FUEL-EFFICIENT FORMATION STRATEGIES FOR CROSS-TRACK MULTISTATIC SAR CONSTELLATIONS

Eduardo Rodrigues Silva Filho, Marc Rodriguez-Cassola, Reuben Katz, Jalal Matar, Alberto Moreira

Microwaves and Radar Institute, DLR

ABSTRACT

The effectiveness of swath reconstruction algorithms in cross-track multistatic synthetic aperture radar (SAR) constellations heavily relies on the precise positioning of individual spacecrafts. Ideally, a static array configuration is desired, where the satellites are evenly distributed in a Local Vertical Local Horizontal (LVLH) frame, emulating a fixed antenna. However, achieving this ideal configuration is not feasible without the implementation of constant active control to counter the relative dynamics between the satellites. Alternatively, it is possible to design natural trajectories that yield the desired formation geometry for a significant portion of the orbital period.

In this paper, we present a comprehensive analysis of the feasibility of a continuous control concept for building a static array in space. Additionally, we propose suitable formation strategies that eliminate the need for continuous control.

***Index Terms*—** Bistatic radar, multistatic radar, formation flying, performance analysis

1. INTRODUCTION

Several distributed SAR concepts have been studied over the last years for their many advantages compared to single-satellite missions [1]. In this context, digital beamforming techniques can be used to achieve reasonable SAR performance with smaller antennas and lower power demands for the individual satellites [2, 3]. The efficiency of the constellation, however, largely depends on the ability to control the relative position of the spacecraft significantly better than the antenna dimensions.

The required nominal distribution of cross-track constellations is sketched in Fig. 1. The satellites must stay at a specific constant distance between each other and at a certain elevation angle in the plane orthogonal to the transmitting satellite velocity in the Earth-centered, Earth-fixed (ECEF) coordinate system [4]. Some tolerance is allowed in the position of individual satellites with respect to the nominal position in the ideal array.

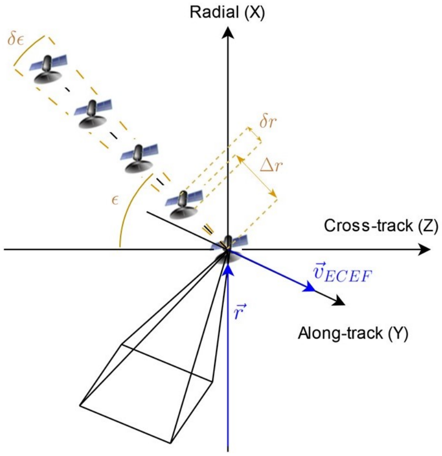


Fig. 1: frame definition for cross-track SAR imaging.

In Fig. 1, the satellites are arranged in an array with a nominal inclination of ϵ, and the individual elements separated by a distance of Δr. The satellites must stay within a tolerance of δr from their nominal radial position in the array and within $\delta\epsilon$ from their nominal angular position in the array. Defining $\vec{r}_i$ as the vector from the lower satellite to the element i of the formation, and $\vec{r}_{\text{ref},i}$ as its reference value, the formation conformity criteria can be expressed using the following equations:

$$\|\vec{r}_{\text{ref},i}\|-\delta r \le \|\vec{r}_i\| \le \|\vec{r}_{\text{ref},i}\|+\delta r\ ,\ \frac{\langle \vec{r}_{\text{i}}, \vec{r}_{\text{ref},i}\rangle}{\|\vec{r}_{\text{i}}\|\cdot\|\vec{r}_{\text{ref},i}\|} \ge \cos(\delta\epsilon) \tag{1}$$

In order to analyze such formations, we establish an orthonormal reference frame. This reference frame, also illustrated in Fig. 1, is centered on a virtual satellite that describes a circular orbit with the same period and mean inclination as the satellites in the formation. This reference frame is defined by the velocity of this virtual satellite in the ECEF frame (Y-axis) and its position vector (X-axis). Throughout this paper we examine the formation geometries in the XY plane of this reference frame, which corresponds to the zero-Doppler plane at a given instant. We define orbit duty cycle as the percentage of the orbital period during which the satellites form an array in the zero-Doppler plane that satisfies the criteria expressed in Eq. 1.

Maintaining the aforementioned satellite formation naturally at all times is not feasible, as the alignment of the formation can only be sustained for a limited duty cycle per orbit. To preserve the formation as it is at all times, constant thrust must be applied to the satellites, which would be highly expensive in terms of fuel consumption. Therefore, natural solutions that temporarily meet the array positioning criteria could be more viable alternative.

In the following sections, we present an analysis of the feasibility of a continuous control concept to establish a static array in space, and propose suitable formation strategies for cross-track multistatic SAR constellations that eliminate the need for continuous control.

2. PROPOSED SOLUTIONS

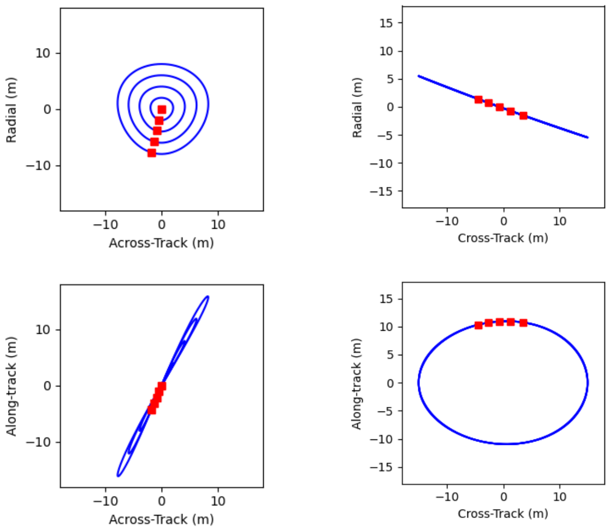
In this study, we examined three solutions: an active solution, where the satellites are continuously maintained in position in the Local-Vertical-Local-Horizontal (LVLH) frame using constant thrust, satisfying all criteria expressed in Eq. 1 at all times; a phased formation solution, where the satellites form a line in the LVLH frame, approximately aligning with the intended array direction, fulfilling the angular criteria for most of the time but the radial distance criteria only during a fraction of the time; and an in-phase formation solution, where the satellites trace an elliptical path, fulfilling the radial requirement for the majority of the time but the angular requirement only during a fraction of the orbit. The two latter solutions follow the natural formation not accounting for orbital disturbances, requiring the propulsion system solely to compensate for such disturbances. Consequently, they are significantly less fuel-intensive compared to the forced solution. The geometries of these natural formations are illustrated in Figure 2. To enhance the orbit duty cycle of the formations, additional satellites beyond the nominal number of elements of the array can be added. Fig. 3 shows the proposed redundant formations.

3. CASE-STUDY

Simulations were conducted to assess the efficacy of each formation. The orbits were designs around a compliant initial state, and subsequently subjected to local optimization. This optimization involved making slight adjustments to the formation amplitude. In the case of the phased formation, the phases between the satellites were also varied to achieve further refinement.

3.1. Array requirements

Table 1 provides the test case's nominal array parameters. In this particular scenario, the array comprises small satellites that are closely spaced, resulting in an array configuration that closely resembles a single fixed antenna.



(a) In-phase formation.

(b) Phased formation.

Fig. 2: Proposed passive formation solutions for cross-track SAR imaging.

Number of array elements	5
Radial separation	2 m
Tilt angle	20 °
Radial tolerance	± 0.5 m
Angular tolerance	± 5 °
Altitude	430 km
Satellite mass	100 kg

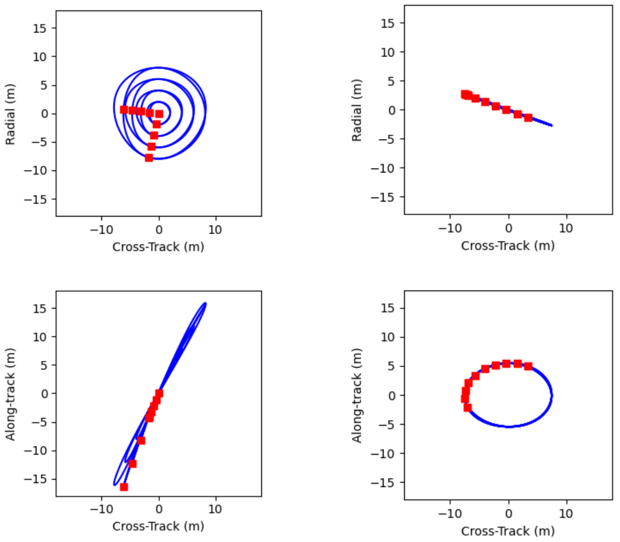
Table 1: Test case nominal array parameters.

The analysis conducted applies to other concepts where satellites are positioned at larger separations as well. However, it is worth noting that as the total length of the array increases, the formation becomes more distorted due to the Earth's rotation. This can pose challenges in constructing a consistent array for longer distances.

3.2. Forced formation feasibility

The required acceleration to maintain all satellites at their nominal positions can be calculated using the Clohessy-Wiltshire equations. This acceleration is directly proportional to the radial and cross-track components of the satellite's position. For the satellite furthest away from the origin in the presented case study, located 8 meters from the lowermost satellite, and assuming the array is centered at the origin of the frame, the calculated acceleration is 1E-5 m/s^2 and corresponds to a force of 1 mN for a 100 kg satellite mass.

For reference, Fig. 4 provides examples of thruster parameters available in the market, along with their respective thrust and specific impulse values.



(a) In-phase formation.

(b) Phased formation.

Fig. 3: Proposed redundant passive formation solutions for cross-track SAR imaging.

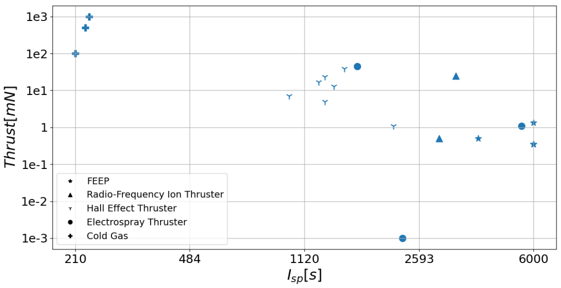


Fig. 4: Thrusters survey for different propulsion technologies.

Several types of propellants are adequate for the precise thrusting required. Cold gas thrusters are usually used for maneuvers with such precision, yet lack a high enough specific impulse for missions of long periods [5]. Sustaining a constant acceleration necessitates significant fuel consumption, which favors the adoption of more the efficient electrical propulsion technologies. In principle, these technologies can still deliver the required thrust magnitude. Nevertheless, electric propulsion has the drawback of demanding substantial power consumption. In our specific case, this power demand would impose a significant burden on the satellite's power budget. The current limitations of existing propulsion technologies render this concept impractical, particularly when considering longer arrays. Therefore, it is essential to explore alternative solutions that are more fuel-efficient and can address these technical challenges.

3.3. Natural formation results

3.3.1. *Natural formations results*

The simulation outcomes are presented in Figs. 5 and 6. These figures illustrate the trajectories of all the satellites within the formation, highlighting the sections of the trajectories that meet the angular, radial, or both criteria.

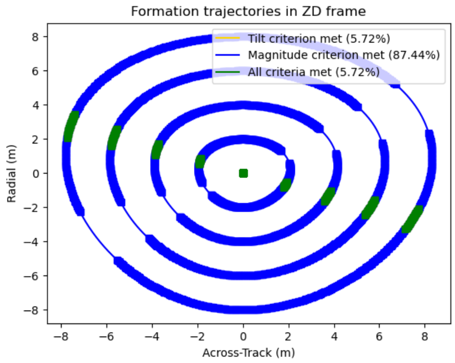


Fig. 5: In-phase formation performance results for the case study.

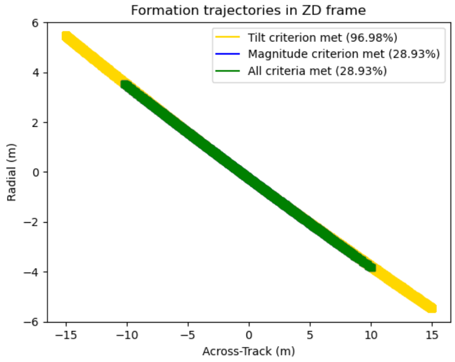


Fig. 6: Phased formation performance results for the case study.

Upon analyzing the results, we can be observed that the in-phase formation predominantly satisfies the magnitude criteria but fulfills the tilt criteria only during certain intervals. Consequently, this formation would be more suitable for mission concepts in which the satellites operate with a low duty cycle or concepts that can accommodate varying tilt ranges. On the other hand, the phased formation demonstrates the opposite behavior, meeting the tilt criteria for the majority of the time but being restricted by the range criteria. Nonetheless, it displays a commendable overall performance, reaching close to 30% duty cycle, which is reasonable for a SAR mission.

Both formations exhibit two regions of conformity separated by half a period. As a result, the formations cover latitude ranges in both hemispheres, separated by 180 degrees.

3.3.2. Redundant natural formations

Expanding the number of satellites beyond the minimum required for forming the array offers a twofold advantage. Firstly, it enables an increased orbit duty cycle, allowing for improved coverage. Secondly, distributing the power consumption among the additional satellites helps to alleviate the power burden on individual satellites within the formation. This approach can contribute to a more efficient and sustainable operation of the satellite array.

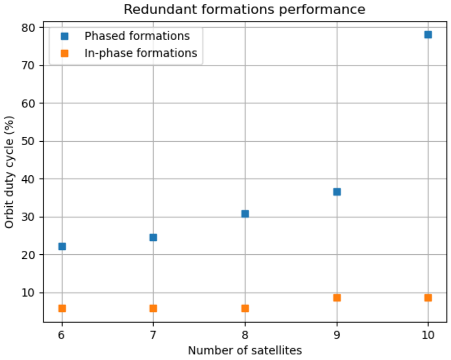


Fig. 7: Performance of redundant formations as a function of the number of satellites.

The in-phase formation requires the insertion of a new set of arrays to approximately double the duty cycle. On the other hand, the phased formation can benefit from the addition of any number of satellites. In that case, since the arrays can be formed using different combinations of satellites, the likelihood of generating an appropriate array increases as the number of satellites rises. Consequently, the gain in coverage is not linear but rather experiences a substantial boost as a larger portion of the common elliptical trajectory is covered, leading to significant improvements in performance.

3.4. Safety considerations

From a safety perspective, the forced solution can be considered one of the most favorable alternatives. In this configuration, the satellites are maintained in formation with different semi-major axes. This implies that in the event of a component failure with subsequent interruption of thrusting, the satellite would naturally drift apart from the others due to the differing periods.

The passive in-phase formation offers constant separation in both the radial and cross-track directions, resulting in a more robust and safer configuration. On the other hand, the passive phased formation carries higher risk due to its reliance on along-track separation. Regardless of the chosen configuration, both would require highly robust autonomous relative navigation and control systems, as well as a reliable Failure Detection, Isolation, and Recovery (FDIR) strategy to ensure safe operation.

4. CONCLUSIONS

This study has successfully demonstrated the feasibility of forming an array in elevation within certain tolerances to achieve a reasonable orbit duty cycle. Two solutions that do not rely on constant thrusting, except for minor perturbation corrections, have been presented. The first solution involves the satellites following synchronized movements along concentric ellipses in the zero-Doppler plane, offering enhanced safety but with compromised performance. The second solution entails the satellites following nearly identical trajectories with slight phase differences, resulting in better performance but with lower safety margins. Both solutions assume the ability to correct for the effects of along-track distance during SAR processing.

The findings of this paper provide a foundation for conducting more realistic analyses of distributed antenna mission concepts. Additionally, it proposes various formation concepts optimized for satellite distribution uniformity in the zero-Doppler plane.

5. REFERENCES

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