

## A wavelet-based three-dimensional Convolutional Neural Network for superresolution of turbulent vorticity

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We examine whether superresolution of small-scale turbulence can well preserve the position and structure of coherent vortices in three-dimensional (3D) turbulence. Convolutional Neural Networks (CNN) are used, because they can well learn characteristic spatial structure. Here we develop a wavelet-based 3D CNN (WCNN3d) for superresolution of coarse-grained data of homogenous isotropic turbulence. Wavelets are well-localized functions in scale and position and yield an efficient orthogonal multi-scale representation of the spatial structure of the vortices using the fast wavelet transform, see e.g. Ref. [1].

Recently, Kim et al.<sup>[2]</sup> applied Cycle GAN to perform superresolution of turbulent flows and showed that turbulent statistics, such as probability density function (PDF) of vorticity and energy spectra in two dimensions are well preserved. However, the positions of vortices are not well kept.

We performed direct numerical simulation (DNS) of turbulence in a periodic box at  $512^3$  grid points. The coarse-grained data are then obtained by applying a Gaussian low pass filter. The  $512^3$  data are divided into  $8^3$  subcubes with  $64^3$  grid points each. Then, we apply the discrete wavelet transform to each subcube and subsequently use 3D CNN. To this end PyWavelets,<sup>[2]</sup> an open source of Python, is used. For the wavelets we choose ‘coif2’ (called Coiflet 12 in Ref. [1]) having four vanishing moments and impose ‘symmetric’ boundary conditions in the wavelet transform of the subcube data.

We assess the performance of our WCNN3d in terms of 3D visualization of vorticity, PDF of vorticity, enstrophy spectra, etc. We find that WCNN3d well reproduces the vorticity statistics and the positions of the vortices from coarse-grained vorticity fields. Moreover, we also show that without wavelets, 3D CNN does neither well reproduce the vorticity statistics nor the vortical structures. We will discuss the influence of choice of low-pass filters on WCNN3d and the influence of the Reynolds number.

### REFERENCES

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