Notes on Aggregation of IVIFS based on transformation techniques

José Carlos R. Alcantud Gustavo Santos-García

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Abstract

The abundant growth of variations in intuitionistic fuzzy sets has posed a challenge in assessing the characteristics of diverse decision-making models. In this context, we will delve into the subject within the context of interval-valued intuitionistic fuzzy sets. Our work has a dual purpose. On a theoretical plane, we establish two theorems that facilitate the conversion of interval-valued intuitionistic fuzzy sets into appropriately correlated pairs of intuitionistic fuzzy sets. On a more pragmatic note, we illustrate how these findings can be harnessed to amalgamate interval-valued intuitionistic fuzzy sets and applied to the collective decision-making process within this framework.

Interval-valued intuitionistic fuzzy sets (IVIFS) are a specialized extension of fuzzy sets that provide a more comprehensive representation of uncertainty and vagueness in decision-making processes. In IVIFS, instead of assigning a single membership value to an element as in traditional fuzzy sets, a range or interval of values is associated with each element. This interval represents the uncertainty in the degree to which the element belongs to the set. Additionally, IVIFS also incorporate an intuitionistic index, which quantifies the hesitation or doubt associated with the membership degree.

The versatility of IVIFS makes them particularly valuable in various fields and applications. Here are some key areas where IVIFS find extensive use:

- Decision Making: IVIFS allow decision-makers to model and manage uncertainty more effectively, making them suitable for applications in multi-criteria decision analysis, risk assessment, and group decision-making.
- Pattern Recognition: In image processing and pattern recognition, IVIFS help handle the inherent ambiguity in classifying objects, making it possible to handle imprecise and uncertain data more robustly.

- Control Systems: IVIFS are applied in control systems to handle uncertain parameters and fuzzy feedback, enabling more stable and adaptive control processes.
- Medicine and Healthcare: In medical diagnosis, where uncertainty and incomplete information are common, IVIFS aid in creating more reliable diagnostic systems.
- Environmental Management: IVIFS are used to assess environmental risks and uncertainties, facilitating better decision-making in environmental management and policy planning.
- Finance and Investment: In portfolio optimization and risk assessment, IVIFS provide a means to model the uncertainty associated with asset returns and financial market dynamics.
- Engineering and Quality Control: IVIFS are employed in quality control and reliability analysis, where they help manage uncertainty in product and process parameters.
- Natural Language Processing: IVIFS can enhance natural language processing tasks, such as sentiment analysis and text categorization, by capturing the uncertainty and vagueness in linguistic data.

In summary, interval-valued intuitionistic fuzzy sets offer a robust framework for dealing with complex and uncertain information across a wide range of applications. Their ability to represent both membership and non-membership information within an interval, along with an intuitionistic index for hesitation, makes them a valuable tool for decision-makers and researchers in diverse fields seeking to handle uncertainty and vagueness more effectively.

These notes encompass the presentation of the paper titled "Aggregation of IVIFS based on transformation techniques," which was delivered by the authors at the 2023 IEEE International Conference on Fuzzy Systems held in Songdo Incheon, Korea, on August 14th, 2023.

Aggregation of interval valued intuitionistic fuzzy sets based on transformation techniques

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José Carlos R. Alcantud   Gustavo Santos-García

BORDA Research Unit and IME, University of Salamanca, Spain

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I. Introduction

The concept of **interval valued intuitionistic fuzzy set** (**IVIFS**) associates each element of a set with an **interval valued intuitionistic fuzzy number** (**IVIFN**).

The development of IVIFSs followed a path similar to the case of **intuitionistic fuzzy sets** (**IFS**). Various authors produced comparison laws based on scores and accuracies, aggregation operators, or decision-making methodologies in the framework of IVIFSs.

As in the case of IFSs, aggregators and comparison laws can be combined to be used in group decision making (**GDM**) and multi-attribute group decision making (**MAGDM**).

Aggregation operators for IFSs and IVIFSs

Aggregation operators for IFSs and IVIFSs are defined separately for each constituent IFNs and IVIFNs. Two basic methodologies can be distinguished:

- The first generates aggregation operators using operational laws for IFNs or IVIFNs.
- The second uses aggregation operators on crisp numbers.

Whatever the case, **the complexity of the IVIFN model produces lengthy formulas whose intuitive meaning is hard to grasp**.

In this work, **we develop a technique that reduces the aggregation of IVIFSs** to the simpler problem of computing aggregate IFSs. We produce an adaptable methodology whose first step consists of a **transformation technique**.

Reducing aggregation of IVIFSs

We produce **two theorems** establishing respective **bijections between the set of all IVIFSs and sets formed by suitable pairs of IFSs**. Both are intuitively appealing and fully operational because they are constructive.

We shall focus on one of these bijections. Then, in the presence of a list of IVIFSs, their aggregate IVIFS can be calculated as follows:

- Each IVIFS is transformed into a pair of IFSs belonging to a certain set (the image of our bijection).
- Now, one can use an aggregation operator on IFSs to compute a pair of IFSs.
- If we can guarantee that after this process, the pair of IFSs is still in the image of our bijection, then its inverse image is a reliable proposal as an aggregate of the list of IVIFSs.
- Hence, we need to pinpoint aggregation operators on IFSs that satisfy the required structural property.

The **applicability** of this technique to decision making is now simple. We can use scores and accuracies for IVIFNs in order to select and prioritize alternatives that are characterized by IVIFSs.

II. Orthopairs and other preliminary concepts

Let X be a fixed set of alternatives and $D[0,1]$ denote the set of closed intervals that are subsets of $\mathcal{I} = [0,1]$.

An **orthopair** is (μ, ν) where $0 \leqslant \mu, \nu \leqslant 1$. When $\mu + \nu \leqslant 1$, it is an intuitionistic fuzzy number (IFN). The set of all IFNs will be denoted as $\mathbb{A}$.

Definition

When (μ_1, ν_1), (μ_2, ν_2) are orthopairs, one can define **the orthopairs** $(\mu_1, \nu_1) \wedge (\mu_2, \nu_2) = (\min\{\mu_1, \mu_2\}, \min\{\nu_1, \nu_2\})$ **and** $(\mu_1, \nu_1) \vee (\mu_2, \nu_2) = (\max\{\mu_1, \mu_2\}, \max\{\nu_1, \nu_2\})$.

The next **two preorders** can be defined on the set of all IFNs: when (μ_1, ν_1), (μ_2, ν_2) are IFNs,
$(\mu_1, \nu_1) \leqslant_L (\mu_2, \nu_2)$ if and only if $\mu_1 \leqslant \mu_2$ and $\nu_1 \leqslant \nu_2$,
$(\mu_1, \nu_1) \preccurlyeq (\mu_2, \nu_2)$ if and only if $\mu_1 \leqslant \mu_2$ and $\nu_2 \leqslant \nu_1$.

Observe that when (μ_1, ν_1), (μ_2, ν_2) are IFNs, and $\mu_1 \leqslant \mu_2$, then either $(\mu_1, \nu_1) \leqslant_L (\mu_2, \nu_2)$ or $(\mu_1, \nu_1) \preccurlyeq (\mu_2, \nu_2)$ (or both) hold true.

For consistency, $(a_1, \ldots, a_n) \leqslant_L (b_1, \ldots, b_n)$ means $a_i \leqslant b_i$ for each i, whenever $(a_1, \ldots, a_n), (b_1, \ldots, b_n) \in \mathcal{I} \times \overset{n}{\ldots} \times \mathcal{I}$.

A. IFSs and related concepts and facts

Definition (Atanassov)

An **intuitionistic fuzzy set** (IFS) A over X is $A = \{\langle x, (\mu_A(x), \nu_A(x))\rangle | x \in X\}$, where $(\mu_A(x), \nu_A(x))$ is an IFN for each $x \in X$. The mappings $\mu_A, \nu_A : X \to [0, 1]$ capture the **membership degree** (**MD**) and **non-membership degree** (**NMD**) of $x \in X$ to A. For simplicity, A will be shortened to $A = \langle \mu_A, \nu_A \rangle$ whenever convenient.

We denote the set of all IFSs over X by IFS(X).

A. IFSs and related concepts and facts (cont.)

This definition can be **extended from IFNs to IFSs** easily. Suppose $A = \{\langle x, (\mu_A(x), \nu_A(x))\rangle | \, x \in X\}$ and $B = \{\langle x, (\mu_B(x), \nu_B(x))\rangle | \, x \in X\}$ are IFSs. Then, **we declare** $A \leqslant_L B$, respectively, $A \preccurlyeq B$, if and only if $(\mu_A(x), \nu_A(x)) \leqslant_L (\mu_B(x), \nu_B(x))$, respectively, $(\mu_A(x), \nu_A(x)) \preccurlyeq (\mu_B(x), \nu_B(x))$, for all $x \in X$. Analogously we can define $A \wedge B$ and $A \vee B$ by pointwise extension.

Conversely, **union and intersection of IFSs**

$$A \cup B = \{\langle x, (\max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\})\rangle | \, x \in X\}$$

$$A \cap B = \{\langle x, (\min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\})\rangle | \, x \in X\}$$

induce union and intersection of IFNs, which always produce IFNs: when (μ_1, ν_1), (μ_2, ν_2) are IFNs,

$$(\mu_1, \nu_1) \cup (\mu_2, \nu_2) = (\max\{\mu_1, \mu_2\}, \min\{\nu_1, \nu_2\}), \quad (1)$$

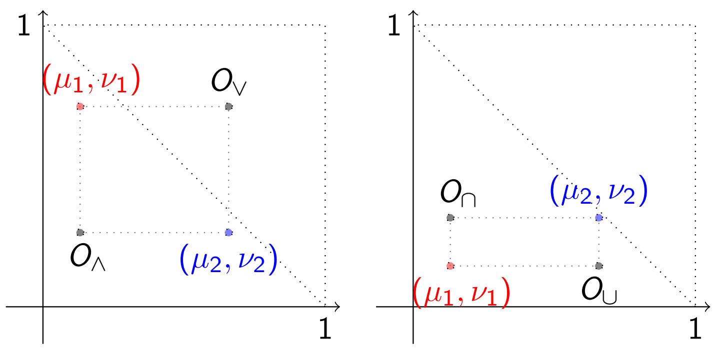
$$(\mu_1, \nu_1) \cap (\mu_2, \nu_2) = (\min\{\mu_1, \mu_2\}, \max\{\nu_1, \nu_2\}). \quad (2)$$

In addition, **subsethood** can be defined as follows:

$$A \subseteq B \Leftrightarrow (\mu_A(x), \nu_A(x)) \preccurlyeq (\mu_B(x), \nu_B(x)) \text{ for all } x \in X.$$

Next figure represents various concepts mentioned above, in two situations where (μ_1, ν_1), (μ_2, ν_2) are IFNs.

A. IFSs and related concepts and facts (cont.)



(a) $(\mu_1, \nu_1) \preccurlyeq (\mu_2, \nu_2)$ (b) $(\mu_1, \nu_1) \leqslant_L (\mu_2, \nu_2)$

Figure: A graphical representation of previous definition. Both (μ_1, ν_1) and (μ_2, ν_2) are IFNs in the two diagrams. The orthopairs $O_\wedge$ and $O_\vee$ respectively represent $(\mu_1, \nu_1) \wedge (\mu_2, \nu_2)$ and $(\mu_1, \nu_1) \vee (\mu_2, \nu_2)$. We show a case where the latter is not be an IFN. $O_\cup$ and $O_\cap$ respectively represent the IFNs $(\mu_1, \nu_1) \cup (\mu_2, \nu_2)$ and $(\mu_1, \nu_1) \cap (\mu_2, \nu_2)$.

B. IVIFSs and related concepts

Definition (Atanassov and Gargov)

An **interval valued intuitionistic fuzzy set** (IVIFS) A over X is $A = \{\langle x, (\mu^A(x), \nu^A(x))\rangle | \, x \in X\}$, where for each $x \in X$, $\mu^A(x) = [\mu_L^A(x), \mu_M^A(x)] \in D[0,1]$, $\nu^A(x) = [\nu_L^A(x), \nu_M^A(x)] \in D[0,1]$ and $\mu_M^A(x) + \nu_M^A(x) \leqslant 1$.
Each pair $([\mu_L^A(x), \mu_M^A(x)], [\nu_L^A(x), \nu_M^A(x)])$ is an interval valued intuitionistic fuzzy number (IVIFN).
We denote the set of all IVIFSs over X by IVIFS(X).

Observe that an IVIFN is $([a,b],[c,d]) \in D[0,1] \times D[0,1]$ such that the orthopair (b,d) is in fact an IFN.
Also, the **score** of $([a,b],[c,d])$ is $\frac{1}{2}(a - c + b - d)$, whereas $\frac{1}{2}(a + b + c + d)$ defines its **accuracy**. **Preference** is given to IVIFNs with higher scores, and when two IVIFNs have the same score, the one with higher accuracy is preferred. **Other formulas** for scores and accuracies have been defined and utilized for multi-attribute decision making.

III. Two practical bijections

Now, we formulate **two theorems**.

Both capture similar intuitions: if we imagine an IVIFN as a rectangle (within the appropriate boundaries), then each IVIFN can be uniquely identified by two opposed 'corners' (which therefore must meet some restrictions).

The choice of the **corners** determines the theorem that we obtain.

First bijection

We shall use the following $\mathcal{O}_1 \subseteq \mathrm{IFS}(X) \times \mathrm{IFS}(X)$:

$$\mathcal{O}_1 = \{(A_1, A_2) \,|\, A_1 \preccurlyeq A_2, A_1 \vee A_2 \in \mathrm{IFS}(X)\}. \quad (3)$$

Theorem

Define $f_1 : \mathrm{IVIFS}(X) \longrightarrow \mathcal{O}_1$ *as follows: when* $A = \{\langle x, (\mu^A(x), \nu^A(x))\rangle \,|\, x \in X\} \in \mathrm{IVIFS}(X)$ *with* $\mu^A(x) = [\mu^A_L(x), \mu^A_M(x)]$, $\nu^A(x) = [\nu^A_L(x), \nu^A_M(x)]$ *for each* $x \in X$*, we define* $f_1(A) = (I^A_1, I^A_2)$ *by the expression:*

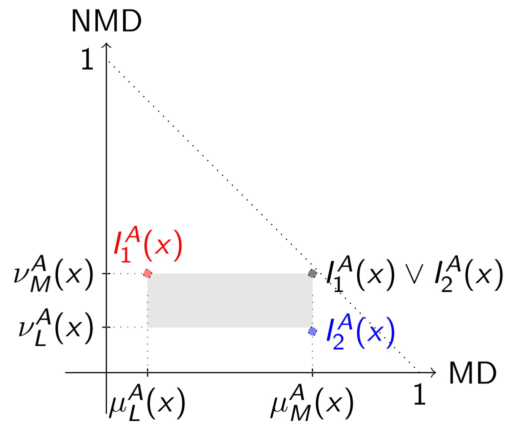
$$I^A_1(x) = (\mu^A_L(x), \nu^A_M(x)),\ I^A_2(x) = (\mu^A_M(x), \nu^A_L(x)) \quad (4)$$

for each $x \in X$*. Then,* f_1 *is a bijection.*

The proof of the theorem will provide an explicit expression for $(f_1)^{-1} : \mathcal{O}_1 \longrightarrow \mathrm{IVIFS}(X)$.

First bijection (cont.)

This figure illustrates the process proving the theorem:
For each $x \in X$, the IVIFN $([\mu_L^A(x), \mu_M^A(x)], [\nu_L^A(x), \nu_M^A(x)])$ represented by the gray box can be uniquely identified by the pair of IFNs formed by its red and blue corners. The collection of such pairs (for all $x \in X$) produces an element from $\mathcal{O}_1$ because the black corner represents an IFN too. This is a reversible process.



Second bijection

We shall use the following $\mathcal{O}_2 \subseteq \mathrm{IFS}(X) \times \mathrm{IFS}(X)$:

$$\mathcal{O}_2 = \{(A_1, A_2) \in \mathrm{IFS}(X)^2 \mid A_1 \leqslant_L A_2\}. \tag{5}$$

Theorem

Define $f_2 : \mathrm{IVIFS}(X) \longrightarrow \mathcal{O}_2$ as follows: when $A = \{\langle x, (\mu^A(x), \nu^A(x))\rangle \mid x \in X\} \in \mathrm{IVIFS}(X)$ with $\mu^A(x) = [\mu^A_L(x), \mu^A_M(x)]$, $\nu^A(x) = [\nu^A_L(x), \nu^A_M(x)]$ for each $x \in X$, we define $f_2(A) = (J^A_1, J^A_2)$ by the expression:

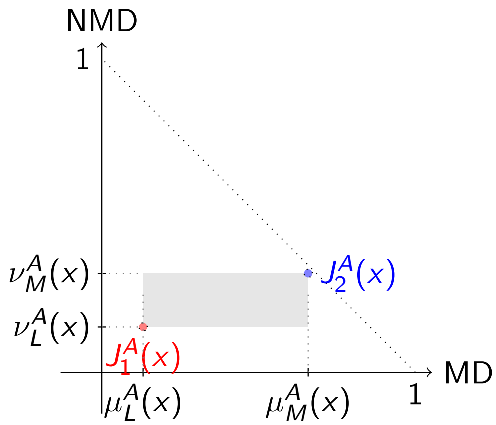
$$J^A_1(x) = (\mu^A_L(x), \nu^A_L(x)),\ J^A_2(x) = (\mu^A_M(x), \nu^A_M(x)) \tag{6}$$

for each $x \in X$. Then, f_2 is a bijection.

The proof of the theorem will provide an explicit expression of $(f_2)^{-1} : \mathcal{O}_2 \longrightarrow \mathrm{IVIFS}(X)$.

Second bijection (cont.)

This figure illustrates the process proving the theorem:
For each $x \in X$, the IVIFN $([\mu_L^A(x), \mu_M^A(x)], [\nu_L^A(x), \nu_M^A(x)])$ represented by the gray box can be uniquely identified by the pair of IFNs formed by its red and blue corners. The collection of such pairs (for all $x \in X$) produces an element from $\mathcal{O}_2$. This is a reversible process.



IV. Application: Aggregation of IVIFSs

Transformation techniques have been used before in the framework of IVIFNs (Chen et al., 2016).

Our proposal is simpler and much more direct. We shall rely on:

- The two bijections defined above, that transform IVIFNs into suitable pairs of IFNs,
- A variation of the concept of aggregation operator on IFNs.

It is noteworthy that most authors concentrate on aggregation of IVIFNs. We shall work with IVIFSs, which are formed by an indexed collection of IVIFNs. This process will lay the groundwork for subsequent applications to MAGDM.

Some auxiliary concepts

We shall be concerned with the next concept:

Definition

A v-**aggregation operator on IFNs** is a mapping $\mathcal{A} : \mathbb{A} \times \overset{n}{\ldots} \times \mathbb{A} \to \mathbb{A}$ that is monotonic with respect to $\leqslant_L$ and such that $\mathcal{A}(0, \overset{n}{\ldots}, 0) = 0$, $\mathcal{A}(1, \overset{n}{\ldots}, 1) = 1$.

Monotonicity with respect to $\leqslant_L$ means that when $(A_1^1, \ldots, A_1^n), (A_2^1, \ldots, A_2^n) \in \mathbb{A} \times \overset{n}{\ldots} \times \mathbb{A}$ and $A_1^i \leqslant_L A_2^i$ for $i = 1, \ldots, n$, then $\mathcal{A}(A_1^1, \ldots, A_1^n) \leqslant_L \mathcal{A}(A_2^1, \ldots, A_2^n)$.

Some auxiliary concepts (cont.)

Naturally, we can **extend this definition to v-aggregation operator on IFSs** (*see the paper for this straightforward process*).

This definition is a variation of the idea of aggregation operators on IFNs (Beliakov et al.), that are required to satisfy monotonicity with respect to $\preccurlyeq$ instead of componentwise-dominance $\leqslant_L$.

Actually, **many aggregation operators on IFSs are v-aggregation operators on IFSs too**. Let us cite some examples that depend on a weighting vector $\omega = (\omega_1, \ldots, \omega_n)$. This concept assumes $\omega_1 + \ldots + \omega_n = 1$ and $\omega_j \in \mathcal{I}$ for all $j = 1, \ldots, n$. Now, when $A_i = \langle \mu_{A_i}, \nu_{A_i} \rangle \in \mathrm{IFS}(X)$, $i = 1, \ldots, n$:

- $\mathrm{IWAM}_\omega(A_1, \ldots, A_n) = \langle \mu, \nu \rangle$ such that for all $x \in X$, $(\mu(x), \nu(x)) = (\sum_{i=1}^{n} \omega_i \mu_{A_i}(x), \sum_{i=1}^{n} \omega_i \nu_{A_i}(x))$ (Beliakov et al.)

Some auxiliary concepts (cont.)

- $\mathrm{IFWA}_\omega(A_1, \ldots, A_n) = \langle \mu, \nu \rangle$ such that for all $x \in X$, $(\mu(x), \nu(x)) = (1 - \prod_{i=1}^{n}(1 - \mu_{A_i}(x))^{\omega_i}, \prod_{i=1}^{n} \nu_{A_i}(x)^{\omega_i})$ (Beliakov et al.)
- $\mathrm{IFWG}_\omega(A_1, \ldots, A_n) = \langle \mu, \nu \rangle$ such that for all $x \in X$, $(\mu(x), \nu(x)) = (\prod_{i=1}^{n} \mu_{A_i}(x)^{\omega_i}, 1 - \prod_{i=1}^{n}(1 - \nu_{A_i}(x))^{\omega_i})$ (Xu and Yager)
- $\mathrm{IFWGA}_\omega(A_1, \ldots, A_n) = \langle \mu, \nu \rangle$ such that for all $x \in X$, $\mu(x) = (1 - \prod_{i=1}^{n}(1 - \mu_{A_i}(x))^{\omega_i}$ and $\nu(x) = (1 - \prod_{i=1}^{n}(1 - \mu_{A_i}(x))^{\omega_i} - \prod_{i=1}^{n}(1 - \mu_{A_i}(x) - \nu_{A_i}(x))^{\omega_i})$ (Chen and Chang)

Respective versions of IFWG_ω and IFWGA_ω that incorporate the spirit of the OWA operator have been defined too. Many other aggregation operators on IFSs have been constructed that are inspired by other ideas.

Some auxiliary concepts (cont.)

It is easy to see that the examples defined above are in fact v-aggregation operators on IFSs too. So, in this section we suppose that $\mathcal{A}$ is a v-aggregation operator on IFNs.

We construct $\bar{\mathcal{A}} : \mathrm{IFS}(X) \times \overset{n}{\ldots} \times \mathrm{IFS}(X) \to \mathrm{IFS}(X)$, a mapping defined as follows. When $I^i = \langle \mu^i, \nu^i \rangle \in \mathrm{IFS}(X)$ for each $i = 1, \ldots, n$, we let $\bar{\mathcal{A}}(I^1, \ldots, I^n) = \langle \mu, \nu \rangle$ with $(\mu(x), \nu(x)) = \mathcal{A}\left((\mu^1(x), \nu^1(x)), \ldots, (\mu^n(x), \nu^n(x))\right)$.

Observe that when $x \in X$, $\mu(x) + \nu(x) \leqslant 1$ because $(\mu^i(x), \nu^i(x)) \in \mathbb{A}$ for all i, and $\mathcal{A} : \mathbb{A} \times \overset{n}{\ldots} \times \mathbb{A} \to \mathbb{A}$.

Definition

With each family $\mathcal{F} = \{(A_1^i, A_2^i)\}_{i=1}^n$ such that $(A_1^i, A_2^i) \in \mathcal{O}_2$ for each $i = 1, \ldots, n$, we associate $\mathcal{A}(\mathcal{F}) = (A_1, A_2) \in \mathcal{O}_2$ such that $A_1 = \bar{\mathcal{A}}(A_1^1, \ldots, A_1^n)$, $A_2 = \bar{\mathcal{A}}(A_2^1, \ldots, A_2^n)$.

Some auxiliary concepts (cont.)

These considerations allow us to define the next mapping:

Definition

Suppose that $\mathcal{A}$ is a v-aggregation operator on IFNs. Then we define $F_{\mathcal{A}} : \mathcal{O}_2 \times \overset{n}{\cdots} \times \mathcal{O}_2 \rightarrow \mathcal{O}_2$ as follows: when $\big((A_1^1, A_2^1), \ldots, (A_1^n, A_2^n)\big) \in \mathcal{O}_2 \times \overset{n}{\cdots} \times \mathcal{O}_2$, we let $F_{\mathcal{A}}\big((A_1^1, A_2^1), \ldots, (A_1^n, A_2^n)\big) = \mathcal{A}(\mathcal{F})$ with $\mathcal{F} = \{(A_1^i, A_2^i)\}_{i=1}^n$.

B. Aggregation of IVIFSs: a general methodology

Algorithm 1 below uses elements defined above to produce a flexible methodology that aggregates IVIFSs:

Algorithm 1 Algorithm for aggregation of IVIFSs.

Input: A finite list of IVIFSs.
Output: Aggregate IVIFS.

Elective element: $\mathcal{A}$, a υ-aggregation operator on IFNs.

1: Apply bijection f_2 (cf., Theorem 2) to all IVIFSs. Each IVIFS produces a pair of IFSs that belongs to $\mathcal{O}_2$.
2: Use Definition 6 to aggregate these pairs into one pair of IFSs that belongs to $\mathcal{O}_2$ using $\mathcal{A}$.
3: Apply inverse of f_2 to this pair of IFSs.
4: **return** The aggregate IVIFS of the initial IVIFSs.

Moreover, we have included *several examples in the paper* to demonstrate **applicability and flexibility of the proposed algorithms**.

V. Application: GDM with IVIFSs

The achievements in the section above can be complemented with the **computation of scores of IVIFNs** in order to establish a flexible GDM methodology for IVIFSs in a routine manner.

Hence, **we can adapt our strategy by selecting an adequate v-aggregation operator on IFNs**, and in the case of the operators described in the previous section and many others, we need to adjust the corresponding weights.

Application: GDM with IVIFSs (cont.)

Algorithm 2 below summarizes this technique.

Algorithm 2 Algorithm for GDM with IVIFSs.

Input: A finite list of IVIFSs. Each IVIFS gives an individual evaluation of a list of alternatives.

Output: Order on the alternatives.

Elective elements: $\mathcal{A}$, a v-aggregation operator on IFNs, and scores and accuracies for IVIFNs.

1: Apply Algorithm 1 to get an aggregate IVIFS.
2: Compute scores and accuracies of the IVIFNs in the aggregate IVIFS.
3: Apply the comparison law corresponding to these scores and accuracies
4: **return** A ranking of the alternatives.

Conclusions

We have been able to **aggregate IVIFSs without having to go through the lengthy process of defining aggregation operators on IVIFSs**.

With the help of scores, we have implemented a new, **customizable GDM technique** in the framework of IVIFSs.

The ability to use **different aggregation operators of IFNs** and **scores of IVIFSs** (and possibly accuracies too) makes it very flexible.

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