

Moving Mesh Methods for Implicit Moving Boundary Problems

Andrea Cangiani[‡], Matthew Hubbard^{†,*}, Thomas Radley[†] and Harry Wells[†]

[†] School of Mathematical Sciences, The University of Nottingham,
University Park, Nottingham, NG7 2RD, United Kingdom.

[‡] SISSA, Via Bonomea, 265, 34136 Trieste, Italy.

* e-mail: matthew.hubbard@nottingham.ac.uk

web page: <https://www.nottingham.ac.uk/mathematics/people/matthew.hubbard>

ABSTRACT

Numerical algorithms for implicit moving boundary problems require the ability to predict both the evolution of the boundary and the dynamics of the solution in the interior. When these dynamics are governed by partial differential equations they are typically simulated on a computational mesh, so fixed-mesh algorithms must be adjusted to allow for a time-dependent computational domain. In this work we choose to define an initial mesh which conforms to the original domain boundary and then move the mesh in a manner which allows it to follow the boundary as it evolves. This approach involves no change in mesh topology and has the potential to work extremely well in situations where the computational domain does not become highly distorted.

In particular, we will consider a velocity-based moving mesh framework, originally presented as a finite element method on unstructured triangular meshes [1, 2] and more recently extended to polygonal meshes using the virtual element method [3]. These methods consist of two stages:

1. Determine mesh velocities using a conservation principle. Basing this on a physical principle, *e.g.* mass conservation, allows us to obtain the correct boundary velocities, even when they are not provided explicitly by the boundary conditions. Mesh velocities in the interior can, if desired, be replaced to preserve equidistribution properties (for example).
2. Move the mesh according to these velocities and update the solution using a standard time-stepping scheme. This is a step required by all velocity-based moving mesh methods and is carried out using the Arbitrary Lagrangian-Eulerian (ALE) approach.

I will outline recent developments of these methods, including: extension to the discontinuous Galerkin framework (to improve robustness for hyperbolic problems); simulation on arbitrary polygonal meshes (for additional flexibility); generalisation to arbitrary orders of accuracy (to efficiently provide very accurate approximations). Numerical results will be presented to demonstrate the effectiveness of these methods for a range of linear and nonlinear problems involving implicit moving boundaries.

REFERENCES

- [1] M.J.Baines, M.E.Hubbard and P.K.Jimack, A moving mesh finite element algorithm for the adaptive solution of time-dependent partial differential equations with moving boundaries, *Appl Numer Math*, **54**, pp. 450–469 (2005).
- [2] M.J.Baines, M.E.Hubbard and P.K.Jimack, Velocity-based moving mesh methods for nonlinear partial differential equations, *Commun Comput Phys*, **10**(3), pp. 509–576 (2011).
- [3] H.Wells, M.E.Hubbard and A.Cangiani, A velocity-based moving mesh virtual element method, submitted to *J Comput Phys* [<http://arxiv.org/abs/2211.13521>].