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Very fast FEM Poisson solvers on lower precision accelerator hardware

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Recently, accelerator hardware in the form of graphics cards including Tensor Cores, specialized for AI, has significantly gained in importance in the domain of high performance computing. For example, NVIDIA's Tesla V100 promises a computing power of up to 125 TFLOP/s achieved by Tensor Cores, but only if half precision floating point format is used and arithmetically intensive operations including dense matrices are performed. When trying to use such lower precision hardware for numerical simulations, i.e., solving partial differential equations with a matrix-based finite element method and a standard multigrid solver, it becomes clear that the peak rates are not nearly reached, not to mention possible problems regarding accuracy. However, if low condition numbers and many dense matrix operations are involved, the indicated high performance can be reached while preserving sufficient accuracy.

The aim of this talk is to present new (semi-)direct methods to solve linear systems arising from Poisson's equation that meet these requirements, based on "prehandling" by means of hierarchical finite elements and an additional Schur complement approach.

Numerical results show that, if the problem is solved in 2D with the direct variant for many right-hand sides simultaneously on the V100, 60 TFLOP/s are reached and a comparative geometric multigrid solver on standard hardware is outperformed by a factor of 600.

Furthermore, a semi-direct variant of the algorithm is outlined that overcomes the memory limitations of the direct one and allows extension to the 3D case.

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