Goal-Oriented Adaptive Finite Element Multilevel Monte Carlo with Convergence Rates

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ABSTRACT

We present an adaptive multilevel Monte Carlo (AMLMC) algorithm [1] for approximating deterministic, real-valued, bounded linear functionals that depend on the solution of a linear elliptic PDE with a lognormal diffusivity coefficient, in a setting where the PDE model is subject to a geometry-induced singularity. This AMLMC algorithm is built on the results of the weak convergence rates in [2] for an adaptive algorithm using isoparametric d-linear quadrilateral finite element approximations and the dual weighted residual error representation in a deterministic setting.

Developed to suit the deterministic, geometric nature of the singularities in the solution, the AMLMC algorithm uses a sequence of deterministic, non-uniform auxiliary meshes as a starting point. The above-mentioned deterministic adaptive algorithm generates these meshes, corresponding to a geometrically decreasing sequence of tolerances. More precisely, given a realization of the diffusivity coefficient and accuracy level, AMLMC constructs its approximate sample using the first mesh in the hierarchy that satisfies the corresponding bias accuracy constraint. This adaptive approach is particularly useful for the lognormal case treated here, which lacks uniform coercivity and thus produces functional outputs that vary over orders of magnitude when sampled.

From the numerical experiments, based on a Fourier expansion of the diffusivity coefficient field, we observe improvements in efficiency compared with both standard Monte Carlo and standard Multilevel Monte Carlo for a problem with a singularity similar to that at the tip of a slit modeling a crack.

This presentation is based on work supported by the Alexander von Humboldt Foundation and the King Abdullah University of Science and Technology (KAUST) office of sponsored research (OSR) under Award No. OSR-2019-CRG8-4033.

REFERENCES

[1] J. Beck, Y. Liu, E. von Schwerin, and R. Tempone, Goal-oriented adaptive finite element multilevel Monte Carlo with convergence rates, *Computer Methods in Applied Mechanics and Engineering*, Vol. **402**, pp. 115582, (2022).

[2] K-S. Moon, E. von Schwerin, A. Szepessy, and R. Tempone, "Convergence rates for an adaptive dual weighted residual finite element algorithm", *BIT Numerical Mathematics*, Vol. **46**, pp. 367–407, (2006).