Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physica A 327 (2003) 76–81
PHYSICA A
www.elsevier.com/locate/physa

Fractal properties of correlated invasion percolation patterns

Raúl H. López[a,*], A.M. Vidales[a], G. Zgrablich[b]

[a] Dpto. de Física, Universidad Nacional de San Luis—CONICET, Chacabuco 917, San Luis 5700, Argentina
[b] Departamento de Química, Universidad Autónoma Metropolitana, Iztapalapa, P.O. Box 55-534, México D.F., Mexico

Abstract

We present results on Monte Carlo simulations for invasion percolation with trapping considering the presence of spatial size correlations, a problem which is relevant to multiphase flow in field scale of porous media. The correlations are generated through the dual site bond model, characterized by a spatial correlation length r_0, which depends on the overlap between site and bond distributions. Our results indicate that in two-dimensional lattices the fractal dimension of the sample-spanning cluster, is non-universal and vary with the correlation. Comparison with other authors recent findings is presented.

PACS: 64.60.Ak; 47.55.Mh

Keywords: Percolation phenomena; Clusters; Fractal dimension

1. Introduction

Although invasion percolation (IP) [1] has been object of intensive study and characterization along many years, it still presents open questions and attracts the attention of researchers. This is founded on the wide variety of natural processes that can be explained, represented or approximated through this kinds of dynamical behavior. Fluid passage through disordered porous media can conveniently be traced by IP models enhancing the understanding of important industrial and technological problems such as oil recovery, gas and underground-water reservoirs, filtration, among many other ones.

* Corresponding author. Fax: +54-2652-430224.
E-mail address: rlopez@unsl.edu.ar (R.H. López).

0378-4371/03/$ - see front matter
doi:10.1016/S0378-4371(03)00454-0

To investigate these phenomena, pore network models have been used to represent the porous media, and the concepts of percolation theory [2,3] have been employed to model slow flow of fluids through the pore space. Basically, the dynamics of the process consists in the displacement of a defending fluid, residing in the porous matrix, by an invading one, creating an interface that evolves in time. When the invading fluid wets the pore walls and gets spontaneously into the pore space, one says that the process is an imbibition one. On the other hand, forced invasion performed by a non-wetting fluid into the porous space is called drainage. During imbibition, the fluid first penetrates the narrowest pores, while in drainage it likes to get first into the widest ones. As the reader knows, there are two main types of IP processes: with and without entrapment of the defending fluid. It was also well established that the value for the fractal dimension D_f of the percolating cluster differs from one type of IP to the other. In two dimensions and for the non-trapping case, it attains the same value as for standard percolation, i.e., 1.89, while for the trapping one, its value is 1.82, indicating a change in the way that porous space is occupied by the invader. In general, the scaling properties for non-trapping IP are believed to be consistent with those in standard percolation. Until the work of Knackstedt et al. [4], it was also understood that those properties for IP with trapping in 2D were different from standard percolation, universal and independent of the lattice type. In their paper, Knackstedt et al. demonstrated that connectivity of the lattice does affect the fractal dimension of the percolating cluster indicating that, contrary to common belief, this dynamical process has non-universal features. Moreover, they found a different behavior from bond to site IP. The fact that connectivity affects the way invasion is performed may be interpreted as a lowering in the probability for entrapment and a consequent increase of D_f. All the processes cited up to here correspond to random networks, i.e., those where the size of one element in the lattice does not depend on the size of any of its neighbors. Thus, considering the role of entrapment and taking into account the fact that size correlations among the elements of a lattice have proven to really affect percolating cluster features in several ways [5,6], one may ask whether the presence of correlations would change the universality of trapping IP, too. The aim of this paper is to answer the above question by proving that D_f is affected by the presence of correlations and also that the observed change in it is completely consistent with the features observed in Ref. [4]. To this end, we simulate an invasion process in a porous matrix performed by a wetting fluid that will displace a defender residing in the porous space. The matrix will be represented by a 2D site-bond square lattice, where sites behave like pores and bonds behave like necks which connect pores. Trapping will be allowed in the IP process. The characteristic size for sites and bonds are sampled, respectively, from two uniform normalized distributions. Bonds have always smaller or at least equal size than sites and we will call this assumption construction principle (CP). When the two size distributions do not overlap, random assignment of sizes to the elements of the network does not violate the CP, thus, a random network is obtained. But, when there exists an overlaping area (Ω) among them, CP fulfillment is the origin of the presence of size correlations among lattice elements. Because distributions are normalized, Ω may take values between 0 and 1. As the overlap Ω increases from 0 (non-correlated case), it is harder to follow CP requirement, thus, space correlations for lattice elements become stronger. The extreme case, $\Omega = 1$, means correlation length going to infinity. The topological organization of lattice elements, because of correlations among their sizes, makes the network appearance like a collection of patches consisting of elements with very similar sizes among each other. The characteristic radius of each patch is a measure of the strength of correlations in the system. In other words, greater values of Ω means greater values of the correlation length, i.e., greater patches of elements with similar sizes. This topological effect has been extensively studied [7] and a relation between correlation length, r_0, and Ω have been stated: $r_0 = 2\Omega^2/(1-\Omega)^2$. Thus, the overlaping area is a main parameter to characterize the way that porous space is connected concerning size segregation patterns through out the lattice. All features and main aspects of this site-bond model with correlations are deeply discussed elsewhere [8–10]. Once porous lattices are simulated using the above described model, invasion of porous space is performed in a standard way [11]. We want to measure the fractal dimension D_f of the percolating cluster just at the point of breakthrough. To this end, we first fix the value for Ω and then we perform series of simulations with different lattice sizes L and measure the mass of the cluster (number of invaded sites and bonds) after breakthrough in each case. Then, a new value for Ω is selected and fixed and every steps are started again. As usually known, the scaling law governing the behavior of mass of the percolating cluster and the size of the lattice being invaded is $M = L^{D_f}$. We use this relation to determine D_f.

2. Results and discussion

All of our calculations were performed using uniform distributions and the different overlappings were attained by keeping the site distribution fixed and shifting the bond distribution. The quantities shown in this section have been averaged over sets ranging from 1000 to 10 000 samples, depending on the size of the network $(32 < L < 1024)$. These quantities were the mean velocity, v_m, defined as the rate L/N, where N is the number of invasion steps needs to reach the breakthrough, the total invaded volume, V_s (associated with network sites), and the fractal dimension, D_f of the spanning cluster. In Fig. 1(a) v_m vs. Ω is shown. An increasing trend is observed as Ω gets greater. There is a maximum around $\Omega = 0.8$ where the velocity begins to decrease. The inset shows the corresponding behavior of V_S, which presents a minimum around $\Omega = 0.8$. On the other hand, Fig. 2 shows snapshots corresponding to porous networks for $\Omega = 0.5$ and 0.8. The "patched structure" observed is due to the presence of correlations. Each patch has a characteristic size of the order of the correlation strength, r_0, which, for $\Omega = 0.5$, is ≈ 3, and, for $\Omega = 0.8$, is ≈ 30. Inside each one of them there are pores of similar radii and, as Ω increases, they organize in this way: smaller elements are concentrated in the central region of the patch, then elements of greater size are disposed in a ring-like structure. This effect is really evident for $\Omega > 0.8$. Thus, for small correlations, there are a lot of small patches. Fluid first selects the smallest element at the interface and the patch

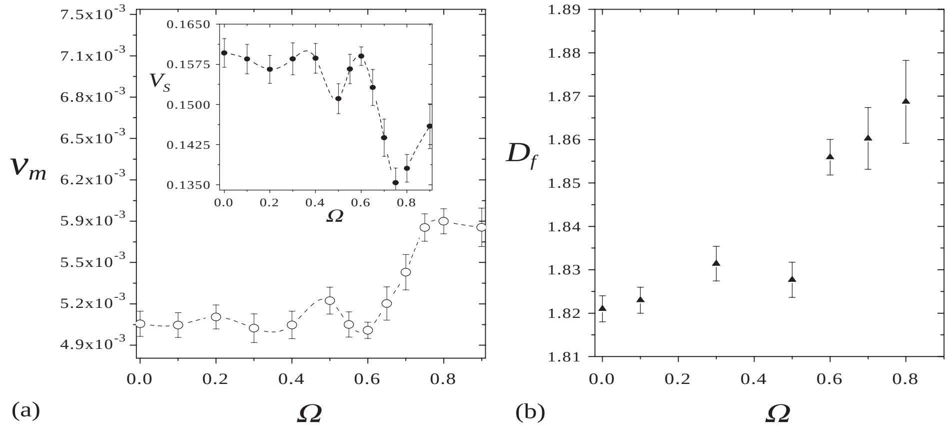

Fig. 1. (a) Mean front velocity up to breakthrough as a function of the correlation degree. In the inset, we show the total invaded pore volume to reach the final state as a function of the correlation degree. (b) Fractal dimension D_f vs. the overlap Ω.

which it belongs to, has a great likelihood to be invaded completely. Then the fluid will invade another patch until breakthrough state is achieved. For higher correlations, there are big patches and like in the previous case, if an element of a patch is invaded, it is highly probable that the whole patch will be, too. But in this case, because of the greater strength of correlations and a low number of patches, the invasion front will get the breakthrough after leaving just a few patches. In this way, invasion is performed faster than for lower correlations and in detriment of invasion efficiency. All above explains the observed behavior in Fig. 1 for v_m and V_S. A detailed study of the way islands are created in the invasion process can be reviewed in Ref. [12]. The way that the dynamic of the invasion process is affected by the presence of correlations has, as a direct consequence, a change in the fractal dimension of the invasion cluster. In Fig. 1(b) we show the dependence of D_f on Ω. As clearly observed, for zero correlations D_f attains the corresponding classical IP value (1.82). As Ω increases, D_f increases until it reaches a value that is close to the one for IP without trapping (1.89). This picture is in agreement with the results in Ref. [4] in the sense that an increase in correlations produces, in many ways, the same effect that an increase in the mean connectivity of the network. It is not yet clearly possible to predict what the value will be for D_f when correlations are extremely high.

3. Concluding remarks

The spatial correlations are most of the time present in real porous media and they are the central matter of the present study in which the sizes of the pores in the

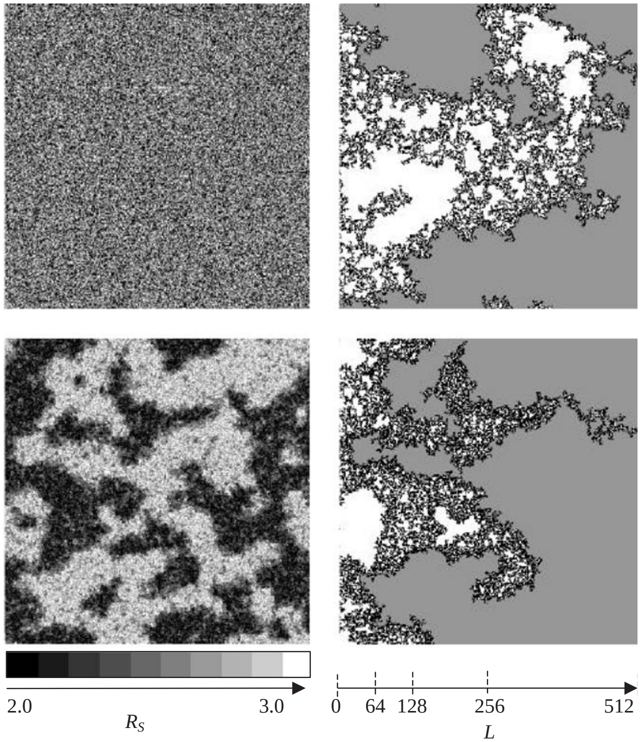

Fig. 2. The first column is a representation of a simulated square lattice for $\Omega = 0.5$ (upper) and 0.8 (lower). The grey scale is proportional to the size site R_S. The second column shows snapshots at breakthrough state. Black points represent the invader fluid, white regions, the trapped defender and grey points represent neither invaded nor isolated elements. The selected patterns were the ones which best represented the average behavior.

medium were assumed to be non-randomly distributed. The correlation are generated through the DSBM characterized by a spatial correlation length, r_0 which depends on the overlap Ω between site and bond distributions. The results indicate that fractal dimension of the sample-spanning cluster, in two-dimensional lattices, is non-universal and vary with the correlation.

References

[1] D. Wilkinson, J.F. Willemsen, J. Phys. A 16 (1983) 3365.
[2] M. Sahimi, Flow and Transport in Porous Media and Fractured Rock, VCH, Weinheim, Germany, 1995.
[3] D. Stauffer, A. Aharony, Introduction to Percolation Theory, Taylor & Francis, London, 1992.
[4] M. Knackstedt, M. Sahimi, A. Sheppard, Phys. Rev. E 65 (2002) 035 101.
[5] I. Kornhauser, R.J. Faccio, J.L. Riccardo, F. Rojas, A.M. Vidales, G. Zgrablich, Fractals 5 (1998) 355.
[6] A. Vidales, R.H. López, G. Zgrablich, Langmuir 15 (1999) 5703.
[7] R.H. López, A.M. Vidales, G. Zgrablich, Langmuir 16 (2000) 3441.

[8] V. Mayagoitia, B. Gilot, F. Rojas, I. Kornhouser, J. Chem. Soc. Faraday Trans. I 84 (1988) 801.
[9] V. Mayagoitia, F. Rojas, J.L. Riccardo, V. Pereyra, G. Zgrablich, Phys. Rev. B 41 (1989) 7150.
[10] J. Riccardo, W. Steele, A. Ramirez, G. Zgrablich, Langmuir 13 (1997) 1064.
[11] A. Sheppard, M. Knackstedt, W. Pinezawski, M. Sahimi, J. Phys. A 32 (1999) 521.
[12] R.H. López, Master Thesis, Universidad Nacional de San Luis, Argentina, 2002 (http://linux0.unsl.edu.ar/∼rlopez/tesis.pdf).