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Prediction of Evaporation Losses in Wet Cooling Towers

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The accurate prediction of all aspects of cooling tower behavior is very important. Accurately predicting evaporation losses is significant because water in cooling towers is cooled primarily through the evaporation of a portion of the circulating water, which causes the concentration of dissolved solids and other impurities to increase. An empirical relation is developed on the basis of ASHRAE's rule of thumb that is simple and accurate with a wide range of applicability. The predicted values are in good agreement with experimental data as well as predictions made by an accurate mathematical model.

INTRODUCTION

Water is commonly used as a heat transfer medium to remove heat; however, water purchased from utilities for use has become expensive in certain areas where it is scarce. Cooling towers are designed to cool a warm water stream mainly through the evaporation of some of the water into an air stream. The water consumption rate of a wet cooling tower system is only about 5% that of a once-through system, making it the least expensive system to operate with purchased water supplies. Furthermore, the ecological effect is reduced because the blowdown is very small [1]. A schematic of a counter-flow wet cooling tower is shown in Figure 1. There are three ways water is lost in a cooling tower: drift, blowdown (also called *bleed off*), and evaporation, which is the most significant of the three. Generally, an efficient eliminator can reduce drift loss to a range of 0.002–0.2% of the water circulation rate [1], and blowdown is typically taken to be 0.5–1% of the water circulation rate to maintain the allowed level of concentration. Cooling tower users are interested to know the amount of water lost under the changing operational conditions, but charts for calculating the makeup water for a wide range of operational conditions are not commonly used. Furthermore, only a few references or standards (e.g., British Standard 4485) contain charts or data that make such predictions possible. According to ASHRAE [1], the rule of thumb is that evaporation loss averages approximately 1% for each 7°C (12.6°F) drop in water temperature. Gosi [2] constructed a chart that can be used

to estimate this loss. It is accurate as well as simple, as no difficult calculations are required, though it is difficult to use on site as it is in the form of a chart. Furthermore, being in graphical form, it requires some time to carry out the calculation, as there are chances of error.

The objective of this paper is to obtain an empirical equation that would enable users of wet cooling towers to predict evaporation loss more accurately, as compared to ASHRAE's rule of thumb, but which is also simple enough to avoid the use of tables or charts.

WET COOLING TOWER MODEL

A mathematical model for wet cooling towers presented here will be used to establish limitations of the proposed empirical equation. The control volume of a counter-flow cooling tower showing the important states is presented in Figure 2. The major assumptions, which are used to derive the basic modeling equations, are summarized in [3, 4].

From steady-state energy and mass balances on an incremental control volume (see Figure 2), one gets [4]

$$\dot{m}_a dh = -[\dot{m}_w - \dot{m}_a(W_o - W)]dh_{f,w} + \dot{m}_a dW h_{f,w} \quad (1)$$

The water energy balance can also be written in terms of the heat- and mass-transfer coefficients, h_c and h_D, respectively, as

$$-\dot{m}_w dh_{f,w} = h_c A_V dV(t_w - t_{db}) + h_D A_V dV(W_{s,w} - W)h_{fg,w} \quad (2)$$

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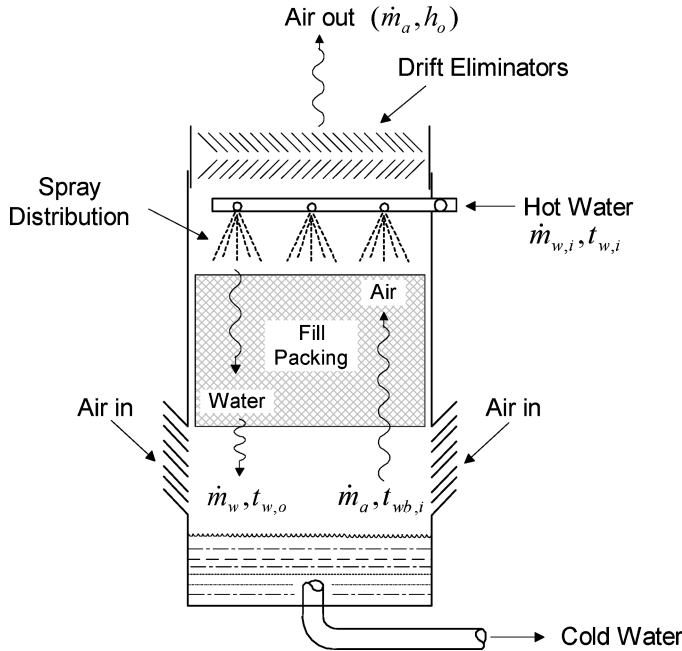
Figure 1 Schematic of a counter-flow wet cooling-tower.

and the air-side water-vapor mass balance as

$$\dot{m}_a dW = h_D A_V dV (W_{s,w} - W) \qquad (3)$$

By substituting the Lewis factor as $Le = h_c/h_D c_{pa}$ in Eq. (2), one gets (after some simplification)

$$-\dot{m}_w dh_{f,w} = h_D A_V dV [Le\, c_{pa}(t_w - t_{db})$$
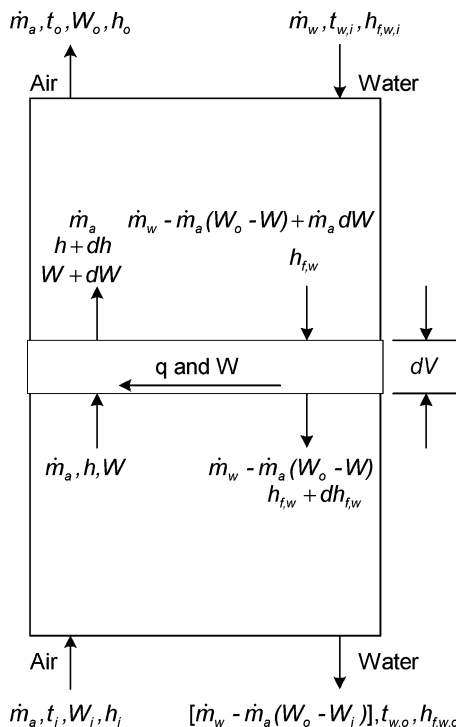$$+ (W_{s,w} - W) h_{fg,w}] \qquad (4)$$

Figure 2 Mass and energy balance of a counter-flow wet cooling-tower.

It should be noted that the Lewis factor is similar to what is used by Braun et al. [5] and Kuehn et al. [3]. Combining Eqs. (1–4), one gets, after some simplification [4]

$$\frac{dh}{dW} = Le\frac{(h_{s,w} - h)}{(W_{s,w} - W)} + \left(h_{g,w} - h_g^0 Le\right) \qquad (5)$$

Equation (5) describes the condition line on the psychrometric chart for the changes in state for moist air passing through the tower. For given water temperatures ($t_{w,i}$, $t_{w,o}$), the Lewis factor (Le), inlet condition of air, and mass flow rates, Eqs. (1), (3), and (5) may be solved numerically for exit conditions of both the air and water streams.

A computer program is written in Engineering Equation Solver (EES) for solving Eqs. (1), (3), and (5). In this program, the properties of the air-water vapor mixture are needed at each step of the numerical calculation. These are obtained from the built-in functions provided in EES. The program gives the dry-bulb temperature, wet-bulb temperature, and humidity ratio of the air as well as the water temperature at each step of the calculation, starting from air-inlet to air-outlet values.

The correlations for heat and mass transfer of cooling towers in terms of physical parameters are not easily available. It is typical to correlate the tower performance data for specific tower designs. For instance, mass transfer data are typically correlated in the form ASHRAE [6]:

$$\frac{h_D A_V V}{\dot{m}_{w,i}} = c \left(\frac{\dot{m}_{w,i}}{\dot{m}_a}\right)^n \qquad (6)$$

where c and n are empirical constants specific to a particular tower design. Multiplying both sides of the above equation by ($\dot{m}_{w,i}/\dot{m}_a$) and considering the definition for NTU gives the empirical value of NTU as

$$NTU_{em} = \frac{h_D A_V V}{\dot{m}_a}\bigg|_{em} = c \left(\frac{\dot{m}_{w,i}}{\dot{m}_a}\right)^{n+1} \qquad (7)$$

The coefficients c and n of the above equation were fit to the measurements of Simpson and Sherwood [7] for four different tower designs over a range of performance conditions given by Braun et al. [5]. In the present calculations, the correlation coefficients for the tower with $c = 1.13$ and $n = -0.612$ were used. In this regard, Eq. (7) is used to calculate the NTU from which the mass transfer coefficient ($h_D A_V$) is determined for the tower specifications.

EMPIRICAL EQUATION

From cooling tower theory [3, 8], ASHRAE [1], and the experience of engineers, it is understood that there are three important quantities involved in evaporation loss within wet cooling towers: the first is the range (i.e., $\Delta t_w = t_{w,i} - t_{w,o}$), the second is the potential of the air to absorb water (i.e., $\Delta t_a = t_{db,i} - t_{wb,i}$) and the third is the maximum possible temperature difference observed in a cooling tower (i.e., $\Delta t_{max} = t_{w,i} - t_{wb,i}$). It is important in this regard to understand that evaporation occurs as

the water cools from the inlet water temperature to the outlet water temperature. The lowest possible temperature that the water can achieve is the inlet wet-bulb temperature, which is currently governed by the inlet dry-bulb temperature and relative humidity. Therefore, the maximum potential for evaporation lies in the difference between the inlet water and inlet wet-bulb temperatures ($t_{w,i} - t_{wb,i}$). For any fixed value of the relative humidity, a higher dry-bulb temperature yields a higher wet-bulb temperature, which clearly indicates a smaller potential for evaporation.

The theoretical and laboratory modeling of wet cooling towers performed by Poppe [9] is considered to be one of the most comprehensive works on this topic. In his experiments, the evaporation loss was determined by measuring the change of water volume over a long period of time precisely using a special laboratory counterflow device. Using Poppe's [9] experimental data of percentage evaporation loss, a regression equation is obtained as a function of the above-mentioned three quantities. The linear regression equation, which fits 99.84% of the experimental data, is given by

$$E = -0.02982 + 0.1665\Delta t_w - 0.006334\Delta t_{max} + 0.009501\Delta t_a \tag{8}$$

It can be seen from Figure 3 that the regression equation provides excellent agreement with the experimental data. In the above regression equation, the 2nd term (on the right hand side) closely represents ASHRAE's rule of thumb. Furthermore, on examining the coefficients, the term associated with each potential indicates that it is also the most important quantity involved, thus verifying the validity of the rule of thumb as well as its importance in evaporation loss prediction.

Now, a regression equation based on the "range" alone is obtained. Again, we can see, from Figure 4, the good agreement between predicted and measured values. The regression
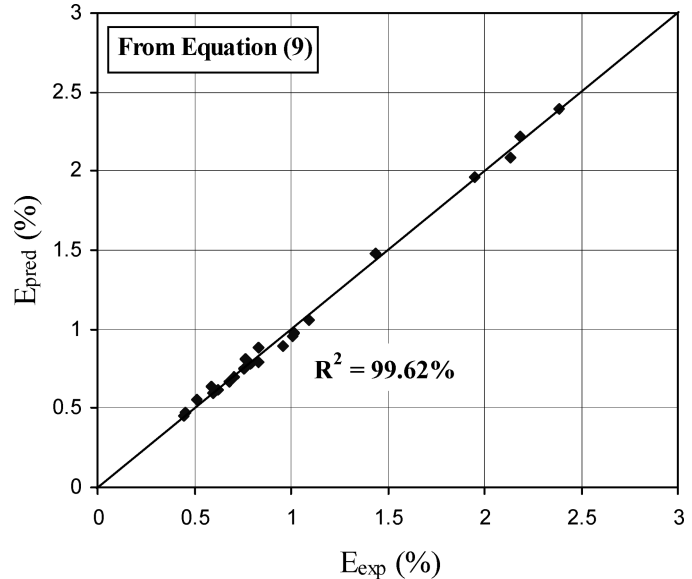

Figure 4   Measured [9] versus predicted values using Eq. (9).

equation, which fits 99.62% of the experimental data, is given by:

$$E = -0.00849 + 0.1544\Delta t_w \tag{9}$$

The small difference of the coefficient of the determination (R^2) value in Figures 3 and 4 assert our understanding that ASHRAE's rule of thumb provides a sound basis for any empirical relation that should be developed to predict percentage evaporation loss.

Comparing the coefficients of Eq. (8), it is evident that Δt_a is more important compared to Δt_{max}. From the physics of the problem, it is apparent that Δt_a has greater significance because it represents the potential of the air to absorb water compared to Δt_{max}, which represents the maximum potential for evaporation. Thus, the latter, in contrast to the former, gives a maximum possible potential that is never completely utilized because of the operating air condition. Keeping this in mind, the following empirical relation was developed, noting that it should be accurate as well as simple so that it can be used by engineers without having to use tables or charts:

$$E = \frac{\Delta t_w}{7 - \left(\Delta t_a^{1.1}/\Delta t_{max}\right)} \tag{10}$$

It should be emphasized that in the denominator of Eq. (10), the term in brackets acts as a correction factor to ASHRAE's rule of thumb [1]. Figure 5 shows that the above empirical relation provides a good prediction of experimental measurements [9] of percentage evaporation loss with a maximum error of 6.6%. Similarly, Figure 6 illustrates the accuracy of Gosi's chart [2] where a maximum error of 6.1% was found.

Table 1 quantifies the error in ASHRAE's rule of thumb [1], Gosi's chart [2], and Eq. (10) when compared to measurements carried out by Poppe [9]. It can be seen from the table that predictions of the empirical relation are almost always more accurate than ASHRAE's rule of thumb, which gives a maximum
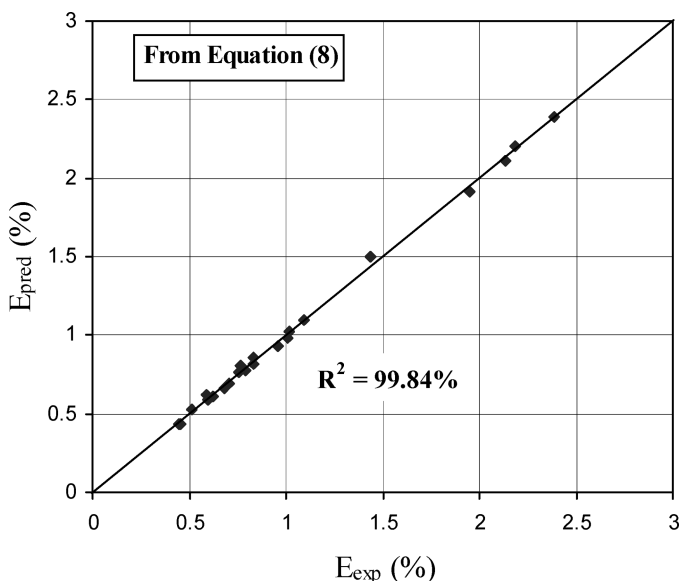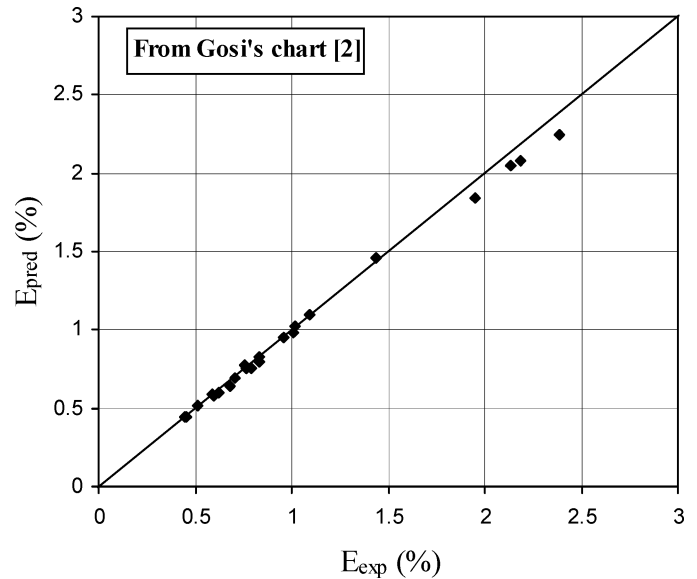

Figure 3   Measured [9] versus predicted values using Eq. (8).

Table 1 Experimental [9] and predicted values of percentage evaporation loss

Serial #	Poppe [9]	ASHRAE	$Error_A$ (%)	Gosi [2]	$Error_G$ (%)	Eq. (10)	$Error_{10}$ (%)
1	0.756	0.697	7.79	0.78	−3.17	0.786	−3.92
2	0.954	0.831	12.85	0.95	0.42	0.945	0.94
3	2.134	1.941	9.02	2.05	3.94	2.068	3.08
4	1.013	0.914	9.74	1.02	−0.69	1.028	−1.51
5	1.436	1.373	4.40	1.46	−1.67	1.487	−3.54
6	1.006	0.889	11.67	0.98	2.58	0.991	1.52
7	0.836	0.736	12.00	0.83	0.72	0.834	0.25
8	1.096	0.986	10.06	1.1	−0.36	1.104	−0.72
9	0.454	0.443	2.45	0.45	0.88	0.468	−3.11
10	0.594	0.563	5.24	0.58	2.36	0.594	0.01
11	0.445	0.429	3.69	0.44	1.12	0.453	−1.84
12	0.514	0.523	−1.72	0.52	−1.17	0.542	−5.47
13	0.679	0.626	7.85	0.64	5.74	0.660	2.74
14	0.585	0.600	−2.56	0.59	−0.85	0.624	−6.61
15	1.95	1.821	6.59	1.84	5.64	1.844	5.41
16	0.833	0.829	0.53	0.8	3.96	0.851	−2.19
17	2.187	2.064	5.61	2.08	4.89	2.090	4.43
18	0.623	0.577	7.36	0.6	3.69	0.612	1.84
19	2.386	2.223	6.84	2.24	6.12	2.252	5.63
20	0.703	0.653	7.13	0.69	1.85	0.698	0.72
21	0.761	0.759	0.32	0.76	0.13	0.793	−4.22
22	0.79	0.726	8.14	0.76	3.80	0.774	1.97

error of 12.85%. Furthermore, it is found that the predictions of Eq. (10) are more accurate than Gosi's chart in half of the values that have been compared. Thus, it can be considered to be as good as Gosi's chart but with the added advantages of requiring only a calculator and a very small amount of time. In general, Eq. (10) seems to offer an overall advantage compared to other methods.

It should be emphasized that the experimental data [9] involved a mass flow ratio not greater than ∼0.55, where low

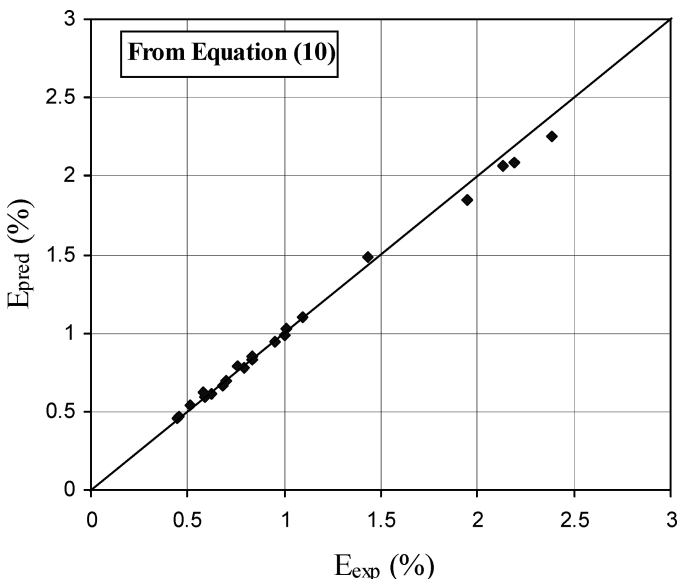**Figure 5**   Measured [9] versus predicted values using Eq. (10).

**Figure 6**   Measured [9] versus predicted values from Gosi's chart [2].

and moderate dry-bulb temperatures (18.65–26.33°C) were involved. Thus, to further ascertain its validity as well as to establish its limitations at higher dry-bulb temperatures (40–50°C range) and different mass flow ratios, the cooling tower model, presented in the previous section, was used to generate evaporation loss values. These were then compared to predictions made by ASHRAE's rule of thumb and Eqs. (8–10).

Figures 7a to 7d are drawn for the following set of input data for a medium-sized tower given in Kuehn et al. [3]: $t_{w,i} = 50°C$, $V = 203.2$ m^3, $Le = 0.9$ and $\dot{m}_a = 93.99$ kg$_a$/s. The water to air mass flow ratio is an important factor as it affects all aspects of the tower performance. Therefore, the calculations are performed for two different flow ratios, 0.5 and 1. Also, depending on the location, the condition of the ambient air can vary from very dry to moist. Therefore, the relative humidity of the incoming air is varied from 0.1 to 0.7 to ascertain the limitation of Eq. (10). It is seen from these figures that Eq. (10) is consistently more accurate compared to ASHRAE's rule of thumb in predicting water loss due to evaporation.

The degree of greatest error is found to be 15% in some extreme cases (combination of high dry-bulb temperature, high relative humidity, and low mass flow ratio), but in a great number of cases, the error is much less. However, in all of these extreme cases, the error of ASHRAE's rule of thumb is calculated to be around 30%. Similarly, it is seen from these figures that Eq. (10) is almost always more accurate compared to Eq. (9), in which the latter gave a maximum error of 25% for extreme cases. Now, when Eq. (10) is compared to Eq. (8), it seems that the former predicts better than the latter except for when the inlet dry-bulb temperature is 40°C. It is noted from Figures 7a to 7d that the investigated cooling range is 5–29°C. Furthermore, it is seen that for the case of incoming moist air ($\phi = 0.7$), predictions made by Eq. (10) are in good agreement with those of the mathematical model, where the maximum calculated error is 15% for the same
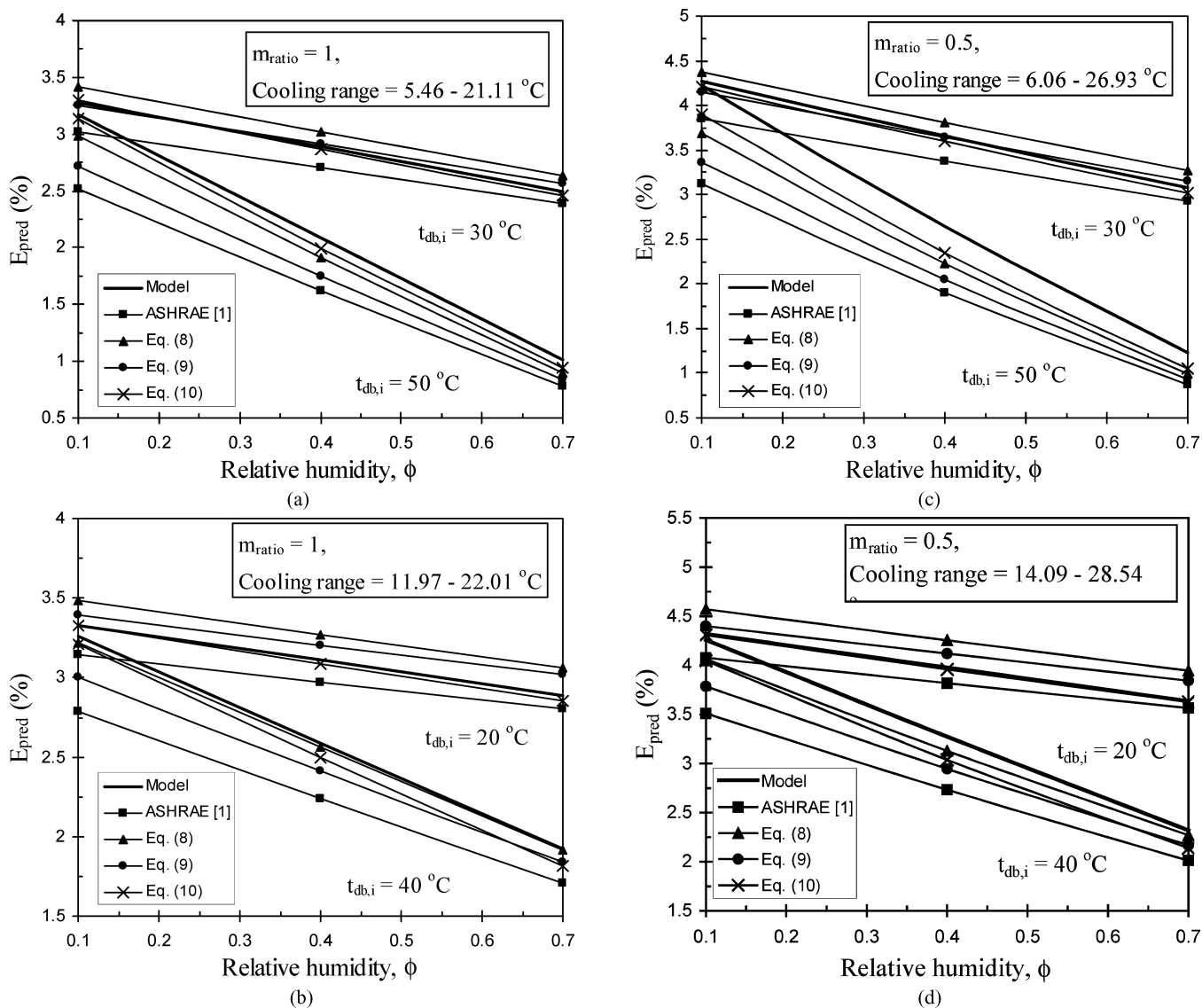
Figure 7 Comparison of evaporation prediction for different operating conditions: (a) mass flow ratio of 1 and dry-bulb temperatures of 50 and 30°C; (b) mass flow ratio of 1 and dry-bulb temperatures of 40 and 20°C; (c) mass flow ratio of 0.5 and dry-bulb temperatures of 50 and 30°C; (d) mass flow ratio of 0.5 and dry-bulb temperatures of 40 and 20°C.

extreme cases previously discussed. In general, the error is much less. For 5–10°C dry-bulb temperature of the entering air (which is not presented in these figures), the relative error in some cases is found to be higher than 20%. Also, Eq. (10) did not predict better than ASHRAE's rule of thumb in the case of cooler dry-bulb temperatures.

CONCLUDING REMARKS

Based on ASHRAE's rule of thumb, the empirical relation developed is comparatively more accurate as well as simpler. It is also very convenient because it only requires the use of a calculator at the site, takes little time, and is valid for a wide range of operating conditions, including the range that is important for

the estimation of evaporative loss. The predicted values obtained from this relation are compared with experimental data as well as with numerical values calculated from an accurate model of cooling towers, which is solved by using the Engineering Equation Solver (EES) program to predict evaporation losses in a medium-sized tower. In both cases, the predictions are in excellent agreement. For accurate prediction, the range of application should be limited so that the smallest cooling range is greater than 5°C, the maximum value of the inlet air relative humidity is 0.7, the dry-bulb temperature of the incoming air is between 20 and 50°C, and the water to air mass flow ratio is between 0.5 and 1.

It is important to note that if the dry-bulb temperature of the entering air is in the above range, the suggested formula produces a very good approximation. The degree of greatest error is found

to be 15% in some extreme operating conditions, but in a great number of cases, the error is much less. On the other hand, in all these extreme cases, the error of ASHRAE's rule of thumb is calculated to be around 30%. In fact, for 5–10°C dry-bulb temperature of the entering air, the relative error in some cases, by using the proposed equation, is found to be higher than 20%. Therefore, the authors believe that it is best to exclude this from the range of application. In the case of higher values of the mass flow rate ratio and cooler dry-bulb temperatures of the incoming air, further investigation is required, as this will add to the general application of the empirical equation.

NOMENCLATURE

A_V	surface area of water droplets per unit volume of the tower, m^2/m^3
c	empirical constant specific to a particular tower design (Eq. 7)
c_p	specific heat at constant pressure, kJ/kg K
E	percentage of water evaporated, %
$Error_A$	percentage error between Poppe's data [9] and ASHRAE's rule of thumb, %
$Error_G$	percentage error between Poppe's data [9] and Gosi [2], %
$Error_{10}$	percentage error between Poppe's data [9] and Eq. (10), %
h	specific enthalpy of moist air, kJ/kg_a
h_c	convective heat transfer coefficient of air, $kW/m^2 K$
h_D	convective mass transfer coefficient, $kg_w/m^2 s$
$h_{f,w}$	specific enthalpy of water evaluated at t_w, kJ/kg_w
$h_{fg,w}$	change-of-phase enthalpy $(h_{fg,w} = h_{g,w} - h_{f,w})$, kJ/kg_w
$h_{g,w}$	specific enthalpy of saturated water vapor evaluated at t_w, kJ/kg_w
h_g^0	specific enthalpy of saturated water vapor evaluated at 0°C, kJ/kg_w
$h_{s,w}$	specific enthalpy of saturated moist air evaluated at t_w, kJ/kg_w
Le	Lewis factor $(Le = h_c/h_D c_{p,a})$
m_{ratio}	mass flow ratio $(= \dot{m}_{w,i}/\dot{m}_a)$
$\dot{m}$	mass flow rate, kg_a/s
n	empirical constant specific to a particular tower design (Eq. 7)
NTU	number of transfer units $(= h_D A_V V/\dot{m}_a)$
q	heat transfer rate, kW
R^2	coefficient of determination
t	temperature, °C
V	volume of tower, m^3
W	humidity ratio of moist air, kg_w/kg_a
$W_{s,w}$	humidity ratio of saturated moist air evaluated at t_w, kg_w/kg_a
ϕ	relative humidity
Δt_a	potential of the air to absorb water $(= t_{db,i} - t_{wb,i})$, °C
Δt_{max}	maximum temperature difference in a cooling tower $(= t_{w,i} - t_{wb,i})$, °C
Δt_w	cooling range $(= t_{w,i} - t_{w,o})$, °C

Subscripts

a	air
db	dry-bulb
em	empirical
exp	experimental value
g, w	vapor at water temperature
i	inlet
max	maximum
o	outlet
$pred$	predicted value
s, w	saturated moist air at water temperature
w	water
wb	wet-bulb

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