

Quantification of time-averaging uncertainties in turbulence simulations

Donnatella Xavier¹, Saleh Rezaeiravesh¹, Ricardo Vinuesa¹ and Philipp Schlatter¹

¹ SimEX/FLOW, KTH Engineering Mechanics, Royal Institute of Technology,
Stockholm SE-100 44, Sweden

dgxavier@mech.kth.se, salehr@kth.se, rvinuesa@mech.kth.se, and pschlatt@mech.kth.se

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When computing the time averages of turbulent flow quantities based on a finite number of time samples, the sample mean estimator (SME) $\hat{\mu}$ is an uncertain estimate of the true ergodic mean, μ . This uncertainty can be quantified using a variance estimator $\sigma^2(\hat{\mu})$, where $\hat{\mu} \sim \mathcal{N}(\mu, \sigma^2)$. The two commonly cited categories of estimators in literature, namely batch means methods and the autoregressive model (ARM)-based estimator (e.g. Ref. [1]) rely on *hyperparameters*, whose crucial role on the accuracy of the estimation has not been investigated before. Our study shows that the hyperparameters in each estimator are linked to the autocorrelation of the underlying turbulent time series and can be expressed in terms of turbulence timescales. This enables deriving novel guidelines for their selection, such as determination of the batch size from the lag-1 or lag-2 correlation of batch means and direct computation of the order of the ARM from the integral timescale. We also introduce a new approach to quantify uncertainties in higher-order turbulence statistics, wherein we solve a UQ forward problem by deriving the uncertainties in complex statistical terms (such as those in turbulent kinetic energy budget), using combinations of arithmetic and differentiation operators applied to the SMEs ($\hat{\mu}$) of the primitive variables. Our proposed method avoids any linearization or approximation, and includes the spatial correlations in the SMEs, to preserve the properties of the physical system. We have already tested our proposed UQ techniques to estimate the uncertainties in the turbulence statistics for flow (i) in a channel, (ii) over periodic hills and (iii) in a periodic pipe. Future work aims to estimate the uncertainties in the turbulent time averages of the flow through the more complicated FDA nozzle [2], which will emphasize the applicability of these UQ techniques to critical problems where there is a vital need to enhance the credibility of computational fluid dynamics.

REFERENCES

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