

# Advanced Experiments on Gaussian Process-based Multi-fidelity Methods over Diverse Mathematical Characteristics

Sihmehmet Yildiz<sup>1</sup>, Hayriye Pehlivan-Solak<sup>1</sup>, Matteo Diez<sup>2</sup>, Omer Goren<sup>1</sup> & Melike Nikbay<sup>1</sup>

<sup>1</sup> Istanbul Technical University, Istanbul, 34469, Turkey  
{yildizsih, pehlivanha, ogoren, nikbay}@itu.edu.tr

<sup>2</sup>CNR-INM, National Research Council- Institute of Marine Engineering, Rome, Italy  
matteo.diez@cnr.it

**Keywords:** *Multi-fidelity, Gaussian Process, CoKriging, NARGP*

Despite the accelerated recent developments in computational sciences and power, processing times needed for engineering uncertainty quantification and optimization studies are still cited as burden for high dimensional problems. Conventional dimensional reduction and surrogate modelling methods have been widely employed and well validated in the literature to enable more affordable computational applications, while new research efforts are more directed towards the development of multi-fidelity strategies to validate and increase their performance and accuracy. Multi-fidelity modelling is a method in which different fidelity levels are simultaneously used together to obtain accurate results within a limited calculation budget [1, 2]. Among these emerging multi-fidelity methods, the Gaussian process-based strategies are numerically more appealing since they are more accurate with limited high-fidelity computational budget and also provide the variance value of the estimation.

The aim of this study is to investigate the Gaussian process-based multi-fidelity methods across selected benchmark problems -specifically chosen to capture diverse mathematical characteristics- by experimenting their learning processes with respect to different performance criteria such as root mean square error (RMSE) and global accuracy of the optimum. Advanced experiments on Forrester[2], Rosenbrock [3], Shifted-rotated Rastigrin [4], ALOS [5], Spring-Mass [3], Discontinuous Forrester, Paciorek[7] functions will be performed to report about the strengths and shortcomings of the proposed methods. In addition, a further investigation on the correlation between the low-fidelity and the high-fidelity models will enhance the mathematical representation of more reliable multi-fidelity modelling.

In this study, Linear-Autoregressive methods with single [2] and multivariate scale factors [8], and Nonlinear-Autoregressive method [9] will be experimented. From our preliminary results, it is observed that as the discrepancy and non-linearity increase between the fidelity levels, the Nonlinear-Autoregressive models provide more accurate results when it is compared to Linear-Autoregressive models.

**REFERENCES**

- [1] M. C. Kennedy and A. O' Hagan, Predicting the output from a complex computer code when fast approximations are available, *Biometrika*, Vol. **87**, pp. 1-13, 2000.
- [2] A. Forrester, A. Sobester and A. Keane, *Engineering design via surrogate modelling: a practical guide*, John Wiley & Sons, 2008.
- [3] M.P. Rumpfkeil and P.S. Beran, Multi-fidelity, gradient-enhanced, and locally optimized sparse polynomial chaos and kriging surrogate models applied to benchmark problems, in AIAA Scitech 2020 Forum, p. 677, 2020.
- [4] H. Wang, Y. Jin and J. Doherty, A generic test suite for evolutionary multifidelity optimization, *IEEE Transactions on Evolutionary Computation*, **22**(6):pp. 836–850, 2017.
- [5] J. Clark, D.L. Bae, K. Global and R. Penmetsa, Engineering design exploration using locally optimized covariance kriging, *AIAA Journal*, **54**(10):pp. 3160–3175, 2016.
- [6] M. Raissi and G.E. Karniadakis, Deep multi-fidelity gaussian processes, *CoRR*, abs/1604.07484, 2016.
- [7] D.J.J. Toal, Some considerations regarding the use of multi-fidelity kriging in the construction of surrogate models, *Structural and Multidisciplinary Optimization*, **51**(6):pp. 1223-1245, December 2014.
- [8] L. Brevault, M. Balesdent and A. Hebbal, Overview of gaussian process based multi-fidelity techniques with variable relationship between fidelities *Aerospace Science and Technology*, Vol. **107**, 2020.
- [9] P. Predikaris, M. Raissi, A. Damianou, N.D. Lawrence and G.E. Karniadakis, Non-linear information fusion algorithms for data-efficient multi-fidelity modelling, *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, Vol. **473**, 2017.