On the use of flux boundary conditions in FE^2 procedures with seamless scale-bridging

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ABSTRACT

We consider discretization-based homogenization of micro-heterogeneous stationary heat conduction problems within the wider framework of Variationally Multiscale methods and study the possibilities to obtain upper and lower bounds on the the overall (two-scale) solution. The strategy presented by Larsson and Runesson[1] is adopted, whereby the solution procedure for the fine-scale fluctuations is based on the local mesh-size in the macroscopic finite element problem. This allows for bridging the extremes – from single scale analysis to computational homogenization – seamlessly within one finite element simulation.

One crucial component in two-scale analysis, often denoted ”Finite Element squared” (FE^2) is the prolongation of the macroscopic fields onto the subscale problem. In computational homogenization on Statistical Volume Elements (SVEs), it is well known that temperature (Dirichlet) and flux (Neumann) boundary conditions result in upper and lower bounds, respectively, on the effective heat conductivity. However, in the standard (primal) variational format, it is only the Dirichlet boundary conditions that satisfy the conformity requirement in the standard (primal) variational form. Hence, other choices of boundary conditions (e.g. flux or periodic boundary conditions) are not directly applicable if one wants to consider connected adjacent SVE's without assuming (infinite) separation of scales.

In order to allow for flux boundary conditions, and thereby studying the solution for the lower bound effective conductivity, while satisfying the conformity requirement, we propose a novel mixed dual variational format. In this format, the finite element multiscale problem, sometimes denoted ”Finite Element squared” (FE^2), (i) incorporates the flux boundary conditions when computational homogenization is adopted, and (ii) is a conforming approximation when the subscale fluctuation is fully resolved within each macroscopic element.

We are thus able to compute the FE^2 approximation using either the (standard) primal format and temperature boundary conditions on SVE's (upper bound) or the dual format and flux boundary conditions on SVE's (lower bound) while allowing for the transition from homogenization to single-scale analysis.

We compare the numerical performance of the two different variational settings (and consequent boundary conditions) in the FE^2 context where the scale separation is successively diminished. In particular, we compare the convergence properties for FE-refinement on both ”scales” and discuss the possibilities to utilize the two approximations in a strategy to control the approximation error.

REFERENCES

[1] Larsson F. and Runesson K. *Adaptive bridging of scales in continuum modeling based on error control*. International Journal for Multiscale Computational Engineering, pp. 371-392, (2008).