Optimal Excitation Control for Power System Stability

Abstract—**A method is presented for finding the closed-loop time optimal excitation control for a power system by applying Pontryagin's minimum principle. Numerical results are presented for single machine-infinite bus problems implementing such control.**

Introduction

Stability from the power system point of view is the ability of synchronous machines to remain in synchronism. Stability may be classified as transient or large disturbance stability and as dynamic or small disturbance stability. With the introduction of high-speed excitation of synchronous generators, it has been found [1], [2] that an auxiliary signal in addition to normal voltage regulator action is necessary to improve dynamic stability. Since the equations governing the power system dynamics are of high order, a closed-loop excitation control is difficult to obtain. A method of deriving a "time optimal control" for ensuring dynamic stability of a single machine infinite-bus power system is described here.

Analysis

For small disturbances, we get a system of linear time-invariant ordinary differential equations

$$\dot{i}_r = \sum_{q=1}^{3} a_{rq}i_q + a_{r4}\delta + b_r u(t), \qquad (r = 1,2,3) \tag{1}$$

$$\dot{\delta} = \omega_0 n \tag{2}$$

$$\dot{n} = a_{51}i_1 + a_{52}i_2 + a_{53}i_3, \tag{3}$$

where i_r $(r = 1, 2, 3)$ are the field, direct axis armature, and quadrature axis armature currents, respectively, δ is the rotor angle (rad), and n and $\dot{n}$ are the normalized velocity and acceleration of the machine, respectively. The control $u(t)$, normalized output voltage of the exciter, is assumed to be a piecewise continuous function of time and constrained in magnitude such that $|u(t)| \leq 1$. The coefficients a_{rq} $(r, q = 1, \cdots, 5)$ depend on synchronous machine and transmission line parameters and also on the operating point. Equations (2) and (3) are known as *swing equations*.

For stable operation of the generator, n and $\dot{n}$ should decrease to zero in minimum time following a disturbance, while the rotor angle δ should remain between 0 and $\pi/2$. Since for small disturbances, rotor angle will not exceed this range, final rotor angle can be considered free.

The optimization problem can be stated as: Find the admissible control $u(t)$ that transfers the system (1)–(3) from the set of given initial states $\{i_1(0), i_2(0), i_3(0), \delta(0), n(0)\}$ to the desired final states $\{n_{(tf)} = 0;\ i_{1(tf)}, i_{2(tf)}, i_{3(tf)}, \delta_{(tf)}$ free$\}$ so as to minimize the cost functional

$$J = \int_{t_0}^{t_f} dt. \tag{4}$$

It is also desired that $\dot{n}(t_f) = 0$. The terminal time $t_f > t_0$ is free.

To the best knowledge of the authors, the available techniques for determining the time optimal control for this system involves iteration, and hence are costly in terms of computing time and implementation.

The disturbances that appear in a power system are generally not known in advance. Some of the coefficients a_{rq} $(r, q = 1, \cdots, 5)$ depend on system resistance, reactance, and also on receiving end voltage. These have to be measured immediately after a disturbance appears on the system. Since the control must act within a very small fraction of a second to stop first-swing instability for large disturbances, a control obtained by standard optimization techniques is not implementable. On small disturbances this time limitation is not as severe. A method of determining the time optimal control directly as a function of the states and other measurable quantities is proposed here.

Manuscript received November 23, 1970; revised January 25, 1971.

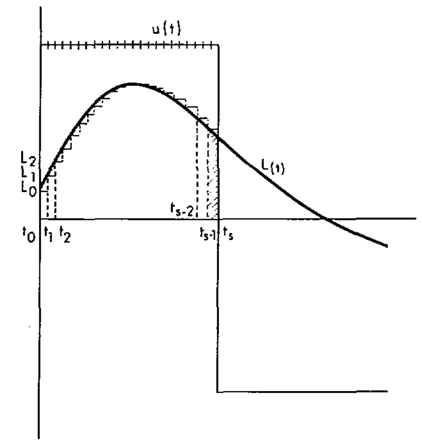

Fig. 1. A portion of $L(t)$ and optimal control $u(t)$.

Step 1: Differentiating (3) with respect to time and substituting (1) in it, we get

$$\begin{aligned}\ddot{n} = {} & a_{51}(a_{11}i_1 + a_{12}i_2 + a_{13}i_3 + a_{14}\delta + b_1u(t)) \\ & + a_{52}(a_{21}i_1 + a_{22}i_2 + a_{23}i_3 + a_{24}\delta + b_2u(t)) \\ & + a_{53}(a_{31}i_1 + a_{32}i_2 + a_{33}i_3 + a_{34}\delta)\end{aligned} \tag{5}$$

or

$$\begin{aligned}\ddot{n} = {} & [(a_{51}a_{11} + a_{52}a_{21} + a_{53}a_{31})i_1 + (a_{51}a_{12} + a_{52}a_{22} + a_{53}a_{32})i_2 \\ & + (a_{51}a_{13} + a_{52}a_{23} + a_{53}a_{33})i_3 + (a_{51}a_{14} + a_{52}a_{24} + a_{53}a_{34})\delta] \\ & + [(a_{51}b_1 + a_{52}b_2)u(t)].\end{aligned} \tag{6}$$

a_{51}, a_{52}, and a_{53} are constants and $b_3 = 0$ for the case considered.

Equation (6) is rewritten as

$$\ddot{n} = L(i_1,i_2,i_3,\delta) + bu(t). \tag{7}$$

Since arguments of L change with respect to time, we write

$$\ddot{n} = L(t) + bu(t) \tag{8}$$

where we assume that

$$|L(t)/b| \leq 1. \tag{9}$$

Step 2: With the known initial values of i_1, i_2, i_3, and δ, we calculate initial value of $L(t)$, say L_0. We assume $L(t)$ will remain constant at this value (Fig. 1). The optimization problem can be restated as follows.

Given the system

$$\ddot{n} = u_1(t), \qquad u_{1\,\min} \leq u_1(t) \leq u_{1\,\max} \tag{10}$$

where

$$u_1(t) = L_0 + bu(t), \tag{10a}$$

find the admissible control that transfers system (8) from the given initial states N_0 (in the $n - \dot{n}$ plane) to the origin in the minimum possible time. The switch curves (Fig. 2) for the double integral plant [3] is given by

$$\gamma^0 = \left\{(n,\dot{n}) \quad : \quad n - \frac{\dot{n}^2}{2[L_0 + b \operatorname{Sgn}\{\dot{n}\}]} = 0, \qquad b < 0\right\}. \tag{11}$$

Step 3: The switch curve (11) decides the optimal dummy control $u_1(t)$ which in turn gives optimal $u(t)$ by relation (10a). The control $u(t)$ is used to solve the system of (1)–(3) for a small increment of time. At the end of the first interval, the value of $L(t)$ is recalculated

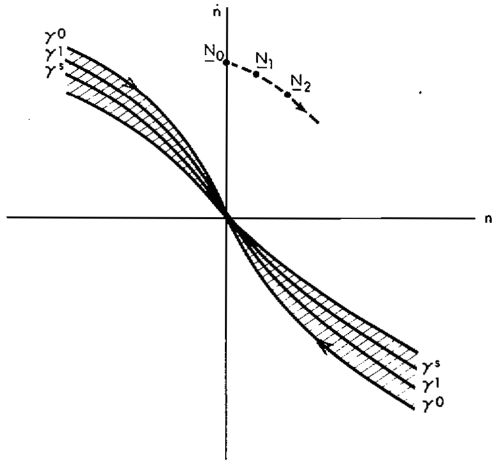

Fig. 2. Moving switch curves in the phase plane.

using the new value of i_1, i_2, i_3, and δ. Suppose that the states of system (8) at $t = t_1$ are N_1 and actual value of $L(t)$ has changed from L_0 to L_1.

Step 4: The time optimal control is now found again using the constant value of L_1 for $L(t)$. The process is continued until the desired final states are reached. At $t = t_p$, the switch curve is given for $L(t) = L_p$ by

$$\gamma^p = \left\{(n,\dot{n}) \;:\; n - \frac{\dot{n}^2}{2[L_p + b \operatorname{Sgn}\{\dot{n}\}]} = 0, \qquad b < 0\right\}. \quad (12)$$

Note that if at any stage $L(t)$ does not change from the previous value, it is not necessary to recalculate γ.

If we consider $L(t)$ $[L(i_1, i_2, i_3, \delta)]$ we find that $L(t)$ is not an explicit function of n. So for a hypothetical double integral plant (8), $L(t)$ can be considered as an additive disturbance to the input $bu(t)$. Oldenburger [4] considered a second-order system similar to (8) with an arbitrary disturbance $L(t)$, and obtained the switching curve (12) L_p $(p = 1, 2, \cdots, P)$ replaced by $L(t)$ with the help of geometrical construction. He remarks that if the term $L(t)$ does not satisfy inequality (8), then the system (8) is uncontrollable. However, he concludes that if (8) is not satisfied for only a small period of time followed by a long controllable section, the scheme will give a suboptimal solution that also will converge. Solution of system (8) with the control scheme obtained by (12) would be suboptimal since $L(t)$ is assumed constant in each subinterval. But (12) shows that only the value of $L(t)$ in the interval $\{t_{s-1}, t_s\}$ (Fig. 1) is needed to determine the switching of the optimal control $u(t)$. Since this is not known in advance, all the switch curves $\gamma^0, \gamma^1, \cdots, \gamma^{s-1}$ are to be calculated. The time-optimal control for system (1)–(3) (which is normal) is bang-bang (by Pontryagin's minimum principle), i.e., except at the switch points where the control is $+1$ or -1. The value of $L(t)$ helps determine the sign of the control at sample points t_0, t_1, etc. So in Fig. 1, irrespective of the variation of $L(t)$ over the subintervals $\{t_0, t_1\} \cdots \{t_{s-2}, t_{s-1}\}$, the time optimal control is $+1$. The correct switching might occur anywhere in the interval $\{t_{s-1}, t_s\}$ due to the variation in $L(t)$. But since the subintervals can be made as small as numerical integration procedures would allow with sufficient accuracy, the error involved in switching is negligible. Again, the solution is time optimal over each subinterval so it will be optimal over the whole interval [5].

Fig. 2 shows a series of switch curves limited to a small region for the variation of $L(t)$. First switching of $u(t)$ occurs when the trajectory reaches γ^s. Fig. 3 shows a deadbeat type of response obtained for a step increase of input torque by the use of the control scheme derived. The system is stabilized in 0.34 s and the control switches twice. Simulation of the system by Runge–Kalta integration

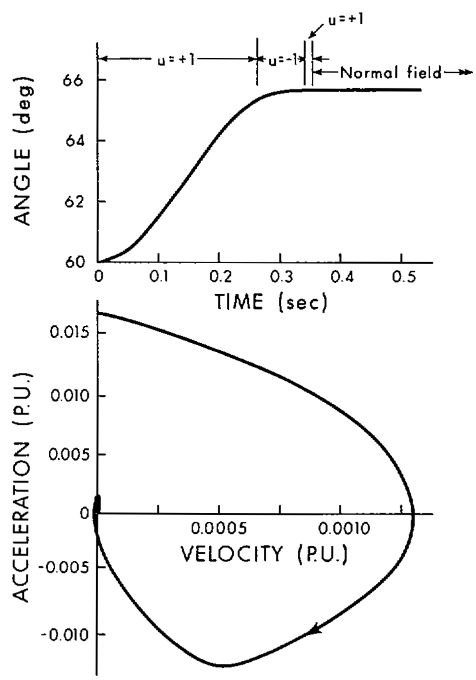

Fig. 3. Linearized system, 10 percent torque step. (a) Rotor angle-time characteristics. (b) Phase plane.

with a step size of 0.0005 s required about 0.85 min of computation time on an IBM/360. In application this, of course, is not needed.

Conclusions

This correspondence shows that a closed-loop optimal excitation control can be realized which is fast and promises considerable advantage in the improvement of reliability and reduction of costs in a power system. The idea may be extended to the nonlinear system also.

Acknowledgment

The authors wish to extend thanks to Dr. V. Gourishankar for his advice.

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A Numerical Solution of the Matrix Equation $P = \phi P \phi^t + S$

***Abstract*—A simple solution for the above matrix equation is given for the case when ϕ is in companion form. If ϕ is not in companion form, then only 5 2/3 n^3 multiplications are necessary for the required transformations and equation solution. Thus this correspondence offers an efficient computational alternative to methods already published.**

The matrix equation $P = \phi P \phi^t + S$, with P and S positive definite matrices, has important applications in the design of linear discrete systems [1]–[3] and thus a number of papers have been published on its numerical solution [4]–[6].

A direct solution can be obtained by rewriting the equation as $n(n + 1)/2$ linear equations in the unknown elements of the symmetric matrix P

$$Ap = s \tag{1}$$

where p is a vector of the unknown elements written in the order

$$p^t = p_{11}, p_{12}, \cdots, p_{1n}, p_{22}, p_{23}, \cdots, p_{2n} \cdots, p_{nn},$$

s is a vector of the corresponding elements of S, and A is an $n(n+1)/2 \times n(n+1)/2$ matrix formed from products of the elements of the ϕ matrix. This matrix is conveniently represented by the partitioned matrix

$$A = \begin{bmatrix} A^{11} & A^{12} & A^{13} \cdots A^{1n} \\ A^{21} & A^{22} & \quad A^{2n} \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ A^{n1} & \cdots & \cdots A^{nn} \end{bmatrix} \tag{2}$$

where the dimension of each A^{IJ} submatrix is $(n - I + 1) \times (n - J + 1)$. The elements $a_{l,m}{}^{IJ}$ of the submatrix A^{IJ} are given by the formulas

$$a^{IJ}{}_{1+i-I,1+j-J} = (\phi_{IJ}\phi_{ij} + \phi_{Ij}\phi_{iJ})K_{I-i,J-j}, \qquad i = I \rightarrow n, \quad j = J \rightarrow n \tag{3}$$

where

$$K_{I-i,J-j} = 1/2, \qquad I = i \text{ or } J = j \text{ (when the two terms in the brackets are equal)}$$
$$= 1, \qquad \text{otherwise.}$$

The method is, however, impractical with large systems as it requires $n^4/4$ storage words; thus an iterative solution given by

$$P_{2k} = \phi^k P_k (\phi^t)^k + P_k \tag{4}$$

is usually resorted to. This has been used extensively by the Cambridge Control Group. Experience has shown that satisfactory convergence is usually obtained within 12 iterations (requiring more than $12n^3$ multiplications) but some problems have required as many as 35 (iterations were terminated when $\|P - P_k\|/\|P\| < 0.001$).

The direct method becomes more attractive if the ϕ matrix is in canonical form; the resulting A matrix will then be sparse, requiring little storage space and few multiplications for solution.

The companion form is attractive as it can be obtained by a series of elementary tranformations (using the method of Danilevsky [7]). An efficient method of utilizing the companion form has been given by Molinari [6]. An alternative method that may in some cases, be more numerically robust follows.

If ϕ is a companion matrix

$$\phi = \begin{bmatrix} 0 & 0 & 0 & \cdot & \cdot & \cdot & \alpha_1 \\ 1 & 0 & \cdot & \cdot & \cdot & & \alpha_2 \\ 0 & 1 & 0 & & & & \cdot \\ \cdot & & \cdot & & & & \cdot \\ \cdot & & & \cdot & & & \cdot \\ \cdot & & & & \cdot & & \cdot \\ 0 & \cdot & \cdot & \cdot & & 1 & \alpha_n \end{bmatrix},$$

then the A matrix in (2) is of the form shown in Fig. 1 for a 5×5 matrix. Thus only $n(n + 1)/2$ multiplications and additional words of storage are required. Moreover, this matrix is easily triangularized into an upper triangular matrix of the partitioned form

$$U = \begin{bmatrix} U^{11} & \cdot & \cdot & \cdot & U^{1n} \\ 0 & U^{22} & & & U^{2n} \\ 0 & 0 & U^{33} & & U^{3n} \\ & & & \cdot & \cdot \\ & & & & \cdot \\ & & & & \cdot \\ 0 & & & & U^{nn} \end{bmatrix} \tag{5}$$

where

$$U^{II} = \begin{bmatrix} 1 & 0 & \cdot & \cdot & \cdot & 0 & u_I{}^{II} \\ & 1 & 0 & & & & u_{I+1}{}^{II} \\ & & & & & & \cdot \\ & & & & & & \cdot \\ & & & & & & \cdot \\ & & & & & & u_{n-1}{}^{II} \\ & & & & & & 1 \end{bmatrix}$$

and for $J > I$

$$U^{IJ} = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 & u_I{}^{IJ} \\ 0 & & & & & & u_{I+1}{}^{IJ} \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ 0 & \cdot & \cdot & \cdot & & & u_n{}^{IJ} \end{bmatrix}.$$

The elements of the partitioned matrices may be obtained from the sequential formulas.

For $I = 1$, $J = 1 \rightarrow n$, $i = 1 \rightarrow n$

$$u^{1J} = -Q(1,i)\alpha_1$$

where

$$Q(1,i) = 1, \qquad J = i - 1$$
$$= \alpha_i, \qquad J = n.$$

For $I = 2 \rightarrow n$, $J = I \rightarrow n$, $i = I \rightarrow n$

$$u^{IJ} = d_i{}^{IJ} L(I,i) \tag{6}$$

where

$$d_i{}^{IJ} = u_{i-1}{}^{I-1,J} + u_n{}^{I-1,J}(\alpha_i - u_{i-1}{}^{I-1,I-1}) - Q(I,i)\alpha_I$$

Manuscript received January 20, 1971.