Sparse recovery problem in a hierarchical Bayesian framework

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ABSTRACT

A common task in inverse problems and imaging is finding a solution that is sparse, in the sense that most of its components vanish. In the framework of compressed sensing, general results guaranteeing exact recovery have been proven. In practice, sparse solutions are often computed combining ℓ_1-penalized least squares optimization with an appropriate numerical scheme to accomplish the task - see, e.g., [1]. A computationally efficient alternative for finding sparse solutions to linear inverse problems is provided by Bayesian hierarchical models, in which the sparsity is encoded by defining a conditionally Gaussian prior model with the prior parameter obeying a generalized gamma distribution [2]. An iterative alternating sequential (IAS) algorithm has been demonstrated to lead to a computationally efficient scheme, and combined with Krylov subspace iterations with an early termination condition, the approach is particularly well suited for large scale problems [3]. Here, we will discuss two hybrid versions of the original IAS that first exploit the global convergence associated with gamma hyperpriors to arrive in a neighborhood of the unique minimizer, then adopt a generalized gamma hyperprior that promote sparsity more strongly. The proposed algorithms will be tested on traditional imaging applications and to problems whose solution allows a sparse coding in an overcomplete system such as composite frames.

REFERENCES

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