*MY PROOF OF THE COLLATZ CONJECTURE*

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***Abstract***

*Mathematicians have long been fascinated by the idea that numbers can be understood through patterns and regularities. The digital root of a number is an interesting tool to explore these patterns.*

*The digital root of an integer is defined as the repeated addition of its digits until only one digit remains. For example, the digital root of 12345 is 1 + 2 + 3 + 4 + 5 = 15 and 1 + 5 = 6.*

*In this paper, the properties of the digital root of numbers are analyzed and a possible connection with the sequences of the Collatz Conjecture is sought. The Collatz Conjecture is a famous unsolved problem in mathematics that refers to a sequence of numbers obtained from an initial number and successively by applying a series of simple operations. It is believed that all sequences of the Collatz Conjecture eventually converge to a cycle of repeating numbers, but this has not yet been proven.*

*This paper focuses on exploring the possible connection between the digital root of numbers and the sequences of the Collatz Conjecture. Specifically, it is investigated if there is any pattern or regularity in the digital root of the numbers in the sequences of the Collatz Conjecture. If a connection is found between these two concepts, it could have important implications for our understanding of numbers and their behavior.*

*In conclusion, this study is an important step in exploring the patterns and regularities of numbers and their possible connection to the Collatz Conjecture. We hope that this study opens new lines of research and contributes to the advancement of knowledge in mathematics.*

***Keywords***

*Digital root, Collatz conjecture, Collatz sequence.*

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## *1. Introduction*

*The digital root of a number is the sum of its digits. If the sum results in a number with two or more digits, the process is repeated until a single digit number is obtained. This last number is the digital root of the original number. For example, the number 123 has a digital root of 6 (1 + 2 + 3 = 6). For the number 12345, its digital root is 6 (1 + 2 + 3 + 4 + 5 = 15, and 1 + 5 = 6). In short, the digital root is the final result of repeating the process of adding the digits until obtaining a single digit number.*

*The digital root of the natural numbers is an efficient way to compress and represent numerical information. It is a useful tool in cryptography and computer security and is used in encryption algorithms to protect sensitive information. It is used to verify the integrity of the transmitted data, detecting unauthorized changes in the information. It is used to generate random numbers and secure keys and is an important component in online identity verification and authentication. It provides a basis for the study of search and ordering algorithms and helps in the study of patterns and regularities in numerical sequences.*

*The digital root can be used in encryption algorithms as a way to protect sensitive information by generating a secure, condensed representation of a number or piece of data. For example, the digital root can be used in RSA encryption, one of the most commonly used encryption algorithms in information security. In this case, the digital root is used to generate public and private keys, which in turn are used to encrypt and decrypt sensitive messages. The digital root can also be used in other encryption algorithms, such as digital* *signatures, to ensure the authenticity and integrity of the information transmitted. In computing, the digital root is an algorithm used in data integrity verification, such as in calculating checksums and error coding.*

*In number theory, the digital root is used to describe the structure of numbers and to investigate their properties. The classification of numbers based on their digital roots is done by separating the numbers into different groups depending on the sum of their digits. The importance is that this classification can be used to analyze and group patterns in numerical data, which can be useful in various applications such as data analysis, mathematical research, and problem solving.*

*In summary, the digital root is a mathematical concept that has a wide range of practical and theoretical applications in different disciplines and fields, and its importance lies in its ability to measure the magnitude of a number and to classify numbers into groups.*

*Digital Root Theorem: The Digital Root Theorem states that the digital root of a number is a number with a single digit if and only if the number is divisible by 9.*

*Modulo 9 Digital Root Property: The modulo 9 digital root property states that the digital root of a number is equal to the remainder obtained by dividing the number by 9.*

*Some of the reasons that the digital root helps find mathematical relationships and properties include:*

*Simplification: The digital root reduces a number to a single digit, which makes it easier to compare and analyze with other numbers.*

*Classification: The digital root allows to classify the numbers in groups according to their properties, which facilitates the identification of patterns and relationships between them.*

*Modularity: The digital root is closely related to calculus modulo 9, which makes it possible to investigate the structure of numbers and find mathematical properties and relationships using number theory techniques.*

**2. The digital root of numbers and its relationship with the sequences of the Collatz Conjecture.**

*The Collatz conjecture is a mathematical problem that refers to an iterative calculation that is applied to an initial natural number. Starting from that number, the conjecture states that one always arrives at 1, regardless of the initial number chosen.*

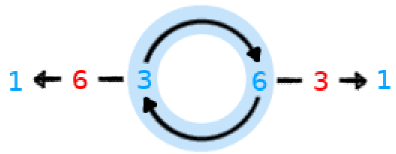
*The Collatz conjecture function:*

$$f\left(n\right) \;=\; \begin{cases} \frac{n}{2} & if \;\; n \;\; is \;\; even \\ 3n+1 & if \;\; n \;\; is \;\; odd \end{cases}$$

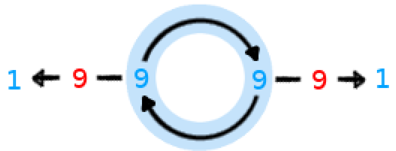
*Next, graphically, the transformations of the numbers when applying the operations of the function and its digital root.*

*The blue numbers are the digital root values of even numbers and the red numbers are the digital root values of odd numbers. The vectors indicate the sense of the transformations of the numbers in each iteration.*

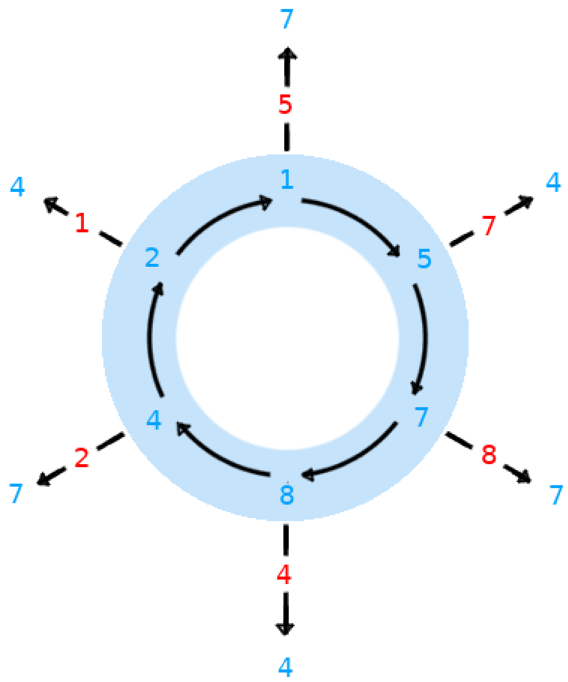
*Group 1 where iterate the numbers dr3, dr6*




*Group 2 where iterate the numbers dr9*




*Group 3 where iterate the numbers dr1, dr2, dr4 dr5, dr7, dr8*




***Axiom 1:*** *All odd and even numbers in groups 1 and 2 will arrive at even numbers with digital root 1.*

***Axiom 2:*** *All odd and even numbers in group 3 will reach even numbers with digital roots 4 and 7.*

***Axiom 3:*** *All even numbers with digital root 7 will arrive at even numbers with digital root 4.*

***Axiom 4:*** *All even numbers with digital root 4 will arrive at odd numbers with digital root 1.*

***Axiom 5:*** *All numbers will arrive at odd numbers with digital root 1.*

***Proof:*** *Inevitably, in one iteration, the sequence will reach the smallest of those numbers and enter loop 4, 2, 1, because there is no other choice and no other different loop is possible.*