How clocks define physical time

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Abstract

It is the prevailing paradigm in contemporary physics to model the dynamical evolution of physical systems in terms of a real parameter conventionally denoted as 't' ('*little tee*'). We typically call such dynamical models 'laws of nature' and t we call 'physical time'. It is common in the philosophy of time to regard t as time itself, and to take the global structure of general relativity as the ultimate guide to physical time, and so consequently the true nature of time. In this paper we defend the idea that physical time, t, is rather better defined as an operational modelling parameter: we measure relations between changing physical quantities using bespoke physical systems—i.e. clocks—that coordinate local coincidences. We argue that the sorts of physical systems that make good clocks—what we call *precision clocks*—are those that exhibit self-sustained oscillations known as limit cycles, which are ubiquitous in open, driven, stable, dissipative systems. We develop the physical and philosophical ramifications of this conception of physical time, particularly the notion that physical time does not track something 'out there' in the world. As a result, we speculate that physical time is perhaps not as different from manifest time as many philosophers of time (and apparently general relativity) seem to suggest.

1 Introduction

Fourteen years after Galileo's death, his former student, Vincenzo Viviani, recounted in his biography of Galileo a story which has become legend in the history of science (Gattei, 2019, p.9). When Galileo was about twenty, he was sat in Pisa Cathedral lost in his own thought, observing an oil lamp hanging at the centre of the nave, gently swinging from side to side in the breeze. According to the legend, by closely observing the time it took for the lamp to complete one trip back and forth Galileo was able to build a hypothesis (which later experiments would confirm) that the period of oscillation of a pendulum is independent of its amplitude.

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If Viviani is right about his account of this episode, then this moment marks a significant revolution in the history of science. To see why, consider how Galileo is supposed to have made the observation of the period of the lamp's oscillation. Ascertaining the period requires a precise measurement of time, but at the end of the 16th century there was very little possibility of finding such a precise clock. As is well known, Galileo's genius was to use his pulse as a clock with which to measure the period of oscillation of the lamp, and so develop his hypothesis concerning the dynamical behaviour of the pendulum.

What was revolutionary about this episode was that it marked the very first use in the history of science of what we will describe below as a *precision clock*—a physical system whose dynamics is described by a *limit cycle*—to formally parameterise the dynamics of a distinct physical system. To be sure, timekeeping devices and practices have existed in a variety of human societies for many millennia, and mechanical clocks predate Galileo's measurement by about 300 years. But while such mechanical clocks were used to mark, for instance, Christian prayer times throughout the day, and so enabled a kind of highly coarse-grained informal parameterisation of a sub-dynamics of the Earth-Sun system, it was the application of such clocks by Galileo to investigate dynamics that marked a watershed moment in science.

As inaccurate as we might today take a human pulse to be as a timekeeping device, not to mention the water clocks and song rhymes variously used by Galileo in his experiments, it is by exploring harmonic systems through such means that ultimately led to the use of such harmonic systems as more precise clocks themselves. Indeed, it was the development of the pendulum as a clock by Christian Huygens in the late 1650s, based on Galileo's pioneering work, that really marked a transformation in precision timing instruments (Glennie and Thrift, 2009, p.1).

1.1 What philosophers say about physical time

The standard means in the physical sciences of formally describing the physical world is through dynamical laws: theories and models parameterising dynamical processes in nature. By definition, such 'dynamical' laws are 'in time': the laws are formulated in terms of a real parameter conventionally denoted as t, or '*little tee*'. It is overwhelmingly the case in physics (although not without exception) that our dynamical laws are formulated as second-order partial differential equations (PDEs), which can be characterised as taking data specifying the state of some physical system at a fixed value of the parameter t (conventionally $t = 0$) and exhaustively generating the states of that system for values $t < 0$ and $t > 0$. The parameter *little tee* we refer to as *physical time*, and we call our antecedent data specifying the state of the system at a fixed t the *initial conditions*, and the states for $t < 0$ and $t > 0$ the past and future of the system, respectively.

Our goal in this work is to explore the notion of physical time. It is common in both physics and the philosophy of time to regard *little tee* as time itself. With regard to philosophy in particular, there is a broad, though perhaps not exhaustive, consensus in the naturalised approach to contemporary philosophy of time that insight on the metaphysics of time will come from establishing the nature of physical time. In the service of this goal, the overwhelming orthodoxy in this approach is to take our best physical theories of reality to be the ultimate guide to the nature of physical time. The common map of the terrain is as follows.

The contemporary philosophical debate about time continues to be generally formulated in terms of the delineation between dynamic and static conceptions of time, which has passed down from the Ancient Greeks. Indeed, much of the debate today remains stalled in an early twentieth century delineation between the so-called A-theory of time, according to which the present moment is in some sense privileged and time flows (and so has a direction), and the B-theory of time, according to which all the events in spacetime form a four-dimensional block, there is no privileged present, and no flow of time. It is typically accepted that the features of time in the B-theory characterise physical time, whereas the features of time in the A-theory characterise what we know as 'manifest time'—time as we experience it.[1] Whilst the sort of evidence that philosophers of time marshal to argue for or against these differing metaphysical conceptions of time includes appeals to our experience of time or the nature of our temporal language, the evidence that typically carries the greatest dialectical weight in the debate are claims concerning the structure of physical time.

The most common source of evidence for the structure of physical time comes from special relativity and what it entails for simultaneity. The logic goes that if special relativity is a reliable guide to the structure of physical time, then as a consequence of the relativity of simultaneity physical time does not support a privileged frame of reference, and so cannot support a privileged present. Moreover, the conformal structure of Minkowski spacetime suggests that all past and future times are equally real, supporting a four-dimensional block picture of time.[2] Perhaps it is due to a combination of the contour of the tradition concerning the philosophy of time, and the accessibility of the relativity of simultaneity as a transparent assumption of special relativity, that this has become the conventional battleground for metaphysicians.[3]

One might suspect, however, that a more complete characterisation of physical time could be drawn from general relativity, but this is not nearly so common in philosophical debates about time. One notable exception to this is Callender (2017), who provides perhaps the most clear, most thorough, and most advanced treatment of the physics of time, including the extension from special relativity to general relativity, and then the consequences for physical time as a result of any physical theory that utilises second-order PDEs to describe reality.[4] As such, Callender provides an excellent exemplar of the current thinking on physical time, and thus it is worth here very briefly surveying some of Callender's analysis.

After reviewing much of the well-known debate concerning special relativity, simultaneity,

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[1] This terminology and concept plays a key role in Callender's (2017) arguments, and is derived from the 'manifest image' of Sellars (1963).

[2] The literature on this is vast; too large to do it justice in one footnote. Popular sources for these main claims include (Rietdijk, 1966; Putnam, 1967; Malament, 1977). For more on the viability of temporal becoming (and thus flow), see (Dieks, 1991, 2006; Ellis, 2007). For an indication of the basic outline of the debate in the contemporary setting, see (Dainton, 2001; Bardon, 2013; Baron and Miller, 2018).

[3] There is, of course, also a sub-strand of naturalised philosophy of time that focuses on the statistical mechanical tradition of Boltzmann, and its consequences for specific questions relevant to the asymmetry of physical time (Albert, 2003; Duplantier, 2013). Whilst we do not say anything explicit here about this tradition, we see our project to be one that reengages philosophical discussions concerning the nature of physical time with the statistical mechanical tradition.

[4] While we will not go into any detail concerning this latter point, one of us (Baron and Evans, 2021) has argued that Callender misses the opportunity to draw the consequence from his analysis concerning the agent-centricity of the direction of time.

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temporal becoming, and the block universe, Callender points out that the argument from the relativity of simultaneity is not so straightforward in general relativity. The general reason for this is that not all solutions to the Einstein field equations (EFEs) are globally hyperbolic (Gödel's solution is one famous example of this). In short, solutions that are not globally hyperbolic cannot be formulated in terms of second-order PDE, which as we saw above are essentially 'ground zero' for establishing the properties of *little tee*. But general relativity can be reformulated as a '3 + 1' (or Hamiltonian) theory (Arnowitt *et al.*, 1962), formally equivalent to its original Lagrangian formulation, such that all solutions of this reformulation are indeed globally hyperbolic. Taking this fact as his starting point, Callender then goes on to provide a masterly appraisal of the possibility of finding evidence for dynamic time—or, more precisely, manifest time—in a range of classes of solution to the EFEs. Callender's conclusion is that the features of manifest time are incompatible with the constraints of relativity (Callender, 2017, p.80).

We wish to set our approach to physical time in contrast to this narrative. While we would certainly agree that the ultimate guide to the nature of time must be the structure of physical time, we contend that the philosophy of time finds itself on a misguided errand chasing physical time in the global structure of general relativity. Of course physical time must be consistent with general covariance, but we wish to emphasise the point here—and this is not a novel point—that general relativity is fundamentally a local theory, about local clocks, rulers, and light signals. Thus, in a sense then, it need not matter whether general relativity is formulated as a 3 + 1 theory, or in its original Einstein-Hilbert form, diffeomorphism invariance renders any arbitrary coordinatisation of the manifold as simply unphysical.[5] This is a well-appreciated detail amongst physicists working with relativity, where coordinate time is often referred to as 'bookkeeper' time. Indeed, recognising that at its heart general relativity relies on local clocks and rulers goes back to Einstein (1970, p.58) himself:

> First, a critical remark on the theory as characterised above. It was noticeable that the theory introduces (besides four-dimensional space) two kinds of physical things, namely 1) rods and clocks, 2) all other things, e.g. the electromagnetic field, the material point, etc. This is in a sense inconsistent; rods and clocks should actually be presented as solutions to the basic equations (objects consisting of moving atomic structures), not as so to speak theoretically self-sufficient beings. However, the procedure is justified by the fact that it was clear from the beginning that the postulates of the theory are not strong enough to deduce from it sufficiently complete equations for physical events sufficiently free of arbitrariness to base a theory of rods and clocks on such a foundation. Unless one wanted to do without a physical interpretation of the coordinates altogether (which in itself would be possible), it was better to allow such inconsistencies—albeit with the obligation to eliminate them at a later stage of the theory. However, one must not legitimise the aforementioned sin to such an extent that one imagines that distances are physical beings of a special kind, essentially different from other physical quantities ("reducing physics to geometry," etc.).

The point Einstein is making here is that it is inconsistent that rods and clocks should be considered primitive entities that are somehow antecedent to the theory of relativity: rods and

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[5]Callender (2017, p.74) points out that for some FLRW solutions, a class of 'fundamental observers' will define coordinate time as proper time, which we agree is certainly physical. We maintain that such fundamental observers are themselves unphysical, and so do not think that these cases alter our overall assessment of the utility of general relativity in the philosophy of time.

clocks are physical entities along with "all other things", and spatial and temporal distances are constructed as part of the theory, and so are not "physical beings of a special kind" that can precede the theory. Given this, what then is physical time? This work addresses this question in the context of Einstein's insight into the relationship between clocks, the measurement of temporal distances, and the nature of time. We claim in this work that it is the measurement of time that is the key to physical time, and we take the measurement of time to be the exclusive remit of clocks; in other words, in so far as the measurement of temporal distance is achieved by clocks, the nature of time is intimately connected to the nature of clocks.

1.2 Outline of the argument

We argue that it is clocks that define physical time. We mean this in the very literal sense that physical time—i.e. *little tee* that appears in our dynamical laws—is a representation of the way in which precision clocks are used to parameterise the behaviour of physical systems in the service of scientific modelling. We characterise a precision clock as a physical system whose dynamics permits a phase space description in terms of a *limit cycle*: a closed periodic trajectory that acts as an attractor to neighbouring phase space trajectories. Only open, driven, nonlinear, dissipative dynamical systems can be described by limit cycles, and thus be stable oscillators, and so precision clocks; nevertheless, there is nothing exotic about a limit cycle: they are generic in nonlinear, dissipative dynamical systems. Such precision clocks are able to discriminate *little tee* with sufficient fine-grain for the purposes of constructing dynamical laws, and so allow the parameterisation of dynamical processes in nature.[6]

We begin in §2 by motivating the relational nature of the measurement of time. In §3 we consider the physics of limit cycles and what this implies for the nature of clocks. In §4 we draw on the expert analysis of Tal (2016) on the use of clocks to measure time to illustrate the features of clocks we emphasise in the preceding sections. The implicit point that we are targeting with our argument is that the measurement of physical time is not tracking some objective feature of the world. In §5 we briefly discuss the process of 'reading out' the time on a precision clock, before concluding in §6 with the speculative thought that physical time and manifest time may not be so different after all.

2 The measurement of time is relational

Our focus in this work on the measurement of time using clocks amounts to an explicit commitment to relationalism about time, of which there is a considerable tradition across the history of the physics and philosophy of time. In this tradition, *little tee* is taken to be a kind of surrogate parameter for a relation between changes in values of physical (typically spatial) variables. Here is Mach (1911) making this point:

> We can eliminate time from every law of nature by putting in its place a phenomenon dependent

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[6]Subsequently, not every physical system can be used as such a clock, as Frank Wilczek has suggested (Harris, 2021).

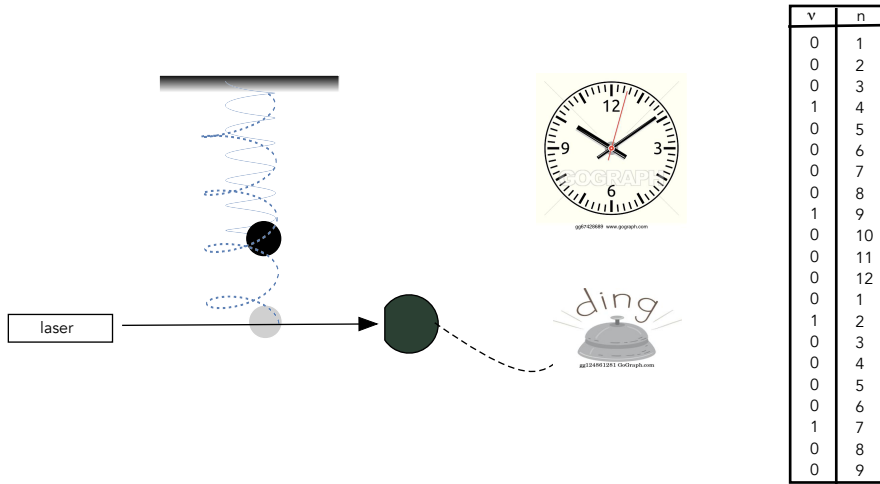| v	n
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Figure 1: The displacement of a simple harmonic oscillator is monitored using a laser beam. When the particle blocks the beam, an alarm goes off—a local event—and, simultaneously, an observer notes the angular displacement of the clock hands as an integer count (mod 12) or (mod 60). On the right is a data table representing the function $\nu(n)$.

> on the earth's angle of rotation. . . If, now, we imagine the. . . temporal positions replaced in the above manner, in the equations in question, we obtain simply *every phenomenon as [a] function of other phenomena*. (p.60, emphasis in original)

> Physics sets out to represent every phenomenon as a function of time. The motion of a pendulum serves as the measure of time. Thus, physics really expresses every phenomenon as a function of the length of the pendulum. . . If one were to succeed in expressing every phenomenon. . . as a function of the phenomenon of pendulum motion, this would only prove that all phenomena are so connected that any one of them can be represented as a function of any other. Physically, then, time is the representability of any phenomenon as a function of any other one. (p.90)

As such, it seems reasonably clear that when we 'measure time', we are really measuring some other, perhaps arbitrary, relation between phenomena.

To make this further apparent, consider a rudimentary account of a clock as essentially a counter that coordinates coincidences between local events: we observe a clock, at the location of some event here and now, and record a natural number, n, as the count. Suppose we observe a recurring physical event (Fig.(1)). The simplest coincidence we can define is the count on a nearby clock when the event occurs. We can represent the occurrence of the event by a single binary number $\nu = \{0, 1\}$ that takes the value 1 if the event occurs, and 0 otherwise at each count of the clock. The coincidences are then represented by a binary function $\nu = \nu(n)$. Despite the fact that n represents the reading on the clock, and so provides in a sense a 'temporal' description of the physical event, the coincidence is essentially between two physical phenomena: the event ν and the count on the clock n.

Typically in classical mechanics, however, it is generally mathematically simpler and more tractable to specify the correlation between ν and n in parametric form, $(\nu(t), n(t))$, where

t is some real number *little tee*.[7] This process of parameterisation is essentially the reverse of the process outlined by Mach in the quotes above: we *introduce* a real parameter. The changing physical phenemenon, ν, is now described by a continuous function of t. The common interpretation of this process is that *little tee* represents the evolution of the system 'in time'. But it is important to remember that introducing *little tee* in this way is nothing more than a mathematical convenience. The physical facts are already fully described by $\nu(n)$.

To see why, consider briefly the typical significance of parameterisation. Ordinarily, upon discovering a correlation between two variables, we often look to find a causal connection of some sort between them; in effect, we look to find which of the two is the dependent and which the independent variable. We can consider the process of parameterisation as introducing a third factor—the parameter—to play the role of the independent variable, and in doing so we 'uncover' a hidden cause of the observed correlation.

However tempting it might be to do so, it is a metaphysical trap to fall into this way of thinking when it comes to the parameterisation of dynamical systems using *little tee*, which simply cannot be the 'cause' of the dynamics any more than our count n above can be the cause of $\nu(n)$. What is more, a good clock must not interact with the system it is coordinating, and so intervening on a clock to change the count can have no causal effect on the phenomenon that one intends to coordinate. We simply cannot manipulate the value of *little tee*. Thus, rather than introducing a new independent variable, the parameterisation of dynamical systems using *little tee* provides nothing more than a surrogate parameter that stands in for whichever local physical system is employed to count coincidences.

3   Precision clocks and limit cycles

Thus far we have yet to articulate anything more than a reminder of time's relational properties. In this section we develop an account of precision clocks as a particular type of physical system, one whose dynamics exhibits a limit cycle. The key take-away from this account is that precision clocks are inescapably subject to the laws of thermodynamics. In this context, we make explicit that the precision of a clock is proportional to the energy it dissipates as heat, and as a result of this, that clocks are ostensibly irreversible.

A precise clock is a physical dynamical system driven far from thermal equilibrium that maintains a stable and periodic oscillation (and coupled to an auxiliary system to count the oscillation, of course (Jespersen and Fitz-Randolph, 2011)). This is achieved when the phase space description of its dynamics exhibits a stable and periodic trajectory, i.e. a limit cycle. The stability of a limit cycle is formally underpinned by the fact that the periodic trajectory acts as an *attractor* to neighbouring phase space trajectories: if a small external impulse to an oscillatory mechanism pushes the system off the limit-cycle trajectory in phase space, then the trajectory quickly decays back onto the limit cycle. Physically, this implies that the clock system must be driven to overcome friction, and so the system must have a constant supply from a

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[7]The mathematical structure of a real parameterisation of integer valued functions is discussed in (Kàrolyi *et al.*, 2008).

source of external energy. As a result, the requirement that precision clocks exhibit limit cycles thus implies that, as dynamical systems, such clocks must be open, driven, nonlinear, dissipative physical systems, and consequently must also be subject to noise. Galileo's clock above—his pulse—can be modelled as just such an electro-chemical limit cycle (Zeeman, 1972).

Let us consider the physical requirements for stability in a bit more detail. Not just any oscillatory phenomenon—for instance, the oil lamp swinging in Pisa Cathedral—can form the basis of a precision clock. Such clocks simply 'wind down' to a stop in the long run due to friction. While some form of energy input, some impulse, is required for the oscillatory system to remain periodic, this is a nontrivial problem in practice. The escapement clock is a good illustration of exactly how this can be achieved: the force of a falling mass transmitted through an escapement provides a consistent input of work which drives a good clock. The escapement mechanism integrates an element of nonlinearity to the linear dynamics of the pendulum, to which the escapement is strongly coupled, and this enables the whole dynamical system to exhibit a limit cycle (Milburn, 2020).

However, there is no such thing as a perfectly precise, perfectly stable clock. Crucial in the formation of a limit cycle is that the work done by the driving force on the oscillatory system is dissipated as heat over one cycle. This means that all precision clocks exhibiting a limit cycle must be dissipative and so, by the fluctuation-dissipation theorem, there is always noise that accompanies the operation of the clock. Due to the stochastic nature of open systems, it is the heat dissipated over one cycle where fluctuations appear, and this results in phase diffusion on the limit cycle and so fluctuations in the clock's period, which consequently becomes a stochastic variable. As such, the time taken to complete a single cycle of the phase-space trajectory will vary from one oscillation to the next. While the average period on a limit cycle is well defined, the variance of the period is nonzero. Good clocks will have a small variance relative to the average period, and this can be decreased further by driving the clock harder, which results in more energy dissipated as heat. It is for this reason that the precision of a clock is proportional to the energy it dissipates as heat. In so far as heat dissipation is the signature of irreversibility, precision clocks are irreversible dynamical systems, and their irreversibility is embodied in the randomness of the period of oscillation of the limit cycle.

It is true that not all clocks are precision clocks; indeed, some clocks are simply not even periodic, such as an hour glass or radio-carbon dating. It is interesting to note that the most important clock of all in the history of humanity—the solar system—is not in fact a precision clock. Despite the fact that we consider the solar system to be an exemplar of the Newtonian 'clockwork universe', describing a deterministic and reversible dynamical world, the solar system is in reality an open dissipative system. The source of this dissipation is the heat generated in each of the planets by tidal forces as they rotate and orbit through the Sun's gravitational field. Unfortunately for Newton's picture, the planets are not point masses and so celestial dynamics is not reversible.

This dissipation notwithstanding, the solar system does indeed appear to be very stable. Definitive proof for this has traditionally been elusive as gravitational dynamics is highly nonlinear, and finding a dynamical solution for more than two bodies interacting only through gravity

is nontrivial. Confirmation that the solar system is not stable, and so not evolving on a stable limit cycle, came only with insight into deterministic chaos in the twentieth century (Laskar, 1994). The reason that the solar system can be used as a relatively good clock is that, for quotidian human affairs, the unstable nature of the solar system is negligible. This suggests that the quality of a clock is always relative to the nature of the changing events it is used to coordinate. For the purposes of describing the dynamical behaviour of the sorts of physical system we take to comprise the domain of terrestrial physics, only precision clocks as we have described them above provide a sufficiently fine grain for this task.

In summary, then, a clock is a stable oscillator equipped with a period counter. Stability requires an open, nonlinear, dissipative system driven far from thermal equilibrium by access to a low entropy source of energy. Like all physical systems, clocks are subject to the laws of thermodynamics (Milburn, 2020). As such, clocks necessarily generate heat and entropy, and depend on the local thermodynamic environment; a clock is not a reversible and deterministic dynamical system. If time is what clocks measure, then the irreversibility of clocks influences our conception of physical time.

4 Case study: UTC

In the preceding sections, we have claimed that the measurement of time is relational, and we have argued that clocks are subject to the laws of thermodynamics and that, to this end, it is in the period of oscillation that we see noise appear. To demonstrate what this means in practice, and connect these features of the measurement of time to the claim that physical time is not tracking something out there in the world, in this section we consider a case study due to Tal (2016) of the determination of coordinated universal time (UTC).

In his paper "Making Time: A Study in the Epistemology of Measurement", Tal develops a model-based approach to the standardisation of measurement. As his case study on measurement, he chooses the measurement of time, and UTC in particular. This is of great benefit for current purposes, as we employ Tal's analysis here as an illustration of the features of clocks we outline above.

The essential problem with the measurement of time is that any assessment of the stability of the period of oscillation of a clock must be made by way of measurement with another clock, whose own stability would be in need of verification. (In the Appendix we given an example of how to use two limit-cycle clocks, of identical design, to establish stability.) As we have seen above, since all clocks are physical systems that obey the laws of thermodynamics, every clock will be subject to noise in the period of oscillation, thus in practice there is no such thing as a perfectly stable clock to play the role of an archetypal standard. In principle, however, an idealised stable period of oscillation has been stipulated as part of the definition of the standard second, which employs a particular transition of the ground state of a caesium atom. As Tal (2016, p.301) points out, this essentially specifies that every period of oscillation of the radiation from this transition is equal to any other from any caesium atom.

In short, UTC is a highly abstract and idealised measure of time built upon a multitude of

actual physical clocks and a hierarchy of bespoke algorithms aimed at coordinating the various measures into a single unified representation of time on Earth. The physical clocks come in two classes, which each make the unavoidable trade-off between stability and accuracy in different ways. The primary frequency standards are designed for optimal accuracy. As of the end of 2021, there were sixteen different primary frequency standards operating in eleven labs around the world (BIPM, 2021). Most of these primary standards are so-called 'caesium fountains', where caesium atoms are released to fall freely in a vacuum to remove the influence of any gravitational redshift from the Earth.[8] However, the technical challenges of operating a caesium fountain render them unsuitable for the task of the ongoing measurement of time (Tal, 2016, p.301).

Secondary frequency standards favour stability over accuracy relative to primary standards. This means that they can operate for much longer stretches, but need to be calibrated against the primary standards from time to time. Secondary standards are typically atomic caesium or rubidium clocks, or hydrogen maser atomic clocks; there are currently more than 450 atomic clocks that contribute to the stability of the measurement of time (BIPM, 2021). But just as with any set of physical clocks, the period of oscillation of each of these secondary standards, as well as the primary standards, will be subject to noise and, as such, together comprise a gradually diverging set of clocks.

It is at this point that we require a specific set of operational procedures on the secondary standards to calculate the abstract measure of time that is UTC. To begin with, a stability-weighted average clock time is established. Since an assessment of the stability of any clock is a function of this calculated average, the weights of each clock themselves are updated based on their past performance in predicting the weighted average. Given our analysis above, clocks with higher than average noise will clearly fail to reach a threshold of stability relative to the average. Such clocks are discarded, which then results in a set of clocks whose periods of oscillation are highly consistent throughout the set.

To this process is added a series of constraints and damping measures on the values of the weights to ensure that abrupt changes in the measure of stability are not brought about by too large changes in the set of weights (Tal, 2016, p.304). And then all of this is coordinated in Paris between participating national laboratories all over the world, who need to transmit their respective measures via GPS satellite. Such transmissions need to be corrected for GPS time, which itself is a derivative of UTC. This whole procedure is directed towards maintaining the stability of UTC.

The next piece of the puzzle is that secondary standards gradually drift against primary standards. As a result, metrologists need to 'steer' the weighted average of secondary standards back towards the official standard of the second set by the primary standards. The steering algorithm employs periodic calibrations with the primary standards, each of which is weighted based on the quality of the calibration, and is then extrapolated to the times in between calibration using

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[8]The other primary standards include a rubidium fountain, a pair of legacy caesium beams operating in Germany, and two optical clocks. As of the end of 2021, optical clocks look set to overtake caesium fountains in accuracy (more on which in §5 below), which would eventually relegate caesium fountains to secondary standards (BIPM, 2021)

statistical models ([Tal](#), [2016](#), p.306). The final part of the calculation of UTC is to adjust this steered weighted average to match the solar day. Any added seconds are referred to as 'leap seconds'.

What is remarkable about this calculation of UTC is the sheer number of modelling assumptions that are employed to generate this idealised measure of time. Moreover, the modelling assumptions are conspicuously dependent upon the outcome of the previous round of modelling. But what is most obvious in this narrative, and hence the reason that we think it is a nice illustration of the arguments in the preceding sections, is how prominent a role that noise plays in the search for a universal measure of time. It is not beyond exaggeration to say that the entire operational procedure is directed towards mitigating the influence of noise on the stability of the period of oscillation of clocks.

However, we wish to make it abundantly clear that at no point in the calculation of UTC is there an objective standard of time that is being tracked. In fact, the entire algorithm for establishing UTC is required *simply because* there is no such objective time that can be tracked. Moreover, it should also be abundantly clear that UTC illustrates the relational nature of time: UTC is established solely through measurements carried out by precision clocks.

So what does this mean for physical time? We think that the calculation of UTC is simply writ large the same process that all physical modelling of time (or dynamical behaviour in time) must go through: time is essentially relational, and the thermodynamics of clocks renders the concept of a perfectly stable periodic oscillator illusory. Consequently, there simply is no objective property of the world that physical time—the *little tee* of our dynamical laws—is tracking.

5 Reading out a clock

Before we conclude our argument, we wish to say a few words concerning the nature of 'reading out' the time on, and so interacting with, a precision clock. Consider how this is achieved for the caesium fountains we discussed above. The process begins by applying a DC voltage to a 'Gunn diode' to create a nonlinear electronic oscillator at microwave frequency, driving a limit cycle in the system. The diode is then used to form an electronic resonator mounted in a microwave cavity to create a 'maser' (like a laser with microwaves). Caesium atoms are then dropped into a high vacuum and passed through the microwave resonator, and used to feedback-lock the phase noise of the microwave source to an atomic transition frequency that is, as we saw above, stipulated to be the same for every caesium atom. The entire system, maser plus atoms, now gives a dynamical limit cycle that has much less phase noise than the microwave source by itself. The locked maser frequency can then be used to generate 'ticks' one per second as a primary frequency standard as above. In an abstract sense, one could think of the maser as like the escapement mechanism and the atom as the pendulum in a driven nonlinear pendulum clock.

This atomic clock system is typically considered a classical clock. The reason that such microwave frequency cavities are classical (as opposed to optical cavities, discussed below, which are quantum) are largely because the energy of a microwave photon is many orders of magnitude less than an optical photon, and so single photon detectors for microwave photons are imprac-

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ticable. As such, the electric field itself must be measured, which requires measuring currents and voltages in the circuit carrying the microwaves. These properties can be described entirely classically in so far as the phase noise is determined by classical circuit parameters (rather than spontaneous emission, as for higher energy photons). As a result, 'reading out' the time on such a clock does not affect the operation or the precision of the clock.

In recent times, the accuracy with which caesium fountains keep time is increasingly being challenged by optical clocks. Optical clocks operate essentially the same as caesium atomic clocks, except instead of a maser at microwave frequencies a laser at optical frequencies is used. It should be no surprise that a laser is a driven, nonlinear, dissipative system that can be described by a limit cycle, and so is a stable oscillator. Moreover, a laser is subject to noise, but in this case quantum noise arises from spontaneous emission (which is the fundamental quantum mechanism for energy loss, and so renders the laser dissipative). This noise still results in phase diffusion of the field emitted from the cavity. To reduce the phase noise, as above, the laser is then feedback-locked to a stable atomic oscillation, this time to an optical transition in some suitable ion held in an ion trap (typically ytterbium or strontium, although a variety of other transitions have been explored). Again, as above, one can then think of the laser as like the escapement mechanism and the atom as the pendulum in a driven nonlinear pendulum clock.

Since optical laser frequencies are in the hundreds of THz range, and the very best electronic counters can only handle hundreds of GHz, counting the cycles in the laser limit cycle poses a challenge. Sophisticated methods have been developed (using so-called 'frequency combs') to generate beat patterns in the carrier frequency of the laser emission so that the beat frequency is in the realm of electronic counters, but we refrain from going into details here. In short, though, in an optical clock we see all the principles of physical clocks operating in a quantum system: far-from-thermal equilibrium, dissipative steady-state oscillations on a limit cycle, phase diffusion, feedback phase stabilisation using atoms, and an explicit counting mechanism in terms of a frequency comb.

However, the significance of optical clocks that we wish to emphasise here arises from their fundamental quantum nature. Above the threshold where stimulated emission dominates spontaneous emission, the intensity of a laser fluctuates very little. Indeed the photon emission rate is very close to a Poisson process, where the average time between emission events is stable. It would be pointless to measure the intensity of the laser to make a clock, as it does not oscillate at all. Instead, a direct measurement of the electric field emitted by the laser is required. In this case, however, there is a direct noise price to pay: the measurement itself leads to phase noise arising purely from the Heisenberg uncertainty principle. In quantum clocks, then, such as laser driven optical clocks, the measurement used to generate the count provides a critical point of difference to classical clocks. We leave the exploration of quantum clocks to future work.

6   Conclusion

Let us finish now where we began, sitting with Galileo in the late 16th century in Pisa Cathedral, lost in thought, watching an oil lamp swinging back and forth. Galileo's insight to employ his

pulse as a relational clock with which to measure the period of oscillation of the lamp now echos quite loudly to us through the intervening centuries. There is a sense in which, one might argue, this was *the* revolution in science. Whether Galileo was aware of it or not, the human pulse is an example of a physical dynamical system that exhibits a limit cycle, and so is a stable oscillator, and thus a precision clock.[9] Such precision clocks are critical to providing the sorts of fine-grained formal parameterisations of the dynamics of physical systems that underpin our scientific descriptions of the world; in short, precision clocks define physical time.

In so far as a naturalised approach to the philosophy of time takes physical time to provide a good guide to the metaphysical nature of time, then we claim that the physics of precision clocks can provide insight into the nature of time. We take our paper to be the first step in exploring the physics of precision clocks and the corresponding philosophical ramifications. We started off setting our approach in contrast to the narrative that general relativity provides the most complete characterisation of physical time. However, we think that so long as we can understand general relativity as a fundamentally local theory, constructed out of local interactions and measurements employing rods and clocks, and where the global spacetime geometry of general relativity is an abstraction from these local interactions, then our characterisation of physical time in this paper will be consistent with that of general relativity.

As a result of this, and as a final thought to point towards future work, we speculate that this characterisation of physical time shows us that at least some parts of what we take to be manifest time are not so foreign to our best physical description of the world. In particular, the thermodynamic irreversibility of clocks points strongly towards physical time having a definite direction. One might also argue that the synchronisation of local coincidences by a relational clock, when combined with the interminable nature of a physical clock's phase diffusion, shows that the future ticks of a clock will almost never resemble its past ticks, and so render local coincidences a special point of reference between past and future. We suspect that supporting such an argument will require the introduction of a physical agent making use of the clock, and as such is beyond the goal of the present work.[10] In so far as introducing the notion of a physical agent might also provide insight into the nature of temporal flow, we see this as an exciting direction for future work.

Appendix

In this appendix we summarise some essential mathematical facts about clocks built from systems exhibiting limit cycles. We will follow the presentation of Aminzare *et al.* (2019). The normal form dynamics of a super-critical Hopf bifurcation leading to a limit cycle, including thermal

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[9]Of course, the period of oscillation of the human pulse varies as a result of a highly changeable set of physical parameters characterising the complex dynamical behaviour of the human cardiovascular system. So long as these are stable, however, the oscillation is stable.

[10]Although see (Evans *et al.*, 2021; Milburn *et al.*, 2023) for work in this direction.

noise, are given by the Ito stochastic differential equations.

$$dx_1 = (\mu x_1 - \omega x_2 - (x_1^2 + x_2^2)x_1)dt + \sigma dW_1(t) \tag{1}$$

$$dx_2 = (\mu x_2 - \omega x_1 - (x_1^2 + x_2^2)x_2)dt + \sigma dW_2(t) , \tag{2}$$

where μ is a linear gain parameter, ω is a linear oscillator frequency, σ^2 is a diffusion constant proportional to temperature, and $dW_k(t)$ are independent Weiner stochastic increments. The nonlinear term describes a nonlinear dissipation process. Without noise this system has a limit cycle centred on the origin with radius $r = \sqrt{\mu}$ and frequency ω.

On the limit cycle $(x_1(t), x_2(t)) = \left(\sqrt{\mu}\cos\theta(t), \sqrt{\mu}\sin\theta(t)\right)$, where the phase on the limit cycle satisfies the Ito stochastic differential equation

$$d\theta = \omega dt + \frac{\sigma}{\sqrt{\mu}}dW(t) , \tag{3}$$

where dW is a Weiner stochastic increment. The most important feature of this reduction is that the phase diffusion rate gets smaller as the size of the limit cycle gets bigger, i.e as the gain rate $\mu >> \sigma^2$.

The period T is defined as the time taken for the phase to change by $\theta(0) - \theta(T) = 2\pi$. However as the phase is a stochastic variable, the period must be a random variable. Sometimes the noise ensures that it gets there sooner rather than later. This is in instance of the *first passage time* problem: finding the time taken for the phase to hit 2π for the first time. It is also know as the inverse diffusion process. The distribution of periods is then given by the Wald or inverse Gaussian distribution that takes the generic form

$$W(t, \alpha, \lambda) = \sqrt{\frac{\lambda}{2\pi}}t^{-3/2}\exp\left[-\frac{\lambda}{2\mu^2 t}(t - \alpha)^2\right] \qquad t \geq 0 , \tag{4}$$

where α, λ are positive real parameters (λ is called the spread paramter). The Wald distribution is completely determined by the mean and variance as

$$\overline{T} = \alpha \tag{5}$$

$$\overline{\Delta T^2} = \frac{\alpha^3}{\lambda} . \tag{6}$$

In the case of the dynamical model considered here we find that

$$\overline{T} = \frac{2\pi}{\omega} \tag{7}$$

$$\overline{\Delta T^2} = \frac{2\pi\sigma^2}{\omega^3\mu} . \tag{8}$$

As expected, the variance in the period decreases inversely proportional to the energy pumping rate μ that determines the size of the limit cycle. As the limit cycle forms when the work done on each cycle equals the energy dissipated on each cycle, the bigger μ becomes the more energy
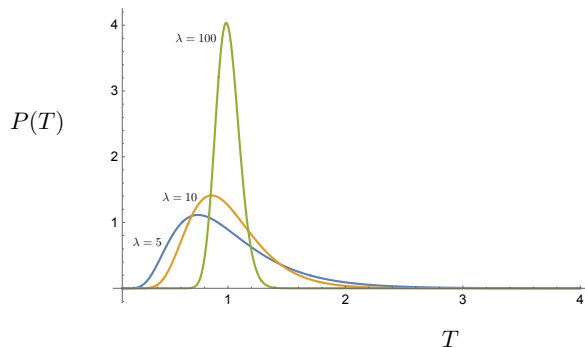
Figure 2: The Wald distribution with a mean period $\bar{T} = 1$ and three different values of the spread parameter, λ. Increasing λ is decreasing noise.

is dissipated. Good clocks are highly dissipative. This is why limit cycles are so important for clock design.

In what follows we choose the units of time such that $\omega = 2\pi$, making the mean period equal to unity. We also define the spread parameter

$$\lambda = \frac{4\pi^2 \mu}{\sigma^2} \ ,\tag{9}$$

which gets larger as the noise (i.e. the temperature) is decreased. The variance can then be written as

$$\overline{\Delta T^2} = \frac{\mu^3}{\lambda} \ .\tag{10}$$

We plot some examples in Fig.(2).

An important feature of the Wald distribution is that each sample period T_k in a ticking clock is independent. This is reflected in the property of divisibility. If T is a random variable with a Wald distribution with mean $\bar{T} = \alpha$ and spread parameter λ, then there exist n pieces of independent and identically distributed random variable $T_1, T_2, \ldots, T_n$ each from the Wald distribution with mean $\frac{\alpha}{n}$ and spread parameter $\frac{\lambda}{n}$ such that $T \equiv T_1 + T_2 + \ldots + T_n$.

The stochastic history of a particular clock is given by the total count N and the list of periods $T_1, T_2, \ldots, T_N$. The total elapsed parametric time is

$$t_N = \sum_{k=1}^{N} T_k \ ,\tag{11}$$

where α, λ are the mean and spread parameters for each T_k. By the property of divisibility, the statistics of T_N is the Wald distribution with mean and spread $N\alpha$, $N\lambda$. The mean of T_N is $N\alpha$ and the variance is

$$\overline{\Delta T_N^2} = N^2 \frac{\alpha^3}{\lambda} \ .\tag{12}$$

If we define the signal to noise ratio as the ratio of the mean to the standard deviation we find

that the signal to noise ratio is

$$SNR = \sqrt{\frac{\lambda}{\alpha}} \, , \tag{13}$$

and is independent of the total count, N.

Let us compare two independent clocks, A and B, with the same values of α, λ. We regard clock A as the relational clock for coordinating the tick events in clock B. Obviously, we could equally well do it the other way around. Due to the noise in each clock, when A records a count of N, the number of events, M, recorded in clock B is not necessarily the same. Define the error function as $\epsilon(M) = (1 - M/N)^2$. The integer M is a random variable making ϵ a random variable.

Let the parametric representation of the count in each clock be $(N_A(t), N_B(t))$. Given a particular time t_N such that $N_A(t_N) = N$, the probability density for this value of t_N is the Wald distribution with mean and spread parameter $N\alpha$, $N\lambda$ respectively. The probability density that clock B records M counts in the same time is the Wald distribution function $W(t_N, M\alpha, M\lambda)$.

We now define the integer valued function $N_A(t)$ as

$$N_A(t) = \left\lfloor \frac{t}{\alpha} \right\rfloor \, , \tag{14}$$

where $\lfloor x \rfloor$ is the integer part of x. If $N_A(t_N) = N$ then we can write $t_N = N\alpha$. The probability that $M = N$ is given by

$$P(M = N) = \int_0^{(N+1)/\mu} dt \, W(t, M\alpha, M\lambda) - \int_0^{(N-1)/\mu} dt \, W(t, M\alpha, M\lambda) \, . \tag{15}$$

In Fig.(3) we plot this probability versus M for three values of λ and $\alpha = 1.0$, $N = 10$. We
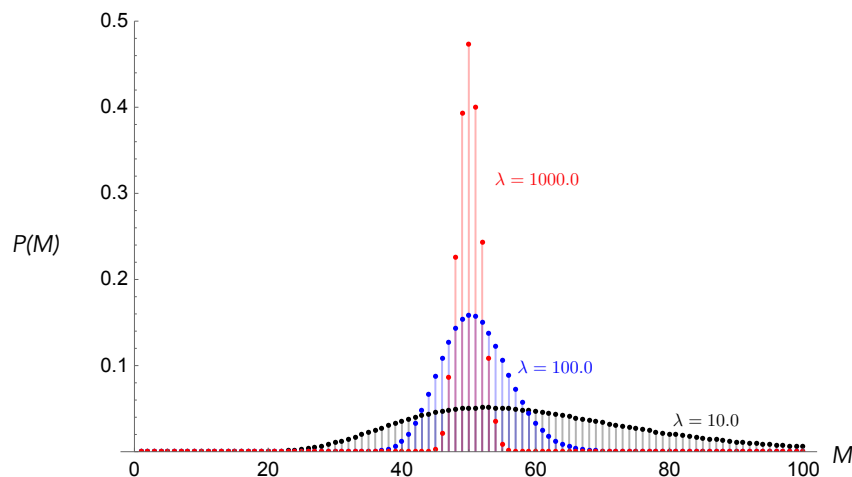

Figure 3: The probability that clock B counts M when clock A has a count of $N = 50$, and various values of the spread parameter. We set $\alpha = 1$.

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assume that both clocks are identical, i.e. they have the same Wald distribution. The average error, when $\alpha = 1$ is found to be

$$\bar{\epsilon} = \frac{1}{\lambda} \ , \tag{16}$$

which decreases as the spread parameter increases; that is to say, the average error decreases as thermal noise decreases.

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