# Notes on Transformation Techniques for IVIFS: Applications to Aggregation and Decision Making

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**Abstract**

We delve into the application of two operational transformation techniques that represent a single interval-valued intuitionistic fuzzy number using two intuitionistic fuzzy numbers in a constructive fashion. These techniques are employed to achieve seamless aggregation of interval-valued intuitionistic fuzzy numbers and facilitate multi-attribute decision-making within this framework. The decision-making and prioritization processes rely on comparison laws that consider the score and accuracy of an interval-valued intuitionistic fuzzy number. We illustrate how these parameters can be derived from the analogous proxies associated with the intuitionistic fuzzy numbers that represent it. To wrap up our exploration, we present a comparative study as the culmination of this research endeavor.

Interval-valued intuitionistic fuzzy sets (IVIFS) are a powerful extension of traditional fuzzy sets that provide a more flexible and expressive framework for handling uncertainty and vagueness in decision-making processes. In IVIFS, instead of assigning a single membership value to an element within a fuzzy set, we assign a closed interval of values, allowing for a more accurate representation of uncertainty and imprecision in real-world scenarios.

The concept of IVIFS has gained significant attention in various fields due to its ability to model and manage complex decision-making situations where the available information is not only incomplete but also imprecise. This unique feature makes IVIFS a valuable tool in applications that require robustness against uncertainties, such as risk assessment, medical diagnosis, financial analysis, and environmental management.

In this context, IVIFS offer a versatile approach to decision making by providing a richer representation of uncertainty, which allows decision-makers to make more informed and reliable choices in situations where traditional fuzzy sets or other uncertainty models may fall short. This brief introduction will delve deeper into the applications of IVIFS in decision-making processes, highlighting their advantages and illustrating how they can enhance the quality of decisions in uncertain environments.

These notes encompass the presentation of the paper titled "Transformation Techniques for Interval-Valued Intuitionistic Fuzzy Sets: Applications to Aggregation and Decision Making," which was delivered by the authors at the Conference of the European Society for Fuzzy Logic and Technology held in Palma de Mallorca, Spain, on September 4th, 2023 [1].

# Transformation techniques for interval-valued intuitionistic fuzzy sets: applications to aggregation and decision making

EUSFLAT 2023, Mallorca

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# Motivation

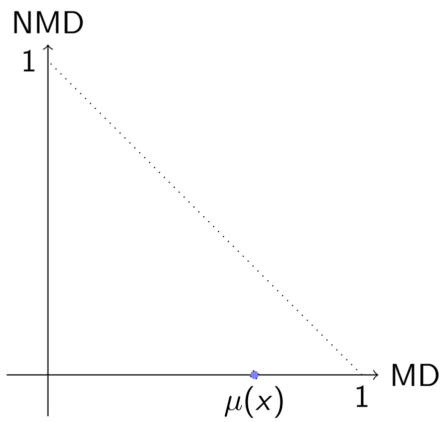



Figure: Fuzzy set associates elements with numbers in $[0, 1]$ – their membership degrees.

## Motivation

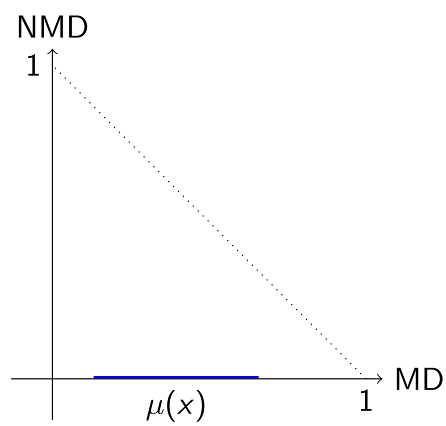



Figure: Interval-valued fuzzy set associates elements with closed subintervals of $[0, 1]$.

## Motivation

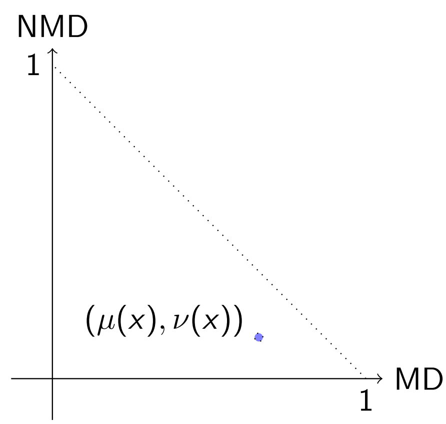



Figure: Intuitionistic fuzzy set associates elements with intuitionistic fuzzy numbers (IFNs) – pairs of membership / non-membership degrees.

## Motivation

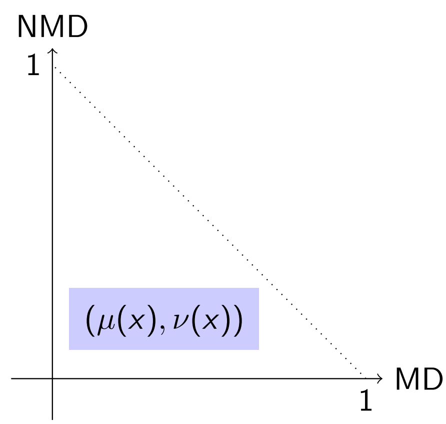



Figure: Interval-valued intuitionistic fuzzy set associates elements with interval-valued intuitionistic fuzzy numbers (IVIFNs).

## Motivation

Transformation techniques exist for **intuitionistic fuzzy sets** (Atanassov and Gargov, FSS 1989) and **interval valued intuitionistic fuzzy sets** (Chen, Cheng and Tsai, INS 2016).

- They are helpful when they allow us to simplify aggregation and decision making.
- The transformation techniques for interval valued intuitionistic fuzzy sets are complicated (they are expressed in terms of pairs of right-angled triangular fuzzy numbers).

## Short summary

We further investigate transformation techniques for **interval valued intuitionistic fuzzy sets**, in continuation of a previous work titled "Aggregation of interval valued intuitionistic fuzzy sets based on transformation techniques" (2023 IEEE International Conference on Fuzzy Systems, Incheon, South Korea).

- **At a theoretical level**, we prove that the two theorems that transform interval valued intuitionistic fuzzy sets into pairs of intuitionistic fuzzy sets produce the same aggregation results.

  We also show that scores and accuracies are "preserved" by these transformations.

- **On a more practical level**, we show how these results can be applied to group decision making in this framework.

  A comparison with an existing methodology is performed in the printed version.

## I. Introduction

**Interval valued intuitionistic fuzzy sets** (**IVIFS**) associate elements of a set with an **interval valued intuitionistic fuzzy number** (**IVIFN**).

**Aggregation operators for IFSs / IVIFSs** are defined separately for each constituent IFNs / IVIFNs. Two basic methodologies:

- The first method uses operational laws for IFNs or IVIFNs.
- The second uses aggregation operators on crisp numbers.

Whatever the case, **the complexity of the IVIFN model produces lengthy formulas whose intuitive meaning is hard to grasp**.

Alternatively, we produced an adaptable methodology whose first step consists of a **transformation technique**.

It allows us to reduce the problem to aggregation of IFNs — much simpler.

## II. Orthopairs and other preliminary concepts

Let $X$ be a fixed set of alternatives and $D[0,1]$ denote the set of closed intervals that are subsets of $\mathcal{I} = [0,1]$.

An **orthopair** is $(\mu, \nu)$ where $0 \leqslant \mu, \nu \leqslant 1$. When $\mu + \nu \leqslant 1$, it is an intuitionistic fuzzy number (IFN). The set of all IFNs will be denoted as $\mathbb{A}$.

### Definition

**Two preorders** are defined on the set of all IFNs: if $(\mu_1, \nu_1)$, $(\mu_2, \nu_2)$ are IFNs,

$(\mu_1, \nu_1) \leqslant_L (\mu_2, \nu_2)$ if and only if $\mu_1 \leqslant \mu_2$ and $\nu_1 \leqslant \nu_2$,
$(\mu_1, \nu_1) \preccurlyeq (\mu_2, \nu_2)$ if and only if $\mu_1 \leqslant \mu_2$ and $\nu_2 \leqslant \nu_1$.

### Definition

When $(\mu_1, \nu_1)$, $(\mu_2, \nu_2)$ are orthopairs, one can define the orthopairs

$(\mu_1, \nu_1) \wedge (\mu_2, \nu_2) = (\min\{\mu_1, \mu_2\}, \min\{\nu_1, \nu_2\})$
$(\mu_1, \nu_1) \vee (\mu_2, \nu_2) = (\max\{\mu_1, \mu_2\}, \max\{\nu_1, \nu_2\})$.

## II.A) IFSs and related concepts and facts

**Definition (Atanassov)**

An **intuitionistic fuzzy set** (IFS) $A$ over $X$ is $A = \{\langle x, (\mu_A(x), \nu_A(x))\rangle | \, x \in X\}$, where $(\mu_A(x), \nu_A(x))$ is an IFN for each $x \in X$. The mappings $\mu_A, \nu_A : X \to [0,1]$ capture the **membership degree** (**MD**) and **non-membership degree** (**NMD**) of $x \in X$ to $A$.

$A$ is often shortened to $A = \langle \mu_A, \nu_A \rangle$.

The set of all IFSs over $X$ is $\mathrm{IFS}(X)$.

The preorders $\leqslant_L$ and $\preccurlyeq$ can be **extended from IFNs to IFSs** easily.

Analogously we can define $A \wedge B$ and $A \vee B$ by elementwise extension.

## II.A) IFSs and related concepts and facts (cont.)

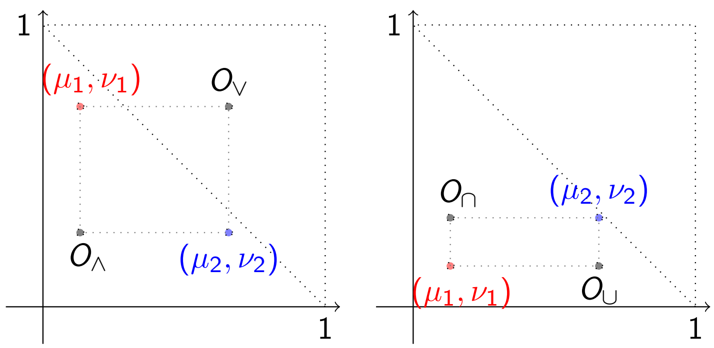
Union and intersection of IFNs always produce IFNs: when $(\mu_1, \nu_1)$, $(\mu_2, \nu_2)$ are IFNs,

$$(\mu_1, \nu_1) \cup (\mu_2, \nu_2) = (\max\{\mu_1, \mu_2\}, \min\{\nu_1, \nu_2\}),$$
$$(\mu_1, \nu_1) \cap (\mu_2, \nu_2) = (\min\{\mu_1, \mu_2\}, \max\{\nu_1, \nu_2\}).$$

It is not always true that $(\mu_1, \nu_1) \vee (\mu_2, \nu_2)$ is an IFN when $(\mu_1, \nu_1)$, $(\mu_2, \nu_2)$ are IFNs.

The next figure represents concepts mentioned above, in two situations where $(\mu_1, \nu_1)$, $(\mu_2, \nu_2)$ are IFNs.

# II.A) IFSs and related concepts and facts (cont.)




(a) $(\mu_1, \nu_1) \preccurlyeq (\mu_2, \nu_2)$ (b) $(\mu_1, \nu_1) \leqslant_L (\mu_2, \nu_2)$

Figure: A graphical representation of definitions. Both $(\mu_1, \nu_1)$ and $(\mu_2, \nu_2)$ are IFNs in the two diagrams. The orthopairs $O_\wedge$ and $O_\vee$ respectively represent $(\mu_1, \nu_1) \wedge (\mu_2, \nu_2)$ and $(\mu_1, \nu_1) \vee (\mu_2, \nu_2)$. $O_\cup$ and $O_\cap$ respectively represent the IFNs $(\mu_1, \nu_1) \cup (\mu_2, \nu_2)$ and $(\mu_1, \nu_1) \cap (\mu_2, \nu_2)$.

## II.B) IVIFSs and related concepts

**Definition (Atanassov and Gargov)**

An **interval valued intuitionistic fuzzy set** (IVIFS) $A$ over $X$ is $A = \{\langle x, (\mu^A(x), \nu^A(x))\rangle | \, x \in X\}$, where for each $x \in X$, $\mu^A(x) = [\mu_L^A(x), \mu_M^A(x)] \in D[0,1]$, $\nu^A(x) = [\nu_L^A(x), \nu_M^A(x)] \in D[0,1]$ and $\mu_M^A(x) + \nu_M^A(x) \leqslant 1$.

Each pair $([\mu_L^A(x), \mu_M^A(x)], [\nu_L^A(x), \nu_M^A(x)])$ is an interval valued intuitionistic fuzzy number (IVIFN).

We denote the set of all IVIFSs over $X$ by $\mathrm{IVIFS}(X)$.

**Observe that** an IVIFN is $([a, b], [c, d]) \in D[0,1] \times D[0,1]$ such that the orthopair $(b, d)$ is in fact an IFN.

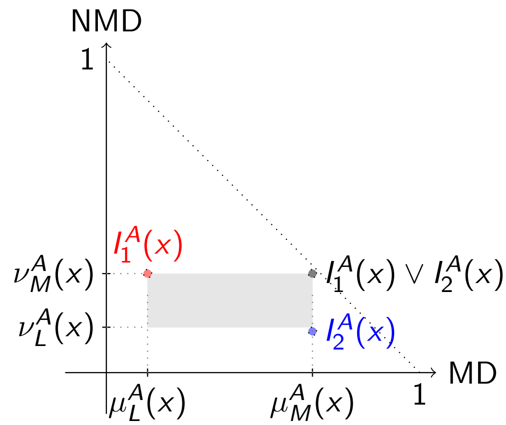
## III. Two practical bijections

Two representation theorems capture similar intuitions:

if we imagine an IVIFN as a rectangle (within the appropriate boundaries), then each IVIFN can be uniquely identified by two opposed ‘corners’ (which therefore must meet some restrictions).

## First bijection (intuitive)

**This figure illustrates the process proving the theorem**:
For each $x \in X$, the IVIFN $([\mu_L^A(x), \mu_M^A(x)], [\nu_L^A(x), \nu_M^A(x)])$ represented by the gray box can be uniquely identified by the pair of IFNs formed by its red and blue corners. The collection of such pairs (for all $x \in X$) produces an element from $\mathcal{O}_1$ because the black corner represents an IFN too. This is a reversible process.



## First bijection (technicalities)

We shall use the following $\mathcal{O}_1 \subseteq \mathrm{IFS}(X) \times \mathrm{IFS}(X)$:

$$\mathcal{O}_1 = \{(A_1, A_2) \,|\, A_1 \preccurlyeq A_2, A_1 \vee A_2 \in \mathrm{IFS}(X)\}. \tag{1}$$

### Theorem

*Define $f_1 : \mathrm{IVIFS}(X) \longrightarrow \mathcal{O}_1$ as follows: when $A = \{\langle x, (\mu^A(x), \nu^A(x))\rangle \,|\, x \in X\} \in \mathrm{IVIFS}(X)$ with $\mu^A(x) = [\mu^A_L(x), \mu^A_M(x)]$, $\nu^A(x) = [\nu^A_L(x), \nu^A_M(x)]$ for each $x \in X$, we define $f_1(A) = (I^A_1, I^A_2)$ by the expression:*

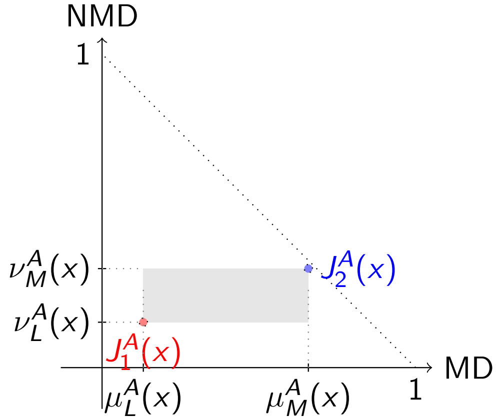
$$I^A_1(x) = (\mu^A_L(x), \nu^A_M(x)),\ I^A_2(x) = (\mu^A_M(x), \nu^A_L(x)) \tag{2}$$

*for each $x \in X$. Then, $f_1$ is a bijection.*

The proof provides an explicit expression for $(f_1)^{-1} : \mathcal{O}_1 \longrightarrow \mathrm{IVIFS}(X)$.

## Second bijection (intuitive)

**The process proving the theorem**:
For each $x \in X$, the IVIFN $([\mu_L^A(x), \mu_M^A(x)], [\nu_L^A(x), \nu_M^A(x)])$ represented by the gray box can be uniquely identified by the pair of IFNs formed by its red and blue corners. The collection of such pairs (for all $x \in X$) produces an element from $\mathcal{O}_2$. This is a reversible process.



## Second bijection (technicalities)

We shall use the following $\mathcal{O}_2 \subseteq \mathrm{IFS}(X) \times \mathrm{IFS}(X)$:

$$\mathcal{O}_2 = \{(A_1, A_2) \in \mathrm{IFS}(X)^2 \,|\, A_1 \leqslant_L A_2\}. \tag{3}$$

**Theorem**

*Define $f_2 : \mathrm{IVIFS}(X) \longrightarrow \mathcal{O}_2$ as follows: when $A = \{\langle x, (\mu^A(x), \nu^A(x))\rangle \,|\, x \in X\} \in \mathrm{IVIFS}(X)$ with $\mu^A(x) = [\mu^A_L(x), \mu^A_M(x)]$, $\nu^A(x) = [\nu^A_L(x), \nu^A_M(x)]$ for each $x \in X$, we define $f_2(A) = (J^A_1, J^A_2)$ by the expression:*

$$J^A_1(x) = (\mu^A_L(x), \nu^A_L(x)),\ J^A_2(x) = (\mu^A_M(x), \nu^A_M(x)) \tag{4}$$

*for each $x \in X$. Then, $f_2$ is a bijection.*

The proof provides an explicit expression of $(f_2)^{-1} : \mathcal{O}_2 \longrightarrow \mathrm{IVIFS}(X)$.

## IV. Application 1: Scores and accuracies of IVIFSs

Xu and Chen (FSKD 2007) defined the **score** of $([a, b], [c, d])$ as $\frac{1}{2}(a - c + b - d)$.

And $\frac{1}{2}(a + b + c + d)$ defines its **accuracy**.

**Use in decision making:** Preference to IVIFNs with higher scores. If two IVIFNs have the same score, the one with higher accuracy is preferred.

### Proposition

*Let $P = ([\mu, \mu'], [\nu, \nu'])$ be an IVIFN.*
*Let $f_1(P) = (A_1^1, A_2^1)$ and $f_2(P) = (A_1^2, A_2^2)$, all $A_1^1$, $A_2^1$, $A_1^2$, $A_2^2$ are IFNs. Then*

$$s(P) = \frac{1}{2}(S(A_1^1) + S(A_2^1)) = \frac{1}{2}(S(A_1^2) + S(A_2^2))$$

*and*

$$h(P) = \frac{1}{2}(H(A_1^1) + H(A_2^1)) = \frac{1}{2}(H(A_1^2) + H(A_2^2)).$$

# V. Application 2: Aggregation of IVIFSs

Our proposal (presented at FUZZ-IEEE 2023) relies on:

- The two bijections defined above.
- A variation of the concept of aggregation operator on IFNs, in the case of the second bijection.

**Definition**

A $v$-**aggregation operator on IFNs** is a mapping $\mathcal{A} : \mathbb{A} \times \overset{n}{\ldots} \times \mathbb{A} \to \mathbb{A}$ that is monotonic with respect to $\leqslant_L$ and such that $\mathcal{A}(\mathbf{0}, \overset{n}{\ldots}, \mathbf{0}) = \mathbf{0}$, $\mathcal{A}(\mathbf{1}, \overset{n}{\ldots}, \mathbf{1}) = \mathbf{1}$.

**Monotonicity** with respect to $\leqslant_L$: if $(A_1^1, \ldots, A_1^n), (A_2^1, \ldots, A_2^n) \in \mathbb{A} \times \overset{n}{\ldots} \times \mathbb{A}$ and $A_1^i \leqslant_L A_2^i$ for $i = 1, \ldots, n$, then $\mathcal{A}(A_1^1, \ldots, A_1^n) \leqslant_L \mathcal{A}(A_2^1, \ldots, A_2^n)$.

## Some auxiliary concepts

Aggregation operators on IFNs (Beliakov et al., INS 2011) are required to satisfy monotonicity with respect to $\preccurlyeq$ instead of $\leqslant_L$.

**Most aggregation operators on IFSs are $v$-aggregation operators**. Examples that depend on a weighting vector $\omega = (\omega_1, \ldots, \omega_n)$. When $A_i = (\mu_i, \nu_i)$ are IFNs, $i = 1, \ldots, n$:

- $\mathrm{IWAM}_\omega(A_1, \ldots, A_n) = (\sum_{i=1}^n \omega_i \mu_i, \sum_{i=1}^n \omega_i \nu_i)$
- $\mathrm{IFWA}_\omega(A_1, \ldots, A_n) = (1 - \prod_{i=1}^n (1-\mu_i)^{\omega_i}, \prod_{i=1}^n \nu_i^{\omega_i})$
- $\mathrm{IFWG}_\omega(A_1, \ldots, A_n) = (\prod_{i=1}^n \mu_i^{\omega_i}, 1 - \prod_{i=1}^n (1-\nu_i)^{\omega_i})$
- $\mathrm{IFWGA}_\omega(A_1, \ldots, A_n) = (\mu, \nu)$ with $\mu = (1 - \prod_{i=1}^n (1-\mu_i)^{\omega_i}$ and $\nu = (1 - \prod_{i=1}^n (1-\mu_i)^{\omega_i} - \prod_{i=1}^n (1-\mu_i-\nu_i)^{\omega_i})$

Respective versions of $\mathrm{IFWG}_\omega$ and $\mathrm{IFWGA}_\omega$ that incorporate the spirit of the OWA operator have been defined too. Many other aggregation operators on IFSs have been constructed that are inspired by other ideas.

## Aggregation of IVIFSs: a general methodology

**Algorithm 1** A flexible procedure for aggregation of IVIFNs. Alternatively: a flexible procedure that encapsulates one IVIFS in one IVIFN.

**Input:** A finite list of IVIFNs (or alternatively: one IVIFS, characterized by one IVIFN associated with each alternative).
*Elective element*: a suitable aggregation operator for IFNs.

1: Apply first/second bijection $f_1/f_2$ to the IVIFNs.
With each IVIFN in the list we get a pair of IFNs.

2: Use an aggregation operator to transform this list of pairs into one pair of IFNs that satisfies the required structural property.
We apply aggregation separately to the first component of the pairs, and then to their second components.

3: Apply $f_1^{-1}/f_2^{-1}$ to this aggregate pair of IFNs.

**Output:** Aggregate IVIFN of the original list of IVIFNs (or alternatively: one IVIFN that encapsulates the information in the IVIFS).

### Proposition

*For a fixed aggregation operator that is also a v-aggregation operator: the choice between $f_1$ and $f_2$ does not affect the output.*

# VI. Application 3: MADM with IVIFSs

Three projects, their characteristics, and respective weights, from Xu and Chen (FSKD 2007)

| | $B_1$ (weight 0.5) | $B_2$ (weight 0.3) | $B_3$ (weight 0.2) |
|---|---|---|---|
| $A_1$ | $([0.5, 0.6], [0.2, 0.3])$ | $([0.3, 0.4], [0.4, 0.6])$ | $([0.4, 0.5], [0.3, 0.5])$ |
| $A_9$ | $([0.2, 0.4], [0.4, 0.5])$ | $([0.6, 0.7], [0.2, 0.3])$ | $([0.5, 0.6], [0.2, 0.3])$ |
| $A_{10}$ | $([0.5, 0.7], [0.1, 0.3])$ | $([0.6, 0.7], [0.1, 0.3])$ | $([0.4, 0.5], [0.2, 0.5])$ |

Aggregate IFNs of the projects in the table above.

| | **Method in Xu and Chen (2007)** | **Algorithm 1** ($\mathrm{IWAM}_\omega$) | **Algorithm 1** ($\mathrm{IFWG}_\omega$) |
|---|---|---|---|
| $A_1$ | $([0.3941, 0.4990], [0.2852, 0.4380])$ | $([0.42, 0.52], [0.28, 0.43])$ | $([0.4102, 0.5123], [0.2855, 0.4467])$ |
| $A_9$ | $([0.3964, 0.5635], [0.2751, 0.3748])$ | $([0.38, 0.53], [0.3, 0.4])$ | $([0.3340, 0.5131], [0.3072, 0.4084])$ |
| $A_{10}$ | $([0.5237, 0.6560], [0.1218, 0.3440])$ | $([0.51, 0.66], [0.12, 0.34])$ | $([0.5051, 0.6544], [0.1210, 0.3456])$ |

# VI. Application 3: MADM with IVIFSs (cont.)

Scores for the projects in previous table:

| | Method in Xu and Chen (2007) | Algorithm 1 ($\mathrm{IWAM}_\omega$) | Algorithm 1 ($\mathrm{IFWG}_\omega$) |
|---|---|---|---|
| $A_1$ | 0.0850 | 0.115 | 0.0952 |
| $A_9$ | 0.1550 | 0.105 | 0.06575 |
| $A_{10}$ | 0.3570 | 0.355 | 0.34645 |

The choice of the decision making mechanism is not innocuous:

- If we use Xu and Chen (2007), the recommendation is $A_{10} \succ A_9 \succ A_1$.
- If we use Algorithm 1 (either with $\mathrm{IWAM}_\omega$ or with $\mathrm{IFWG}_\omega$), the recommendation becomes $A_{10} \succ A_1 \succ A_9$.

The decision between $A_1$ and $A_9$ is different.

# Conclusions

We have been able to **aggregate IVIFSs without having to go through the lengthy process of defining aggregation operators on IVIFSs**.

With the help of scores, we have implemented a new, **customizable MADM technique** in the framework of IVIFSs.

The ability to use **different aggregation operators of IFNs** and **scores of IVIFSs** (and possibly accuracies too) makes it very flexible.

## Future work

Utilization of **other scores** in decision-making.

Production of **novel scores** for IVIFNs.

**Extensions** for the analysis of temporal information.

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