On the Use of Neural Networks for Inverse Problems

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ABSTRACT

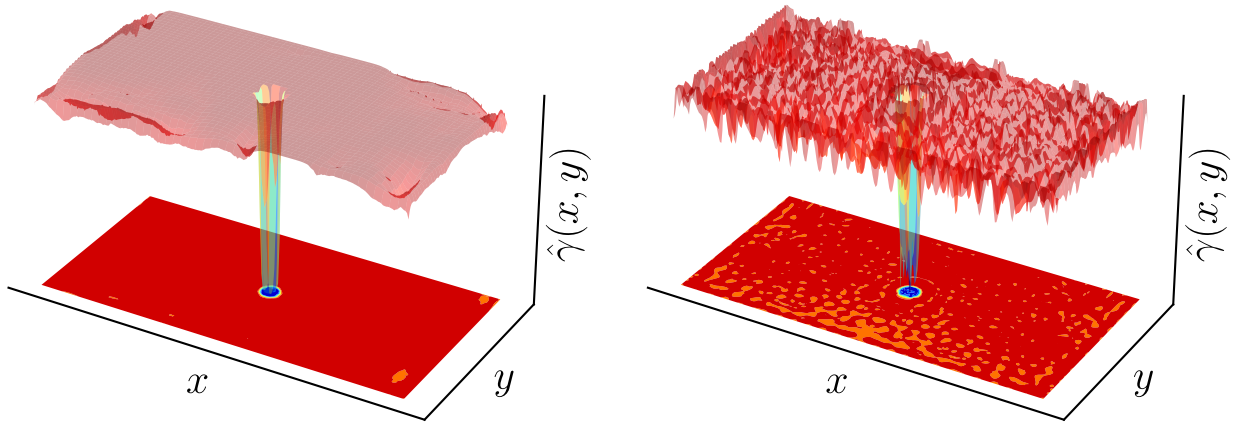
Physics-informed neural networks (PINNs) [1] have recently arisen as a promising solution methodology for inverse problems. The solution is approximated with a neural network trained by minimizing the residual of a partial differential equation. This work aims to pinpoint the strengths and weaknesses of PINNs in relation to the classical adjoint optimization. We present an incremental comparison of PINNs w.r.t. the classical adjoint optimization in the context of inverse problems. To this end, we consider the three key ingredients (a) the forward solver, (b) the Ansatz space of the optimization variable, and (c) the sensitivity computation. The empirical investigation is performed for full waveform inversion, where the unknown is a scaling function of the density field to locate internal voids [2]. PINN-based approaches, as presented in [3], represent both the solution and the scaling function with separate neural networks and perform a nested minimization of the emerging residuals.

For the incremental comparison, we first replace the neural network responsible for the forward solution with a non-learnable forward operator, i.e., a classical finite difference scheme as forward solver (a). Next, the importance of the discretization of the scaling function for the density field is investigated by comparing a discretization using a neural network, as in [3], with a discretization using piecewise polynomials (b). Lastly, the sensitivity computation with automatic differentiation is partially substituted with the continuous adjoint method (c). These aspects are studied using two-dimensional benchmark problems [2] and complex three-dimensional cases based on CT scans of rare drill cores [4].

The investigation led to two main insights. Firstly, the fully PINN-based approach is not the most efficient, as the optimizer needs to learn both the forward and inverse solution simultaneously. Instead, it is more advantageous to restrict the optimization to the scaling function of the density field and use a conventional method, e.g. the finite difference method, as forward solver. This modification reduces the number of required epochs from 400'000-600'000 as reported in [3] to 50, while the time per epoch only increases by a factor of five. The second insight is that using neural networks to discretize the scaling function of the density field is highly beneficial. Piece-wise polynomials lead to numerous oscillatory artifacts, whereas neural networks recover both smoother and sharper solutions (Fig. 1).

Due to these insights and the the continuous adjoint method being faster than automatic differentiation, a hybrid approach is proposed. It enables an efficient integration of neural networks with classical forward solvers. Its idea is to discretize the scaling function of the density field with a neural network, after which a classical forward solver estimates the forward solution. The sensitivity is then computed in two steps. First, the sensitivity of the measurement cost w.r.t. the inverse field is computed using the continuous adjoint method. Next, the sensitivity of the inverse field w.r.t. the network parameters is obtained with automatic differentiation. The hybrid approach retains the advantages of a neural network discretization while achieving similar wall-clock-times as the classical adjoint method.

We will conclude the presentation with a generalization of the proposed approach for other types of inverse problems.



(a) Neural network discretization.

(b) Piece-wise constant discretization.

Figure 1: Surface plot of the predicted material distribution for a plate with hole after 50 epochs, illustrating the smoother and sharper inversion with neural networks.

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