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Identification of families among highly inclined asteroids

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Abstract

A dataset of 3652 high-inclination numbered asteroids was analyzed to search for dynamical families. A fully automated multivariate data analysis technique was applied to identify the groupings. Thirteen dynamical families and twenty-two clumps were found. When taxonomic information is available, the families show cosmochemical consistency and support an interpretation based on a common origin from a single parent body. Four families and three clumps found in this work show a size distribution which is compatible with a formation due to a cratering event on the largest member of the family, and also three families have B- or related taxonomic types members, which represents a 14% of the B-types classified by Bus and Binzel [2002. Icarus 158, 146–177].

Keywords: Asteroids; Asteroids, dynamics

1. Introduction

Asteroid families consist of swarms of fragments generated after energetic collisions between asteroids, resulting either in the formation of large craters on the target bodies or in their complete breakup. Families are very important for asteroid studies because they represent a constraint for any attempt at modeling the overall collisional history of the asteroid belt and provide useful information about the physics of breakup events from which they originated.

The families are recognized by searching for clusters in the three-dimensional space of proper elements, parameters characterizing the asteroid orbits which are very close to invariants of motion and originally remember the initial proximity of the orbits generated by a single fragmentation event. The foundation of this work was laid by Hirayama (1918, and following papers) who was the first author to use proper elements to search for families and discovered the most populous families still known today as the *Hirayama families*: Eos, Themis, Koronis, Flora, and Maria. It is now understood that these clusters are subject to slow spreading and dispersal due to resonance (Nesvorný et al., 2002a), close encounters with large asteroids (Carruba et al., 2003), or Yarkovsky effect (Bottke et al., 2001), thus an old family do not reflect the immediate outcome of a collision.

This pioneer work was followed by many others (Brouwer, 1951; Arnold, 1969; Lindblad and Southworth, 1971; Carusi and Massaro, 1978; Williams, 1979, 1989; Kozai, 1979), but no uniform agreement has been achieved and the discrepancies were generally very large due to different datasets and classification methods. Moreover, the disagreement on the number of families proposed by different authors and their memberships was almost complete, situation that became worse due to cosmochemical inconsistencies in some of the proposed families (Chapman et al., 1989). Thus, only the classical and very prominent *Hirayama families* could be taken seriously into consideration as the result of the disruption of a common parent body.

However, the family identification techniques has improved significantly during the last decade. The family memberships have been redetermined with much improved reliability due to the availability of better catalogues of many thousands of asteroid orbits (Knežević and Milani, 2003), to the development of refined secular perturbation theories for the computation of

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proper elements (Milani and Knežević, 1994; Knežević and Milani, 2000; see also Knežević et al., 2002), and to the application of powerful automated clustering techniques to identify the statistically significant groupings in the proper element space. Automated techniques present several advantages with respect to techniques based mostly on visual and subjective inspection of the data: they are much less prone to subjectivity, the results are more easily reproducible, it is possible to use different metrics to produce alternative results, robustness tests of the results can be easily performed, and the statistical significance of the identified groupings can be verified. There are many different identification techniques available, but nowadays the Hierarchical Clustering Method (HCM; Zappalà et al., 1990, 1994) is the preferred method to find asteroid families (for example, Mothé-Diniz et al., 2005; Nesvorný et al., 2005).

The family searches mentioned above were based on datasets of proper elements in which high-inclination and high-eccentricity objects were not included. This is due mainly to the reason that the analytical theory used to obtain proper elements (Milani and Knežević, 1994) is very accurate for asteroids with small inclination and eccentricity, but it is not accurate for objects with $\sin i$ and/or $e > 0.3$. Based on previous work by Williams (1969, 1979, 1989), Lemaître and Morbidelli (1994) proposed a semianalytical theory to calculate proper elements which is particularly suited for these highly inclined and eccentric asteroids. Later, Knežević and Milani (2000) adopted an approach similar to that used for the outer planets by Carpino et al. (1987) to obtain "synthetic" proper elements for a large number of asteroids using only numerical techniques. The results obtained by these authors shown that the proper elements have a better time stability with respect to the elements computed by an analytical theory and can be applied to high-inclination and high-eccentricity objects. In spite of these efforts, there is not published any systematic search for families in this region of the main belt using the automatic techniques described above.

Since most of the previous work on asteroid families deals only with the cases having $\sin i < 0.3$, in this paper a search for significant groupings among highly inclined asteroids using the HCM is presented. In the following section the database and the clustering technique are described, but also the method to quantitatively assess the significance of families is explained. In Section 3 the list of the families identified is presented and briefly discussed. The main conclusions are outlined in the final section.

2. The Clustering Method

The basic dataset was the list of 3697 asteroid synthetic proper elements (proper semimajor axis a', proper eccentricity e', and proper inclination i') taken from the Asteroid Dynamics Site (AstDyS—http://hamilton.dm.unipi.it/astdys, March 2005 version; Knežević and Milani, 2000). For this work only asteroids with sine of the proper inclination greater than 0.3 were used. Figs. 1 and 2 show the distributions of the dataset in the (a', e') and $(a', \sin i')$ planes, respectively.

The HCM has been described by Zappalà et al. (1990, 1994, 1995) and it will not be explained here. Following these authors, a standard metric have been adopted and distances in proper element space are defined as $d = n\langle a'\rangle[k_a(\delta a'/\langle a'\rangle)^2 + k_b(\delta e')^2 + k_c(\delta \sin i')^2]^{1/2}$, where $\delta a'$, $\delta e'$, and $\delta \sin i'$ are proper elements

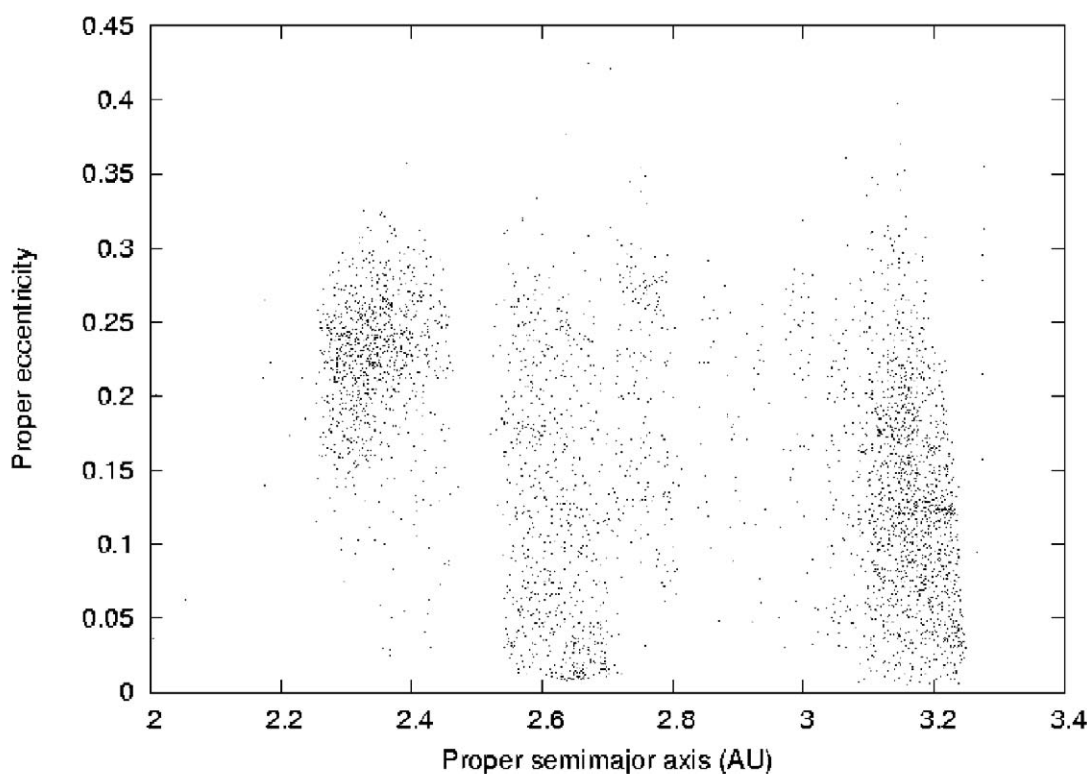

Fig. 1. Distribution in the (a', e') plane for the 3652 highly inclined asteroids used for family identification.

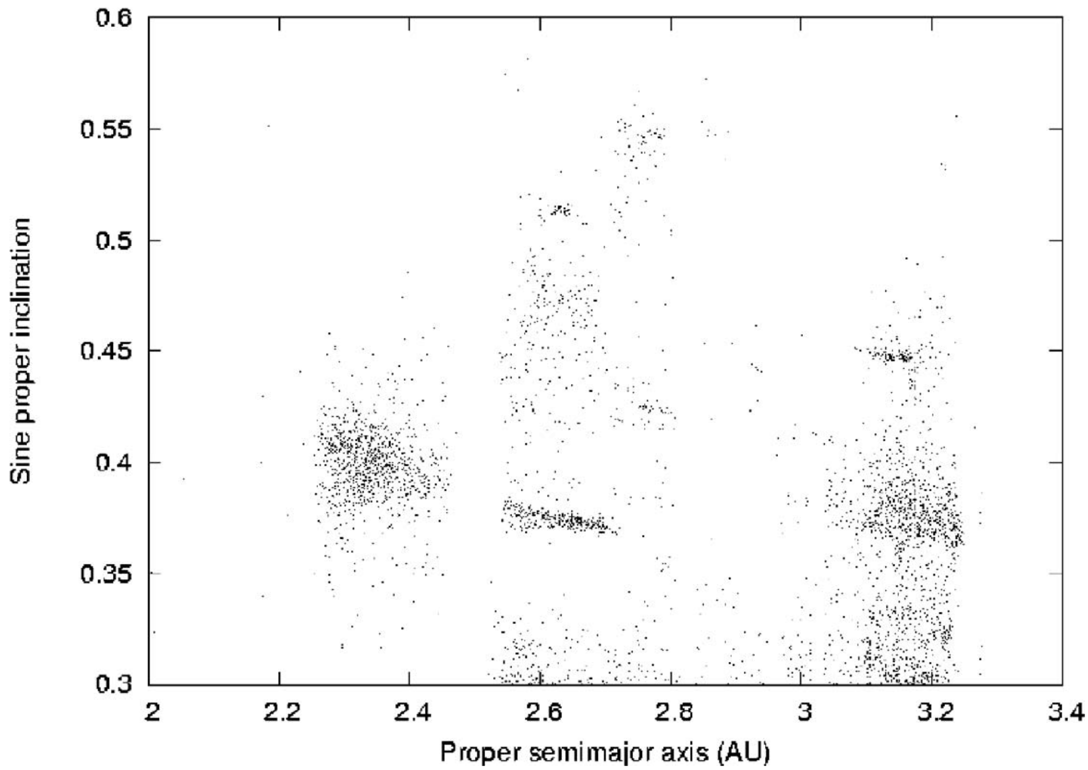

Fig. 2. Distribution in the $(a', \sin i')$ plane for the 3652 highly inclined asteroids used for family identification.

differences, $\langle a' \rangle$ and n are the average proper semimajor axis and mean motion of the two orbits, and $k_a = 5/4$, $k_b = 2$, $k_c = 2$ are constants (Zappalà et al., 1990, 1994). Other possible values for these constants produce similar results.

The choice of the cutoff distance at which family members are defined, d_0, is critical. One possibility is to use the nominal cutoff d_{nom} of the corresponding region of the asteroid belt. This value represents the statistical limit to differentiate real cluster from background groupings, and it may be computed as the average minimum distance between all the neighboring asteroids in the same region of the asteroid belt (Beaugé and Roig, 2001). A second possibility consists into detection of all the clusters at different values of the cutoff d_0 and apply some criterion to determine which families can be defined at each cutoff (Nesvorný et al., 2002b).

In principle, the use of a nominal cutoff would be enough to find asteroid families, but the analysis of the family structure at different cutoff levels may help to determine the presence of interlopers (Migliorini et al., 1995), and may also help to differentiate subfamilies inside families with complex structure. Thus, the identification of families was done varying the value of cutoff in steps of 10 $\text{m}\,\text{s}^{-1}$ within two extreme values, d_{min} and d_{max}. The minimum corresponds to the smallest cutoff at which the first cluster started to be detected, while the maximum corresponds to approximately $d_{\text{nom}} - 10\ \text{m}\,\text{s}^{-1}$.

Another relevant parameter involved in the HCM is the minimum number of objects, N_{min}, for a cluster to be considered significant. This may be determined by computing the average number of orbits, N_0, within a sphere of radius d_0 at every point of the proper element space, and defining $N_{\text{min}} = N_0 + 2\sqrt{N_0}$ (Zappalà et al., 1995). On the other hand, the limiting number, N_{lim}, such that clusters may be considered nominal families (clusters related to the breakup of a large asteroid) or clumps (clusters not necessarily related to breakup events) if they have a number of members larger or smaller than this limit, respectively, is defined as $N_{\text{lim}} = 2.5 N_{\text{min}}$.

Table 1
Properties of the different zones considered

Parameter	Zone A	Zone B	Zone C
a_{min} (AU)	2.065	2.501	2.825
a_{max} (AU)	2.501	2.825	3.278
N_{tot}	1000	950	1702
d_{nom}	163.05	184.84	141.18
d_{min}	80.00	45.00	35.00
d_{max}	150.00	175.00	130.00
d_0	140.00	165.00	125.00
N_{min}	7	10	7
N_{lim}	18	25	18

In order to accomplish the family search in a more efficient way, the main asteroid belt with $\sin i > 0.3$ has been divided in three semimajor axis zones limited by the 3:1, 5:2, and 2:1 mean motion resonances with Jupiter. For each zone, the total population, the nominal cutoff (d_{nom}), the minimum and maximum cutoff values (d_{min} and d_{max}), the adopted cutoff (d_0), the minimum number of objects to be considered significant (N_{min}), and the limiting number (N_{lim}) are listed in Table 1. Since the families found are well defined at cutoff levels smaller than d_{nom}, the adopted cutoff values for each zone are $d_0 \approx d_{\text{nom}} - 20\ \text{m}\,\text{s}^{-1}$.

3. Results

A total of 1339 asteroids (36.7%) in the sample dataset of 3652 (45 objects were excluded from the original dataset because their semimajor axis are larger than 3.278 AU) were classified by the procedure described in the previous section in thirteen families and twenty-two clumps, which are listed in Table 2.

For each family or clump this table gives an identification (zone and family in that zone), the least-numbered member of the family, the number of members at cutoff d_0, the number of members with known taxonomy (Bus and Binzel, 2002; Bus, 1999; Lazzaro et al., 2004; Tholen, 1989), and the main taxonomic type in the Bus taxonomy. Figs. 3 and 4 show the location in the proper element space of the families identified in this work. Tables with proper element ranges of variation and the complete memberships of all the families and clumps listed in Table 2 could be obtained from the author upon request.

The three families found in zone A are in the Phocaea group. This region of the belt is separated from the adjacent belt asteroids by a secular resonance and commensurability gaps, and also crossed by secondary secular resonances, so any concentration of objects in this region should not necessarily by interpreted as a family with a common origin from a single parent body and it might be a stability island (Knežević and Milani, 2003). Nevertheless, if this stable dynamical island contains the parent body of a true family the fragments could or could not cross the stability boundaries making the detection of such a family very difficult. Thus, these families are included in this work because they appear as compact clusters of asteroids in proper element space and it is not easy to prove if they are real families or not. The family of 25 Phocaea (A1) appears clearly at $d_0 \approx 120$ m s^{-1} after many subgroups formed at smaller d_0 joined together. Other subgroups joined this family at $d_0 \approx 130$ m s^{-1}. The objects with known taxonomy in this family are mainly S-type. The family of 1660 Wood (A2) is formed by two similar subgroups joined at $d_0 \approx 140$ m s^{-1} and appears at $d_0 \approx 110$ m s^{-1}. This family shows fairly homogeneous in composition, S-types being clearly predominant, but has the peculiar feature of including several bodies of similar size. These two families appear to reside at opposite extremes of the proper element space centered around 25 Phocaea, the largest object in the region, and adjacent each other, so perhaps they are two subgroups of the same family divided by dynamical effects.

The first family found in zone B is 2 Pallas (B1), which was originally suggested by Williams (1992) (family 129) and Lemaître and Morbidelli (1994), and appears at $d_0 \approx 145$ m s^{-1}. The largest asteroid in this family (2 Pallas) is a B-type object, as other three small members (2382, 5222, and 5234), but 4969 is a C-type. The largest family found in this zone is 480 Hansa (B2), originally proposed by Hergenrother et al. (1996). The core of this family is formed at $d_0 \approx 55$ m s^{-1}, but many large subgroups were included at $d_0 \approx 105$, and 155 m s^{-1}. The only two objects with a taxonomic class are 480 and 4880, and both are S-types. These two families are composed by a large object and many small asteroids, some of them with similar taxonomic types, so they are compatible with an origin due to a cratering event on the largest member. In the case of family B1, Pallas resides in the large $(a', e', \sin i')$ extreme of the family while the smaller members are distributed along a narrow region of proper element space to smaller values of $(a', e', \sin i')$. On the other hand, family B2 shows a proper element distribution which seems produced by a conic velocity field centered on 480 Hansa. The last family in this zone, 686 Gersuind (B3), has two objects with known taxonomic class in the center of the family proper element space, but they are of different types (686 is an S-type and 1609 is a D-type). This does not make cosmochemical sense, so one of these asteroids may be an interloper.

The largest number of families were found in zone C. The family associated with 31 Euphrosyne (C1) is formed at $d_0 \approx 95$ m s^{-1} when two other subgroups and that which contains this asteroid joined together. The other members of this family are all small objects with radius of few kilometers, so it looks like this family could also be the result of a cratering event on the large Cb object Euphrosyne. The largest family

Table 2

Id	Name	N	N_{spec}	Tax
	(a) Nominal families identified			
A1	25 Phocaea	293	19	S
A2	1660 Wood	26	5	S
A3	5247 Krylov	24	0	
B1	2 Pallas	22	5	B
B2	480 Hansa	265	2	S
B3	686 Gersuind	28	2	
C1	31 Euphrosyne	48	1	
C2	702 Alauda	134	5	B–Cg
C3	1303 Luthera	40	0	
C4	1901 Moravia	118	2	B
C5	4152 Weber	57	0	
C6	6051 Anaximenes	32	1	
C7	16708 1995 SP1	39	0	
	(b) Clumps identified			
A4	326 Tamara	13	1	
A5	2860 Pasacentennium	7	0	
A6	6179 Brett	9	0	
A7	6446 Lomberg	7	0	
A8	6487 Tonyspear	8	0	
A9	7779 Susanring	8	0	
B4	148 Gallia	12	1	
B5	945 Barcelona	16	1	
C8	181 Eucharis	8	1	
C9	276 Adelheid	11	1	
C10	1444 Pannonia	9	2	
C11	2892 Filipenko	9	1	
C12	4379 Snelling	9	0	
C13	6534 1995 DT1	10	0	
C14	7838 Feliceierman	9	0	
C15	15848 1995 YJ4	7	0	
C16	18568 Thuillot	12	0	
C17	18895 2000 GJ108	7	0	
C18	19907 4220 T-3	15	0	
C19	24794 1993 UB7	7	0	
C20	30956 1994 QP	10	0	
C21	56286 1999 LG9	10	0	

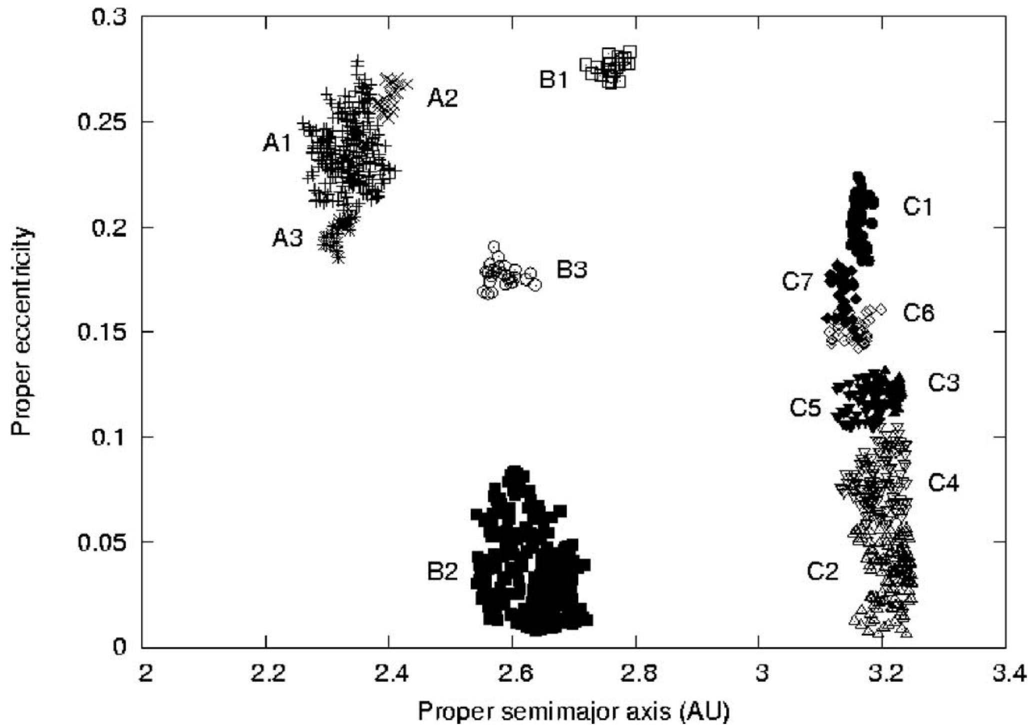

Fig. 3. Distribution in the (a', e') plane for the 13 families identified. The families are labeled with the identifications listed in Table 2.

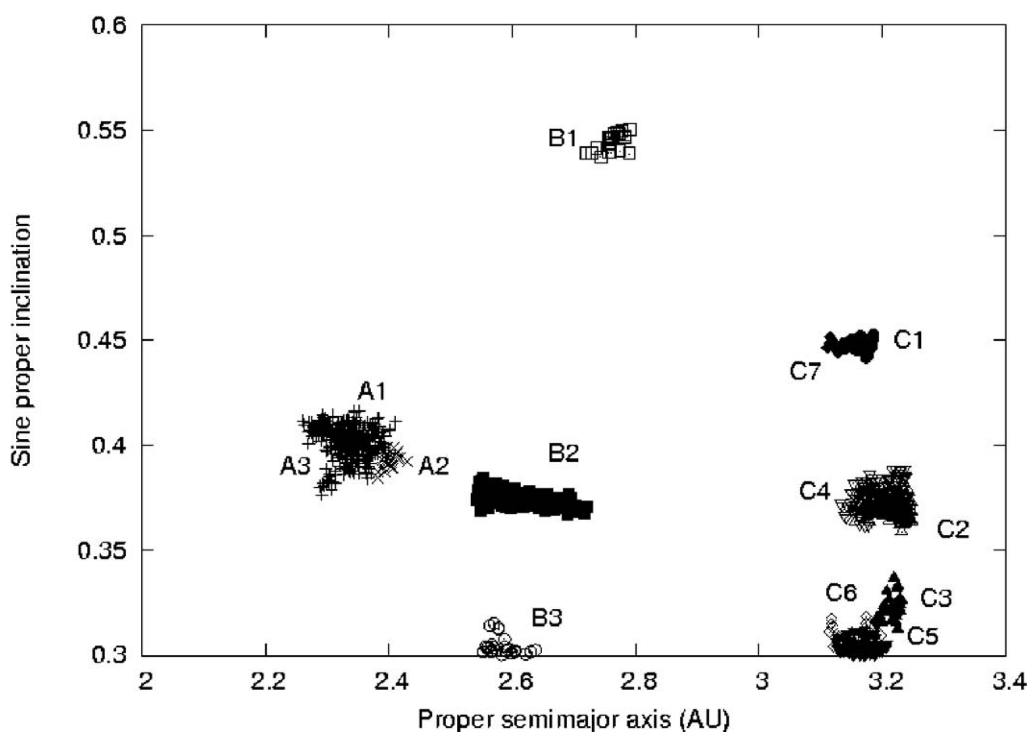

Fig. 4. Distribution in the $(a', \sin i')$ plane for the 13 families identified. The families are labeled with the identifications listed in Table 2.

in this zone, 702 Alauda (C2) appears at $d_0 \approx 105\ \mathrm{m\,s^{-1}}$ but a large subgroup of 88 asteroids was included in this family at $d_0 \approx 115\ \mathrm{m\,s^{-1}}$. This family appears to be fairly homogeneous in composition, since five of their members have a taxonomic classification which are consistent with a breakup event (702, 1101, and 3139 are B-types, 1838 is a Cgh-type, and 3246 is a Cg-type). These families were also found by Foglia and Masi (2004) using a different clustering method. The family associated with 1303 Luthera (C3) appears at $d_0 \approx 45\ \mathrm{m\,s^{-1}}$ and it is another example of a family formed by a relative large object and a large number of smaller ones. Another large family is 1901 Moravia (C4), which is clearly defined at $d_0 \approx 115\ \mathrm{m\,s^{-1}}$.

The only two objects in this family with a taxonomic classification, 1901 and 3036, are both B-types.

In addition to these families, four potentially interesting clumps were also found. The clumps B4 (148 Gallia, with a diameter of 97.7 km), C8 (181 Eucharis, 106 km) and C9 (276 Adelheid, 121.6 km) show absolute magnitude distributions which are consistent with a cratering event on the largest member of the clump. On the other hand, the clump associated with the C-type Asteroid 326 Tamara (A4) appears in a region dominated by S-types objects. These small groupings with peculiar taxonomic types or absolute magnitude distributions are worth some more physical observations to assess their significance. On the other hand, the clump associated with 945 Barcelona (B5) was previously proposed by Foglia and Masi (2004).

4. Conclusions

Thirteen asteroid families constituted by 1126 objects (30.8% of the total sample) and twenty-two clumps with high-inclination orbits are presented here. When taxonomic information is available, the families found in this work show cosmochemical consistency and support an interpretation based on a common origin from a single parent body. However, it is difficult to reach a conclusion about if they are a result of a breakup event or not without a deeper knowledge of the physical and chemical properties of all family members. Nevertheless, from the point of view of the dynamical structure the interpretation is straightforward and these families are reliable enough to be used as tests for theories on composition and evolution of asteroids.

The most significant families are those associated with 25 Phocaea (A1), 480 Hansa (B1), 702 Alauda (C2), and 1901 Moravia (C4), that collectively include about 75% of the population in families. It is important to mention that the "A" region is separated from the adjacent belt asteroids and crossed by resonances, so any concentration of objects in this region should not necessarily by interpreted as a family.

Four of the families found in this work (2 Pallas, 480 Hansa, 31 Euphrosyne, and 1303 Luthera) and three clumps (148 Gallia, 181 Eucharis, and 276 Adelheid) show an absolute magnitude distribution which is compatible with a formation due to a cratering event on the largest member of the family. It is worth noting that a priori the crater-derived families are expected to be seen preferentially in the case of large parent bodies, as it is the case with these seven groupings, because otherwise the large size gap between the cratered parent and the ejected fragments would make it difficult to detect; but this type of family could be common in this case of high-inclination objects because they could encounter projectiles at systematically higher velocities, producing more harm to targets with smaller projectile sizes. To test this hypothesis, the collisional velocity distribution of the largest members of these families against a population of particles, which must be representative of the belt population, are obtained and shown in Figs. 5a and 5b.

In order to find these distributions the formulation of Greenberg (1982), Bottke and Greenberg (1993) was used and the population of particles was formed using only the osculating orbital elements of the 682 largest objects in the main belt. The mean collision velocity and intrinsic collisional probability found for these asteroids are $\langle V_{\rm col}\rangle = 11.38\ {\rm km\,s^{-1}}$ and $\langle P_i\rangle = 1.82\times10^{-18}\ {\rm km^{-2}\,yr^{-1}}$ for 2 Pallas, $\langle V_{\rm col}\rangle = 7.83\ {\rm km\,s^{-1}}$ and $\langle P_i\rangle = 4.66\times10^{-18}\ {\rm km^{-2}\,yr^{-1}}$ for 480 Hansa, $\langle V_{\rm col}\rangle = 9.14\ {\rm km\,s^{-1}}$ and $\langle P_i\rangle = 1.75\times10^{-18}\ {\rm km^{-2}\,yr^{-1}}$ for 31 Euphrosyne, $\langle V_{\rm col}\rangle = 6.50\ {\rm km\,s^{-1}}$ and $\langle P_i\rangle = 2.14\times10^{-18}\ {\rm km^{-2}\,yr^{-1}}$ for 1303 Luthera, $\langle V_{\rm col}\rangle = 8.62\ {\rm km\,s^{-1}}$ and $\langle P_i\rangle = 2.77\times10^{-18}\ {\rm km^{-2}\,yr^{-1}}$ for 148 Gallia, $\langle V_{\rm col}\rangle = 7.01\ {\rm km\,s^{-1}}$ and $\langle P_i\rangle = 1.99\times10^{-18}\ {\rm km^{-2}\,yr^{-1}}$ for 181 Eucharis, and $\langle V_{\rm col}\rangle = 7.28\ {\rm km\,s^{-1}}$ and $\langle P_i\rangle = 2.60\times10^{-18}\ {\rm km^{-2}\,yr^{-1}}$ for 276 Adelheid (for comparison, the mean values for the main belt are $\langle V_{\rm col}\rangle = 5.3\ {\rm km\,s^{-1}}$ and $\langle P_i\rangle = 2.86\times10^{-18}\ {\rm km^{-2}\,yr^{-1}}$; see, for example, Bottke et al., 1994). Their collision velocity distributions show that these asteroids have a significant probability to collide with a projectile at high velocities (larger that $\sim 10\ {\rm km\,s^{-1}}$) but, on the other hand, these high collision velocities lower the overall collision probability per encounter. Then, the collisional evolution of these high-inclination objects may be significantly different from those of other main belt asteroids with similar size, so the formation of these families deserves a more careful study and a comprehensive analysis of each particular case.

Finally, three families (2 Pallas, 702 Alauda, and 1901 Moravia) show quite homogeneous spectrophotometric properties, with all of its members with known taxonomy belonging to the low-albedo B class. This number represents a 14% of the B-types classified by Bus and Binzel (2002) for the main belt.

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References

Arnold, L.R., 1969. Asteroid families and jets streams. Astron. J. 74, 1235–1242.

Beaugé, C., Roig, F., 2001. A semianalytical model for the motion of the trojan asteroids: Proper elements and families. Icarus 153, 391–415.

Bottke, W.F., Greenberg, R., 1993. Asteroidal collision probabilities. Geophys. Res. Lett. 20, 879–881.

Bottke, W.F., Nolan, M.C., Greenberg, R., Kolvoord, R., 1994. Velocity distributions among colliding asteroids. Icarus 107, 255–268.

Bottke, W.F., Vokrouhlický, D., Brož, M., Nesvorný, D., Morbidelli, A., 2001. Dynamical spreading of asteroid families by the Yarkovsky effect. Science 294, 1693–1696.

Brouwer, D., 1951. Secular variations of the orbital elements of minor planets. Astron. J. 56, 9–32.

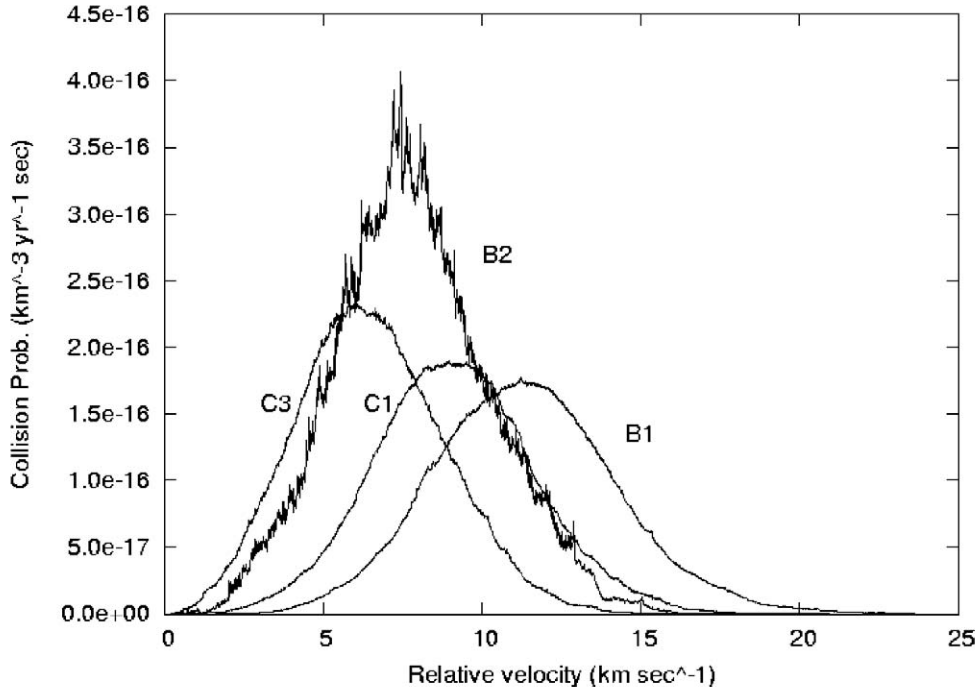
Bus, S.J., 1999. Compositional structure in the asteroid belt: Results of a spectroscopic survey. Ph.D. thesis, MIT.

Bus, S.J., Binzel, R.P., 2002. Phase II of the small main-belt asteroid spectroscopic survey. Icarus 158, 146–177.

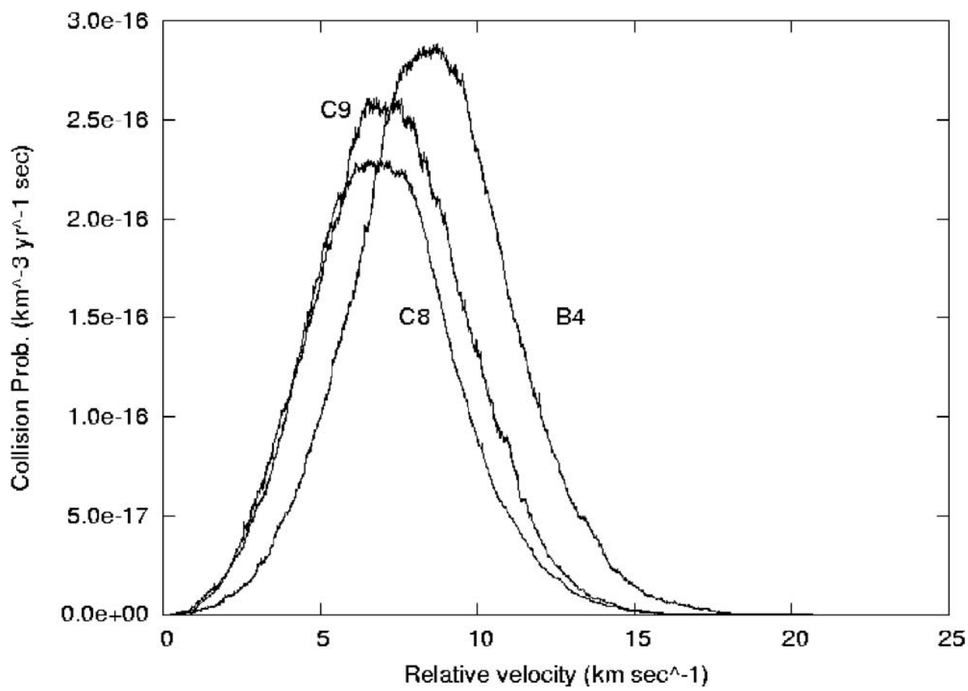
Carpino, M., Milani, A., Nobili, A.N., 1987. Long-term numerical integrations and synthetic theories for the motion of the outer planets. Astron. Astrophys. Suppl. 181, 182–194.

Carusi, A., Massaro, E., 1978. Statistic and mapping of asteroidal concentration in the proper element space. Astron. Astrophys. Suppl. 34, 81–90.

Carruba, V., Burns, J.A., Bottke, W., Nesvorný, D., 2003. Orbital evolution of the Gefion and Adeona asteroid families: Close encounters with massive asteroids. Icarus 162, 308–327.



(a)



(b)

Fig. 5. Collision velocity distribution for the largest members of the proposed families formed by a cratering collision. (a) B1 (2 Pallas), B2 (480 Hansa), C1 (31 Euphrosyne), and C3 (1303 Luthera); (b) B4 (148 Gallia), C8 (181 Eucharis), and C9 (276 Adelheid).

Chapman, C.R., Paolicchi, P., Zappalà, V., Binzel, R.P., Bell, J.F., 1989. Asteroid families: Physical properties and evolution. In: Binzel, R.P., Gehrels, T., Matthews, M.S. (Eds.), Asteroids II. Univ. of Arizona Press, Tucson, pp. 386–415.

Foglia, S., Masi, G., 2004. New cluster for highly inclined main-belt asteroids. Minor Planet Bull. 31, 100–102.

Greenberg, R., 1982. Orbital interactions: A new geometrical formalism. Astron. J. 87, 184–195.

Hergenrother, C.W., Larson, S.M., Spahr, T.B., 1996. The Hansa family: A new high inclination asteroid family. Bull. Am. Astron. Soc. 28, 1097.

Hirayama, K., 1918. Groups of asteroids probably of common origin. Astron. J. 31, 185–188.

Knežević, Z., Milani, A., 2000. Synthetic proper elements for outer main belt asteroids. Celest. Mech. 78, 17–46.

Knežević, Z., Milani, A., 2003. Proper element catalogs and asteroid families. Astron. Astrophys. 403, 1165–1173.

Knežević, Z., Lemaître, A., Milani, A., 2002. The determination of asteroid proper elements. In: Bottke Jr., W.F., Cellino, A., Paolicchi, P., Binzel, R.P. (Eds.), Asteroids III. Univ. of Arizona Press, Tucson, pp. 603–612.

Kozai, Y., 1979. The dynamical evolution of the Hirayama families. In: Gehrels, T. (Ed.), Asteroids. Univ. of Arizona Press, Tucson, pp. 197–210.

Lazzaro, D., Angeli, C., Carvano, J.M., Mothé-Diniz, T., Duffard, R., 2004. S^3OS^2: The visible spectroscopic survey of about 800 asteroids. Icarus 172, 179–220.

Lemaître, A., Morbidelli, A., 1994. Proper elements for highly inclined asteroidal orbits. Celest. Mech. 60, 29–56.

Lindblad, B.A., Southworth, R.B., 1971. A study of asteroid families and streams by computer techniques. In: Gehrels, T. (Ed.), Physical Studies of Minor Planets, NASA SP-267. National Aeronautics and Space Administration, Washington, DC, pp. 337–352.

Migliorini, F., Zappalá, V., Vio, R., Cellino, A., 1995. Interlopers within asteroid families. Icarus 118, 271–291.

Milani, A., Knežević, Z., 1994. Asteroid proper elements and the dynamical structure of the asteroid main belt. Icarus 107, 219–254.

Mothé-Diniz, T., Roig, F., Carvano, J.M., 2005. Reanalysis of asteroid families structure through visible spectroscopy. Icarus 174, 54–80.

Nesvorný, D., Morbidelli, A., Vokrouhlický, D., Bottke, W.F., Brož, M., 2002a. The Flora family: A case of the dynamically dispersed collisional swarm? Icarus 157, 155–172.

Nesvorný, D., Bottke, W.F., Dones, L., Levison, H.F., 2002b. The recent breakup of an asteroid in the main-belt region. Nature 417, 720–771.

Nesvorný, D., Jedicke, R., Whiteley, R.J., Ivezić, Ž., 2005. Evidence for asteroid space weathering from the Sloan Digital Sky Survey. Icarus 143, 132–152.

Tholen, D.J., 1989. Asteroid taxonomic classifications. In: Gehrels, T. (Ed.), Asteroids. Univ. of Arizona Press, Tucson, pp. 1139–1150.

Williams, J.G., 1969. Secular perturbations in the Solar System. Ph.D. thesis, University of California, Los Angeles.

Williams, J.G., 1979. Proper orbital elements and family memberships of the asteroids. In: Gehrels, T. (Ed.), Asteroids. Univ. of Arizona Press, Tucson, pp. 1040–1063.

Williams, J.G., 1989. Asteroid family identification and proper elements. In: Binzel, R.P., Gehrels, T., Matthews, M.S. (Eds.), Asteroids II. Univ. of Arizona Press, Tucson, pp. 1034–1072.

Williams, J.G., 1992. Asteroid families—An initial search. Icarus 96, 251–280.

Zappalà, V., Cellino, A., Farinella, P., Knežević, Z., 1990. Asteroid families. I. Identification by hierarchical clustering and reliability assessment. Astron. J. 100, 2030–2046.

Zappalà, V., Cellino, A., Farinella, P., Milani, A., 1994. Asteroid families. II. Extension to unnumbered multiopposition asteroids. Astron. J. 107, 772–801.

Zappalà, V., Bendjoya, Ph., Cellino, A., Farinella, P., Froeschlé, C., 1995. Asteroid families: Search of a 12487 asteroid sample using two different clustering techniques. Icarus 116, 291–314.