# Comparison of Spider-Robot Information Models 

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#### Abstract

The paper deduces a mathematical model of a spider-robot with six three-link limbs. Many limbs with a multilink structure greatly complicate the process of synthesizing a model, since in total the robot has twenty-four degrees of freedom, i.e., three coordinates of the center of mass of the body in space, three angles of rotation of the body relative to its center of mass and three degrees of freedom for each limb, to describe the position of the links. The derived mathematical model is based on the Lagrange equations with a further transformation of the equations to the Cauchy normal form in a matrix form. To test the resulting model in a SimInTech environment, an information model is synthesized and two simple experiments ar carried out to simulate the behavior of real spiders: moving forward in a straight line and turning in place at a given angle. The experimental results demonstrate that the synthesized information model can well cope with the tasks and the mathematical model underlying it can be used for further research.


Keywords-Hexapod; Spider-Robot; Mobile Robots; Mathematical Model; Lagrange Equations of the Second Kind.

## I. Introduction

The main task of this research is $\sim--$ synthesis of control of a group of robotic spiders. Group control, using such advantages as a large radius of coverage, a theoretically unlimited set of functions and higher reliability, allows you to solve many practical problems: the study of hard-to-reach surfaces, difficult work in hostile environments, participation in rescue operations, etc. The paper considers the first stage of solving the main problem - the synthesis and study of a spider robot model.

A spider-robot belongs to a class of walking robots. Its increased cross-country ability requires complex algorithms for maintaining balance and high energy consumption, which is greatly complicated by the lack of compact and energyintensive power sources today. Nevertheless, research in this area is well underway and over time and many walking robots have been developed to accomplish various tasks [1].

Spider robots can be used in adverse conditions [2], [3], hazardous industries [4], hard-to-reach places [5]-[7], [34] and in open space [8], [9]. They can be used for moving cargo [10], rescue operations [11], [12], etc.

Most scientists take natural analogues of spiders or other animals [9], [13]-[21]. Therefore, most of such robots tend to have an even number of limbs, four [6], [16], six [22]-[25] or eight [3], [5], [26]-[30]. In addition to the number of limbs, researchers are trying to adopt some features of the
movement of spiders, for example, the ability to jump [22], [31] and run [22], move over rough terrain [24], [26], [32], [33], grip [2], [9], [34], and flip from the back [35]. Part of the research is devoted to finding optimal algorithms for moving limbs [29], finding stable support points [36]-[39] or considering rolling as another way to move limbs [40].

The studies [9], [13], [30], [41] are devoted to the simulation of spider robots. For example, in [42], kinematics and dynamics are derived by using a modified DenavitHartenberg method and Newton-Euler approach. Sometimes only a part of a spider-robot is modeled, for example, in the work [43], a hydraulic mechanism is considered that can control a robot's limb, and in [44], forward and inverse kinematics of a limb. In [31], the authors create a three-legged apparatus to demonstrate a jumping mechanism.

In [6], [24], [25], the physical implementation of spider robots is considered. In [45], the authors use the LabVIEW program for robot design. The researchers assemble a fullfledged spider-robot based on an ATMEGA microcontroller and control it using MATLAB capabilities [3]. In the Simulink environment, the authors in [23] develop a fast gait model for a spider-robot. The work [26] uses Theo Jansen Mechanism to move limbs, which is one way to mimic the gait of animals. Scientists in Japan have created a prototype spider-robot that can walk and drive by switching to wheels embedded in limbs [46]. Researchers from Germany offer a prototype of a cheap robot for 70 euro [47].

In [14], [17], [18], [27]-[29], [45], the synthesis of control systems is investigated. The stability [18], possibility of autonomous activities [45], actions when colliding with obstacles [45], climbing [14], and the use of machine learning to optimize movement on different locality [17], [27]-[29] are studied. The above studies show that most of the research is in the field of information modeling based on ready-made solutions offered by software packages. It is practically impossible to find mathematical justifications for an entire system, and neither for individual parts. That the derivation and validation of a rigorous mathematical model in the form of Cauchy are expected to complement the overall picture of walking robots and contribute to robotics research.

## II. Derivation of a Mathematical Model

### 2.1 Problem Statement

Our research is to derive a mathematical model of a spider-robot specified as a control object with six three-link limbs as shown in Fig. 1.

$$
\begin{equation*}
q=\left(\xi, \eta, \zeta, \psi, \theta, \gamma, \ldots \varphi_{i 1}, \varphi_{i 2}, \varphi_{i 3}, \ldots\right)^{T} \tag{1}
\end{equation*}
$$

where $i=\{1-6\}, \xi, \eta$ and $\zeta$ - coordinates of the center of mass $O$ spider-robot body, $\psi, \theta$ and $\gamma-$ angles of rotation of the body relative to its center of mass $O, \varphi_{i 1}$ - the angle of rotation of the $i$-th limb relative to the body $\left(i_{1}\right), \varphi_{i 2}$ and $\varphi_{i 3}$ - angles of position of its joints ( $i_{2}$ and $i_{3}$ );


Fig. 1. Spider-robot schematic

### 2.2 Lagrange Equations

It is assumed that the orthogonal coordinate system $O x y z$ is rigidly connected to the body, and the system $O \xi \eta \zeta$ moves translationally together with the point $O$, and its axes at each moment of time are respectively parallel to the axes of the absolute coordinate system [48].

The following notation are introduced in this work:

$$
\begin{gathered}
\Delta \varphi_{i 0}=\Delta \varphi_{i 0}\left(\varphi_{i 1}\right)=\varphi_{i 0}-\varphi_{i 1}, \\
\Delta \varphi_{i}=\Delta \varphi_{i}\left(\varphi_{i 2}, \varphi_{i 3}\right)=\varphi_{i 2}-\varphi_{i 3}, \\
\alpha_{i 0}=\alpha_{i 0}(\psi, \theta) \approx \psi \sin \varphi_{i 0}+\theta \cos \varphi_{i 0}, \\
\alpha_{i}=\alpha_{i}\left(\psi, \theta, \varphi_{i 1}\right) \approx \psi \sin \left(\varphi_{i 0}-\varphi_{i 1}\right)+\theta \cos \left(\varphi_{i 0}-\varphi_{i 1}\right), \\
\alpha_{i}^{\prime}=\alpha_{i}^{\prime}\left(\psi, \theta, \varphi_{i 1}\right) \approx \psi \cos \left(\varphi_{i 0}-\varphi_{i 1}\right)-\theta \sin \left(\varphi_{i 0}-\varphi_{i 1}\right), \\
\alpha_{i 2}=\alpha_{i 2}\left(\psi, \theta, \varphi_{i 1}, \varphi_{i 2}\right)=\varphi_{i 2}+\alpha_{i}, \\
\beta_{i}=\beta_{i}\left(\psi, \theta, \varphi_{i 1}, \varphi_{i 2}, \varphi_{i 3}\right)=\pi-\varphi_{i 2}-\varphi_{i 3}-\alpha_{i},
\end{gathered}
$$

where $\varphi_{i 0}$ - angle from the positive semiaxis $O x$, on which the attachment points of the $i$-th limb is located, i.e., first hinge.

The mathematical model of the spider robot was synthesized in the form of a system of Lagrange equations of the second kind [48]-[50]:

$$
\frac{d \mathcal{L}}{d t}\left(\frac{d \mathcal{L}}{d \dot{q}_{\mathrm{i}}}\right)-\frac{d \mathcal{L}}{d q_{\mathrm{i}}}=N Q_{i}^{n}
$$

where $i=\{1-24\}, L-$ lagrangian, $Q n i-$ generalized nonpotential forces acting along the $i$-th coordinate. These equations allow us to describe the motion of a mechanical system with ideal holonomic constraints [51]. Let's represent the system in matrix form in equation (2).

$$
\begin{equation*}
A(q) \ddot{q}+B(q, \dot{q})=U \tag{2}
\end{equation*}
$$

The matrix $A(q)$ is represented as follows:

$$
A=\left(\begin{array}{lllll}
A_{1} & & & & \\
& A_{2} & & & \\
& & \ddots & & \\
& \mathbb{O} & & & A_{3 i} \\
& & & & \ddots
\end{array}\right)
$$

Where

$$
\begin{gathered}
A_{1}=\left(\begin{array}{lll}
m & & \\
& m & \\
& & m
\end{array}\right), A_{2}=\left(\begin{array}{ccc}
I & & \\
& I & \\
& & I
\end{array}\right), \\
A_{3 i}=\left(\begin{array}{ccc}
\frac{1}{2} m_{1} l_{1}^{2}+2 I_{1} & \\
& \left(\frac{1}{2} m_{2}+m_{3}\right) l_{2}^{2}+2 I_{2} & m_{3} l_{2} l_{3} \cos \Delta \varphi_{i} \\
& m_{3} l_{2} l_{3} \cos \Delta \varphi_{i} & \frac{1}{2} m_{3} l_{3}^{2}+2 I_{3}
\end{array}\right),
\end{gathered}
$$

where $i=\{1-6\}$, $m$-body weight, $m_{j}$-mass of the $j$-th joint of the limb, $I$ - body moment of inertia, $I_{j}$ - moment of inertia of the $j$-th articulation of the limb, $l_{j}$ - the length of the $j$-th articulation of the limb.
Vector $B(q, \dot{q})$ :

$$
B==\left(\begin{array}{lllllllllll}
0 & 0 & \frac{\partial \mathcal{L}}{\partial \zeta} & \frac{\partial \mathcal{L}}{\partial \psi} & \frac{\partial \mathcal{L}}{\partial \theta} & 0 & \cdots & \frac{\partial \mathcal{L}}{\partial \varphi_{i 1}} & \frac{\partial \mathcal{L}}{\partial \varphi_{i 2}} & \frac{\partial \mathcal{L}}{\partial \varphi_{i 3}} & \cdots
\end{array}\right)^{T},
$$

where $i=\{1-6\}$,

$$
\begin{gathered}
\frac{\partial \mathcal{L}}{\partial \zeta}=\left(\begin{array}{l} 
\\
\frac{\partial \mathcal{L}}{\partial \psi}=g \sum_{i=1}^{6}\left\{m _ { 1 } \left(l_{i 0}\right.\right. \\
\end{array}\right)=\begin{aligned}
& \left.\cos \alpha_{i 0} \sin \varphi_{i 0}+\frac{1}{2} l_{1} \cos \alpha_{i} \sin \Delta \varphi_{i 0}\right) \\
& +m_{2}\left(l_{i 0} \cos \alpha_{i 0} \sin \varphi_{i 0}\right. \\
& \left.+\left(l_{1} \cos \alpha_{i}+\frac{1}{2} l_{2} \cos \alpha_{i 2}\right) \cdot \sin \Delta \varphi_{i 0}\right) \\
& +m_{3}\left(l_{i 0} \cos \alpha_{i 0} \sin \varphi_{i 0}\right. \\
& +\left(l_{1} \cos \alpha_{i}+l_{2} \cos \alpha_{i 2}\right. \\
& \left.\left.\left.+\frac{1}{2} l_{3} \cos \beta_{i}\right) \sin \Delta \varphi_{i 0}\right)\right\} \\
\frac{\partial \mathcal{L}}{\partial \theta}=g \sum_{i=1}^{6}\left\{m _ { 1 } \left(l_{i 0}\right.\right. & \left.\cos \alpha_{i 0} \cos \varphi_{i 0}+\frac{1}{2} l_{1} \cos \alpha_{i} \cos \Delta \varphi_{i 0}\right) \\
& +m_{2}\left(l_{i 0} \cos \alpha_{i 0} \cos \varphi_{i 0}\right. \\
& \left.+\left(l_{1} \cos \alpha_{i}+\frac{1}{2} l_{2} \cos \alpha_{i 2}\right) \cdot \cos \Delta \varphi_{i 0}\right) \\
& +m_{3}\left(l_{i 0} \cos \alpha_{i 0} \cos \varphi_{i 0}\right. \\
& +\left(l_{1} \cos \alpha_{i}+l_{2} \cos \alpha_{i 2}\right. \\
& \left.\left.\left.+\frac{1}{2} l_{3} \cos \beta_{i}\right) \cos \Delta \varphi_{i 0}\right)\right\} \\
\frac{\partial \mathcal{L}}{\partial \varphi_{i 1}}=g\left\{-\frac{1}{2} m_{1} l_{1}\right. & \cos \alpha_{i}-m_{2}\left(l_{1} \cos \alpha_{i}+\frac{1}{2} l_{2} \cos \alpha_{i 2}\right) \\
& -m_{3}\left(l_{1} \cos \alpha_{i}+l_{2} \cos \alpha_{i 2}\right. \\
& \left.\left.+\frac{1}{2} l_{3} \cos \beta_{i}\right)\right\} \alpha_{i}^{\prime}
\end{aligned} \\
\end{gathered}
$$

$$
\left.\begin{array}{rl}
\frac{\partial \mathcal{L}}{\partial \varphi_{i 2}}=-m_{3} l_{2} l_{3} \sin \Delta \varphi_{i} \dot{\varphi}_{i 2} \dot{\varphi}_{i 3} \\
& +g\left\{\frac{1}{2} m_{2} l_{2} \cos \alpha_{i 2}\right. \\
& \left.+m_{3}\left(l_{2} \cos \alpha_{i 2}+\frac{1}{2} l_{3} \cos \beta_{i}\right)\right\}
\end{array}\right\}
$$

where $l_{i 0}$ - distance from the point $O$ to the attachment point of the $i$-th limb.

In order for the apparatus to stand and walk, the vector $U$ is composed of the following components:

- Support reaction force applied to the supporting limbs $p \in$ $P$ (hanging in the air are denoted by $p \in P$ ) and evenly distributed between them;
- Torques of the body, which are formed from the reaction force of the bearing;
- Spinning frictional forces, expressed by the moments that are applied to the joints of the limbs and which inhibit the rotation of the joints, preventing them from gaining speed indefinitely;
- Sliding frictional forces that slow down the limb when walking and prevent it from slipping.
The following are the elements of vector $U$ :

$$
\begin{gathered}
Q_{\xi}=N_{O \xi}+F_{O \xi}=\sum_{P} N_{\xi}^{p}+\sum_{P} F_{\xi}^{p}, \\
Q_{\eta}=N_{O \eta}+F_{O \eta}=\sum_{P} N_{\eta}^{p}+\sum_{P} F_{\eta}^{p}, \\
Q_{\zeta}=N_{O \zeta}+F_{O \zeta}=\sum_{P} N_{\zeta}^{p}+\sum_{P} F_{\zeta}^{p}, \\
Q_{\psi}=M_{\psi}=\left(\sum_{P} r_{p}^{f} \times N^{p}\right)_{\psi}, \\
Q_{\theta}=M_{\theta}=\left(\sum_{P} r_{p}^{f} \times N^{p}\right)_{\theta}, \\
Q_{\gamma}=M_{\gamma}=\left(\sum_{P} r_{p}^{f} \times N^{p}\right)_{\gamma}, \\
Q_{\varphi_{s 1}}=m_{r o t}^{s l}-m_{f r}^{s l}, \\
Q_{\varphi_{s 2}}=m_{r o t}^{s 2}-m_{f r}^{s 2}, \\
Q_{\varphi_{s 3}}=m_{r o t}^{s 3}-m_{f r}^{s 3}, \\
Q_{\varphi_{p 1}}=m_{r o t}^{p 1}-m_{f r}^{p 1}+\left(N_{x}^{p} \cos \varphi_{p 1}+N_{y}^{p} \sin \varphi_{p 1}\right) \\
\cdot\left(l_{3} \cos \beta_{p}+l_{2} \cos \left(\varphi_{p 2}+\alpha_{p}\right)+l_{l} \cos \alpha_{p}\right) \\
Q_{\varphi_{p 2}}=m_{r o t}^{p 2}-m_{f r}^{p 2}+\left(-N_{x}^{p} \sin \varphi_{p 1}+N_{y}^{p} \cos \varphi_{p 1}\right) . \\
\left(l_{3} \sin \beta_{p}-l_{2} \sin \left(\varphi_{p 2}+\alpha_{p}\right)\right)+N_{z}^{p}\left(l_{3} \cos \beta_{p}+l_{2} \cos \left(\varphi_{p 2}+\right.\right. \\
\left.\left.\alpha_{p}\right)\right), \\
Q_{\varphi_{p 3}}=m_{r o t}^{p 3}-m_{f r}^{p 3}+\left(-N_{x}^{p} \sin \varphi_{p 1}+N_{y}^{p} \cos \varphi_{p 1}\right) l_{3} \sin \beta_{p}+ \\
N_{z}^{p}\left(l_{3} \cos \beta_{p}\right),
\end{gathered}
$$

where $M_{\psi}\left(M_{\theta}, M_{\gamma}\right)$ - moment of forces acting on the body along $O x(O y, O z), r_{p}^{f}$ - a radius vector from the center of mass of the body to the point of contact of the $p$-th limb of the earth, $N_{\xi}^{p}\left(N_{\eta}^{p}, N_{\zeta}^{p}\right)$ - projection of the reaction force of the support applied to the $p$-th limb onto the axis $O \xi(O \eta$, $O \zeta), N_{x}^{p}\left(N_{y}^{p}, N_{z}^{p}\right)$ - projection of the reaction force of the support applied to the $p$-th limb onto the axis $O x(O y, O z)$, $m_{f r}^{i j}$ - moment created by the spinning friction force in the $j$ ом шарнире $i$ - th hinge of the i-th limb:

$$
m_{f r}^{i j}=k_{f r} N^{i j}
$$

where $k_{f r} \in \mathbb{R}$-rolling friction coefficient, and $N^{i j}$ - support reaction force acting on the $j$-th hinge of the $i$-th limb

### 2.3 Cauchy Normal Form

The transformation of the resulting system of the above equations to the Cauchy normal form is performed by transferring higher derivatives to the left side of the equation (1) and changing the variables.

$$
v=\dot{q} \Rightarrow \dot{v}=\ddot{q}
$$

Let the state vector be, $i=\{1-6\}$ :

$$
v=\left(\begin{array}{lllllllllll}
v_{\xi} & v_{\eta} & v_{\zeta} & v_{\psi} & v_{\theta} & v_{\gamma} & \cdots & v_{i 1} & v_{i 2} & v_{i 3} & \cdots
\end{array}\right)^{T},
$$

and the control vector, $i=\{1-6\}$ :

$$
u=\left(\begin{array}{lllll}
\cdots & m_{\mathrm{rot}}^{i 1} & m_{\mathrm{rot}}^{i 2} & m_{\mathrm{rot}}^{i 3} & \cdots
\end{array}\right)^{T}
$$

Then we represent system (1) in a Cauchy normal form:

$$
\dot{v}=\binom{\mathbb{O}_{6 \times 24}}{H_{18 \times 24}^{v}} v+\binom{\mathbb{O}_{6 \times 18}}{F_{18 \times 18}^{u}} u+\binom{C_{6 \times 1}^{O}}{C_{18 \times 1}^{\varphi}}
$$

where $i=\{1-6\}, \mathbb{O}_{n \times m}$ is a null matrix,

$$
\begin{aligned}
& H^{v}=\left(\left.\mathbb{O}_{18 \times 6}\right|^{h_{1}^{v}} \begin{array}{llll} 
& & & \\
& h_{2}^{v} & & \\
& & \ddots & \\
& & & h_{6}^{v}
\end{array}\right), \\
& h_{i}^{v}=h_{i}^{v}\left(\varphi_{i 2}, \varphi_{i 3}\right)=\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & c_{812}^{\prime}() & c_{83}^{\prime}() \\
0 & c_{91}^{\prime}() & c_{923}^{\prime}()
\end{array}\right), \\
& F^{u}=\left(\begin{array}{llll}
f_{1}^{u} & & & \\
& f_{2}^{u} & & \\
& & \ddots & \\
& & & f_{6}^{u}
\end{array}\right), \\
& f_{i}^{u}=f_{i}^{u}\left(\varphi_{i 2}, \varphi_{i 3}\right)=\left(\begin{array}{ccc}
c_{72} & 0 & 0 \\
0 & c_{85}() & c_{86}() \\
0 & c_{95}() & c_{96}()
\end{array}\right),
\end{aligned}
$$

$$
\begin{gathered}
C^{0}=\left(\begin{array}{ccc}
c_{1}() & c_{2}() & c_{3}() \\
c_{4}()+M_{4}() & \left.c_{5}()+M_{5}() \quad M_{6}()\right)^{T}, \\
\vdots \\
C^{\varphi}=\left(\begin{array}{c} 
\\
c_{71}()+N_{7}()-c_{72} m_{f r}^{i l} \\
c_{84}()+N_{8}()-c_{85}() m_{f r}^{i 2}-c_{86}() m_{f r}^{i 3} \\
c_{94}()+N_{9}()-c_{95}() m_{f r}^{i 2}-c_{96}() m_{f r}^{i 3} \\
\vdots
\end{array}\right)
\end{array}\right.
\end{gathered}
$$

### 2.4 Calculation of the Reaction Force of the Support

An important part of calculating the support reaction force is the distribution of the load among the limbs. For example, if all the left limbs and one right limb are supporting, then additional pressure is exerted on it for the two "hanging" limbs of the right side. To take these features into account, a formula for calculating the load distribution coefficients is derived:

$$
c_{k}^{f}=c_{k}^{f}-\left(1-\frac{\Delta_{k l}}{\Delta_{0}}\right),
$$

where $l \in\{1-6\}$ - the number of the supporting limb on which the current iteration of calculations is going on, $k \in$ $\{1-6\}$ - the number of the supporting limb, which is to the left or to the right of $l, \triangle_{0} \in R$ - the difference rotation angles between the supporting limbs, at which the distribution of the load is uniform, $\triangle_{k l} \in R$ - the difference rotation angles between $k$ and $l$. To obtain the last parameter, the following algorithm is developed:

1) Assign each limb a support coefficient according to the principle: supporting limb? Yes: $c^{f}=1$, if limb supports the robot, and 0 otherwise;
2) Iterating limbs over $k$ counterclockwise:
2.1) If $c_{k}^{f} \neq 0$, then go to step 2.2, otherwise $k=k+$ 1 and repeat this step;
2.2) Iterating limbs over 1 starting from $k+$ 1 counterclockwise:
2.2.1) If $c_{l}^{f} \neq 0$, then use (2), otherwise $l=l+1$ and repeat this step;
2.3) Iterating limbs over $l$ starting from $k-1$ : counterclockwise:
2.3.1) If $c_{l}^{f} \neq 0$, then use (2); and otherwise, $l=l+1$ and repeat this step. Next, the support reaction force for each supporting limb is calculated via:

$$
N_{i}=c_{i}^{f} \frac{\left(m+6\left(m_{1}+m_{2}+m_{3}\right)\right) g}{6}, \quad i=\{1-6\} .
$$

## III. Testing the Proposed Model

Based on the proposed mathematical model in a Cauchy normal form, in the SimInTech [52] dynamic modeling environment, an information model of a spider-robot is synthesized as shown in Fig. 2. Model parameters are shown in Table I and Table II.


Fig. 2. Information model of robot-spider
TABLE I. Body and Limb Parameters

| Parameter1 | Body | Joint 1 | Joint 2 | Joint 3 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{m}, \mathrm{kg}$ | 0.0942 | 0.0848 | 0.0105 | 0.0383 |
| $\mathrm{l}, \mathrm{cm}$ | - | 3.0000 | 7.6200 | 11.0300 |
| $\mathrm{I}, \mathrm{kg} \cdot \mathrm{mm}^{2}$ | 486.8409 | 52.6469 | 12.3215 | 27.0795 |

${ }^{1} m$ - weight, $l$ - the length of the articulation of the limb, $I$ - moment of inertia.

TABLE II. Parameters of the Location of Attachments of Limbs TO THE BODY

| Parameter1 | Limb |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1 .} \mathbf{A R 2}$ | 2. AL | 3. MR | 4. ML | 5. RR | 6. RL |  |
| $\varphi \mathrm{i} 0, \mathrm{rad}$ | $\pi / 3$ | $2 \pi / 3$ | 0 | $\pi$ | $5 \pi / 3$ | $4 \pi / 3$ |  |
| $\mathrm{l} i 0, \mathrm{~mm}$ | 101 | 101 | 70 | 70 | 101 | 101 |  |

${ }^{1} \varphi_{i} 0$ - angle from the positive semiaxis $O x$, on which the attachment point of the $i$-th limb is located, i.e. first hinge, $l i 0$ - distance from the point $O$ to the attachment point of the $i$-th limb.
${ }^{2} \mathrm{AR}$ - anterior right, AL - anterior left, MR - middle right, ML - middle left, RR - rear right, RL - rear left.
solved: the positions of the hinges for different phases of motion are included in the program in advance.

### 3.1 Spider-Robot Forward Movement

The forward movement of the spider-robot is set by the standard combination of limb movements, presented in Table III, where RA - raise, LO - lower, TU — turn, -TU — turn back. The sign of the limb rotation angle at the TU stage sets the direction of the robot's movement: forward or backward. Steps -2 to 0 are preparatory and only used when starting a movement, while steps 1 to 6 are looped. The designation of the limbs in Table III: AR - anterior right, MR - middle
right, RR — rear right, AL — anterior left, ML — middle left, and RL - rear left.

TABLE III. Spider-Robot Movement Algorithm

| Step | Limb |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1. AR | 2. AL | 3. MR | 4. ML | 5. RR | 6. RL |  |
| -2 | RA |  |  | RA | RA |  |  |
| -1 | TU |  |  | TU | TU |  |  |
| 0 | LO |  |  | LO | LO |  |  |
| 1 |  | RA | RA |  |  | RA |  |
| 2 | - TU | TU | TU | - TU | - TU | TU |  |
| 3 |  | LO | LO |  |  | LO |  |
| 4 | RA |  |  | RA | RA |  |  |
| 5 | TU | - TU | - TU | TU | TU | - TU |  |
| 6 | LO |  |  | LO | LO |  |  |

Let the spider-robot go from point $\mathrm{A}(0 ; 0)$ to point $\mathrm{B}(0$; 20). The distance between $A$ and $B$ is 0.2 m . The error is 0.05 m . Fig. 3 show the results of the experiment. The steps in robot movement are clearly visible in both figures. Algorithm steps $-1,2$ and 5 occur in time intervals $[2,3],[5,6]$ and $[8,9]$, respectively, which is expressed by the curves of changes in both coordinates and distance to the target. Before these stages, one of the two triplets of limbs is raised (steps $-2,1$ and 4), after lowering (steps 0, 3 and 6 ); each lasting one second. As a result, the spider-robot reaches the target within 8 seconds from the start of movement, i.e., starting from the 1st second.


Fig. 3. (a) Changing the coordinates of the center of mass of the body; (b) Change in distance to target B

### 3.2 Rotating the Spider-Robot in Place

The rotation around its axis is made according to the scheme given in Table IV. The algorithm is looped, and every 10 steps the robot turns through some angle until it reduces the error to zero.

TABLE IV. Spider-Robot Movement Algorithm

| Step | Limb |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1. AR | 2. AL | 3. MR | 4. ML | 5. RR | 6. RL |  |
| 1 | RA |  |  |  |  | RA |  |
| 2 | TU |  |  |  |  | TU |  |
| 3 | LO |  |  |  |  | LO |  |
| 4 |  | RA |  |  | RA |  |  |
| 5 |  | TU |  |  | TU |  |  |
| 6 |  | LO |  |  | LO |  |  |
| 7 |  |  | RA | RA |  |  |  |
| 8 |  |  | TU | TU |  |  |  |
| 9 |  |  | LO | LO |  |  |  |
| 10 | - TU | - TU | - TU | - TU | - TU | - TU |  |

Let the robot turn around its axis by $\pi / 6$. Accuracy is 0.1 radians. Fig. 4 the results of the experiment are presented.


Fig. 4. (a) Changing the angle of rotation of the spider-robot; (b) Change the difference between the current and target rotation angle

Both graphs clearly show the stages of the rotation of the apparatus. From small fluctuations, one can understand that a change in the position of three pairs of limbs, i.e., steps 1-3, $4-6$ and $7-9$ occurs in time intervals $[1,7]$ and $[8,14]$. At the 10th step, the return of the limbs to their original position occurs in the intervals $[7,8]$ and $[14,15]$, which is expressed by the curves of the change in the angle of rotation of the robot and the magnitude of the error. As a result, the robot turns to the required angle within 14 seconds from the start of movement [53]-[60].

## IV. CONCLUSIONS

As a result of the synthesis work, a mathematical model of a spider robot with six three-link limbs in the form of Cauchy. Based on the obtained mathematical model in the dynamic modeling environment SimInTech, an information model of a spider robot is synthesized. The information model has been tested. The results obtained in this work make it possible to apply mathematical tools to study walking robots, in particular, study of stability, observability, etc. The
results of this work will be taken as the basis for solving the problem of group control of spider robots.

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