

# ACCELERATED NONLINEAR PDE-CONSTRAINED OPTIMIZATION BY REDUCED ORDER MODELLING

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We present a framework to accelerate optimization of problems where the objective function is governed by a nonlinear PDE using projection-based reduced order models (ROMs) and trust region methods. To reduce the cost of objective function and gradient evaluations by several orders of magnitude, we replace the underlying full order model (FOM) with a series of hyperreduced ROMs constructed on-the-fly during optimization. Each hyperreduced ROM is equipped with an online-efficient *a posteriori* error estimator, which is used to define a trust region. The ROM is updated when the boundary of the trust region is reached. Hyperreduction is performed following a goal-oriented empirical quadrature procedure, which is trained to guarantee first order consistency of the hyperreduced ROM with the FOM at the trust region center, which ensures the optimizer converges to a local minimum. In addition, we control the FOM error by adaptive mesh refinement. We demonstrate the framework through optimization of an FFD-parametrized airfoil under Reynolds-averaged Navier-Stokes flow in two different contexts: pressure-matching inverse design and lift-constrained drag minimization.

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