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APPLICATION OF LEVEL-SET TYPE TOPOLOGY OPTIMIZATION ANALYSIS FOR CAVITY SHAPE ESTIMATION PROBLEM IN STRUCTURES BASED ON NON-DESTRUCTIVE HAMMERING TEST

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Abstract. In this study, we present application of the level-set type topology optimization analysis for the cavity shape estimation problem in structures based on the non-destructive hammering test. The cavity shape is identified so as to minimize a performance function. The performance function is defined as the square sum of the residual between computed and the observed displacements on structure surface. In this study, accuracy of identified cavity shape is investigated by changing numerical parameters in the topology optimization.

1 INTRODUCTION

There are a lot of concrete structures exceeding service life in Japan, and it is necessary to perform inspections for the structures. It is desired that we can appropriately find a cavity in structures, because there is a possibility that a cavity causes collapse accident. In addition, it is important to know shape of cavity accurately due to difference of stress singularity on crack boundary line. Therefore, the level-set type topology optimization analysis for cavity in structures is carried out in this study. The displacement response data on surface in the hammering test is employed for the identification of the cavity shape. In the level-set type optimization analysis, sensitivity for the level-set function is calculated based on the adjoint variable method [1,2], and iterative computation for estimation of cavity shape is conducted by using a reaction diffusion equation with respect to the level-set function. In this study, numerical experiments are carried out by changing the value of the regularization parameter in the reaction diffusion equation.

2 TOPOLOGY OPTIMIZATION USING LEVEL-SET FUNCTION

The performance function is defined as shown in Eq.(1). Here, $\boldsymbol{u}$ and $\boldsymbol{u}_{obs.}$ indicate the computational displacement and the observed displacement, respectively. The parameter $\boldsymbol{Q}$

denotes the weighting diagonal matrix. The diagonal component is given as 1 at the measurement point, and is given as 0 at the other nodal points. t_0 and t_f mean initial and terminal time. The purpose is to find the distribution of level-set function, i.e., determination of cavity region, so as to minimize the performance function J.

$$J = \frac{1}{2}\int_{t_0}^{t_f} (\boldsymbol{u} - \boldsymbol{u_{obs.}})^T \boldsymbol{Q} (\boldsymbol{u} - \boldsymbol{u_{obs.}}) dt \tag{1}$$

The Lagrange function shown in Eq. (2) is obtained by introducing the adjoint variable and the constraint condition, i.e., equation of motion[3] due to obtain surface displacement response by the hammering test. Here, the dot mark and $\boldsymbol{\lambda}$ indicate differentiation with respect to time and the adjoint variable vector. In addition, the material and the cavity regions are determined by the sign of the level-set function. Therefore, regarding the coefficient matrices of finite element equation, i.e., $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$, as the level-set function ϕ, the matrices are represented by $\boldsymbol{M}(\phi)$, $\boldsymbol{C}(\phi)$ and $\boldsymbol{K}(\phi)$. $\boldsymbol{f}$ denotes the external force vector.

$$J^* = J + \int_{t_0}^{t_f} \boldsymbol{\lambda}^T (\boldsymbol{M}(\phi)\ddot{\boldsymbol{u}} + \boldsymbol{C}(\phi)\dot{\boldsymbol{u}} + \boldsymbol{K}(\phi)\boldsymbol{u} - \boldsymbol{f}) dt \tag{2}$$

The first variation of the Lagrange function is calculated in order to obtain the stationary condition. Consequently, the equation of motion shown in Eq.(3) is obtained by the gradient of the Lagrange function with respect to the adjoint variable. In addition, the adjoint equation shown in Eq.(4) is obtained by the gradient of the Lagrange function with respect to the displacement. The Newmark β method is applied to discretize Eqs. (3) and (4) in time.

$$\boldsymbol{M}(\phi)\ddot{\boldsymbol{u}} + \boldsymbol{C}(\phi)\dot{\boldsymbol{u}} + \boldsymbol{K}(\phi)\boldsymbol{u} = \boldsymbol{f} \tag{3}$$

$$\boldsymbol{M^T}(\phi)\ddot{\boldsymbol{\lambda}} - \boldsymbol{C^T}(\phi)\dot{\boldsymbol{\lambda}} + \boldsymbol{K^T}(\phi)\boldsymbol{\lambda} = -\boldsymbol{Q^T}(\boldsymbol{u} - \boldsymbol{u_{obs.}}) \tag{4}$$

Here, it is assumed that the value of level-set function is propagated by the reaction diffusion equation shown in Eq.(5). In Eq.(5), the comma mark indicates differentiation. Applying the finite element method for Eq.(5) to discretize in space, the finite element equation shown in Eq.(6) is consequently obtained. Here, the parameters $\kappa(\phi)$, C, τ, $\tilde{t}$ and $\boldsymbol{N_e}$ denote the positive parameter, the parameter adjusting magnitude of $J^*_{e,\phi}$, the regularization parameter, the fictitious time for optimization and the shape function vector, respectively. For differentiation with respect to the fictitious time $\tilde{t}$, the backward difference method is employed.

$$\phi_{,\tilde{t}} - \kappa(\phi)\tau\left(\phi_{,xx} + \phi_{,yy} + \phi_{,zz}\right) = -\kappa(\phi) C J^*_{,\phi} \tag{5}$$

$$\boldsymbol{M}_e \boldsymbol{\phi}_{e,\tilde{t}} + \kappa(\phi)\tau \boldsymbol{S}_e \boldsymbol{\phi}_e = -\kappa(\phi) C J^*_{e,\phi} \int_{\Omega_e} \boldsymbol{N}_e d\Omega \tag{6}$$

In Eqs.(5) and (6), $J^*_{e,\phi}$ is obtained by differentiation of coefficient matrices $\boldsymbol{M}(\phi)$, $\boldsymbol{C}(\phi)$ and $\boldsymbol{K}(\phi)$ with the level-set function ϕ (See Eq.(7).). In this study, coefficient matrices $\boldsymbol{M}(\phi)$, $\boldsymbol{C}(\phi)$ and $\boldsymbol{K}(\phi)$ are given by $\phi * \boldsymbol{M}$, $\phi * \boldsymbol{C}$ and $\phi * \boldsymbol{K}$, respectively. Consequently, Eq. (8) is obtained from Eq.(7). Here, $H_e(\phi)$ indicates the Heaviside step function.

$$J^*_{e,\phi} = \int_{t_0}^{t_f} \boldsymbol{\lambda}_e^T (\boldsymbol{M}_{e,\phi}\ddot{\boldsymbol{u}}_e + \boldsymbol{C}_{e,\phi}\dot{\boldsymbol{u}}_e + \boldsymbol{K}_{e,\phi}\boldsymbol{u}_e) dt \tag{7}$$

$$J^*_{e,\phi} = \int_{t_0}^{t_f} \boldsymbol{\lambda}_e^T (\boldsymbol{M}_e\ddot{\boldsymbol{u}}_e + \boldsymbol{C}_e\dot{\boldsymbol{u}}_e + \boldsymbol{K}_e\boldsymbol{u}_e) H_e(\phi) dt \tag{8}$$

Here, the computational flow is shown as follows.

1. Select the finite element mesh, the boundary conditions, the initial conditions, the numerical parameters and initial the value of number of iteration : k=0.
2. Solve the equation of motion, i.e., Eq.(3), and calculate the performance function, i.e., Eq.(1).
3. If the judgement equation $|J^{(k)} - J^{(k-1)}/J^0|$ is lower than convergence criterion ε , this calculation finalizes. Otherwise, go to next step.
4. Solve the adjoint equation, i.e., Eq.(4), and calculate Eq.(6) for each element.
5. Compute distribution of the level-set function for each node by Eq.(6) superposed for all elements.
6. If the level-set function is positive value, the node exists in body region. And, if the level-set function is negative value or zero, the node exists in cavity region or exists on the boundary between body and cavity regions, respectively. Based on the above, update of topology of structure, and return to step 2.

3 NUMERICAL EXPERIMENTS

The computational model used in numerical experiments are shown in Fig.1, and numerical parameters are show in Tab.1. The hammering force is give by the Gaussian pulse wave, i.e., $F(t)=F_{max}\exp(-(t-t_{peak})^2/s^2)$, and F_{max}, t_{peak} and s are respectively given as 2000N, 10^{-3}s and 10^{-4}s. In this study, the numerical result of displacement for the right hand side target shape shown in Fig.2 is used as the measurement value. The topology optimization analysis is carried out from the left hand side initial shape shown in Fig.2. The regularization parameter τ is adjusted as Case-A : τ=0.001, Case-B : τ=0.005 and Case-C : τ=0.010, and numerical experiments were carried out. As the measurement result, the displacement for z direction is only employed.

The variation of performance function is shown in Fig.3. It is found that if τ is big value, the value of performance function converges to small value. Fig.4 shows the estimated cavity shape. It is seen that if τ is big value, the size of cavity region become to small. From this result, it appears that if we'd like to obtain appropriate cavity shape, we have to pay attention not only for setting of the regularization parameter τ but also position of the observation points.

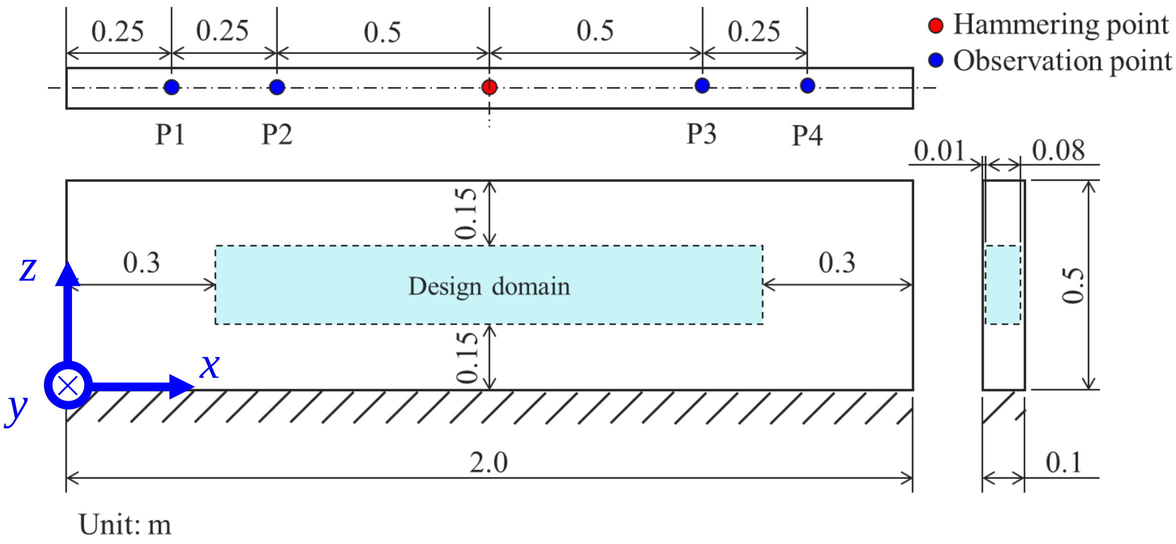
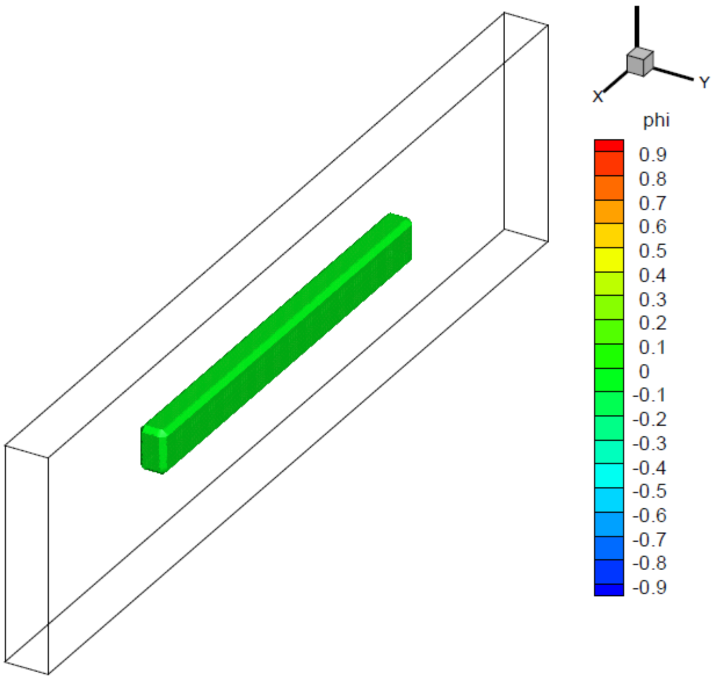


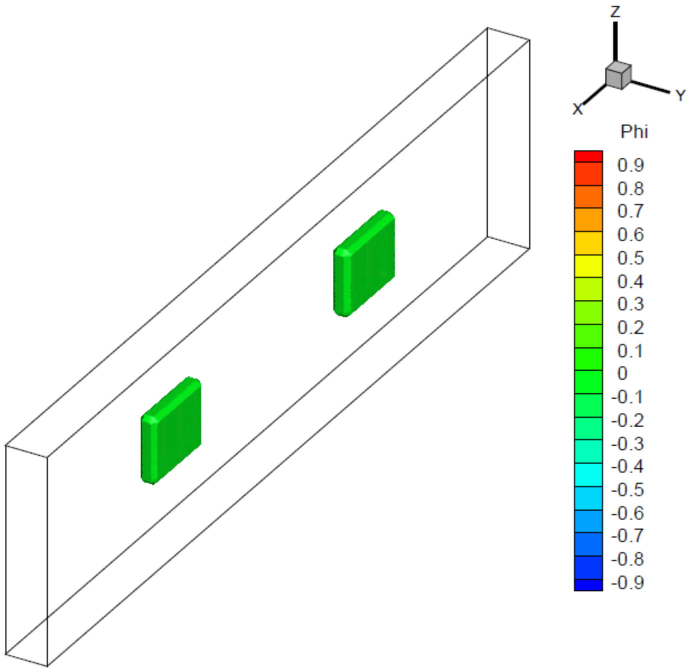
Figure 1: Setup of design domain and size of computational domain

Table 1: Numerical conditions

Number of nodes	112761
Number of elements	100000
Number of time steps	256
Time increments Δt	39.0625
Young's modulus E, GPa	35.096
Poisson's ratio ν	0.16
Mass density ρ, kg/m^3	2300
Damping coefficient c_M, c_K (Reyleigh damping)	90.0, 1.0×10^{-6}
Weight parameters q_u, q_v, q_w (Diagonal component in matrix $\mathbf{Q}$)	0, 0, 1.0×10^{9}
Virtual time increment $\Delta\tilde{t}$	1.0×10^{-8}
Convergence criterion ε	1.0×10^{-3}



Initial shape



Target shape

Figure 2: Initial and target shapes of cavity

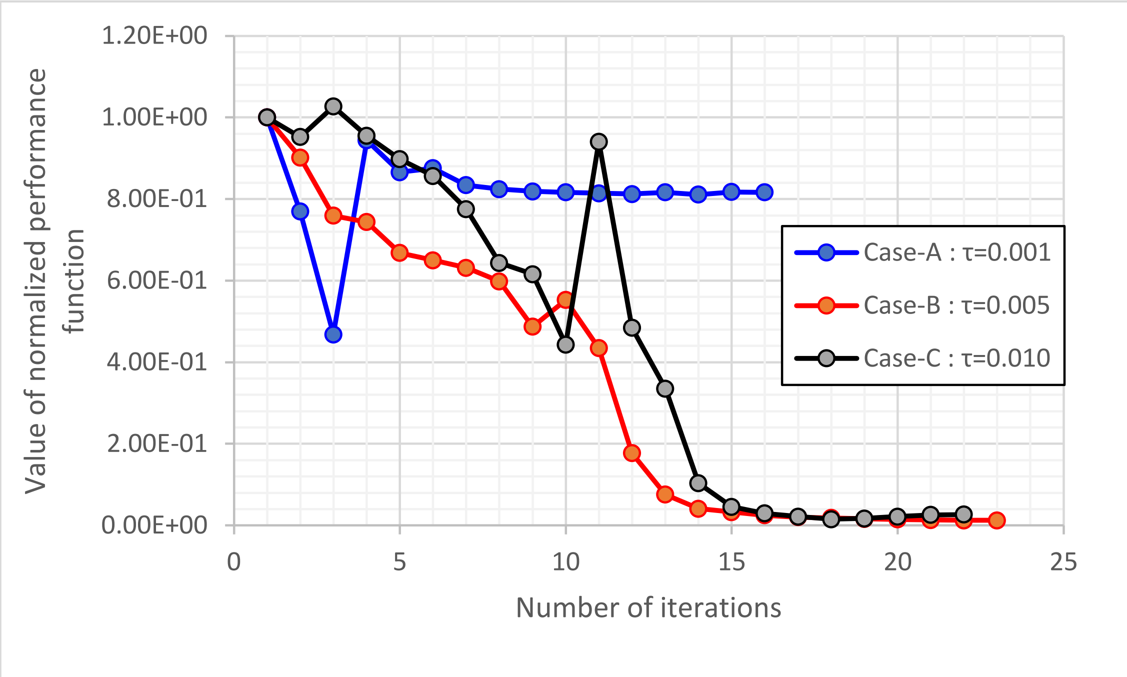
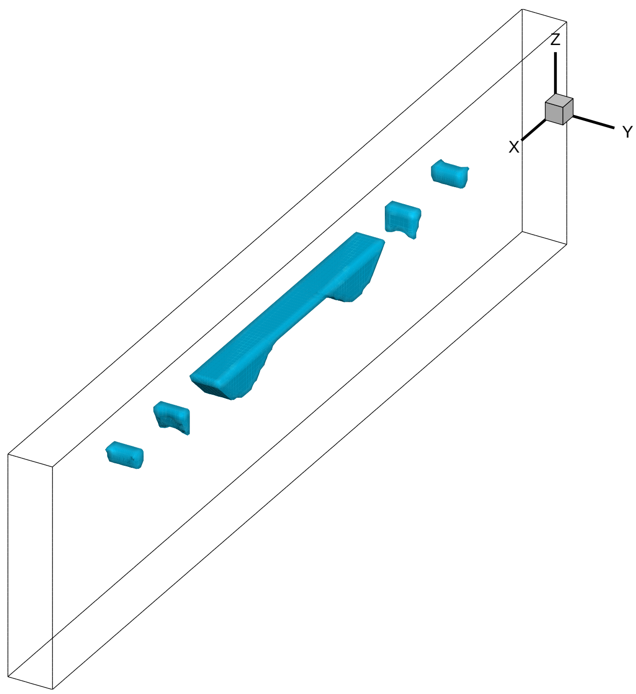
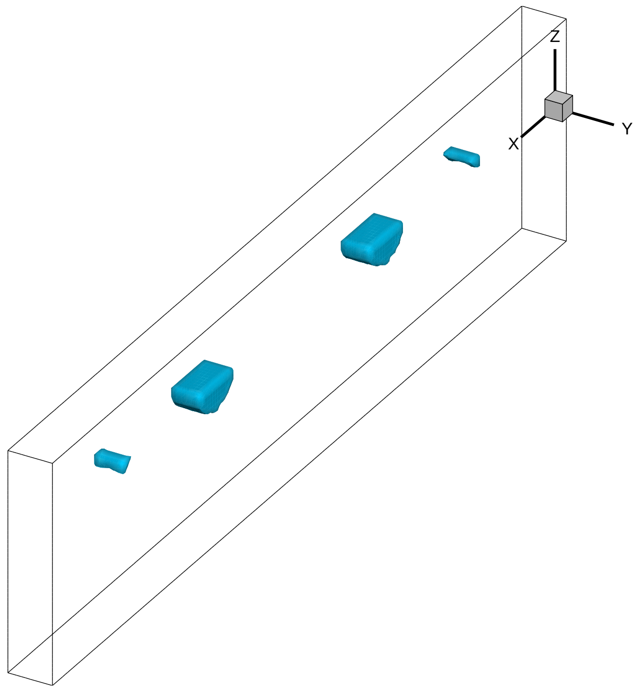


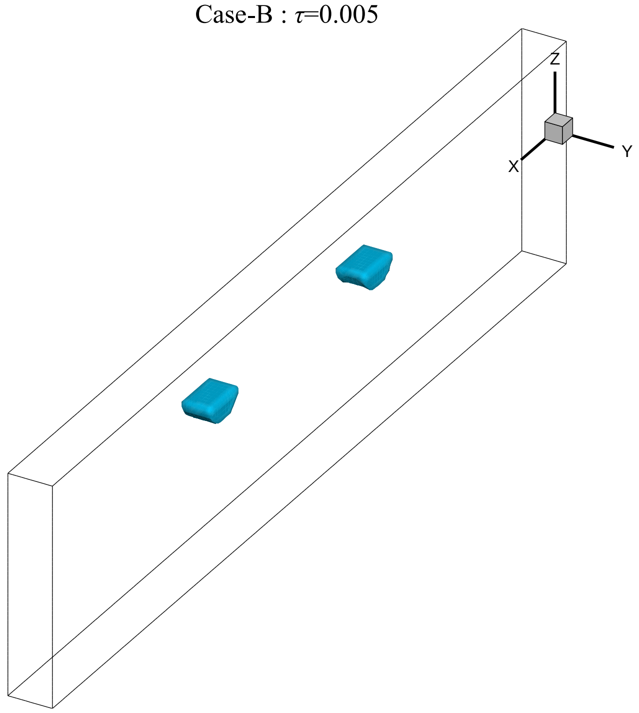
Figure 3: Variation of normalized performance function



Case-A : τ=0.001



Case-B : τ=0.005



Case-C : τ=0.010

Figure 4: Comparison of cavity shape for each regularization parameter τ

4 CONCLUSIONS

In this study, we presented application of the topology optimization analysis using the level set function for judgement of cavity shape in the hammering test. The finite element method was applied to discretize the governing equation in space, and the discretized equation was employed as the constraint condition of the performance function. The Newmark β method was applied to discretize the finite element equations for the state and the adjoint variables in time.

According to numerical results, it was found that if we'd like to obtain appropriate cavity shape, we have to pay attention not only for setting of the regularization parameter τ but also position of the observation points. In future, this analysis will be applied to practical problems.

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