begin{Bmatrix} A + R + c.y, If\ y > L_1 \\ A + c.y, If\ L_2 \le\ y \le L_1 \\ A + c.y \qquad , If\ y < L_2 \end{Bmatrix} \tag{3.1}$$

Where C is the cost function of product; A is the inspection cost per unit; c is the production cost per item; R is the rework cost per unit. Hence, the expected cost of a product is:

$$E(C) = \int_{L_1}^{\infty} (A + R + c.y) f(y) dy + \int_{L_2}^{L_1} (A + c.y)\ f(y) dy$$

$$+ \int_{-\infty}^{L_2} (A + c.y)\ f(y) dy \tag{3.2}$$

$$E(C) = A + R\int_{L_1}^{\infty} f(y)dy + c.\mu \tag{3.3}$$

Where:

$f(y) = \frac{1}{\sqrt{2\pi}} e^{\frac{1}{2\sigma}(y-\mu)^2}$ is the normal distribution density function with mean μ and standard deviation σ. Let $z = \frac{y-\mu}{\sigma}$ then,

$\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{z^2}$is the standard normal distribution density function. Now consider the following:

$\int_{-\infty}^{y} f(y)dy = \int_{-\infty}^{\frac{y-\mu}{\sigma}} \varphi(z)\, dz = \Phi(z)$ the standard normal cumulative distribution function.

Now let's define the following:

$\alpha = \frac{L_1-\mu}{\sigma}$, $\delta = \frac{L_2-\mu}{\sigma}$

$\beta = \Phi\left(\frac{L_1-\mu}{\sigma}\right) = \Phi(\alpha)$, $\gamma = \Phi\left(\frac{L_2-\mu}{\sigma}\right) = \Phi(\delta)$

Standardizing the normal distribution function to standard normal using the transformation $z = \frac{y-\mu}{\sigma}$ and β.

$$\therefore E(C) = A + R[1-\beta] + c.\mu \tag{3.4 }$$

The probability of the product is shipped to the customers is(β). Hence, the expected total cost of modified EMQ model with perfect production process quality and inspection process including the set-up cost , the holding cost, and the production cost is

$$ETC1 = S_t \,.\frac{D}{Q.\beta} + 0.5\,.Q.\beta.\left(1-\frac{O}{I}\right).h + D\,.E(C) \tag{3.5}$$

Let p is the percentage of rework items that would be sold in a primary market, and (*1-p*) is the percentage of rework items that would be sold in a secondary market. The revenue function of a product is:

$$V = \left\{ \begin{array}{cc} p.a + (1-p).r & ,If\ y > L_1 \\ a & ,If\ L_2 \leq\ y \leq L_1 \\ r & ,If\ y < L_2 \end{array} \right\} \qquad (3.6)$$

Where V is the revenue function of product under perfect measurement. Hence, the expected revenue of a product is:

$$E(V) = \int_{L_1}^{\infty} (p.a + (1-p).r) f(y)dy + a \int_{L_2}^{L_1} f(y)dy$$

$$+ r \int_{-\infty}^{L_2} f(y)dy \qquad (3.7)$$

Standardizing the normal distribution function to standard normal using the transformation $= \frac{y-\mu}{\sigma}$, β and γ.

$$E(V) = [p.a + (1-p).r].[1-\beta] + a.[\beta - \gamma] + r.\gamma \qquad (3.8)$$

The expected total profit is the sum of the revenues from selling final products (primary and secondary markets units) minus the setup, holding and product costs.Hence, the expected total profit is:

$$ETP1_{(Q,\mu)} = D.E(V) - ETC1 \qquad (3.9)$$

Substitute Eqs(3.5) and Eqs(3.8) into Eqs(3.9). Eq.(3.9) can be written as:

$$ETP1_{(Q,\mu)} = D.([p.a + (1-p).r].[1-\beta] + a.[\beta - \gamma] + r.\gamma) - S_t.\frac{D}{Q.\beta}$$

$$-0.5.Q.\beta.\left(1 - \frac{O}{I}\right).h - D.E(C) \quad (3.10)$$

3.3.2 EMQ MODEL WITH UNIFORMITY PENALTY

The cost function of product under uniformity penalty is:

$$C_I = \begin{Bmatrix} A + R + c.y + k(y-t)^2, If\ y > L_1 \\ A + c.y + k(y-t)^2, If\ L_2 \leq y \leq L_1 \\ A + c.y + k(y-t)^2 \qquad , If\ y < L_2 \end{Bmatrix} \quad (3.11)$$

where C_I is the cost function of product under uniformity penalty Hence, the expected cost of a product is:

$$E(C_I) = \int_{L_1}^{\infty} (A + R + c.y - k(y-t)^2) f(y)dy + \int_{L_2}^{L_1} (A + c.y - k(y-t)^2)\, f(y)dy$$

$$+ \int_{-\infty}^{L_2} (A + c.y + k(y-t)^2)\, f(y)dy \quad (3.12)$$

$$E(C_I) = A + R\int_{L_1}^{\infty} f(y)dy + c.\mu + \int_{-\infty}^{\infty} (k(y-t)^2) f(y)dy \quad (3.13)$$

Using the standard normal distribution and the notations defined in the previous section the Eq.(3.13) can be written as:

$$E(C_I) = A + R[1-\beta] + c.\mu$$

$$+ \int_{-\infty}^{\infty} (k(y-t)^2) f(y)dy \quad (3.14)$$

where :

$$\int_{-\infty}^{\infty}(k(y-t)^2)f(y)dy = (\mu-t)^2+\sigma^2 \tag{3.15}$$

$$\therefore E(C_I) = E(C) + (\mu-t)^2+\sigma^2 \tag{3.16}$$

The probability of the product is shipped to the customers is (β).Hence, the expected total cost of modified EMQ model with perfect production process quality, inspection process and uniformity penalty including the set-up cost , the holding cost, and the production cost is

$$ETC2 = S_t \cdot \frac{D}{Q.\beta} + 0.5 \cdot Q.\beta.\left(1-\frac{O}{I}\right).h + D \cdot E(C_I) \tag{3.17}$$

Let *p* is the percentage of rework items that would be sold in a primary market, and (*1-p*) is the percentage of rework items that would be sold in a secondary market, Hence ,the revenue function of a product is

$$V_I = \begin{Bmatrix} p.a+(1-p).r & ,If\ y > L_1 \\ a & ,If\ L_2 \le y \le L_1 \\ r & ,If\ y < L_2 \end{Bmatrix} \tag{3.18}$$

Where V_I is the revenue function of product under perfect measurement system and uniformity penalty .Hence, the expected revenue of a product is:

$$E(V_I) = \int_{L_1}^{\infty}(p.a+(1-p).r)f(y)dy + a\int_{L_2}^{L_1} f(y)dy + r\int_{-\infty}^{L_2} f(y)dy$$

$$= E(V) \tag{3.19}$$

The expected total profit is the sum of the revenues from selling final products (primary and secondary markets units) minus the setup, holding and product costs. Hence, the expected total profit is:

$$ETP2_{(Q,\mu)} = D.E(V) - ETC2 \tag{3.20}$$

Substitute Eqs(3.17) and Eqs(3.19) into Eqs(3.20). Eq.(3.20) can be written as:

$$ETP2_{(Q,\mu)} = D.(a.[\beta - \gamma] + r.\gamma + [p.a + (1-p).r].[1-\beta]) - S_t.\frac{D}{Q.\beta} - 0.5.Q.\beta.\left(1 - \frac{O}{I}\right).h - D\,E(C_I) \tag{3.21}$$

3.4 RESULTS AND SENSITIVITY ANALYSIS

In this section, an illustrative example for the model developed above is presented using parameters from the literature. This is followed by sensitivity analysis. For the numerical analysis, 'NLPSolve' command of Maple 12 software is used.

3.4.1 NUMERICAL EXAMPLE

Consider a production process, which produces products that have a normally distributed quality characteristic y with standard deviation $\sigma = 1.3$, a product whose quality characteristic falls between the two limits (10< y <15), then it sold in a primary market at a regular price $80 ,a product with quality characteristic below lower specification limit (y <10), then it sold in a secondary market at reduced price $67.5, a product whose quality characteristic fall above the upper specification limit (y >15) need to be reworked with cost $2.The processing cost of an item is $30, and the inspection cost of a product is

0.2. Knowing that: I=100,O=80,S_t=20,h=1,D=2000,p=0.8.Table 3.1 below summarizes the obtained results

Table 3.1 The optimum combination (Q*,μ*) *and* ETPvalues

	Profit model without uniformity penalty	**Profit model with uniformity penalty**
$Q*$	632	633
$\mu*$	10	10.41
$ETP_{(Q,\mu)}$	4.68907 10^5	4.53027 10^5

From the table above, it is clear that the expected total profit in the modified EMQ model without uniformity penalty is bigger than a modified EMQ model with uniformity penalty. The reason is that a Taguchi quadratic loss function is added in the production cost term in second model which increase process mean and economic manufacturing quantity, consequently, decrease the expected total profit.

3.4.2 SENSITIVITY ANALYSIS FOR THE PARAMETERS

In this section, the effect of the process standard deviation σ, demand quantity and the cost parameters, on the target meant value, economic manufacturing quantity value and the expected profit values is studied.

Table 3.2 The sensitivity analysis of the process standard deviation on the modified EMQ model

σ		EMQ model without uniformity penalty				EMQ model with uniformity penalty			
		μ	Q	$ETP1$	Change percentage	μ	Q	$ETP2$	Change percentage
1.95	+%50	10.65	636	4.50760 10^5	-3.87%	10.92	636	4.35531 10^5	-3.86%
1.625	+%25	10.31	634	4.59059 10^5	-2.21%	10.63	634	4.43058 10^5	-2.20%
1.3	original	10	632	4.68907 10^5	0	10.41	633	4.53027 10^5	0
0.975	-%25	10	631	4.86256 10^5	3.69%	10.35	632	4.69752 10^5	3.69%
0.65	-%50	10	628	5.05950 10^5	7.89%	10.26	631	4.88878 10^5	7.90%

From the table above, we have following conclusions. First, it is clear that as the process standard deviation increase, the process mean and economic manufacturing quantity increases also, because as standard deviation increase. Hence, the demand is constant, the proportion of a primary market decrease, while the proportion of a secondary market and rework increase. So the process mean will shift to the right side to compensate the increase which occurs in the secondary market proportion. Second, as the process standard deviation increase, the expected total profit decrease. This is because, the proportion of a primary market decrease.

Now, the effect of the demand quantity on the modified EMQ model is stated on table 3-3 below.

Table 3.3 The sensitivity analysis of the D on the modified EMQ model

D		EMQ model without uniformity penalty				EMQ model with uniformity penalty			
		μ	Q	$ETP1$	Change percentage	μ	Q	$ETP2$	Change percentage
3000	+%50	10	775	7.03325 10^5	49.99%	10.41	776	6.79505 10^5	50.0%
2500	+%25	10	707	5.86117 10^5	24.99%	10.41	708	5.66267 10^5	24.99%
2000	original	10	632	4.68907 10^5	0	10.41	633	4.53027 10^5	0
1500	-%25	10	547	3.51694 10^5	-24.99%	10.41	548	3.39784 10^5	-33.32%
1500	-%50	10	447	2.34479 10^5	-49.99%	10.41	448	2.26539 10^5	-49.99%

From Table 3-3 it is clear that as the demand quantity increase, consequently the economic manufacturing quantity and the expected profit increase also. The demand quantity *D*, do not affect the process mean. This is obvious, because of a fact that, the process mean depends mainly on the process itself (e.g. cost parameters), not on the external parameters such as a demand.

Now, the effect of the three cost parameters (c, A and R) on the modified EMQ model is stated on tables 3-4 and 3-5 below.

Table 3.4 The sensitivity analysis of the A,R and c the modified EMQ model

SENSITIVITY		**EMQ model without uniformity penalty**			
PARAMETER	CHANGE	μ	Q	$ETP1$	CHANGE PERCENTAGE
A=0.3 c=45 R=3	+%50	10	629	$4.56678\ 10^5$	-2.60%
A=0.25 c=37.5 R=2.5	+%25	10	630	$4.59986\ 10^5$	-1.90%
A=0.2 c=30 R=2	original	10	632	$4.68907\ 10^5$	0
A=0.15 c=22.5 R=1.5	-%25	10.2	634	$4.78953\ 10^5$	-2.14%
A=0.1 c=15 R=1	-%50	10.35	635	$4.81305\ 10^5$	-2.64%

Table 3.5 The sensitivity analysis of the A,R and c the modified EMQ model with uniformity penalty

SENSITIVITY		**EMQ model with uniformity penalty**			
PARAMETER	CHANGE	μ	Q	$ETP1$	CHANGE PERCENTAGE
A=0.3 c=45 R=3 k=0.48	+%50	10.32	631	$4.40043\ 10^5$	-2.86%
A=0.25 c=37.5 R=2.5 k=0.4	+%25	10.37	632	$4.47843\ 10^5$	-1.14%
A=0.2 c=30 R=2 k=0.32	original	10.41	633	$4.53027\ 10^5$	0
A=0.15 c=22.5 R=1.5 k=0.24	-%25	10.86	634	$4.63027\ 10^5$	2.20%
A=0.1 c=15 R=1 k=0.16	-%50	11.07	635	$4.66024\ 10^5$	2.86%

From the table above, it is clear that as the inspection cost, rework cost, and processing cost increase, the process mean economic manufacturing quantity decrease and the expected total profit decrease.

3.5 CONCLUSION

In this chapter, a modified EMQ model with perfect measurement system is developed for a process targeting problem. Two cases are considered. In the first case the non-uniformity of the product is addressed. While, in the second case the uniformity of product is considered. The solutions were generated for an example contains some data from the process targeting and economic manufacturing quantity literature. Sensitivity analysis for the process standard deviation and the cost parameters was conducted for each case. In the model developed in this chapter, inspection is assumed to be error free. This assumption is relaxed in chapter 4.

CHAPTER 4

A MODEFIED (EMQ) MODEL WITH 100% ERROR-PRONE INSPECTION SYSTEM

4.1 PERFACE

The purpose of this chapter is to extend the model in chapter 3 to the situation where the inspection system is error prone. The literature demonstrated that inspection is error prone **Duffuaa and Siddiqui (2003)**. The motivation behind this extension is the fact that measurement system can cause considerable loss due to misclassification of products. This loss can be either a loss in profit due to misclassifying a higher quality product as a lower quality product, or vice versa. The loss per item due to this error may seem small; however, the overall loss may be in millions (considering millions of items produced per year). The rest of the assumptions and conditions under which the model has been developed are the same as chapter three for the same production process described in chapter two (Section 2.4). This chapter is organized as follows: the problem description is presented in section 4.2, and the model development in section 4.3. Results and sensitivity analysis for the model's parameters in section 4.4. The chapter is concluded in section 4.5.

4.2 STATEMENT OF PROBLEM

Consider a production process that is mentioned in chapter two (figure 2-2). The quality characteristic of product is normally distributed with unknown μ and the known standard deviation σ. A product whose quality falls between the two limits ($L_2 < y < L_1$) is accepted and sold in a primary market at a regular price \$$a$, a product with quality characteristic below lower specification limit ($y < L_2$), is sold in a secondary market at reduced price \$$r$ where$(a > r)$. Finally the product whose quality characteristic fall above the upper specification limit ($y > L_1$) need to be reworked. Now consider the case where the inspection system is error prone. Thus, it tends to misclassify the produced items according to their quality characteristic level. Hence, the measured quality characteristic has an observed value (i.e. x) which is different from the actual value (i.e. y) due to the presence of inspection error. Both quality characteristics (the observed X and the actual Y) are normally distributed and the relation between them is the following

$$X = Y + \varepsilon \qquad (4.1)$$

Where ε is a random variable which represents the inspection error has a normal distribution with mean 0 and known standard deviation $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. The correlation coefficient between the actual and observed quality characteristics ρ is given by the formula

$$\rho = 1 - \frac{\sigma_\varepsilon^2}{\sigma_x^2} = \frac{\sigma_y^2}{\sigma_x^2} \qquad (4.2)$$

Since, the actual and observed quality characteristics are both normally distributed; then, their joint distribution is bivariate normal distribution which is given by

$$h(x,y) = \frac{1}{2\pi\sigma_y\sigma_x\sqrt{1-\rho^2}} e^{\frac{-1}{2(1-\rho^2)}*\left(\frac{(y-\mu)^2}{\sigma_y^2} + \frac{(x-\mu)^2}{\sigma_x^2} - \frac{2*\rho*(x-\mu)*(y-\mu)}{\sigma_y*\sigma_x}\right)} \tag{4.3}$$

To reduce the effect of the inspection error, instead of using the original limits (L_1 and L_2) for inspection, we based the inspection on new limits (cut off points) and use these new limits as the classification criteria (figure 4-1).

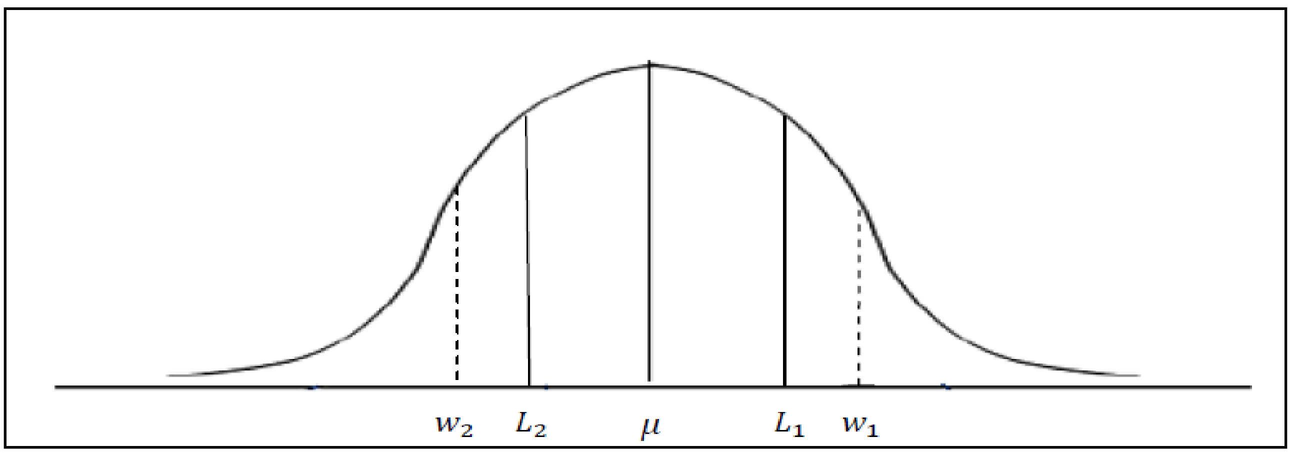

Figure 4.1 Cut off points for the inspection error

The location of these cut off points depends on many factors, such as: the loss in profit due to misclassifying a higher quality product into a lower quality, the penalty associated with misclassifying a lower quality product with a higher quality, the value of the mean, the value of the standard deviation…etc.

Prior to model development, the types of losses and penalties associated with misclassification of the items will be described. As shown in table below.

Table 4.1 Penalties Associated with Misclassifications

OBSERVED	ACTUAL		
	Primary Market	Secondary Market	Rework
Primary Market	-	b_{ps}	b_{pr}
Secondary Market	Implicit a-r	-	b_{sr}
Rework	Implicit a-R	Implicit r-R	-

First, there are three type of loss in profit due to misclassify a higher quality product as a lower quality product (table 4-2).

Table 4.2 Loss in profit due to product misclassifications

Loss in profit	Due to
a-r	Classify a primary market item as a secondary market item
a	Classify a primary market item as rework item
r	Classify a secondary market item as rework item

Also, there are three types of penalties associated with misclassifying a lower quality product as a higher quality product. These penalties reflect on replacement and warranty costs and loss of good will and customer dissatisfaction (table 4-3).

Table 4.3 Penalties due to product misclassifications

Penalty	Due to
b_{sr}	Classify a secondary market item as a rework item
b_{ps}	Classify a primary market item as a secondary item
b_{pr}	Classify a primary market item as a rework item

The problem we are trying to solve here is to develop an integrated EMQ with process targeting model that provides the optimum EMQ, process target mean and cut off points.

4.3 MODEL DEVELOPMENT

In this section, the modified of EMQ model with error-prone will be presented. The modified of EMQ model will be solved to find the optimum value of the, economic manufacturing quantity, process target mean and the value of the two cut off points too.

4.3.1 EMQ MODEL WITH NON-UNIFORMITY PENALTY AND ERROR-PRONE

The cost function of product under non-uniformity penalty and error inspection is:

$$C_e = \begin{cases} A + c.y + R & x \geq w_1, y > L_1 \\ A + c.y + R & x \geq w_1,\ L_2 \leq y \leq L_1 \\ A + c.y + R & x \geq w_1, y < L_2 \\ & \\ A + c.y + b_{pr} & w_2 \leq x \leq w_1, y > L_1 \\ A + c.y & w_2 \leq x \leq w_1,\ L_2 \leq y \leq L_1 \\ A + c.y + b_{ps} & w_2 \leq x \leq w_1, y < L_2 \\ & \\ A + c.y + b_{sr} & x < w_2, y > L_1 \\ A + c.y & x < w_2,\ L_2 \leq y \leq L_1 \\ A + c.y & x < w_2, y < L_2 \end{cases} \tag{4.4}$$

Where C_e is the cost function of product under non-uniformity penalty and error inspection. Hence, the expected cost function of product under non-uniformity penalty and error inspection is:

$$\begin{aligned} E(C_e) = & \int_{w_1}^{\infty}\int_{L_1}^{\infty}[A + R + c.y]h(x,y)\,dydx + \int_{w_1}^{\infty}\int_{L_2}^{L_1}[A + c.y + R]h(x,y)\,dydx \\ & + \int_{w_1}^{\infty}\int_{-\infty}^{L_2}[A + c.y + R\,]h(x,y)\,dydx \\ & + \int_{w_2}^{w_1}\int_{L_1}^{\infty}\left[A + c.y + b_{pr}\right]h(x,y)\,dydx \\ & + \int_{w_2}^{w_1}\int_{L_2}^{L_1}[A + c.y]h(x,y)\,dydx \\ & + \int_{w_2}^{w_1}\int_{-\infty}^{L_2}\left[A + c.y + b_{ps}\right]h(x,y)\,dydx \\ & + \int_{-\infty}^{w_2}\int_{L_1}^{\infty}[A + c.y + b_{sr}]h(x,y)\,dydx \\ & + \int_{-\infty}^{w_2}\int_{L_2}^{L_1}[A + c.y]h(x,y)\,dydx \\ & + \int_{-\infty}^{w_2}\int_{-\infty}^{L_2}[A + c.y]h(x,y)\,dydx \end{aligned} \tag{4.5}$$

$$E(C_e) = A\int_{w_1}^{\infty}\int_{-\infty}^{\infty} h(x,y)\,dydx + c\int_{w_1}^{\infty}\int_{-\infty}^{\infty} y.h(x,y)\,dydx + dydx$$

$$+ R\int_{w_1}^{\infty}\int_{-\infty}^{\infty} h(x,y)\,dydx + A\int_{w_2}^{w_1}\int_{-\infty}^{\infty} h(x,y)\,dydx$$

$$+ c\int_{w_2}^{w_1}\int_{-\infty}^{\infty} y.h(x,y)\,dydx + b_{pr}\int_{w_2}^{w_1}\int_{L_1}^{\infty} h(x,y)\,dydx$$

$$+ b_{ps}\int_{w_2}^{w_1}\int_{-\infty}^{L_2} h(x,y)\,dydx + A\int_{-\infty}^{w_2}\int_{-\infty}^{\infty} h(x,y)\,dydx$$

$$+ c\int_{-\infty}^{w_2}\int_{-\infty}^{\infty} y.h(x,y)\,dydx \ + \ b_{sr}\int_{-\infty}^{w_2}\int_{L_1}^{\infty} h(x,y)\,dydx \qquad (4.6)$$

Now, add the similar term together

$$E(C_e)$$

$$= A + c.\mu + R\int_{w_1}^{\infty} f(x)\,dx + b_{pr}\int_{w_2}^{w_1}\int_{L_1}^{\infty} h(x,y)\,dydx + b_{ps}\int_{w_2}^{w_1}\int_{-\infty}^{L_2} h(x,y)\,dydx$$

$$+ \ b_{sr}\int_{-\infty}^{w_2}\int_{L_1}^{\infty} h(x,y)\,dydx \qquad (4.7)$$

Now consider the following notations:

Let $f(y) = \frac{1}{\sqrt{2\pi}} e^{\frac{1}{2\sigma}(y-\mu)^2}$ is the normal distribution density function with mean μ and standard deviation σ. Let $z = \frac{y-\mu}{\sigma}$ then,

$\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{z^2}$is the standard normal distribution density function. Now consider the following:

$\int_{-\infty}^{y} f(y)dy = \int_{-\infty}^{\frac{y-\mu}{\sigma}} \varphi(z)\,dz = \Phi(z)$ the standard normal cumulative distribution function.

Now let's define the following:

$\alpha = \frac{L_1-\mu}{\sigma}$, $\delta = \frac{L_2-\mu}{\sigma}$

$\beta = \Phi\left(\frac{L_1-\mu}{\sigma}\right) = \Phi(\alpha)$, $\gamma = \Phi\left(\frac{L_2-\mu}{\sigma}\right) = \Phi(\delta)$

$\omega = \frac{w_1-\mu}{\sqrt{\sigma_y^2+\sigma_e^2}}$, $\eta = \frac{w_2-\mu}{\sqrt{\sigma_y^2+\sigma_e^2}}$

$\Omega = \Phi\left(\frac{w_1-\mu}{\sqrt{\sigma_y^2+\sigma_e^2}}\right) = \Phi(\omega)$, $\xi = \Phi\left(\frac{w_2-\mu}{\sqrt{\sigma_y^2+\sigma_e^2}}\right) = \Phi(\eta)$

Accordingly equation (3) can be written as:

$$E(C_e) = A + c.\mu + R.(1-\Omega) + b_{pr}\int_{w_2}^{w_1}\int_{L_1}^{\infty} h(x,y)\,dydx + b_{ps}\int_{w_2}^{w_1}\int_{-\infty}^{L_2} h(x,y)\,dydx + b_{sr}\int_{-\infty}^{w_2}\int_{L_1}^{\infty} h(x,y)\,dydx \tag{4.8}$$

The probability of the product is shipped to the customers is (Ω) .Hence, the expected total cost of modified EMQ model with imperfect production process quality and inspection process including the set-up cost , the holding cost, and the production cost is

$$ETC3 = S_t \,.\frac{D}{Q.\Omega} + 0.5\,.Q.\Omega.\left(1-\frac{O}{I}\right).h + D\,.E(C_e) \tag{4.9}$$

Let p is the percentage of rework items that would be sold in a primary market, and (*1*-*p*) is the percentage of rework items that would be sold in a secondary market. Hence, the revenue function of a product is

$$V_e = \begin{Bmatrix} p.a + (1-p).r & x \geq w_1, y > L_1 \\ p.a & x \geq w_1,\ L_2 \leq\ y \leq L_1 \\ (1-p).r & x \geq w_1, y < L_2 \\ \\ a & w_2 \leq\ x \leq w_1, y > L_1 \\ a & w_2 \leq\ x \leq w_1,\ L_2 \leq\ y \leq L_1 \\ a & w_2 \leq\ x \leq w_1, y < L_2 \\ \\ r & x < w_2, y > L_1 \\ a - r & x < w_2,\ L_2 \leq\ y \leq L_1 \\ r & x < w_2, y < L_2 \end{Bmatrix} \tag{4.10}$$

Where V_{Ie} is the revenue function of product under non-uniformity penalty and error inspection .Hence, the expected revenue of a product is:

$$\begin{aligned} E(V_e) = {} & [p.a + (1-p).r] \int_{w_1}^{\infty} \int_{L_1}^{\infty} h(x,y)\, dydx + p.a \int_{w_1}^{\infty} \int_{L_2}^{L_1} h(x,y)\, dydx \\ & + (1-p).r \int_{w_1}^{\infty} \int_{-\infty}^{L_2} h(x,y)\, dydx + a \int_{w_2}^{w_1} \int_{L_1}^{\infty} h(x,y)\, dydx \\ & + a \int_{w_2}^{w_1} \int_{L_2}^{L_1} h(x,y)\, dydx + a \int_{w_2}^{w_1} \int_{-\infty}^{L_2} h(x,y)\, dydx \\ & + r \int_{-\infty}^{w_2} \int_{L_1}^{\infty} h(x,y)\, dydx + (a-r) \int_{-\infty}^{w_2} \int_{L_2}^{L_1} h(x,y)\, dydx \\ & + r \int_{-\infty}^{w_2} \int_{-\infty}^{L_2} h(x,y)\, dydx \end{aligned} \tag{4.11}$$

$$\begin{aligned} E(V_e) = {} & a(\Omega - \xi) + p.a \int_{w_1}^{\infty} \int_{L_2}^{\infty} h(x,y)\, dydx + (1-p).r \int_{w_1}^{\infty} \int_{L_1}^{\infty} h(x,y)\, dydx \\ & + (1-p).r \int_{w_1}^{\infty} \int_{-\infty}^{L_2} h(x,y)\, dydx - r \int_{-\infty}^{w_2} \int_{L_2}^{L_1} h(x,y)\, dydx \\ & + r \int_{-\infty}^{w_2} \int_{L_1}^{\infty} h(x,y)\, dydx \\ & + r \int_{-\infty}^{w_2} \int_{-\infty}^{L_2} h(x,y)\, dydx \end{aligned} \tag{4.12}$$

The expected total profit is the sum of the revenues from selling final products (primary and secondary markets units) minus the setup, holding and product costs. Hence, the expected total profit is:

$$ETP3_{(Q,\mu,w_1,w_2)} = D.E(V_e) - ETC3 \qquad (4.13)$$

4.3.2 EMQ MODEL WITH UNIFORMITY PENALTY AND ERROR-PRONE

The cost function of product under uniformity penalty and error inspection is:

$$C_{Ie} = \begin{cases} A + R + c.y + k(y-t)^2 & x \geq w_1, y > L_1 \\ A + c.y + R + k(y-t)^2 & x \geq w_1,\ L_2 \leq\ y \leq L_1 \\ A + c.y + R + k(y-t)^2 & x \geq w_1, y < L_2 \\ \\ A + c.y + b_{pr} + k(y-t)^2 & w_2 \leq\ x \leq w_1, y > L_1 \\ A + c.y + k(y-t)^2 & w_2 \leq\ x \leq w_1,\ L_2 \leq\ y \leq L_1 \\ A + c.y + b_{ps} + k(y-t)^2 & w_2 \leq\ x \leq w_1, y < L_2 \\ \\ A + c.y + b_{sr} + k(y-t)^2 & x < w_2, y > L_1 \\ A + c.y + k(y-t)^2 & x < w_2,\ L_2 \leq\ y \leq L_1 \\ A + c.y + k(y-t)^2 & x < w_2, y < L_2 \end{cases} \qquad (4.14)$$

Where C_{Ie} is the cost function of product under uniformity penalty and error inspection.

The expected cost function of product under uniformity penalty and error inspection is:

$$E(C_{Ie}) = \int_{w_1}^{\infty}\int_{L_1}^{\infty}[A - R - c.y - k(y-t)^2]h(x,y)\,dydx$$

$$+ \int_{w_1}^{\infty}\int_{L_2}^{L_1}[A + c.y + R + k(y-t)^2]h(x,y)\,dydx$$

$$+ \int_{w_1}^{\infty}\int_{-\infty}^{L_2}[A + c.y + R + k(y-t)^2\,]h(x,y)\,dydx$$

$$+ \int_{w_2}^{w_1}\int_{L_1}^{\infty}\left[A + c.y + b_{pr} + k(y-t)^2\right]h(x,y)\,dydx$$

$$+ \int_{w_2}^{w_1}\int_{L_2}^{L_1}[A + c.y + k(y-t)^2]h(x,y)\,dydx$$

$$+ \int_{w_2}^{w_1}\int_{-\infty}^{L_2}\left[A + c.y + b_{ps} + k(y-t)^2\right]h(x,y)\,dydx$$

$$+ \int_{-\infty}^{w_2}\int_{L_1}^{\infty}[A + c.y + b_{sr} + k(y-t)^2]h(x,y)\,dydx$$

$$+ \int_{-\infty}^{w_2}\int_{L_2}^{L_1}[A + c.y + k(y-t)^2]h(x,y)\,dydx$$

$$+ \int_{-\infty}^{w_2}\int_{-\infty}^{L_2}[A + c.y + k(y-t)^2]h(x,y)\,dydx \qquad (4.15)$$

$$E(C_{Ie}) = A\int_{w_1}^{\infty}\int_{-\infty}^{\infty} h(x,y)\,dydx + c\int_{w_1}^{\infty}\int_{-\infty}^{\infty} y.\,h(x,y)\,dydx$$

$$+\int_{w_1}^{\infty}\int_{-\infty}^{\infty}[k(y-t)^2]h(x,y)\,dydx + R\int_{w_1}^{\infty}\int_{-\infty}^{\infty} h(x,y)\,dydx$$

$$+A\int_{w_2}^{w_1}\int_{-\infty}^{\infty} h(x,y)\,dydx + c\int_{w_2}^{w_1}\int_{-\infty}^{\infty} y.\,h(x,y)\,dydx$$

$$+\int_{w_2}^{w_1}\int_{-\infty}^{\infty}[k(y-t)^2]h(x,y)\,dydx + b_{pr}\int_{w_2}^{w_1}\int_{L_1}^{\infty} h(x,y)\,dydx$$

$$+b_{ps}\int_{w_2}^{w_1}\int_{-\infty}^{L_2} h(x,y)\,dydx + A\int_{-\infty}^{w_2}\int_{-\infty}^{\infty} h(x,y)\,dydx$$

$$+c\int_{-\infty}^{w_2}\int_{-\infty}^{\infty} y.\,h(x,y)\,dydx + \int_{-\infty}^{w_2}\int_{-\infty}^{\infty}[k(y-t)^2]h(x,y)\,dydx$$

$$+b_{sr}\int_{-\infty}^{w_2}\int_{L_1}^{\infty} h(x,y)\,dydx \qquad (4.16)$$

Now, add the similar term together

$$E(C_{Ie}) = A + c.\mu + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[k(y-t)^2]h(x,y)\,dydx + R\int_{w_1}^{\infty}\int_{-\infty}^{\infty} h(x,y)\,dydx$$

$$+b_{pr}\int_{w_2}^{w_1}\int_{L_1}^{\infty} h(x,y)\,dydx + b_{ps}\int_{w_2}^{w_1}\int_{-\infty}^{L_2} h(x,y)\,dydx$$

$$+b_{sr}\int_{-\infty}^{w_2}\int_{L_1}^{\infty} h(x,y)\,dydx \qquad (4.17)$$

$$E(C_{Ie}) = A + c.\mu + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[k(y-t)^2]h(x,y)\,dydx + R\int_{w_1}^{\infty}f(x)dx$$
$$+ b_{pr}\int_{w_2}^{w_1}\int_{L_1}^{\infty}h(x,y)\,dydx + b_{ps}\int_{w_2}^{w_1}\int_{-\infty}^{L_2}h(x,y)\,dydx$$
$$+ b_{sr}\int_{-\infty}^{w_2}\int_{L_1}^{\infty}h(x,y)\,dydx \qquad (4.18)$$

Using the standard normal distribution and the notations defined in the previous section the Eq.(4.18) can be written as

$$E(C_{Ie}) = A + c.\mu + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[k(y-t)^2]h(x,y)\,dydx + R(1-\Omega)$$
$$+ b_{pr}\int_{w_2}^{w_1}\int_{L_1}^{\infty}h(x,y)\,dydx + b_{ps}\int_{w_2}^{w_1}\int_{-\infty}^{L_2}h(x,y)\,dydx$$
$$+ b_{sr}\int_{-\infty}^{w_2}\int_{L_1}^{\infty}h(x,y)\,dydx \qquad (4.19)$$

The probability of the product is shipped to the customers is (Ω).Hence, the expected total cost of modified EMQ model with imperfect production process quality and inspection process including the set-up cost , the holding cost, and the production cost is

$$ETC4 = S_t\,.\frac{D}{Q.\Omega} + 0.5\,.Q.\Omega.\left(1-\frac{O}{I}\right).h + D\,.E(C_{Ie}) \qquad (4.20)$$

Let p is the percentage of rework items that would be sold in a primary market, and (*1-p*) is the percentage of rework items that would be sold in a secondary market, Hence ,the revenue function of a product is

$$V_{Ie} = \begin{cases} p.a + (1-p).r & x \geq w_1, y > L_1 \\ p.a & x \geq w_1,\ L_2 \leq\ y \leq L_1 \\ (1-p).r & x \geq w_1, y < L_2 \\ \\ a & w_2 \leq\ x \leq w_1, y > L_1 \\ a & w_2 \leq\ x \leq w_1,\ L_2 \leq\ y \leq L_1 \\ a & w_2 \leq\ x \leq w_1, y < L_2 \\ \\ r & x < w_2, y > L_1 \\ a-r & x < w_2,\ L_2 \leq\ y \leq L_1 \\ r & x < w_2, y < L_2 \end{cases} \tag{4.21}$$

Where V_{Ie} is the revenue function of product under uniformity penalty and error inspection. Hence, the expected revenue of a product is

$$E(V_{Ie})$$
$$= [p.a + (1-p).r]\int_{w_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dydx + p.a\int_{w_1}^{\infty}\int_{L_2}^{L_1} h(x,y)\,dydx$$
$$+ (1-p).r\int_{w_1}^{\infty}\int_{-\infty}^{L_2} h(x,y)\,dydx + a\int_{w_2}^{w_1}\int_{L_1}^{\infty} h(x,y)\,dydx + a\int_{w_2}^{w_1}\int_{L_2}^{L_1} h(x,y)\,dyx$$
$$+ a\int_{w_2}^{w_1}\int_{-\infty}^{L_2} h(x,y)\,dydx + r\int_{-\infty}^{w_2}\int_{L_1}^{\infty} h(x,y)\,dydx + (a-r)\int_{-\infty}^{w_2}\int_{L_2}^{L_1} h(x,y)\,dydx$$
$$+ r\int_{-\infty}^{w_2}\int_{-\infty}^{L_2} h(x,y)\,dydx$$
$$= E(V_e) \tag{4.22}$$

The expected total profit is the sum of the revenues from selling final products (primary and secondary markets units) minus the setup, holding and product costs. Hence, the expected total profit is:

$$ETP4_{(Q,\mu,w_1,w_2)} = D.E(V_e) - ETC4 \tag{4.23}$$

4.4 RESULTS AND SENSITIVITY ANALYSIS

In this section, an illustrative example for the model developed above is presented using parameters from the literature. This is followed by sensitivity analysis. For the numerical analysis, ‘NLPSolve’ command of Maple 12 is used.

4.4.1 NUMERICAL EXAMPLE

Consider a production process, which produces products that have a normally distributed quality characteristic y, a product will be inspected for determining if it is sold to a primary market, or secondary market, or rework. A product whose quality characteristic falls between the two limits (10< y <15), then it sold in a primary market at a regular price $\$80$,a product with quality characteristic below lower specification limit (y <10), then it sold in a secondary market at reduced price $\$67.5$, a product whose quality characteristic fall above the upper specification limit (y >15) need to be reworked with cost \$2.The inspection system tends to make some classification error, if a secondary market product is classified as a primary market product, then the producer compensates the customer with penalty$b_{ps} = a - r$, if a rework item is classified as a primary market product, then the producer compensates the customer with penalty$b_{pr} = a$, finally, if a secondary market product is classified as a rework, then the producer compensates the customer with penalty$b_{sr} = r$. The processing cost of an item is \$30, and the inspection cost per item of material is \$0.2. The process standard deviation is 1.3 .The error in the measuring system is represented by the correlation co-efficient having the value ρ=0.85 , i.e.σ_{ε}=0.557 and σ_x=1.4143. The uniform search is conducted over w_2∈[8,12] and

$w_1 \in$[13,17]. Knowing that: *I=100,O=80,S_t=20,h=1,D=2000.*Table 4.4 below summarizes the obtained results

Table 4.4 The optimum values of the modified EMQ model with measurement error model

	EMQ model without uniformity penalty	**EMQ model with uniformity penalty**
$Q*$	632	633
$\mu*$	10	10.67
w_1^*	14	14
w_2^*	9	9
ETP	4.02364 10^5	3.86391 10^5

From the table above, it is clear that the expected total profit in a modified EMQ model without uniformity penalty is bigger than a modified EMQ model with uniformity penalty. The reason is that a taguchi quadratic loss function is added in the production cost term in second model which increase process mean and economic manufacturing quantity, consequently, decrease the expected total profit. The performance of the system under measurement error is less than when the system error-free (Chapter 3), because the total expected profit under measurement error is less than when the system under error-free , due to inspection error.

4.4.2 SENSITIVITY ANALYSIS FOR THE PARAMETERS

In this section, the sensitivity analysis for the correlation coefficient ρ and the penalty costs is conducted, to study their effect on the model and the results

First, the effect of the correlation coefficient between actual quality characteristic y and the observed quality characteristic x is studied. Table 4-5, below show the effect of the correlation coefficient on the modified EMQ model.

Table 4.5 The sensitivity analysis of the correlation coefficient on the modified EMQ model with measurement error

ρ	**EMQ model without uniformity penalty**						**EMQ model with uniformity penalty**					
	w_1	w_2	μ	Q	$ETP3$	Change percentage	w_1	w_1	μ	Q	$ETP4$	Change percentage
0.95	14	9	10	636	$4.56891\ 10^5$	13.55%	14	9	10.67	636	$4.27937\ 10^5$	13.56%
0.9	14	9	10	634	$4.32474\ 10^5$	6.96%	14	9	10.67	634	$4.13824\ 10^5$	7.1%
0.85(original)	**14**	**9**	**10**	**632**	$\mathbf{4.02364\ 10^5}$	**0%**	**14**	**9**	**10.67**	**633**	$\mathbf{3.86391\ 10^5}$	**0%**
0.8	14	9	10	631	$3.70024\ 10^5$	-8.03%	14	9	10.67	632	$3.55093\ 10^5$	-8.0%
0.75	14	9	10	630	$3.25544\ 10^5$	-19.09%	14	9	10.67	631	$3.13324\ 10^5$	-18.91%

It is clear that as the correlation coefficient ρ increases the error standard deviation decreases as well. Therefore, as the correlation coefficient value increased the deviation between the actual and observed quality characteristics is decreased and approaches zero. Hence, the model tends to be closer to the model in chapter three with no inspection error. The higher the value of the correlation coefficient, the higher value for the expected total profit, because, if the correlation coefficient value is high then, more produced items are classified correctly according to their quality characteristic values therefore, no more penalty cost is going to be paid. While the small value of the correlation coefficient means more produced items are misclassified due to the high deviation between the actual and observed quality characteristics. Hence, more penalties are going to be paid which resulting in more loss which reduce the net profit and more variability between the produced items.

Now, we come to the sensitivity analysis of the penalty cost parameters (table 4-6). These penalties associated with classifying and selling a lower quality product as a higher quality one. In the original model the producer compensates the customer by what the customer has paid for the higher quality. Four cases are tested; table 4-6 summarize the results of the conducted sensitivity analysis on the penalty cost parameters.

Table 4.6 The sensitivity analysis of the penalties on the modified EMQ model with measurement error

penalties	EMQ model without uniformity penalty						EMQ model with uniformity penalty					
	w_1	w_1	μ	Q	$ETP3$	Change percentage	w_1	w_1	μ	Q	$ETP4$	Change percentage
+%50	14	9	10	632	$3.57659\ 10^5$	-11.11%	14	9	10.67	633	$3.43076\ 10^5$	-11.21%
+%25	14	9	10	632	$3.86721\ 10^5$	-3.88%	14	9	10.67	633	$3.70548\ 10^5$	-4.1%
original	**14**	**9**	**10**	**632**	$\mathbf{4.02364\ 10^5}$	**0%**	**14**	**9**	**10.67**	**633**	$\mathbf{3.86391\ 10^5}$	**0%**
-%25	14	9	10	632	$4.35182\ 10^5$	8.15%	14	9	10.67	633	$4.17688\ 10^5$	8.10%
-%50	14	9	10	632	$4.67518\ 10^5$	16.19%	14	9	10.67	633	$4.48213\ 10^5$	16.0%

It is clear that, as well the penalty cost values increases the expected total profit values decrease, since the producer pays more if the items quality is misclassified.

4.5 CONCLUSION

In this chapter, a modified EMQ model is developed for the case of measurement systems with error. The concept of cut-off points is used to reduce the impact of the error. Two cases are considered. In the first case the non-uniformity of the product is addressed. While, in the second case the uniformity of product is considered. The solutions were generated for an example contains some data from the economic manufacturing quantity and process targeting literature. The solutions were generated for an example contains some data from the economic manufacturing quantity and process targeting literature. Sensitivity analysis for the correlation coefficient between the actual and observed quality characteristics and the penalty cost parameters for each case was conducted, to study their effect on the optimal manufacturing quantity, process target mean and the expected total profit values.

CHAPTER 5

CHEN (2006) MODEL WITH 100% ERROR-PRONE INSPECTION SYSTEM

5.1 PERFACE

The purpose of this chapter is to extend **Chen (2006)** model where the inspection system (manually or automated) is error prone. This assumption is more realistic assumption as conformed in the literature. The motivation behind this work stems from the fact that neglecting the effect of measurement error should have overestimated the expected total cost. To reduce the effect of the inspection error, instead of using the original limits (L_1 and L_2) for inspection, the inspection is based on new limits (cut off points),as mention in chapter 4, and these new limits are used for the classification criteria, see figure 5-1 . Let ω be the cut off value on X, the location of these cut off points depends on many factors, such as: the value of the mean, the value of the standard deviation…etc. The problem addressed in this chapter is to determine the optimal EMQ, the process target and the optimal inspection limits (cut off points).

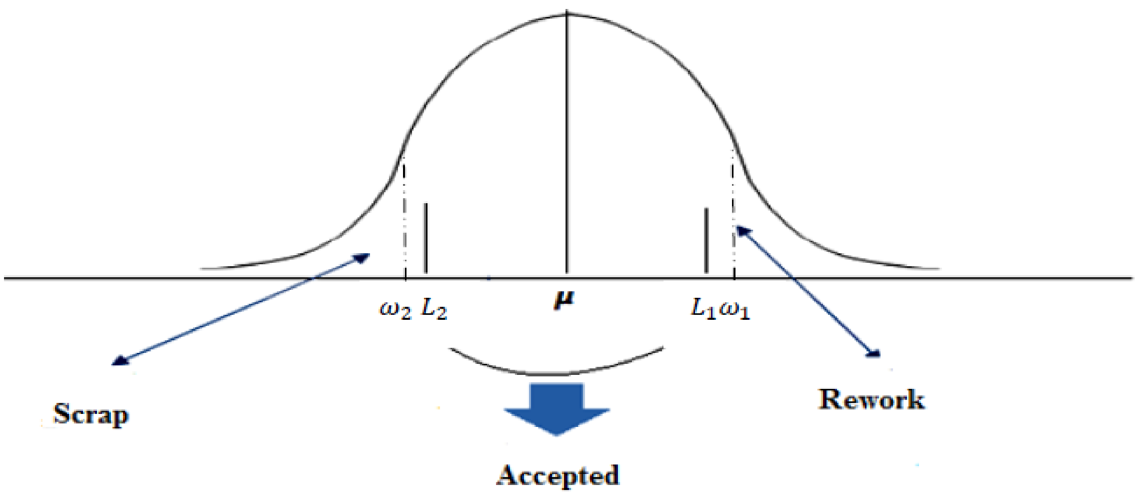

Figure 5.1 Cut off points for the inspection error

Prior to model development, the types of losses and penalties associated with misclassification of the items will be described. As shown in table below.

Table 5.1 Penalties Associated with Misclassifications

Penalty	Due to
b_{RA}	Classify rework item as accepted item
b_{RS}	Classify rework item as scrap item
b_{AR}	Classify accepted item as rework item
b_{AS}	Classify accepted item as scrap item
b_{SA}	Classify scrap item as accepted item
b_{SR}	Classify scrap item as rework item

Chen (2006) model consider the problem of EMQ model with imperfect quality. In the production process, a product will be inspected for determining if it is accepted, scrapped, or rework. Let y denote the normal quality characteristic of the product, the L_2 denote the lower specification limit (LSL) of product, and the L_1 denote the upper specification limit (USL) of product. A product will be accepted if $L_2 < y < L_1$, will be scrapped if $y < L_2$, and need to be reworked if $y > L_1$. The producer will ship the conforming units to the customers. For the rework process, we consider the perfect and imperfect cases. For the perfect rework, the product is reworked only once and the product of the rework will be conformance. For the imperfect rework, the product may be reworked once more and the quality characteristic of rework is the same as that of production. The quality loss of conformance will be measured by Taguchi quadratic quality loss function. We consider the following two modified EMQ model with the perfect and imperfect rework.

The rest of the rest of the chapter is organized as follows: in the section 5.1.1, Chen (2006) model with perfect rework will be discussed, followed by Chen (2006) model with imperfect rework (section 5.1.2). The modified Chen (2006) model under measurement error will be discussed in section 5.2, followed by the illustrative example and sensitivity analysis in section 5.3. Finally, the conclusion will be presented in section 5.4

5.1.1 CHEN (2006) MODEL WITH PERFECT REWORK

Assume that the quality characteristic of product is normally distributed with unknown μ and the known standard deviation σ. In the perfect rework, the product is reworked only once and the product of the rework will be conformance. The product quality of rework is truncated normally distributed. Hence, the cost function of product with perfect rework is

$$C_I = \begin{Bmatrix} A + R + k(y-t)^2, If\ y > L_1 \\ A + k(y-t)^2, If\ L_2 \leq\ y \leq L_1 \\ A + S, If\ y < L_2 \end{Bmatrix} \tag{5.1}$$

Hence, the expected cost of a product is:

$$E(C_I) = \int_{L_1}^{\infty} (A+R)f(y)dy + \int_{L_1}^{\infty} f(y)dy . \frac{\int_{L_2}^{L_1} k(y-t)^2 f(y)dy}{\int_{L_2}^{L_1} f(y)dy}$$

$$+ \int_{L_2}^{L_1} (A + k(y-t)^2)f(y)dy + \int_{-\infty}^{L_2} (A+S)f(y)dy \tag{5.2}$$

Where C_I is the cost function of product under the perfect rework; *A* is the inspection cost per unit; *k* is the quality loss coefficient (= $\frac{R}{\Delta^2}$); Δ is the tolerance (=| $L_1 - t$ |=| $L_2 - t$ |); *R* is the rework cost; *t* is the target value; *S* is the scrap cost per unit.

$$E(C_I) = A + R \int_{L_1}^{\infty} f(y)dy + \int_{L_1}^{\infty} f(y)dy . \frac{\int_{L_2}^{L_1} k(y-t)^2 f(y)dy}{\int_{L_2}^{L_1} f(y)dy} + \int_{L_2}^{L_1} (k(y-t)^2)f(y)dy$$

$$+ \int_{-\infty}^{L_2} Sf(y)dy \tag{5.3}$$

Where:

$f(y) = \frac{1}{\sqrt{2\pi}} e^{\frac{1}{2\sigma}(y-\mu)^2}$ is the normal distribution density function with mean μ and standard deviation σ. Let $z = \frac{y-\mu}{\sigma}$ then,

$\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{z^2}$is the standard normal distribution density function. Now consider the following:

$\int_{-\infty}^{y} f(y)dy = \int_{-\infty}^{\frac{y-\mu}{\sigma}} \varphi(z)\, dz = \Phi(z)$ the standard normal cumulative distribution function.

Now let's define the following:

$\alpha = \frac{L_1-\mu}{\sigma}$, $\delta = \frac{L_2-\mu}{\sigma}$

$\beta = \Phi\left(\frac{L_1-\mu}{\sigma}\right) = \Phi(\alpha),$ $\gamma = \Phi\left(\frac{L_2-\mu}{\sigma}\right) = \Phi(\delta)$

Standardizing the normal distribution function to standard normal using the transformation $z = \frac{y-\mu}{\sigma}$ and β,γ

$$\therefore E(C_I) = A + [1-\beta]\left[R + \frac{\int_{L_2}^{L_1} k(y-t)^2 f(y)dy}{\beta-\gamma}\right] + \int_{L_2}^{L_1} (k(y-t)^2)f(y)dy + S.\gamma \qquad (5.4)$$

Where:

$$\int_{L_2}^{L_1} (k(y-t)^2)f(y)dy$$

$$= k\{[(\mu-t)^2+\sigma^2] + [\beta-\gamma] - \sigma[(\mu-2t+L_1)\varphi(\alpha) - (\mu-2t+L_2)\varphi(\delta)]\} \qquad (5.5)$$

The probability of the product is shipped to the customers is$(1-\gamma)$.Hence, the modified EMQ model with perfect rework including the set-up cost, the holding cost, and the production cost is

$$TC_{(I)} = S_t \cdot \frac{D}{Q[1-\gamma\,]} + 0.5\,.\,Q.\,[1-\gamma]\left(1-\frac{O}{I}\right).\,h + D\,E(C_I) \tag{5.6}$$

5.1.2 CHEN (2006) MODEL WITH IMPERFECT REWORK

In the imperfect rework, the product may be reworked once more and the quality characteristic of rework is the same as that of production. Hence, the cost function of product with imperfect rework is

$$C_{II} = \begin{Bmatrix} R + E(C_{II}), If\ y > L_1 \\ A + k(y-t)^2, If\ L_2 \leq\ y \leq L_1 \\ A + S, If\ y < L_2 \end{Bmatrix} \tag{5.7}$$

Where C_{II} is the cost function of product under the imperfect rework; and $E(P_{II})$ is the expected cost of a product. Hence, the expected cost of a product is

$$E(C_{II}) = \int_{L_1}^{\infty} (R + E(C_{II}))f(y)dy + \int_{L_2}^{L_1} (A + k(y-t)^2)f(y)dy + \int_{-\infty}^{L_2} (A+S)f(y)dy \tag{5.8}$$

$$E(C_{II}) = R\int_{L_1}^{\infty} f(y)dy + E(C_{II})\int_{L_1}^{\infty} f(y)dy + A\int_{L_2}^{L_1} f(y)dy + \int_{L_2}^{L_1} (k(y-t)^2)f(y)dy + (A+S)\int_{-\infty}^{L_2} f(y)dy \tag{5.9}$$

Using the standard normal distribution and the notations defined in the previous section the Eq. (5.14) can be written as:

$$\therefore E(C_{II}) = E(C_{II})[1-\beta)] + A.\beta + R[1-\beta] + \int_{L_2}^{L_1} (k(y-t)^2)f(y)dy$$

$$+ S.\gamma \qquad (5.10)$$

Hence,

$$E(C_{II}) = A + \frac{R[1-\beta] + \int_{L_2}^{L_1}(k(y-t)^2)f(y)dy + S.\gamma}{\beta} \qquad (5.11)$$

The probability of the product scrapped is

$$\Phi\left(\frac{L_2-\mu}{\sigma}\right) + \left[1-\Phi\left(\frac{L_1-\mu}{\sigma}\right)\right]\Phi\left(\frac{L_2-\mu}{\sigma}\right) + \left[1-\Phi\left(\frac{L_1-}{\sigma}\right)\right]^2 \Phi\left(\frac{L_2-\mu}{\sigma}\right) + \cdots$$

$$= \frac{\Phi\left(\frac{L_2-\mu}{\sigma}\right)}{\Phi\left(\frac{L_1-\mu}{\sigma}\right)} = \frac{\gamma}{\beta} \qquad (5.12)$$

Hence, the probability of the product that is shipped to the customers is

$$1 - \frac{\Phi\left(\frac{L_2-\mu}{\sigma}\right)}{\Phi\left(\frac{L_1-\mu}{\sigma}\right)} = \frac{\Phi\left(\frac{L_1-\mu}{\sigma}\right) - \Phi\left(\frac{L_2-\mu}{\sigma}\right)}{\Phi\left(\frac{L_1-\mu}{\sigma}\right)} = \frac{\beta-\gamma}{\beta} \qquad (5.13)$$

The modified EMQ model with imperfect rework including the set-up cost, the holding cost, and the production cost is

$$TC_{(II)} = S_t \cdot \frac{D}{Q\left[\frac{\beta-\gamma}{\beta}\right]} + 0.5\,.\,Q.\left[\frac{\beta-\gamma}{\beta}\right]\left(1-\frac{O}{I}\right).\,h$$

$$+ D.E(C_{II}) \qquad (5.14)$$

5.2 CHEN (2006) MODEL WITH PERFECT REWORK AND MEASUREMENT ERROR

The cost function of product under measurement error is:

$$P_{Ie} = \left\{ \begin{array}{ll} A + R + k(y - t)^2 & x \geq w_1, y > L_1 \\ A + b_{RA} & x \geq w_1,\ L_2 \leq\ y \leq L_1 \\ A + b_{RS} & x \geq w_1, y < L_2 \\ & \\ A + b_{AR} & w_2 \leq\ x \leq w_1, > L_1 \\ A + k(y - t)^2 & w_2 \leq\ x \leq w_1,\ L_2 \leq\ y \leq L_1 \\ A +\ b_{AS} & w_2 \leq\ x \leq w_1, y < L_2 \\ & \\ A + S +\ \ b_{SR} & x < w_2, y > L_1 \\ A + S + b_{SA} & x < w_2,\ L_2 \leq\ y \leq L_1 \\ A + S & x < w_2, y < L_2 \end{array} \right\} \tag{5.15}$$

Where P_{Ie}, is the cost of a product under measurement error. Hence, the expected cost function of product under measurement error is:

$$E(P_{Ie}) = \int_{\omega_1}^{\infty}\int_{L_1}^{\infty}[A+R]h(x,y)\,dyd + \int_{\omega_1}^{\infty}\int_{L_1}^{\infty}h(x,y)\,dydx\,.\frac{\int_{L_2}^{L_1}[k(y-t)^2]f(y)dy}{\int_{L_2}^{L_1}f(y)dy}$$

$$+\int_{\omega_1}^{\infty}\int_{L_2}^{L_1}[A+b_{RA}]h(x,y)\,dyd + \int_{\omega_1}^{\infty}\int_{-\infty}^{L_2}[A+b_{RS}]h(x,y)\,dyd$$

$$+\int_{\omega_2}^{\omega_1}\int_{L_1}^{\infty}[A+b_{AR}]h(x,y)\,dydx$$

$$+\int_{\omega_2}^{\omega_1}\int_{L_2}^{L_1}[A+k(y-t)^2]h(x,y)\,dydx$$

$$+\int_{\omega_2}^{\omega_1}\int_{-\infty}^{L_2}[A+\ b_{AS}]h(x,y)\,dydx$$

$$+\int_{-\infty}^{\omega_2}\int_{L_1}^{\infty}[A+S+\ \ b_{SR}]h(x,y)\,dydx$$

$$+\int_{-\infty}^{\omega_2}\int_{L_2}^{L_1}[A+S+b_{SA}]h(x,y)\,dydx$$

$$+\int_{-\infty}^{\omega_2}\int_{-\infty}^{L_2}[A+S]h(x,y)\,dydx \qquad (5.16)$$

Now, add the similar term together

$$E(P_{Ie}) = A + S\int_{-\infty}^{\omega_2}\int_{-\infty}^{\infty} h(x,y)\,dydx + R\int_{\omega_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dydx$$

$$+ b_{RA}\int_{\omega_1}^{\infty}\int_{L_2}^{1} h(x,y)\,dydx + b_{RS}\int_{\omega_1}^{\infty}\int_{-\infty}^{L_2} h(x,y)\,dyd$$

$$+ b_{AR}\int_{\omega_2}^{\omega_1}\int_{L_1}^{\infty} h(x,y)\,dydx + b_{AS}\int_{\omega_2}^{\omega_1}\int_{-\infty}^{L_2} h(x,y)\,dydx$$

$$+ b_{SA}\int_{-\infty}^{\omega_2}\int_{L_2}^{L_1} h(x,y)\,dydx + b_{SR}\int_{-\infty}^{\omega_2}\int_{L_1}^{\infty} h(x,y)\,dydx$$

$$+ \int_{\omega_2}^{\omega_1}\int_{L_2}^{L_1} [A + k(y-t)^2] h(x,y)\,dydx$$

$$+ \int_{\omega_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dydx \,.\frac{\int_{L_2}^{L_1}[k(y-t)^2] f(y)dy}{\int_{L_2}^{L_1} f(y)dy} \tag{5.17}$$

Using the standard normal distribution and the notations defined in the previous section the Eq.(5.17) can be written as

$$E(P_{Ie})$$

$$= A + S.\xi + R.\int_{\omega_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dydx + b_{RA}\int_{\omega_1}^{\infty}\int_{L_2}^{L_1} h(x,y)\,dydx$$

$$+ b_{RS}\int_{\omega_1}^{\infty}\int_{-\infty}^{L_2} h(x,y)\,dyd + b_{AR}\int_{\omega_2}^{\omega_1}\int_{L_1}^{\infty} h(x,y)\,dydx + b_{AS}\int_{\omega_2}^{\omega_1}\int_{-\infty}^{L_2} h(x,y)\,dydx$$

$$+ b_{SA}\int_{-\infty}^{\omega_2}\int_{L_2}^{L_1} h(x,y)\,dydx + b_{SR}\int_{-\infty}^{\omega_2}\int_{L_1}^{\infty} h(x,y)\,dydx$$

$$+ \int_{\omega_2}^{\omega_1}\int_{L_2}^{L_1} [k(y-t)^2] h(x,y)\,dydx$$

$$+ \int_{\omega_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dydx.\frac{\int_{L_2}^{L_1}[k(y-t)^2] f(y)dy}{\beta - \gamma} \tag{5.18}$$

The probability of the product is shipped to the customers is $(1-\xi)$.Hence, the expected total cost of modified EMQ model with imperfect production process quality and inspection process including the set-up cost , the holding cost, and the production cost is

$$ETC1_{(Q,\mu,\omega_1,\omega_2)} = S_t \cdot \frac{D}{Q.(1-\xi)} + 0.5 \,.\, Q.(1-\xi).\left(1-\frac{O}{I}\right).h + D\,E(P_{Ie}) \qquad (5.19)$$

5.2.1 CHEN (2006)MODEL WITH IMPERFECT REWORK AND MEASUREMENT ERROR.

The cost function of product under imperfect rework and measurement is:

$$P_{IIe} = \left\{\begin{array}{ll} A+R+E(P_{IIe}) & x \geq w_1, y > L_1 \\ A+b_{RA} & x \geq w_1,\ L_2 \leq y \leq L_1 \\ A+b_{RS} & x \geq w_1, y < L_2 \\ & \\ A+b_{AR} & w_2 \leq x \leq w_1, y > L_1 \\ A+k(y-t)^2 & w_2 \leq x \leq w_1,\ L_2 \leq y \leq L_1 \\ A+b_{AS} & w_2 \leq x \leq w_1, y < L_2 \\ & \\ A+S+b_{SR} & x < w_2, y > L_1 \\ A+S+b_{SA} & x < w_2,\ L_2 \leq y \leq L_1 \\ A+S & x < w_2, y < L_2 \end{array}\right\} \qquad (5.20)$$

Where P_{IIe}, is the cost of product under imperfect rework and measurement. Hence, the expected cost function of product under imperfect rework and measurement error is:

$$E(P_{IIe}) = \int_{\omega_1}^{\infty}\int_{L_1}^{\infty}[A + R + E(P_{IIe})]h(x,y)\,dyd + \int_{\omega_1}^{\infty}\int_{L_2}^{L_1}[R + b_{RA}]h(x,y)\,dyd$$

$$+ \int_{\omega_1}^{\infty}\int_{-\infty}^{L_2}[R + b_{RS}]h(x,y)\,dyd + \int_{\omega_2}^{\omega_1}\int_{L_1}^{\infty}[A + b_{AR}]h(x,y)\,dydx$$

$$+ \int_{\omega_2}^{\omega_1}\int_{L_2}^{L_1}[A + k(y-t)^2]h(x,y)\,dydx$$

$$+ \int_{\omega_2}^{\omega_1}\int_{-\infty}^{L_2}[A + \ b_{AS}]h(x,y)\,dydx$$

$$+ \int_{-\infty}^{\omega_2}\int_{L_1}^{\infty}[A + S + \ \ b_{SR}]h(x,y)\,dydx$$

$$+ \int_{-\infty}^{\omega_2}\int_{L_2}^{L_1}[A + S + b_{SA}]h(x,y)\,dydx$$

$$+ \int_{-\infty}^{\omega_2}\int_{-\infty}^{L_2}[A + S]h(x,y)\,dydx \qquad (5.21)$$

Now, add the similar term together

$$E(P_{IIe}) = A\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)\,dydx + R\int_{\omega_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dydx$$

$$+ E(P_{IIe})\int_{\omega_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dyd + b_{RA}\int_{\omega_1}^{\infty}\int_{L_2}^{L_1} h(x,y)\,dydx$$

$$+ b_{RS}\int_{\omega_1}^{\infty}\int_{-\infty}^{L_2} h(x,y)\,dyd + \int_{\omega_2}^{\omega_1}\int_{L_2}^{L_1}[k(y-t)^2]h(x,y)\,dydx$$

$$+ b_{AR}\int_{w_2}^{w_1}\int_{L_1}^{\infty} h(x,y)\,dydx + b_{AS}\int_{\omega_2}^{\omega_1}\int_{-\infty}^{L_2} h(x,y)\,dydx$$

$$+ S\int_{-\infty}^{\omega_2}\int_{-\infty}^{\infty} h(x,y)\,dydx + b_{SA}\int_{-\infty}^{\omega_2}\int_{L_2}^{L_1} h(x,y)\,dydx$$

$$+ b_{SR}\int_{-\infty}^{\omega_2}\int_{L_1}^{\infty} h(x,y)\,dydx \qquad (5.22)$$

$$E(P_{IIe})$$

$$= \frac{A + R\int_{\omega_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dydx + b_{RA}\int_{\omega_1}^{\infty}\int_{L_2}^{L_1} h(x,y)\,dydx + b_{RS}\int_{\omega_1}^{\infty}\int_{-\infty}^{L_2} h(x,y)\,dyd}{\left[1 - \int_{\omega_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dyd\right]}$$

$$+ \frac{\int_{\omega_2}^{\omega_1}\int_{L_2}^{L_1}[k(y-t)^2]h(x,y)\,dydx + b_{AR}\int_{\omega_2}^{\omega_1}\int_{1}^{\infty} h(x,y)\,dydx + b_{AS}\int_{\omega_2}^{\omega_1}\int_{-\infty}^{L_2} h(x,y)\,dydx}{\left[1 - \int_{\omega_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dyd\right]}$$

$$+ \frac{S\int_{-\infty}^{\omega_2}\int_{-\infty}^{\infty} h(x,y)\,dydx + b_{SA}\int_{-\infty}^{\omega_2}\int_{L_2}^{L_1} h(x,y)\,dydx + b_{SR}\int_{-\infty}^{\omega_2}\int_{L_1}^{\infty} h(x,y)\,dydx}{\left[1 - \int_{\omega_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dyd\right]} \qquad (5.23)$$

Using the standard normal distribution and the notations defined in the previous section the Eq.(5.23) can be written as

$E(P_{IIe})$

$$= \frac{A + S.\xi + R.\int_{\omega_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dydx + b_{RA}\int_{\omega_1}^{\infty}\int_{L_2}^{L_1} h(x,y)\,dydx + b_{RS}\int_{\omega_1}^{\infty}\int_{-\infty}^{L_2} h(x,y)\,dyd}{\left[1 - \int_{\omega_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dyd\right]}$$

$$+ \frac{\int_{\omega_2}^{\omega_1}\int_{L_2}^{L_1}[k(y-t)^2]h(x,y)\,dydx + b_{AR}\int_{\omega_2}^{\omega_1}\int_{L_1}^{\infty} h(x,y)\,dydx + b_{AS}\int_{\omega_2}^{\omega_1}\int_{-\infty}^{L_2} h(x,y)\,dydx}{\left[1 - \int_{w_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dyd\right]}$$

$$+ \frac{b_{SA}\int_{-\infty}^{\omega_2}\int_{L_2}^{L_1} h(x,y)\,dydx + b_{SR}\int_{-\infty}^{\omega_2}\int_{L_1}^{\infty} h(x,y)\,dydx}{\left[1 - \int_{\omega_1}^{\infty}\int_{L_1}^{\infty} h(x,y)\,dyd\right]} \tag{5.24}$$

The probability of the product is shipped to the customers is ($\frac{\Omega-\xi}{\xi}$).Hence, the expected total cost of modified EMQ model with imperfect production process quality and inspection process including the set-up cost , the holding cost, and the production cost is

$$ETC2_{(Q,\mu,\omega_1,\omega_2)} = S_t \,.\, \frac{D}{Q.\left(\frac{\Omega-\xi}{\xi}\right)} + 0.5\,.\,Q.\left(\frac{\Omega-\xi}{\xi}\right).\left(1-\frac{O}{I}\right).\,h + D.E(P_{IIe}) \tag{5.25}$$

5.3 RESULTS AND SENSITIVITY ANALYSIS

In this section, an illustrative example for the model developed above is presented using parameters from the literature. This is followed by sensitivity analysis. For the numerical analysis, ‘NLPSolve’ command of Maple 12 is used.

5.3.1 NUMERICAL EXAMPLE

Consider a production process, which produces a product that have a normally distributed quality characteristic y with unknown mean μ. A product will be inspected for determining if it is accepted, scrap, or rework. A product whose quality characteristic falls between the two limits (10< y <15) is accepted, while a product with quality characteristic below lower specification limit (y <10) is scrap with cost \$1. Finally, a product whose quality characteristic fall above the upper specification limit (y >15) need to be reworked with cost \$2.The process standard deviation is 1.3.The inspection cost per item A=0.2.The error in the measurement system is represented by the correlation coefficient having the value ρ=0.85,i.e.σ_{ε}=0.557 and σ_{x}=1.414.The uniform search over $\omega_2 \in [8,12]$and $\omega_1 \in [13,17]$ is conducted. Knowing that:

I=100,O=80,S_t=*20*,h=*1*,D=2000.The chosen of penalties take a lot of our consideration because we aim to prevent our product from customer loss cost, so the penalties associated with classifying accepted item as rework or scrap items is chosen more higher cost than other penalties. Table (5-2), show all penalties associated with misclassifications

Table 5.2 Penalties Associated with Misclassifications

Penalty	Due to	Cost(\$)
b_{RA}	Classify rework item as accepted item	10
b_{RS}	Classify rework item as scrap item	10
b_{AR}	Classify accepted item as rework item	70
b_{AS}	Classify accepted item as scrap item	70
b_{SA}	Classify scrap item as accepted item	70
b_{SR}	Classify scrap item as rework item	60

Tables(5-3) and (5-3) , summarize the results.

Table 5.3 Chen (2006) model without error

	perfect rework	imperfect rework
$Q*$	657	653
$\mu*$	12.35	12.43
ETC	1515	1576

Table 5.4 Chen (2006) model with error

	(perfect rework)	(imperfect rework)
$Q*$	700	695
$\mu*$	12.30	12.41
ω_1^*	14.910	14.921
ω_2^*	10.112	10.120
ETC	**33,385.7**	**36,171.3**

It is clear that the expected total cost under measurement error is greater than when error-free **Chen (2006)** model, the reason is a penalty cost is added when the system under measurement error, which lead to increasing economic manufacturing quantity .The economic manufacturing quantity of perfect rework model, is larger than or equal to that of imperfect one. The reason is that the probability of the product is shipped to the customer for the perfect rework model is larger than that of imperfect one. The process mean and the expected total inventory cost of perfect rework model are smaller than or equal to that imperfect one, because the possibility of imperfect rework increases the incurred cost and raises the manufacturing target value

5.3.2 SENSITIVITY ANALYSIS FOR THE PARAMETERS

In this section, the sensitivity analysis for the correlation coefficient ρ and the penalty costs is conducted, to study their effect on the model and its optimal values.

First, the effect of the correlation coefficient between actual quality characteristic y and the observed quality characteristic x is studied. Tables 5-5, below shows the effect of the correlation coefficient on the modified **Chen (2006)** model.

Table 5.5 The sensitivity analysis of the correlation coefficient on the Chen (2006) model with measurement error

ρ	**Chen (2006) model with perfect rework and measurement error.**						**Chen (2006) model with imperfect rework and measurement error.**					
	ω_1	ω_2	μ	Q	$ETC1$	Change percentage	ω_1	ω_1	μ	Q	$ETC2$	Change percentage
0.95	14.910	10.112	12.30	696	26,932.2	-19.33%	14.921	10.120	12.41	689	28,890.0	-20.13%
0.9	14.910	10.112	12.30	698	30,310.8	-9.21%	14.921	10.120	12.41	692	32,800.1	-9.32%
0.85(original)	**14.910**	**10.112**	**12.30**	**700**	**33,385.7**	**0%**	**14.921**	**10.120**	**12.41**	**695**	**36,171.3**	**0%**
0.8	14.910	10.112	12.30	702	36,280.2	8.67%	14.921	10.120	10.67	697	39,622.0	9.54%
0.75	14.910	10.112	12.30	705	39,805.7	19.23%	14.921	10.120	10.67	699	43,456.2	20.14%

It is clear that as the correlation coefficient ρ increases the error standard deviation decreases as well. Therefore, as the correlation coefficient value increased the deviation between the actual and observed quality characteristics is decreased and approaches zero. Hence, the model tends to be closer to the model in chapter three with no inspection error.

The higher the value of the correlation coefficient, the lower value for the expected total cost, because, if the correlation coefficient value is high then, more produced items are classified correctly according to their quality characteristic values therefore, no more penalty cost is going to be paid. While the small value of the correlation coefficient means more produced items are misclassified due to the high deviation between the actual and observed quality characteristics. Hence, more penalties are going to be paid which resulting in more loss which increase the expected total cost and more variability between the produced items.

Now, we come to the sensitivity analysis of the penalty cost parameters (table 5-6). These penalties associated with classifying a lower quality product as a higher quality one. In the original model the producer compensates the customer by what the customer has paid for the higher quality. Four cases are tested; table 4-6 summarize the results of the conducted sensitivity analysis on the penalty cost parameters.

Table 5.6 The sensitivity analysis of the penalties on the Chen (2006) model with measurement error

penalties	**Chen (2006) model with perfect rework and measurement error.**						**Chen (2006) model with imperfect rework and measurement error.**					
	ω_1	ω_1	μ	Q	$ETC1$	Change percentage	ω_1	ω_1	μ	Q	$ETC2$	Change percentage
+%50	14.910	10.112	12.30	700	39,174.7	17.34%	14.921	10.120	12.41	695	42,363.8	17.12%
+%25	14.910	10.112	12.30	700	36,230.1	8.52%	14.921	10.120	12.41	695	39,177.1	8.31%
original	**14.910**	**10.112**	**12.30**	**700**	**33,385.7**	**0%**	**14.921**	**10.120**	**12.41**	**695**	**36,171.3**	**0%**
-%25	14.910	10.112	12.30	700	30,537.8	-8.53%	14.921	10.120	12.41	695	33,093.1	-8.51%
-%50	14.910	10.112	12.30	700	27,606.6	-17.31%	14.921	10.120	12.41	695	29,982.4	-17.11%

It is clear that, as well the penalty cost values increases the expected total cost values increase, since the producer pays more if the items quality is misclassified.

5.4 CONCLUSION

In this chapter, Chen (2006) model is developed for the case of measurement systems with error. The solutions were generated for an example contains some data from the Chen (2006) paper. Sensitivity analysis for the correlation coefficient between the actual and observed quality characteristics and the penalty cost parameters was conducted, to study their effect on the optimal economic manufacturing quantity, process target mean and the expected total cost values.

CHAPTER 6

CONCLUSIONS

6.1 PREFACE

This chapter concludes the work conducted in this thesis. A brief summary of the models developed in the thesis is provided in section 6.2. And further research directions are provided in section 6.3.

6.2 SUMMARY

In this thesis, the joint problem of the optimum economic manufacturing quantity and process target has been formulated in a joint formula. The literature in the area was reviewed in chapter 2.In chapter 3; the model developed assumes an error-free 100% inspection policy for product quality control. This assumption has been relaxed in chapter 4.Sensitivity analysis was presented in chapters 3 and 4.In chapter 5, Chen (2006) model is further extended for the case of measurement systems with error and sensitivity analysis was also presented on the model. The major contributions of this thesis are:

- In this thesis six new models have been developed.
- The first model is developed for the stated problem where product quality is controlled by 100% error-free inspection system without uniformity penalty (Model 1)
- The second model is developed for the stated problem where product quality is controlled by 100% error-free inspection system while the uniformity penalty has been addressed (Model 2)
- The third model is developed for the above stated problem where product quality is controlled by 100% error-prone inspection system without uniformity penalty (Model 1)
- The fourth model is developed for the above stated problem where product quality is controlled by 100% error-prone inspection system with uniformity penalty (Model 2)
- The fifth model is developed for Chen (2006) model under perfect rework where product quality is controlled by 100% error-prone inspection
- The final model extended Chen (2006) model for the case of imperfect rework where product quality is controlled by 100% error-prone inspection
- Examples from the literature are solved using the six developed EMQ models.
- Sensitivity analysis for all developed EMQ models has been conducted to study the effect of changing the models' parameters, on the economic manufacturing quantity, optimal target mean and the expected total profit/cost values.

6.3 FUTURE RESEARCH

The work done in this thesis can be extended in several directions. The following points list some of the possible extensions:

- Extend the modified EMQ models to situations where the shortage is allowed.
- Develop the modified EMQ model under the assumption that the process deteriorates and shift over time. Different drift functions (e.g. linear, quadratic etc) and distribution functions (e.g. exponential, weibull etc) can be used for that purpose.
- Extend the modified EMQ model where the production process has multi-stage processes in series.
- Extend the modified EMQ model where the product has multiple quality characteristics either dependent or independent.
- Generalize the modified EMQ models to the case that the product has n-class screening classification.
- Extend the modified EMQ models where the production process parameters are unknown (e.g. USL, LSL, σ etc.), and determine as decision variables of the optimization models.

APPENDICES

APPENDIX A

NOMENCLATURE

TC = Total inventory cost per unit time.

D = Demand quantity in units per unit time.

Q = Economic manufacturing quantity.

S_t = Set-up cost for each production run.

O = Demand rate in units per day.

I = Production rate in units per day.

h = Holding cost per unit item per unit time.

c = Production cost per item.

A = Inspection cost per unit.

R = Rework cost.

L_1 = Upper specification limit (LSL) of product.

L_2 = Lower specification limit (LSL) of product.

k = Quality loss coefficient (= $\frac{\Delta}{\Delta^2}$).

Δ = The tolerance ($=| L_1 - t |=| L_2 - t |$).

t = Target value.

S = Scrap cost per unit.

σ = Standard deviation of the process.

σ_e = Standard deviation of the measurement error.

Y = Quality characteristic to be measured.

X = Observed value of 'Y'.

a = Selling price of primary market.

r = Selling price of secondary market, (a>r).

w_1 = Cut off value of 'X' for primary market.

w_2 = Cut off value of 'X' for secondary market.

ω_1 = Cut off value of 'X' for upper specification limit for Chen (2006) model.

ω_2 = Cut off value of 'X' for lower specification limit for Chen (2006) model.

p = Percentage of rework items that would be sold in a primary market.

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VITA

Name: Ahmed Adnan Zaid

Nationality: Jordanian

Permanent Address: Qalqilia, Palestaine.

Tel:+970-599426835

Email Address: engahmad86@hotmail.co.uk

Education:

B.Sc.degree in Industrial Engineering,An-Najah National University, Palestaine,August 2009

M.Sc. degree in Industrial and System Engineering ,King Fahd University of Petroleum and Minerals ,Dhahran, Saudia Arabia, January 2012

Current Address: Dhahran, Saudia Arabia 31261. P.O. Box 8633

Tel:+966598221971