## Research paper

# Optimal menu when agents make mistakes 

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#### Abstract

This paper studies how an optimal menu chosen by a social planner depends on whether agents receive imperfect signals about their true tastes (imperfect self-knowledge) or the properties of available alternatives (imperfect information). Under imperfect self-knowledge, it is not optimal to offer fewer alternatives than the number of different tastes present in the population, unless noise is infinite (agents have no clue about their true preferences). As noise increases, the social planner offers menu items that are closer together (more similar). However, under imperfect information, as noise increases, it could be optimal to construct a menu with more distinct alternatives, restrict the number of options, or, for some finite noise, offer a single item.


## 1. Introduction

In everyday life, we often face choices from discrete menus; for example, when choosing an insurance plan, a school for our children, or a pension fund. When confronted with these important decisions, we often make mistakes for two potential reasons. First, we may misperceive the true properties of alternatives, i.e., we have imperfect information. Second, we may misperceive our own tastes, i.e., we have imperfect self-knowledge. For example, consider two individuals who are looking to buy a car. The first person wants to buy a particular type of car, a used minivan, but cannot distinguish between a high-quality car and a "lemon", which leads to a potential mistake due to her imperfect knowledge about the alternatives. The second person wants to buy her first new car from the dealership, which discloses truthful information about the car's condition and properties, but she lacks experience about what type of car she would enjoy the most, resulting in a potential mistake due to her imperfect self-knowledge about her own taste.

There exist many well-documented possible mechanisms that explain imperfect information and imperfect self-knowledge. Imperfect information could be related to ignorance or uncertainty. For example, individuals can be uninformed and underestimate potential cost savings from changing prescription drug plans (Kling et al., 2012), not be fully informed about crucial aspects of an insurance plan (Handel and Kolstad, 2015), and, when choosing a car, may think of fuel costs as scaling linearly in miles per gallon instead of gallons per mile (Allcott, 2013). Imperfect self-knowledge can be because people vary in their ability to retrieve

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or memorize relevant information about themselves, engage more or less in reflecting on who they are, or that some individuals simply lack experience in a particular choice situation. For example, when choosing a gym contract, individuals overestimate their attendance and their likelihood of canceling automatically renewed memberships (DellaVigna and Malmendier, 2006)². In general, we are myopic in decision-making, can lack the skill to predict our own tastes and risk preferences, and can be led to erroneous choices thought by fallible memory and incorrect evaluation of past experiences (Kahneman, 1994; Heckman et al., 2021) ${ }^{3}$.

In the examples above, a government or other social planner can regulate the size of the menu from which consumers choose and the properties of alternatives within it. The social planner cannot possibly know the individual tastes of a particular agent and, hence, is not able to provide the best alternative for each agent. However, knowing the characteristics of the overall population, including probabilities of mistakes and distribution of tastes, he can construct a menu of alternatives, referred to as an optimal menu, that maximizes the sum of the expected utilities of agents.

I analyze the choice of a planner who should choose the optimal menu under the assumption that agents misperceive either the true properties of available alternatives or their own tastes. Due to either of these two types of misperception, an agent could make a mistake, i.e., choose an alternative with a property that is not the best match for her taste. In two extreme cases, when the misperception is insignificant or agents choose an alternative randomly, the optimal menus are identical under both types of misperception. For the intermediate degrees of rationality, the dependence of the optimal choice set on the precision of choice is complex. I use a binary model and numerical calculations to obtain a solution for this intermediate case.

The results are the following. When agents misperceive the available options, it is optimal to limit choices when the probability of making mistakes is moderately high. Further, it could be optimal to construct a menu with more distinct alternatives. In contrast, when agents misperceive their own tastes, it is optimal to limit choice only when agents choose randomly, and to propose alternatives that are more similar when there is a greater probability of a mistake.

The intuition behind the results is that, when agents misperceive the properties of alternatives, every additional alternative on the menu has the benefit of providing more choice (matching the agents' taste more precisely) at the cost of increasing the probability and magnitude of mistakes. Thus, the more similar the alternatives are, the more difficult it is for the agent to differentiate between them. Therefore, it could be optimal to construct a menu with more distinct alternatives to decrease the probability of a mistake. When the probability of a mistake is large, it becomes optimal to remove options that induce a large utility loss, and for some finitely large probability of a mistake, it is optimal to leave one option that matches the mean taste of the population.

In contrast, when agents have imperfect self-knowledge, the misperception of taste distorts the distance between the true taste and the properties of the options in the same way for all options. Thus, the probability of a mistake depends only on the midpoints between the properties of the two closest alternatives. It would not be decreased if alternatives were differentiated as long as the midpoint between their properties is the same. Moreover, while the particular optimal property of a new additional alternative would depend on the distributions of tastes and mistakes, the social planner can always add an item with a property that matches an existing alternative, and it will not decrease the utility of the agents. Therefore, adding a new alternative to the menu is weakly beneficial for the social planner.

The discussion about individuals misperceiving the true properties of alternatives and accordingly failing to choose the best one goes back at least as far as Luce (1959), who analyzes agent choice subject to random noise. Mirrlees $(1987,2017)$ and Sheshinski (2003b,a, 2010, 2016) study the welfare maximization problem when agents misperceive the true properties of alternatives. They show that, while the choice should not be limited when the agents are completely rational, the optimum choice set is a singleton when the probability of a mistake is relatively high. In contrast, this paper focuses on comparing optimal menu allocations in two situations: when the agent misperceives either the true properties of alternatives or her own taste. Thus, if agents misperceive the true properties of alternatives, the optimal menu differs significantly from the one when agents misperceive their own tastes. This study highlights the importance of taking into account not only the demand for a particular alternative but also the probability and source of mistakes when designing a menu set.

In addition, this paper proposes a new explanatory insight into the choice paradox (Schwartz, 2004), i.e., the effect when a larger choice set sometimes decreases the satisfaction of individuals and ultimately can lead to the rejection of an offer. This phenomenon has been observed, for example, when consumers purchased jam and chocolate (Iyengar and Lepper, 2001) and when they made more important decisions such as a choice of 401 k pension plans (Iyengar et al., 2004), or decided on participation in an election (Nagler, 2015) ${ }^{4}$. Several studies suggest that the existence of the choice paradox and the efficiency of corresponding interventions, such as the categorization of goods, depends on whether consumers are familiar with products or not (Chernev, 2003; Mogilner et al., 2008). There are numerous models that attempt to explain this evidence (Irons and Hepburn, 2007; Sarver, 2008; Ortoleva, 2013; Kuksov and Villas-Boas, 2010). While my study does not focus on a particular mechanism, it suggests that the existence of this phenomenon and relevant interventions depend on the source of mistakes in the decision-making process. Thus, when agents misperceive the true properties of alternatives, we can observe choice overload, and limiting the menu size could be a

[^1]welfare-maximizing intervention. However, when agents have imperfect self-knowledge, we would not observe the choice overload and, hence, should not limit the choice.

The rest of the paper is organized as follows. The next section presents the model setup. Section 3 discusses a simple model with two agents to illustrate the intuition behind the results, and then provides numerical simulations with populations of agents. The last section concludes.

## 2. Model

A population of $M \geq 2$ agents chooses from a set of $N \geq 2$ alternatives. The utility of the agent $i \in\{1, \ldots, M\}$ from the alternative $j \in\{1, \ldots, N\}$ is $U_{i}^{j}=-\left(t_{i}-v^{j}\right)^{2}$, where $t_{i} \in \mathbb{R}$ is the taste (bliss point) of $i$ and $v^{j} \in \mathbb{R}$ is the property of $j$. $T \geq 2$ is the number of unique tastes in the population. The agent misperceives the parameters of the model. I describe two versions of the model:

- with misperceived true properties of alternatives: for any alternative $j$, the agent $i$ observes a signal $\vartheta_{i}^{j}=v^{j}+e_{i}^{j}$, where $v^{j}$ is the true property of the option, and noise $e_{i}^{j}$ is a random variable drawn from a distribution with mean zero and variance $\sigma_{i}^{j}$. She chooses the alternative with the signal that is the closest match to her taste ${ }^{5}$, i.e., solves the following problem:

$$
\max _{j \in\{1, \ldots, N\}}-\left(t_{i}-\vartheta_{i}^{j}\right)^{2}
$$

- with misperceived own true taste: the agent $i$ observes a signal $\tau_{i}=t_{i}+e_{i}$, where $t_{i}$ is the true taste of the agent, and noise $e_{i}$ is a random variable drawn from a distribution with mean zero and variance $\sigma_{i}$. She chooses the alternative with the property that is the closest match to the signal of her taste, i.e., solves the following problem:

$$
\max _{j \in\{1, \ldots, N\}}-\left(\tau_{i}-v^{j}\right)^{2}
$$

In both versions of the model, if there are several alternatives that solve the agent's problem, then the agent chooses randomly between them with equal probabilities.

The social planner maximizes overall welfare by choosing a number and properties of available alternatives, i.e., the optimal menu:

$$
\max _{N, v^{j} \forall j \in\{1, \ldots, N\}} \sum_{i=1}^{M} \sum_{j=1}^{N} P_{i}^{j} U_{i}^{j},
$$

where $P_{i}^{j}$ is the probability that the agent $i$ chooses option $j$. I assume that $N \leq T$ : the maximum number of options that the social planner could propose is equal to the number of tastes in the population. ${ }^{6}$

The problem has the following time-line:

1. The social planner observes (i) distributions of mistakes, (ii) what the tastes in the population are, and (iii) the number of agents with each taste.
2. He chooses the optimal menu.
3. Agents observe signals.
4. They choose an alternative from the menu.

## 3. Solution

The solution to the welfare maximization problem depends on the size of the noise. Regardless of the source of mistakes, when there is no noise, the social planner creates a menu with alternatives that match tastes perfectly; when noise is infinite, it is optimal to limit choices and provide only one alternative that matches the mean taste in the population. This result is formalized in Propositions 1 and 2.

Proposition 1. If $\sigma_{i}^{j}=0$ or $\sigma_{i}=0 \forall(i, j)$, then $N=T, v^{j}=t_{i}$.
Proof. Since $U_{i} \leq 0 \forall i \Rightarrow \max \left(\sum_{i=1}^{M} \sum_{j=1}^{N} P_{i}^{j} U_{i}^{j}\right)=0$ which is obtained when $N=T, v^{j}=t_{i}$.
Proposition 2. If $\sigma_{i}^{j}=+\infty$ or $\sigma_{i}=+\infty \forall(i, j)^{7}$, then $N=1$ and $v^{j}=\frac{\sum t_{i}}{M}$.
Proof. If $\sigma_{i}^{j}=+\infty$ or $\sigma_{i}=+\infty$, then all alternatives are a priori the same for agents and by the assumption $P_{i}^{j}=\frac{1}{N}$. Therefore, the solution to the welfare maximization problem is $N=1$ and $v^{j}=\frac{\sum t_{i}}{M}$.

[^2]In the next subsection, I illustrate the solution to the model for the intermediate cases using a model with uniformly distributed noise and two agents. Then, I show that the results obtained are valid for the larger population of agents with a continuous distribution of noise using numerical simulations.

### 3.1. Two agents

There are two agents, $i \in\{1,2\}$, with tastes symmetrically allocated around zero, $t_{1}=-t_{2}<0 .{ }^{8}$ The social planner could propose at most two options, $j \in\{1,2\}$. I assume that $v^{1} \leq v^{2}$. The situation when $v^{1}=v^{2}$ is identical to the situation when the social planner proposes only one alternative and limits the agents' choice.

I assume that the noise is uniformly distributed, $e_{i}^{j}$ and $e_{i} \sim U(-b,+b)$. Therefore, the social planner expects that agent 1 chooses the first option with probability $P_{1}^{1}$ and the second option with probability $P_{1}^{2}$. Agent 2 chooses similarly.

In the case of misperceived true properties of alternatives, the probabilities are as follows:

$$
\begin{aligned}
& P_{1}^{2}=\min \left(1, \max \left(0,0.5 *\left(\frac{\left.v^{1}-v^{2}+2 b\right)}{2 b}\right)^{2}\right)\right), \\
& P_{1}^{1}=1-P_{1}^{2}, \\
& P_{2}^{1}=\min \left(1, \max \left(0,0.5 *\left(\frac{\left.v^{1}-v^{2}+2 b\right)}{2 b}\right)^{2}\right)\right), \\
& P_{2}^{2}=1-P_{2}^{1} .
\end{aligned}
$$

In the case of misperceived true own tastes, the probabilities are as follows:

$$
\begin{aligned}
& P_{1}^{2}=\min \left(1, \max \left(0, \frac{t_{1}+b-\frac{v^{1}+v^{2}}{2}}{2 b}\right)\right), \\
& P_{1}^{1}=1-P_{1}^{2}, \\
& P_{2}^{1}=\min \left(1, \max \left(0, \frac{\frac{v^{1}+v^{2}}{2}-\left(t_{2}-b\right)}{2 b}\right)\right), \\
& P_{2}^{2}=1-P_{2}^{1} .
\end{aligned}
$$

The solution to the welfare maximization problem is formalized in Propositions 3 and 4.
Proposition 3. In the case of misperceived true values of alternatives, the welfare maximization problem has the following solution:

- small noise ( $b \leq\left|t_{i}\right|$ ): $v^{1}=-v^{2}=t_{1}$;
- medium noise $\left(\left|t_{i}\right|<b<4\left|t_{i}\right|\right): v^{1}=-v^{2}=\frac{-b^{2}-4 b t_{1}}{3 t_{1}}$;
- large noise $\left(b \geq 4\left|t_{i}\right|\right): v^{1}=v^{2}=0$.

Proof. See Appendix A.
Proposition 4. In the case of misperceived true own tastes, the welfare maximization problem has the following solution:

- small noise ( $b \leq\left|t_{i}\right|$ ): $v^{1}=-v^{2}=t_{1}$;
- medium and large noise $\left(b>\left|t_{i}\right|\right): v^{1}=-v^{2}=-\frac{t_{1}^{2}}{b}$.


## Proof. See Appendix B.

Accordingly, when the noise is small ( $b \leq\left|t_{i}\right|$ ), in both cases the social planner proposes options that match the tastes of the agents perfectly, and they choose the option closest to their true taste with certainty. When the noise is significantly large $\left(b>\left|t_{i}\right|\right)$, then the solution depends on the source of mistakes. If agents misperceive the true properties of alternatives, it is optimal to limit the choice when the noise is finitely large. However, when agents misperceive their tastes, it is optimal to propose two alternatives with different properties for any finite noise.

In addition, if agents misperceive the true properties of alternatives, there exists noise ( $\left|t_{i}\right|<b<2\left|t_{i}\right|$ ) when the difference in the properties of proposed alternatives increases in the noise, i.e., the property of the first item decreases ( $\frac{\partial v^{1}}{\partial b}<0$ ) and the property of the second item increases $\left(\frac{\partial v^{2}}{\partial b}>0\right)$ with the noise. However, if agents misperceive their tastes, the social planner always proposes alternatives that are more similar as the noise becomes greater. Fig. 1 illustrates these results for given parameters.

[^3]

Fig. 1. Optimal properties of alternatives when agents have imperfect information (on the top) or imperfect self-knowledge (on the bottom) for different noises ( $b=2$ on the left and $b=4$ on the right) and $t_{1}=-1$.


Fig. 2. Optimal property of the first alternative as a function of $b$ and $t_{1}=-1$.

### 3.1.1. Intuition

The results are driven by the fact that if a taste is unclear, the distance between the true taste and the properties of the options is distorted in the same way for all options, while if the properties of the options are unclear, this distortion is different for any option. Consider the probability that the agent makes the wrong choice (i.e., she chooses the alternative that is not the closest to her true taste): Eqs. (1)-(2) in the case of misperceived true properties of alternatives, and Eqs. (3)-(4) in the case of misperceived true own tastes.

When the noise originates from the misperception of alternatives, the probability of a mistake does not depend on the individual taste and is equal for both agents. Therefore, placing options close to each other increases the probability that agents make the wrong choice, which is a nonlinear function of $v_{1}$ and $v_{2}$. Thus, there is an inverted U-shaped curvilinear relationship between the optimal property of the alternative and the size of the noise, as depicted in Fig. 2 for optimal $v_{1}=-v_{2}=v$. When the noise is significant, but still small $\left(\left|t_{i}\right|<b<2\left|t_{i}\right|\right)$, the social planner wants to distance the properties of alternatives from each other. In this situation, the loss from the decrease in utility, if the correct choice is made, is smaller than the gain from the decrease in the probability of making the wrong choice. However, when the noise is moderately large ( $2\left|t_{i}\right| \leq b<4\left|t_{i}\right|$ ), it is not profitable to distance the properties of alternatives farther apart. The loss from the decrease in utility in the case of the correct choice outweighs the gain from the decrease in the probability of the wrong choice. Therefore, the social planner chooses properties of alternatives closer to each other. When the probability of making the wrong choice is significantly high ( $b \geq 4\left|t_{i}\right|$ ), it is optimal to propose alternatives with identical properties.

However, when agents misperceive their tastes, the probability of making a mistake depends linearly on individual tastes and the midpoint between properties of alternatives $\left(\frac{v_{1}+v_{2}}{2}\right)$. Moving the midpoint would decrease the probability of a mistake for one agent, but equally increase it for another agent. Therefore, differentiating the properties of alternatives cannot decrease the overall probability of making the wrong choice. Given that the utilities of the agents are convex in the loss from the mismatch, there are no incentives for the social planner to propose items with asymmetrical properties or with properties that are more distinct than the tastes of agents. Accordingly, the social planner chooses $v$ by equalizing the marginal gain of locating an option closer to the center for the second agent (reducing the loss in the case of making the wrong choice) and the marginal loss for the first agent (reducing the gain in the case of making the correct choice).

### 3.2. Many agents

In this section, I solve the model for the larger population of agents with a continuous distribution of noise using numerical simulations. This example aims to provide suggestive evidence that the results described in the previous section are not driven by the binary model setup and could be observed in a more complex setting too.


Fig. 3. Optimal menu allocation when agents misperceive the true properties of alternatives or their own tastes, and $\lambda=0.1$. The red lines indicate the optimal properties of alternatives. The histogram shows the distribution of agents.

### 3.2.1. Setup

There is a single-peaked population of agents with a variety of tastes $T=7$. When agents misperceive the true properties of alternatives, $e_{i}^{j}$ is assumed to be identically and independently Gumbel distributed. The Gumbel distribution has fatter tails than a Normal distribution; however, the difference between them is often indistinguishable empirically (Train, 2002). At the same time, the difference between Gumbel distributed variables, which is used to calculate the probabilities of an agent's choices, follows the Logistic distribution. This significantly simplifies the numerical simulation. Therefore, the probability that agent $i$ chooses option $j$ is:

$$
P_{i}^{j}=\frac{\exp \left(U_{i}^{j} / \lambda\right)}{\sum_{i}^{N} \exp \left(U_{i}^{j} / \lambda\right)}
$$

When agents misperceive their own true tastes, $e_{i}$ is assumed to be identically and independently Logistic distributed. ${ }^{9}$ In this case, the probability that agent $i$ chooses option $j$ is:

$$
P_{i}^{j}=\int_{\frac{v^{j-1}+v^{j}}{2}}^{\frac{v^{j}+v^{j+1}}{2}} \frac{\exp \left(\frac{t_{i}-v^{j}}{0.5 \lambda}\right)}{0.5 \lambda\left(1+\exp \left(\frac{t_{i}-v^{j}}{0.5 \lambda}\right)\right)^{2}} d v^{j}
$$

In both situations, higher values of $\lambda$ correspond to larger variance and, hence, to a greater probability of making a mistake. I solve for every possible menu size and then select the one that maximizes welfare. ${ }^{10}$

### 3.2.2. Results

The solution with the optimal number of alternatives and optimal menu allocation is presented in Figs. 3-6 for different $\lambda$. The gray bars (histogram) correspond to the number of agents with a particular taste. The optimal properties of alternatives are defined by vertical lines. The optimal number of options is stated above the graphs. In some situations, there are fewer vertical lines than the optimal number of alternatives, because there are several identical options that match the same taste. Intuitively, additional options with repeated values increase the probability that agents will choose a particular alternative. Thus, when one taste is more salient in the population, it is beneficial to highlight the alternative that matches this taste. ${ }^{11},{ }^{12}$

Fig. 3 shows that, when the noise is small, it is optimal to provide alternatives that match tastes perfectly under both kinds of mistakes.

Figs. 4 and 5 show the optimal menus for the situation when the noise is significantly large. When agents misperceive the true properties of alternatives, it is optimal to limit the choice (Figs. 4). When the probabilities of making mistakes increase, the social planner decreases the menu size. When agents misperceive their own taste, it is not optimal to limit their choice (Figs. 5). Thus, the social planner proposes 7 alternatives with unique properties for any noise. When the probabilities of making mistakes increase, he allocates alternatives closer to each other and to the mean taste in the population.

[^4]

Fig. 4. Optimal menu allocation when agents misperceive the true properties of alternatives for different noise ( $\lambda=1$ on the left and $\lambda=2$ on the right graph). The red lines indicate the optimal properties of alternatives. The histogram shows the distribution of agents.


Fig. 5. Optimal menu allocation when agents misperceive their own tastes for different noise ( $\lambda=1$ on the left and $\lambda=2$ on the right graph). The red lines indicate the optimal properties of alternatives. The histogram shows the distribution of agents.

It is worth noticing that the effect of the decrease in the inequality of tastes is similar to the decrease in noise. Fig. 6 shows the optimal menu allocation for different populations of agents with the same variety of tastes $T=7$, but with a lower density of agents with the most frequent (mode) taste $t_{\text {mode }}=0$. In this situation, when agents misperceive the true properties of alternatives (left graph, Fig. 6), the social planner proposes more alternatives to agents, compared to the optimal menu for a population with a higher density of agents with mode taste (left graph, Fig. 4). Similarly, when agents misperceive their own tastes (right graph, Fig. 6), the social planner proposes 7 alternatives, but allocates them further away from each other and from the mean taste in the population, compared to the optimal menu for a population with a higher density of agents with mode taste (left graph, Fig. 5).

## 4. Conclusion

This study demonstrates the significance of considering the origin of a mistake while constructing a menu. When agents make decisions blindly, the social planner should limit the choice and offer only one option to agents, regardless of the source of the agents' mistakes. However, in more realistic situations, in which the decision is imperfect but not random, if agents misperceive the true properties of alternatives, the optimal menu could differ dramatically from one where agents misperceive their own tastes.

The present paper assumes that all agents are identical in their precision and that the mistakes they make are independent. However, the perceived properties of alternatives or tastes can be correlated in many real-life decision situations, and some agents are better than others at distinguishing alternatives and knowing their own tastes. People might, for instance, consistently misperceive the properties of alternatives in one particular direction because they are risk-averse or pessimistic. Individuals with extreme preferences might be experts who are knowledgeable about both their own tastes and the properties of alternatives, whereas people with more moderate preferences might find it more difficult to distinguish between options on the menu and be less aware of their own preferences. In addition, this paper is agnostic about the mechanisms behind the origin of mistakes. For example, consider the repeatable choice and assume that the misperception of tastes is caused by a fallible memory. Then, in contrast to the situation where a choice is made only once, it may be best to limit the menu, because doing so may reduce the likelihood of future mistakes. All in all, it would be interesting and insightful to explore further generalizations for the dimensions mentioned above.


Fig. 6. Optimal menu allocation when agents misperceive the true properties of alternatives (left graph) or their own tastes (right graph) and $\lambda=1$. The red lines indicate the optimal properties of alternatives. The histogram shows the distribution of agents.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Appendix A. Proof of Proposition 3

I denote $t_{1}=t<0$. If $b<|t|$, then the probability of a mistake equals zero and the first best allocation is optimal. Therefore, I consider a situation when $b \geq|t|$ and $0 \leq P_{i}^{j} \leq 1 \forall i, j$. Then, the welfare maximization problem is the following:

$$
\begin{aligned}
\max _{v^{1}, v^{2}} W\left(v^{1}, v^{2}\right)= & \left\{\left(1-0.5 *\left(\frac{\left.v^{1}-v^{2}+2 b\right)}{2 b}\right)^{2}\right) \cdot\left(-\left(t-v^{1}\right)^{2}-\left(-t-v^{2}\right)^{2}\right)\right. \\
& \left.0.5 *\left(\frac{\left.v^{1}-v^{2}+2 b\right)}{2 b}\right)^{2} \cdot\left(-\left(t-v^{2}\right)^{2}-\left(-t-v^{1}\right)^{2}\right)\right\} .
\end{aligned}
$$

The derivative with respect to $v^{1}$ is:

$$
\frac{t\left(-1.5\left(v^{1}\right)^{2}-4 b v^{1}+3 v^{1} v^{2}-1.5\left(v^{2}\right)^{2}+4 b v^{2}\right)-2 b^{2} v^{1}}{b^{2}}=0 .
$$

The derivative with respect to $v^{2}$ is:

$$
\frac{t\left(-1.5\left(v^{1}\right)^{2}+4 b v^{1}-3 v^{1} v^{2}+1.5\left(v^{2}\right)^{2}-4 b v^{2}\right)-2 b^{2} v^{1}}{b^{2}}=0 .
$$

This system of equations has two solutions:

$$
\begin{aligned}
& v^{1}=v^{2}=0 \\
& v^{1}=-v^{2}=\frac{-b^{2}-4 b t}{3 t} .
\end{aligned}
$$

Since $v^{1} \leq 0$, the second solution exists only for $b \leq 4|t|$. Moreover, when $b=4|t|$, then $v=0$ and the two solutions coincide. In this situation, the welfare is $W(b=4|t|)=-2 t^{2}$. At the same time, if one substitutes $v^{1}=\frac{-b^{2}-4 b t}{3 t}$ into the maximization problem, then $W(b=|t|)=0$ and $W>-2 t^{2}$ for any $|t|<b<4|t|$. Therefore, for $b<4|t|$ the welfare is maximized when $v^{1}=-v^{2}=\frac{-b^{2}-4 b t}{3 t}$; for $b \geq 4 t$ it is optimal to provide the menu with two identical alternatives $v^{1}=v^{2}=0$.

## Appendix B. Proof of Proposition 4

If the $b<|t|$, then the probability of a mistake equals zero and the first best allocation is optimal. Therefore, I consider a situation when $b \geq|t|$ and $0 \leq P_{i}^{j} \leq 1 \forall i, j$. Then the welfare maximization problem is the following:

$$
\begin{aligned}
\max _{v^{1}, v^{2}} W\left(v^{1}, v^{2}\right)= & \left\{\frac{t+b-\frac{v^{1}+v^{2}}{2}}{2 b} \cdot-\left(t-v^{2}\right)^{2}+\left(1-\frac{t+b-\frac{v^{1}+v^{2}}{2}}{2 b}\right) \cdot-\left(t-v^{1}\right)^{2}\right. \\
& \left.\frac{\frac{v^{1}+v^{2}}{2}-(-t-b)}{2 b} \cdot-\left(-t-v^{1}\right)^{2}+\left(1-\frac{\frac{v^{1}+v^{2}}{2}-(-t-b)}{2 b}\right) \cdot-\left(-t-v^{2}\right)^{2}\right\} .
\end{aligned}
$$

The derivative with respect to $v^{1}$ is:

$$
-\frac{4 t^{2}+4 b v_{1}+3 v_{1}^{2}+2 v_{1} v_{2}-v_{2}^{2}}{2 b}=0 .
$$

The derivative with respect to $v^{2}$ is:

$$
\frac{4 t^{2}-v_{1}^{2}-4 b v_{2}+2 v_{1} v_{2}+3 v_{2}^{2}}{2 b}=0 .
$$

This system of equations has three solutions:

$$
\begin{align*}
& v^{1}=v^{2}=-\frac{t^{2}}{b}  \tag{5}\\
& v^{1}=1 / 2\left(-b-\sqrt{2} \sqrt{b^{2}-2 t^{2}}, v^{2}=1 / 2\left(b-\sqrt{2} \sqrt{b^{2}-2 t^{2}}\right) ;\right.  \tag{6}\\
& v^{1}=1 / 2\left(-b+\sqrt{2} \sqrt{b^{2}-2 t^{2}}, v^{2}=1 / 2\left(b+\sqrt{2} \sqrt{b^{2}-2 t^{2}}\right)\right. \tag{7}
\end{align*}
$$

Solutions (6) and (7) exist only for $b^{2}>2 t^{2}$. Then, given $b^{2}>2 t^{2}$, by using the second derivative test, for both of these solutions:

$$
\frac{\partial^{2} W}{\partial v_{1}^{2}} \frac{\partial^{2} W}{\partial v_{2}^{2}}-\frac{\partial^{2} W}{\partial v_{1} \partial v_{2}}=4-\frac{8 t^{2}}{b^{2}}<0 .
$$

Therefore, solutions (6) and (7) are not the maxima. Since $b \geq|t|$, for the solution (5):

$$
\frac{\partial^{2} W}{\partial v_{1}^{2}}=\frac{\partial^{2} W}{\partial v_{2}^{2}}=-2+\frac{2 t^{2}}{b^{2}}<0 .
$$

Therefore, the welfare is maximized when $v^{1}=-v^{2}=-\frac{t^{2}}{b}$.

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[^1]:    ${ }^{2}$ Overconfidence is a leading explanation for the results in DellaVigna and Malmendier (2006). Note, however, that other assumptions about consumer preferences and beliefs can also explain the empirical findings. For example, the agents could be risk-averse or prefer to have a commitment device to make them go to the gym. In these situations, the agents act according to their preferences and do not make a mistake.
    ${ }^{3}$ See also Falk et al. (2021) who study how accounting for differences in self-knowledge could significantly increase the explanatory power of regression models.
    ${ }^{4}$ Further discussion on empirical evidence when choice opportunities can harm consumer can be found, for example, in Scheibehenne et al. (2010) or Chernev et al. (2015).

[^2]:    ${ }^{5}$ For discussion on when this behavior is optimal for the agent, see Weibull et al. (2007).
    6 I make this assumption because the welfare function is not monotone in the number of options: for example, if for a given distribution, the optimal number of alternatives is 4 , then the solution to the welfare maximization problem automatically includes any number that is divisible by 4 .
    7 Here I abuse notation and denote the situation when $k \rightarrow+\infty$ as $\sigma_{i}^{j}=+\infty$ and $\sigma_{i}=+\infty \forall(i, j)$, where $k=1,2, \ldots$ is the sequence of models, which are equivalent in all respects except $x$, and $\lim _{k \rightarrow+\infty} x_{k}=+\infty$, where $x$ is either $\sigma_{i}^{j}$ or $\sigma_{i}$.

[^3]:    ${ }^{8}$ It is without loss of generality, because, for any two distinct tastes one always can re-scale tastes to be symmetrically allocated around zero.

[^4]:    ${ }^{9}$ In this case, I do not use the Gumbel distribution, since it is asymmetric. The asymmetry property skews the optimal menu, complicating the visual comparison. However, the qualitative results of the welfare analysis with the Gumbel distribution are identical to the analysis with the Logistic distribution.
    10 Calculations are performed in R using the "optimx" package.
    11 Mirrlees (2017) refers to such manipulation as "advertising". One possible type of "advertising" is nudges. For example, it was shown that setting an option as a default increases the probability that this alternative will be chosen. See Thaler and Sunstein (2008) for additional discussion on the topic.
    12 One way to avoid the presence of identical options in the menu is to introduce the following probability function: $P_{i}^{j}=\frac{m(j) P_{i}^{j}}{\int m(y) P_{i}^{y} d y}$, where $m(j)$ is a density of alternatives with identical properties (Mirrlees, 2017). This formula relates to the modified multinomial logit model by Matějka and McKay (2015). Accordingly, another possible explanation for the "advertising" effect is prior knowledge of agents about options in a menu.

