

Attenuation and localization of waves in taut cables with suspended masses

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Keywords: *Metamaterials, Cable dynamics, Homogenization, Wave localization*

In this contribution, we treat the problem of wave propagation in taut cables presenting a periodic array of scatter elements, that is composed of a discrete set of masses hanging by means of elastic or rigid connections. The cable is pre-stretched between two fixed ends and, by definition, is considered as infinitely flexible. We assume small-displacements around the equilibrium configuration of the system. A linear-elastic in-plane vibration analysis is carried out. We show how the dynamics of such systems is inherently connected with the typical behavior of mechanical metamaterials characterized by the capability of attenuating waves.

Inspired by the homogenization techniques commonly applied to continuous composites, we present a method that involves the definition of an effective mass, to be lumped at the positions of the hanging elements. By placing the two extremities of the cable at $\pm\infty$, the system becomes of infinite dimensions and the spectrum continuous. We find that the dispersion relation obtained by using the lumped effective mass predicts the presence of band gaps in the spectrum, *i.e.* intervals of frequencies where waves are attenuated, and that the eigenvalues of the finite dimensional case (corresponding to superimposed non-attenuated propagating waves) can be found only outside these bands.

Introducing a defect of periodicity, we finally study the problem of wave localization [1, 2, 3]. At specific frequencies inside the band gaps, the motion of the system becomes very large in correspondence of the defect and decays exponentially outside it.

The approach followed can be generalized to deal with various structures composed by a continuous 1D element with a discrete and periodic array of scatterers of different kind.

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