# Optimal Design of Reinforced Concrete Frames

by

Mostafa A. Hassanain

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

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DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

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This thesis, written by MOSTAFA AHMED MOSTAFA AHMED HASSANAIN under the direction of his Thesis Advisor and approved by his Thesis Committee, has been presented to and accepted by the Dean of the College of Graduate Studies, in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in CIVIL ENGINEERING.

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كلية الدراسات العليا
COLLEGE OF GRADUATE STUDIES

الله

بسم الله الرحمن الرحيم

سُبْحَانَكَ لَا عِلْمَ لَنَا إِلَّا مَا عَلَّمْتَنَا ۖ إِنَّكَ أَنْتَ الْعَلِيمُ الْحَكِيمُ

صدق الله العظيم

**To my parents**

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First and foremost, praise and thanks be to Almighty Allah, the Most Gracious, the Most Merciful, and peace be upon His Prophet.

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## THESIS ABSTRACT

Student Name : **MOSTAFA AHMED MOSTAFA AHMED HASSANAIN**

Title of Study : **OPTIMAL DESIGN OF REINFORCED CONCRETE FRAMES**

Major Field : **CIVIL ENGINEERING**

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Analysis and design of reinforced concrete (RC) frames is formulated as a nonlinear programming problem according to the ACI Code provisions. Second-order analysis is employed in which second-order influences pertaining to RC frames, are incorporated.

Concrete dimensions and steel areas for columns and beams are the design variables. For each story, the design variables pertaining to the concrete sections are linked, meaning that the column widths are assigned the same design variable as well as each of the column depths, beam widths, and beam depths. The design variables pertaining to the steel, however, are varied. The objective function is the sum of all the costs for each column and beam. Constraints consist of requirements of the ACI Code, and explicit bounds on the design variables.

Modern optimization algorithms and software are utilized to develop an optimization system that is capable of analyzing and designing economical RC rectangular frames of moderate height. The capabilities of the system are demonstrated. Design guidelines for RC frame members are developed and tested. Moreover, some behavioral studies are performed.

# خلاصة الرسالة


إسم الطالـــب : مصطفى احمد مصطفى احمد حســـانين
عنوان الدراسة : التصميم الأمثل للهياكل الخرسانية المسلحة
التخصـــص : هندسة مدنية
تاريخ الشهادة : ذو الحجة ١٤١٢هـ ، الموافق ( يونيه ١٩٩٢م )

تناولـــت الرسالة التحليل والتصميم الأمثل للهياكل الخرسانية المسلحة في إطار برمجة غير خطية عامة ، وفقاً للمواصفات الأمريكية ACI 318 - 83 ، باستخدام طرق التحليل من الدرجة الثانية التي تشمل التأثيرات غير الخطية المتعلقة بتلك الهياكل .

وقد إعتبرت قياسات الأعمدة والكمرات الخرسانية ، وكميات حديد التسليح متغيرات للتصميم ، مع الأخذ في الاعتبار أن كلاً من سمك الأعمدة وعرضها لها نفس القياسات في الطابق الواحد ، وكذلك الأمر بالنسبة للكمرات ، أما بالنسبة لكميات حديد التسليح فإنها تختلف من عمود إلى آخر ومن كمرة إلى أخرى حسب متطلب التصميم . وبهذا فإن كل عمود أو كمرة سيحتوي على أقل كمية من الحديد تحقق القيود المفروضة على التصميم . كما تم إعتبار التكلفة الإجمالية للأعمدة والكمرات هي الدالة التي يراد الوصول إلى حدها الأدنـــى ، وقد تم التعبير عنها في صورة دالة عناصرها : حجم الخرسانة ، وزن حديد التسليح ، والمساحة السطحية لخشب التسليح ، إضافة إلى وحدات التكلفة الخاصة بكل من هذه العناصر . وتتكون القيود المفروضة على التصميم من متطلبات المواصفات القياسية الأمريكية ، والحدود الموضوعة على متغيرات التصميم .

وتم استخدام طرق وبرامج الحاسب الآلي الحديثة الخاصة بالتصميم الأمثل لتطوير نظام تصميم يمكن من خلاله تحليل وتصميم هياكل خرسانية إقتصادية ، ذات أشكال معتادة متوسطة الإرتفاع ، مع مراعاة العديد من الإعتبارات العملية . وقد تم توضيح إمكانات النظام من خلال دراسة عدة حالات ، كما تم تطوير واختبار بعض إرشادات التصميم للهياكل الخرسانية المسلحة الإقتصادية ، بالإضافة إلى ذلك فقد تم إجراء بعض الدراسات التحليلية لبعض الهياكل الخرسانية تحت تأثيرات مختلفة .


درجة الماجستير في العلوم

جامعة الملك فهد للبترول والمعادن

الظهران - المملكة العربية السعودية

يونية ١٩٩٢م

*CHAPTER 1*

# INTRODUCTION

## 1.1 GENERAL

### 1.1.1 Background

Reinforced concrete is the most widely used material in construction. It has been in use for more than a century for almost all structures, great or small-buildings, bridges, dams, tunnels, tanks, and so on. In recent years, reinforced concrete has been utilized with increasing sophistication in the construction of multistory tall buildings, nuclear power plants, gravity-type offshore platforms, defense installations, etc. Since the advent of reinforced concrete, many new structural engineering concepts have emerged; one of these is the concept of numerical structural optimization.

For the last three decades, the trend in structural design has been towards improving the final design to the maximum degree possible without impairing the functional purposes the structure is supposed to serve. This trend in the design process can be attributed to several reasons. Firstly, the advent of relatively low-cost, high-power computers has made both the analysis and design processes, regardless of complexity, manageable with relative ease. Secondly, the developments in mathematical programming techniques and structural analysis methods, and thirdly, the competitive market and the sheer desire to encompass all the possibilities for the final design have contributed to design progress. These three main factors have led to the emergence of the concept of numerical structural optimization, or simply, structural optimization.

Considerable work was devoted at the beginning to the optimal design of homogeneous (steel) structures for a variety of optimality and design criteria. Research in the optimization of nonhomogeneous (reinforced concrete) structures progressed at a slower rate because of the difficulties posed by their behavior.

When loaded for a few hours at a low stress, a reinforced concrete structure behaves very nearly elastically. However, when the loading is sustained at high stresses (greater than fifty percent of ultimate values), the behavior of concrete can depart very considerably from a linear stress-strain relationship. Microcracks spread, and result in internal displacements which are nonrecoverable. Under sustained load at a high stress, concrete may weaken and eventually fail by the extension of microcracks.

The inelastic (nonlinear) behavior of reinforced concrete structures has been recognized for a long time. In 1924, G. P. Manning wrote

> Our knowledge of reinforced concrete structures has been very seriously hindered and obscured by the development of the theory of the elastic structure. The absolute mathematical certainty of the figured results when the material is assumed to be elastic (in the full scientific meaning of the term) appears to have fascinated many academical writers on the subject and has completely blinded them to the fact that the resemblance between a concrete structure and a perfect elastic structure is very slight. [1]

Four decades later, Winter [2] and Cohn [3] discussed the need for developing inelastic analysis and design methods for reinforced concrete structures. Cohn stated that "an analysis or design method ... has to reflect as closely as possible the actual behavior of the structure."

Recently, Vecchio and Balopoulou [4] studied the factors contributing to the nonlinear behavior of reinforced concrete frames under short-term loading conditions through a detailed experimental investigation of a large-scale reinforced concrete plane frame. The test results indicated that frame behavior can be significantly affected by second-order influences such as material nonlinearities, geometric nonlinearities, concrete shrinkage, shear deformations, tension stiffening effects, and membrane action.

### 1.1.2 Analysis of Reinforced Concrete Structures

Since it is known that reinforced concrete does not respond elastically to loads of more than about half the ultimate, there is a certain inconsistency in designing reinforced concrete cross sections based on inelastic (ultimate strength) behavior when the moments, shears, and thrusts for which those sections are being designed have been found by linear elastic analysis. Although this presently accepted procedure by which elastic analysis is coupled with inelastic design is not consistent, it is practical, safe, and conservative [5].

To overcome this inconsistency, many researchers have proposed methods for the inelastic analysis of reinforced concrete structures. One of the earlier approaches was to use the method of perfectly plastic limit analysis [6]. However, the limits of the reliability and usefulness of the plastic collapse predictions to reinforced concrete have become clear, particularly in the light of extensive experimental work [7,8].

Another approach was to use the imposed rotations method of Macchi [9]. This method is based on the interpretation of inelastic phenomena as the cumulative effect of elastic responses due to the applied loads and inelastic strains. The method was originally oriented to hand computations. Later, a mathematical programming model based on this method was developed and applications to reinforced concrete beams and frames were reported [7,10,11]. The model was reformulated by Kaneko [12] to improve computational efficiency. Based on this refined formulation, computer programs were developed [13,14] and their applicability to large frames was demonstrated [13].

Finite element analysis offers a powerful numerical tool for rigorous analysis of concrete structures under various stages of loading up to failure considering all the nonlinear effects. Much work has been done in this area since Ngo and Scordelis [15] reported the first application of the finite element analysis to reinforced concrete. However, due to the enormous computational effort involved, most applications have been restricted to simple structures. Krishnamoorthy and Panneerselvam [16,17], for instance, proposed a finite element model for the nonlinear analysis of reinforced concrete framed structures under various stages of loading. A computer program to perform the analysis was presented. The authors analyzed two simply supported beams, a two-span continuous beam, and a two-hinge portal frame. Nonlinear finite element analysis is still quite expensive and further research work is required to refine the finite element models and to improve the solution algorithms.

The ACI Code [18] recognizes the aforementioned inconsistency by permitting a certain amount of moment redistribution. In this process, the bending moments of flexural members obtained from an elastic analysis are adjusted to take into account the plastic behavior of reinforced concrete.

### 1.1.3 Optimization of Reinforced Concrete Structures

The objective in reinforced concrete optimization is usually to find the concrete cross-sectional dimensions and the corresponding amounts of reinforcing steel. The selection of an objective function whose least value is sought can be one of the most important decisions in the whole optimal design process.

In general, the objective function represents the most important single property of a design. Weight is the most commonly used objective function due to the fact that it is readily quantified. In reinforced concrete optimization, however, minimum weight is not always the cheapest. Cost is of wider practical importance than weight, although it is often difficult to obtain sufficient data for the construction of a real cost function. A general cost function may include the costs involved in the design and construction as well as maintenance costs, repair costs, insurance, etc. However, it is not always desirable to consider a function which is as general as possible. The result might be a 'flat' function which is not sensitive to variations in the design variables and the optimization process, practically, will not improve the design. Thus, from a practical viewpoint, it is desired to introduce such an objective function that is both sensitive to variations in the design variables and representative of the most important cost components [19].

Another approach is to consider both the initial cost of the structure as well as the failure costs which depend upon the probabilities of failure. Failure costs include such items as additional replacement costs, damage to property, casualties, business interruption, and legal services. The assumption is that the failure cost is given by the damage cost associated with a particular failure multiplied by its probability of occurrence. It is, however, recognized that answering the moral question of what constitutes an appropriate failure damage cost is likely to be as difficult as estimating the probability of failure of a structure [19].

The objective function is minimized under a set of conditions called constraints. The most important of them are the design constraints such as the ACI Code requirements. Other constraints are given by the limits being placed on the design variables. If the constraints are too restrictive, there may not be any feasible solution for the problem. In such a case, the constraints must be relaxed by allowing larger resource limits for them [20].

## 1.2 LITERATURE REVIEW

One of the earliest papers that dealt with cost optimization of reinforced concrete buildings was that of Louis A. Hill, Jr. [21]. He showed that large, nonrepetitive structures can be automatically designed by computers for less cost, with less tendency toward error, and with readily checked output. As a specific test case, the automated optimum design of a four-bay, four-story reinforced concrete building was shown. Analysis was performed using moment distribution. Ultimate strength design methods were used exclusively. Design variables included the width and depth of all beams, girders and columns, as well as the spacing of intermediate beams. Also, the amount, placing, bending, and cutoff points of all steel reinforcing were design variables. The structure was designed to be safe against collapse at ultimate loads, whether caused by wind, earthquake, or live loadings. Also, it had to perform adequately under elastic loading conditions; thus, precluding undue vibrations and excessive deflections. Successful completion of this work proved that old objections to the use of computers in the area of large civil-type structures were not valid. A problem with this study, however, was the enormous computational time required to design a practical-sized building frame. The author reported that it took 46 minutes of computer time to design the building frame he considered.

Cohn and Grierson [22] obtained the optimal design of reinforced concrete beams and frames of given concrete sizes such that under any possible load combination, certain specified minimum load factors could be guaranteed against both the collapse of the structure and the first yield of its critical sections. By linearizing the merit function and developing a method to generate all limit equilibrium constraints, the problem was solved with the help of linear programming and computer techniques. In this work, it was possible to produce optimal designs which simultaneously satisfy limit equilibrium (plastic limit stage), serviceability (elastic limit stage), and optimality (minimum material consumption) criteria. The authors, however, stated that "an optimal solution verifying the three conditions above would still have to be checked for compatibility", and this was left for a separate investigation. A five-span continuous beam and a two-bay, one-story frame were investigated.

Cohn [23] presented a general formulation to the problem of concrete frame optimal design on the basis of the serviceability (as opposed to compatibility) approach. It was suggested that design solutions can be found in a way similar to the limit design of steel structures if some modifications are applied to the assumptions on which the limit design method is based. With given geometry and loading, frames were designed for minimum longitudinal steel consumption with adequate safety against both the plastic collapse of the structure and the first yield of its critical sections. Two frame examples were solved.

A more general formulation of the optimal frame problem was presented later by Grierson and Cohn [24] wherein design plastic moments, member stiffnesses, and frame geometry were all treated as design variables and were found for simultaneous satisfaction of optimality, limit equilibrium, serviceability, plastic compatibility, and elastic compatibility. This is a nonlinear programming problem of considerable complexity and the authors illustrated the expected trends for a simple two-span continuous beam example in which various simplifying assumptions were made in order to linearize the problem.

Rozvany and Cohn [25] combined existing serviceability methods [22,23] with the lower-bound approach to optimal design and applied the resulting method to reinforced concrete frames and slabs which are subject to several alternative loading conditions. The adopted design objective was the minimization of the reinforcing steel volume. The effect of axial forces in the members and the nonlinearity of the moment-steel area relation were investigated.

Cohn [26] explained the concept of multi-criteria optimal design of frames. The optimal design problem consisted of finding member proportions for a minimum-weight structure under any type of static loading, so that adequate safety against plastic and/or incremental collapse, loss of stability, and/or serviceability be provided. Some examples of applications to steel and reinforced concrete structures were presented. For reinforced concrete structures, the optimality criterion (objective function) was taken to be the weighted cost of the materials if both concrete and steel sections were to be found. A simpler optimality criterion was used if only the steel is allowed to vary, by assuming that the concrete sizes were assigned from conventional elastic design considerations.

Munro et al. [27] presented a linear programming formulation of the optimal design problem for reinforced concrete frames with compatibility, limited ductility, equilibrium, and serviceability constraints. The simplex algorithm was used to solve the linear programming problem and to determine the values of the design bending moments at all the critical sections such that the total volume of steel reinforcement is minimized. Four multibay, multistory frames were investigated to illustrate the procedure and comparisons were made with ultimate strength designs based on elastic solutions for factored loads.

Gerlein and Beaufait [28] proposed a linear programming procedure for the preliminary strength design of multibay, multistory, moment resisting reinforced concrete frames. The design was based on a rigid plastic collapse theory. A reduced set of collapse mechanisms were used to define the kinematic constraints. Special constraints were defined in order to satisfy ACI Code requirements and practical design considerations. The objective function to be minimized was taken as the total volume of reinforcing steel required by the members. Once the optimization process has been completed, the member sizes and reinforcement at critical sections can be selected to complete the preliminary design. These can be used to carry out an accurate analysis in order to determine the internal actions that the members are to be designed to resist. A computer program to perform the optimization process was given. A three-bay, three-story frame was investigated.

Krishnamoorthy and Mosi [29], and Krishnamoorthy [30] presented a formulation for the optimal design of reinforced concrete frames as a nonlinear programming problem incorporating the method of imposed rotations to perform the inelastic analysis. A trilinear moment-rotation law for the critical sections was assumed. Cross-sectional dimensions and reinforcement areas for beams and columns were taken as design variables. The authors used a design member linking strategy to group identical members with a restriction that beam members and column members cannot be mixed. The design variables, including the reinforcement areas, were kept uniform for all the members in a group. The objective function consisted of the sum of all the material costs for beam and column groups; this included costs of concrete, steel, and formwork. The minimum cost of the whole structure was the sum of the minimized costs of the groups. Strength constraints, reinforcement constraints, and size limitation constraints were imposed on beam and column groups separately. A computer program was developed and applied to some multistory frame examples.

Gurujee and Agashe [31] presented a general nonlinear programming method for the optimal design of reinforced concrete frames. A hexalinear moment-rotation law for the critical sections was adopted. The objective function and the constraints were formulated in a manner similar to that of references [29,30]. A computer program was prepared and applied to the same examples investigated in those references.

The optimum design problem of reinforced concrete frames is highly nonlinear. The problem size (number of design variables and constraints) is relatively large even for simple structures. While it might be possible to optimize simultaneously all design variables, the large size of the problem and the different natures of variables and constraints favor a multilevel approach in which the highly nonlinear problem is divided into subproblems. Such an approach will be economical in the computational effort since, in many optimal design procedures, this effort is an exponential function of the problem size.

Kirsch [32] proposed a multilevel-formulation approach to the optimal design of reinforced concrete structures. Based on this approach, an integrated problem can be decomposed by dividing it into three optimization levels. In the third (system) level, the design moments are optimized considering the results of an elastic analysis. In the second (element) level, the concrete dimensions of each element are optimized for the given design moments. In the first (cross section) level, the amounts of reinforcing steel in each critical cross section are optimized independently for the given concrete dimensions and design moments. The author stated that this approach "is suitable for different types of structures such as beams, frames, plates, etc." The discussion was limited, however, to continuous beams.

Huanchun and Zheng [33] treated the highly nonlinear optimum design problem of reinforced concrete frames in two levels corresponding to global constraints and local constraints respectively, with iterations in each level. The global constraints were those relevant to all design variables such as displacement constraints, size constraints, etc., and the local constraints were those related to the design variables of a single member only such as strength, size, percentage of reinforcement, and all other Chinese Code requirements. In the first level, the top horizontal displacement of the frame was taken as the objective function, and was maximized to satisfy all global constraints. The optimum solution of this level was used to obtain the lower bounds of the sectional sizes of members. In the second level, using these values from the first level, the original objective function (the cost of total frame material) was minimized satisfying all local constraints. In this study, heights and widths of member sections were all specified to be integers to meet the requirements of a modular system. A two-bay, five-story frame subjected to the action of dead loads, live loads, wind loads, and a seven-degree earthquake load was investigated using this approach.

Choi and Kwak [34] adopted the concept of optimum reinforced concrete structure as an assembly of optimum individual reinforced concrete members. Instead of using a sophisticated optimization model that requires many design variables and complicated descriptive functions, they proposed an algorithm that uses a more effective direct search method to find the optimum member sections from a predetermined section data base. Thus, the optimization of the entire structure is accomplished through the individual member optimization. This approach can reduce design efforts and yield practical optimum designs. A two-bay, ten-story office building was tested using this approach. However, the authors did not answer the question of whether the assemblage of optimum members is always the absolute optimum structural design and the question of how close it is to the optimum.

Ali and Grierson [35] developed a nonlinear design method that simultaneously and explicitly accounts for the basic strength and deformability criteria for reinforced concrete frames. The method is based on a limit design formulation that simultaneously satisfies the basic conditions of equilibrium, compatibility, and the constitutive law of reinforced concrete at both service and ultimate load levels, with no need for subsequent check for any of these conditions. The authors aimed for optimal solutions that satisfy all basic design criteria. The design solution included the set of member cross-section dimensions and steel percentages for which the members had sufficient plastic moment and rotation capacities at the specified ultimate load level while ensuring adequate serviceability at the specified service load level, and for which the total cost of concrete and steel was minimum. A two-bay, one-story frame example was solved to demonstrate a practical application of the method. The method developed in this work required the identification of the critical collapse mechanisms and critical deformation states for the structure. This may lead to considerable complexity for even moderately large frames and may be numerically and computationally prohibitive as the problem size increases.

Later, Ali and Grierson [36] developed an alternate nonlinear design method in which the explicit design conditions in reference [35], i.e., limit equilibrium, limit compatibility, and material serviceability, were implicitly satisfied. An optimal limit design formulation was obtained. Suitable algorithms and related computer codes were used to solve the design problem having a nonlinear cost function and explicit nonlinear constraints. The method was applied to a continuous beam and a practical-sized building frame.

A nonlinear programming formulation for the optimal design of tall reinforced concrete framed tube buildings was presented by Spires and Arora [37,38]. Cross-sectional dimensions and reinforcement areas for beams and columns were taken as design variables. However, they were kept uniform in each story. The objective function to be minimized consisted of the sum of all the costs for each beam and column. This included costs of concrete, reinforcing steel, and formwork. Constraints on the design consisted of two type: structural constraints (Building drift, and fundamental frequency), and member constraints (ACI Code requirements, size limitations, and serviceability criteria).

In this formulation, the story height and column spacing were kept constant throughout the structure. Thus, limiting the generality of the formulation. Two frame examples were presented to test the validity of the formulation and to ascertain the influence of various design variables on the overall cost of framed tubes.

Structures are usually subjected to external loadings that are complex and continuously changing with time. In design practice, the environment is usually replaced by a finite number of distinct loading conditions which may be evaluated based on deterministic or probabilistic design philosophies. If any of the quantities involved in the design (loadings, material properties, etc.) are treated as random variables, the formulation is classified as probability or reliability based. If all the quantities are treated as deterministic (in a nonstatistical fashion), like the case in all of the above work, then the formulation is so classified [19]. In reliability-based optimum design, the optimization procedure involves the determination of the optimal level of safety which best satisfies the design criteria and constraints. This is accomplished through the minimization of an objective function, which takes into consideration the probability of failure as well as the consequences of failure in terms of cost.

Surahman and Rojiani [39] presented a reliability-based optimum design formulation of reinforced concrete frames. The objective function to be minimized was the total cost of failure, which included the initial structural cost as well as the loss due to failure. Two failure modes were considered for each member: for beams, failure due to bending and due to shear and, for columns, failure due to combined axial load and bending and due to shear. The optimization procedure consisted of two steps. The first step was the minimization of the initial structural cost for a given risk level expressed in terms of the probability of failure. Designs were obtained for several values of the probability of failure. The second step was the minimization of the sum of the initial structural cost and the cost of failure. The probability of failure which resulted from the minimization of this total cost was the optimal probability of failure. Two reinforced concrete multibay, multistory frames were designed using this procedure.

Simoes [40] described a mathematical programming technique which minimizes the total average volume of steel reinforcement of a reinforced concrete frame for a specified failure probability. The structural material is assumed to exhibit a perfectly-plastic behavior so that plastic collapse is the only possible failure mode. The technique consists of solving, in an iterative process, a reliability assessment problem, which incorporates recent developments in large-scale constrained concave quadratic programming and an optimal sizing problem (convex minimization) until the best reliability-based design against collapse is found.

Hoit et al. [41] presented a formulation that combines reliability and optimization techniques by adding the global displacements to the set of design variables. The formulation addressed the possibility of using a universal procedure for obtaining optimal reinforced concrete frame designs independently of local code restrictions. Design variables were the section sizes and the areas of longitudinal reinforcement. The objective function was the total cost of concrete and steel. The adequacy of the frames was guaranteed by imposing constraints representing the maximum probability of failure of the members considering practices involving current structural design codes, and the global displacements allowed, combined with a prescribed limited system probability of failure. Two frames were chosen from the literature to evaluate and compare the results obtained with this procedure. Both frames have been optimized using the theory of optimal limit design. A comparison of current ACI Code safety requirements and reliability constraints was presented.

It has been found that the majority of the investigators have analyzed their frames based on methods that are not acknowledged by the ACI Code. Moreover, almost all of them have ignored the inclusion of any provisions to take care of geometric nonlinearity (P-Δ effect) in the analysis process. Some investigators have included such provisions. However, they have utilized approximate methods to accomplish that. And there is a certain inconsistency in using approximate methods when one seeks an optimal design.

In regard to optimization, some investigators have minimized the amount of reinforcing steel used in beams and columns assuming given concrete dimensions. This will not minimize the cost of the whole structure. The rest of the investigators have minimized the cross-sectional dimensions and reinforcement areas for beams and columns. However, they have linked the design variables so that beam and column dimensions as well as reinforcement areas were kept uniform in each story or in each group of similar members. This will result in some members having more steel than the minimum required to satisfy the constraints. Thus, increasing the overall cost of the structures. The majority of the investigators have not included the cost of formwork into their objective functions. For the average concrete structure, the cost of the formwork is often more than the cost of both the concrete and the reinforcing steel.

Neglecting it will greatly affect the optimization process. Almost all of the investigators have limited the generality of their formulations in some way or another. Some have assumed constant story height and constant column spacing throughout the structure. Others have used solid rectangular sections for beams at locations where T-sections may be required. Thus, there is a need for a general optimization system that will overcome those limitations.

## 1.3 OBJECTIVES

The central problem in optimization for engineers is the formulation and execution of problems, rather than the mathematical techniques themselves. This is the central theme of this research. In this approach, optimization is considered as a new perspective to the process of design, and one that gives considerable insight into the engineering design process. Mathematical techniques are not treated in detail.

The main objective of this research is to develop an optimization system that can be used to optimize reinforced concrete frames according to the ACI 318-83 Code provisions. The system will be capable of analyzing and designing economical reinforced concrete rectangular frames of moderate height taking into account practical considerations such as uniform beam and column dimensions in each story, use of straight reinforcing steel bars instead of bent bars, etc [42,43,44]. A second-order analysis will be employed in which second-order influences pertaining to reinforced concrete frames, namely geometric nonlinearity and material nonlinearity, will be incorporated. Design will be based on the ACI ultimate strength design method. Structured FORTRAN 77 programming will be used to develop the proposed system.

## 1.4 PROCEDURE

The specific tasks involved in the development of the proposed optimization system are as follows:

1. Develop a FORTRAN 77 analysis program for the problem according to the direct stiffness method [45]. Second-order influences mentioned previously in section 1.3 will be taken into account in the analysis as follows:

   a. In regard to geometric nonlinearity (P-Δ effect), iterative method [45,46,47] will be used. In this method the analysis is complicated by the fact that the axial forces in the members, which are not known in advance, are related to the displacements, which are also unknowns, through the stiffness matrix. Therefore, the analysis must be conducted in a cyclic fashion, reevaluating the terms in the stiffness matrix after each cycle to account for changes in axial loads. This process is repeated until two successive analyses yield approximately the same results. This method has been called "exact" by MacGregor [48].

   b. In regard to material nonlinearity, the ACI moment redistribution method will be used. In this method, the negative moments at the supports of continuous flexural members are modified by a factor that is a function of the percentage of reinforcement. The modified negative moments are used, thereafter, to calculate moments at midspans.

2. Formulate the optimal design problem of reinforced concrete frames and create its mathematical model. This requires identification of

design variables, objective function, and design constraints.

3. Develop subroutines for the design of reinforcement, ties, and stirrups according to the ACI ultimate strength design method.

4. Integrate the analysis program developed in '1' above and the design subroutines developed in '3' with the general purpose optimizer IDESIGN [49,50,51] to create the proposed system.

5. Evaluate the performance of the system through several case studies.

6. Perform some parametric studies and develop some simple rules for estimating economical cross-sectional dimensions and reinforcement areas for columns and beams.

*CHAPTER 2*

# ANALYSIS OF REINFORCED CONCRETE FRAMES

## 2.1 GENERAL

The basic objective of frame analysis is to determine the bending moment, shear, thrust, and other load effects at each critical section in columns and beams. Final design of the structural components in the building frame is based on such effects.

A number of methods has been developed over the years for the analysis of building frames. The so-called classical methods, such as the method of slope deflection and the method of moment distribution, provided the basic analytical tools for the analysis of frames for many years. Computer-based matrix methods of analysis have worked their way into the structural engineering profession at a steady rate over the past thirty years, initially against some strong opposition from those who preferred to keep the developing electronic computer at a safe distance. Physically the distance was considerable in the days of solitary mainframes but it became more immediate with the increasing availability of time-sharing, high-speed computers.

Use of matrix theory makes it possible to reduce the detailed numerical operations required in the analysis of building frames to systematic processes of matrix manipulation which can be performed automatically and rapidly by computer. As a consequence, an 'exact' determination of bending moments, shears, and thrusts throughout the entire frame can be obtained quickly and at small expense. Provided that computer facilities are available, highly refined frame analyses are possible at lower cost than for the classical methods of analysis previously employed.

In this study, the powerful, computer-oriented *direct stiffness method* [45] will be utilized to perform the analysis of the building frames. Derivation of the basic equations is given in the cited reference and is not presented here.

## 2.2 STRUCTURAL MODELING

### 2.2.1 Introduction

It is seldom possible for the engineer to analyze an actual complex indeterminate structure. Almost without exception, certain idealizations must be made in devising an analytical model, so that the analysis will be practically possible. Therefore, it is prudent to define a structural model for reinforced concrete frames in terms of geometry and loading that could represent a wide array of realistic conditions.

### 2.2.2 Geometry Parameters

Truthfully speaking, all building frames are three-dimensional (Fig. (2.1)). Because of the repetition of frames in the *z*-direction, designers typically divide the structure into simpler two-dimensional frames and assume them to act independently, even though they are actually linked and interact. This reduces the amount of time, and consequently the amount of money, needed for the analysis, while at the same time, permits the computation of displacements and

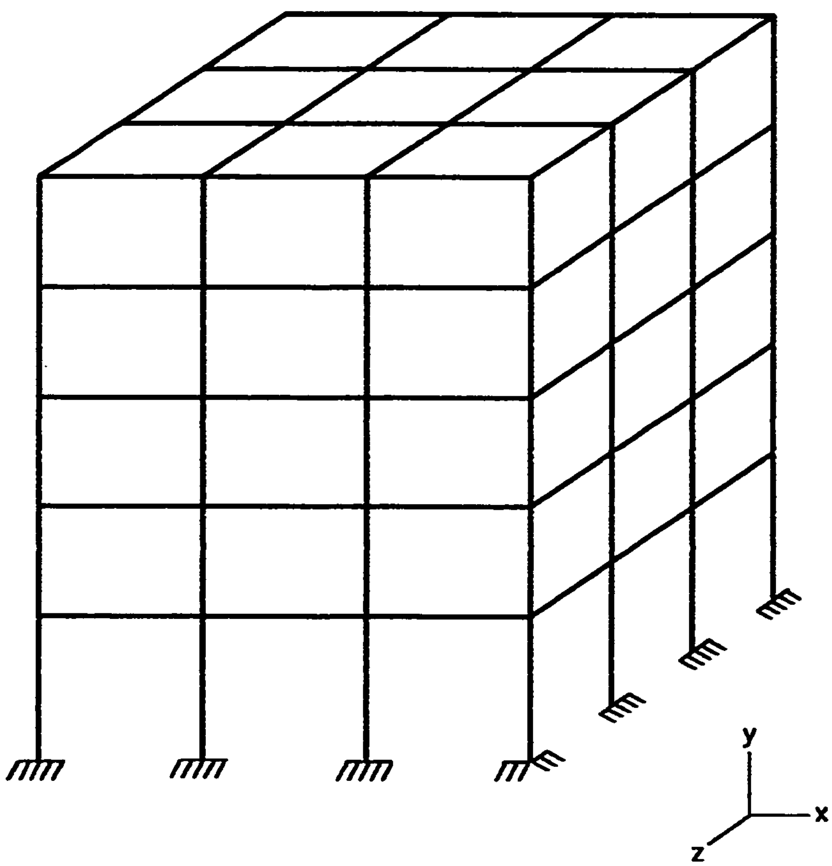



**Figure 2.1** Three-dimensional frame.

actions with sufficient accuracy. A two-dimensional (plane) frame is shown in Fig. (2.2).

In this study, only regular rectangular reinforced concrete plane frames are considered. These represent the most common framing scheme used in construction. Columns and beams are represented by straight lines, generally coincident with their actual centroidal axes. They frame into joints which are, according to a nearly universal assumption, considered to be rigid. That is, the angular relationship between columns and beams framing into a joint remains constant throughout the loading history of the frame. In this context, the term *joints* refers to points of intersection of columns with beams, as well as points of support. Supports may be fixed, or pinned, or there may be roller supports.

The critical sections for bending moments in columns are located at their tops and bottoms near the joints. For beams, the critical sections for positive and negative bending moments are assumed to be located at midspans and near the joints, respectively. To obtain moments at midspans, each beam is divided at its midpoint into two portions. For the purpose of structural analysis, each one of these two portions is called a *member*. Moreover, each column is also called a member. The points at which members meet are called *nodes*. Thus, each joint is a node but not every node is a joint.

Stiffness calculations for frame analysis are based on the gross concrete cross section and not on the cracked section. The contribution of the reinforcement is neglected, which compensates to some extent for the neglect of the influence of cracks [5]. In most cases in frame analysis, it is only the ratio of stiffness which influences the result, and not the absolute values of the stiffnesses. Stiffness ratios are but little affected by different assumptions (e.g.,

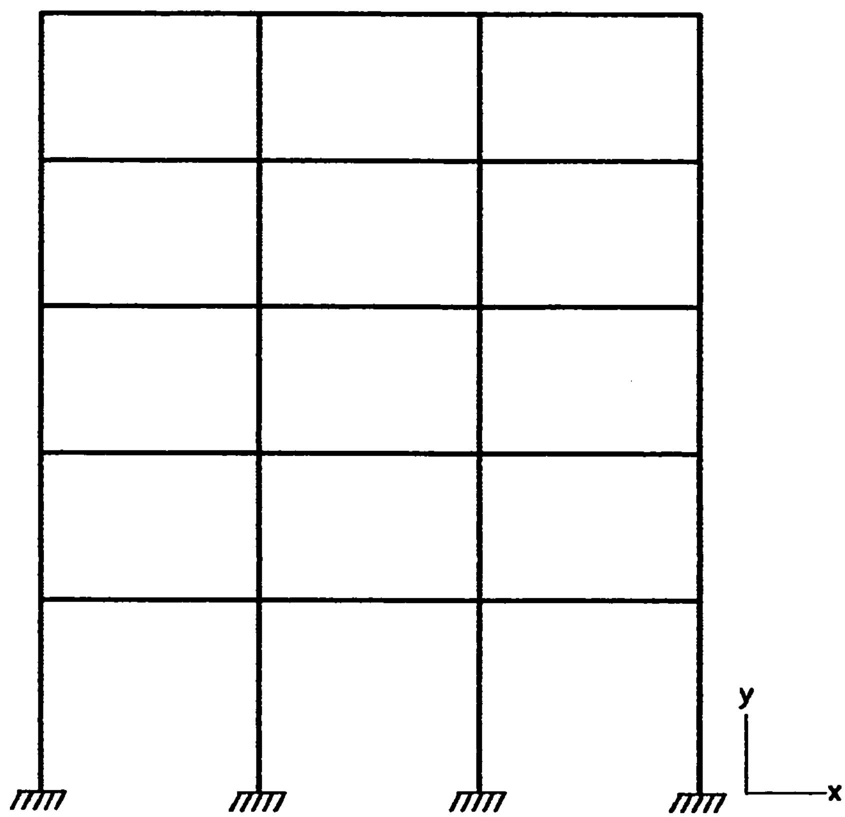



**Figure 2.2** Two-dimensional (plane) frame.

use of gross section vs. cracked section) in computing the moment of inertia, provided there is consistency for all members. In recognition of this fact, it is sufficiently accurate to base stiffness calculations on the full concrete cross section. There may be some justification for including the transformed area of the steel in computing moments of inertia of columns, because, unlike beams, often the column stress will be entirely compressive with no cracking to compensate for the neglect of the steel. In addition, steel percentages are considerably higher for columns than for beams. However, it is the usual practice to compute moments of inertia of columns just as for beams, considering only the cross section of the concrete.

The number of stories in a building is a significant parameter in analysis and design. Most reinforced concrete buildings are in the low- to medium-rise range, perhaps less than ten to fifteen stories. The focus of this study is on low- to medium-rise reinforced concrete buildings, thus excluding special design considerations that are particularly important for high rises. The range of stories dealt with here is from five to ten stories.

### 2.2.3 Loading Parameters

Vertical (gravity) loads which include dead plus live loads are assumed to be uniformly distributed over the whole beam. Service live loads are assumed not to exceed three-quarters of service dead loads. This is a reasonable assumption in many cases and it eliminates the need to consider loading patterns in structural analysis [52,53]. Only one loading pattern will be considered, with full live load on all spans for maximum positive and negative bending moments.

Lateral loads, e.g., wind forces, are assumed to be resisted by the building frame, with no stiffening assistance from the floors, walls, and partitions. This is a very common assumption in frame analysis methods. Figure (2.3) illustrates the loading parameters for a typical multibay, multistory building frame.

## 2.3 FIRST-ORDER ELASTIC ANALYSIS

### 2.3.1 Introduction

The general approach during the stiffness analysis of a structure is to generate a set of nodal equilibrium equations for the structure which relate the nodal displacements to the nodal actions. By solving these equations for the nodal displacements, and then combining the member deformations with the member stiffnesses, the end actions for each individual member can be computed.

The equations mentioned above can be expressed in matrix form as:

$$\mathbf{A} = \mathbf{S}\,\mathbf{D} \tag{2.1}$$

where **A** and **D** are the vectors that contain the nodal actions and nodal displacements, respectively, for all of the nodes in the structure. The symbol **S** represents the overall structure stiffness matrix which is assembled from the stiffness matrices of the individual members. The assembly of the structure stiffness matrix, assuming *m* members, may be stated as:

$$\mathbf{S} = \sum_{i=1}^{m} \mathbf{S}_{\mathrm{MS}i} \tag{2.2}$$

where $\mathbf{S}_{\mathrm{MS}i}$ is the *i*th member stiffness matrix with end displacements and end actions taken in the directions of structural (global) coordinates.

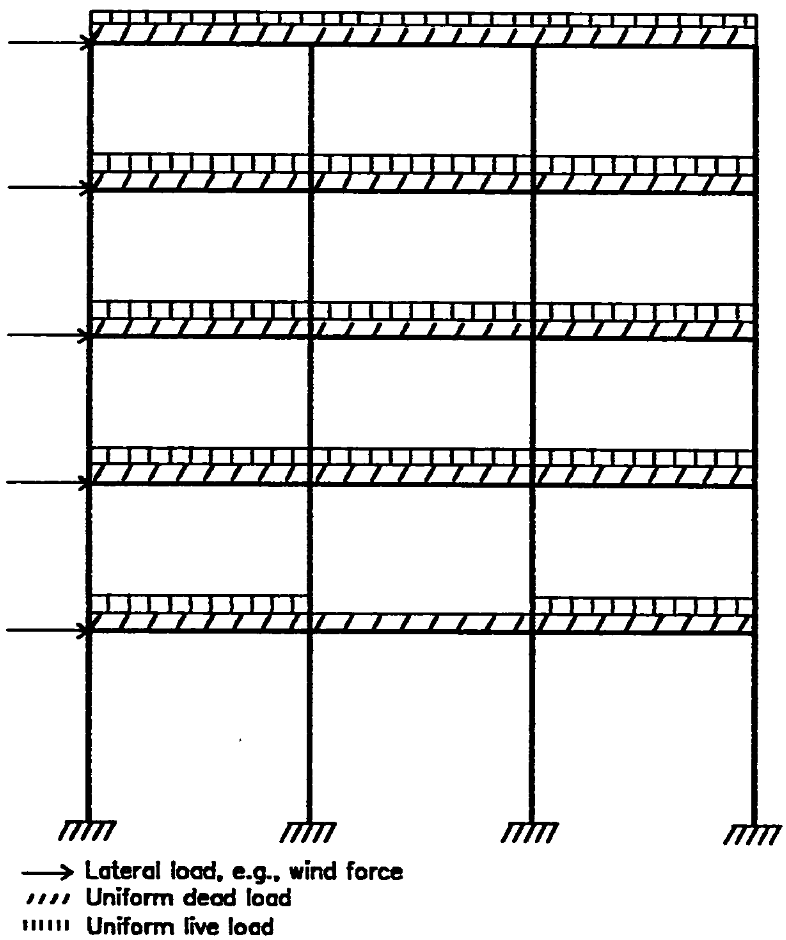



**Figure 2.3** Loading parameters for a typical multibay, multistory frame.

### 2.3.2 Method of Analysis

Figure (2.4a) shows a typical member $i$ within a plane frame. The nodes on the member are denoted as $j$ and $k$. The orthogonal set of axes $x, y$ and $z$ shown in Fig. (2.4a) are reference axes for the structure. The plane frame lies in the $x$-$y$ plane, which is assumed to be a principal plane of bending for all the members. Orthogonal member-oriented axes $x_M, y_M$ and $z_M$ appear in Fig. (2.4b) with the origin located at node $j$. The $x_M$ axis coincides with the centroidal axis of the member and is positive in the sense from $j$ to $k$. The possible displacements of the member ends are indicated, in their positive senses, in Fig. (2.4b) for the member-oriented axes. They consist of translations in the $x_M$ and $y_M$ directions, and rotations in the $z_M$ sense. The member axes are rotated from the structure axes $x_S$ and $y_S$ about the $z_M$ axis through the angle $\gamma$.

If $A$ denotes the cross-sectional area of member $i$, $I$ denotes its moment of inertia, $L$ denotes its length, and $E$ denotes the modulus of elasticity of its material, then the $i$th member stiffness matrix for member (local) coordinates can be written as follows:

$$\mathbf{S}_{Mi} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \tag{2.3}$$

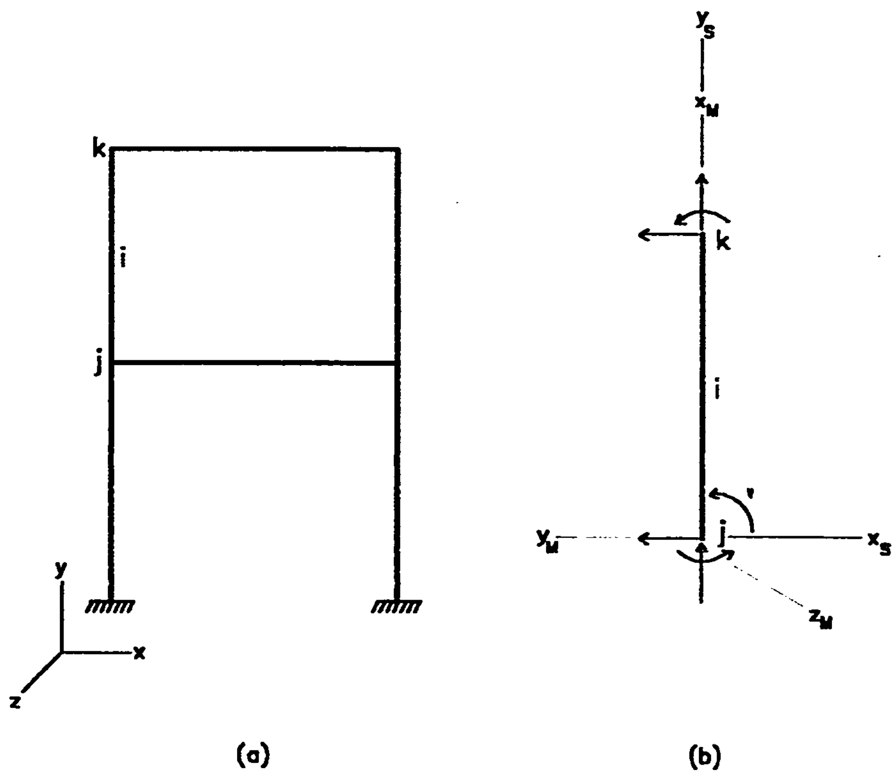



**Figure 2.4** Plane frame member.

It is assumed here that deformations due to shear forces are negligible compared to those due to bending. This is a reasonable assumption since members are long and slender. Consequently, the stiffness matrix in Eq. (2.3) does not include the effect of shear deformations.

In order to perform the assembly process stated in Eq. (2.2), it is necessary to transform the member stiffness matrices, from the individual member coordinate systems, to the structural coordinate system. This can be accomplished by introducing the rotation transformation matrix $\mathbf{R}_T$ which is defined as:

$$\mathbf{R}_T = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 & 0 & 0 & 0 \\ -\sin\gamma & \cos\gamma & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\gamma & \sin\gamma & 0 \\ 0 & 0 & 0 & -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where $\gamma$ is positive if measured counterclockwise.

Having the rotation transformation matrix on hand, one may then calculate the member stiffness matrices for structure coordinates using the following equation (omitting the subscript $i$):

$$\mathbf{S}_{MS} = \mathbf{R}_T^T \mathbf{S}_M \mathbf{R}_T$$

in which $\mathbf{R}_T^T$ is the transpose of $\mathbf{R}_T$. The next step is to assemble the member stiffness matrices for structure coordinates into the overall structure stiffness matrix as stated in Eq. (2.2).

In the next phase of the analysis, arrays associated with loads on the frame are formed. External actions applied at nodes constitute the vector $\mathbf{A}_N$. Actions at the ends of restrained members, with respect to member-oriented axes, due to loads constitute the matrix $\mathbf{A}_{ML}$. This matrix is an array of order $6 \times m$, in which each column consists of the end actions of a given member. These are two forces in the $x_M$ and $y_M$ directions and a moment in the $z_M$ sense, applied at both ends. Fixed-end actions $\mathbf{A}_{MS}$, in the directions of structure axes, can be computed using the relationship

$$\mathbf{A}_{MS} = \mathbf{R}_T^T \mathbf{A}_{ML}$$

Similar to the assembly of the structure stiffness matrix, an equivalent nodal load vector $\mathbf{A}_E$ can be constructed from member contributions, as follows:

$$\mathbf{A}_E = -\sum_{i=1}^{m} \mathbf{A}_{MSi}$$

where $\mathbf{A}_{MSi}$ is a vector of fixed-end actions at both ends of member $i$.

Addition of the vectors $\mathbf{A}_N$ and $\mathbf{A}_E$ produces the combined nodal load vector $\mathbf{A}_C$, as follows:

$$\mathbf{A}_C = \mathbf{A}_N + \mathbf{A}_E$$

It is useful to rearrange and partition the structure stiffness matrix $\mathbf{S}$ and the nodal load vector $\mathbf{A}_C$ so that terms pertaining to the free displacements come before those pertaining to the restrained (support) displacements. Thus, the expanded form of Eq. (2.1) is:

$$\begin{Bmatrix} \mathbf{A}_{FC} \\ \mathbf{A}_{RC} \end{Bmatrix} = \begin{bmatrix} \mathbf{S}_{FF} & \mathbf{S}_{FR} \\ \mathbf{S}_{RF} & \mathbf{S}_{RR} \end{bmatrix} \begin{Bmatrix} \mathbf{D}_F \\ \mathbf{D}_R \end{Bmatrix} \tag{2.4}$$

In this expression, the subscripts F and R refer to free and restrained displacements, respectively.

Having all of the required arrays on hand, one may complete the solution by calculating the free nodal displacements $\mathbf{D}_F$. For convenience in solution, Eq. (2.4) will be rewritten as two separate matrix equations:

$$\mathbf{A}_{FC} = \mathbf{S}_{FF}\,\mathbf{D}_F + \mathbf{S}_{FR}\,\mathbf{D}_R \tag{2.5a}$$

$$\mathbf{A}_{RC} = \mathbf{S}_{RF}\,\mathbf{D}_F + \mathbf{S}_{RR}\,\mathbf{D}_R \tag{2.5b}$$

The first of these equations may be solved (symbolically) for the unknown nodal displacements $\mathbf{D}_F$ (which are expanded later into the vector $\mathbf{D}$), as follows:

$$\mathbf{D}_F = \mathbf{S}_{FF}^{-1}\,(\mathbf{A}_{FC} - \mathbf{S}_{FR}\,\mathbf{D}_R)$$

Support reactions may now be calculated from Eq. (2.5b) as:

$$\mathbf{A}_R = -\mathbf{A}_{RC} + \mathbf{S}_{RF}\,\mathbf{D}_F$$

However, they are of no immediate importance in this study.

As the final step in the analysis, the member end actions $\mathbf{A}_M$ are calculated using the relationship

$$\mathbf{A}_M = \mathbf{A}_{ML} + \mathbf{S}_M\,\mathbf{R}_T\,\mathbf{D}$$

Writing the stiffness matrix as in Eq. (2.3) implies an assumption that is very common in frame analysis methods. This assumption is that the structure behaves linearly under the applied loads. That is, the computed nodal displacements and member end actions are directly proportional to the applied loads. If the loads are doubled, then these quantities also double. In addition, the principle of superposition holds, i.e., for a building having lateral wind loads (causing axial forces in members) and vertical gravity loads (causing bending moments) applied simultaneously, it is assumed that the individual loads act independently, and the total effect of all loads can be obtained by a linear superposition of the effects of each individual load. In another word, there is no interaction between axial forces and bending moments in a structural member. A frame analysis incorporating these assumptions is referred to as a *first-order elastic analysis*.

## 2.4 SECOND-ORDER ELASTIC ANALYSIS

### 2.4.1 Introduction

If the axial forces in the members are large or if the members are slender, the analysis will have to take into account the additional bending moments that are produced in the members by the axial forces when the members deflect laterally. To illustrate this, reference is made to Fig. (2.5). Figure (2.5a) shows a frame member, often known as a *beam-column*, axially loaded by $P$ and bent under the action of the end moments $M_e$. If no axial load were present, the moment $M_0$ in the member would be constant throughout and equal to the end moments $M_e$. This is shown in Fig. (2.5b). In this situation, i.e., in simple bending without axial compression, the member deflects as shown by the dashed curve of Fig. (2.5a), where $\Delta_0$ represents the deflection caused by bending alone. When $P$ is applied, the moment increases by an amount equal to $P$ times its lever arm $\Delta_a$. The increased moment causes additional deflection, so that the deflection curve under the simultaneous action of $P$ and $M_0$ is the solid curve of Fig. (2.5a). The final deflection at midspan is:

$$\Delta = \Delta_0 + \Delta_a$$

and the total moment is now:

$$M = M_0 + P\Delta$$

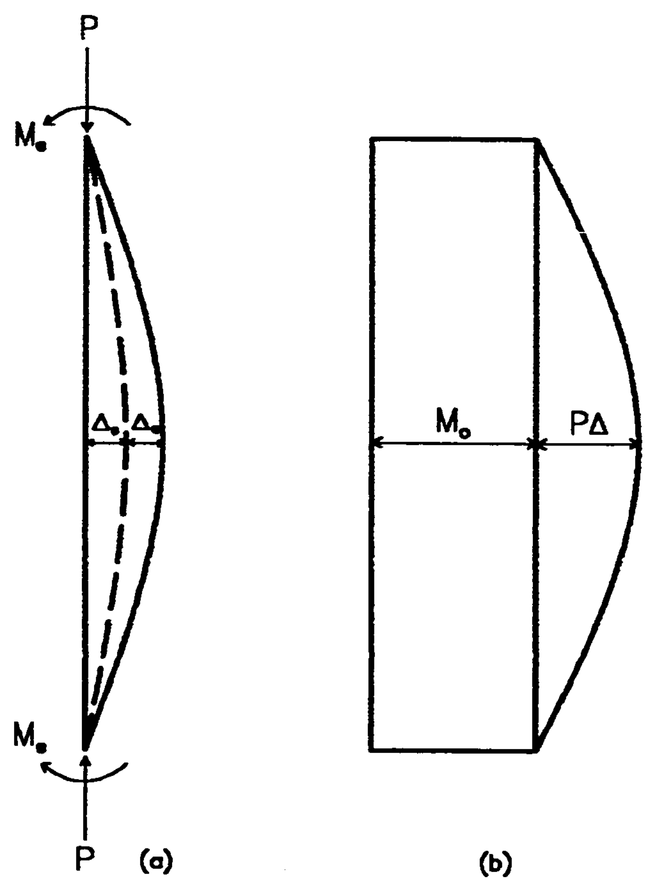



**Figure 2.5** Moments in slender members due to compression plus bending.

i.e., the total moment consists of the primary moment $M_0$ which acts in the absence of $P$ and the additional secondary moment caused by $P$, equals $P$ times the deflection $\Delta$. This axial-flexural interaction is referred to as the *P-Δ effect.* When this affect is taken into account in the structural analysis of the building, the analysis is referred to as a *second-order elastic analysis.*

### 2.4.2 Method of Analysis

A number of methods for second-order analysis of building frames have evolved in the past two decades. Most of them are based on simplified approximate hand calculations. Moreover, some of them cannot be considered as second-order analysis methods in the first place, because they merely apply some modifications to first-order analysis results in order to incorporate second-order effects. This can lead to inaccuracies for large structures. A review of some of those methods is presented in [48].

Since the structural analysis in this study is based on matrix theory, it would seem logical to incorporate P-Δ effect in the analysis using the same theory [45,46,47]. This will lead to an 'exact' determination of displacements and actions in the building [48].

The procedure is based primarily on modifying the member stiffness matrix of Eq. (2.2) by introducing the so-called *stability stiffness functions*: $s_1$, $s_2$, $s_3$ and $s_4$. These functions account for the change in bending stiffness of the member due to the presence of an axial force. Each term in the member stiffness matrix will be expressed as the product of the stiffness without P-Δ effect and a stability stiffness function. The modified member stiffness matrix will be:

$$\mathbf{S}_M = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3}s_1 & \frac{6EI}{L^2}s_2 & 0 & -\frac{12EI}{L^3}s_1 & \frac{6EI}{L^2}s_2 \\ 0 & \frac{6EI}{L^2}s_2 & \frac{4EI}{L}s_3 & 0 & -\frac{6EI}{L^2}s_2 & \frac{2EI}{L}s_4 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3}s_1 & -\frac{6EI}{L^2}s_2 & 0 & \frac{12EI}{L^3}s_1 & -\frac{6EI}{L^2}s_2 \\ 0 & \frac{6EI}{L^2}s_2 & \frac{2EI}{L}s_4 & 0 & -\frac{6EI}{L^2}s_2 & \frac{4EI}{L}s_3 \end{bmatrix} \tag{2.6}$$

The axial force $P$ may be either compression or tension, but compression is usually of greater interest than tension because of the possibility of buckling. For the case when $P$ is a compressive axial force

$$s_1 = \frac{(kL)^3 \sin kL}{12\varphi_c}$$

$$s_2 = \frac{(kL)^2(1 - \cos kL)}{6\varphi_c}$$

$$s_3 = \frac{kL(\sin kL - kL\cos kL)}{4\varphi_c}$$

$$s_4 = \frac{kL(kL - \sin kL)}{2\varphi_c}$$

in which

$$\varphi_c = 2 - 2\cos kL - kL\sin kL$$

and

$$k = \left(\frac{P}{EI}\right)^{\frac{1}{2}}$$

while for the case when $P$ is a tensile axial force

$$s_1 = \frac{(kL)^3 \sinh kL}{12\varphi_t}$$

$$s_2 = \frac{(kL)^2(\cosh kL - 1)}{6\varphi_t}$$

$$s_3 = \frac{kL(kL \cosh kL - \sinh kL)}{4\varphi_t}$$

$$s_4 = \frac{kL(\sinh kL - kL)}{2\varphi_t}$$

in which

$$\varphi_t = 2 - 2\cosh kL + kL \sinh kL$$

If the axial force in the member is zero, it can be shown by applying L'Hospital rule successively that $s_1 = s_2 = s_3 = s_4 = 1$. All of the expressions given for the stability stiffness functions can be derived by elementary beam analysis considering the presence of the axial force.

In addition to the modifications performed on the member stiffness matrix, other modifications have to be performed on the member fixed-end moments in order to account for the presence of the axial force. The modified member fixed-end moment at each end can be expressed as the product of the fixed-end moment at that end in the absence of the axial force and a factor $F$ that depend on whether the axial force is compressive or tensile [54]. For uniformly loaded members, it can be easily shown that the fixed-end moments at both ends are equal to $wL^2/12$ (with the appropriate sign) where $w$ is the magnitude of the uniform load. For the case when $P$ is a compressive axial force

$$F = \frac{12}{u^2}\left[1 - \frac{u}{2}\cot\frac{u}{2}\right]$$

in which

$$u = kL$$

while for the case when $P$ is a tensile axial force

$$F = \frac{12}{u^2}\left[\frac{u}{2}\coth\left(\frac{u}{2}\right) - 1\right]$$

The stiffness matrix expressed in Eq. (2.6) is a function of the axial force in the member which, in turn, depends on the displaced configuration of the structure that is not known in advance. As a result, the solution can only be obtained through an iterative process. An initial solution can be obtained by neglecting the effect of the axial forces, i.e., by setting the stability stiffness functions in Eq. (2.6) equal to unity. The resulting solution is the first-order theory prediction. The axial forces found from the first iteration can then be used to calculate the stability stiffness functions. One can then evaluate the modified member stiffness matrices (Eq. (2.6)) and solve for a new set of axial forces; the second iteration is then completed. The latter values are then used to calculate a new set of the stability stiffness functions which gives new values for the axial forces. The process is repeated until the axial loads found in one iteration are close to the values computed in the previous iteration, i.e., until a predetermined convergence criterion is fulfilled. Since second-order effects will not change the axial loads in the members significantly, the process will usually converge rapidly so that two iterations are generally sufficient.

The iterative method of analysis described above can be used to determine the critical (buckling) load for a frame. The loads on the frame can be gradually increased until the stiffness matrix $\mathbf{S}_{FF}$ becomes singular. This singularity is the criterion for obtaining the magnitude of loading that causes elastic instability in the frame.

The ACI Building Code [18] permits the use of second-order analysis in the design of reinforced concrete buildings. In addition, the Code allows direct design for column forces if such analysis is performed. Since the second order P-Δ effects are now directly accounted for in a second-order structural analysis, there is no need to indirectly account for them by calculating moment magnifiers. Moreover, there is no need to differentiate between sway-prevented (braced) and sway-permitted (unbraced) frames since the method considers that all buildings sway. Columns are designed with bending moments obtained from the second-order analysis, with the effective length factor $K$ always conservatively taken as 1.0 [55]. These simplifications represent significant reduction in effort for the optimization process, since there are fewer constraints and gradients to evaluate for each column in the building frame.

## 2.5 INELASTIC ANALYSIS

### 2.5.1 Introduction

Since it is known that reinforced concrete does not respond elastically to loads of more than about half the ultimate, there is a certain inconsistency in designing reinforced concrete cross sections based on inelastic (ultimate strength) behavior when the moments, shears, and thrusts for which those sections are being designed have been found by elastic analysis. Although this presently accepted procedure by which elastic analysis is coupled with inelastic design is inconsistent, it is safe and conservative [5]. It has been shown that a frame so analyzed and designed will not fail at a lower load than anticipated. On the other hand, it is known that an indeterminate frame will not fail when the ultimate moment capacity of just one critical section is reached if adjacent, less stressed sections can pick up additional load. The ability to shift load to adjacent sections in an indeterminate structure, termed *moment redistribution*, is used in an ultimate-strength-design method called *plastic design.* Although plastic design has been used to size steel members for many years, its use is restricted by the current ACI Code because the ductility of underreinforced concrete members is limited.

### 2.5.2 Inelastic Analysis Under the ACI Code

A limited amount of moment redistribution is permitted under the ACI Code, depending upon a rough measure of available ductility. The Code allows the negative moments at the supports of continuous flexural members, calculated by elastic theory (not by an approximate analysis), to be increased or decreased by not more than

$$20\left[1 - \frac{\rho - \rho'}{\rho_b}\right] \quad \%$$

where $\rho$ = ratio of tension reinforcement

$\rho'$ = ratio of compression reinforcement

$\rho_b$ = reinforcement ratio producing balanced strain condition

The Code states that the net reinforcement ratio $\rho - \rho'$ at the cross section where the moment is reduced must not exceed $0.50\rho_b$. Redistribution for steel ratios above $0.50\rho_b$ is conservatively prohibited.

As mentioned above, the adjustment of the negative moments may be either an increase or decrease so long as the positive moments are also adjusted to satisfy static equilibrium. The envelope of adjusted moments would then be used to design the sections by the strength method. The net effect on the envelope is a reduction for both negative and positive moments. This is not actually a reduction in the safety factor below that implied in code safety provisions; rather, it means a reduction in the excess strength which would otherwise be present in the structure because of the actual redistribution of moments that would occur before failure [5].

## 2.6 MOMENTS AT THE FACE OF SUPPORTS

The ACI Building Code [18] and Commentary [56] require moments obtained at the centerlines of columns to be reduced to the moments at the face of columns for design of beam members. Because the slope of the moment diagram for the beam is usually quite steep in the region of the support, there will be a substantial difference between the column centerline moment and face moment. If the former were used in proportioning the member, an unnecessarily large section would result. Moments at the face of column can be computed as shown in Fig. (2.6). The amount of reduction in moments between the column centerline and its face is equal to the area under the shear diagram between those two points.

It is common practice to conservatively disregard the reduction in column moments from the centerline of the beam to top or bottom face of the beam. The shear in columns is usually much less than that in beams, and the shape of the moment curve in the vicinity of supports is much flatter; hence, the reduction in moment in column would be much smaller than in beams and can be disregarded [5,53].

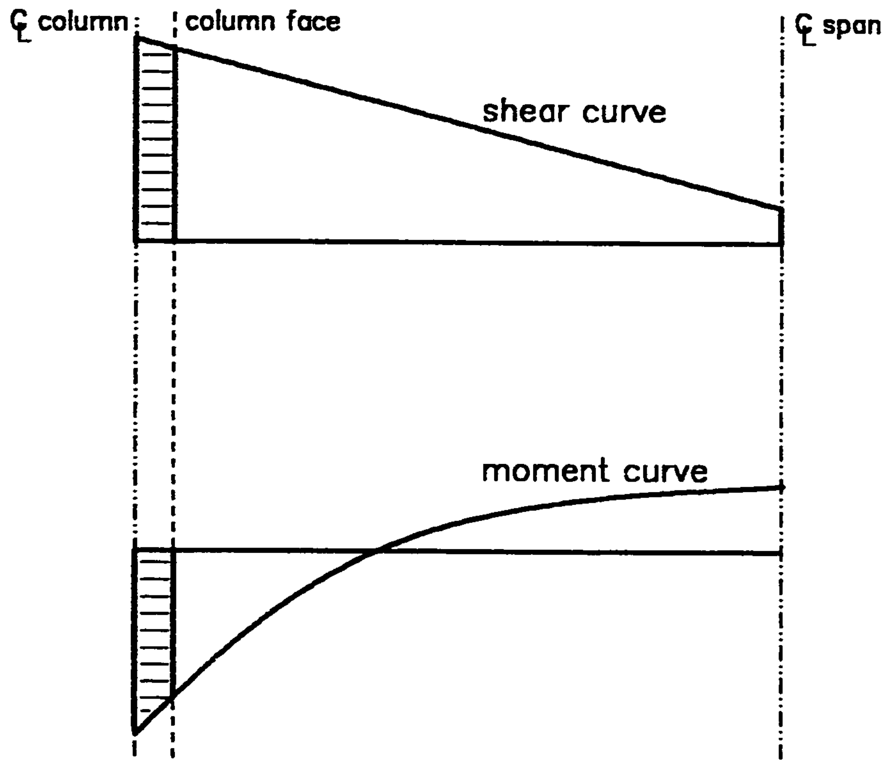



**Figure 2.6** Reduction in beam moment at face of column.

*CHAPTER 3*

# FORMULATION OF THE OPTIMAL DESIGN PROBLEM

## 3.1 GENERAL

Formulation of the optimal design problem requires identification of design variables for the structural system, objective function that needs to be minimized, and design constraints that must be imposed on the system.

Once the design problem has been formulated, it is transcribed into the following standard nonlinear constrained optimization model:

Find the set of $n$ design variables contained in the vector $\mathbf{b}$ that will minimize an objective function

$$f(\mathbf{b}) \tag{3.1}$$

subject to the constraints

$$g_i(\mathbf{b}) \leq 0, \quad i = 1,\ldots,m \tag{3.2}$$

$$h_i(\mathbf{b}) = 0, \quad i = 1,\ldots,k \tag{3.3}$$

$$b_i^l \leq b_i \leq b_i^u, \quad i = 1,\ldots,n \tag{3.4}$$

where $m$ = number of inequality constraints

$k$ = number of equality constraints

$b_i^l$ = lower bound on the $i$th design variable

$b_i^u$ = upper bound on the $i$th design variable

## 3.2 DESIGN VARIABLES

Usually, the cost of formwork represents about fifty percent of the total cost of a reinforced concrete frame [52,53]. Thus, it is obvious that any efforts made to improve the economy of a concrete structural frame should be primarily concentrated on reducing formwork costs. The cost of formwork is minimized by simplifying and repeating the shapes to be formed as much as possible. Using one column size for each story or for a number of stories and varying the amount of reinforcement will simplify form construction. For a line of continuous beams, keeping the beam size constant, even when loads and spans differ, and varying the amount of reinforcement from span to span will also simplify the construction of forms. In both cases, less labor will be used, fewer supervisors and inspectors will be needed, and costs will be lower. These ideas are employed in this study as the basis of a general strategy for frame economy.

For the present formulation, cross-sectional dimensions and reinforcement areas for columns and beams are taken as design variables. Specifically, for columns there exist three design variables: the width, $b_c$, the effective depth, $d_c$, and the longitudinal reinforcing steel area, $A_{st}$. Also, for beams there exist three design variables: the width and the effective depth of the web, $b_w$ and $d_b$, respectively, and the tensile reinforcing steel area, $A_s$. For each story, the design variables pertaining to the concrete sections are linked, meaning that the column widths are assigned the same design variable as well as each of the column effective depths, beam widths, and beam effective depths. This gives practical designs facilitating the use of repetitive formwork. For each story, there are only two design variables pertaining to the reinforcing steel. These are the longitudinal reinforcing steel area in the most critical column and the tensile reinforcing steel area at the most critical location of bending moment in beams. The steel areas in other columns and at other locations in beams are calculated based on the respective moments and the widths and depths which are design variables. As a result, each member will contain only the minimum amount of steel required to satisfy the imposed constraints.

Therefore, there is a total of six design variables for each story. These are arranged in the following order:

$b_n$ = width of columns

$b_{n+1}$ = effective depth of columns

$b_{n+2}$ = longitudinal reinforcing steel area in the most critical column

$b_{n+3}$ = width of beam webs

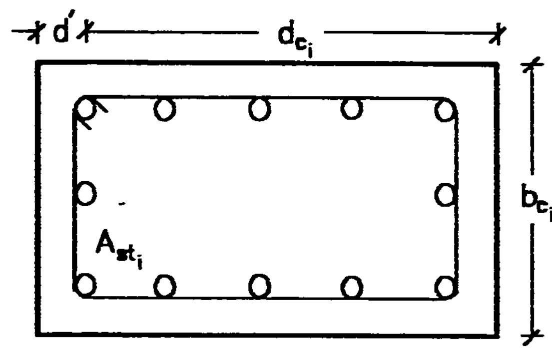
$b_{n+4}$ = effective depth of beam webs

$b_{n+5}$ = tensile reinforcing steel area at the most critical location of bending moment in beams

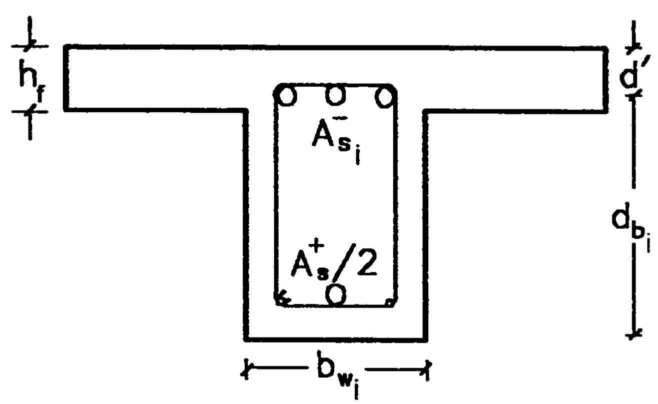
where $n$ is a counter on the number of design variables. It starts with 1 for the first story and increases by 6 for each consecutive story. Fig. (3.1) shows the design variables for columns and beams at the $i$th story level.

### 3.3 OBJECTIVE FUNCTION

The objective function to be minimized is the total cost of the frame. It is expressed in terms of concrete volume, steel weight, formwork surface area as well as their unit costs. The total cost of a reinforced concrete plane frame can be expressed as




(a)




(b)

**Figure 3.1** Design variables of reinforced concrete frame members: (a) Typical column (b) Typical beam.

$$Cost = C_{columns} + C_{beams}$$

where $C_{columns}$ = cost of columns for the whole frame

$C_{beams}$ = cost of beams for the whole frame

and these can be written as

$$C_{columns} = C_c n_c \sum_{i=1}^{N_s} (V_{cc} - V_{cs} - V_t)_i + C_s n_c \gamma_s \sum_{i=1}^{N_s} (V_{cs} + V_t)_i + C_f n_c \sum_{i=1}^{N_s} (A_{cf})_i \quad (3.5)$$

and

$$C_{beams} = C_c n_b \sum_{i=1}^{N_s} (V_{bc} - V_{bs} - V_v)_i + C_s n_b \gamma_s \sum_{i=1}^{N_s} (V_{bs} + V_v)_i + C_f n_b \sum_{i=1}^{N_s} (A_{bf})_i \quad (3.6)$$

where $C_c$ = cost of concrete per unit volume

$C_s$ = cost of steel, ties, and stirrups per unit weight

$C_f$ = cost of formwork per unit surface area

$N_s$ = number of stories

$n_c$ = number of columns per story

$n_b$ = number of beams per story

$\gamma_s$ = unit weight of steel

$V_{cc}$ = volume of concrete in a column

$V_{cs}$ = volume of longitudinal reinforcing steel in a column

$V_t$ = volume of lateral ties in a column

$A_{cf}$ = surface area of formwork for a column

$V_{bc}$ = volume of concrete in a beam

$V_{bs}$ = volume of tensile reinforcing steel in a beam

$V_v$ = volume of stirrups in a beam

$A_{bf}$ = surface area of formwork for a beam

For the present formulation, the number of columns per story as well as the number of beams per story are kept constant assuming rectangular frames which are the most common type in frame construction. The formulation, however, can be generalized to treat variable number of columns and number of beams per story.

The formulation of the objective function incorporates the concept of multi-criteria optimal design. The following optimization alternatives may be obtained as particular cases:

- minimum total cost design: $C_c = C_c$, $C_s = C_s$, $C_f = C_f$
- minimum concrete volume design: $C_c = 1.0$, $C_s = C_f = 0.0$
- minimum steel weight design: $C_c = C_f = 0.0$, $C_s = 1.0$
- minimum total volume design: $C_c = 1.0$, $C_s = 1.0/\gamma_s$, $C_f = 0.0$
- minimum total weight design: $C_c = \gamma_c$, $C_s = 1.0$, $C_f = 0.0$
- minimum formwork surface area design: $C_c = C_s = 0.0$, $C_f = 1.0$

where $\gamma_c$ denotes the unit weight of concrete.

The various quantities in Eqs (3.5) and (3.6) are obtained below with reference to Figs (3.2) and (3.3). All equations are expressed in Inch-Pound units.

***a. Columns:***

The volume of concrete in a column can be expressed as

$$V_{cc} = A_{gc} L_u$$

$$= b_c (d_c + d') \left( L_c - \frac{(d_{b1} + d_{b2}) + 2d'}{2} \right)$$

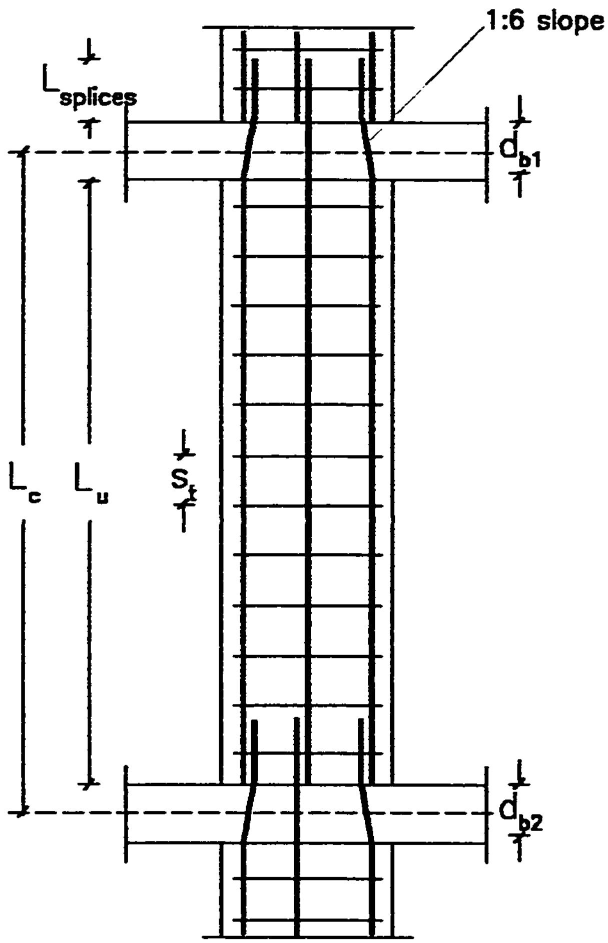



**Figure 3.2** Typical column details.

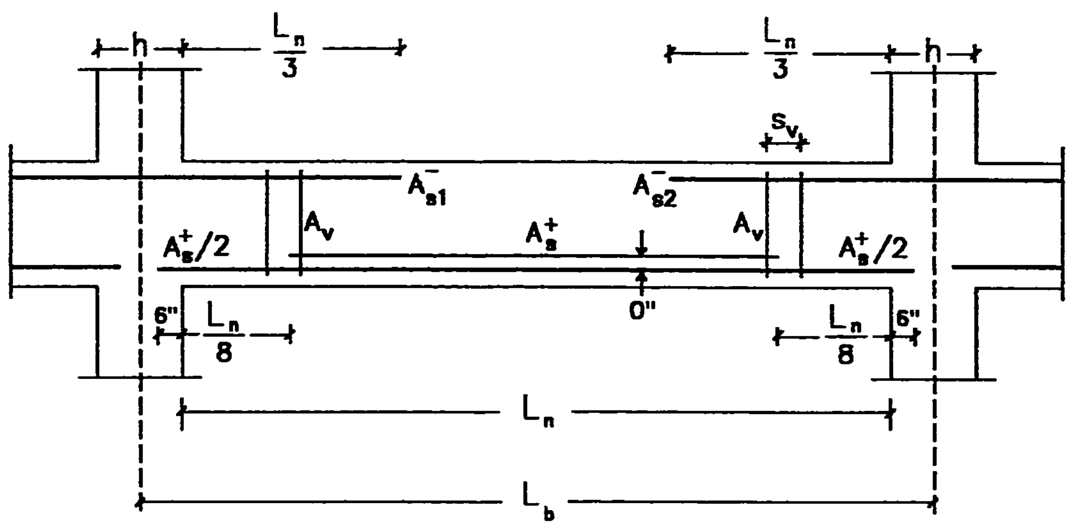



**Figure 3.3** Typical beam details.

where $A_{gc}$ = gross cross-sectional area of column

$L_u$ = unsupported length (clear height) of column

$d'$ = concrete cover (to center of reinforcing steel bars)

$L_c$ = length of column between beam center lines

$d_{b1}$ = effective depth of beam web in the current story

$d_{b2}$ = effective depth of beam web in the previous (lower) story

The volume of longitudinal reinforcing steel in a column can be written as

$$\begin{aligned} V_{cs} &= A_{st} L_{bars} \\ &= A_{st} (L_u + L_{inclined} + L_{splices}) \\ &= A_{st} \left[ \left( L_c - \frac{(d_{b1} + d_{b2}) + 2d'}{2} \right) + 1.014(d_{b1} + d') + 30 d_{sl} \right] \end{aligned} \tag{3.7}$$

where $L_{bars}$ = length of longitudinal reinforcing steel bars

$L_{inclined}$ = length of the inclined portion of the steel bars

$L_{splices}$ = length of splices

$d_{sl}$ = diameter of longitudinal reinforcing steel bars

Eq. (3.7) is formulated for the most critical column in a story, i.e., the column for which $A_{st}$ is a design variable. For other columns in the same story, the longitudinal reinforcing steel area is calculated based on the respective moments in those columns and is then substituted in Eq. (3.7).

For the present formulation, spiral columns are avoided in favor of tied columns. The weight of spirals is two or three times as much as the weight of ties in a comparable column, and the cost of spiral steel is about twice the cost of tie steel. Also, bars and machinery suitable for making spirals are not found in every fabrication shop so delivery of spirals in small quantities may be delayed [43]. The volume of lateral ties in a column can be computed as follows:

$$V_t = A_t L_{tie} n_t$$

$$= A_t \left[ 2(b_c + d_c + d') - 8(d' - \frac{d_{sl}}{2} - d_t) \right] \left[ \frac{L_u}{s_t} + 1 \right]$$

$$= A_t \left[ 2(b_c + d_c) - 6d' + 4(d_{sl} + 2d_t) \right] \left[ \frac{1}{s_t} \left( L_c - \frac{(d_{b1} + d_{b2}) + 2d'}{2} \right) + 1 \right]$$

where $A_t$ = cross-sectional area of bars used for ties

$L_{tie}$ = length of one tie

$n_t$ = number of ties in one column

$d_t$ = diameter of bars used for ties

$s_t$ = vertical spacing of ties which shall not exceed 16 longitudinal bar diameters, 48 tie bar diameters, or the least dimension of the column [18].

Finally, the surface area of formwork for a column can be written as

$$A_{cf} = 2(b_c + d_c + d')L_u$$

$$= 2(b_c + d_c + d') \left( L_c - \frac{(d_{b1} + d_{b2}) + 2d'}{2} \right)$$

***b. Beams:***

The volume of concrete in a beam can be expressed as

$$V_{bc} = A_{gb} L_b$$

$$= b_w (d_b + d') L_b$$

where $A_{gb}$ = gross cross-sectional area of beam

$L_b$ = length of beam between column center lines

For the present formulation, the CRSI [57] recommended details for flexural reinforcement are used. All recommended bar details use straight bars and meet the ACI Code requirements for development of flexural reinforcement. Truss-bent bars are eliminated because bending increases fabrication and placing costs. The volume of flexural reinforcing steel in a beam can be computed as follows:

$$V_{bs} = \frac{A_s^+}{2}\left(L_n - \frac{L_n}{8} - \frac{L_n}{8}\right) + \frac{A_s^+}{2}(L_n + 12'') + (A_{s1}^- + A_{s2}^-)\left(\frac{L_n}{3} + \frac{d_c + d'}{2}\right)$$

$$= A_s^+\left[0.875\{L_b - (d_c + d')\} + 6''\right] + (A_{s1}^- + A_{s2}^-)\left(\frac{L_b}{3} - \frac{d_c + d'}{6}\right)$$

where $A_s^+$ = positive flexural reinforcing steel area

$A_{s1}^-, A_{s2}^-$ = negative flexural reinforcing steel areas (at the two ends of the span: left and right, respectively)

$L_n$ = clear span length

On one hand, $A_s^+$, $A_{s1}^-$, or $A_{s2}^-$ could be the design variable $b_{n+5}$ depending on the magnitudes of the bending moment at the critical section locations throughout the beam span length. On the other hand, none of them could be that design variable if the maximum bending moment in the beam is not the most critical among all beams in the story.

The volume of stirrups in a beam can be written as

$$V_v = A_v L_v n_s$$

$$= A_v\left[2(b_w + d_b + d') - 8\left(d' - \frac{d_{sf}}{2} - d_v\right)\right]\left(\frac{l}{s_v} + 1\right) \qquad (3.8)$$

where $A_v$ = cross-sectional area of bars used for stirrups

$L_v$ = length of one stirrup

$n_s$ = number of stirrups in one beam

$d_{sf}$ = diameter of flexural reinforcing steel bars

$d_v$ = diameter of bars used for stirrups

$l$ = distance over which stirrups are distributed

$s_v$ = longitudinal spacing of stirrups

The magnitudes of $s_v$ and $l$ in Eq. (3.8) depend on the shear design category. If minimum shear reinforcement is required, then $s_v$ will be the smallest of $A_v f_y/50b_w$ (in which $f_y$ is the yield strength of stirrup steel), $d_b/2$ or 24 inches, and $l$ will be the distance from the face of the column to the point beyond which shear reinforcement theoretically is no longer required, i.e., the point at which $V_u = \varphi V_c/2$, where $V_u$ is the shear force produced by the factored loads, $\varphi$ is the strength reduction factor, equals 0.85, and $V_c$ is the nominal shear strength provided by concrete, equals $2\sqrt{f_c'}b_w d_b$ with $f_c'$ being the specified compressive strength of concrete. If, however, shear reinforcement must be provided, then each of $s_v$ and $l$ will consist of two portions: $s_{max}$ and $s_{min}$, and $l_{max}$ and $l_{min}$. According the ACI Code, if $V_s$, the nominal shear strength provided by the stirrups, is not greater than $4\sqrt{f_c'}b_w d_b$, then $s_{max}$, the maximum spacing of stirrups, will be the smallest of $A_v f_y/50b_w$, $d_b/2$ or 24 inches. When $V_s$ exceeds $4\sqrt{f_c'}b_w d_b$, these maximum spacings are halved. In no case, is $V_s$ to exceed $8\sqrt{f_c'}b_w d_b$. The minimum spacing of stirrups, $s_{min}$, which is required in the vicinity of a distance $d_b$ from the face of the column, is calculated from the following relation with $s$ equals $s_{min}$:

$$s = \frac{\varphi A_v f_y d_b}{\varphi V_s} \tag{3.9}$$

The calculated $s_{\min}$ should be neither so small that placement problems would result nor so large that maximum spacing criteria would control.

Knowing the maximum spacing of stirrups, Eq. (3.9) can be solved for the excess shear, $\varphi V_s$, for which a value for $V_u$, call it $V_u^*$, can be calculated. The distance from the face of the column to the point at which $V_u = V_u^*$ (referring to Fig. (3.4)) is $l_{\min}$, the distance over which stirrups are spaced with $s_{\min}$. And the distance from the point at which $V_u = V_u^*$ to the point at which $V_u = \varphi V_c/2$ is $l_{\max}$, the distance over which stirrups are spaced with $s_{\max}$.

Finally, the surface area of formwork for a beam can be computed as follows:

$$\begin{aligned} A_{bf} &= b_w L_n + 2(d_b + d') L_b \\ &= b_w \{L_b - (d_c + d')\} + 2(d_b + d') L_b \end{aligned}$$

## 3.4 DESIGN CONSTRAINTS

### 3.4.1 Introduction

The constraints on the design consist of two types: structural constraints, such as code requirements and serviceability criteria, and size limitation constraints. Structural constraints are in accordance with the ACI 318-83 Code provisions [18]. Except for the explicit size limitations, all other constraints depend on the state variables (member forces) and are therefore implicit

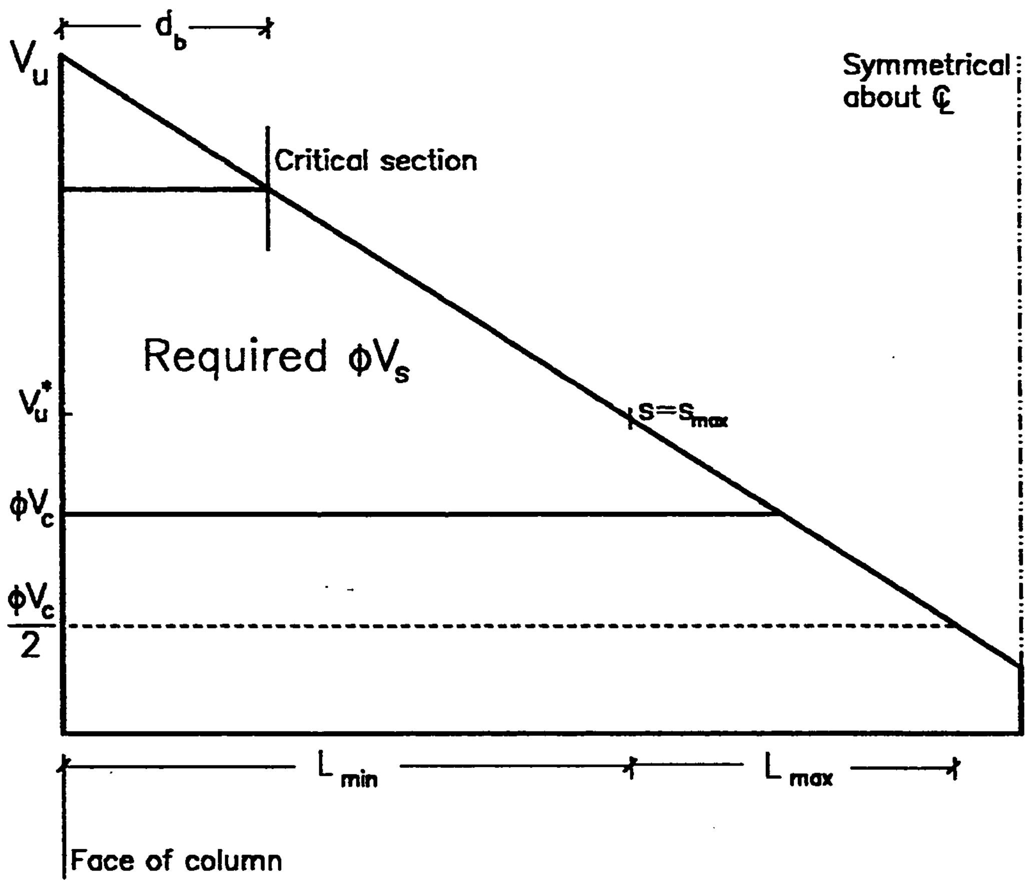



**Figure 3.4** Design of shear reinforcement.

functions of the design variables. Each critical column has four constraints, while each critical beam has nine constraints. In addition, there is one compatibility constraint between beam web widths and column widths. Therefore, there is a total of fourteen constraints for each story. All structural constraints are expressed as inequality constraints. There are no equality constraints in this formulation. All constraints are presented in the normalized form, i.e., in the form of Eq. (3.2).

### 3.4.2 Structural Constraints

#### 3.4.2.1 Column Constraints

*a. Geometric Constraint:*

For the present formulation, columns may be square or rectangular. In order to ensure that the width of the column will not exceed its depth (which is assumed to be in the direction of bending), the column dimensions are constrained by

$$\frac{b_c}{d_c + d'} - 1.0 \leq 0$$

*b. Strength Constraint:*

The ACI Code establishes adequate safety margins for columns by applying overload factors to the service loads and strength reduction factors to the nominal ultimate strengths. Therefore, $P_u \leq \varphi P_n$ is a basic safety criteria, where $P_u$ is the factored axial load at a given eccentricity, $P_n$ is the nominal axial strength at a given eccentricity, and $\varphi$ is a strength reduction factor, equals 0.70 for tied columns.

For columns with very small or zero calculated eccentricities, the ACI Code recognizes that accidental construction misalignments and other unforeseen factors may produce actual eccentricities in excess of these small design values. Also concrete strength under high, sustained axial loads may be somewhat smaller than the short-term cylinder strength. Therefore, regardless of the magnitude of the calculated eccentricities, the ACI Code limits the maximum design strength to $0.80\varphi P_0$ for tied columns. Here, $P_0$ is the nominal strength of the axially loaded column with zero eccentricity. This results in

$$P_u \leq (0.80)(0.70)P_0$$

where

$$\begin{aligned} P_0 &= 0.85 f_c' \, (A_{gc} - A_{st}) + A_{st} f_y \\ &= 0.85 f_c' \, \{b_c(d_c + d') - A_{st}\} + A_{st} f_y \end{aligned}$$

Thus, the foregoing constraint can be rewritten as

$$P_u \leq (0.80)(0.70)\left[(0.85) f_c' \{b_c(d_c + d') - A_{st}\} + A_{st} f_y\right]$$

or, in the normalized form

$$1.0 - \frac{0.476 f_c' \{b_c(d_c + d') - A_{st}\} + 0.56 A_{st} f_y}{P_u} \leq 0$$

This constraint establishes the minimum size of the column. Here, $P_u$ is the factored axial load on the most critical column in a story and is obtained from the analysis. The sign of $P_u$ is corrected if tension exists in the column because after it is obtained from the analysis, its absolute value is taken and used in the foregoing constraint. Thus, the compression constraint remains applicable.

*c. Minimum Reinforcing Steel Area Constraint:*

Columns that are concentrically compressed occur rarely, if ever, in reinforced concrete construction. They mainly carry loads in compression, but simultaneous bending is almost always present. Bending moments are caused by continuity, i.e., by the fact that columns are parts of monolithic frames in which the support moments of the beams are partly resisted by the abutting columns; by unbalanced floor loads, and by transverse loads such as wind forces. Even when design calculations show a column to be loaded purely axially, inevitable imperfections of construction will introduce eccentricities and consequent bending in the column as built. For this reason, columns must be designed on the basis of the interaction between combined bending and axial load. However, since the axial load has direct influence on the moment capacity of the column, and vice versa, there is no simple way of uncoupling the two effects.

Fig. (3.5) shows a strength interaction diagram typical for reinforced concrete columns. Since a single curve represents only one reinforcing steel ratio, a family of curves can be drawn for varying quantities of steel and placed on the same diagram. This is done for 2 percent and 6 percent steel (different steel quantity curves fall roughly linearly between these two). The ACI Code allows the steel ratio to vary from a minimum of 1 percent to a maximum of 8 percent.

To establish the constraint for minimum steel area required to resist the applied moments, one must resort to the interaction diagram, Fig. (3.5). Since the P-Δ effect is directly included in structural analysis, the moment magnification concept is not employed in the following procedure. To establish the required constraint, it is necessary to write the required steel ratio $\rho_g$ in

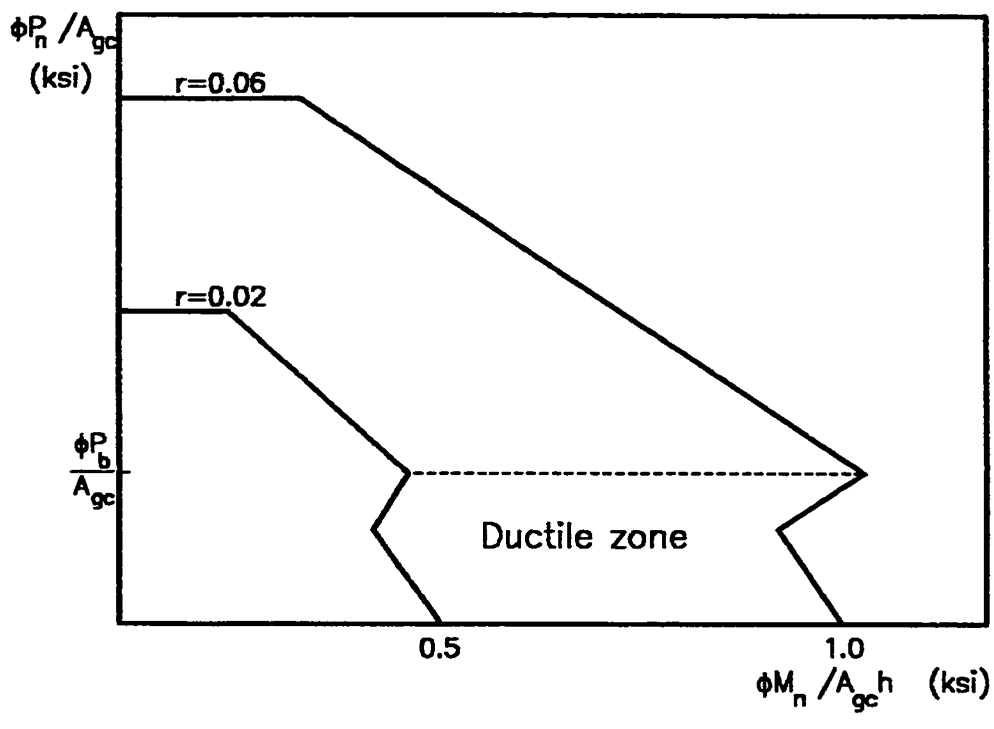



**Figure 3.5** Schematic of an interaction diagram typical for reinforced concrete columns (r = reinforcing steel ratio, $\rho_g$).

terms of the applied factored moment $M_u$. This can be done [37,38] by observing that in the ductile range for typical interaction diagrams the required steel ratio varies linearly with the applied moment. Since most interaction diagrams express the applied moment as a nondimensional quantity, this can be expressed as

$$\rho_g = 0.02 + \frac{0.06 M_u}{1.25 A_{gc} h}$$

where $h$ is the column dimension in the direction of bending, equals $d_c + d'$. But $\rho_g$ can be easily expressed in terms of the design variables as

$$\rho_g = \frac{A_{st}}{b_c h}$$

Therefore, the final constraint is given as

$$1.0 - \frac{A_{st}}{0.02 b_c (d_c + d') + 0.048 \dfrac{M_u}{d_c + d'}} \leq 0$$

Here, $M_u$ is the factored bending moment applied on the most critical column in a story and is obtained from the analysis.

### *d. Maximum Reinforcing Steel Area Constraint:*

To ensure that the column steel ratio $\rho_g$ does not exceed the ACI Code limit $\rho_{g\,max}$ of 8 percent, the following constraint is imposed:

$$\rho_g \leq \rho_{g\,max}$$

Expressing both of them in terms of the design variables and putting in the normalized form, the final constraint can be written as

$$\frac{A_{st}}{0.08b_c(d_c+d')} - 1.0 \leq 0$$

### 3.4.2.2 Beam Constraints

***a. Geometric Constraints:***

Reinforced concrete beams may be wide and shallow requiring compression steel, or relatively narrow and deep with no compression steel. Consideration of maximum material economy often leads to proportions with effective depth $d_b$ in the range from about 1.5 to 2.0 times the web width $b_w$. This results in the following constraints:

$$1.0 - \frac{d_b}{1.5b_w} \leq 0$$

and

$$\frac{d_b}{2.0b_w} - 1.0 \leq 0$$

***b. Flexural Capacity Constraint:***

All beams are designed to ensure that the moment produced by factored loads $M_u$ does not exceed the available flexural design strength $\varphi M_n$ of the cross section at any point along the length of the beam. Here, $\varphi$ equals 0.90, and $M_n$ is the nominal moment capacity of the cross section. Mathematically, this can be written as

$$M_u \leq \varphi M_n \tag{3.10}$$

According to the ACI Code, the design strength in flexure of a cross section (without compression reinforcement) may be expressed as

$$\varphi M_n = \varphi\left[A_s f_y\left(d_b - \frac{a}{2}\right)\right] \tag{3.11}$$

where $a$ is the depth of the equivalent rectangular stress block, and can be computed by

$$a = \frac{A_s f_y}{0.85 f'_c b_w} \tag{3.12}$$

Substituting Eqs (3.11) and (3.12) into Eq. (3.10) gives the following constraint:

$$M_u \le 0.9\left[A_s f_y \left(d_b - \frac{A_s f_y}{1.7 f'_c b_w}\right)\right]$$

or, in the normalized form

$$1.0 - \frac{0.9}{M_u}\left[A_s f_y \left(d_b - \frac{A_s f_y}{1.7 f'_c b_w}\right)\right] \le 0$$

Here, $M_u$ is the factored negative bending moment at the most critical location in beams in a story and is obtained from the analysis.

### *c. Minimum Reinforcing Steel Area Constraint:*

On occasion, architectural or functional considerations may require beam dimensions to be set much larger than those required for flexural strength. Because of the large arm between the components of the internal couple, a beam of this type may require a very small area of reinforcement. As a result, its nominal flexural strength may be less than the cracking moment of the cross section. If the cracking moment in a beam of this type is ever exceeded, e.g., by accidental overload, the beam will fail suddenly by rupture of the steel. To prevent such brittle failure, a lower limit is established for the steel ratio. The

ACI Code sets this limit to

$$\rho_{min} = \frac{200}{f_y}$$

Thus, the following constraint can be imposed on the beam steel ratio ρ:

$$\rho \geq \rho_{min}$$

Expressing ρ in terms of the design variables and putting in the normalized form, the final constraint can be written as

$$1.0 - \frac{A_s^- f_y}{200 b_w d_b} \leq 0$$

*d. Maximum Reinforcing Steel Area Constraint:*

In order to have reasonable assurance that concrete beams fail in a ductile manner under flexure, the ACI Code limits the amount of tension steel to not more than 75 percent of the amount in the balanced strain, that is,

$$\rho_{max} = 0.75 \rho_b$$

where

$$\rho_b = 0.85 \frac{f_c'}{f_y} \beta_1 \left[ \frac{87{,}000}{87{,}000 + f_y} \right]$$

in which

$$\beta_1 = 0.85 - 0.05 \left[ \frac{f_c' - 4000}{1000} \right] \geq 0.65$$

for $f_c'$ greater than 4000 psi, and 0.85 for $f_c'$ less than 4000 psi. This limitation will provide adequate ductile behavior for most designs. One condition where greater ductile behavior is required is in design for redistribution of moments in frames. Since moment redistribution is dependent on adequate ductility in hinge regions, the ACI 318-83 Commentary [56] limits the amount of tension steel in hinging regions by

$$\rho_{max} = 0.5\rho_b$$

Thus, the following constraint can be imposed:

$$\rho \leq \rho_{max}$$

Expressing $\rho$ in terms of the design variables and putting in the normalized form, the final constraint can be expressed as

$$\frac{A_s^-}{0.5\rho_b b_w d_b} - 1.0 \leq 0$$

*e. **Shear Strength Requirement Constraint:***

According to the ACI Code procedures, the design of beams for shear is to be based on the relation

$$V_u \leq \varphi V_n \tag{3.13}$$

in which $V_n$ is the nominal shear strength of the cross section, equals to the sum of the contributions of the concrete and the shear reinforcement if present.

The designer may wish to establish the beam web size to achieve a certain maximum nominal shear stress. This may be desirable for economical stirrup size and spacings. A practical guideline for ordinary design is to use $V_s = 4\sqrt{f_c'}b_w d_b$, typically permitting stirrup spacing from 3 inches to a maximum of $d_b/2$.

Thus, the total $V_n$ is

$$V_n = V_c + V_s$$

$$= 2\sqrt{f_c'}b_w d_b + 4\sqrt{f_c'}b_w d_b$$

$$= 6\sqrt{f_c'}b_w d_b \tag{3.14}$$

If Eq. (3.14) is substituted into Eq. (3.13), the following constraint can be imposed:

$$V_u \leq \varphi 6\sqrt{f_c'}b_w d_b$$

or, in the normalized form

$$1.0 - \frac{5.1\sqrt{f_c'}b_w d_b}{V_u} \leq 0$$

Here, $V_u$ is the factored shear force in the most critical beam in a story and is obtained from the analysis.

### *f. Beam Crack Width Constraint:*

As a reinforced concrete beam deflects under flexure, the tension side of the beam cracks wherever the low tensile strength of the concrete is exceeded. The more the beam deflects, the greater the length and width of cracks. Although cracking cannot be prevented, it is possible by careful detailing of the steel to produce beams that develop narrow, closely spread cracks in preference to a few wide cracks.

Excessive cracking of the concrete that covers the reinforcement is of considerable concern: the protection of the reinforcement from the environment depends on the integrity of the concrete cover. The acceptable width of flexural cracks in service depends mostly on the conditions of exposure.

To control beam crack width under flexure, the ACI Code requires cross sections of maximum positive and negative moment to be so proportioned that the quantity $z$ given by

$$z = f_s (d_c A)^{1/3} \tag{3.15}$$

does not exceed 175 kips/inches for interior exposure and 145 kips/inches for exterior exposure. These values correspond to maximum crack widths of 0.016 inches and 0.013 inches, respectively. In Eq. (3.15), $f_s$ is the steel stress to be taken as 60 percent of the specified yield strength $f_y$, $d_c$ is the thickness of concrete cover measured from the tension face to the center of bar closest to that face, and $A$ is the concrete area surrounding one bar, equals to total effective tension area of concrete surrounding the tension reinforcement and having the same centroid as that reinforcement, divided by the number of bars.

For beams with main flexural reinforcement in one layer, a convenient design aid can be developed, based on Eq. (3.15). With reference to Fig. (3.6), the total tensile area of concrete equals $2d_c b_w$. Because only one layer of steel exists, $d_c$ equals $d'$. Thus the tensile area per bar is

$$A = \frac{2d'b_w}{n_{min}}$$

where $n_{min}$ is the minimum number of bars in the single layer of reinforcement at top or bottom of beam, set by the designer. The *CRSI Handbook* [57] provides tables that can assist the designer to choose a suitable $n_{min}$.

Eq. (3.15) can then be rewritten as

$$z = 0.0006 f_y \left[ \frac{2(d')^2 b_w}{n_{min}} \right]^{1/3}$$

Using the limit for exterior exposure which is more critical, the following

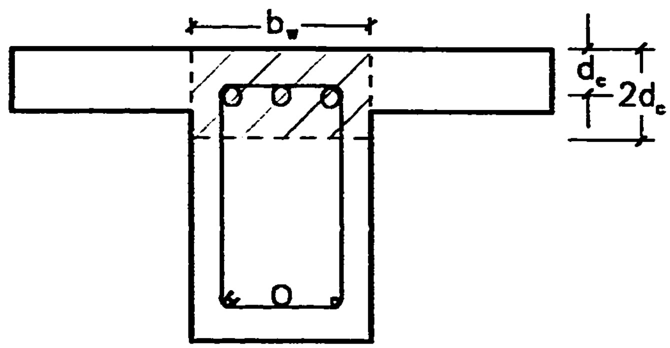



**Figure 3.6** Dimensional notation for beam crack width constraint.

constraint can be imposed:

$$\frac{0.0006}{145} f_y \left[ \frac{2(d')^2 b_w}{n_{min}} \right]^{1/3} - 1.0 \leq 0$$

*g. Minimum Beam Width Constraint:*

In regard to the placement of steel bars within the beam width, it is necessary to maintain a certain minimum distance between adjacent bars in order to ensure proper placement of concrete around them. Air pockets below the steel are to be avoided, and full surface contact between the bars and the concrete is desirable to optimize bond strength. The ACI Code specifies that the minimum clear distance between adjacent bars shall not be less than the nominal diameter of the bars, or 1 inch.

To ensure that the steel area required for flexure will fit in the beam width within the aforementioned standard spacing requirements, an approximate relation between steel area and beam width was derived [37,38] from standard rebar data. This relation can be written as follows:

$$b_r = 6.0 + 2(n_{max} - 1) \left[ \frac{4.2 A_s^-}{\pi \, n_{max}} \right]^{1/2}$$

where $b_r$ is the width required to accommodate the steel area within the standard spacing requirements, and $n_{max}$ is the maximum number of bars at top or bottom of beam, set by the designer. The *ACI Detailing Manual* [58] provides tables that can assist the designer to choose a suitable $n_{max}$.

Knowing $b_r$, the following constraint can be imposed:

$$b_r \le b_w$$

or, in the normalized form

$$\frac{6.0 + 2(n_{max} - 1)\left[\dfrac{4.2A_s^-}{\pi\, n_{max}}\right]^{1/2}}{b_w} - 1.0 \le 0$$

***h. Deflection Requirement Constraint:***

To be designed properly, reinforced concrete beams must have adequate stiffness as well as strength. Under service loads, deflections must be limited so that attached nonstructural elements, e.g., partitions, doors, windows, pipes, and plaster ceilings, will not be damaged or rendered inoperative by large deflections. Obviously floor beams that sag excessively or vibrates as live loads are applied are not satisfactory. It is important, therefore, to maintain control of deflections, in one way or another, so that members designed mainly for strength at prescribed overloads will also perform well in normal service.

There are two approaches to deflection control, both acceptable under the provisions of the ACI Code. The first is indirect, and consists of setting suitable upper limits on the span-depth ratio. In the second, it is essential to calculate deflections, and to compare those predicted values with specific limitations that are imposed by the code.

In reinforced concrete design, there is no such thing as exact calculated deflections. Calculations can, at best, provide a guide to probable actual deflections. This is so because of uncertainties regarding material properties, effects of cracking, and load history for the member under consideration. Extreme precision in deflection calculations, therefore, is never justified, because highly accurate results are unlikely.

This leaves us with the first approach in which the ACI Code limits deflections by placing restrictions on the minimum depth of the beam. This approach eliminates the need to compute deflections and controls deflections by requiring that beam depths not be less than a specified fraction of the span length. This is true only for those cases where partitions, ceilings, and other nonstructural elements are not being supported. Otherwise, deflections must be computed. It is assumed here that only walls are being supported. Minimum depths for a variety of common support conditions are given in the ACI Building Code [18].

For continuous beams, the ACI Code sets the minimum thickness to $L_b/18.5$ if one end only is continuous, and $L_b/21$ if both ends are continuous. For reinforcement having a yield point other than 60,000 psi, these values are multiplied by $0.4 + f_y/100{,}000$ with $f_y$ in psi. Since each story contains one-end as well as both-end continuous beams, it is required to determine a suitable ratio, $R$, that takes these variations into account. Let us denote the span length of the leftmost beam in a story by $l_1$, the span length of the rightmost beam by $l_2$, and the maximum span length among intermediate beams by $l_3$. According to the aforementioned prescribed limits, the greater of $l_1$ and $l_2$ should be divided by 18.5, and $l_3$ should be divided by 21. Now, the greater of these represents the required ratio $R$. Thus, the final constraint is given as

$$1.0 - \frac{(d_b + d')}{R\left[0.4 + \frac{f_y}{100{,}000}\right]} \leq 0$$

### 3.4.2.3 Compatibility Constraint

An important compatibility constraint is imposed to ensure that the width of the columns at a given story is not less than the corresponding beam width to allow continuation of beam reinforcing steel bars through the columns. In the normalized form, this constraint can be written as

$$1.0 - \frac{b_c}{b_w} \leq 0$$

### 3.4.3 Size Limitation Constraints

These are upper and lower bounds imposed on beam and column dimensions, and reinforcing steel areas, based on architectural and/or geometrical criteria. Such constraints have many different names in the literature, such as the side constraints, technological constraints, and simple bounds. All these names explain clearly the nature of this type of constraints. Mathematically, they can be represented by Eq. (3.4).

*CHAPTER 4*

# RCFOPT SOFTWARE

## 4.1 GENERAL

Based on the structural analysis methods discussed in Chapter 2 and the formulation presented in Chapter 3, an optimization system for reinforced concrete frames has been developed. The system is called RCFOPT (Reinforced Concrete Frame OPTimization system). It is capable of analyzing and designing economical reinforced concrete rectangular frames of moderate height according to the ACI 318-83 Code provisions [18].

The general flow chart shown in Fig. (4.1) illustrates the structure of RCFOPT. At the heart of the system are the USER subroutines which contain the objective function and the constraint function expressions. These subroutines represent the link between the two main phases of the optimization process: the analysis phase (RCFRAME) and the optimal design phase (IDESIGN). The optimization process as a whole consists of cycling between those two phases in an iterative fashion until the optimum is reached.

## 4.2 ANALYSIS PHASE (RCFRAME)

### 4.2.1 Introduction

The analysis phase operates through the program RCFRAME (Reinforced Concrete FRAME analysis). RCFRAME consists of a main program that uses

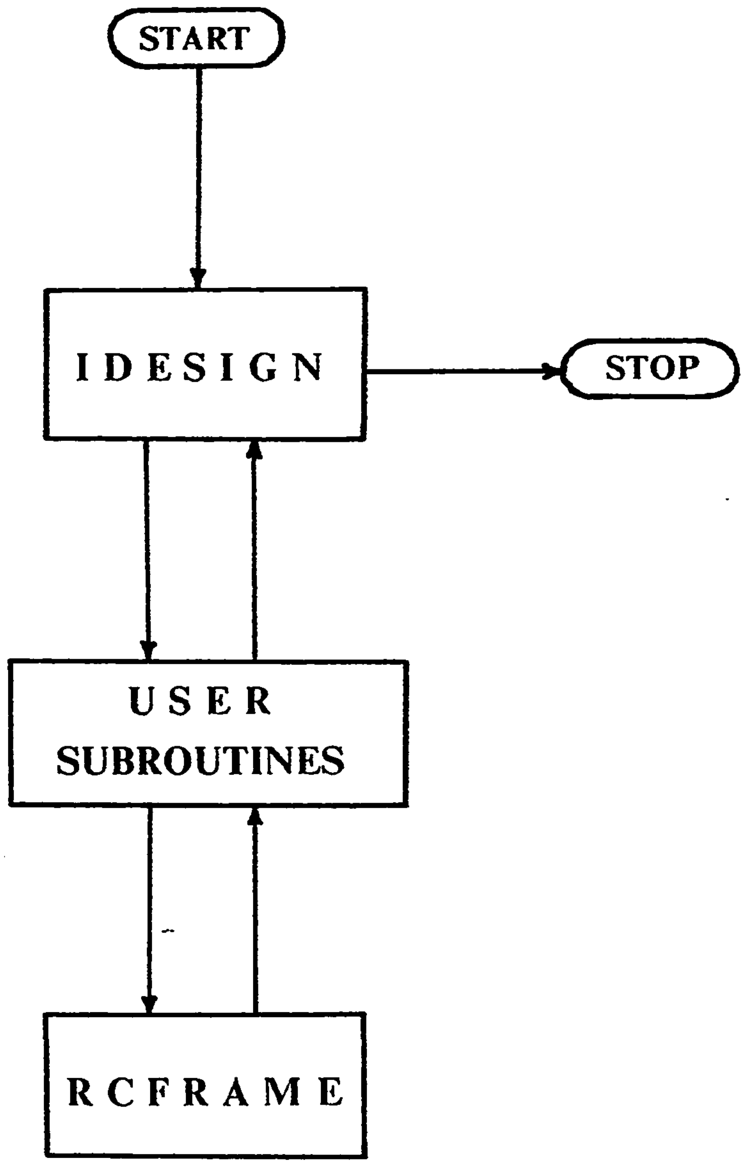



**Figure 4.1** General structure of RCFOPT program.

a series of subprograms to perform the detailed calculations. It is written in structured, double precision FORTRAN 77. The program was developed on the IBM 3090-150E mainframe computer of the Data Processing Center at King Fahd University of Petroleum and Minerals. However, it is completely system independent.

RCFRAME can be used aside from the optimal design phase to analyze a given framed structure. The program can handle building frames of the size which is commonly encountered in engineering practice. Up to ten-bay, ten-story frames can be analyzed. The program can perform first-order elastic analysis, second-order elastic analysis, and second-order inelastic analysis. A variable called IELAST is set in the input data file to 1, 2, or 3 to designate those analyses, respectively.

### 4.2.2 Input Data

RCFRAME works in a batch mode of computation. The user has to prepare an input data file describing the material properties and costs, and the geometric layout of the frame as well as the loading. RCFRAME makes no unit conversions during the analysis. All input data must have consistent units. The data can be input in either SI or US customary units. However, the latter must be used when RCFRAME is employed in the optimization system RCFOPT because the objective function as well as the constraint functions are all expressed in terms of US customary units. The input data file has a free format, i.e., all READ statements in the program are not formatted except when reading characters. This makes it easier to input the data and eliminates a lot of potential errors. A sample input data file is presented in Appendix 1.

### 4.2.3 Program Operation

#### 4.2.3.1 Introduction

The primary mathematical task involved in the stiffness analysis of framed structures consists of solving a set of $n$ simultaneous linear algebraic equations for $n$ unknowns. For a plane frame with $NN$ nodes, the number of equations which must be solved is $3NN$, since there are three degrees of freedom at each node. If the full structure stiffness matrix is stored in computer memory in a square array, the total number of elements which must be stored will be equal to the square of the number of equations. This number increases very rapidly with an increase in the size of the structure. For example, for a plane frame with 50 nodes the structure stiffness matrix will contain 22,500 elements, while for a frame with 100 nodes this number increases to 90,000. Since eight bytes are required in FORTRAN to store a double precision real number in memory, in either personal computers or mainframe computers, the corresponding memory requirements are 180,000 and 720,000 bytes, respectively.

On the IBM PS/2 and compatible systems, FORTRAN compilers limit the amount of memory for any dimensioned array or any COMMON area to a maximum of 64K bytes or 65,536 bytes. With these limits, one will start running into storage problems for a plane frame with just 31 nodes. Even on many mainframe computers one cannot do a great deal better. Therefore, a computer program which stores the full structure stiffness matrix in memory is severely limited in the size of structure it can handle. Since it is the objective of this study to develop a program that can be used to analyze and design building frames of the size which is commonly encountered in practice, it is necessary to devise a way to conserve computer memory during program execution.

It was shown in Chapter 2 that the primary unknowns in stiffness analysis, i.e., the free nodal displacements, can be solved for as follows:

$$\mathbf{D}_F = \mathbf{S}_{FF}^{-1} (\mathbf{A}_{FC} - \mathbf{S}_{FR}\mathbf{D}_R)$$

While it is symbolically convenient to imply inversion of the stiffness matrix $\mathbf{S}_{FF}$ in the above equation, it is not efficient to actually calculate $\mathbf{S}_{FF}^{-1}$ in a computer program. The number of arithmetic steps involved in the process of matrix inversion increases very rapidly with the size of the matrix. For large framed structures, the time spent to invert the matrix can be impractical, particularly on small computers. Therefore, a more efficient scheme for solving the equations is needed.

Fortunately, the structure stiffness matrix has two properties which can help in solving the storage and equation-solving problems discussed above. First, the matrix is always symmetric. Therefore, valuable computer memory space would be wasted if all of the elements in the matrix were stored. By storing elements on the main diagonal and above, the memory size requirements will be greatly reduced. This can lead to significant savings for large frames.

The second property of the structure stiffness matrix which can be advantageous is that, for many structures, the majority of the matrix elements are with zero magnitude. In addition, the nonzero elements can often be grouped into a relatively narrow band around the main diagonal by proper numbering of the degrees of freedom. Therefore, considerable computer memory can be saved by only storing the upper half-band of the matrix.

Some storage schemes and corresponding equation-solving procedures have been developed to take advantage of the symmetric banded property of the structure stiffness matrix. The scheme employed in RCFRAME is the rectangular upper half-band storage scheme.

### 4.2.3.2 Rectangular Upper Half-Band Storage Scheme

Figure (4.2a) shows the elements in the upper half-band of an arbitrary stiffness matrix in their original positions in a square array, while Fig. (4.2b) shows the same elements stored in a narrow rectangular array. Although the new storage array still has the same number of rows, the number of columns has been reduced by more than three times. This latter number is usually called the *half-band width* or simply the *band width* of the matrix. The actual reduction percentage which can be achieved depends upon the connectivity of the members and the numbering scheme used for the degrees of freedom at the two end of the member.

During the analysis of any structure, the analyst has no control over the connectivity of the members, but there is complete control over the numbering of the degrees of freedom. The minimum half-band width will be obtained by numbering the nodes so that the absolute difference in the node numbers at the two ends of each member is a minimum. This is generally achieved by numbering the nodes sequentially across that side of the structure which has a smaller number of nodes. The half-band width associated with any member which connects the two nodes $N1$ and $N2$ is

$$MHBW = (IABS\,(N2 - N1) + 1) * NDFN$$

where $MHBW$ is the member half-band width and $NDFN$ is the number of degrees of freedom per node. The matrix half-band width will be the largest

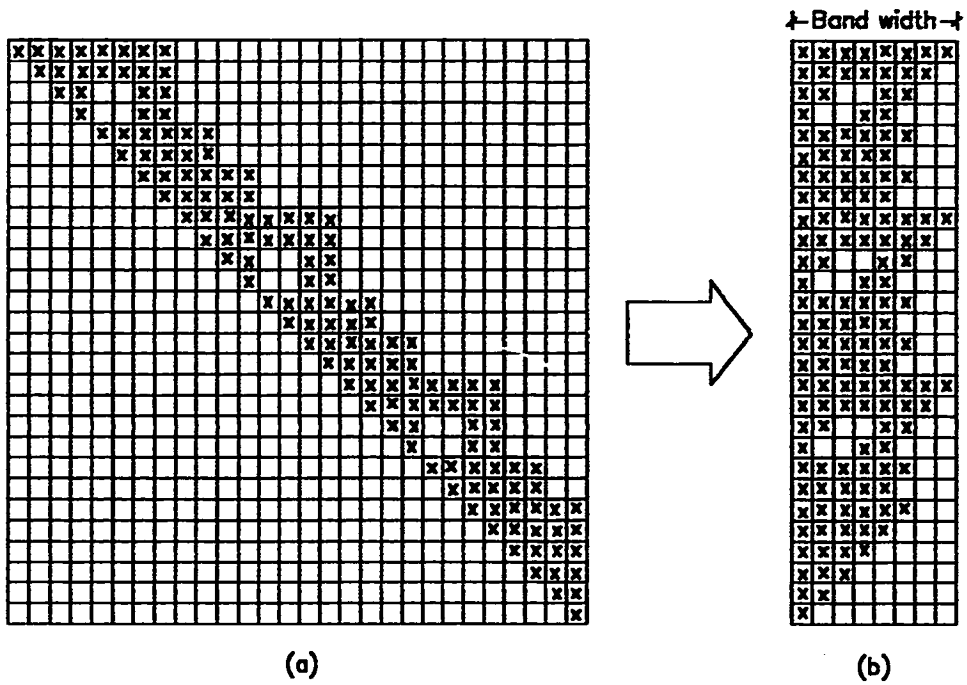



**Figure 4.2** Rectangular upper half-band storage scheme.

number obtained for *MHBW* after considering all of the members in the structure.

The rectangular array storage scheme results in the main diagonal of the matrix being stored in the first column of the array. The elements to the right of the main diagonal, in each matrix row, are stored sequentially in each row in the array. This results in the elements in each column of the matrix, above the main diagonal, being stored in a sloped arrangement extending upward to the right from the left column of the rectangular array. One of the tasks involved in writing a computer program which uses this storage scheme is to develop a bookkeeping procedure which can be used to relate the locations of the elements in the matrix to the locations of the elements in the rectangular storage array. To simplify the bookkeeping, the unrestrained stiffness matrix $\mathbf{S}_{FF}$ is initially generated. Only the elements corresponding to the unrestrained degrees of freedom are computed. The matrix is then adjusted so that the computed displacements at the restrained supports are zero. The location of the elements in the rectangular array are then determined and the assembly process is performed. The matrix is checked after it has been totally generated, to ensure that all elements on the main diagonal are nonzero. If a zero diagonal element is found, the matrix is considered to be singular and execution of RCFRAME is terminated.

The stiffness matrix generated in the form of Fig (4.2b) is factored and the solution for $\mathbf{D}_F$ is obtained in forward and backward sweeps. The topics of factorization and solution are covered in [45].

After the free nodal displacements have been calculated, the member end actions $A_M$ are obtained from the relationship

$$A_M = A_{ML} + S_M R_T D$$

and the analysis is then complete.

### 4.2.4 Output Data

RCFRAME generates an output file that contains a summary of the input data as well as tables listing final member sizes, recommended reinforcement, and, if desired, member end actions. A sample output file is presented in Appendix 3.

## 4.3 OPTIMAL DESIGN PHASE (IDESIGN)

It has been stated before that the optimal design problem of reinforced concrete frames is highly nonlinear. Both the objective function as well as the constraints are nonlinear functions of the design variables.

Many numerical methods have been developed to solve the general nonlinear programming problem. The methods start from an initial design provided by the user which is iteratively improved until the optimum is reached. Many of these methods have been incorporated into general-purpose design optimization software packages. One such package is IDESIGN (Interactive DESIGN optimization of engineering systems) [49,50,51]. IDESIGN consists of a main program and several standard subroutines that need not be changed by the user. It is written in structured, double precision FORTRAN 77.

In order to solve a problem through IDESIGN, the user must describe it by coding the following four FORTRAN subroutines:

USERMF: Minimization (cost) Function evaluation subroutine

USERCF: Constraint Functions evaluation subroutine

USERMG: Minimization (cost) function Gradient evaluation subroutine

USERCG: Constraint function Gradient evaluation subroutine

A fifth subroutine, USEROU, may also be provided by the user to perform post-optimality analyses for the optimal solution and obtain more OUtput. The user can call his own subroutines through the above subroutines. All of the user-supplied subroutines must be compiled and linked to IDESIGN to create an executable code with IDESIGN controlling the flow.

IDESIGN can solve any general nonlinear programming problem formulated as given in Eqs (3.1) to (3.4), linear programming problems and unconstrained problems. It can treat equality, inequality, and design variable bound constraints. The following algorithms are available:

1. Cost function bounding (CFB) algorithm.
2. Pshenichny's linearization method (LINRM).
3. Sequential quadratic programming (SQP) algorithm that generates and uses approximate second-order information for the Lagrange function; this algorithm has been also called recursive quadratic programming (RQP) algorithm in the literature.
4. A hybrid method that combines the cost function bounding and the sequential quadratic programming algorithms.
5. Conjugate gradient method for unconstrained problems.

IDESIGN is used in this study to solve the nonlinear programming problem. The optimization process as a whole consists of cycling between two distinct phases defined as analysis and optimal design in an iterative fashion until the optimum is reached. Figure (4.3) shows the structure of IDESIGN.

IDESIGN can be used in an interactive or batch mode of computation. It has been designed to accommodate both beginners and experienced users. The beginner can respond to one menu at a time as guided by the on-line instruction. The expert can prepare an input data file and thus bypass immediate menus. The program requires minimal input data for the problem; the user must provide the initial design, lower and upper limits on design variables, problem parameters, and the parameter values to invoke various options available in the program. A sample input data file is presented in Appendix 2.

The following capabilities to evaluate gradients and check gradient expressions are available:

1. If the user does not program gradient expressions in USERMG and USERCG subroutines, the program has an option to automatically calculate them. The finite difference method (forward, backward, or central) is employed using the specified value of $\delta$ (input data).
2. An option is available to determine the optimum value of $\delta$ for the finite difference gradient evaluation of cost and constraint functions.
3. If the user has programmed gradient expressions in USERMG and USERCG subroutines, an option is available to verify them, i.e., the gradient evaluation is checked using the finite difference approach. If the gradient expressions are in error, an option is available to either stop the program or continue its execution.

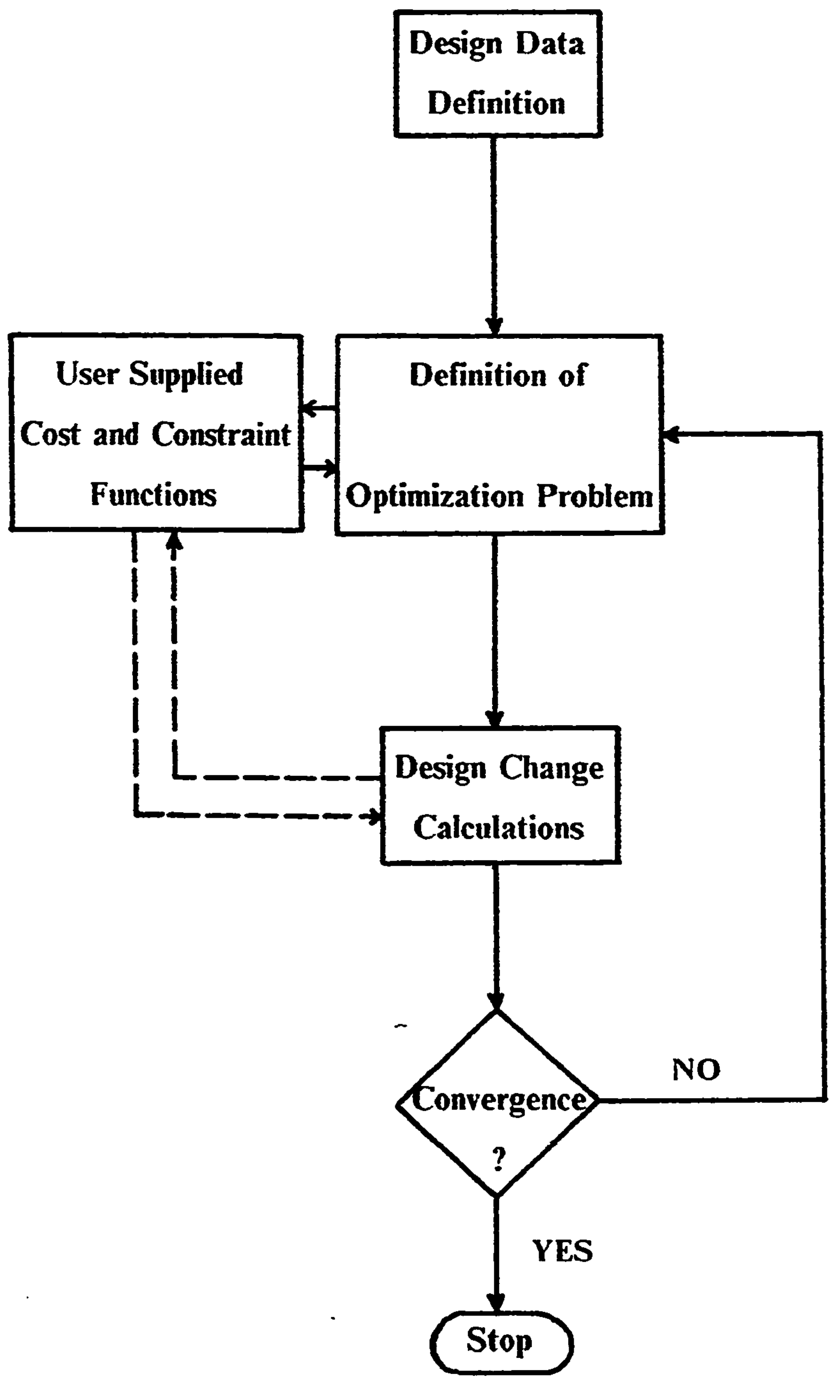



**Figure 4.3** Structure of IDESIGN program.

These options have proven to be extremely useful in many practical applications.

The complexity of the problem under investigation makes it very cumbersome to calculate the gradients analytically. Numerical evaluation of the gradients is commonly used and practically justified in such a problem with adequate accuracy being obtained. Therefore, USERMG and USERCG subroutines are provided empty.

Several levels of output can be obtained from IDESIGN. This is specified in the input data. The minimum output giving the final design, design variables and constraint activities, and histories of cost function, convergence parameter and maximum constraint violation, can be obtained. More detailed information at each iteration can also be obtained. The detailed output is used primarily for debugging the program. A sample minimum output data file is presented in Appendix 4.

*CHAPTER 5*

# DESIGN EXAMPLES

## 5.1 GENERAL

To exploit the various capabilities of RCFOPT program, a three-bay, five-story reinforced concrete frame is studied. The frame is designed according to the different objective function criteria available in the program. The minimum total cost design criterion is adopted as a reference case (Case 1). Other cases are obtained and are compared with Case 1. All cases, except the last one (Case 8), are solved using second-order elastic analysis with no moment redistribution, i.e., using IELAST to be equal to 2.

All cases are solved using the recursive (sequential) quadratic programming algorithm available in the IDESIGN software package [49,50,51]. The CPU times reported are for double precision calculations on IBM 3090-150E mainframe computer in time sharing environment at the Data Processing Center, King Fahd University of Petroleum and Minerals.

## 5.2 PROBLEM DESCRIPTION

The three-bay, five-story reinforced concrete frame in Fig. (5.1) is to be designed for the loading shown. The load values indicated in the figure are assumed to define the factored-load level in accordance with the American National Standards Institute (ANSI) *Minimum Design Loads for Buildings and*

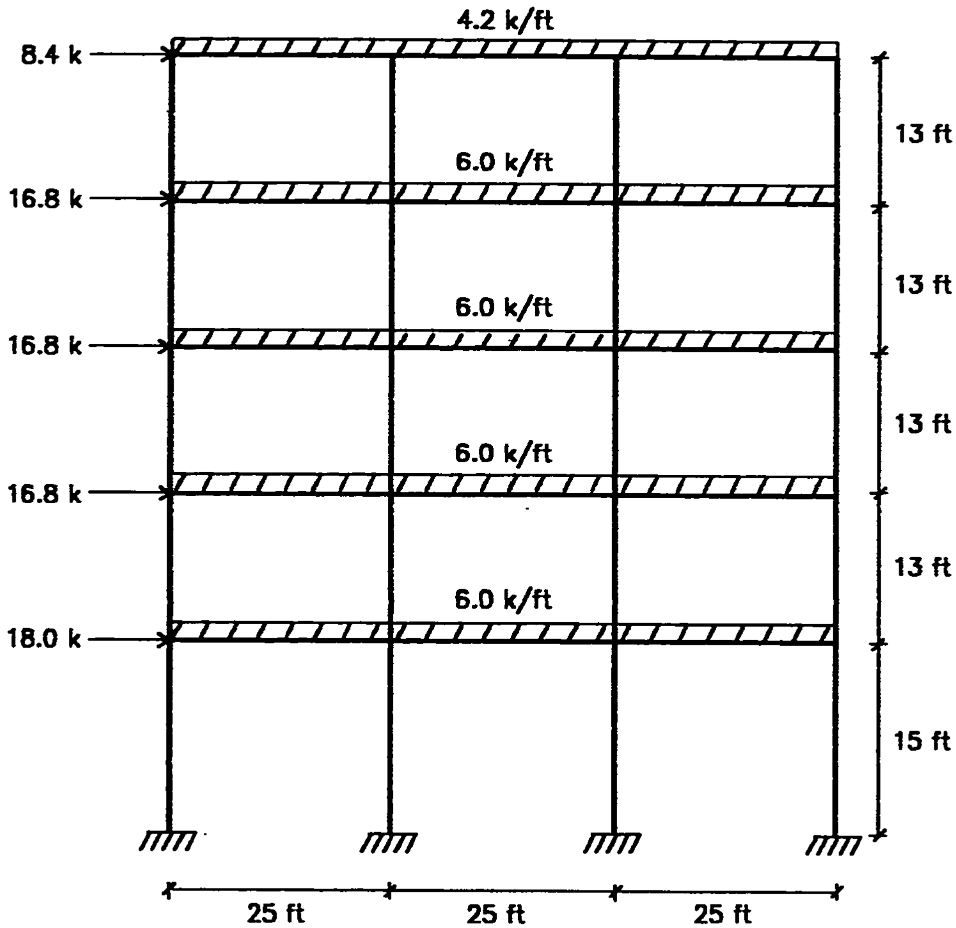



**Figure 5.1** Geometry and loading of the three-bay, five-story frame.

*Other Structures* [59]. Material unit costs are based on recent average market prices [60]. Complete input data files are presented in Appendices 1 and 2.

## 5.3 PROBLEM SOLUTION

The number of design variables for this problem is 30, and the number of constraints (excluding 60 explicit design variable bound constraints) is 70. The frame is designed according to the different objective function criteria available in RCFOPT program. The following case studies are performed:

*Case 1*: Minimum total cost design

*Case 2*: Minimum concrete volume design

*Case 3*: Minimum steel weight design

*Case 4*: Minimum total volume design

*Case 5*: Minimum total weight design

*Case 6*: Minimum formwork surface area design

*Case 7*: Zero-cost formwork design; IELAST=2

*Case 8*: Zero-cost formwork design; IELAST=3

For the sake of comparison, optimum column and beam design variables for the different cases are presented in Tables (5.1) and (5.2), respectively. Below are some observations on each case.

### *Case 1*

The frame is designed according to the minimum total cost design criterion, i.e., $C_c=80.0$ \$/yd$^3$, $C_s=0.385$ \$/lb, and $C_f=2.8$ \$/ft$^2$. Second-order elastic

**Table 5.1** Optimum column design variables for different cases.

| Story No. | Design variable | Case 1 | Case 2 | Case 3 | Case 4 |
|---|---|---|---|---|---|
| 1 | $b_c$ | 14.137 | 14.147 | 13.146 | 14.147 |
| | $d_c$ | 11.836 | 11.647 | 17.879 | 11.647 |
| | $A_{st}$ | 12.495 | 15.602 | 11.848 | 12.570 |
| 2 | $b_c$ | 13.726 | 13.729 | 13.160 | 13.729 |
| | $d_c$ | 11.226 | 11.229 | 11.518 | 11.229 |
| | $A_{st}$ | 12.148 | 15.092 | 11.362 | 12.891 |
| 3 | $b_c$ | 13.386 | 13.385 | 12.611 | 13.385 |
| | $d_c$ | 10.886 | 10.885 | 10.111 | 10.885 |
| | $A_{st}$ | 11.025 | 13.659 | 9.969 | 11.641 |
| 4 | $b_c$ | 13.095 | 13.094 | 12.374 | 13.094 |
| | $d_c$ | 10.595 | 10.594 | 9.874 | 10.594 |
| | $A_{st}$ | 10.124 | 12.765 | 8.725 | 10.606 |
| 5 | $b_c$ | 12.096 | 12.095 | 12.048 | 12.095 |
| | $d_c$ | 9.596 | 9.595 | 9.548 | 9.595 |
| | $A_{st}$ | 8.538 | 11.266 | 8.280 | 8.625 |

$b_c$ and $d_c$ are in inches
$A_{st}$ is in square inches

**Table 5.1(Cont.)** Optimum column design variables for different cases.

| Story No. | Design variable | Case 5 | Case 6 | Case 7 | Case 8 |
|---|---|---|---|---|---|
| 1 | $b_c$ | 14.147 | 14.147 | 13.542 | 13.019 |
| | $d_c$ | 11.647 | 11.647 | 14.599 | 16.115 |
| | $A_{st}$ | 12.536 | 12.589 | 11.928 | 11.860 |
| 2 | $b_c$ | 13.729 | 13.729 | 13.636 | 13.206 |
| | $d_c$ | 11.229 | 11.229 | 11.136 | 10.706 |
| | $A_{st}$ | 12.162 | 12.346 | 11.744 | 11.680 |
| 3 | $b_c$ | 13.385 | 13.385 | 13.400 | 12.858 |
| | $d_c$ | 10.885 | 10.885 | 10.900 | 10.358 |
| | $A_{st}$ | 11.028 | 11.032 | 11.029 | 10.849 |
| 4 | $b_c$ | 13.094 | 13.094 | 12.985 | 12.378 |
| | $d_c$ | 10.594 | 10.594 | 10.485 | 9.878 |
| | $A_{st}$ | 10.126 | 10.171 | 9.904 | 9.413 |
| 5 | $b_c$ | 12.095 | 12.095 | 12.104 | 11.751 |
| | $d_c$ | 9.595 | 9.595 | 9.604 | 9.251 |
| | $A_{st}$ | 8.541 | 8.755 | 8.495 | 8.363 |

$b_c$ and $d_c$ are in inches
$A_{st}$ is in square inches

**Table 5.2** Optimum beam design variables for different cases.

| Story No. | Design variable | Case 1 | Case 2 | Case 3 | Case 4 |
|---|---|---|---|---|---|
| 1 | $b_w$ | 14.137 | 14.147 | 13.146 | 14.147 |
| | $d_b$ | 21.208 | 21.221 | 24.592 | 21.221 |
| | $A_s^-$ | 5.502 | 5.517 | 4.244 | 5.517 |
| 2 | $b_w$ | 13.726 | 13.729 | 13.160 | 13.729 |
| | $d_b$ | 20.619 | 20.594 | 23.722 | 20.594 |
| | $A_s^-$ | 4.960 | 4.965 | 4.261 | 4.965 |
| 3 | $b_w$ | 13.386 | 13.385 | 12.611 | 13.385 |
| | $d_b$ | 20.079 | 20.078 | 24.868 | 20.078 |
| | $A_s^-$ | 4.534 | 4.533 | 3.633 | 4.533 |
| 4 | $b_w$ | 13.095 | 13.094 | 12.374 | 13.094 |
| | $d_b$ | 19.643 | 19.641 | 23.879 | 19.641 |
| | $A_s^-$ | 4.184 | 4.183 | 3.377 | 4.183 |
| 5 | $b_w$ | 12.096 | 12.095 | 12.048 | 12.095 |
| | $d_b$ | 18.144 | 18.143 | 18.420 | 18.143 |
| | $A_s^-$ | 3.092 | 3.091 | 3.043 | 3.091 |

$b_w$ and $d_b$ are in inches
$A_s^-$ is in square inches

**Table 5.2(Cont.)** Optimum beam design variables for different cases.

| Story No. | Design variable | Case 5 | Case 6 | Case 7 | Case 8 |
|---|---|---|---|---|---|
| 1 | $b_w$ | 14.147 | 14.147 | 13.542 | 13.019 |
| | $d_b$ | 21.221 | 21.221 | 23.420 | 22.662 |
| | $A_s^-$ | 5.517 | 5.517 | 4.727 | 4.094 |
| 2 | $b_w$ | 13.729 | 13.729 | 13.636 | 13.206 |
| | $d_b$ | 20.594 | 20.594 | 21.098 | 20.784 |
| | $A_s^-$ | 4.965 | 4.965 | 4.846 | 4.315 |
| 3 | $b_w$ | 13.385 | 13.385 | 13.400 | 12.858 |
| | $d_b$ | 20.078 | 20.078 | 20.100 | 20.422 |
| | $A_s^-$ | 4.533 | 4.533 | 4.551 | 3.909 |
| 4 | $b_w$ | 13.094 | 13.094 | 12.985 | 12.378 |
| | $d_b$ | 19.641 | 19.641 | 20.287 | 20.911 |
| | $A_s^-$ | 4.183 | 4.183 | 4.056 | 3.381 |
| 5 | $b_w$ | 12.095 | 12.095 | 12.104 | 11.751 |
| | $d_b$ | 18.143 | 18.143 | 18.155 | 17.627 |
| | $A_s^-$ | 3.091 | 3.091 | 3.099 | 2.749 |

$b_w$ and $d_b$ are in inches
$A_s^-$ is in square inches

analysis is used with no moment redistribution. The optimal design and the history of the iterative design process are given in Appendices 3 and 4.

Starting from an infeasible design with a maximum constraint violation of 52.5%, the program takes 18 iterations to converge to an optimal design. At the optimal point, the objective function value is $18953.2. Almost all of the design variables pertaining to concrete dimensions are close to their lower bounds. The active constraints on columns are those imposed on column geometry and its minimum reinforcing steel area. Strength constraints do not have much effect on columns for this case. For beams, the active constraints are those imposed on the geometry, flexural capacity, crack width, and minimum width for steel accommodation. The column/beam compatibility constraint is always active. The CPU time for this case is 203 seconds.

*Case 2*

The frame is designed according to the minimum concrete volume design criterion, i.e., $C_c = 1.0$, and $C_s = C_f = 0.0$. Second-order elastic analysis is used with no moment redistribution.

At the optimal point, the objective function value is 38.4 $yd^3$. The actual cost of this design is $19264.6, i.e., more than the cost of Case 1 design by 1.6%. Almost all of the design variables pertaining to concrete dimensions are close to their lower bounds as in Case 1. Therefore, there is not much change in the concrete dimensions for this case compared to Case 1. The amount of steel, however, increases by 25% with the major portion of this increase happening in columns. Because of this increase, the constraint imposed on the minimum column reinforcing steel area is no more active and is replaced by the constraint on the maximum reinforcing steel area with the geometrical constraint being still active. The active constraints on beams are the same as in Case 1. Moreover, the column/beam compatibility constraint is always active as in Case 1.

*Case 3*

The frame is designed according to the minimum steel weight design criterion, i.e., $C_c = C_f = 0.0$, and $C_s = 1.0$. Second-order elastic analysis is used with no moment redistribution.

At the optimal point, the objective function value is 19566.8 lb. The actual cost of this design is \$19461.3, i.e., more than the cost of Case 1 design by 2.7%. The amount of steel decreases by 8% in columns and 16% in beams. This is accompanied by an increase of 4% in column cross sectional areas (mainly in the first story) and 11% in beam cross sectional areas. The active constraints on columns and beams are not much changed from Case 1. The column/beam compatibility constraint is always active as in Case 1.

*Case 4*

The frame is designed according to the minimum total volume design criterion, i.e., $C_c = 1.0$, $C_s = 1.0/\gamma_s = 7.6 \times 10^{-5}$ yd$^3$/lb, and $C_f = 0.0$. Second-order elastic analysis is used with no moment redistribution.

At the optimal point, the objective function value is 40.0 yd$^3$. The actual cost of this design is \$18997.6, i.e., approximately equal to the cost of Case 1 design. Almost all of the design variables pertaining to concrete dimensions are close to their lower bounds as in Case 1. Therefore, there is not much change in the concrete dimensions for this case compared to Case 1. The amount of steel increases by 4% with the major portion of this increase happening in columns. The active constraints on columns and beams are not much changed from Case 1. The column/beam compatibility constraint is always active as in Case 1.

*Case 5*

The frame is designed according to the minimum total weight design criterion, i.e., $C_c=\gamma_c=4050.0$ lb/yd$^3$, $C_s=1.0$, and $C_f=0.0$. Second-order elastic analysis is used with no moment redistribution.

At the optimal point, the objective function value is 175976.6 lb. The actual cost of this design is \$18956.4, i.e., approximately equal to the cost of Case 1 design. The design variables are close to those obtained in Case 1. The active constraints are the same as in Case 1.

*Case 6*

The frame is designed according to the minimum formwork surface area design criterion, i.e., $C_c=C_s=0.0$, and $C_f=1.0$. Second-order elastic analysis is used with no moment redistribution.

At the optimal point, the objective function value is 2880.5 ft$^2$. The actual cost of this design is \$19424.4, i.e., more than the cost of Case 1 design by 2.5%. Almost all of the design variables pertaining to concrete dimensions are close to their lower bounds as in Case 1. Therefore, there is not much change in the concrete dimensions for this case compared to Case 1. The amount of steel increases by 5% with the major portion of this increase happening in columns. The active constraints on columns and beams are not much changed from Case 1. Moreover, the column/beam compatibility constraint is always active as in Case 1.

*Case 7*

The frame is designed while neglecting the cost of formwork, i.e., $C_c$=80.0 \$/yd$^3$, $C_s$=0.385 \$/lb, and $C_f$=0.0. Second-order elastic analysis is used with no moment redistribution.

At the optimal point, the objective function value is \$10831.0, i.e., the cost of the frame decreases by 43% which shows the effect of formwork on the total cost of reinforced concrete frames. The amount of steel decreases by 2% in columns and 4% in beams. This is accompanied by an increase of 4% in column cross sectional areas (mainly in the first story with square columns being replaced by rectangular columns) and 3% in beam cross sectional areas. the active constraints on columns and beams are not much changed from Case 1. The column/beam compatibility constraint is always active as in Case 1.

*Case 8*

All of the seven cases above have been performed using second-order elastic analysis with no moment redistribution. In this case, moment redistribution will be utilized in the analysis and design of the frame in order to assess its effect on the optimal design. The frame is designed while neglecting the cost of formwork (similar to Case 7) in order to eliminate the effect of formwork on the design variables.

At the optimal point, the objective function value is $10455.0, i.e., less than the cost of Case 7 design by only 3.5%. However, the program takes 51% more CPU time than Case 7 in order to converge to an optimal design.

Because the net effect of moment redistribution is a reduction in both negative and positive bending moments, it is logical to have a reduction in beam dimensions and reinforcement areas. Geometrical configurations of columns have slight changes while their reinforcement areas are not much changed. The active constraints are the same as in Case 7.

## 5.4 CONVERGENCE OF THE PROBLEM

To observe the effect of different starting designs on the convergence of the problem, two widely separated starting designs are used to start the iterative design process. For the first starting point all the design variables are at their lower bounds and for the second point they are all at their upper bounds. Both starting points converge to the same optimum solution with almost the same values for design variables and objective function. The active constraints are the same as in Case 1.

Table (5.3) gives the number of iterations, CPU times, and costs of the two designs. The convergence (cost history) is shown in Fig. (5.2).

**Table 5.3** Convergence of different starting designs.

| Starting design | No. of iterations | CPU time (seconds) | Cost (dollars) |
|---|---|---|---|
| LB | 18 | 203 | 18953.50 |
| UB | 24 | 266 | 18955.21 |

LB means lower bounds
UB means upper bounds

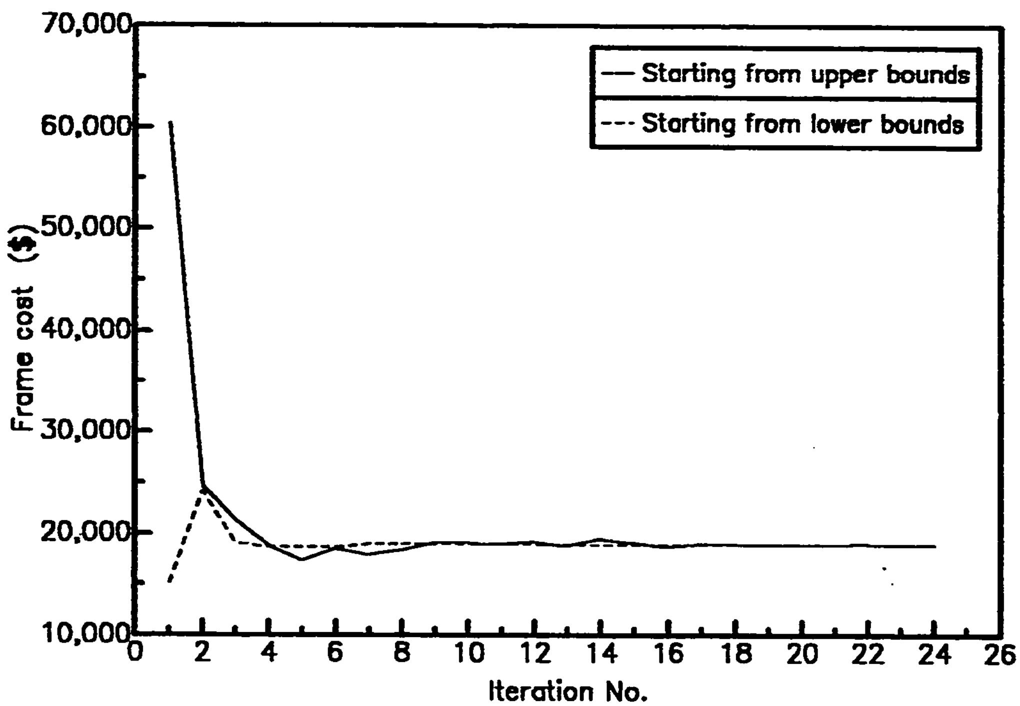



**Figure 5.2** Cost history of the three-bay, five-story frame.

## 5.5 DISCUSSION

The capability of RCFOPT program to generate different designs for a particular reinforced concrete frame has been investigated. It has been proven that the minimum total cost criterion is the most economical design criterion for reinforced concrete frames. All other criteria have generated designs that are more expensive than the design generated by the minimum total cost criterion.This conclusion supports the choice of this criterion for the problem formulation in this study.

The effect of formwork on the total cost of reinforced concrete frames has been investigated. It has been proven that the cost of formwork comprises the major portion of the total cost of frames. Therefore, any efforts made to improve the economy of a reinforced concrete frames should be primarily concentrated on reducing formwork cost. This has been the basis of a general strategy for frame economy in this study.

Two widely separated starting designs have been tried to observe their effect on the convergence of the problem. For the first starting point all the design variables were at their lower bounds and for the second point they were all at their upper bounds. Both starting points have converged to the same optimum solution with almost the same values for design variables and objective functions. However, the number of iterations, and consequently the CPU times were different. This shows that the starting design estimate for the iterative process affects the rate of convergence of an optimization algorithm, while it should not affect the optimum solution. A good starting design can be obtained after some preliminary analyses. It is desirable to use such a starting design for the optimization algorithm because this can save substantial computational effort especially for large-scale problems.

While analysis and design using moment redistribution generates lower-cost reinforced concrete frames than with no moment redistribution, it is believed that the reduction in cost does not justify the extra computational effort to reach an optimum solution. For the case of the three-bay, five-story reinforced concrete frame studied above, there was a 3.5% reduction in cost while the CPU time increased by 51%.

*CHAPTER 6*

# APPLICATIONS OF RCFOPT SOFTWARE

## 6.1 GENERAL

In addition to being a design optimization package for reinforced concrete frames, RCFOPT can be used in two other ways. It can be used to develop design aids in the form of equations and graphs for the preliminary design of reinforced concrete frame members. Furthermore, RCFOPT can be used as a research tool for studying the behavior of reinforced concrete frames at the optimal point under different geometric conditions, material properties, and loadings.

This chapter is divided into two main parts. The first part presents the findings of twenty five optimal design cases performed on a three-bay, five-story reinforced concrete frame in order to study the variations of column and beam dimensions and reinforcement ratios under different combinations of beam span lengths and service live loads. Based on those findings, some general guidelines for the preliminary design of reinforced concrete frame members are developed and tested. In the second part of the chapter, a behavioral study is performed on a four-bay, ten-story reinforced concrete frame in order to study the actual configuration of the frame at the optimal design and to assess the optimal variations of the design variables under a multitude of conditions involving changing number of stories, lateral (wind) loads, and aspect ratios. Finally, the effect of the number of bays on he the optimum cost of reinforced concrete frames is investigated.

The recursive (sequential) quadratic programming algorithm available in IDESIGN is used exclusively. The CPU times reported are for double precision calculations on IBM 3090-150E mainframe computer at the Data Processing Center, King Fahd University of Petroleum and Minerals.

## 6.2 GENERAL DESIGN GUIDELINES

### 6.2.1 Introduction

In the previous chapter, it has been shown that RCFOPT program can take any frame design specified by the user and iteratively refine it to obtain an optimal design. The efficiency of this iterative process depends on the starting preliminary design. A satisfactory preliminary design saves many analysis and design cycles and consequently reduces the time required to reach an optimal solution. Therefore, it is prudent to develop some guidance as to how to obtain good starting points so that designers, even those who have no access to the program, can generate economical framed structures in less time.

### 6.2.2 Model Description

The same three-bay, five-story reinforced concrete frame utilized in the previous chapter and shown in Fig. (5.1) is used to develop the aforementioned guidelines. For many years, most reinforced concrete buildings have been in the low- to medium-rise range. Figure (6.1) shows the percentages of buildings constructed in 1983 in the United States in terms of different building height

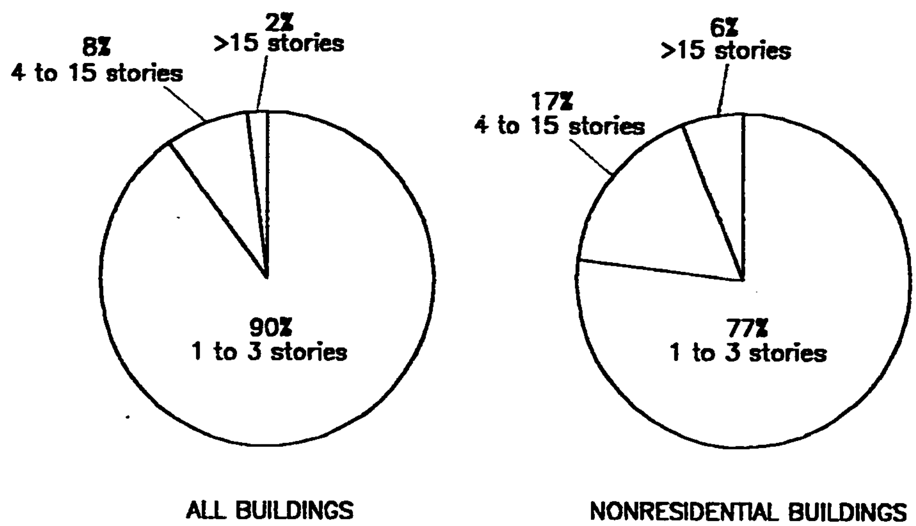



**Figure 6.1** Percentages of buildings constructed in 1983 in the United States in terms of different building height categories.

categories [52]. From this it can be readily seen that the vast majority of the volume of construction is in the one- to three-story range. There is no indication that this trend has changed since then. Therefore, a five-story building seems reasonable for the development of general design guidelines.

Concrete compressive strength and steel yield strength are 4 ksi and 60 ksi, respectively. Both material strengths are readily available in the market place and will result in members that are durable and perform well structurally. Design live loads are in accordance with the American National Standards Institute *Minimum Design Loads for Buildings and Other Structures* [59]. Material unit costs are based on recent average market prices [60]. The cost of concrete is taken as 80.0 $\$/yd^3$, the cost of steel is 0.385 \$/lb, and the cost of formwork is 2.8 $\$/ft^2$.

Beam span lengths are varied from 15 ft to 35 ft with 5 ft increments. Service Live loads are varied from 50 psf to 150 psf with 25 psf increments. Therefore, a total of 25 optimal design cases are studied.

### 6.2.3 Development of the Guidelines

#### 6.2.3.1 Introduction

This section presents the findings of the twenty five optimal design cases mentioned above. Based on these findings, the proposed guidelines are developed for beams and then for columns. Their accuracies are checked and they are applied to a four-bay, four-story reinforced concrete frame in order to ascertain their generality.

#### 6.2.3.2 Optimal Beam Depth Expression

Table (6.1) summarizes the variations of the optimal beam span to depth ratio $L_b/d_b$ with the change of span length for different service live loads. The results are plotted in Fig. (6.2). It can be readily seen that, for span range from 20 ft to 35 ft, the variations are essentially linear. Case studies on the 15-ft span have shown that beam dimensions are controlled mainly by geometric rather than strength constraints at such short span. For this span and for low live loads, the beam dimensions hit the lower bounds on the design variables making the $L_b/d_b$ ratio always constant. Because of its effect on the general variation of the curves, the 15-ft span case is disregarded.

The variations in Fig. (6.2) can now be expressed in the form of linear equations, i.e.,

$$\frac{L_b}{d_b} = C_1 L_b + C_2$$

where $C_1$ and $C_2$ are constants. The constants $C_1$ and $C_2$ can be determined by linearly fitting the points of Fig. (6.2). Figure (6.3) presents results of that fitting for various live loads. For the live load of 50 psf as an example, $C_1$ and $C_2$ are 0.28223 and 9.6101, respectively. Therefore,

$$\frac{L_b}{d_b} = 0.28223\, L_b + 9.6101$$

is the equation of optimal beam depth for that live load. Obviously, the constants $C_1$ and $C_2$ will be different for different live loads. Thus, there is a different expression for each live load. For the live load of 75 psf, the equation is:

**Table 6.1** Values of $L_b/d_b$ as obtained from computer results.

| $L_b$ | Live load (psf) | | | | |
|---|---|---|---|---|---|
| | 50 | 75 | 100 | 125 | 150 |
| 15′ | 12.000 | 12.000 | 12.000 | 12.000 | 11.921 |
| 20′ | 15.094 | 14.724 | 14.458 | 14.201 | 13.954 |
| 25′ | 16.854 | 16.484 | 16.129 | 15.790 | 15.464 |
| 30′ | 18.182 | 17.734 | 17.391 | 17.143 | 16.744 |
| 35′ | 19.355 | 18.919 | 18.502 | 18.182 | 17.872 |

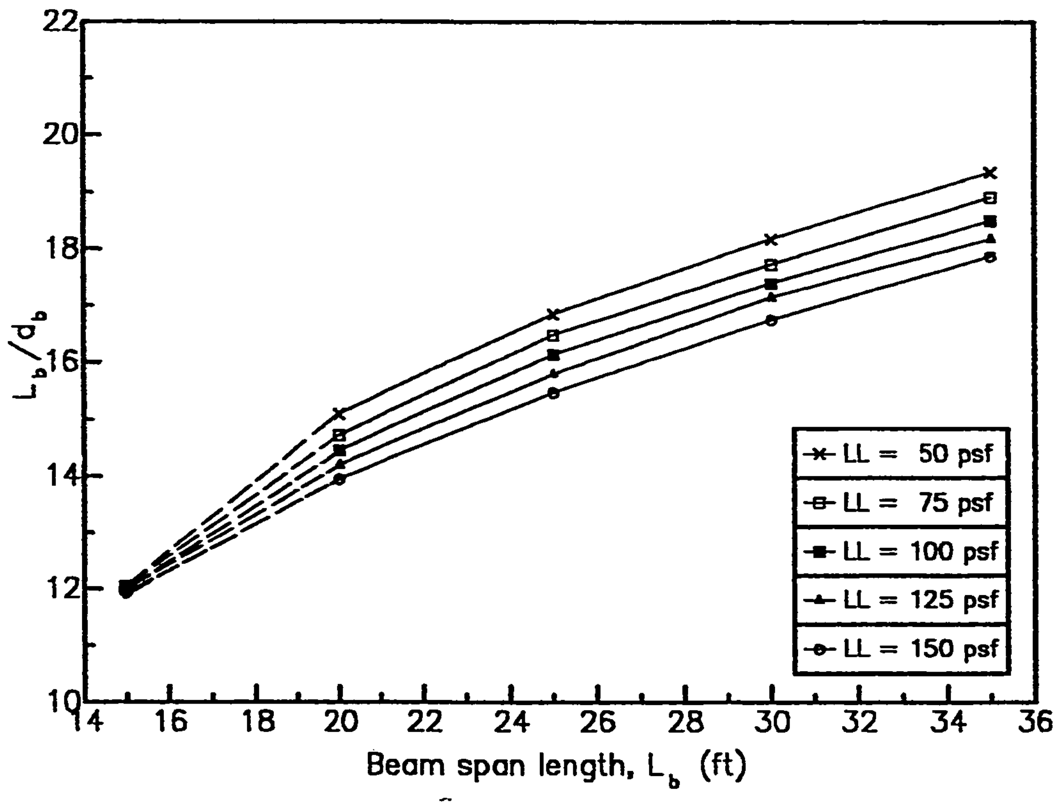



**Figure 6.2** Variations of optimal $L_b/d_b$ ratio with the change of beam span length for different live loads.

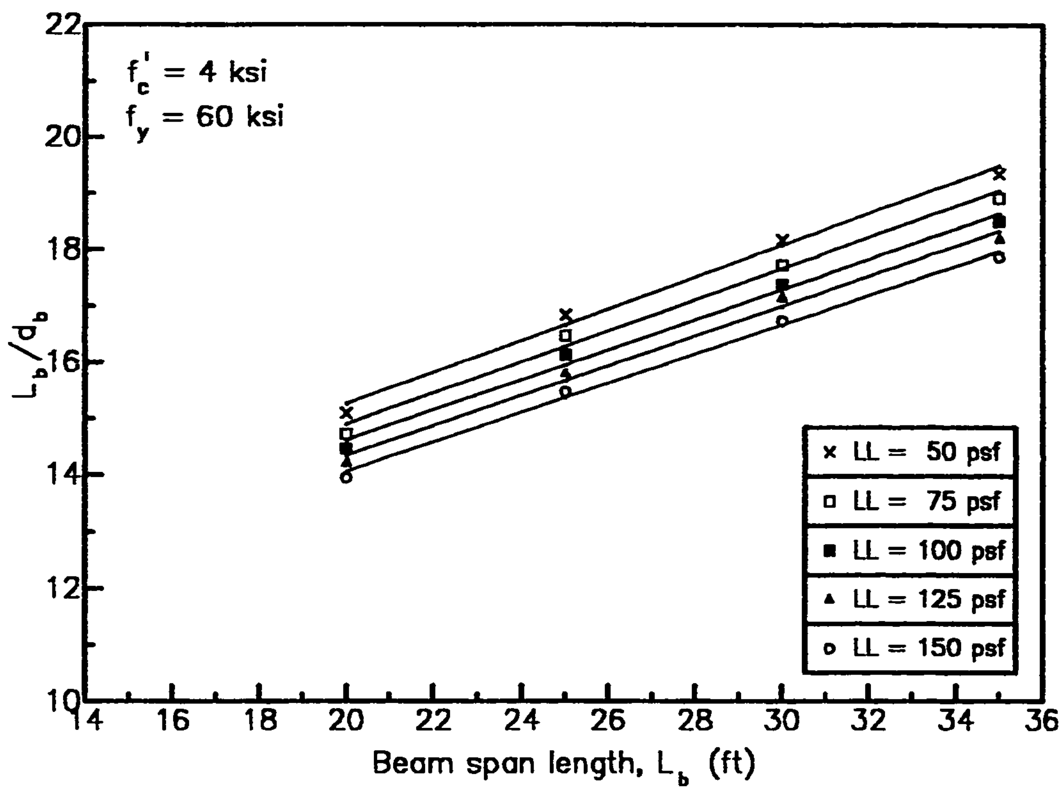



**Figure 6.3** Linear fittings of optimal $L_b/d_b$ ratio with respect to beam span length for different live loads.

$$\frac{L_b}{d_b} = 0.27670\, L_b + 9.3560$$

For the live load of 100 psf, the equation is:

$$\frac{L_b}{d_b} = 0.26788\, L_b + 9.2532$$

For the live load of 125 psf, the equation is:

$$\frac{L_b}{d_b} = 0.26592\, L_b + 9.0161$$

For the live load of 150 psf, the equation is:

$$\frac{L_b}{d_b} = 0.26068\, L_b + 8.8398$$

Each pair of $C_1$ and $C_2$ is plotted with respect to service live load and fitted linearly as shown in Figs (6.4) and (6.5). The figures give the expressions for each of $C_1(LL)$ and $C_2(LL)$ as functions of service live load.

The expression for optimal beam depth is now:

$$\frac{L_b}{d_b} = C_1(LL)\, L_b + C_2(LL)$$

With the values from linear fittings shown in Figs (6.4) and (6.5), the following expression can be obtained:

$$\frac{L_b}{d_b} = (\, - \,2.2 \times 10^{-4}\, LL + 0.29223\,)\, L_b + (\, - \,7.5 \times 10^{-3}\, LL + 9.96720\,)$$

This expression can be written in a simpler form as:

$$\frac{L_b}{d_b} = (\, - \,2.2 \times 10^{-4}\, LL + 0.29\,)\, L_b + (\, - \,7.5 \times 10^{-3}\, LL + 9.97\,)$$

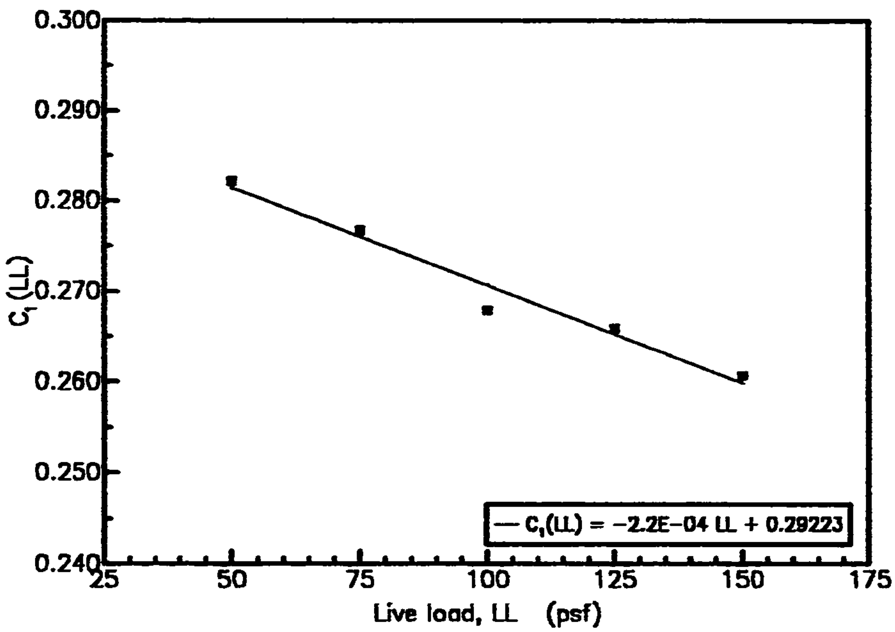



**Figure 6.4** Constant $C_1$ for optimal beam depth expression as a linear function of the live load.

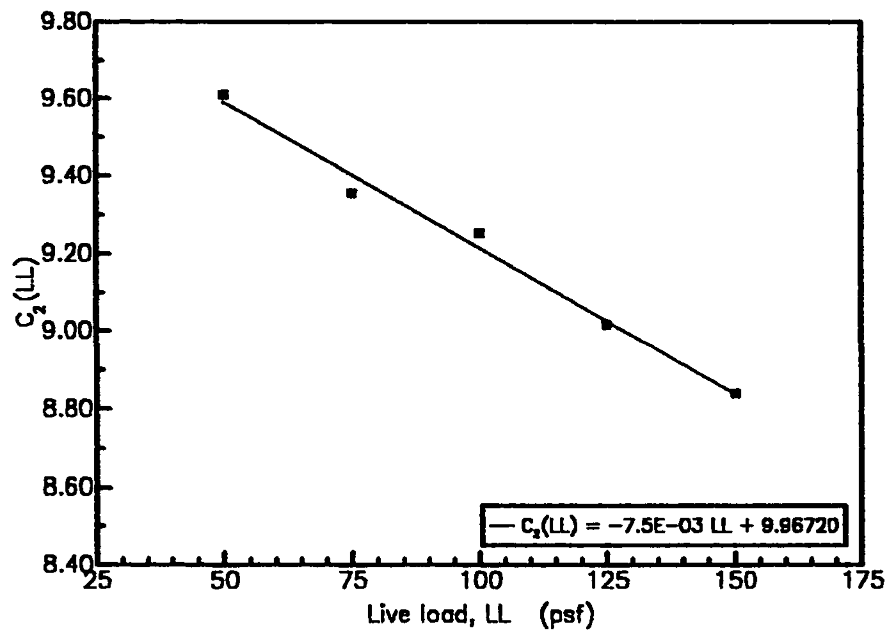



**Figure 6.5** Constant $C_2$ for optimal beam depth expression as a linear function of the live load.

where $L_b/d_b$ is dimensionless, $LL$ is in psf, and $L_b$ is in ft.

The above expression is applicable when the spacing between frames is 12 ft. In order to generalize that expression to be applicable for any spacing, a factor of $S/12$ should be incorporated, where $S$ is the spacing between frames in ft. Thus, the final form of the optimal beam depth expression can be written as:

$$\frac{L_b}{d_b} = \left( -2.2 \times 10^{-4}\, LL \left(\frac{S}{12}\right) + 0.29 \right) L_b + \left( -7.5 \times 10^{-3}\, LL \left(\frac{S}{12}\right) + 9.97 \right) \qquad (6.1)$$

### 6.2.3.3 Accuracy of Optimal Beam Depth Expression

The accuracy of the optimal beam depth expression is checked by comparing some $L_b/d_b$ values picked up from the computer results with the values calculated from Eq. (6.1). Comparison of the values of optimal $L_b/d_b$ ratio for two different live loads and for three different span lengths is given in Table (6.2). The maximum error between values obtained from the equation and exact optimal values is 1.5%. For the entire range of live loads and span lengths, values from the equation and values picked up from the computer results agree fairly closely. Thus, Eq. (6.1) is fairly accurate and can be used for the calculation of optimal beam depth in the preliminary design stage.

### 6.2.3.4 Optimal Beam Reinforcement Ratio Expression

Table (6.3) summarizes the variations of the optimal beam reinforcement ratio $\rho$ with the change of span length for different service live loads. The

**Table 6.2** Accuracy of beam depth expression.

| $L_b$ | 75 psf | | | 125 psf | | |
|---|---|---|---|---|---|---|
| | EQ | CR | ERR | EQ | CR | ERR |
| 20′ | 14.878 | 14.724 | 1.1% | 14.283 | 14.201 | 0.6% |
| 25′ | 16.245 | 16.484 | -1.5% | 15.595 | 15.790 | -1.2% |
| 30′ | 17.613 | 17.734 | -0.7% | 16.908 | 17.143 | -1.4% |

EQ means $L_b/d_b$ is obtained from equation
CR means $L_b/d_b$ is obtained from computer results
ERR means percetage error between the two values

**Table 6.3** Values of ρ (%) as obtained from computer results.

| $L_b$ | Live load (psf) | | | | |
|---|---|---|---|---|---|
| | 50 | 75 | 100 | 125 | 150 |
| 15′ | 0.667 | 0.667 | 0.757 | 0.845 | 0.908 |
| 20′ | 1.035 | 1.113 | 1.171 | 1.221 | 1.265 |
| 25′ | 1.362 | 1.419 | 1.478 | 1.543 | 1.597 |
| 30′ | 1.656 | 1.707 | 1.756 | 1.819 | 1.870 |
| 35′ | 1.897 | 1.948 | 2.023 | 2.066 | 2.102 |

results are plotted in Fig. (6.6). It can be readily seen that, for span range from 20 ft to 35 ft, the variations are essentially linear. The 15-ft span case is disregarded because of its effect on the general variation of the curves especially at lower live loads.

The variations in Fig. (6.6) can now be expressed in the form of linear equations, i.e.,

$$\rho = C_1 L_b + C_2$$

where $C_1$ and $C_2$ are constants. The constants $C_1$ and $C_2$ can be determined by linearly fitting the points of Fig. (6.6). Figure (6.7) presents results of that fitting for various live loads. For the live load of 50 psf, the equation of optimal beam reinforcement ratio is:

$$\rho = 5.8 \times 10^{-2} L_b - 9.6 \times 10^{-2}$$

For the live load of 75 psf, the equation is:

$$\rho = 5.6 \times 10^{-2} L_b + 9.9 \times 10^{-3}$$

For the live load of 100 psf, the equation is:

$$\rho = 5.7 \times 10^{-2} L_b + 4.8 \times 10^{-2}$$

For the live load of 125 psf, the equation is:

$$\rho = 5.6 \times 10^{-2} L_b + 0.11626$$

For the live load of 150 psf, the equation is:

$$\rho = 5.6 \times 10^{-2} L_b + 0.17798$$

Each pair of $C_1$ and $C_2$ is plotted with respect to service live load as shown in Figs (6.8) and (6.9). The plots can be approximated by two straight lines for a reasonable fit. The figures give the expressions for each of $C_1(LL)$ and $C_2(LL)$

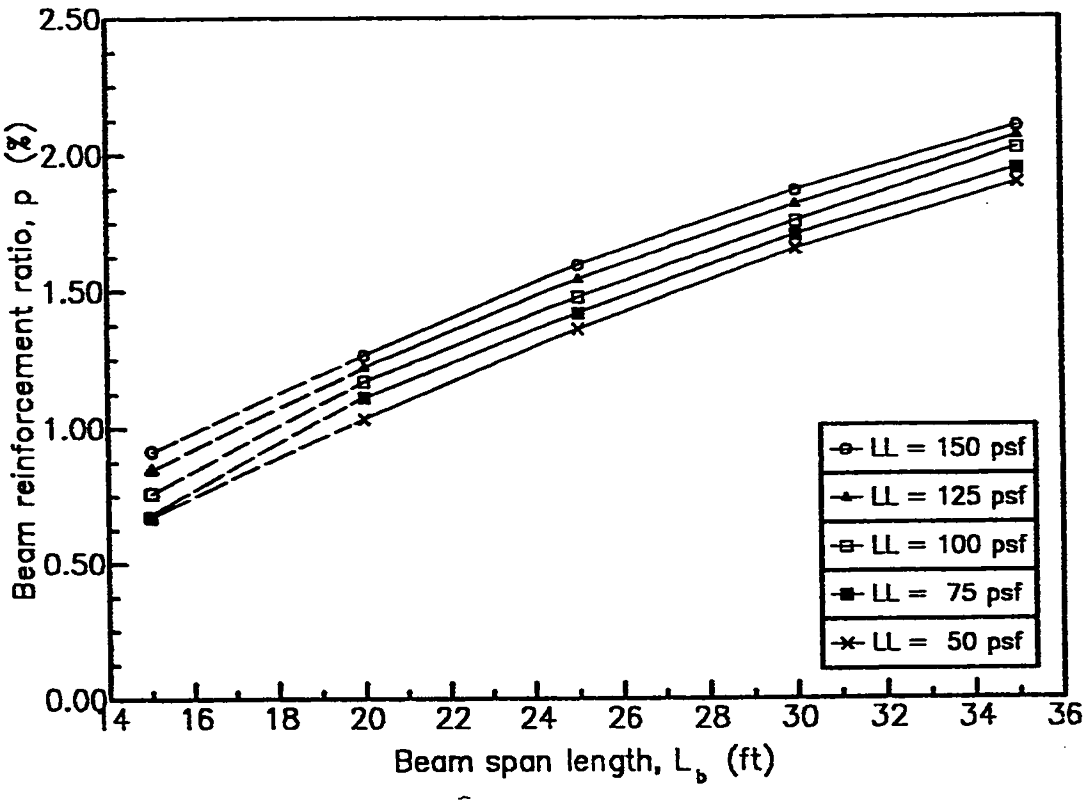



**Figure 6.6** Variations of optimal beam reinforcement ratio with the change of beam span length for different live loads.

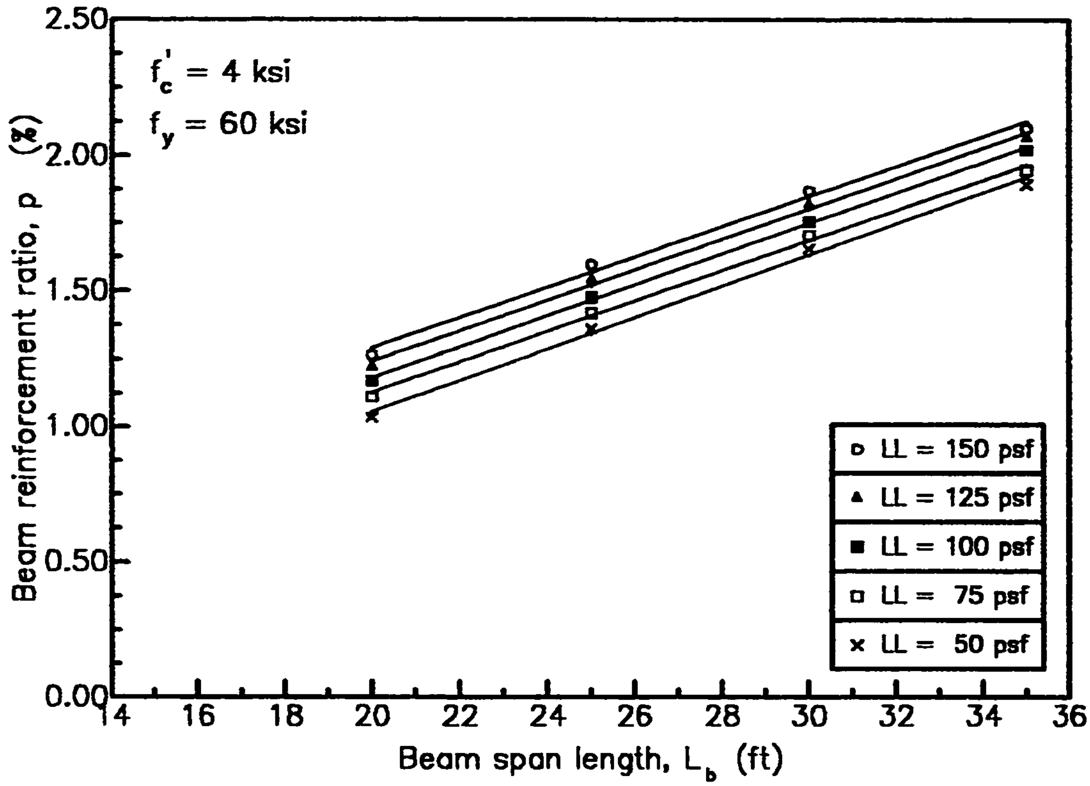



**Figure 6.7** Linear fittings of optimal beam reinforcement ratio with respect to beam span length for different live loads.

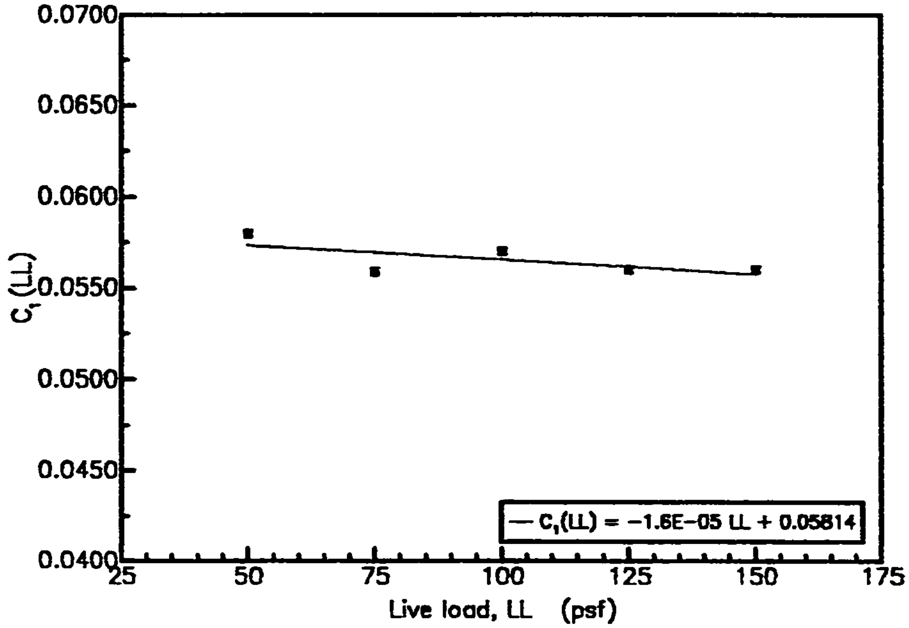



**Figure 6.8** Constant $C_1$ for optimal beam reinforcement ratio expression as a linear function of the live load.

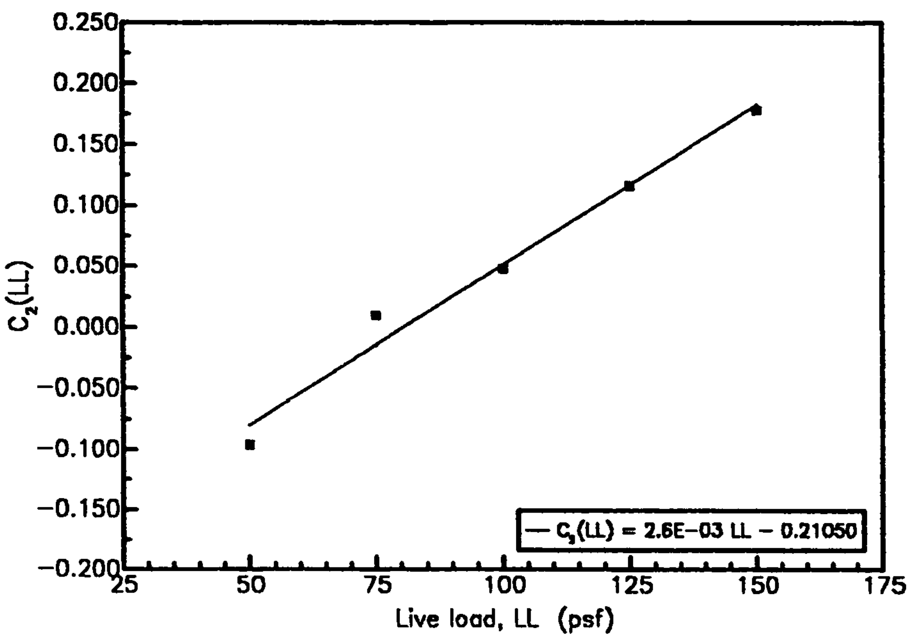



**Figure 6.9** Constant $C_2$ for optimal beam reinforcement ratio expression as a linear function of the live load.

as functions of service live load.

The expression for optimal beam reinforcement ratio is now:

$$\rho = C_1(LL)\,L_b + C_2(LL)$$

Using values from the linear fitting shown in Figs (6.8) and (6.9), the following expression can be obtained:

$$\rho = (\,-\,1.6 \times 10^{-5}\,LL + 0.05814\,)\,L_b + (\,2.6 \times 10^{-3}\,LL - 0.21050\,)$$

Generalizing for any frame spacing, the above expression can be written in a simpler form as:

$$\rho = (\,-\,1.6 \times 10^{-5}\,LL\left(\frac{S}{12}\right) + 0.06\,)\,L_b + (\,2.6 \times 10^{-3}\,LL\left(\frac{S}{12}\right) - 0.21\,) \qquad (6.2)$$

where $\rho$ is a percentage, $LL$ is in psf, and both of $L_b$ and $S$ are in ft.

#### 6.2.3.5 Accuracy of Optimal Beam Reinforcement Ratio Expression

The accuracy of the optimal beam reinforcement ratio expression is checked by comparing some $\rho$ values picked up from the computer results with values calculated from Eq. (6.2). Comparison of the values for two different live loads and for three different span lengths is given in Table (6.4). For the entire range of live loads and span lengths, the error between values from the equation and values picked up from the computer results is quite small. The maximum error is 4.4%. Thus, Eq. (6.2) can be used for the calculation of optimal beam reinforcement ratio.

**Table 6.4** Accuracy of beam reinforcement ratio expression.

| $L_b$ | 75 psf | | | 125 psf | | |
|---|---|---|---|---|---|---|
| | EQ | CR | ERR | EQ | CR | ERR |
| 20′ | 1.161 | 1.113 | 4.3% | 1.275 | 1.221 | 4.4% |
| 25′ | 1.455 | 1.419 | 2.5% | 1.565 | 1.543 | 1.4% |
| 30′ | 1.749 | 1.707 | 2.5% | 1.855 | 1.819 | 2.0% |

EQ means $\rho$ is obtained from equation
CR means $\rho$ is obtained from computer results
ERR means percetage error between the two values

#### 6.2.3.6 Determination of Beam Web Width

Having determined the beam depth and its reinforcement ratio, what remains to complete the design is to determine the beam web width. This can be achieved by observing the active constraints on beams at the optimal point. In almost all of the twenty five optimal design cases, it has been noted that the constraint on beam crack width which is

$$\frac{0.0006}{145} f_y \left[ \frac{2(d')^2 b_w}{n_{min}} \right]^{1/3} - 1.0 \leq 0$$

and the constraint on minimum beam width for steel accommodation which is

$$\frac{6.0 + 2(n_{max} - 1)\left[ \frac{4.2 A_s^-}{\pi\, n_{max}} \right]^{1/2}}{b_w} - 1.0 \leq 0$$

are always active. These constraints are directly related to the beam web width.

The web width can be directly obtained from the former constraint and then be checked with the latter constraint to see which one of them controls. In all cases, the $d_b/b_w$ ratio should be maintained between 1.5 and 2.0, preferably on the lower side.

#### 6.2.3.7 Column Design Chart

The main design aid adopted by the ACI Code [18] for the proportioning of columns are the interaction curves. The designer *estimates* the gross cross sectional area of the column, carry out the structural analysis and determine the axial load and the maximum moment in the column. Using the interaction curves, the designer then reads the required value of the reinforcement ratio $\rho_g$. If $\rho_g$ falls within the 1 to 8 percent limits established by the ACI Code, the design is complete. Otherwise, the dimensions must be readjusted and the analysis and design repeated.

Selecting a suitable initial estimate for the gross area depends primarily on the experience of the designer. Even experienced designers cannot estimate the *minimum* required gross area needed to satisfy the ACI Code requirements. Therefore, a design chart for columns that is based on optimal design concepts will be very valuable for designers.

All of the twenty five optimal design cases performed to develop these guidelines have generated columns that are square rather than rectangular. In addition, the column/beam compatibility constraint has been active in all of the cases making column widths identical to beam widths. Since beam widths are already known from before (Section 6.2.3.6), therefore column widths are also known. And since all columns are square, one ends up with the dimensions of columns at hand.

What remains to complete the design of columns is to determine the required values of the reinforcement ratio $\rho_g$. In order to achieve that, the results of the twenty five optimal design cases are utilized. Each case generates five different combinations of $P_u$ and $M_u$ making the total number of combinations 125 each of which could represent specific $\rho_g$ at specific $P_u$ for certain eccentricity $e$, where $e = M_u / P_u$. By lumping several combinations of $P_u$ and $e$ for the same values of $\rho_g$, normalizing $e$ by dividing it by $h$, and plotting the results, one ends up with the curves shown in Fig. (6.10). Figure (6.10)

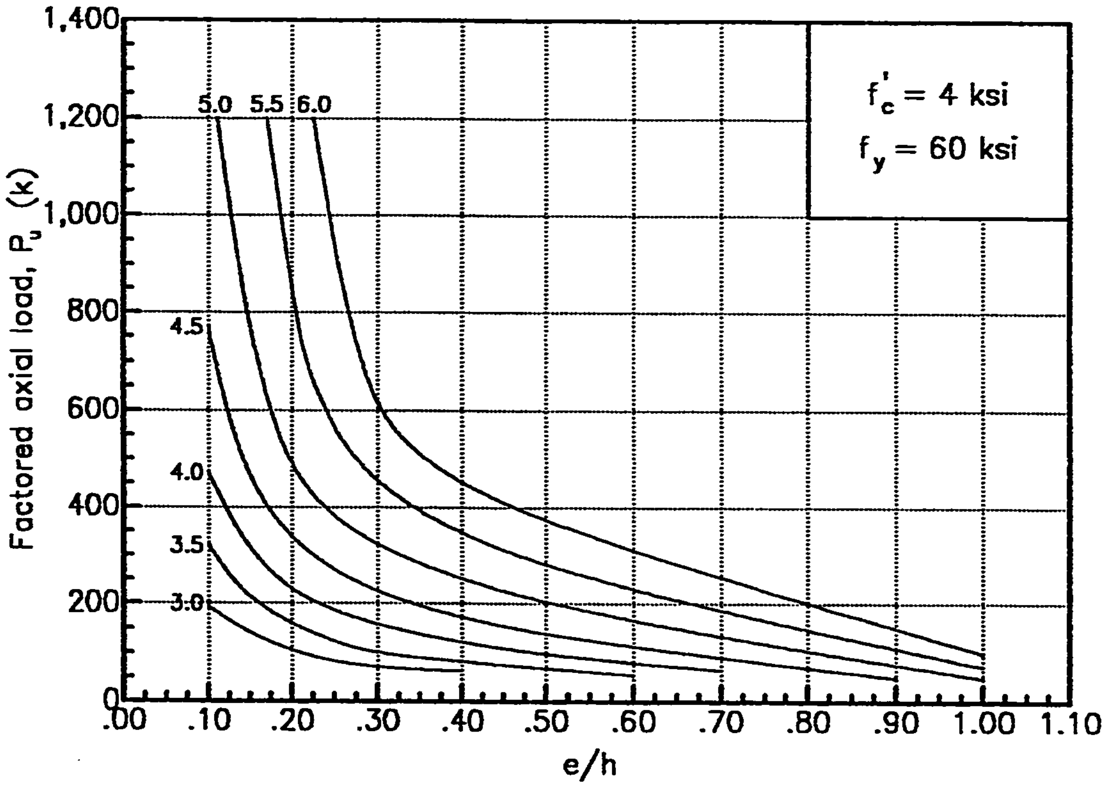



**Figure 6.10** Design chart for reinforced concrete columns.

gives the column reinforcement ratio in percentage for a given factored axial load at a given *e/h*. And now the column design is complete.

Several points have been checked and compared between Fig. (6.10) and the ACI interaction curves. The saving in the amount of steel achieved by using Fig. (6.10) has been found to be in the range from 10% to 16%.

### 6.2.4 Design Example

The above design guidelines have been developed using a three-bay, five-story reinforced concrete frame. In order to ascertain the generality of these guidelines, a four-bay, four-story reinforced concrete frame is designed using the same material strengths, unit costs, and column heights. Beam span lengths are taken to be 22 ft. The applied service live load is 85 psf, and the spacing between frames is 14 ft.

Comparison between values obtained from the computer results and those obtained using the guidelines have shown the following facts:

1. The maximum percentage difference in the value of the beam effective depth is 2.5%.
2. The maximum percentage difference in the value of the beam reinforcement ratio is 3.0%.
3. The maximum percentage difference in the value of the column reinforcement ratio is 3.8%.

Therefore, the developed design guidelines are fairly accurate and can be used for the preliminary design of reinforced concrete frame members.

## 6.3 BEHAVIORAL STUDY

### 6.3.1 Introduction

In order to demonstrate the utilization of RCFOPT software as a research tool, a behavioral study is performed on a four-bay, ten-story reinforced concrete frame. The configuration of the frame at the optimal design is studied. Then, the optimal variations of the design variables are investigated under a multitude of conditions involving changing number of stories, lateral (wind) loads, and aspect ratios. All studies are performed using second-order elastic analysis with no moment redistribution.

### 6.3.2 Model Description

Figure (6.11) shows the layout of the four-bay, ten-story reinforced concrete frame which is used for this study. The geometric dimensions and the loadings are similar to those of the three-bay, five-story frame shown in Fig. (5.1). Material strengths and costs are also the same.

The number of design variables for this problem is 60, and the number of constraints (excluding 120 explicit design variable bound constraints) is 140. The program takes 20 iterations to converge to an optimal design; the convergence (cost history) is shown in Fig. (6.12). The CPU time is 2424 seconds.

### 6.3.3 Optimal Frame Configuration

To study the actual configuration of the ten-story frame at the optimal design, one needs only to investigate the design variables. Tables (6.5) and (6.6)

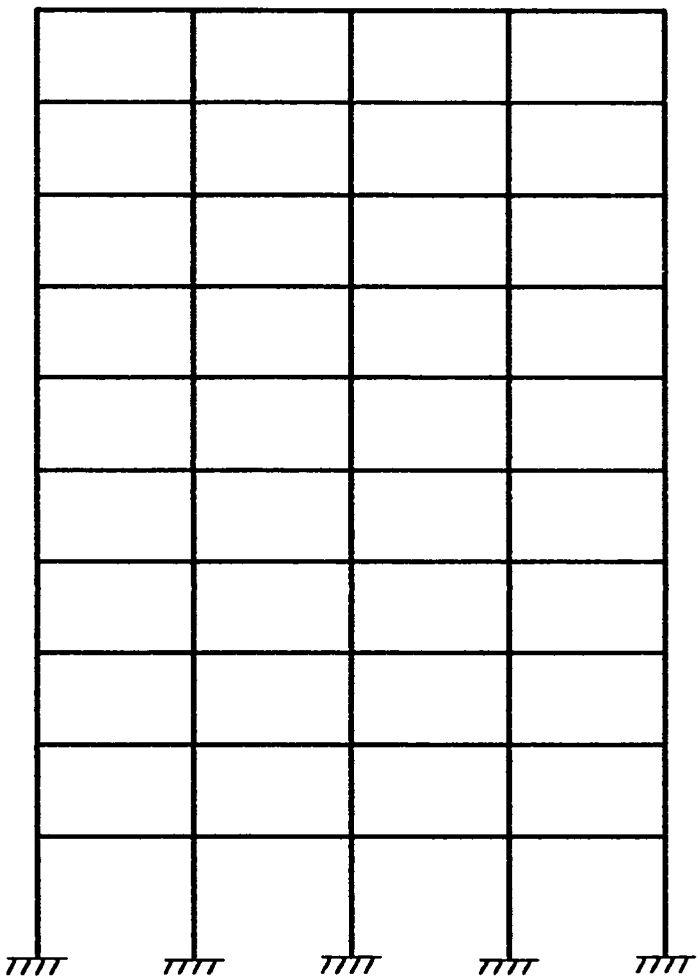


**Figure 6.11** Layout of the four-bay, ten-story frame.

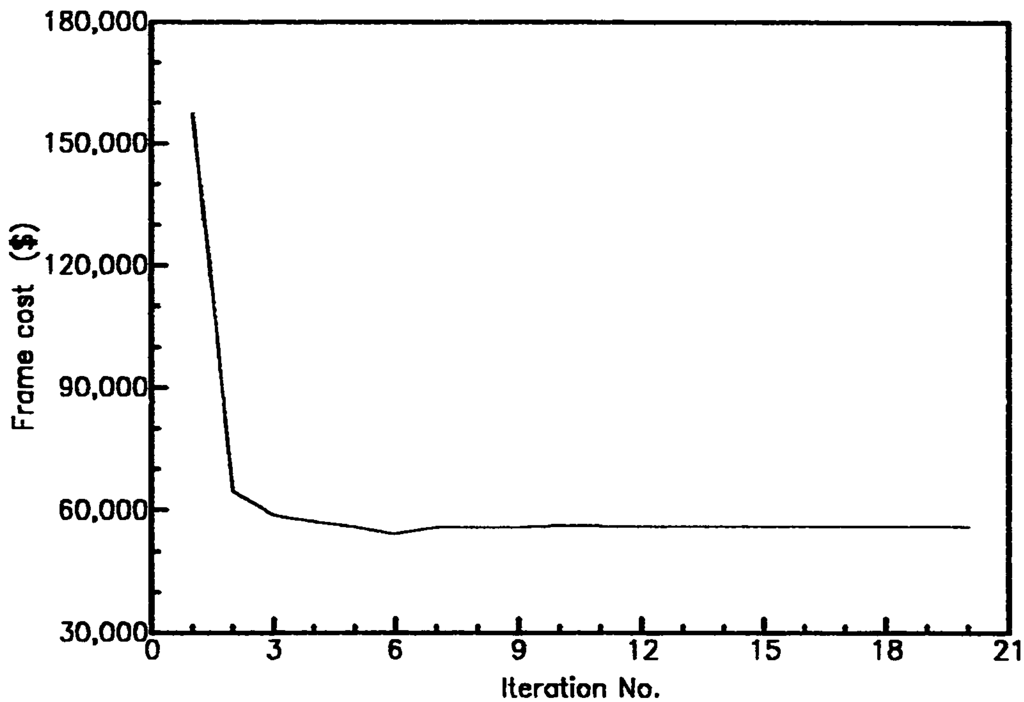



**Figure 6.12** Cost history of the ten-story frame problem.

**Table 6.5** Column design variable variations for the ten-story frame.

| Story No. | $b_c$ | $d_c$ | $A_{st}$ |
|---|---|---|---|
| 1 | 14.4 | 23.2 | 29.617 |
| 2 | 14.6 | 20.1 | 26.456 |
| 3 | 14.5 | 17.6 | 23.365 |
| 4 | 14.3 | 15.3 | 20.293 |
| 5 | 14.0 | 13.0 | 17.123 |
| 6 | 13.9 | 11.4 | 13.433 |
| 7 | 13.7 | 11.2 | 12.633 |
| 8 | 13.4 | 10.9 | 11.692 |
| 9 | 13.2 | 10.7 | 10.936 |
| 10 | 12.1 | 9.6 | 9.374 |

$b_c$ and $d_c$ are in inches
$A_{st}$ is in square inches

**Table 6.6** Beam design variable variations for the ten-story frame.

| Story No. | $b_w$ | $d_b$ | $A_s^-$ |
|---|---|---|---|
| 1 | 14.4 | 22.4 | 5.651 |
| 2 | 14.6 | 22.0 | 6.179 |
| 3 | 14.5 | 21.8 | 6.038 |
| 4 | 14.3 | 21.4 | 5.708 |
| 5 | 14.0 | 21.4 | 5.345 |
| 6 | 13.9 | 21.0 | 5.158 |
| 7 | 13.7 | 20.5 | 4.897 |
| 8 | 13.4 | 20.2 | 4.603 |
| 9 | 13.2 | 19.9 | 4.298 |
| 10 | 12.1 | 18.1 | 3.072 |

$b_w$ and $d_b$ are in inches
$A_s^-$ is in square inches

summarize the variations of these variables with story level for columns and beams, respectively. These variations are plotted in Figs (6.13) to (6.16).

Figure (6.13) shows column dimension variations with story level. It can be readily seen that the changes in column depths are much more than the corresponding changes in column widths up to the fifth story after which they take the same trend. This behavior agrees with what is done in practice where column widths are usually fixed and depths are changed to meet strength requirements. One can also see that rectangular columns dominates in the lower five stories after which columns become square. Column reinforcement area variations are shown in Fig. (6.14). It can be noted that the changes in reinforcement areas correspond closely to the changes in depths. All design variables move to their lower bounds at the roof level. This configuration of the columns implies that in the lower five stories, column dimensions are governed mainly by strength requirements, while in the upper five stories, columns can be proportioned on the basis of geometric considerations alone.

Figure (6.15) shows beam dimension variations with story level. It can be noted that the ratio of beam depths to beam web widths is almost always constant due to the geometric constraints imposed on this ratio. Moreover, beam web widths and column widths (shown before in Fig. (6.13)) vary identically due to the compatibility constraint which is almost always active. The variations in beam reinforcement areas are shown in Fig. (6.16). It can be readily seen that beam reinforcement peaks at the second story because of higher bending moments in the beams compared to the first story. This behavior has been obtained before by Spires and Arora [37,38]. All design variables move to their lower bounds at the roof level.

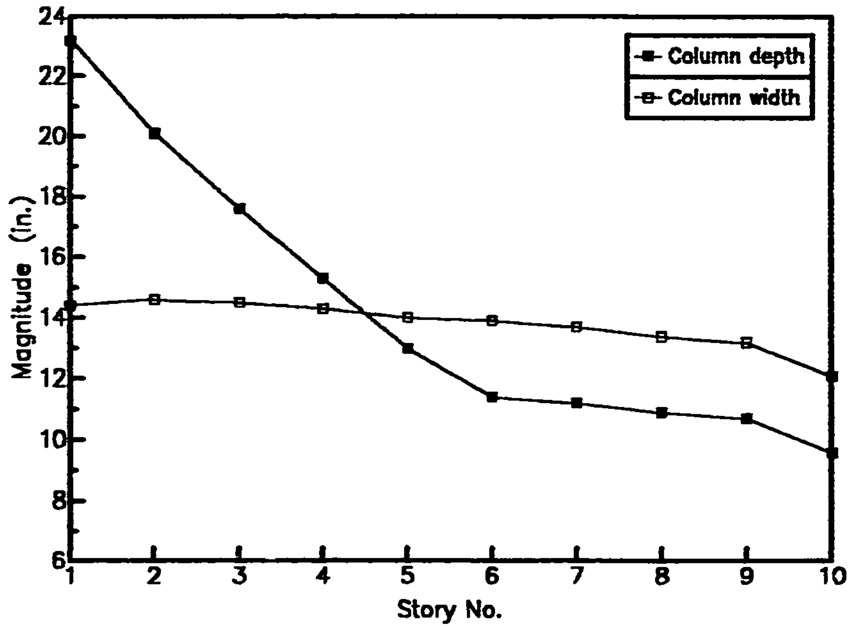



**Figure 6.13** Column dimension variation for ten-story frame.

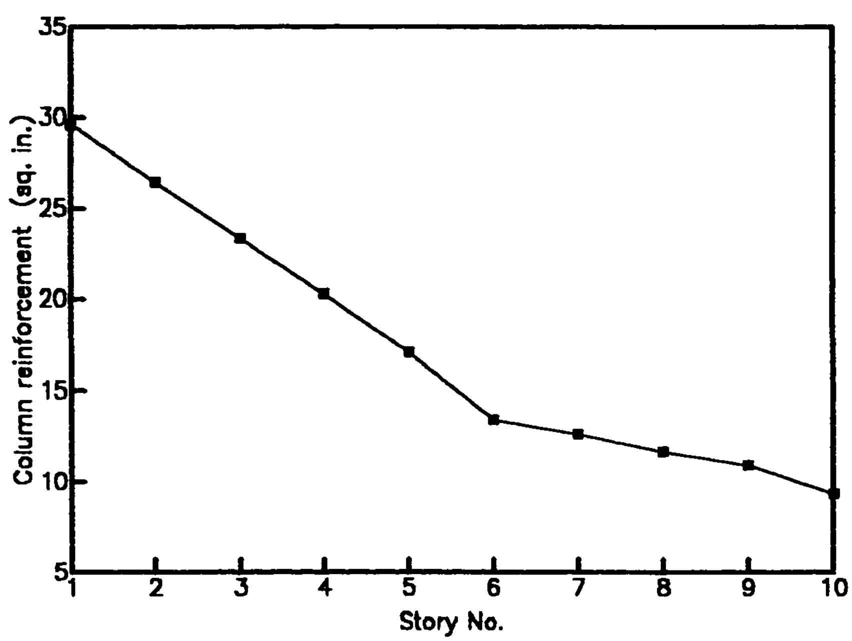



**Figure 6.14** Column reinforcement variation for ten-story frame.

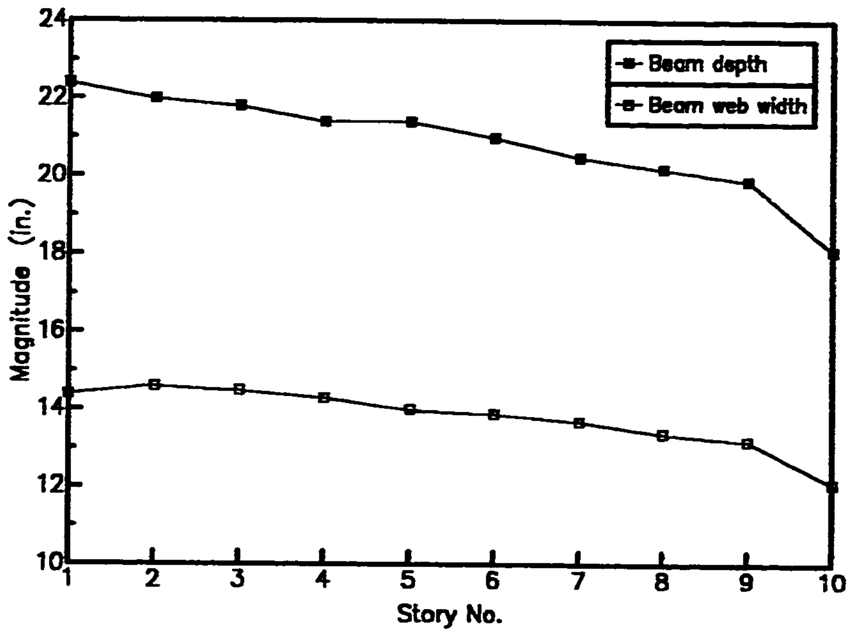



**Figure 6.15** Beam dimension variation for ten-story frame.

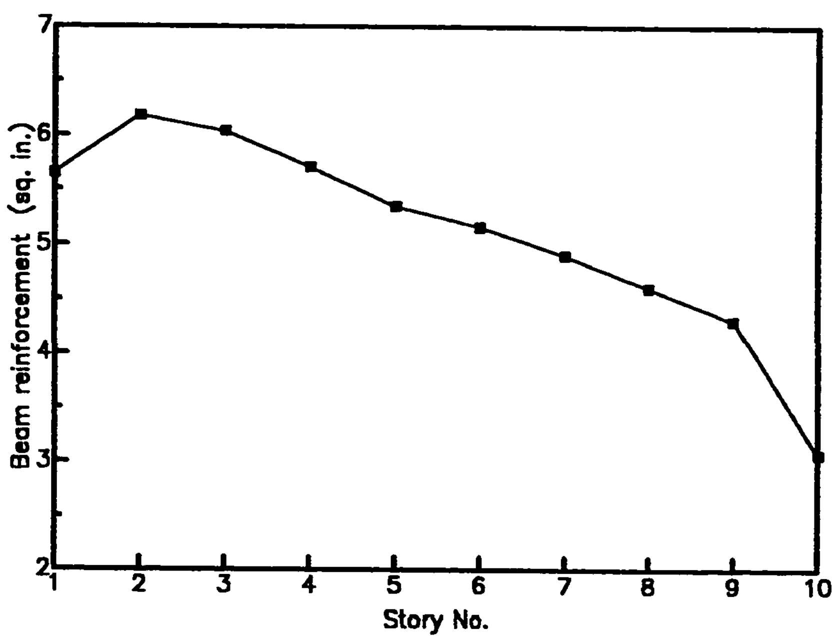



**Figure 6.16** Beam reinforcement variation for ten-story frame.

### 6.3.4 Effect of Number of Stories on Columns

To study the effect of different number of stories on the variations of column design variables, four reinforced concrete frames of different number of stories are investigated: a four-story frame, a six-story frame, an eight-story frame, and a ten-story frame. Tables (6.7) through (6.9) summarize the variations of the design variables with story level.

Figure (6.17) shows column depth variations for the four frames. It can be seen that as the number of stories decreases, the change in the magnitude of depths between any two stories also decrease. Moreover, the changes in the magnitude of depths between any two stories in the lower stories are much more than the changes in the upper stories for all frames except for the four-story frame where the changes are almost equal. This suggests that column dimensions are governed by strength requirements at lower stories in higher frames, while at upper stories, geometric considerations control the sizes of columns. For lower frames, such as the four-story frame, geometry controls column dimensions in all stories. A similar behavior is exhibited by column reinforcement areas as shown in Fig. (6.18). The variations in column reinforcement ratios are plotted in Fig. (6.19).

Additional insight regarding the variations of column design variables can be gained by studying the active column constraints at the optimal design. Figure (6.20) shows the activity of various column constraints. The variation of the constraints conforms with the structural response of frames under uniform and lateral loads. As the number of stories in a frame increases, axial forces and bending moments on the lower columns also increase, making the constraints on strength and maximum reinforcement ratio active. On the other hand, it can be

**Table 6.7** Column depth variations for frames of different number of stories.

| Story No. | Number of Stories | | | |
|---|---|---|---|---|
| | 4 | 6 | 8 | 10 |
| 1 | 11.2 | 14.6 | 18.4 | 23.2 |
| 2 | 10.9 | 11.5 | 15.6 | 20.1 |
| 3 | 10.5 | 10.9 | 13.8 | 17.6 |
| 4 | 9.9 | 10.5 | 12.5 | 15.3 |
| 5 | ---- | 10.3 | 10.9 | 13.0 |
| 6 | ---- | 9.7 | 10.8 | 11.4 |
| 7 | ---- | ---- | 10.5 | 11.2 |
| 8 | ---- | ---- | 9.6 | 10.9 |
| 9 | ---- | ---- | ---- | 10.7 |
| 10 | ---- | ---- | ---- | 9.6 |

Column depths are in inches

**Table 6.8** Column reinforcement variations for frames of different number of stories.

| Story No. | Number of stories | | | |
|---|---|---|---|---|
| | 4 | 6 | 8 | 10 |
| 1 | 10.269 | 17.106 | 23.828 | 29.617 |
| 2 | 10.066 | 14.135 | 20.626 | 26.456 |
| 3 | 9.085 | 11.366 | 17.067 | 23.365 |
| 4 | 8.222 | 10.377 | 13.002 | 20.293 |
| 5 | ---- | 9.479 | 11.901 | 17.123 |
| 6 | ---- | 8.643 | 11.128 | 13.433 |
| 7 | ---- | ---- | 10.265 | 12.633 |
| 8 | ---- | ---- | 9.010 | 11.692 |
| 9 | ---- | ---- | ---- | 10.936 |
| 10 | ---- | ---- | ---- | 9.374 |

Column reinforcement areas are in square inches

**Table 6.9** Column reinforcement ratio (%) variations for frames of different number of stories.

| Story No. | Number of stories | | | |
|---|---|---|---|---|
| | 4 | 6 | 8 | 10 |
| 1 | 5.471 | 7.145 | 7.973 | 8.003 |
| 2 | 5.606 | 7.316 | 8.025 | 8.018 |
| 3 | 5.376 | 6.330 | 7.533 | 8.017 |
| 4 | 5.347 | 6.140 | 6.374 | 7.972 |
| 5 | ---- | 5.786 | 6.628 | 7.891 |
| 6 | ---- | 5.807 | 6.291 | 6.953 |
| 7 | ---- | ---- | 6.074 | 6.731 |
| 8 | ---- | ---- | 6.154 | 6.512 |
| 9 | ---- | ---- | ---- | 6.276 |
| 10 | ---- | ---- | ---- | 6.403 |

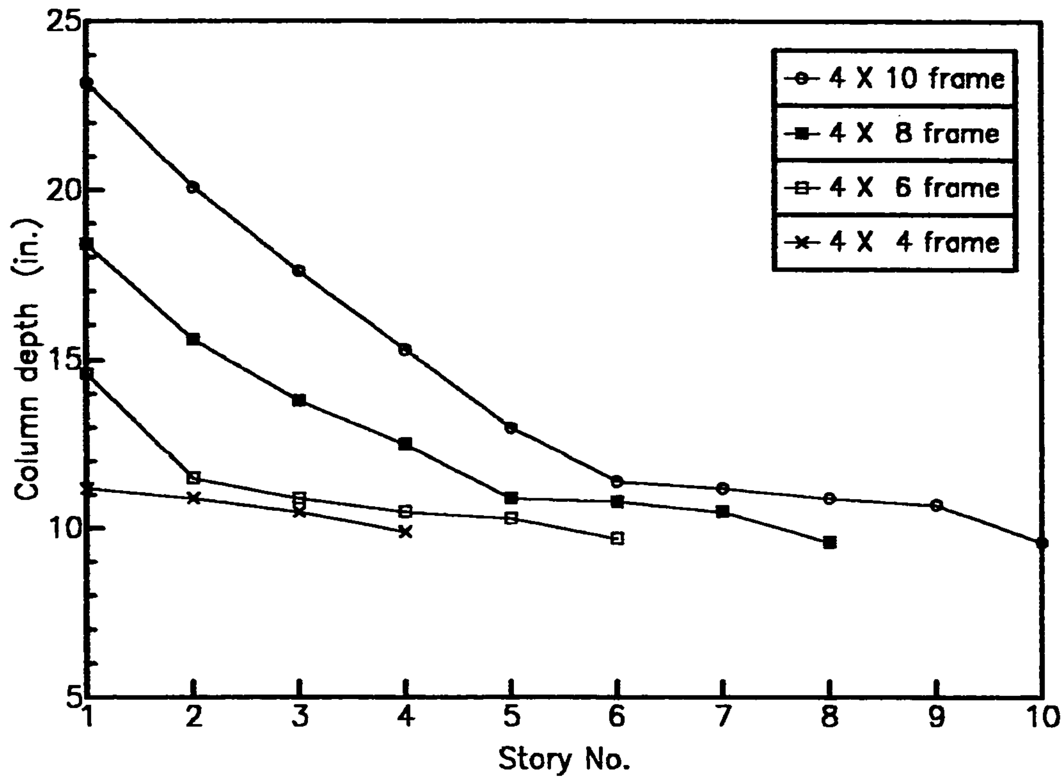



**Figure 6.17** Column depth variations for frames of different number of stories.

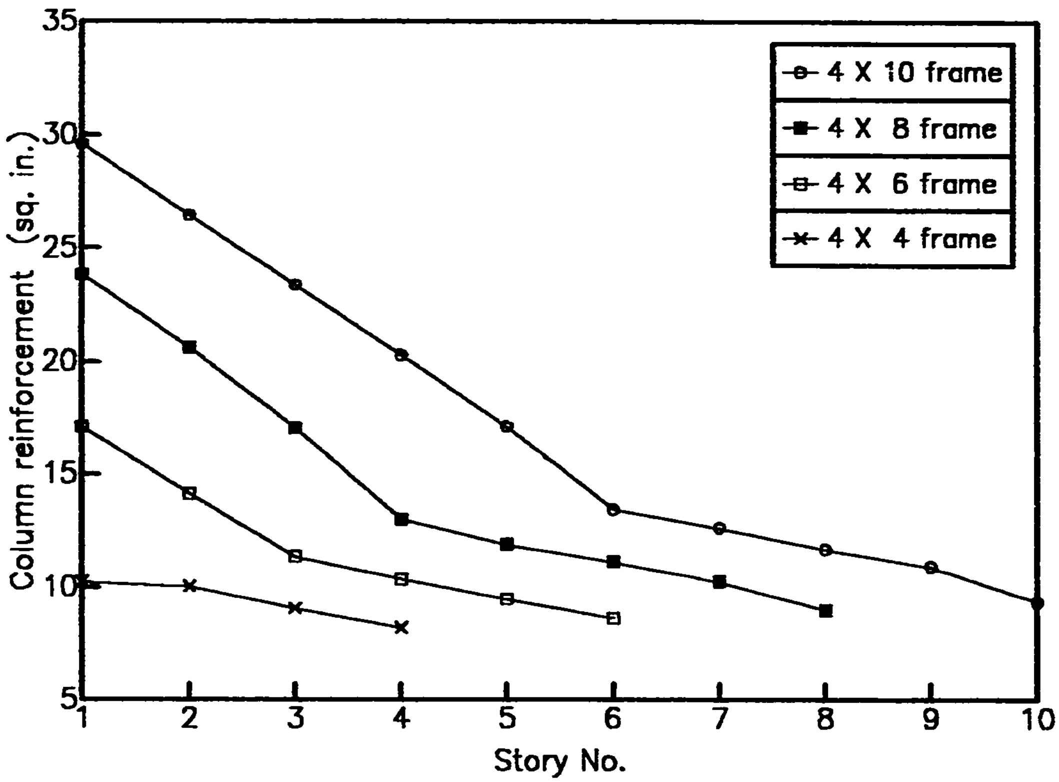



**Figure 6.18** Column reinforcement variations for frames of different number of stories.

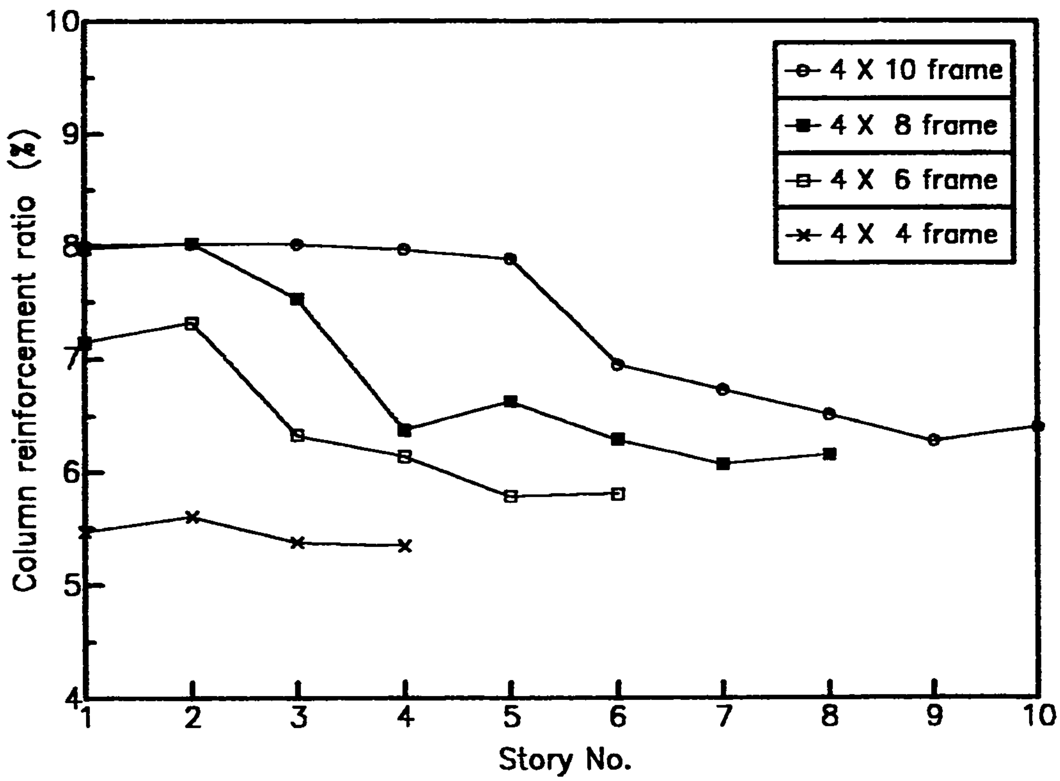



**Figure 6.19** Column reinforcement ratio variations for frames of different number of stories.

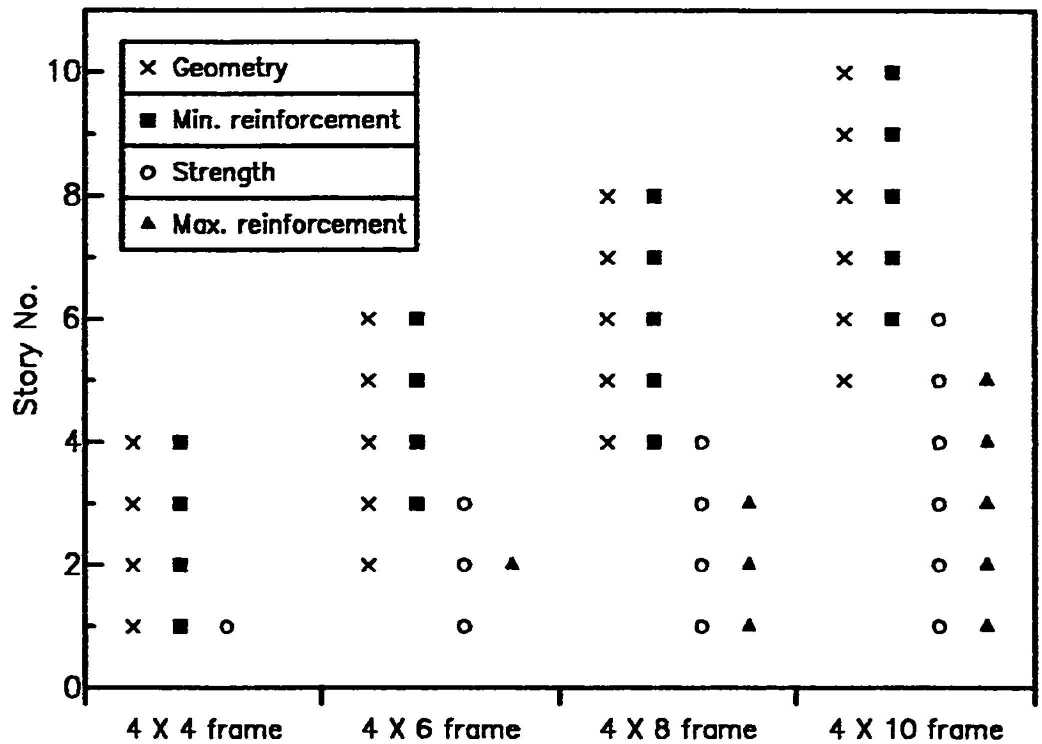



**Figure 6.20** Active column constraints for frames of different number of stories.

seen from the figure that the constraints on the geometry and the minimum reinforcing steel ratio form a band going up as the number of stories increase which implies that, due to the decrease in axial forces and bending moments at upper stories, geometry and minimum reinforcement ratio constraints start to control the proportioning of columns.

### 6.3.5 Effect of Lateral (Wind) Load on Beams

To study the effect of lateral (wind) load on the variations of beam design variables, the same four-bay, ten-story frame of Section (6.2.2) is utilized. The original lateral load is taken as the reference load (Wind Load Factor, WLF=1.0). Three other cases are investigated: WLF=0.0, WLF=0.5, and WLF=1.5. Tables (6.10) through (6.12) summarize the variations of the design variables with story level.

Figure (6.21) shows beam depth variations. Although the case of WLF=0.0, in which it is assumed that no wind forces act on the ten-story building, is purely theoretical, it does raise an interesting point. In the figure, it can be seen that in all the cases, except when WLF=0.0, beam depths expectedly decrease with increasing story number. When WLF is taken to be 0.0, beam depths actually increase with story number. Some thought into the structural response of the frame explains this behavior. Because the frame is subjected to uniform loads only with no lateral loads, axial forces decrease rapidly with height causing column depths to decrease while widths are almost constant. Consequently, the stiffness ratio of columns decrease with height, while joints are still subjected to the same moments induced by the uniform loads. Since distribution of loading at joints occurs according to the stiffness

**Table 6.10** Effect of lateral (wind) load on beam depth variations for the ten-story frame.

| Story No. | Wind load factor | | | |
|---|---|---|---|---|
| | 0.0 | 0.5 | 1.0 | 1.5 |
| 1 | 19.5 | 20.6 | 22.4 | 24.8 |
| 2 | 19.3 | 20.7 | 22.0 | 23.5 |
| 3 | 19.3 | 20.6 | 21.8 | 22.9 |
| 4 | 19.4 | 20.4 | 21.4 | 22.5 |
| 5 | 19.4 | 20.3 | 21.4 | 22.0 |
| 6 | 19.6 | 20.5 | 21.0 | 21.9 |
| 7 | 19.9 | 20.1 | 20.5 | 21.0 |
| 8 | 19.7 | 19.8 | 20.2 | 20.5 |
| 9 | 19.8 | 19.8 | 19.9 | 20.1 |
| 10 | 18.0 | 18.0 | 18.1 | 18.2 |

Beam depths are in inches

**Table 6.11** Effect of lateral (wind) load on beam reinforcement variations for the ten-story frame.

| Story No. | Wind load factor | | | |
|---|---|---|---|---|
| | 0.0 | 0.5 | 1.0 | 1.5 |
| 1 | 3.427 | 4.446 | 5.651 | 6.686 |
| 2 | 3.474 | 4.779 | 6.179 | 7.311 |
| 3 | 3.618 | 4.758 | 6.038 | 7.162 |
| 4 | 3.727 | 4.689 | 5.708 | 6.765 |
| 5 | 3.750 | 4.625 | 5.345 | 6.222 |
| 6 | 3.840 | 4.461 | 5.158 | 5.572 |
| 7 | 3.840 | 4.403 | 4.897 | 5.307 |
| 8 | 3.925 | 4.300 | 4.603 | 4.880 |
| 9 | 3.923 | 4.108 | 4.298 | 4.420 |
| 10 | 2.987 | 3.028 | 3.072 | 3.131 |

Beam reinforcement areas are in square inches

**Table 6.12** Effect of lateral (wind) load on beam reinforcement ratio (%) variations for the ten-story frame.

| Story No. | Wind load factor | | | |
|---|---|---|---|---|
| | 0.0 | 0.5 | 1.0 | 1.5 |
| 1 | 1.352 | 1.564 | 1.752 | 1.797 |
| 2 | 1.395 | 1.673 | 1.924 | 2.020 |
| 3 | 1.453 | 1.686 | 1.910 | 2.044 |
| 4 | 1.489 | 1.690 | 1.865 | 2.004 |
| 5 | 1.498 | 1.675 | 1.784 | 1.924 |
| 6 | 1.496 | 1.636 | 1.767 | 1.792 |
| 7 | 1.508 | 1.647 | 1.744 | 1.805 |
| 8 | 1.545 | 1.645 | 1.701 | 1.738 |
| 9 | 1.536 | 1.596 | 1.636 | 1.653 |
| 10 | 1.383 | 1.402 | 1.403 | 1.422 |

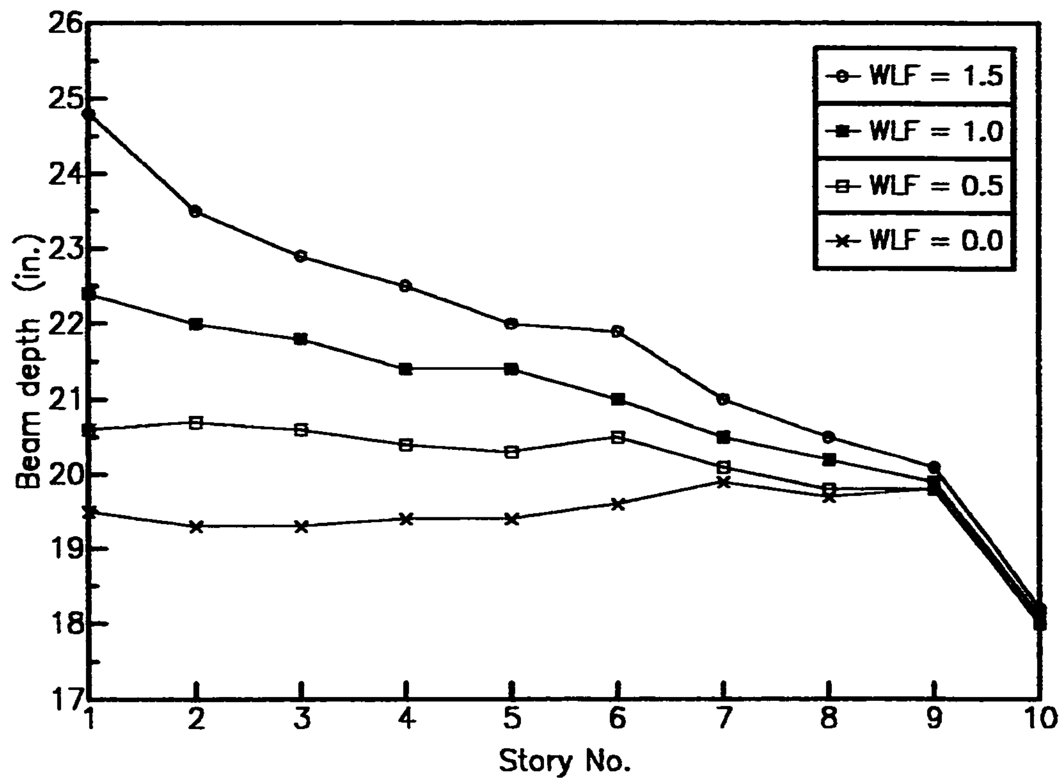



**Figure 6.21** Effect of lateral (wind) load on beam depth variations for the ten-story frame (WLF = Wind Load Factor).

ratio, beam depths and reinforcement areas (shown in Fig. (6.22)) have to increase to take care of the difference in moments. Beam reinforcement ratios are shown in Fig. (6.23); they take the same trend as the reinforcement areas.

### 6.3.6 Effect of Aspect Ratio on Optimal Frame Configuration

The effect of the aspect ratio, or ratio of building height to its width, on optimal frame configuration is studied in this section. A four-bay, ten-story reinforced concrete frame is used in this study. The building height is kept constant at 132 ft while the width is varied by varying beam span lengths from 15 ft to 35 ft with 5 ft increments. Therefore, five different aspect ratios are considered: 2.20, 1.65, 1.32, 1.10, and 0.94. Tables (6.13) to (6.15) and (6.16) to (6.18) summarize the variations of the design variables with story level for columns and beams, respectively.

Column width and beam width variations are shown in Figs (6.24) and (6.28), respectively. They vary almost identically due to the column/beam compatibility constraint. It can be noted that at high aspect ratios, the changes in widths are much more than the changes at lower aspect ratios.

Column depth variations are shown in Fig. (6.25). It can be seen that as the aspect ratio increases, i.e., as the stiffness of the building decreases, the changes in the magnitude of depths between any two stories decrease. Moreover, the changes in the lower stories are much more than the changes in the upper stories. A similar behavior is exhibited by column reinforcement areas as shown in Fig. (6.26). Such behavior can be explained by studying the active column constraints at the optimal points. Figure (6.27) shows the activity of various

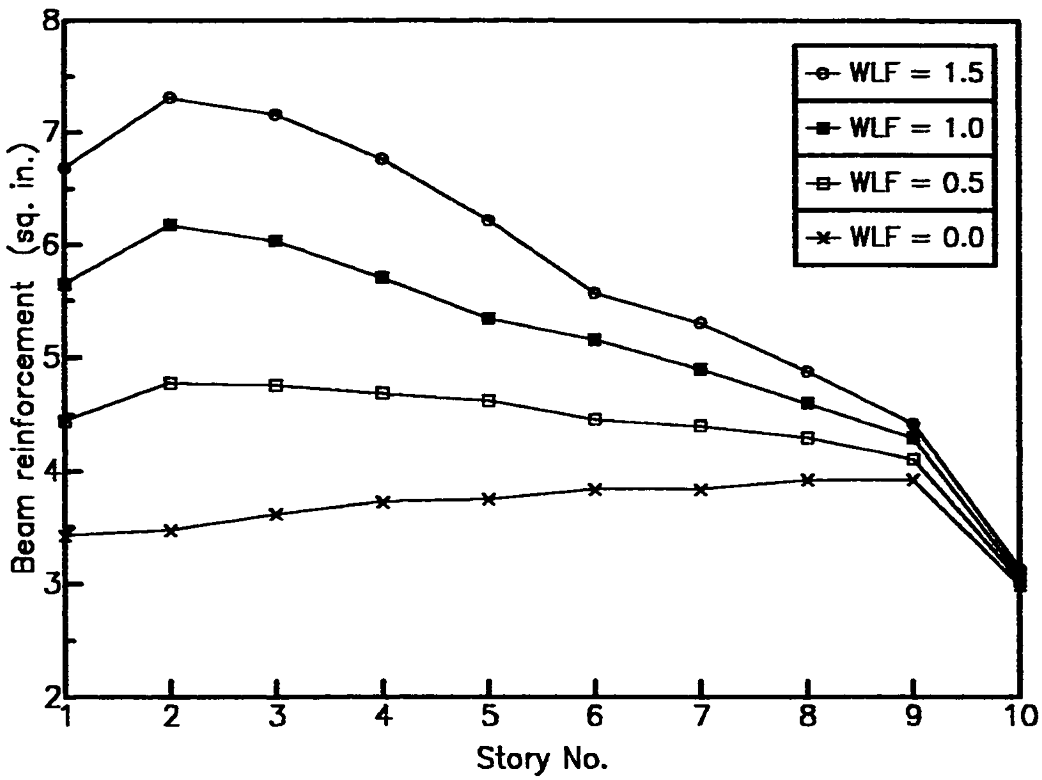



**Figure 6.22** Effect of lateral (wind) load on beam reinforcement variations for the ten-story frame.

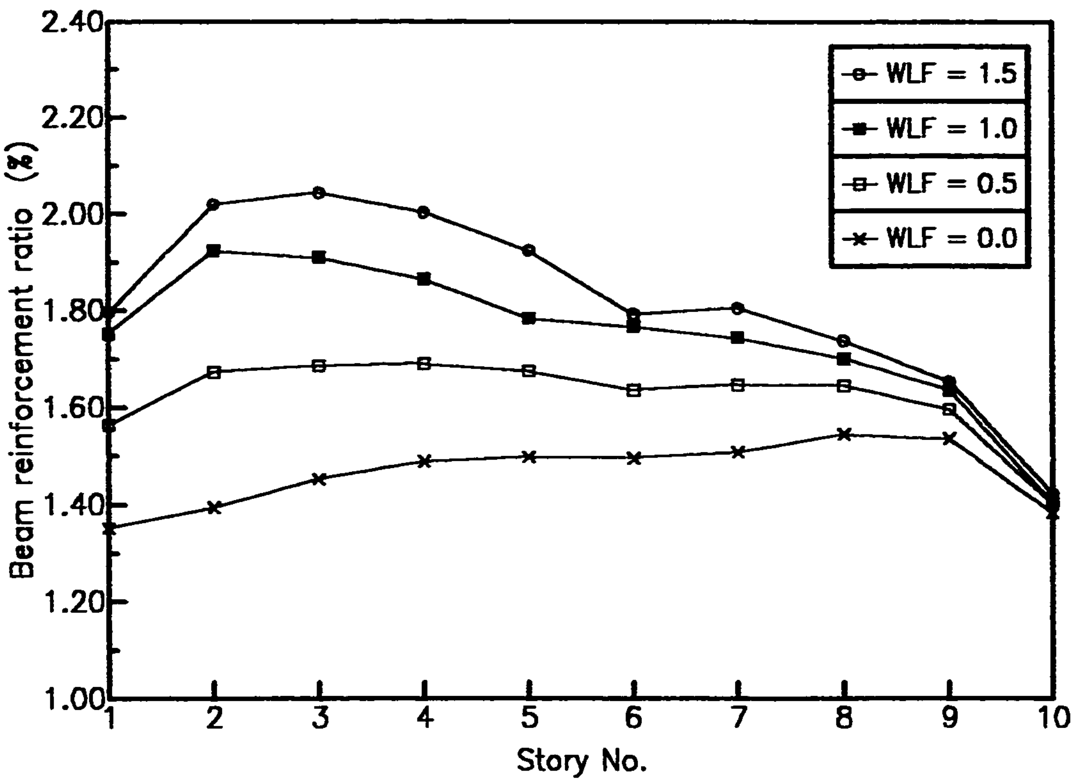



**Figure 6.23** Effect of lateral (wind) load on beam reinforcement ratio variations for the ten-story frame.

**Table 6.13** Effect of aspect ratio on column width variations for the ten-story frame.

| Story No. | Aspect ratio | | | | |
|---|---|---|---|---|---|
| | 2.20 | 1.65 | 1.32 | 1.10 | 0.94 |
| 1 | 13.5 | 13.8 | 14.4 | 15.7 | 16.2 |
| 2 | 13.3 | 13.8 | 14.6 | 15.7 | 15.7 |
| 3 | 13.1 | 13.7 | 14.5 | 15.6 | 15.7 |
| 4 | 12.9 | 13.5 | 14.3 | 15.6 | 15.7 |
| 5 | 12.5 | 13.2 | 14.0 | 15.4 | 15.7 |
| 6 | 12.1 | 12.8 | 13.9 | 14.7 | 15.6 |
| 7 | 11.6 | 12.4 | 13.7 | 14.3 | 15.7 |
| 8 | 11.0 | 12.1 | 13.4 | 14.1 | 15.7 |
| 9 | 10.5 | 11.8 | 13.2 | 14.0 | 15.4 |
| 10 | 10.0 | 10.8 | 12.1 | 13.4 | 14.9 |

Column widths are in inches

**Table 6.14** Effect of aspect ratio on column depth variations for the ten-story frame.

| Story No. | Aspect ratio | | | | |
|---|---|---|---|---|---|
| | 2.20 | 1.65 | 1.32 | 1.10 | 0.94 |
| 1 | 15.7 | 19.8 | 23.2 | 27.9 | 34.2 |
| 2 | 14.9 | 18.6 | 20.1 | 25.9 | 31.5 |
| 3 | 13.7 | 15.5 | 17.6 | 21.7 | 27.6 |
| 4 | 12.4 | 14.1 | 15.3 | 18.5 | 23.7 |
| 5 | 11.3 | 12.4 | 13.0 | 15.7 | 19.9 |
| 6 | 10.1 | 11.2 | 11.4 | 15.5 | 18.6 |
| 7 | 9.1 | 9.9 | 11.2 | 11.8 | 13.6 |
| 8 | 8.5 | 9.6 | 10.9 | 11.6 | 13.2 |
| 9 | 8.0 | 9.3 | 10.7 | 11.5 | 12.9 |
| 10 | 7.5 | 8.3 | 9.6 | 10.9 | 12.4 |

Column depths are in inches

**Table 6.15** Effect of aspect ratio on column reinforcement variations for the ten-story frame.

| Story No. | Aspect ratio | | | | |
|---|---|---|---|---|---|
| | 2.20 | 1.65 | 1.32 | 1.10 | 0.94 |
| 1 | 14.769 | 18.984 | 29.617 | 38.142 | 47.630 |
| 2 | 13.172 | 16.001 | 26.456 | 32.981 | 42.667 |
| 3 | 12.543 | 14.756 | 23.365 | 30.194 | 37.773 |
| 4 | 11.798 | 12.214 | 20.293 | 26.272 | 32.913 |
| 5 | 10.849 | 11.746 | 17.123 | 22.356 | 28.069 |
| 6 | 9.719 | 11.098 | 13.433 | 16.459 | 20.877 |
| 7 | 8.602 | 10.187 | 12.633 | 14.236 | 17.092 |
| 8 | 7.380 | 9.151 | 11.692 | 13.645 | 16.918 |
| 9 | 6.114 | 8.143 | 10.936 | 12.815 | 15.766 |
| 10 | 4.649 | 6.459 | 9.374 | 12.713 | 17.015 |

Column reinforcement areas are in square inches

**Table 6.16** Effect of aspect ratio on beam width variations for the ten-story frame.

| Story No. | Aspect ratio | | | | |
|---|---|---|---|---|---|
| | 2.20 | 1.65 | 1.32 | 1.10 | 0.94 |
| 1 | 13.5 | 13.8 | 14.4 | 15.7 | 15.7 |
| 2 | 13.3 | 13.8 | 14.6 | 15.7 | 15.7 |
| 3 | 13.1 | 13.7 | 14.5 | 15.6 | 15.7 |
| 4 | 12.9 | 13.5 | 14.3 | 15.6 | 15.7 |
| 5 | 12.5 | 13.2 | 14.0 | 15.4 | 15.7 |
| 6 | 12.1 | 12.8 | 13.9 | 14.7 | 15.6 |
| 7 | 11.6 | 12.4 | 13.7 | 14.3 | 15.7 |
| 8 | 11.0 | 12.1 | 13.4 | 14.1 | 15.7 |
| 9 | 10.5 | 11.8 | 13.2 | 14.0 | 15.4 |
| 10 | 10.0 | 10.8 | 12.1 | 13.4 | 14.9 |

Beam widths are in inches

**Table 6.17** Effect of aspect ratio on beam depth variations for the ten-story frame.

| Story No. | Aspect ratio | | | | |
|---|---|---|---|---|---|
| | 2.20 | 1.65 | 1.32 | 1.10 | 0.94 |
| 1 | 20.8 | 20.7 | 22.4 | 23.9 | 28.2 |
| 2 | 20.0 | 20.7 | 22.0 | 24.1 | 28.6 |
| 3 | 19.7 | 20.5 | 21.8 | 23.9 | 28.5 |
| 4 | 19.3 | 20.2 | 21.4 | 23.4 | 28.0 |
| 5 | 18.8 | 19.7 | 21.4 | 23.4 | 27.4 |
| 6 | 18.2 | 19.1 | 21.0 | 24.4 | 27.8 |
| 7 | 17.4 | 18.6 | 20.5 | 25.2 | 28.1 |
| 8 | 16.5 | 18.2 | 20.2 | 24.7 | 27.5 |
| 9 | 15.8 | 17.7 | 19.9 | 24.3 | 27.6 |
| 10 | 15.0 | 16.2 | 18.1 | 20.1 | 22.3 |

Beam depths are in inches

**Table 6.18** Effect of aspect ratio on beam reinforcement variations for the ten-story frame.

| Story No. | Aspect ratio | | | | |
|---|---|---|---|---|---|
| | 2.20 | 1.65 | 1.32 | 1.10 | 0.94 |
| 1 | 4.641 | 5.024 | 5.651 | 6.587 | 6.854 |
| 2 | 4.406 | 5.081 | 6.179 | 6.986 | 7.267 |
| 3 | 4.209 | 4.925 | 6.038 | 6.982 | 7.380 |
| 4 | 3.914 | 4.635 | 5.708 | 6.878 | 7.387 |
| 5 | 3.554 | 4.266 | 5.345 | 6.717 | 7.505 |
| 6 | 3.105 | 3.798 | 5.158 | 6.027 | 7.159 |
| 7 | 2.610 | 3.434 | 4.897 | 5.621 | 6.987 |
| 8 | 2.063 | 3.122 | 4.603 | 5.475 | 6.914 |
| 9 | 1.707 | 2.806 | 4.298 | 5.280 | 6.676 |
| 10 | 1.016 | 1.910 | 3.072 | 4.551 | 6.561 |

Beam reinforcement areas are in square inches

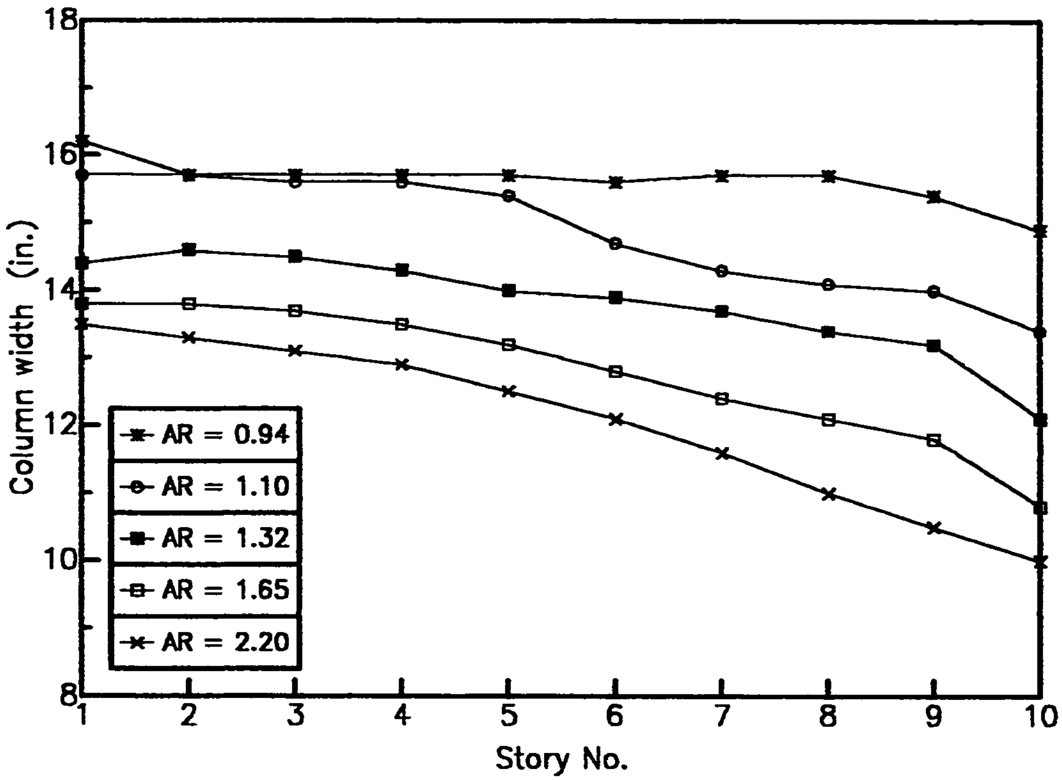



**Figure 6.24** Effect of aspect ratio on column width variations for the ten-story frame (AR = Aspect Ratio).

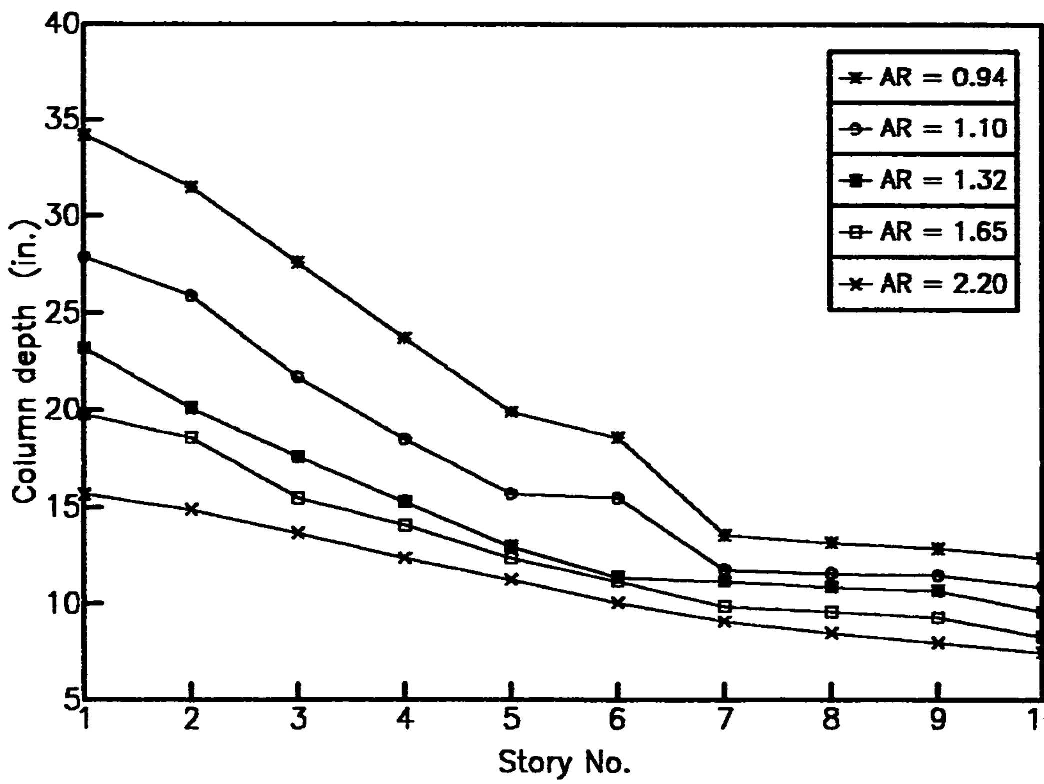



**Figure 6.25** Effect of aspect ratio on column depth variations for the ten-story frame.

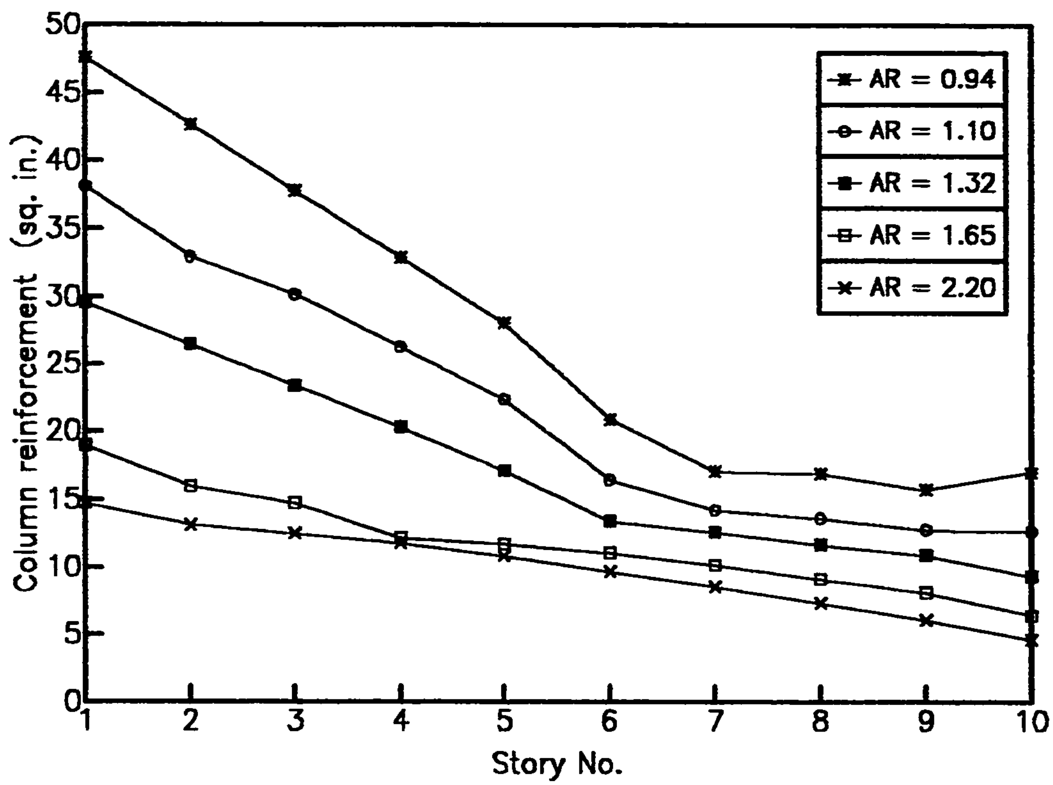



**Figure 6.26** Effect of aspect ratio on column reinforcement variations for the ten-story frame.

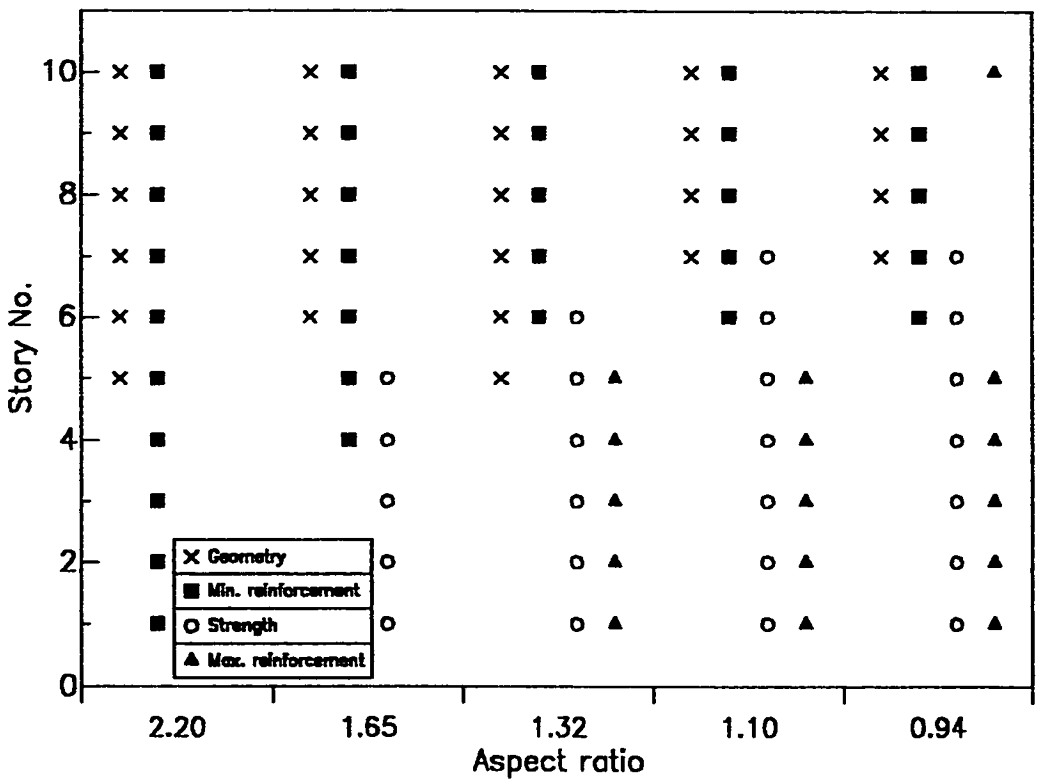



**Figure 6.27** Active column constraint variations for the ten-story frame under deifferent aspect ratios.

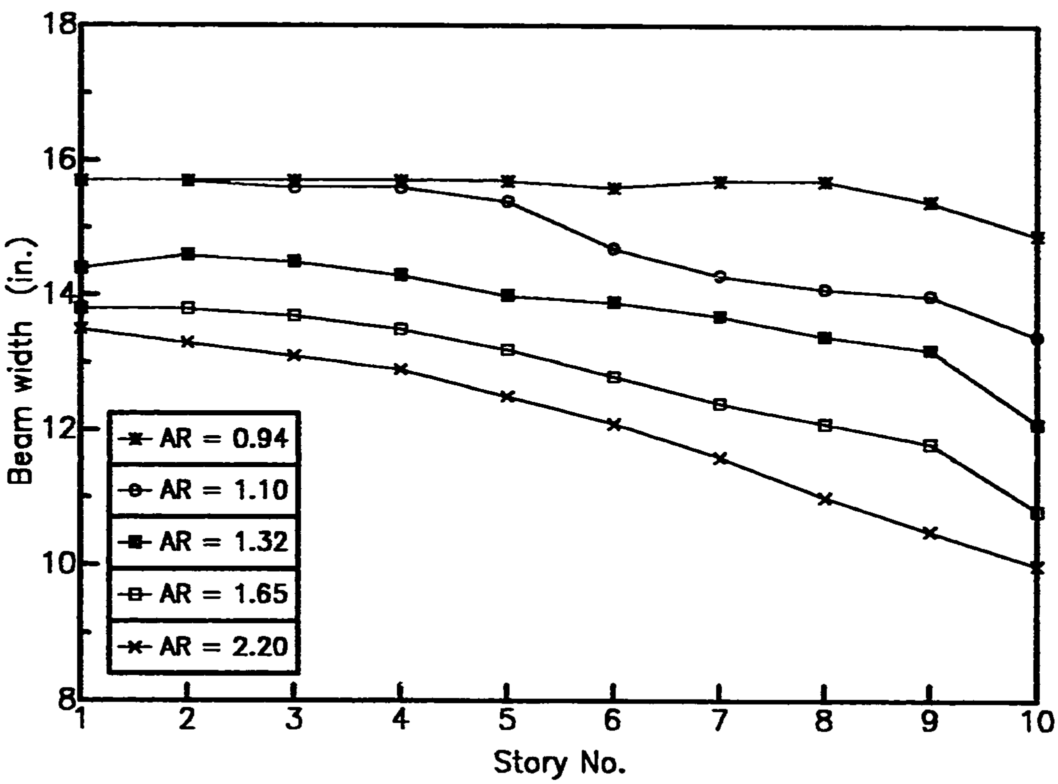



**Figure 6.28** Effect of aspect ratio on beam width variations for the ten-story frame.

column constraints. It can be seen that at higher aspect ratios, the constraints on the geometry and the minimum reinforcing steel ratio dominate, and they shift upward as buildings become stiffer. The constraints on the strength and the maximum reinforcing steel ratio are more active for stiffer buildings where axial forces and bending moments coming from the longer beams to columns are much more than for buildings of lower stiffness.

Figure (6.29) shows beam depth variations for the different aspect ratios. Beam reinforcement area variations are shown in Fig. (6.30). It can be noted that as buildings become stiffer, the change in the magnitude of beam reinforcement areas between any two stories decreases which is expected because the effect of wind forces on stiffer buildings is much less than their effect on buildings of lower stiffness.

## 6.4 EFFECT OF NUMBER OF BAYS ON FRAME COST

To study the effect of different number of bays on the cost of reinforced concrete frames, a one-story frame that could be an industrial building is considered. The total width of the frame is kept constant at 60 ft and is divided into different number of bays (equal span lengths). The total cost of the frame as well as its components (beam cost and column cost) are plotted in Fig. (6.31) with respect to the number of bays.

It can be noted from the figure that as the number of bays increase, the total cost of the frame decreases until the number of bays becomes 7 at which the frame cost starts to increase. Beam cost decreases with increasing number of bays because shorter beams are less expensive than longer beams. Column cost

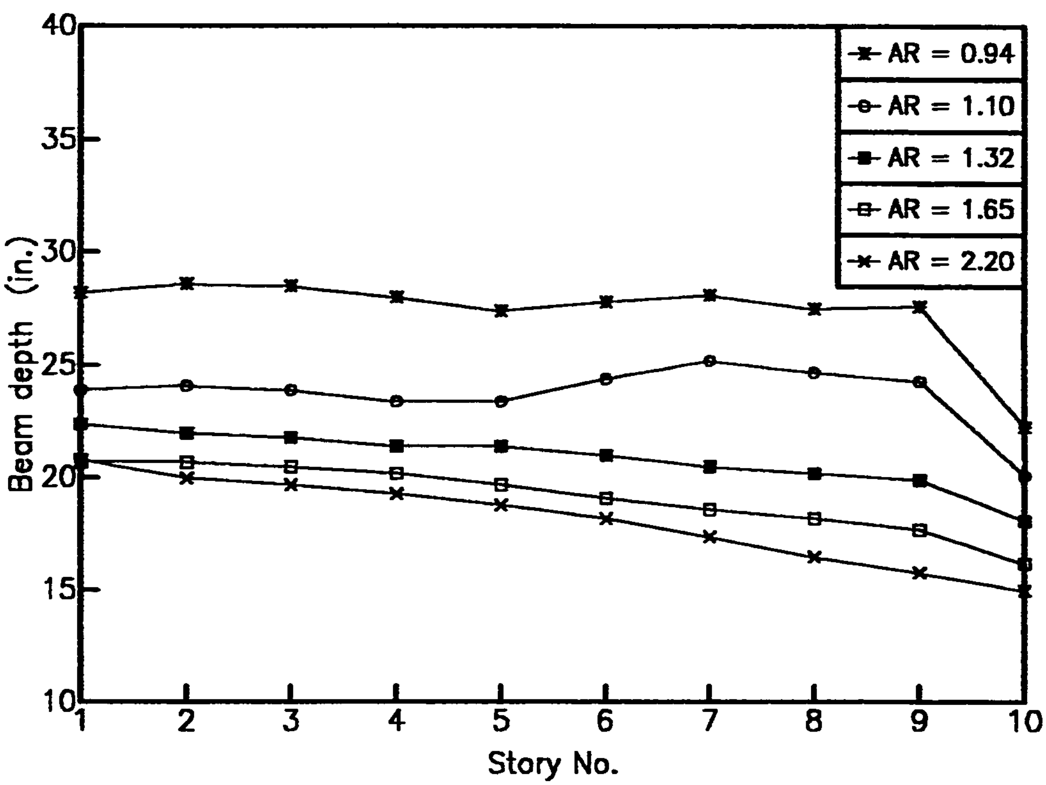



**Figure 6.29** Effect of aspect ratio on beam depth variations for the ten-story frame.

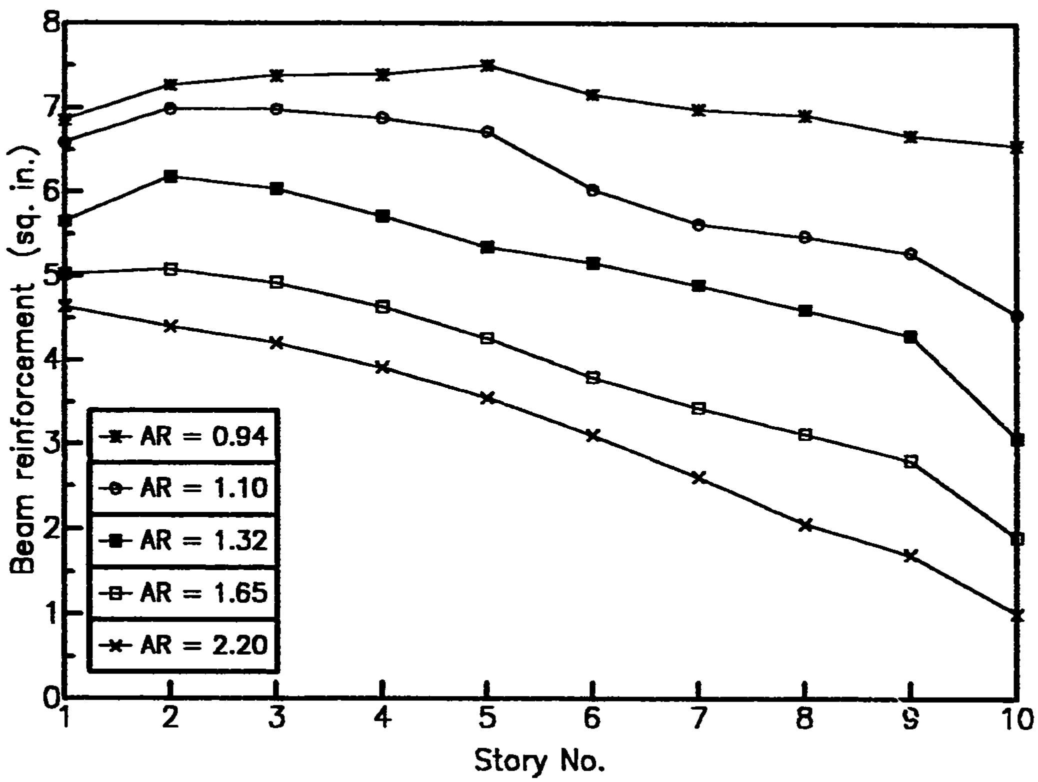



**Figure 6.30** Effect of aspect ratio on beam reinforcement variations for the ten-story frame.

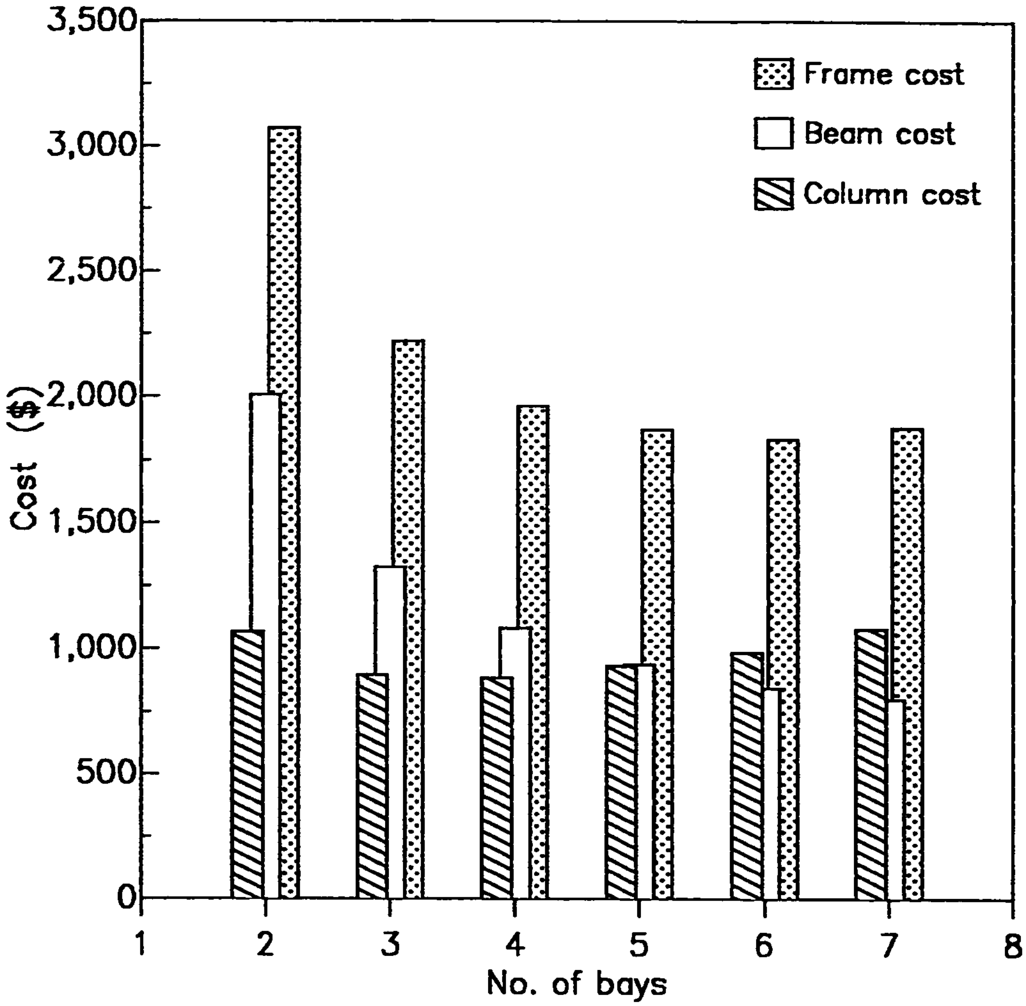



**Figure 6.31** Costs of columns and beams for a 60-ft-width frame for different number of bays (Equal span lengths).

decreases with increasing number of bays until the number of columns becomes more than what is needed, at span lengths in the order of 12 to 15 ft, at which the column cost starts to increase. In this range, the cost of many small columns exceeds the cost of a few larger columns.

From Fig. (6.31) it can be concluded that the optimum spacing is in the order of 12 to 15 ft. Any increase in column spacing beyond this will increase the cost of the frame significantly.

*CHAPTER 7*

# CONCLUSIONS AND RECOMMENDATIONS

## 7.1 GENERAL

An optimization system for reinforced concrete frames has been developed. The system can be used to analyse and design economical reinforced concrete rectangular frames of moderate height according to the ACI 318-83 Code provisions taking into account practical considerations. A second-order analysis has been employed in which second-order influences pertaining to reinforced concrete frames have been incorporated.

The capabilities of the system have been demonstrated through several case studies. Some general design guidelines for reinforced concrete frame members have been developed and tested. Also, some behavioral studies have been performed.

## 7.2 CONCLUSIONS

Many design examples have been presented in this study to demonstrate the application of RCFOPT software. From the findings of those examples, the following conclusions are outlined:

1. An optimization system for the design of reinforced concrete frames has been developed and tested successfully.

2. Minimum total cost is the most economical design criterion for reinforced concrete frames.

3. Cost of formwork affects the optimal design of reinforced concrete frames, especially dimensions and reinforcement areas of columns and beams in the lower stories.

4. The saving in the cost of reinforced concrete frames using moment redistribution was only 3.5% . However, the CPU time increased by 51%.

5. Columns of reinforced concrete frames within five stories are mostly governed by geometric rather than strength considerations.

6. At moderate values of wind load (about 30 miles/hour), beam depths can be kept constant in all stories.

7. Most economical column spacing is in the order of 12 to 15 ft. Any increase in column spacing beyond this will increase frame cost significantly.

8. Some design guidelines in the form of equations and charts have been developed for the economical design of reinforced concrete frame members.

## 7.3 RECOMMENDATIONS FOR FUTURE RESEARCH

The optimization system developed in this study forms the foundation of a general, powerful system that can be used to optimize reinforced concrete frames. Further study in this area could be performed to:

1. Expand the formulation to include general three-dimensional reinforced concrete frames.

2. Consider the cracked section behavior in the analysis and compare the obtained results with those of this study.

3. Include the various material strengths in the developed design guidelines.

4. Perform more parametric studies to investigate the variations of the design variables under a variety of conditions involving different cost parameters, different material strengths, and different loadings.

Sufficient data for specific cases have been presented in this study which point the way for further research. Ultimately, a very powerful optimization system could be developed, one that would lead to substantial cost savings for reinforced concrete frame projects everywhere.

# APPENDIX 1
# RCFRAME Input Data File

FRAME NO. 1

STRUCTURAL AND LOAD PARAMETERS

| NBAY | NSTORY | NJ | NR | NRJ | NSJ | NLJ | NLM | IELAST | IPRINT |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 5 | 39 | 12 | 4 | 0 | 5 | 30 | 2 | 2 |

SPACING OF FRAMES

144.0

JOINT COORDINATES

| JOINT | X | Y |
|---|---|---|
| 1 | 0.00 | 0.00 |
| 2 | 0.00 | 180.00 |
| 3 | 0.00 | 336.00 |
| 4 | 0.00 | 492.00 |
| 5 | 0.00 | 648.00 |
| 6 | 0.00 | 804.00 |
| 7 | 150.00 | 180.00 |
| 8 | 150.00 | 336.00 |
| 9 | 150.00 | 492.00 |
| 10 | 150.00 | 648.00 |
| 11 | 150.00 | 804.00 |
| 12 | 300.00 | 0.00 |
| 13 | 300.00 | 180.00 |
| 14 | 300.00 | 336.00 |
| 15 | 300.00 | 492.00 |
| 16 | 300.00 | 648.00 |
| 17 | 300.00 | 804.00 |
| 18 | 450.00 | 180.00 |
| 19 | 450.00 | 336.00 |
| 20 | 450.00 | 492.00 |
| 21 | 450.00 | 648.00 |
| 22 | 450.00 | 804.00 |
| 23 | 600.00 | 0.00 |
| 24 | 600.00 | 180.00 |
| 25 | 600.00 | 336.00 |
| 26 | 600.00 | 492.00 |
| 27 | 600.00 | 648.00 |
| 28 | 600.00 | 804.00 |
| 29 | 750.00 | 180.00 |
| 30 | 750.00 | 336.00 |
| 31 | 750.00 | 492.00 |
| 32 | 750.00 | 648.00 |
| 33 | 750.00 | 804.00 |
| 34 | 900.00 | 0.00 |
| 35 | 900.00 | 180.00 |
| 36 | 900.00 | 336.00 |
| 37 | 900.00 | 492.00 |
| 38 | 900.00 | 648.00 |
| 39 | 900.00 | 804.00 |

MEMBER INFORMATION

| MEMBER | J | K |
|---|---|---|
| 1 | 1 | 2 |
| 2 | 2 | 3 |
| 3 | 3 | 4 |
| 4 | 4 | 5 |
| 5 | 5 | 6 |
| 6 | 6 | 11 |
| 7 | 11 | 17 |
| 8 | 5 | 10 |
| 9 | 10 | 16 |
| 10 | 4 | 9 |
| 11 | 9 | 15 |
| 12 | 3 | 8 |
| 13 | 8 | 14 |
| 14 | 2 | 7 |
| 15 | 7 | 13 |
| 16 | 12 | 13 |
| 17 | 13 | 14 |
| 18 | 14 | 15 |
| 19 | 15 | 16 |
| 20 | 16 | 17 |
| 21 | 17 | 22 |
| 22 | 22 | 28 |
| 23 | 16 | 21 |
| 24 | 21 | 27 |
| 25 | 15 | 20 |
| 26 | 20 | 26 |
| 27 | 14 | 19 |
| 28 | 19 | 25 |
| 29 | 13 | 18 |
| 30 | 18 | 24 |
| 31 | 23 | 24 |
| 32 | 24 | 25 |
| 33 | 25 | 26 |
| 34 | 26 | 27 |
| 35 | 27 | 28 |
| 36 | 28 | 33 |
| 37 | 33 | 39 |
| 38 | 27 | 32 |
| 39 | 32 | 38 |
| 40 | 26 | 31 |
| 41 | 31 | 37 |
| 42 | 25 | 30 |
| 43 | 30 | 36 |
| 44 | 24 | 29 |
| 45 | 29 | 35 |
| 46 | 34 | 35 |
| 47 | 35 | 36 |
| 48 | 36 | 37 |
| 49 | 37 | 38 |
| 50 | 38 | 39 |

JOINT RESTRAINTS

| JOINT | JR1 | JR2 | JR3 |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 12 | 1 | 1 | 1 |
| 23 | 1 | 1 | 1 |
| 34 | 1 | 1 | 1 |

JOINT LOADS

| JOINT | AJ1 | AJ2 | AJ3 |
|---|---|---|---|
| 2 | 18000.0 | 0.0 | 0.0 |
| 3 | 16800.0 | 0.0 | 0.0 |
| 4 | 16800.0 | 0.0 | 0.0 |
| 5 | 16800.0 | 0.0 | 0.0 |
| 6 | 8400.0 | 0.0 | 0.0 |

MEMBER LOADS

| MEMBER | UDL |
|---|---|
| 6 | -350.0 |
| 7 | -350.0 |
| 8 | -500.0 |
| 9 | -500.0 |
| 10 | -500.0 |
| 11 | -500.0 |
| 12 | -500.0 |
| 13 | -500.0 |
| 14 | -500.0 |
| 15 | -500.0 |
| 21 | -350.0 |
| 22 | -350.0 |
| 23 | -500.0 |
| 24 | -500.0 |
| 25 | -500.0 |
| 26 | -500.0 |
| 27 | -500.0 |
| 28 | -500.0 |
| 29 | -500.0 |
| 30 | -500.0 |
| 36 | -350.0 |
| 37 | -350.0 |
| 38 | -500.0 |
| 39 | -500.0 |
| 40 | -500.0 |
| 41 | -500.0 |
| 42 | -500.0 |
| 43 | -500.0 |
| 44 | -500.0 |
| 45 | -500.0 |

COLUMN DATA

| COLUMN | MEMBER |
|---|---|
| 1 | 1 |
| 2 | 16 |
| 3 | 31 |
| 4 | 46 |
| 5 | 2 |

| | |
|---|---|
| 6 | 17 |
| 7 | 32 |
| 8 | 47 |
| 9 | 3 |
| 10 | 18 |
| 11 | 33 |
| 12 | 48 |
| 13 | 4 |
| 14 | 19 |
| 15 | 34 |
| 16 | 49 |
| 17 | 5 |
| 18 | 20 |
| 19 | 35 |
| 20 | 50 |

BEAM DATA

| BEAM | MEMBERS | |
|---|---|---|
| 1 | 14 | 15 |
| 2 | 29 | 30 |
| 3 | 44 | 45 |
| 4 | 12 | 13 |
| 5 | 27 | 28 |
| 6 | 42 | 43 |
| 7 | 10 | 11 |
| 8 | 25 | 26 |
| 9 | 40 | 41 |
| 10 | 8 | 9 |
| 11 | 23 | 24 |
| 12 | 38 | 39 |
| 13 | 6 | 7 |
| 14 | 21 | 22 |
| 15 | 36 | 37 |

STORY DATA

| STORY | COLUMNS | | | | BEAMS | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 1 | 2 | 3 |
| 2 | 5 | 6 | 7 | 8 | 4 | 5 | 6 |
| 3 | 9 | 10 | 11 | 12 | 7 | 8 | 9 |
| 4 | 13 | 14 | 15 | 16 | 10 | 11 | 12 |
| 5 | 17 | 18 | 19 | 20 | 13 | 14 | 15 |

CONCRETE DATA

| FC | COVER | HF |
|---|---|---|
| 8000.0 | 2.5 | 11.0 |

STEEL DATA

| FY | NMIN | NMAX | CBD | TID | BBD | SRD |
|---|---|---|---|---|---|---|
| 60000.0 | 3 | 4 | 1.410 | 0.500 | 1.000 | 0.375 |

UNIT COST DATA

| UCOSTC | UCOSTS | UCOSTF |
|---|---|---|
| 80.0 | 0.385 | 2.8 |

# APPENDIX 2
# IDESIGN Input Data File

```
                  FRAME NO. 1    (3 X 5)
 30    0   70   50    0   -1    1    0    2    5
1.0000D-03  1.0000D-03  1.0000D-03  1.0000D-03
  1      1.2000D+01     1.0000D+01     3.6000D+01
  2      1.3000D+01     7.5000D+00     3.6000D+01
  3      1.0000D+01     1.0000D+00     2.5000D+01
  4      1.2000D+01     1.0000D+01     3.6000D+01
  5      2.2000D+01     1.5000D+01     5.0000D+01
  6      3.5000D+00     1.0000D+00     2.0000D+01
  7      1.2000D+01     1.0000D+01     3.6000D+01
  8      1.4000D+01     7.5000D+00     3.6000D+01
  9      7.5000D+00     1.0000D+00     2.5000D+01
 10      1.2000D+01     1.0000D+01     3.6000D+01
 11      1.8000D+01     1.5000D+01     5.0000D+01
 12      2.5000D+00     1.0000D+00     2.0000D+01
 13      1.2000D+01     1.0000D+01     3.6000D+01
 14      1.0000D+01     7.5000D+00     3.6000D+01
 15      7.0000D+00     1.0000D+00     2.5000D+01
 16      1.2000D+01     1.0000D+01     3.6000D+01
 17      1.8000D+01     1.5000D+01     5.0000D+01
 18      2.5000D+00     1.0000D+00     2.0000D+01
 19      1.2000D+01     1.0000D+01     3.6000D+01
 20      9.0000D+00     7.5000D+00     3.6000D+01
 21      6.0000D+00     1.0000D+00     2.5000D+01
 22      1.2000D+01     1.0000D+01     3.6000D+01
 23      2.0000D+01     1.5000D+01     5.0000D+01
 24      2.0000D+00     1.0000D+00     2.0000D+01
 25      1.2000D+01     1.0000D+01     3.6000D+01
 26      9.5000D+00     7.5000D+00     3.6000D+01
 27      5.5000D+00     1.0000D+00     2.5000D+01
 28      1.2000D+01     1.0000D+01     3.6000D+01
 29      1.9000D+01     1.5000D+01     5.0000D+01
 30      1.5000D+00     1.0000D+00     2.0000D+01
```

# APPENDIX 3
# RCFRAME Output Data File

R C F O P T

REINFORCED CONCRETE FRAME
OPTIMIZATION SYSTEM

MOSTAFA HASSANAIN

DATE: THU, 07 MAY, 1992
TIME: 180334

FRAME NO. 1

*** SUMMARY OF INPUT DATA ***

STRUCTURAL AND LOAD PARAMETERS

| NBAY | NSTORY | NJ | NR | NRJ | NSJ | NLJ | NLM | IELAST | IPRINT |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 5 | 39 | 12 | 4 | 0 | 5 | 30 | 2 | 2 |

SPACING OF FRAMES

144.0

JOINT COORDINATES

| JOINT | X | Y |
|---|---|---|
| 1 | 0.00 | 0.00 |
| 2 | 0.00 | 180.00 |
| 3 | 0.00 | 336.00 |
| 4 | 0.00 | 492.00 |
| 5 | 0.00 | 648.00 |
| 6 | 0.00 | 804.00 |
| 7 | 150.00 | 180.00 |
| 8 | 150.00 | 336.00 |
| 9 | 150.00 | 492.00 |
| 10 | 150.00 | 648.00 |
| 11 | 150.00 | 804.00 |
| 12 | 300.00 | 0.00 |
| 13 | 300.00 | 180.00 |
| 14 | 300.00 | 336.00 |
| 15 | 300.00 | 492.00 |

| | | |
|---|---|---|
| 16 | 300.00 | 648.00 |
| 17 | 300.00 | 804.00 |
| 18 | 450.00 | 180.00 |
| 19 | 450.00 | 336.00 |
| 20 | 450.00 | 492.00 |
| 21 | 450.00 | 648.00 |
| 22 | 450.00 | 804.00 |
| 23 | 600.00 | 0.00 |
| 24 | 600.00 | 180.00 |
| 25 | 600.00 | 336.00 |
| 26 | 600.00 | 492.00 |
| 27 | 600.00 | 648.00 |
| 28 | 600.00 | 804.00 |
| 29 | 750.00 | 180.00 |
| 30 | 750.00 | 336.00 |
| 31 | 750.00 | 492.00 |
| 32 | 750.00 | 648.00 |
| 33 | 750.00 | 804.00 |
| 34 | 900.00 | 0.00 |
| 35 | 900.00 | 180.00 |
| 36 | 900.00 | 336.00 |
| 37 | 900.00 | 492.00 |
| 38 | 900.00 | 648.00 |
| 39 | 900.00 | 804.00 |

MEMBER INFORMATION
------------------

| MEMBER | J | K |
|---|---|---|
| 1 | 1 | 2 |
| 2 | 2 | 3 |
| 3 | 3 | 4 |
| 4 | 4 | 5 |
| 5 | 5 | 6 |
| 6 | 6 | 11 |
| 7 | 11 | 17 |
| 8 | 5 | 10 |
| 9 | 10 | 16 |
| 10 | 4 | 9 |
| 11 | 9 | 15 |
| 12 | 3 | 8 |
| 13 | 8 | 14 |
| 14 | 2 | 7 |
| 15 | 7 | 13 |
| 16 | 12 | 13 |
| 17 | 13 | 14 |
| 18 | 14 | 15 |
| 19 | 15 | 16 |
| 20 | 16 | 17 |
| 21 | 17 | 22 |
| 22 | 22 | 28 |
| 23 | 16 | 21 |
| 24 | 21 | 27 |

| | | |
|---|---|---|
| 25 | 15 | 20 |
| 26 | 20 | 26 |
| 27 | 14 | 19 |
| 28 | 19 | 25 |
| 29 | 13 | 18 |
| 30 | 18 | 24 |
| 31 | 23 | 24 |
| 32 | 24 | 25 |
| 33 | 25 | 26 |
| 34 | 26 | 27 |
| 35 | 27 | 28 |
| 36 | 28 | 33 |
| 37 | 33 | 39 |
| 38 | 27 | 32 |
| 39 | 32 | 38 |
| 40 | 26 | 31 |
| 41 | 31 | 37 |
| 42 | 25 | 30 |
| 43 | 30 | 36 |
| 44 | 24 | 29 |
| 45 | 29 | 35 |
| 46 | 34 | 35 |
| 47 | 35 | 36 |
| 48 | 36 | 37 |
| 49 | 37 | 38 |
| 50 | 38 | 39 |

JOINT RESTRAINTS
----------------

| JOINT | JR1 | JR2 | JR3 |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 12 | 1 | 1 | 1 |
| 23 | 1 | 1 | 1 |
| 34 | 1 | 1 | 1 |

JOINT LOADS
-----------

| JOINT | AJ1 | AJ2 | AJ3 |
|---|---|---|---|
| 2 | 0.180D+05 | 0.000D+00 | 0.000D+00 |
| 3 | 0.168D+05 | 0.000D+00 | 0.000D+00 |
| 4 | 0.168D+05 | 0.000D+00 | 0.000D+00 |
| 5 | 0.168D+05 | 0.000D+00 | 0.000D+00 |
| 6 | 0.840D+04 | 0.000D+00 | 0.000D+00 |

MEMBER LOADS
------------

| MEMBER | UDL |
|---|---|
| 6 | -0.350D+03 |
| 7 | -0.350D+03 |
| 8 | -0.500D+03 |
| 9 | -0.500D+03 |
| 10 | -0.500D+03 |

| | |
|---|---|
| 11 | -0.500D+03 |
| 12 | -0.500D+03 |
| 13 | -0.500D+03 |
| 14 | -0.500D+03 |
| 15 | -0.500D+03 |
| 21 | -0.350D+03 |
| 22 | -0.350D+03 |
| 23 | -0.500D+03 |
| 24 | -0.500D+03 |
| 25 | -0.500D+03 |
| 26 | -0.500D+03 |
| 27 | -0.500D+03 |
| 28 | -0.500D+03 |
| 29 | -0.500D+03 |
| 30 | -0.500D+03 |
| 36 | -0.350D+03 |
| 37 | -0.350D+03 |
| 38 | -0.500D+03 |
| 39 | -0.500D+03 |
| 40 | -0.500D+03 |
| 41 | -0.500D+03 |
| 42 | -0.500D+03 |
| 43 | -0.500D+03 |
| 44 | -0.500D+03 |
| 45 | -0.500D+03 |

COLUMN DATA
-----------

| COLUMN | MEMBER |
|---|---|
| 1 | 1 |
| 2 | 16 |
| 3 | 31 |
| 4 | 46 |
| 5 | 2 |
| 6 | 17 |
| 7 | 32 |
| 8 | 47 |
| 9 | 3 |
| 10 | 18 |
| 11 | 33 |
| 12 | 48 |
| 13 | 4 |
| 14 | 19 |
| 15 | 34 |
| 16 | 49 |
| 17 | 5 |
| 18 | 20 |
| 19 | 35 |
| 20 | 50 |

BEAM DATA
---------

| BEAM | MEMBERS | |
|---|---|---|
| 1 | 14 | 15 |
| 2 | 29 | 30 |
| 3 | 44 | 45 |
| 4 | 12 | 13 |
| 5 | 27 | 28 |
| 6 | 42 | 43 |
| 7 | 10 | 11 |
| 8 | 25 | 26 |
| 9 | 40 | 41 |
| 10 | 8 | 9 |
| 11 | 23 | 24 |
| 12 | 38 | 39 |
| 13 | 6 | 7 |
| 14 | 21 | 22 |
| 15 | 36 | 37 |

STORY DATA
----------

| STORY | COLUMNS | | | | BEAMS | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 1 | 2 | 3 |
| 2 | 5 | 6 | 7 | 8 | 4 | 5 | 6 |
| 3 | 9 | 10 | 11 | 12 | 7 | 8 | 9 |
| 4 | 13 | 14 | 15 | 16 | 10 | 11 | 12 |
| 5 | 17 | 18 | 19 | 20 | 13 | 14 | 15 |

CONCRETE DATA
-------------

| FC | COVER | HF |
|---|---|---|
| 8000.0 | 2.5 | 11.0 |

STEEL DATA
----------

| FY | NMIN | NMAX | CBD | TID | BBD | SRD |
|---|---|---|---|---|---|---|
| 60000.0 | 3.0 | 4.0 | 1.410 | 0.500 | 1.000 | 0.375 |

UNIT COST DATA
--------------

| UCOSTC | UCOSTS | UCOSTF |
|---|---|---|
| 80.000 | 0.385 | 2.800 |

*** RESULTS OF ANALYSIS ***

*** AT OPTIMAL DESIGN ***

COLUMN ACTIONS
--------------

| COLUMN | AMCOLM1 | AMCOLM2 | AMCOLM3 | AMCOLM4 | AMCOLM5 | AMCOLM6 |
|---|---|---|---|---|---|---|
| 1 | 0.331D+06 | 0.126D+05 | 0.156D+07 | 0.331D+06 | 0.126D+05 | 0.716D+06 |
| 2 | 0.821D+06 | 0.248D+05 | 0.229D+07 | 0.821D+06 | 0.248D+05 | 0.217D+07 |
| 3 | 0.799D+06 | 0.226D+05 | 0.216D+07 | 0.799D+06 | 0.226D+05 | 0.191D+07 |
| 4 | 0.405D+06 | 0.277D+05 | 0.246D+07 | 0.405D+06 | 0.277D+05 | 0.252D+07 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 5 | 0.268D+06 | 0.377D+04 | 0.342D+06 | 0.268D+06 | 0.377D+04 | 0.246D+06 |
| 6 | 0.641D+06 | 0.217D+05 | 0.170D+07 | 0.641D+06 | 0.217D+05 | 0.169D+07 |
| 7 | 0.630D+06 | 0.179D+05 | 0.137D+07 | 0.630D+06 | 0.179D+05 | 0.143D+07 |
| 8 | 0.311D+06 | 0.301D+05 | 0.230D+07 | 0.311D+06 | 0.301D+05 | 0.240D+07 |
| 9 | 0.200D+06 | 0.706D+04 | 0.560D+06 | 0.200D+06 | 0.706D+04 | 0.542D+06 |
| 10 | 0.466D+06 | 0.147D+05 | 0.113D+07 | 0.466D+06 | 0.147D+05 | 0.117D+07 |
| 11 | 0.461D+06 | 0.125D+05 | 0.957D+06 | 0.461D+06 | 0.125D+05 | 0.999D+06 |
| 12 | 0.222D+06 | 0.258D+05 | 0.195D+07 | 0.222D+06 | 0.258D+05 | 0.208D+07 |
| 13 | 0.128D+06 | 0.117D+05 | 0.924D+06 | 0.128D+06 | 0.117D+05 | 0.909D+06 |
| 14 | 0.294D+06 | 0.903D+04 | 0.683D+06 | 0.294D+06 | 0.903D+04 | 0.726D+06 |
| 15 | 0.293D+06 | 0.702D+04 | 0.529D+06 | 0.293D+06 | 0.702D+04 | 0.567D+06 |
| 16 | 0.136D+06 | 0.225D+05 | 0.169D+07 | 0.136D+06 | 0.225D+05 | 0.183D+07 |
| 17 | 0.531D+05 | 0.138D+05 | 0.104D+07 | 0.531D+05 | 0.138D+05 | 0.112D+07 |
| 18 | 0.123D+06 | 0.414D+04 | 0.295D+06 | 0.123D+06 | 0.414D+04 | 0.351D+06 |
| 19 | 0.123D+06 | 0.129D+04 | 0.106D+06 | 0.123D+06 | 0.129D+04 | 0.957D+05 |
| 20 | 0.548D+05 | 0.171D+05 | 0.126D+07 | 0.548D+05 | 0.171D+05 | 0.141D+07 |

**BEAM ACTIONS**

------------

| BEAM | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 0.210D+04 | 0.581D+05 | 0.236D+05 | 0.210D+04 | 0.230D+05 | 0.302D+07 |
| | 0.210D+04 | 0.104D+06 | 0.579D+07 | | | |
| 2 | 0.369D+03 | 0.711D+05 | 0.223D+07 | 0.369D+03 | 0.999D+04 | 0.185D+07 |
| | 0.369D+03 | 0.911D+05 | 0.509D+07 | | | |
| 3 | 0.303D+04 | 0.735D+05 | 0.199D+07 | 0.303D+04 | 0.756D+04 | 0.244D+07 |
| | 0.303D+04 | 0.887D+05 | 0.415D+07 | | | |
| 4 | 0.139D+05 | 0.643D+05 | 0.368D+06 | 0.139D+05 | 0.165D+05 | 0.279D+07 |
| | 0.139D+05 | 0.973D+05 | 0.510D+07 | | | |
| 5 | 0.806D+04 | 0.731D+05 | 0.248D+07 | 0.806D+04 | 0.767D+04 | 0.194D+07 |
| | 0.806D+04 | 0.885D+05 | 0.468D+07 | | | |
| 6 | 0.374D+04 | 0.762D+05 | 0.241D+07 | 0.374D+04 | 0.463D+04 | 0.244D+07 |
| | 0.374D+04 | 0.854D+05 | 0.374D+07 | | | |
| 7 | 0.124D+05 | 0.682D+05 | 0.101D+07 | 0.124D+05 | 0.123D+05 | 0.273D+07 |
| | 0.124D+05 | 0.929D+05 | 0.455D+07 | | | |
| 8 | 0.757D+04 | 0.754D+05 | 0.284D+07 | 0.757D+04 | 0.507D+04 | 0.195D+07 |
| | 0.757D+04 | 0.856D+05 | 0.429D+07 | | | |
| 9 | 0.288D+04 | 0.792D+05 | 0.283D+07 | 0.288D+04 | 0.131D+04 | 0.249D+07 |
| | 0.288D+04 | 0.818D+05 | 0.321D+07 | | | |
| 10 | 0.149D+05 | 0.712D+05 | 0.149D+07 | 0.149D+05 | 0.912D+04 | 0.271D+07 |
| | 0.149D+05 | 0.895D+05 | 0.412D+07 | | | |
| 11 | 0.104D+05 | 0.776D+05 | 0.318D+07 | 0.104D+05 | 0.266D+04 | 0.194D+07 |
| | 0.104D+05 | 0.830D+05 | 0.395D+07 | | | |
| 12 | 0.516D+04 | 0.828D+05 | 0.329D+07 | 0.516D+04 | 0.249D+04 | 0.258D+07 |
| | 0.516D+04 | 0.778D+05 | 0.257D+07 | | | |
| 13 | 0.223D+05 | 0.501D+05 | 0.815D+06 | 0.223D+05 | 0.696D+04 | 0.212D+07 |
| | 0.223D+05 | 0.641D+05 | 0.284D+07 | | | |
| 14 | 0.182D+05 | 0.563D+05 | 0.254D+07 | 0.182D+05 | 0.783D+03 | 0.129D+07 |
| | 0.182D+05 | 0.579D+05 | 0.277D+07 | | | |
| 15 | 0.170D+05 | 0.624D+05 | 0.265D+07 | 0.170D+05 | 0.540D+04 | 0.207D+07 |
| | 0.170D+05 | 0.517D+05 | 0.110D+07 | | | |

*** OPTIMAL DESIGN ***

| COLUMN NO. | B | D | AS | NO. OF TIES | SPACING OF TIES |
|---|---|---|---|---|---|
| 1 | 14.1 | 11.8 | 9.277 | 13 | 14.1 |
| 2 | 14.1 | 11.8 | 11.710 | 13 | 14.1 |
| 3 | 14.1 | 11.8 | 11.273 | 13 | 14.1 |
| 4 | 14.1 | 11.8 | 12.495 | 13 | 14.1 |
| 5 | 13.7 | 11.2 | 4.965 | 11 | 13.7 |
| 6 | 13.7 | 11.2 | 9.700 | 11 | 13.7 |
| 7 | 13.7 | 11.2 | 8.760 | 11 | 13.7 |
| 8 | 13.7 | 11.2 | 12.148 | 11 | 13.7 |
| 9 | 13.4 | 10.9 | 5.591 | 11 | 13.4 |
| 10 | 13.4 | 10.9 | 7.767 | 11 | 13.4 |
| 11 | 13.4 | 10.9 | 7.164 | 11 | 13.4 |
| 12 | 13.4 | 10.9 | 11.025 | 11 | 13.4 |
| 13 | 13.1 | 10.6 | 6.815 | 12 | 13.1 |
| 14 | 13.1 | 10.6 | 6.089 | 12 | 13.1 |
| 15 | 13.1 | 10.6 | 5.507 | 12 | 13.1 |
| 16 | 13.1 | 10.6 | 10.124 | 12 | 13.1 |
| 17 | 12.1 | 9.6 | 7.358 | 13 | 12.1 |
| 18 | 12.1 | 9.6 | 4.320 | 13 | 12.1 |
| 19 | 12.1 | 9.6 | 3.347 | 13 | 12.1 |
| 20 | 12.1 | 9.6 | 8.538 | 13 | 12.1 |

| BEAM NO. | B | D | AS1 (-) | AS2 (+) | AS3 (-) | STIRRUPS SMIN | L | SMAX | L |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 14.1 | 21.2 | 0.999 | 5.302 | 5.502 | 5.5 | 59.5 | 10.6 | 83.8 |
| 2 | 14.1 | 21.2 | 5.502 | 5.302 | 4.783 | 7.9 | 35.3 | 10.6 | 83.8 |
| 3 | 14.1 | 21.2 | 4.783 | 5.302 | 3.844 | 8.6 | 30.8 | 10.6 | 83.8 |
| 4 | 13.7 | 20.6 | 0.943 | 5.155 | 4.960 | 5.9 | 52.0 | 10.3 | 81.7 |
| 5 | 13.7 | 20.6 | 4.960 | 5.155 | 4.521 | 7.6 | 35.7 | 10.3 | 81.8 |
| 6 | 13.7 | 20.6 | 4.521 | 5.155 | 3.560 | 8.4 | 30.0 | 10.3 | 81.8 |
| 7 | 13.4 | 20.1 | 0.947 | 5.020 | 4.534 | 6.0 | 48.2 | 10.0 | 80.0 |
| 8 | 13.4 | 20.1 | 4.534 | 5.020 | 4.259 | 7.5 | 34.7 | 10.0 | 80.0 |
| 9 | 13.4 | 20.1 | 4.259 | 5.020 | 3.117 | 8.5 | 27.6 | 10.0 | 80.0 |
| 10 | 13.1 | 19.6 | 1.439 | 4.911 | 4.184 | 6.1 | 45.4 | 9.8 | 78.6 |
| 11 | 13.1 | 19.6 | 4.184 | 4.911 | 4.000 | 7.4 | 33.3 | 9.8 | 78.6 |
| 12 | 13.1 | 19.6 | 4.000 | 4.911 | 2.537 | 7.5 | 32.9 | 9.8 | 78.6 |
| 13 | 12.1 | 18.1 | 0.847 | 4.536 | 3.092 | 0.0 | 0.0 | 9.1 | 99.9 |
| 14 | 12.1 | 18.1 | 3.092 | 4.536 | 3.008 | 0.0 | 0.0 | 9.1 | 99.9 |
| 15 | 12.1 | 18.1 | 3.008 | 4.536 | 1.152 | 0.0 | 0.0 | 9.1 | 99.9 |

# APPENDIX 4
# IDESIGN Output Data File

```
**************************************************
        WELCOME TO PROGRAM IDESIGN3.5
  INTERACTIVE DESIGN OPTIMIZATION OF ENGINEERING
  SYSTEMS TO SOLVE THE PROBLEM:  MINIMIZE  F(B)
  SUBJECT TO  HI(B) = 0, AND  GI(B) <= 0
  REPORT ANY PROBLEMS TO PROFESSOR J.S.ARORA
**************************************************
```

INPUT DATA IS ECHOED

FRAME NO. 1 (3 X 5)

```
  30    0   70   50    0   -1    1    0    2    5
1.0000D-03  1.0000D-03  1.0000D-03  1.0000D-03
```

| | | | |
|---|---|---|---|
| 1 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 2 | 1.3000D+01 | 7.5000D+00 | 3.6000D+01 |
| 3 | 1.0000D+01 | 1.0000D+00 | 2.5000D+01 |
| 4 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 5 | 2.2000D+01 | 1.5000D+01 | 5.0000D+01 |
| 6 | 3.5000D+00 | 1.0000D+00 | 2.0000D+01 |
| 7 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 8 | 1.4000D+01 | 7.5000D+00 | 3.6000D+01 |
| 9 | 7.5000D+00 | 1.0000D+00 | 2.5000D+01 |
| 10 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 11 | 1.8000D+01 | 1.5000D+01 | 5.0000D+01 |
| 12 | 2.5000D+00 | 1.0000D+00 | 2.0000D+01 |
| 13 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 14 | 1.0000D+01 | 7.5000D+00 | 3.6000D+01 |
| 15 | 7.0000D+00 | 1.0000D+00 | 2.5000D+01 |
| 16 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 17 | 1.8000D+01 | 1.5000D+01 | 5.0000D+01 |
| 18 | 2.5000D+00 | 1.0000D+00 | 2.0000D+01 |
| 19 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 20 | 9.0000D+00 | 7.5000D+00 | 3.6000D+01 |
| 21 | 6.0000D+00 | 1.0000D+00 | 2.5000D+01 |
| 22 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 23 | 2.0000D+01 | 1.5000D+01 | 5.0000D+01 |
| 24 | 2.0000D+00 | 1.0000D+00 | 2.0000D+01 |
| 25 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 26 | 9.5000D+00 | 7.5000D+00 | 3.6000D+01 |
| 27 | 5.5000D+00 | 1.0000D+00 | 2.5000D+01 |
| 28 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 29 | 1.9000D+01 | 1.5000D+01 | 5.0000D+01 |
| 30 | 1.5000D+00 | 1.0000D+00 | 2.0000D+01 |

```
****************************************************************
*                  FRAME NO. 1   (3 X 5)                       *
****************************************************************
```

```
NUMBER OF DESIGN VARIABLES         =      30
NUMBER OF EQUALITY CONSTRAINTS     =       0
NUMBER OF INEQUALITY CONSTRAINTS   =      70
MAXIMUM NUMBER OF ITERATIONS       =      50
PREVIOUS ITERATIONS RUN            =       0
PRINTING CODE                      =      -1
GRADIENT CALCULATION INDICATOR     =       1
PROBLEM TYPE (2=LP, 0,1=NLP)       =       0
ALGORITHM INDICATOR                =       2
NO. OF CONSECUTIVE ITER. FOR ACT   =       5
TOL. IN CONSTR. VIOL. AT OPT.      =  1.0000D-03
CONVERGENCE PARAMETER VALUE        =  1.0000D-03
DEL FOR F. D. GRAD. CALCULATION    =  1.0000D-03
ACCEPTABLE CHANGE IN COST FUNC.    =  1.0000D-03
```

::::::: STARTING DESIGN AND ITS LIMITS :::::::

******************************************************

| NO. | DESIGN | LOWER LIM | UPPER LIM |
|---|---|---|---|
| 1 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 2 | 1.3000D+01 | 7.5000D+00 | 3.6000D+01 |
| 3 | 1.0000D+01 | 1.0000D+00 | 2.5000D+01 |
| 4 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 5 | 2.2000D+01 | 1.5000D+01 | 5.0000D+01 |
| 6 | 3.5000D+00 | 1.0000D+00 | 2.0000D+01 |
| 7 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 8 | 1.4000D+01 | 7.5000D+00 | 3.6000D+01 |
| 9 | 7.5000D+00 | 1.0000D+00 | 2.5000D+01 |
| 10 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 11 | 1.8000D+01 | 1.5000D+01 | 5.0000D+01 |
| 12 | 2.5000D+00 | 1.0000D+00 | 2.0000D+01 |
| 13 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 14 | 1.0000D+01 | 7.5000D+00 | 3.6000D+01 |
| 15 | 7.0000D+00 | 1.0000D+00 | 2.5000D+01 |
| 16 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 17 | 1.8000D+01 | 1.5000D+01 | 5.0000D+01 |
| 18 | 2.5000D+00 | 1.0000D+00 | 2.0000D+01 |
| 19 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 20 | 9.0000D+00 | 7.5000D+00 | 3.6000D+01 |
| 21 | 6.0000D+00 | 1.0000D+00 | 2.5000D+01 |
| 22 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 23 | 2.0000D+01 | 1.5000D+01 | 5.0000D+01 |
| 24 | 2.0000D+00 | 1.0000D+00 | 2.0000D+01 |
| 25 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 26 | 9.5000D+00 | 7.5000D+00 | 3.6000D+01 |
| 27 | 5.5000D+00 | 1.0000D+00 | 2.5000D+01 |
| 28 | 1.2000D+01 | 1.0000D+01 | 3.6000D+01 |
| 29 | 1.9000D+01 | 1.5000D+01 | 5.0000D+01 |
| 30 | 1.5000D+00 | 1.0000D+00 | 2.0000D+01 |

REQUIRED DIMENSION OF ARRAY A= 15351

DESIGN VARIABLES AT ITERATION = 1 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.2000D+01 | 2 | 1.3000D+01 | 3 | 1.0000D+01 | 4 | 1.2000D+01 | 5 | 2.2000D+01 |
| 6 | 3.5000D+00 | 7 | 1.2000D+01 | 8 | 1.4000D+01 | 9 | 7.5000D+00 | 10 | 1.2000D+01 |
| 11 | 1.8000D+01 | 12 | 2.5000D+00 | 13 | 1.2000D+01 | 14 | 1.0000D+01 | 15 | 7.0000D+00 |
| 16 | 1.2000D+01 | 17 | 1.8000D+01 | 18 | 2.5000D+00 | 19 | 1.2000D+01 | 20 | 9.0000D+00 |
| 21 | 6.0000D+00 | 22 | 1.2000D+01 | 23 | 2.0000D+01 | 24 | 2.0000D+00 | 25 | 1.2000D+01 |
| 26 | 9.5000D+00 | 27 | 5.5000D+00 | 28 | 1.2000D+01 | 29 | 1.9000D+01 | 30 | 1.5000D+00 |

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 5.2530D-01 | 1.0000D+00 | 1.7621D+04 |

DESIGN VARIABLES AT ITERATION = 2 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.3641D+01 | 2 | 1.3551D+01 | 3 | 1.1424D+01 | 4 | 1.3642D+01 | 5 | 2.3184D+01 |
| 6 | 4.8143D+00 | 7 | 1.4192D+01 | 8 | 1.4499D+01 | 9 | 1.2346D+01 | 10 | 1.4194D+01 |
| 11 | 2.1288D+01 | 12 | 4.8828D+00 | 13 | 1.3941D+01 | 14 | 1.1503D+01 | 15 | 1.0841D+01 |
| 16 | 1.3943D+01 | 17 | 2.0912D+01 | 18 | 4.6643D+00 | 19 | 1.3428D+01 | 20 | 1.0849D+01 |
| 21 | 9.7126D+00 | 22 | 1.3429D+01 | 23 | 2.1396D+01 | 24 | 3.9510D+00 | 25 | 1.1747D+01 |
| 26 | 9.2468D+00 | 27 | 7.9379D+00 | 28 | 1.1747D+01 | 29 | 1.9413D+01 | 30 | 2.8716D+00 |

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 3.3822D-02 | 7.8894D+01 | 1.9501D+04 |

DESIGN VARIABLES AT ITERATION = 3 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.3646D+01 | 2 | 1.3532D+01 | 3 | 1.1439D+01 | 4 | 1.3647D+01 | 5 | 2.3171D+01 |
| 6 | 4.8210D+00 | 7 | 1.4194D+01 | 8 | 1.4407D+01 | 9 | 1.2331D+01 | 10 | 1.4197D+01 |
| 11 | 2.1291D+01 | 12 | 4.8877D+00 | 13 | 1.3940D+01 | 14 | 1.1501D+01 | 15 | 1.0851D+01 |
| 16 | 1.3941D+01 | 17 | 2.0909D+01 | 18 | 4.6636D+00 | 19 | 1.3419D+01 | 20 | 1.0840D+01 |
| 21 | 9.6795D+00 | 22 | 1.3386D+01 | 23 | 2.1607D+01 | 24 | 3.9049D+00 | 25 | 1.1749D+01 |
| 26 | 9.2485D+00 | 27 | 7.9343D+00 | 28 | 1.1748D+01 | 29 | 1.9402D+01 | 30 | 2.8730D+00 |

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 3.3751D-02 | 3.5025D+01 | 1.9502D+04 |

DESIGN VARIABLES AT ITERATION = 4 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.3965D+01 | 2 | 1.2082D+01 | 3 | 1.2511D+01 | 4 | 1.3967D+01 | 5 | 2.2700D+01 |
| 6 | 5.2310D+00 | 7 | 1.4411D+01 | 8 | 7.5000D+00 | 9 | 1.1209D+01 | 10 | 1.4413D+01 |
| 11 | 2.1617D+01 | 12 | 5.2642D+00 | 13 | 1.3810D+01 | 14 | 1.1360D+01 | 15 | 1.1238D+01 |
| 16 | 1.3811D+01 | 17 | 2.0715D+01 | 18 | 4.5993D+00 | 19 | 1.3253D+01 | 20 | 1.0688D+01 |
| 21 | 9.6510D+00 | 22 | 1.3226D+01 | 23 | 2.1604D+01 | 24 | 3.8300D+00 | 25 | 1.1859D+01 |
| 26 | 9.3594D+00 | 27 | 8.2703D+00 | 28 | 1.1859D+01 | 29 | 1.8677D+01 | 30 | 2.9614D+00 |

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 4.4111D-01 | 2.7804D+00 | 1.9309D+04 |

DESIGN VARIABLES AT ITERATION = 5 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.4139D+01 | 2 | 1.1619D+01 | 3 | 1.2771D+01 | 4 | 1.4140D+01 | 5 | 2.1231D+01 |
| 6 | 5.5021D+00 | 7 | 1.3763D+01 | 8 | 1.0113D+01 | 9 | 1.1368D+01 | 10 | 1.3763D+01 |
| 11 | 2.0645D+01 | 12 | 5.0327D+00 | 13 | 1.2707D+01 | 14 | 1.0202D+01 | 15 | 1.0434D+01 |
| 16 | 1.2706D+01 | 17 | 2.3495D+01 | 18 | 3.7309D+00 | 19 | 1.2703D+01 | 20 | 1.0205D+01 |
| 21 | 9.4133D+00 | 22 | 1.2703D+01 | 23 | 2.1559D+01 | 24 | 3.7539D+00 | 25 | 1.2078D+01 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 26 | 9.5778D+00 | 27 | 8.5092D+00 | 28 | 1.2078D+01 | 29 | 1.8133D+01 | 30 | 3.0710D+00 |

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 9.1169D-02 | 2.7576D+00 | 1.8975D+04 |

DESIGN VARIABLES AT ITERATION = 6 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.3997D+01 | 2 | 1.4377D+01 | 3 | 1.1921D+01 | 4 | 1.3996D+01 | 5 | 2.0995D+01 |
| 6 | 5.3129D+00 | 7 | 1.3755D+01 | 8 | 1.1161D+01 | 9 | 1.2083D+01 | 10 | 1.3755D+01 |
| 11 | 2.0633D+01 | 12 | 4.9987D+00 | 13 | 1.3060D+01 | 14 | 1.0560D+01 | 15 | 1.0660D+01 |
| 16 | 1.3060D+01 | 17 | 2.1972D+01 | 18 | 4.1317D+00 | 19 | 1.2762D+01 | 20 | 1.0262D+01 |
| 21 | 9.4407D+00 | 22 | 1.2762D+01 | 23 | 2.1528D+01 | 24 | 3.8000D+00 | 25 | 1.2103D+01 |
| 26 | 9.6035D+00 | 27 | 8.4550D+00 | 28 | 1.2103D+01 | 29 | 1.8155D+01 | 30 | 3.0960D+00 |

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 2.6556D-02 | 1.9037D+01 | 1.9029D+04 |

DESIGN VARIABLES AT ITERATION = 7 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.3981D+01 | 2 | 1.4674D+01 | 3 | 1.1923D+01 | 4 | 1.3980D+01 | 5 | 2.0971D+01 |
| 6 | 5.2917D+00 | 7 | 1.3743D+01 | 8 | 1.1150D+01 | 9 | 1.2049D+01 | 10 | 1.3743D+01 |
| 11 | 2.0701D+01 | 12 | 4.9826D+00 | 13 | 1.3081D+01 | 14 | 1.0581D+01 | 15 | 1.0679D+01 |
| 16 | 1.3081D+01 | 17 | 2.1863D+01 | 18 | 4.1563D+00 | 19 | 1.2765D+01 | 20 | 1.0265D+01 |
| 21 | 9.4448D+00 | 22 | 1.2765D+01 | 23 | 2.1523D+01 | 24 | 3.8034D+00 | 25 | 1.2105D+01 |
| 26 | 9.6047D+00 | 27 | 8.4517D+00 | 28 | 1.2105D+01 | 29 | 1.8157D+01 | 30 | 3.0972D+00 |

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 2.6267D-02 | 6.8630D+00 | 1.9034D+04 |

DESIGN VARIABLES AT ITERATION = 8 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.3833D+01 | 2 | 1.4508D+01 | 3 | 1.2027D+01 | 4 | 1.3832D+01 | 5 | 2.1802D+01 |
| 6 | 5.0961D+00 | 7 | 1.3605D+01 | 8 | 1.1059D+01 | 9 | 1.1711D+01 | 10 | 1.3605D+01 |
| 11 | 2.1260D+01 | 12 | 4.8053D+00 | 13 | 1.3682D+01 | 14 | 1.1183D+01 | 15 | 1.1425D+01 |
| 16 | 1.3683D+01 | 17 | 1.8431D+01 | 18 | 4.8679D+00 | 19 | 1.2794D+01 | 20 | 1.0294D+01 |
| 21 | 9.6477D+00 | 22 | 1.2795D+01 | 23 | 2.1400D+01 | 24 | 3.8367D+00 | 25 | 1.2107D+01 |
| 26 | 9.6071D+00 | 27 | 8.4421D+00 | 28 | 1.2107D+01 | 29 | 1.8161D+01 | 30 | 3.0997D+00 |

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 1.0196D-01 | 2.6296D+00 | 1.9007D+04 |

DESIGN VARIABLES AT ITERATION = 9 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.4124D+01 | 2 | 1.1878D+01 | 3 | 1.2313D+01 | 4 | 1.4124D+01 | 5 | 2.1208D+01 |
| 6 | 5.4781D+00 | 7 | 1.3642D+01 | 8 | 1.1141D+01 | 9 | 1.2046D+01 | 10 | 1.3642D+01 |
| 11 | 2.1048D+01 | 12 | 4.8530D+00 | 13 | 1.3377D+01 | 14 | 1.0877D+01 | 15 | 1.1099D+01 |
| 16 | 1.3377D+01 | 17 | 2.0112D+01 | 18 | 4.5174D+00 | 19 | 1.2769D+01 | 20 | 1.0269D+01 |
| 21 | 9.5929D+00 | 22 | 1.2769D+01 | 23 | 2.1402D+01 | 24 | 3.8078D+00 | 25 | 1.2094D+01 |
| 26 | 9.5937D+00 | 27 | 8.4812D+00 | 28 | 1.2094D+01 | 29 | 1.8141D+01 | 30 | 3.0862D+00 |

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 1.5779D-02 | 2.3187D-01 | 1.8984D+04 |

DESIGN VARIABLES AT ITERATION = 10 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.4123D+01 | 2 | 1.2110D+01 | 3 | 1.2467D+01 | 4 | 1.4123D+01 | 5 | 2.1184D+01 |
| 6 | 5.4835D+00 | 7 | 1.3649D+01 | 8 | 1.1149D+01 | 9 | 1.2049D+01 | 10 | 1.3649D+01 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 2.1013D+01 | 12 | 4.8627D+00 | 13 | 1.3395D+01 | 14 | 1.0895D+01 | 15 | 1.1115D+01 |
| 16 | 1.3395D+01 | 17 | 2.0093D+01 | 18 | 4.5455D+00 | 19 | 1.2776D+01 | 20 | 1.0276D+01 |
| 21 | 9.6102D+00 | 22 | 1.2776D+01 | 23 | 2.1359D+01 | 24 | 3.8164D+00 | 25 | 1.2094D+01 |
| 26 | 9.5937D+00 | 27 | 8.4833D+00 | 28 | 1.2094D+01 | 29 | 1.8140D+01 | 30 | 3.0861D+00 |

***** DESIGN AT THIS ITERATION IS USABLE *****

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 1.7826D-04 | 6.4244D-01 | 1.8990D+04 |

DESIGN VARIABLES AT ITERATION = 11 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.4087D+01 | 2 | 1.2752D+01 | 3 | 1.2348D+01 | 4 | 1.4087D+01 | 5 | 2.1131D+01 |
| 6 | 5.4358D+00 | 7 | 1.3690D+01 | 8 | 1.1190D+01 | 9 | 1.2051D+01 | 10 | 1.3690D+01 |
| 11 | 2.0829D+01 | 12 | 4.9148D+00 | 13 | 1.3399D+01 | 14 | 1.0899D+01 | 15 | 1.1100D+01 |
| 16 | 1.3399D+01 | 17 | 2.0099D+01 | 18 | 4.5505D+00 | 19 | 1.2831D+01 | 20 | 1.0331D+01 |
| 21 | 9.6790D+00 | 22 | 1.2831D+01 | 23 | 2.1085D+01 | 24 | 3.8782D+00 | 25 | 1.2098D+01 |
| 26 | 9.5978D+00 | 27 | 8.4818D+00 | 28 | 1.2098D+01 | 29 | 1.8147D+01 | 30 | 3.0903D+00 |

***** DESIGN AT THIS ITERATION IS USABLE *****

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 1.5045D-03 | 2.5505D-01 | 1.8984D+04 |

DESIGN VARIABLES AT ITERATION = 12 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.4063D+01 | 2 | 1.2813D+01 | 3 | 1.2342D+01 | 4 | 1.4063D+01 | 5 | 2.1231D+01 |
| 6 | 5.4028D+00 | 7 | 1.3697D+01 | 8 | 1.1197D+01 | 9 | 1.2048D+01 | 10 | 1.3697D+01 |
| 11 | 2.0791D+01 | 12 | 4.9240D+00 | 13 | 1.3398D+01 | 14 | 1.0898D+01 | 15 | 1.1086D+01 |
| 16 | 1.3398D+01 | 17 | 2.0098D+01 | 18 | 4.5491D+00 | 19 | 1.2877D+01 | 20 | 1.0377D+01 |
| 21 | 9.7526D+00 | 22 | 1.2877D+01 | 23 | 2.0830D+01 | 24 | 3.9309D+00 | 25 | 1.2098D+01 |
| 26 | 9.5983D+00 | 27 | 8.4895D+00 | 28 | 1.2098D+01 | 29 | 1.8147D+01 | 30 | 3.0909D+00 |

***** DESIGN AT THIS ITERATION IS USABLE *****

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 1.4463D-04 | 7.0393D-01 | 1.8976D+04 |

DESIGN VARIABLES AT ITERATION = 13 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.4122D+01 | 2 | 1.2109D+01 | 3 | 1.2423D+01 | 4 | 1.4122D+01 | 5 | 2.1183D+01 |
| 6 | 5.4816D+00 | 7 | 1.3731D+01 | 8 | 1.1231D+01 | 9 | 1.2133D+01 | 10 | 1.3731D+01 |
| 11 | 2.0598D+01 | 12 | 4.9677D+00 | 13 | 1.3392D+01 | 14 | 1.0892D+01 | 15 | 1.1062D+01 |
| 16 | 1.3392D+01 | 17 | 2.0088D+01 | 18 | 4.5410D+00 | 19 | 1.2989D+01 | 20 | 1.0489D+01 |
| 21 | 9.9453D+00 | 22 | 1.2989D+01 | 23 | 2.0194D+01 | 24 | 4.0585D+00 | 25 | 1.2097D+01 |
| 26 | 9.5966D+00 | 27 | 8.5168D+00 | 28 | 1.2097D+01 | 29 | 1.8145D+01 | 30 | 3.0891D+00 |

***** DESIGN AT THIS ITERATION IS USABLE *****

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 1.7165D-03 | 2.3066D-01 | 1.8967D+04 |

DESIGN VARIABLES AT ITERATION = 14 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.4135D+01 | 2 | 1.1878D+01 | 3 | 1.2489D+01 | 4 | 1.4135D+01 | 5 | 2.1202D+01 |
| 6 | 5.4997D+00 | 7 | 1.3714D+01 | 8 | 1.1214D+01 | 9 | 1.2131D+01 | 10 | 1.3714D+01 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 2.0677D+01 | 12 | 4.9455D+00 | 13 | 1.3389D+01 | 14 | 1.0889D+01 | 15 | 1.1053D+01 |
| 16 | 1.3389D+01 | 17 | 2.0083D+01 | 18 | 4.5375D+00 | 19 | 1.3006D+01 | 20 | 1.0506D+01 |
| 21 | 9.9801D+00 | 22 | 1.3006D+01 | 23 | 2.0104D+01 | 24 | 4.0792D+00 | 25 | 1.2095D+01 |
| 26 | 9.5953D+00 | 27 | 8.5235D+00 | 28 | 1.2095D+01 | 29 | 1.8143D+01 | 30 | 3.0878D+00 |

***** DESIGN AT THIS ITERATION IS USABLE *****

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 2.4073D-04 | 4.6222D-01 | 1.8965D+04 |

DESIGN VARIABLES AT ITERATION = 15 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.4144D+01 | 2 | 1.1640D+01 | 3 | 1.2536D+01 | 4 | 1.4144D+01 | 5 | 2.1240D+01 |
| 6 | 5.5124D+00 | 7 | 1.3715D+01 | 8 | 1.1215D+01 | 9 | 1.2148D+01 | 10 | 1.3715D+01 |
| 11 | 2.0665D+01 | 12 | 4.9465D+00 | 13 | 1.3385D+01 | 14 | 1.0885D+01 | 15 | 1.1025D+01 |
| 16 | 1.3385D+01 | 17 | 2.0077D+01 | 18 | 4.5322D+00 | 19 | 1.3092D+01 | 20 | 1.0592D+01 |
| 21 | 1.0125D+01 | 22 | 1.3092D+01 | 23 | 1.9642D+01 | 24 | 4.1797D+00 | 25 | 1.2095D+01 |
| 26 | 9.5948D+00 | 27 | 8.5411D+00 | 28 | 1.2095D+01 | 29 | 1.8142D+01 | 30 | 3.0873D+00 |

***** DESIGN AT THIS ITERATION IS USABLE *****

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 5.2290D-04 | 4.3245D-02 | 1.8955D+04 |

DESIGN VARIABLES AT ITERATION = 16 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.4145D+01 | 2 | 1.1683D+01 | 3 | 1.2530D+01 | 4 | 1.4145D+01 | 5 | 2.1223D+01 |
| 6 | 5.5130D+00 | 7 | 1.3719D+01 | 8 | 1.1219D+01 | 9 | 1.2150D+01 | 10 | 1.3719D+01 |
| 11 | 2.0648D+01 | 12 | 4.9514D+00 | 13 | 1.3385D+01 | 14 | 1.0885D+01 | 15 | 1.1025D+01 |
| 16 | 1.3385D+01 | 17 | 2.0077D+01 | 18 | 4.5327D+00 | 19 | 1.3094D+01 | 20 | 1.0594D+01 |
| 21 | 1.0126D+01 | 22 | 1.3094D+01 | 23 | 1.9641D+01 | 24 | 4.1827D+00 | 25 | 1.2095D+01 |
| 26 | 9.5951D+00 | 27 | 8.5404D+00 | 28 | 1.2095D+01 | 29 | 1.8143D+01 | 30 | 3.0876D+00 |

***** DESIGN AT THIS ITERATION IS USABLE *****

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 6.8127D-06 | 2.6830D-01 | 1.8954D+04 |

DESIGN VARIABLES AT ITERATION = 17 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.4138D+01 | 2 | 1.1817D+01 | 3 | 1.2499D+01 | 4 | 1.4138D+01 | 5 | 2.1209D+01 |
| 6 | 5.5036D+00 | 7 | 1.3725D+01 | 8 | 1.1225D+01 | 9 | 1.2148D+01 | 10 | 1.3725D+01 |
| 11 | 2.0622D+01 | 12 | 4.9594D+00 | 13 | 1.3386D+01 | 14 | 1.0886D+01 | 15 | 1.1025D+01 |
| 16 | 1.3386D+01 | 17 | 2.0079D+01 | 18 | 4.5341D+00 | 19 | 1.3095D+01 | 20 | 1.0595D+01 |
| 21 | 1.0124D+01 | 22 | 1.3095D+01 | 23 | 1.9643D+01 | 24 | 4.1839D+00 | 25 | 1.2096D+01 |
| 26 | 9.5959D+00 | 27 | 8.5383D+00 | 28 | 1.2096D+01 | 29 | 1.8144D+01 | 30 | 3.0884D+00 |

***** DESIGN AT THIS ITERATION IS USABLE *****

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 8.3464D-05 | 1.5296D-01 | 1.8953D+04 |

DESIGN VARIABLES AT ITERATION = 18 (RQP ALGORITHM USED)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.4137D+01 | 2 | 1.1836D+01 | 3 | 1.2495D+01 | 4 | 1.4137D+01 | 5 | 2.1207D+01 |
| 6 | 5.5023D+00 | 7 | 1.3726D+01 | 8 | 1.1226D+01 | 9 | 1.2148D+01 | 10 | 1.3726D+01 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 11 | 2.0619D+01 | 12 | 4.9604D+00 | 13 | 1.3386D+01 | 14 | 1.0886D+01 | 15 | 1.1025D+01 |
| 16 | 1.3386D+01 | 17 | 2.0079D+01 | 18 | 4.5343D+00 | 19 | 1.3095D+01 | 20 | 1.0595D+01 |
| 21 | 1.0124D+01 | 22 | 1.3095D+01 | 23 | 1.9643D+01 | 24 | 4.1840D+00 | 25 | 1.2096D+01 |
| 26 | 9.5960D+00 | 27 | 8.5380D+00 | 28 | 1.2096D+01 | 29 | 1.8144D+01 | 30 | 3.0885D+00 |

***** DESIGN AT THIS ITERATION IS USABLE *****

| MAX VIO | CONV PARM | COST |
|---|---|---|
| 7.3675D-05 | 1.5299D-01 | 1.8953D+04 |

***** CONVERGENCE CRITERIA SATISFIED *****

HISTORIES OF (1) MAXIMUM CONSTRAINT VIOLATION - THIS SHOULD BE NEARLY ZERO FOR FEASIBLE DESIGN (2) CONVERGENCE PARAMETER THIS SHOULD BE NEARLY ZERO FOR OPTIMUM DESIGN (3) COST

| I | MAX. VIO. | CONV. PARM | COST |
|---|---|---|---|
| 1 | 5.25296D-01 | 1.00000D+00 | 1.76211D+04 |
| 2 | 3.38217D-02 | 7.88943D+01 | 1.95012D+04 |
| 3 | 3.37507D-02 | 3.50247D+01 | 1.95020D+04 |
| 4 | 4.41110D-01 | 2.78038D+00 | 1.93092D+04 |
| 5 | 9.11691D-02 | 2.75761D+00 | 1.89751D+04 |
| 6 | 2.65559D-02 | 1.90369D+01 | 1.90291D+04 |
| 7 | 2.62666D-02 | 6.86296D+00 | 1.90341D+04 |
| 8 | 1.01958D-01 | 2.62957D+00 | 1.90074D+04 |
| 9 | 1.57794D-02 | 2.31867D-01 | 1.89843D+04 |
| 10 | 1.78261D-04 | 6.42438D-01 | 1.89903D+04 |
| 11 | 1.50453D-03 | 2.55046D-01 | 1.89838D+04 |
| 12 | 1.44629D-04 | 7.03935D-01 | 1.89761D+04 |
| 13 | 1.71651D-03 | 2.30665D-01 | 1.89668D+04 |
| 14 | 2.40735D-04 | 4.62220D-01 | 1.89645D+04 |
| 15 | 5.22897D-04 | 4.32447D-02 | 1.89549D+04 |
| 16 | 6.81274D-06 | 2.68296D-01 | 1.89544D+04 |
| 17 | 8.34644D-05 | 1.52958D-01 | 1.89533D+04 |
| 18 | 7.36755D-05 | 1.52994D-01 | 1.89532D+04 |

CONSTRAINT ACTIVITY

| NO. | ACTIVE | VALUE | LAGR. MULT. |
|---|---|---|---|
| 1 | YES | -1.39365D-02 | 0.00000D+00 |
| 2 | | -3.93885D-01 | 0.00000D+00 |
| 3 | YES | 7.36755D-05 | 3.12289D+02 |
| 4 | | -2.29319D-01 | 0.00000D+00 |
| 5 | YES | -1.24271D-04 | 2.89887D+02 |
| 6 | | -2.49907D-01 | 0.00000D+00 |
| 7 | YES | 4.52442D-06 | 7.30734D+02 |
| 8 | | -4.50591D+00 | 0.00000D+00 |
| 9 | | -4.38727D-01 | 0.00000D+00 |
| 10 | | -3.13095D-01 | 0.00000D+00 |
| 11 | YES | -3.39962D-02 | 0.00000D+00 |
| 12 | YES | 1.77475D-07 | 1.89876D+03 |

| | | | |
|---|---|---|---|
| 13 | | -4.61962D-01 | 0.00000D+00 |
| 14 | YES | -4.87164D-07 | 1.03136D+03 |
| 15 | YES | -4.72792D-07 | 4.69919D+02 |
| 16 | | -6.84417D-01 | 0.00000D+00 |
| 17 | YES | 1.19867D-06 | 2.57400D+02 |
| 18 | | -1.93992D-01 | 0.00000D+00 |
| 19 | YES | -1.49447D-03 | 3.56277D+02 |
| 20 | | -2.48879D-01 | 0.00000D+00 |
| 21 | YES | -9.95025D-07 | 8.60263D+02 |
| 22 | | -4.25822D+00 | 0.00000D+00 |
| 23 | | -4.63977D-01 | 0.00000D+00 |
| 24 | | -3.26658D-01 | 0.00000D+00 |
| 25 | YES | -4.34502D-02 | 0.00000D+00 |
| 26 | YES | -6.27820D-07 | 2.41363D+03 |
| 27 | | -4.25679D-01 | 0.00000D+00 |
| 28 | YES | 5.78113D-07 | 1.38877D+03 |
| 29 | YES | -8.07154D-08 | 4.56312D+02 |
| 30 | | -1.16769D+00 | 0.00000D+00 |
| 31 | YES | 2.48323D-06 | 2.33680D+02 |
| 32 | | -2.30956D-01 | 0.00000D+00 |
| 33 | YES | 9.92752D-08 | 2.99418D+02 |
| 34 | | -2.50000D-01 | 0.00000D+00 |
| 35 | YES | -2.52524D-06 | 7.83992D+02 |
| 36 | | -4.06080D+00 | 0.00000D+00 |
| 37 | | -4.84102D-01 | 0.00000D+00 |
| 38 | | -3.20181D-01 | 0.00000D+00 |
| 39 | YES | -5.13980D-02 | 0.00000D+00 |
| 40 | YES | -8.07862D-08 | 2.25383D+03 |
| 41 | | -3.92397D-01 | 0.00000D+00 |
| 42 | YES | 9.92752D-08 | 1.31444D+03 |
| 43 | YES | -1.17068D-07 | 5.51664D+02 |
| 44 | | -2.24502D+00 | 0.00000D+00 |
| 45 | YES | 1.75219D-06 | 2.14493D+02 |
| 46 | | -2.62074D-01 | 0.00000D+00 |
| 47 | YES | 1.30928D-07 | 3.24757D+02 |
| 48 | | -2.50000D-01 | 0.00000D+00 |
| 49 | YES | -2.58819D-06 | 7.94428D+02 |
| 50 | | -3.87974D+00 | 0.00000D+00 |
| 51 | | -5.02559D-01 | 0.00000D+00 |
| 52 | | -3.11311D-01 | 0.00000D+00 |
| 53 | YES | -5.83223D-02 | 0.00000D+00 |
| 54 | YES | -1.26072D-07 | 2.36926D+03 |
| 55 | | -3.65479D-01 | 0.00000D+00 |
| 56 | YES | 1.44123D-07 | 1.40006D+03 |
| 57 | YES | -6.04266D-08 | 5.54526D+02 |
| 58 | | -5.57407D+00 | 0.00000D+00 |
| 59 | YES | 1.96772D-06 | 1.80435D+02 |
| 60 | | -2.70575D-01 | 0.00000D+00 |
| 61 | YES | 7.61699D-08 | 3.58571D+02 |
| 62 | | -2.50000D-01 | 0.00000D+00 |
| 63 | YES | -2.32163D-06 | 6.70012D+02 |
| 64 | | -3.22179D+00 | 0.00000D+00 |

| | | | |
|---|---|---|---|
| 65 | | -5.69631D-01 | 0.00000D+00 |
| 66 | | -5.61180D-01 | 0.00000D+00 |
| 67 | YES | -8.29106D-02 | 0.00000D+00 |
| 68 | YES | -6.11161D-08 | 2.21587D+03 |
| 69 | | -2.73049D-01 | 0.00000D+00 |
| 70 | YES | 7.61699D-08 | 1.31048D+03 |

DESIGN VARIABLE ACTIVITY

| NO. | ACTIVE | DESIGN | LOWER | UPPER | LAGR. MULT. |
|---|---|---|---|---|---|
| 1 | LOWER | 1.41366D+01 | 1.00000D+01 | 3.60000D+01 | 0.00000D+00 |
| 2 | | 1.18364D+01 | 7.50000D+00 | 3.60000D+01 | 0.00000D+00 |
| 3 | | 1.24954D+01 | 1.00000D+00 | 2.50000D+01 | 0.00000D+00 |
| 4 | LOWER | 1.41366D+01 | 1.00000D+01 | 3.60000D+01 | 0.00000D+00 |
| 5 | LOWER | 2.12075D+01 | 1.50000D+01 | 5.00000D+01 | 0.00000D+00 |
| 6 | | 5.50227D+00 | 1.00000D+00 | 2.00000D+01 | 0.00000D+00 |
| 7 | LOWER | 1.37256D+01 | 1.00000D+01 | 3.60000D+01 | 0.00000D+00 |
| 8 | LOWER | 1.12256D+01 | 7.50000D+00 | 3.60000D+01 | 0.00000D+00 |
| 9 | | 1.21476D+01 | 1.00000D+00 | 2.50000D+01 | 0.00000D+00 |
| 10 | LOWER | 1.37256D+01 | 1.00000D+01 | 3.60000D+01 | 0.00000D+00 |
| 11 | LOWER | 2.06191D+01 | 1.50000D+01 | 5.00000D+01 | 0.00000D+00 |
| 12 | | 4.96042D+00 | 1.00000D+00 | 2.00000D+01 | 0.00000D+00 |
| 13 | LOWER | 1.33863D+01 | 1.00000D+01 | 3.60000D+01 | 0.00000D+00 |
| 14 | LOWER | 1.08863D+01 | 7.50000D+00 | 3.60000D+01 | 0.00000D+00 |
| 15 | | 1.10245D+01 | 1.00000D+00 | 2.50000D+01 | 0.00000D+00 |
| 16 | LOWER | 1.33863D+01 | 1.00000D+01 | 3.60000D+01 | 0.00000D+00 |
| 17 | LOWER | 2.00794D+01 | 1.50000D+01 | 5.00000D+01 | 0.00000D+00 |
| 18 | | 4.53429D+00 | 1.00000D+00 | 2.00000D+01 | 0.00000D+00 |
| 19 | LOWER | 1.30953D+01 | 1.00000D+01 | 3.60000D+01 | 0.00000D+00 |
| 20 | LOWER | 1.05953D+01 | 7.50000D+00 | 3.60000D+01 | 0.00000D+00 |
| 21 | | 1.01235D+01 | 1.00000D+00 | 2.50000D+01 | 0.00000D+00 |
| 22 | LOWER | 1.30953D+01 | 1.00000D+01 | 3.60000D+01 | 0.00000D+00 |
| 23 | LOWER | 1.96429D+01 | 1.50000D+01 | 5.00000D+01 | 0.00000D+00 |
| 24 | | 4.18404D+00 | 1.00000D+00 | 2.00000D+01 | 0.00000D+00 |
| 25 | LOWER | 1.20960D+01 | 1.00000D+01 | 3.60000D+01 | 1.89532D+04 |
| 26 | LOWER | 9.59603D+00 | 7.50000D+00 | 3.60000D+01 | 1.52994D-01 |
| 27 | | 8.53800D+00 | 1.00000D+00 | 2.50000D+01 | 0.00000D+00 |
| 28 | LOWER | 1.20960D+01 | 1.00000D+01 | 3.60000D+01 | 7.36755D-05 |
| 29 | LOWER | 1.81440D+01 | 1.50000D+01 | 5.00000D+01 | 0.00000D+00 |
| 30 | | 3.08853D+00 | 1.00000D+00 | 2.00000D+01 | 0.00000D+00 |

COST FUNCTION AT OPTIMUM = 1.895316D+04

NO. OF CALLS FOR COST FUNCTION EVALUATION (USERMF) .................. = 575
NO. OF CALLS FOR EVALUATION OF COST FUNCTION GRADIENT (USERMG) ..... = 0
NO. OF CALLS FOR CONSTRAINT FUNCTION EVALUATION (USERCF) ............ = 575
NO. OF CALLS FOR EVALUATION OF CONSTRAINT FUNCTION GRADIENTS (USERCG) = 0
NO. OF TOTAL GRADIENT EVALUATIONS .................................. = 610

CPU TIME = 203.140 SECONDS

# NOTATION

$a$ = depth of the equivalent rectangular stress block

$A$ = cross-sectional area of a member; concrete area surrounding one bar

$\mathbf{A}$ = vector of end actions for all members in a frame

$A_{bf}$ = surface area of formwork for a beam

$\mathbf{A}_C$ = vector of combined nodal loads

$A_{cf}$ = surface area of formwork for a column

$\mathbf{A}_E$ = vector of equivalent nodal loads

$\mathbf{A}_F$ = vector of actions at free nodes

$\mathbf{A}_{FC}$ = vector of combined nodal loads corresponding to $\mathbf{D}_F$

$A_{gb}$ = gross cross-sectional area of a beam

$A_{gc}$ = gross cross-sectional area of a column

$\mathbf{A}_M$ = matrix of member end actions

$\mathbf{A}_{ML}$ = matrix of fixed-end actions, with respect to member-oriented axes, due to loads for all members in a frame

$\mathbf{A}_{MS}$ = matrix of fixed-end actions, in the direction of structure axes, at both ends of a member

$\mathbf{A}_N$ = vector of actions applied at nodes

$\mathbf{A}_R$ = vector of reactions at restrained nodes (supports)

$\mathbf{A}_{RC}$ = vector of combined nodal loads corresponding to $\mathbf{D}_R$

$A_s^-$ = tensile reinforcing steel area at the most critical location of negative bending moment in beams in a story

$A_{s1}^-$ = negative flexural reinforcing steel area at the left end of a span

$A_{s2}^-$ = negative flexural reinforcing steel area at the right end of a span

$A_s^+$ = positive flexural reinforcing steel area

$A_{st}$ = longitudinal reinforcing steel area in a column

$A_t$ = cross-sectional area of bars used for ties in columns

$A_v$ = cross-sectional area of bars used for stirrups in beams

$AR$ = aspect ratio

$\mathbf{b}$ = vector of design variables

$b_c$ = width of columns in a story

$b_i$ = $i$th design variable

$b_i^l$ = lower bound on the $i$th design variable

$b_i^u$ = upper bound on the $i$th design variable

$b_r$ = web width required to accommodate the steel area within standard spacing requirements

$b_w$ = width of beam web in a story

$C_1, C_2$ = constants used in developing beam design guidelines

$C_{beams}$ = cost of all beams in a frame

$C_c$ = cost of concrete per unit volume

$C_{columns}$ = cost of all columns in a frame

$C_f$ = cost of formwork per unit surface area

$Cost$ = total cost of a reinforced concrete plane frame

$C_s$ = cost of longitudinal steel, ties, and stirrups per unit weight

$d'$ = concrete cover (to center of reinforcing steel bars)

$d_b$ = effective depth of beam web in a story

$d_{b1}$ = effective depth of beam web in the current story

$d_{b2}$ = effective depth of beam web in the previous (lower) story

$d_c$ = depth of columns in a story; thickness of concrete cover measured from the tension face to center of bar closest to that face

$d_{sf}$ = diameter of flexural reinforcing steel bars

$d_{sl}$ = diameter of longitudinal reinforcing steel bars in columns

$d_t$ = diameter of bars used for ties in columns

$d_v$ = diameter of bars used for stirrups in beams

$\mathbf{D}$ = vector of end displacements for all members in a frame

$\mathbf{D}_F$ = vector of free nodal displacements

$\mathbf{D}_R$ = vector of restrained (support) nodal displacements

$e$ = column eccentricity

$E$ = modulus of elasticity

$f_c'$ = compressive strength of concrete, measured at 28 days after casting

$f_s$ = stress in tension steel

$f_y$ = yield strength of steel

$f(\,)$ = objective function

$F$ = factor used to modify fixed-end moments of members in order to account for the presence of axial forces

$g(\,)$ = inequality constraint function

$h(\,)$ = equality constraint function

$h$ = column dimension in the direction of bending, equals $d_c + d'$

$i$ = member number; story number; counting index

$I$ = moment of inertia of a member

$j$ = right end of a member

$k$ = left end of a member; number of equality constraints; $\sqrt{P/EI}$

$K$ = effective length factor

$l$ = distance over which stirrups are distributed (required)

$l_1$ = span length of the leftmost beam in a story

$l_2$ = span length of the rightmost beam in a story

$l_3$ = maximum span length among intermediate beams in a story

$l_{max}$ = distance over which stirrups are spaced with $s_{max}$

$l_{min}$ = distance over which stirrups are spaced with $s_{min}$

$L$ = length of a member

$L_b$ = length of beam between column center lines

$L_{bars}$ = length of longitudinal reinforcing steel bars in a column

$L_c$ = length of column between beam center lines

$L_{inclined}$ = length of the inclined portion of steel bars in a column

$L_n$ = clear span length

$L_{splices}$ = length of splices in a column

$L_{tie}$ = length of one tie in a column

$L_u$ = unsupported length (clear height) of a column

$L_v$ = length of one stirrup in a beam

$LL$ = service live load

$m$ = number of members in a frame; number of inequality constraints

$M_0$ = nominal moment strength for a member subjected to bending alone

$M_e$ = end moment

$M_n$ = nominal moment capacity of a cross section

$M_u$ = moment due to factored loads

$MHBW$ = member half-band width

$n$ = number of design variables; counter on the number of design variables

$n_b$ = number of beams per story

$n_c$ = number of columns per story

$n_{max}$ = maximum number of steel bars at top or bottom of a beam

$n_{min}$ = minimum number of steel bars in a single layer of tension reinforcement at top or bottom of a beam

$n_s$ = number of stirrups in a beam

$n_t$ = number of ties in a column

$N1$, $N2$ = nodes connecting left and right ends of a member, respectively

$N_s$ = number of stories in a frame

$NDFN$ = number of degrees of freedom per node, equals 3 for a plane frame

$NN$ = total number of nodes in a plane frame

$P$ = unfactored axial (concentrated) load applied to a column or a beam

$P_0$ = nominal strength of an axially loaded column with zero eccentricity

$P_n$ = nominal axial strength at a given eccentricity

$P_u$ = factored axial load at a given eccentricity

$R$ = ratio used for beam deflection constraint

$\mathbf{R}_T$ = rotation transformation matrix

$s_1, s_2, s_3, s_4$ = stability stiffness functions

$s_{max}$ = maximum spacing of stirrups

$s_{min}$ = minimum spacing of stirrups

$s_t$ = vertical spacing of ties in a column

$s_v$ = longitudinal spacing of stirrups in a beam

$S$ = spacing between frames

$\mathbf{S}$ = structure stiffness matrix

$\mathbf{S}_M$ = member stiffness matrix in the direction of member axes

$\mathbf{S}_{MS}$ = member stiffness matrix in the direction of structure axes

$\mathbf{S}_{FF}$ = stiffness submatrix corresponding to actions $\mathbf{A}_F$ due to unit displacements $\mathbf{D}_F$

$\mathbf{S}_{FR}$ = stiffness submatrix corresponding to actions $\mathbf{A}_F$ due to unit displacements $\mathbf{D}_R$

$\mathbf{S}_{RF}$ = stiffness submatrix corresponding to actions $\mathbf{A}_R$ due to unit displacements $\mathbf{D}_F$

$\mathbf{S}_{RR}$ = stiffness submatrix corresponding to actions $\mathbf{A}_R$ due to unit displacements $\mathbf{D}_R$

$u$ = $kl$

$V_{bc}$ = volume of concrete in a beam

$V_{bs}$ = volume of tensile reinforcing steel in a beam

$V_c$ = nominal shear strength provided by concrete

$V_{cc}$ = volume of concrete in a column

$V_{cs}$ = volume of longitudinal reinforcing steel in a column

$V_n$ = nominal shear strength of a cross section

$V_s$ = nominal shear strength provided by stirrups

$V_t$ = volume of lateral ties in a column

$V_u$ = shear strength produced by factored loads

$V_u^*$ = factored shear force corresponding to the amount of excess shear

$V_v$ = volume of stirrups in a beam

$WLF$ = wind load factor

$x$ = horizontal axis

$x_M$ = member-oriented horizontal axis

$x_S$ = structure-oriented horizontal axis

$y$ = vertical axis

| | | |
|---|---|---|
| $y_M$ | = | member-oriented vertical axis |
| $y_S$ | = | structure-oriented vertical axis |
| $z$ | = | out-of-plane axis; factor of the ACI Code relating to crack width |
| $z_M$ | = | member-oriented out-of-plane axis |
| $\beta_1$ | = | ratio of depth of the rectangular stress block to the distance between neutral axis and extreme compression fiber |
| $\gamma$ | = | angle from member-oriented axes to structure-oriented axes measured counterclockwise |
| $\gamma_c$ | = | unit weight of concrete |
| $\gamma_s$ | = | unit weight of steel |
| $\delta$ | = | parameter used in IDESIGN for calculation of gradients of various functions by the finite difference method. |
| $\Delta$ | = | maximum deflection |
| $\Delta_0$ | = | maximum first-order deflection caused by bending alone |
| $\Delta_a$ | = | additional deflection caused by axial load alone |
| $\rho$ | = | ratio of tension reinforcement |
| $\rho'$ | = | ratio of compression reinforcement |
| $\rho_b$ | = | reinforcement ratio producing balanced strain condition |
| $\rho_g$ | = | ratio of longitudinal reinforcement in a column |
| $\rho_{g\,max}$ | = | ACI Code maximum ratio of longitudinal reinforcement in a column |
| $\rho_{max}$ | = | ACI Code maximum ratio of tension reinforcement |
| $\rho_{min}$ | = | ACI Code minimum ratio of tension reinforcement |
| $\varphi$ | = | ACI Code strength reduction factor |
| $\varphi_c$ | = | $2 - 2\cos kL - kL\sin kL$ |
| $\varphi_t$ | = | $2 - 2\cosh kL + kL\sinh kL$ |

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