The neural network multigrid solver for the Navier-Stokes equations and its application to 3D simulation

Nils Margenberg[1], Christian Lessig[2] and Thomas Richter[3]

[1] Helmut Schmidt University, Holstenhofweg 85, 22043 Hamburg, margenbn@hsu-hh.de
[2] Institute for Simulation and Graphics, University of Magdeburg
Universitätsplatz 2, 39104 Magdeburg, Germany, christian.lessig@ovgu.de
[3] Institute for Analysis and Numerics, University of Magdeburg
Universitätsplatz 2, 39104 Magdeburg, Germany, thomas.richter@ovgu.de

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Our work extends the Deep Neural Network Multigrid Solver (DNN-MG) introduced in [1] by an online learning approach and the application to 3D flow simulation. DNN-MG is a novel method for machine learning enhanced simulations, which improves computational efficiency by a combination of a geometric multigrid solver and a recurrent neural network. The multigrid solver is used in DNN-MG for the coarse levels while the neural network corrects interpolated solutions on fine levels, avoiding the increasingly expensive computations. DNN-MG uses the neural network to correct local patches of the mesh domain. This greatly facilitates generalizability and allows one to use a network trained on one mesh domain also on a different one. The locality also results in a compact neural network with a small number of parameters, which reduces training time, data and the costs for network evaluation in a simulation. The memory of the network and the coarse multigrid solutions provide a "guide" for the corrections. In [1] we have demonstrated the efficacy and generalizability of DNN-MG for variations of the 2D laminar flow around an obstacle. DNN-MG improved the solutions as well as lift and drag functionals while requiring less than half the computation time of a full multigrid solution [3].

Although DNN-MG provides solutions with high accuracy and is computationally highly efficient in 2D, it is unclear how the method scales to 3D, where the computational effort is still a critical factor and the flow is much more complex. We investigate the scaling of DNN-MG to 3D flows. We also address the separation between testing and training time which includes data generation by employing an online learning approach and give an outlook on ways to incorporate unsupervised learning into the method.

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