An Adaptive Trefftz Method to Analyze the Influence of the Midfield Propagation Conditions on Environmental Railway Noise

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ABSTRACT

Despite being considered as an environmentally friendly means of transportation, railway transportation is a problematic source of noise in its surroundings. In response to this issue, industrials and academic actors are trying to develop numerical tools capable of modelling acoustic propagation for railway applications, with an emphasis on virtual certification.

The relevant bandwidth for railway acoustics (20 – 20000 Hz) is in the mid-frequencies. Traditional acoustic numerical methods dedicated to low frequencies (FEM [1], BEM [2,3]) and high frequencies (SEA [4,5]) are insufficient in this range [6]. The overall acoustic response spectrum shows a high modal density and sensitivity to uncertainties while maintaining a localized response, therefore dedicated mid-frequency numerical methods must be considered.

The Variational Theory of Complex Rays (VTCR) [7], a Trefftz method [8], has been implemented for mid-frequency acoustic propagation. This method relies on choosing a priori shape functions as functions satisfying the equilibrium equation. In acoustics, it corresponds to solving the Helmholtz equation and modelling the pressure field as a sum of plane waves (Herglotz functions) over the angular sector, with each radial direction being called a VTCR ray.

Using the variational formulation [9,10] and a Galerkin method with these shape functions to obtain a discretized model, a 2D VTCR acoustic propagation code was implemented. The shape functions are defined over the whole computational domain, allowing us to mesh only the domain boundaries, resulting in a much smaller linear system to solve and no meshing needed inside the domain. The formulation of VTCR also rids the computations of certain interpolation and numeric dispersion errors intrinsic to FEM [11], while facilitating the coupling of different methods and physics.

Error measurements and convergence of VTCR in acoustics have been shown for bounded and unbounded 2D domains by comparison with analytical solutions [12]. However no systematic error estimators for VTCR have been implemented yet, therefore showing the interest in the elaboration of an a posteriori error estimator [13].

The novelty of the proposed work is the implementation of an adaptive procedure for VTCR for acoustic railway applications. Using a goal-oriented approach and considering the pressure field over a relevant domain as a quantity of interest, an a posteriori error estimator based on dual weighted residuals has been implemented, allowing for an adaptive code. As opposed to FEM, VTCR does not consider adaptivity in terms of mesh refinement or interpolation order but rather with the number of VTCR rays needed to reconstruct the pressure field. If the error threshold is not met, the algorithm with continue to enrich solution by computing additional VTCR rays.

In the presentation, the performance of adaptive VTCR will be shown for examples of railway acoustic problems and the relevance of VTCR for virtual certification in railway applications will be studied.

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