07

A decade of the Wits Maths Connect-Primary project (2010–2020): Design research moving promising interventions to scale

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Abstract

In this chapter, we trace the movement – over the course of the 2010–2020 decade – of interventions developed within the Wits Maths Connect-Primary project (WMC-P), and the scaling-up of the project from ten schools to provincial and national contexts. The focus of key interventions, the rationales for them, and the ways in which this approach to expanding scale differs from the larger-scale policy interventions are discussed. Learning outcome data, usually in a pre-/post-test design model, from all the interventions in the WMC-P project are included. We discuss this evidence of impact, and reflect on what the outcomes suggest as critical areas for focus in the next decade. In-service capacity-building through work with subject advisors and pre-service primary teacher education form particularly important thrusts within the emphases going forward.

KEYWORDS
early number learning, multiplicative reasoning, primary teachers' mathematical content knowledge, design research, South Africa

1 Introduction

A little over a decade ago, amidst concerns in South Africa about the lack of impact that research was having on improving learning outcomes in the basic education sector, a call was put out by a consortium that included the National Research Foundation (NRF) and the First Rand Foundation, seeking Research and Development Chairs in Numeracy and Literacy in 2010. The call document noted the requirement to work with at least ten public schools serving historically disadvantaged populations of learners, and stated explicitly that the goal was:

> *To research sustainable and practical solutions to the challenges of improving mathematics, numeracy and literacy education in schools (NRF 2010).*

We successfully applied for one of these Chairs with a proposal that had two key strands:

- a focus on improving number teaching and number learning in primary schools through the design, development, implementation, and study of research-based interventions,
- and attention to models and course-content for improving primary teachers' mathematical and pedagogical content knowledge.

A decade later, now in the third five-year phase of the Chair project, we reflect on the interventions that have now come to implementation at provincial and national levels, and on the models, evidence base and collaborations that have brought these interventions – that began at a much smaller scale – to these larger scales. We also reflect on the ways in which this iterative design research approach differs from the approach taken in the broader policy terrain, and note the key aspects that we feel remain important to address through research and development in early grade mathematics.

In this chapter, our attention is focused on how the trajectory of the WMC-P project worked with key problems that have been highlighted in the international mathematics education literature, and which are markedly present in the South African context. These problems are summarised briefly here. Firstly, while there is certainly evidence that the quality of mathematics teaching is important in ways that impact on learning outcomes (Hill et al. 2005), there are also caveats pointing to the difficulties with tracking how interventions that support professional development and that focus on improving both mathematical and pedagogic content knowledge translate into learning gains, even in small-scale studies (Darling-Hammond et al. 2017). Secondly, national and international evidence points to the fact that scaling up initiatives successfully is a key challenge that the mathematics education research and policy community have to grapple with (Cobb & Jackson 2015). While promising results from small-scale studies are abundant, the capacity for doing this at systemic level is challenging, and appears – at best – to substantially dilute the strength of success at larger scales. We state these problems here as research questions that frame the story shared in this chapter:

1. How can primary teachers' competence, confidence, and enthusiasm for teaching mathematics be supported in ways that can be studied alongside associated learning outcomes in early mathematics?
2. How can promising small-scale outcomes be taken to a large scale?

We begin with a brief background to the rationales for the initial foci within the WMC-P project, and then go on to consider key ways in which our two strands of focus – improving number teaching and number learning, and improving primary teachers' mathematical content and pedagogic content knowledge – played through into the design and implementation of an intervention trajectory that addressed the research problems identified above. This trajectory has involved two five-year phases: Phase 1, 2011–2015, and Phase 2, 2016–2020, with a third five-year phase now under way (2021–2025). A key part of this trajectory lay in dealing with the two strands in integrated ways rather than as separate strands of knowledge and practice. Our approaches to doing this, the success of initial interventions, and scaling up these successes are then discussed, with a summary of the outcomes achieved. We conclude with reflections on what this 'long' model of research and development has achieved, and why we feel that this patient pathway to change has been both necessary and important. We comment, too, on what we see as critical emphases for ongoing development of primary mathematics teaching in South Africa in the decade ahead.

2 2011: The rationale for initial foci: a literature review

When the WMC-P project was launched in 2011, we were aware of widespread evidence of highly inefficient counting-based approaches to working out the answers to number problems in learners' work in South Africa (Schollar 2008). Ensor et al. (2009) traced this back to a teaching method that kept learners in concrete counting approaches through insisting on work with counters and cubes to evaluate quantities, with limited evidence of moves to working with symbolic number representations and number relationships. Staying with teaching, there was also evidence of teachers' poor conceptual knowledge (Taylor & Vinjevold 1999), limited understanding of progression and pacing, and poor coverage of the curriculum (Reeves & Muller 2005).

Given that number as a topic makes up more than half of Foundation Phase (FP) curriculum content, together with the evidence of early number learning being a strong predictor of later mathematical performance (Geary 2011), improving early number teaching and learning was an obvious priority in deciding what to focus on in order to improve mathematics outcomes in the early grades.

The WMC-P project began work in 2011 with a set of baseline observations of teaching in Grade 2 classrooms in ten partner public schools across suburban and township settings. All of these schools had been identified as underperforming in district-level monitoring of mathematics outcomes. Drawing on the approaches of Bob Wright and his research team in Australia on early mathematical remediation (Wright et al. 2006), we adapted their model of individual task-based interviews. We interviewed learners with the help of people who spoke the African home languages of the children interviewed. In each partner school, we interviewed a cross-attainment sample of six learners, gathering in-depth data on their talking, gesture, work with manipulatives, and their writing in relation to a range of early number tasks that we offered to them orally in a conversational setting.

The set of data resulting from the learner interviews confirmed the prevalence of counting-based working in early addition and subtraction problems, with 75% of our cross-attainment sample of early Grade 2 learners across the ten schools working in these ways. To illustrate this way of working, when offered a problem involving, say, finding the total number of counters, with four counters in one hand and three in the other, the majority of the learner sample proceeded to count out four fingers on one hand, and then count out three fingers on the other hand, put the two hands together and count out all the open fingers from one to seven to get the total.

Alongside this, classroom observation data provided evidence of teachers ignoring the potential offered by artefacts like abaci and hundred charts to leverage the patterns in the decimal number system for more efficient calculation, and instead, they simply used these for counting in ones in the same way that counters had previously been used (Venkat & Askew 2012). More generally, all calculations were repeatedly worked with from first principles; results were rarely treated as 'established' and usable for deriving further results (Venkat & Naidoo 2012). These empirical data confirmed the need to focus on early teaching of number.

3 Two key intervention foci: primary school teachers' mathematical content knowledge and early number teaching

We focused our intervention projects on primary teachers' knowledge of mathematical content, and on teaching to support number learning. The former involved the design, implementation and study of a 20-day year-long course focused on primary mathematics knowledge from the perspective of teaching. The latter focused on two key projects: the Structuring Number Starters (SNS) project and the Multiplicative Reasoning (MR) project. The SNS project, launched in 2011, aimed to improve mental mathematical working in the CAPS-mandated 'mental starter' section of lessons. It began with the WMC-P team working directly with FP teachers in the ten partner schools in grade cohorts, but has grown over time and been rolled out at provincial level. The MR project began in the context of a series of postgraduate student studies with a single or a few classes using Askew's (2005) multiplicative word-problem teaching materials. It grew – over time – to provincial-level mediation of the approach through the work of FP district subject advisors, working with teachers. A further project has involved a collaboration with the Rhodes Numeracy Chair project led by

Mellony Graven and the DBE. This project's journey to national policy implementation is shared in Venkat and Graven's (2022) chapter, focusing on mental mathematics, in Volume 3 of this series. All of the teaching for number-learning projects has included the development of curriculum-linked teaching and learning materials that teachers are supported to use in classrooms.

Across the two main sections that follow, we outline the interventions to improve content knowledge first, and then the early number-teaching interventions. Across both foci, our work in the first five-year phase involved direct work with teachers. In the second five-year phase, as our attention turned to scaling up promising initiatives, we developed models to support district subject advisors when they mediate initiatives with teachers.

3.1 Interventions to improve knowledge of mathematical and pedagogic content

In Phase 1, we developed and implemented three cycles of the one-year 20-day Connecting Primary Maths (CPM) course, with groups of teachers drawn from the ten partner schools. The 20 days were made up of 16 contact days spread across the academic year in eight two-day blocks, with eight half-days of independent working on homework and school-based tasks for use in classrooms making up the remaining four days. As its name suggests, the emphasis in this course was on developing an understanding of key primary mathematics concepts from a pedagogic perspective. This meant paying extensive attention to mathematics as 'reason-able' in the sense that steps in mathematical working have reasons that need to be understood and communicated in teaching. This involved, in turn, sharing and discussing work, using representations and explanations that were key to understanding the concepts. We do not focus on the outcomes of this course here; these are reported in Venkat et al. (2016). We do note, however, that the 12–14 percentage point pre- to post-test gains that were produced across the three cohorts of teachers in each year is substantially higher than the level of gains noted within the subsequent PrimTEd study (Bowie et al. 2019) that was based on first-year and fourth-year pre-service teacher cohorts. Tests across both the WMC-P and PrimTEd projects shared several overlapping items. There was evidence of teachers' take-up of broader representational forms and inclusion of explanations – both of which were highlighted in the CPM course – as important ways of working constructively with mathematics when teaching.

These outcomes produced proof that it is possible to improve primary teachers' mathematical content knowledge from a pedagogic perspective, but a key limitation was that the model was difficult to scale up in the primary sector, because the generalist orientation to primary teaching makes it difficult to reach the large numbers of teachers involved in teaching mathematics. This pointed to the need to broaden capacity for supporting primary mathematics teaching within the system.

Prior research has also identified shortcomings in the content and pedagogic content knowledge of district subject advisors – the layer in place to support and monitor subject teaching (Taylor 2013). Subject advisors, in general, each need to support about 100 primary schools, which means heavy workloads and full diaries.

Such challenges have also been raised as working against capacity-building through extensive additional professional development (Metcalfe & Witten 2019). However, this layer also offered the most cost-effective route for support, as personnel are already in the system with the mandate to support teaching. This led us to design an intervention based on topic-specific materials packages that included pre-tests, a short sequence of lesson plans (usually four lessons for use once a week across four weeks), and post-tests. Collaboration with provinces led to our working with provincial cohorts of FP subject advisors on a package of two to three training sessions for them, that integrated attention to content knowledge and ways of working with teachers in schools. The focus in the training sessions was on the selected topic in the package: aspects of additive reasoning, multiplicative reasoning, and base-ten thinking have all been rolled out through this approach. Training was followed by subject advisors familiarising themselves with the materials before, and through, working with Grade 2 or Grade 3 teachers in a school in their districts. Their work included working alongside teachers to administer and mark pre-tests, run the lesson sequence, and then administer and mark the post-tests, usually over a six-week period. Outcomes of these scale-up trials in North West that sought to develop content and pedagogic content knowledge in order to build capacity for supporting primary mathematics teaching are reported in the following sections.

Thus, in order to scale up attention to primary mathematics content knowledge from the perspective of teaching, this focus was shifted to teaching interventions in which teachers were supported by district subject advisors and provincial leaders, rather than by our small research and development team. The Phase 2 focus on development of mathematical content knowledge is therefore integrated into the section on teaching interventions that follows.

3.2 Improving early number teaching: initial models, outcomes, and expansions of scale

3.2.1 The Structuring Number Starters project

As we have noted, evidence seen in the baseline data collected from our ten partner schools in Gauteng showed highly inefficient counting-in-ones strategies for solving addition and subtraction tasks. In the Structuring Number Starters (SNS) project, we drew on an international research base that highlighted the importance of mental flexibility with early number, founded on developing strong number sense as a critical foundation for all mathematical working, while also highlighting the insufficient emphasis on early number sense that exists in our schools (Baroody & Dowker 2003).

Phase 1 – a focus on number structure: To address the problem of learners using inefficient counting-in-ones strategies for additive tasks, we developed and trialled a package of materials and training for teachers, for use with cohorts of learners as they moved across Grades 1 to 3 in the ten public primary schools with which the WMC-P project was partnered. The materials and activities developed in this part of the broader WMC-P project aimed to develop learners' understanding of number structure during the mental mathematics segment of the lesson, and thus was called the Structuring

Number Starters project.

By ‘number structure’, we refer to a range of number relationships and properties. In our first phase, between 2011 and 2015, this focus included ordinal relationships involving counting on and counting back in ones from various numbers, because this fluency was important in breaking the prevalence of counting out all quantities from one. Number relationships also included number combinations or bonds, and a range of relationships involving tens as ‘friendly numbers’ to work with in the decimal number system. This kind of ‘base-ten thinking’ involves seeing and using ten as a unit when solving arithmetic tasks – which implies moving away from counting-in-ones (Wright et al. 2006). Gervasoni et al. (2010, 316), drawing on a wide range of literature in the field of early number learning, state that:

> *Children’s success with solving 2-digit by 2-digit problems relies heavily on their understanding of ten as both a collection of ten ones and as a single unit of ten that can be counted, decomposed, traded, and exchanged for units of different value.*

Task presentations also included key representations that promote and support attention to number structure, such as number lines and part–part–whole diagrams. Examples of tasks and representations that work across these aspects of number structure by drawing attention to number relationships rather than to counting are presented in Figures 1 to 5.

The location of this work in lesson starters was linked to the introduction in the Curriculum and Assessment Policy Statement (CAPS) of a far more tightly prescribed sequence of coverage for the main activity within mathematics lessons, with schools’ progress with this sequence being monitored by district subject advisors. The mental starters lesson segment, also prescribed in the CAPS document, therefore provided an opening for intervention in ways that dovetailed with the policy.

Phase 1 – Outcomes: We took the Chairs’ mandate to function as linked research and development projects seriously. For us, this meant ensuring that research evidence included learning outcome data linked with the interventions we had designed and implemented. This was important to the design research orientation of the project, where the efficacy of intervention models needed to be understood, and where adaptations to these models were data-driven. It was also important in the broader context of critique: a lack of rigorous data in many education interventions in South Africa had been noted (Mouton et al. 2013). In the longitudinal Structuring Number Starters project, we repeated the early Grade 2 baseline interviews of 2011 with a parallel set of interviews in 2014, having worked in the interim with teachers in the partner schools on the Structuring Number Starters materials and training package. In repeating the task-based interviews with the 2014 early Grade 2 cohort, we noted an important difference: over half the learner sample (56%) were now able to work with what are described as ‘count-on’ approaches (see Venkat et al. 2021 for more detail on the sample and outcomes). Now, arriving at the total of the ‘four counters in one hand and three in the other hand’ task usually produced a version of this kind of response:

> *The learner gestures towards the interviewer’s hand with four counters and says the word ‘four’. She then opens three fingers, one at a time, counting alongside this with the words: ‘five, six, seven’. She stops and says: ‘Seven’.*

Figure 1: Splitting numbers into different combinations

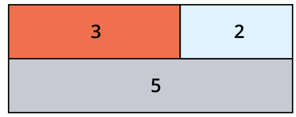
I can split a set of 5 stickers into 3 stickers and 2 stickers	
Here, we see the quantity 5 as made up of '3 and 2' *There is a relationship between the 3 quantities. 5 = 3 + 2*	

Figure 2: Which is closer?

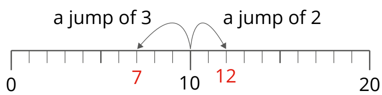
Which number is closer to 10, 7 or 12?	
To answer this question, we have to picture the position of 7 and 12 in relation to 10. *A number line is useful for showing this – so, 12 is closer to 10.*	

Figure 3: Working with doubles

6 + 7 is 13 because 6 and 6 is 12, so 6 and 7 is one more than that.
Here, 6 + 7 is seen in relation to 6 + 6, which is an easy 'double' fact. Rather than working out 6 + 7 by counting, the learner uses a known fact and then adapts this result for the new problem.

Figure 4: Subtraction on a number line

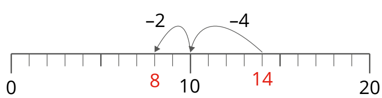
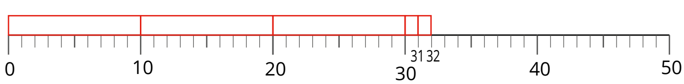
I work out 14 – 6 by taking away 4 to get to 10 and then taking away the remaining 2.	
Here, 14 is seen in relation to the '10' that comes before it. 6 is broken down into 4 and 2, as this combination allows for a jump back from 14 to 10 as a first step.	

Figure 5: Adding two-digit numbers

32 can be broken down into 30 and 2, so when I calculate '54 + 32' I can first add 30 to 54 (54 + 30 = 84) and then add the 2 to the interim result (84 + 2 = 86) to get my answer.	
We can break down numbers into 'tens' and 'units'. Here, we see the number as made up of a 'multiple of tens' in value and as a set of units.	OR alternatively 

Source: Authors.

In 'counting-on' from four, the triple count seen in count-all approaches is reduced to a single count of the second part of the quantity. It is therefore substantially more efficient than count-all strategies, but retains some counting in ones. This result was important to us as it indicated improvements in learners' fluency with counting-on and counting-down-from a range of numbers, in comparison with what we had seen in 2011.

Phase 2 – Emphasising base-ten number structure: In the WMC-P's second five-year phase, we followed up on a request from the Gauteng Department of Education to broaden our work into another district. This led to a partner school group comprising four new schools in a second district, and six schools from Phase 1 continuing into Phase 2. In terms of our focus, the 2014 outcomes reflected increases in the uptake of working with the Structuring Number Starters tasks and materials amongst teachers – seen in our observations of more coherent and more progression-oriented teaching over time (Askew et al. 2019a). This led to some rethinking of the emphases within our tasks, materials, and training. Specifically, emerging findings from Morrison's (2018) doctoral study showed that improving children's work with base-ten relationships was particularly important within improving their overall performance on early number. The improvements in counting fluencies seen in Phase 1 allowed us to shift our focus entirely to working with tasks oriented to number relationships, and base-ten tasks within this, replacing our earlier inclusion of tasks that focused on counting fluency. In Phase 2, we worked with the types of tasks shown in Figures 1–4 in this chapter, from Grade 1 onwards, and incorporated Figure 5 task-types with Grades 2 and 3.

Towards the end of Phase 2, the pack of materials for Grade 3 was formalised into a learners' workbook and a parallel teachers' guide (Morrison 2020a & b) that included details on points to draw attention to in instructional talk, alongside the presentation of the key base-ten structural representations highlighted above. This packaging of the materials into the workbook and teachers' guide offered two key advantages. Firstly, the teachers' guide included key 'educative' elements (Davis & Krajcik 2005, 3) in the form of representations and explanations that were important for supporting number sense. This brought in the elements of content and pedagogic content knowledge that we had identified as critical to linking the growth of content/pedagogic knowledge with classroom pedagogy. Secondly, this packaging of educative materials was important to being able to scale up the project to a provincial level.

Phase 2 – Outcomes: Outcomes in the 2018 round of early Grade 2 interviews (based on the six schools that had participated across Phases 1 and 2) pointed back to the attention to number relationships and base-ten thinking in interesting ways. In 2011, only 2.8% of the learner sample showed any competence with using number relationship and/or base-ten-oriented strategies that moved beyond counting in ones. By 2014, despite the substantial shift from counting all, to counting on, the proportions using number relationships/base-ten strategies increased by a more limited margin (2.8 percentage points) to 5.6%. In the 2018 data set, a much larger shift was seen in this marker, with 25% of the early Grade 2 learners in the interview sample able to use efficient number relationships or strategies in their working with early addition and subtraction. This result was particularly important, as it showed that it was possible – relatively early in the FP – to use the materials and training combination in ways that enabled a much larger proportion of learners to get to the point of using efficient calculation strategies (see Venkat et al. 2021 for more details).

Phase 3 – The Base-Ten Thinking project in Grade 2: As noted already, a consequence of this evidence of promising shifts in learning outcomes over time was that materials were collated. In 2020, Gauteng shared these materials with subject advisors and lead teachers in 150 schools for use in Grade 3. Although roll-out was curtailed by the onset of the Covid-19 pandemic, interim feedback indicated positive responses to the materials themselves and the possibilities for their use in the mental starter section of lessons. In Phase 3, there has been a Grade 2 roll-out called The Base-Ten Thinking project in Gauteng and KwaZulu-Natal that involves subject advisors and heads of department in 75 schools (225 classes) and 36 schools (108 classes) respectively, across the two provinces. In this trial, we have included training for the subject advisors and heads of department that is geared towards building awareness in the system of important aspects of early number progression. Thus, in the ten-year period the Structuring Number Starters project has moved from trials in FP classes in ten schools, to provincial buy-in and roll-out, with ongoing research studies linked to the various stages of scaling up.

3.2.2 The Multiplicative Reasoning project

Multiplicative reasoning (MR) refers to the kinds of thinking underlying situations that are underpinned by a multiplicative structure, which involves an implicit sense of ratio, even though this is not acknowledged in most early grade teaching. For example, in a problem such as “If there are three apples on each plate, how many apples are on four plates altogether?” the implicit ratio is 1:3, a ratio of one plate to three apples. This is in marked contrast to the additive structure within “If there are three apples on one plate, and three on another, how many apples are there altogether?” (See Askew [2018] for discussion of the distinction between these two structures). The subtle shift in structure marked by the difference between ‘each plate’ and ‘one plate’ was at the centre of the MR projects. The focus on multiplicative structure follows the broad consensus in the mathematics education literature base on MR as a foundational pillar upon which much of mathematics in the Intermediate Phase and beyond (e.g. fractions, percentages, ratio and proportion, gradients, and trigonometry) is built.

The international literature points to children commonly finding it difficult to distinguish multiplicative situations from additive situations, and reverting to addition/subtraction in problems that call for multiplication/division (Anghileri 2000). This problem is seen in South African evidence too, with the 2019 Trends in Mathematics and Science Study (TIMSS) assessment data providing examples of this (Bowie et al. 2022). The South African evidence shows, however, that learners not only find it difficult to distinguish between additive and multiplicative situations: they also have difficulty in correctly carrying out multiplication and division calculations. Schollar’s (2008) data from Grade 5 and Grade 7 learners show ongoing use of the counting-in-ones approach described earlier, in the context of two-digit by two-digit multiplication problems, with this highly inefficient approach shifting – at best – to counting in multiples of the multiplier or divisor, rather than fluency being developed in the algorithms for multi-digit multiplication and division.

There were thus two issues to think about: building fluency with basic multiplication facts, and supporting teachers who were teaching children how to recognise multiplicative situations. Both of these features underpinned the design of the MR project.

Phase 1 – Models and outcomes: Early in Phase 1, we had looked at Askew's (2005) teaching and learning materials which included lesson sequences focused on MR. Using simple story situations, Askew directed teachers' and learners' attention towards enacting and making representations of these situations, with teaching focused on drawing attention to key features that are common to multiplicative situations, and specifically, the occurrence of iterations of 'equal groups' as a way of making explicit the implicit ratio structure of multiplicative situations. From 2012 to 2014, a number of postgraduate students conducted sequences of intervention lessons, topped and tailed with pre- and post-tests, with small numbers of classes within a grade in schools. Early results from these studies indicated that this model – of a short-run sequence of four to six carefully designed lessons – could produce promising learning gains (e.g. see Dlamini 2014; Hansa 2015).

Sharing results from these studies led to an invitation in 2015 from a Phase 1 partner school to try out this short-run intervention model across the school's entire FP. In this study, WMC-P team members developed a sequence of four intervention lessons tailored across Grades 1–3 (see Askew 2015), and worked with all the FP teachers as they implemented these lessons once a week, topped and tailed, as before, by pre- and post-tests. These materials, once again, included learners' tasks and teachers' notes. Reporting on this study, Askew et al. (2019b) noted high levels of gains: Grade 1 learners had a mean score average increase of 22 percentage points between the pre-test and delayed post-test, with Grades 2 and 3 learners having mean increases of 10 and 9 percentage points, respectively.

Phase 2 – Expansions and outcomes: The positive outcomes in the initial trials led to a scaling up to all ten Phase 2 partner schools in a staggered arrangement in 2017, and we dipped into studying MR in the Intermediate Phase. Keeping the model of a pre-test, followed by four intervention lessons and a post-test, WMC-P team members worked in this design iteration with one teacher in each Grade 4, 5 and 6 class in six schools, with Grade 7 teachers also involved in two schools. Promising outcomes were seen in Grades 5 and 6, with the results in these two grades being statistically significant. In Grade 5 intervention classes (n = 234) there was a pre- to post-test increase of 7 percentage points, compared with only a 1 percentage point increase in the control classes (n = 142). In Grade 6 there was a 5 percentage point gain in the intervention classes (n = 209) compared with a 2 percentage point decrease in control classes (n = 111). In Grade 7 there was a 14 percentage point gain (n = 143). Although no control class data was gathered for Grade 7, Venkat and Mathews (2019) illustrated the beginnings of the gains across the Grade 7 classes in improvements in learners' capacity to set up appropriate models of MR situations, and in moves towards more efficient calculation. Of interest, and in spite of the test items being pitched at lower number ranges than those specified in CAPS, the data showed that Grade 4 learners did not demonstrate any benefit from the intervention. There were also substantial differences between the schools in terms of the success of their outcomes.

Across the MR interventions detailed thus far, the WMC-P team worked directly with and alongside the teachers, implementing the structured intervention lesson plans. While providing useful insights into implementation, in depth, this model was clearly limited in its potential to be scaled up. Thus, the next iteration of the MR lesson sequence model looked at broadening the capacity for supporting teachers with their teaching of MR within the education system, and set out to understand whether learning gains were possible to effect through working with this 'intermediary' model. In 2019, a provincial group of FP subject advisors was brought together and an implementation model was developed that included training sessions run by the WMC-P team. The focus was on a combination of MR content knowledge, key representations and why they were useful, given the South African evidence, and also on working with teachers collaboratively rather than evaluatively. This latter aspect was important in the context of evidence that subject advisors in South Africa tended to view their work primarily as monitoring teachers and checking that policy mandates for coverage (of content, etc.) were adhered to, rather than providing teaching support.

Reporting on the outcomes of this scaled-up MR iteration, Venkat and Askew (2021) noted two important findings. First, outcomes based on pre- and post-tests administered by the subject advisors suggested substantial pre- to post-test improvement at the learner level. Changes in performance, when assessed on ten multiplicative items, showed mean marks increasing from 31.7% on the pre-test, to a post-test mean of 46.9%, an increase of 15.2 percentage points (n = 1022 Grade 2 learners). This finding encourages us to think that a multi-level model of support – the university team working with mathematics subject advisors, who, in turn work with teachers, and teachers working with learners – has potential for building both advisors' and teachers' capacities and to raise standards. Second, observations and reflections from the subject advisors indicated their increased awareness of both the need for, and skills in implementing more dialogic conversations with teachers, conversations that focus on mathematics and its teaching and learning. In follow-up meetings with the subject advisors, they noted that this was a marked move away from the more usual conversations they had with teachers, that were more one-sided (advisors telling teachers) and more focused on policy and curriculum implementation, irrespective of individual school, teacher, and learner needs.

In Phase 3, the provincial model of working via subject advisors has been replicated in a second province with results similar to those shared in this chapter (Morrison 2021).

4 Reflection on expanding the scale

It is interesting that a project that began in ten schools in one district has been able – in a ten-year period – to bring interventions to provincial and national scales. Incorporating the collection of data on learners' understanding of the content area in our focus has been an important part of building a grounded data-corpus revealing change over time, and directing attention to what is still to be achieved. Short-run design research iterations have allowed us to gather detail on children's additive and multiplicative understandings in the early grades, alongside a focus on teachers' ways of working with these topics in classrooms. We have analysed data for differences over time at the levels of teaching and learning. Supporting changes in in-service teaching on a larger scale is now being looked at through interventions by subject advisors, who can build capacity at the levels of knowledge and practice for this more collaborative way of working on the ground. Current evidence suggests that this is possible to do, with our sense that a multi-year model of working with subject advisors on classroom-based interventions on key topics may well be the most productive way to start entrenching both capacity for supporting primary mathematics teaching and learning, and an orientational shift towards this being the core function of district-level support.

The iterative design-based research model carries a key caveat: the lack of comparable control schools. In the first two phases of the project, the aims and the available funding were closely linked to ten schools and explorations within them of models that showed promise in terms of learning gains. In moving to provincial-level interventions now, some of our externally funded projects are now starting to include parallel control schools. We note that in the South African terrain of such low outcomes in mathematics, this deferral of experimental designs may well be useful (and cost-effective), given the more pressing priority to simply develop and understand interventions that succeed in raising learning outcomes.

While much has been achieved within this trajectory of work, an area that we are only starting to move into now is pre-service primary mathematics teacher education. Bachelor of Education programmes, with their four-year timelines, should provide important spaces for building more principled education systems, and breaking the vicious cycle of poor learners being taught by teachers with gaps and limitations in their own mathematical understandings. We are armed with some evidence now on how primary teachers can be supported in order to teach for learning and for progression. Thus, as we begin our third five-year cycle, scaling up in the in-service terrain by working with subject advisors on the roll-out of interventions, and in the pre-service terrain by working with higher education institutions become our key points of focus.

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