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GEOMECHANICAL MODELING USING VARIABLE ORDER SPECTRAL ELEMENT METHOD AT NON-CONFORMAL MESHES

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Abstract: The paper considers an overview of the problems of mathematical modeling of geomechanical processes occurring in rocks during the geological exploration and development of reservoirs and well boring process. The mathematical formulation is based on the theory of repeated superposition of large deformations. A numerical discretization of the posed boundary problems of interacting solids is performed using a discontinuous spectral element method and multi-point constraints at non-matching mesh interfaces between interacting solid rock structures.

Several industrial applications of the developed approach are considered. Seismic wave propagation in the heterogeneous media with initial geomechanical stresses is considered. A modelling of an induced anisotropy is performed by the superposition of dynamic deformations onto initial generally finite strains. Use of variable order spectral elements at non-conformal meshes allows one to simplify the process of unstructured mesh generation for the discretization of complex geological models and to set the local spatial order of the SEM discretization depending on the speed of seismic waves in geological structures, which

significantly reduces the computational costs when conducting numerical modeling and lowers the requirements to the model preprocessing and mesh quality.

The considered approach allows predicting in more detail the behavior of the rock during reservoir development, taking into account different stages of the field deformations. In particular, the redistribution of accumulated deformations during multistep loading and / or changes in the structure (topology) of the loaded body, as well as contact conditions of adhesion / sliding at the interlayer boundaries and bonded contacts are taken into account.

These problems were solved using CAE Fidesys software, which allows solving static and dynamic problems of geomechanics and geophysics using finite (FEM) and spectral (SEM) element methods of a variable approximation order in space at non-conformal unstructured meshes.

1 INTRODUCTION.

During the geological exploration and development of underground reservoirs a number of problems arise [3, 4, 16]. The paper considers an overview of such problems of mathematical modeling of geomechanical processes. The mathematical formulation is based on the theory of repeated superposition of large deformations [13-15]. A numerical discretization of the posed boundary problems of interacting solids [12, 22, 23] is performed using a discontinuous spectral element method and multi-point constraints [1, 2, 5-7] at non-matching mesh interfaces between interacting solid rock structures.

Geomechanical models which take into account real geological layers are often complex to build finite element meshes containing only hexahedron elements, the base elements in SEM [10]. In common cases, the 3D finite element mesh is unstructured, hybrid, and non-conformal and contains elements with different orders [11]. The advantage of SEM method for solving problems of seismic wave propagation over FEM was shown as in [25, 26].

The problem of non-conformal meshes was solved by using tied contact on boundaries with such meshes [22]. This approach was used in previous works [21] and showed good results for seismic wave simulation too.

The problems were solved using CAE Fidesys software [8, 11, 24], which allows solving static and dynamic problems of geomechanics and geophysics using finite (FEM) and spectral (SEM) element methods [9, 10, 23] of a variable approximation order in space at non-conformal unstructured meshes [17-21].

2 MATHEMATICAL MODEL

Theory for solving problems of redistribution of large strains in a body, i.e., the problems of repeated changing of boundaries and (or) boundary conditions) was presented in [28, 29]. So mathematical model consists of:

Equations of motion: $\overset{n}{\nabla} \cdot [(1 + \Delta_{0,n})^{-1} \overset{n}{\Sigma}_{0,n+1}(t) \cdot \Psi_{n,n+1}(t)] - \rho_n \frac{\partial^2 u}{\partial t^2} = 0$ (1)

Initial conditions: $u(0) = u_n; \frac{du}{dt}(0) = 0$ (2)

Boundary conditions: $\sigma_{0,n+1}(t)\big|_\infty = \sigma_{0,n+1}^\infty(t)$ (3)

$$\begin{aligned}(\overset{n}{N}_{n+1}(t) \cdot \overset{n}{\Sigma}_{0,n+1}(t)\Big|_{\overset{n}{\Gamma}_{n+1}(t)})|_{hole} = 0(\overset{n}{N}_{n+1}(t) \cdot \overset{n}{\Sigma}_{0,n+1}(t)\Big|_{\overset{n}{\Gamma}_{n+1}(t)})|_{matrix} \\ = (\overset{n}{N}_{n+1}(t) \cdot \overset{n}{\Sigma}_{0,n+1}(t)\Big|_{\overset{n}{\Gamma}_{n+1}(t)})|_{inclusion}(u(t)|_{\overset{n}{\Gamma}_{n+1}(t)})|_{matrix} \\ = (u(t)|_{\overset{n}{\Gamma}_{n+1}(t)})|_{inclusion}\end{aligned} \tag{4}$$

Constitutive relations

$$\overset{n}{\Sigma}_{0,n+1}(t) = (1 + \Delta_{0,n+1}(t)) \cdot {\Psi^*_{n,n+1}}^{-1}(t) \cdot \sigma_{0,n+1}(t) \cdot \Psi^{-1}_{n,n+1}(t) \tag{5}$$

$$\sigma_{0,n+1}(t) = (1 + \Delta_{0,n+1}(t))^{-1} \Psi^*_{0,n+1}(t) \cdot \overset{0}{\Sigma}_{0,n+1}(t) \cdot \Psi_{0,n+1}(t) \tag{6}$$

$$E_{0,n+1}(t) = \frac{1}{2}(\Psi_{0,n+1}(t) \cdot \Psi^*_{0,n+1}(t) - I) \tag{7}$$

Geometrical relations

$$1 + \Delta_{0,n+1}(t) = \det \Psi_{0,n+1}(t) \tag{8}$$

$$\Psi_{0,n+1}(t) = \Psi_{0,n} \cdot \Psi_{n,n+1}(t) \tag{9}$$

$$\Psi_{n,n+1}(t) = I + \overset{n}{\nabla} u(t) \tag{10}$$

$$\rho_n = \frac{\rho_0}{1 + \Delta_{0,n}} \tag{11}$$

The mathematical formulation is based on the theory of repeated superposition of large deformations [28, 29].

3 NUMERICAL MODEL

1.1 Spectral element method (SEM)

A numerical discretization of the purposed boundary problem of interacting solids is based on spectral element method [9, 10, 23].

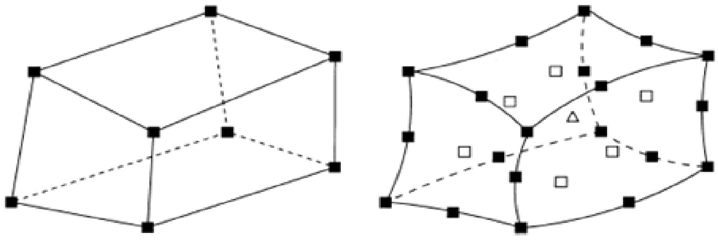

Figure 1: Element and nodes (Geometry)

Basis functions of SEM— Legendre polynomials (12), provide high order of approximation in space.

$$f\left(\boldsymbol{x}(\xi,\eta,\zeta)\right) \approx \sum_{\alpha,\beta,\gamma=0}^{n_l} f^{\alpha\beta\gamma} l_\alpha(\xi) l_\beta(\eta) l_\gamma(\zeta)\ \nabla f\left(\boldsymbol{x}(\xi,\eta,\zeta)\right)$$
$$\approx \sum_{i=1}^{3} \hat{x}_i \left[\sum_{\alpha=0}^{n_l} f^{\alpha\beta'\gamma'} l'_\alpha(\xi_{\alpha'})\partial_i\xi + \sum_{\beta=0}^{n_l} f^{\alpha'\beta\gamma'} l'_\beta(\eta_{\beta'})\partial_i\eta + \sum_{\gamma=0}^{n_l} f^{\alpha'\beta'\gamma} l'_\gamma(\zeta_{\gamma'})\partial_i\zeta\right] \quad (12)$$

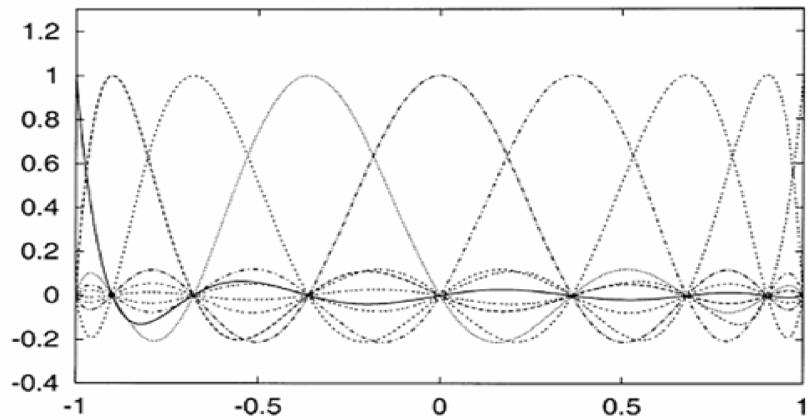

Figure 2: SEM basis function.

1.2 Isoparametric SEM

Spectral convergence (superconvergence) [27] at the smooth solutions with minimal dispersion [10]: $\|\boldsymbol{u}-\boldsymbol{u_h}\| \leq \boldsymbol{C}\boldsymbol{h}^{\boldsymbol{N}}\boldsymbol{e}^{-\boldsymbol{N}}$ in H^1 norm can be achieved if isoparametric elements are used.

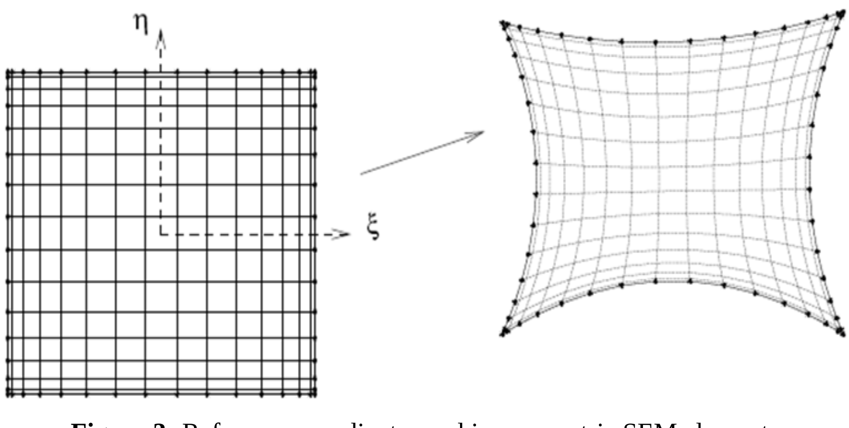

Figure 3: Reference coordinates and isoparametric SEM elements

$$\boldsymbol{x}(\boldsymbol{\xi}) = \sum_{a=1}^{n_a} l_a(\boldsymbol{\xi})\boldsymbol{x}_a\,; \quad \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{\xi}} = \sum_{a=1}^{n_a} \frac{\partial l}{\partial \boldsymbol{\xi}}\boldsymbol{x}_a; \quad J_b = \left|\frac{\partial(x,y)}{\partial(\xi,\eta)}\right| \tag{13}$$

1.3 Tied contact

Contact conditions [23] for tied surfaces include a continuity of displacements:

$$x^1|_{AB} = x^2|_{AB} \tag{14}$$

And a continuity of the traction: $t^1|_{AB} = -t^2|_{AB}$ (15)

Here x^i are coordinates in the current (deformed) state and t^i is the traction at the interface between the tied solids. Discrete Multipoint Constraints (MPC) for displacements have form:

$$x_s = F(x_m) = \sum_{i=0}^{n} N_i(\xi)\, x_m^i \tag{16}$$

Here $N_i(\xi)$ value of surface form function at projection of slave node to master face, ξ reference element system coordinates of this point.

$$u_s - \sum_j N_j(\xi) u_m^j = 0 \tag{17}$$

Weak form of traction continuity:

$$\int_{\Gamma^s} N_i^s \sigma^m n^s d\Gamma^s + \int_{\Gamma^m} N_i^m \sigma^s n^m d\Gamma^m = 0 \tag{18}$$

When the contact pair is active next conditions must be fulfilled:

- slave node and master face belong to different solids;
- a distance from the slave node to its projection onto the master face is less than a given threshold;
- a projection of the slave node onto the master face is inside the face;
- an angle between the master and slave surfaces' normal is more than a given threshold (180 degrees for the plane contact).

As some nodes could be simultaneously both master and slave for the multibody contact it's necessary to convert a system of MPC equations into the RREF (upper trapezoidal) SLAE in order to obtain a fundamental solution i.e. to choose global master and slave DOFs as well as to get rid of overdetermined system due to linearly dependent constraints. Further the obtained fundamental solution is used for the direct constraint elimination [20].

4 RESULTS.

4.1 Tied contact: beam bending

The developed algorithm was used for the linear static analysis in order to provide continuous displacements and stresses in assemblies with gaps and intersections. An example of such problem of a split beam with gaps and intersections is presented below. Geometry's dimensions are 1m x 1m, material properties: Young's modulus 11.7 GPa, Poisson ration 0.35. Tied contact between beam's parts is set.

Boundary conditions: fixed displacements along X and Y directions on the left border and constant pressure on the top border of 1 GPa.
A coarse non-conformal mesh was generated for both parts with 4-th and 3-rd orders of spectral elements in them correspondingly. Figures 4 and 5 shows mesh and continuous displacements and stresses through part's boundary under bending.

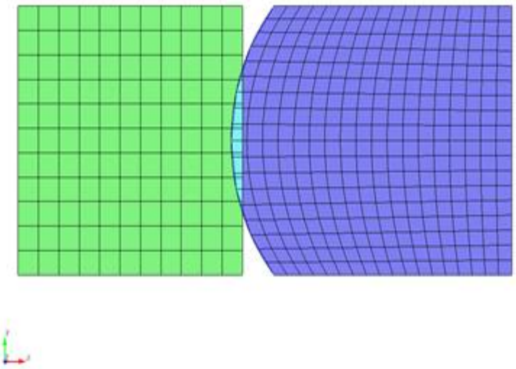

Figure 4: Mesh with gaps and overlap.

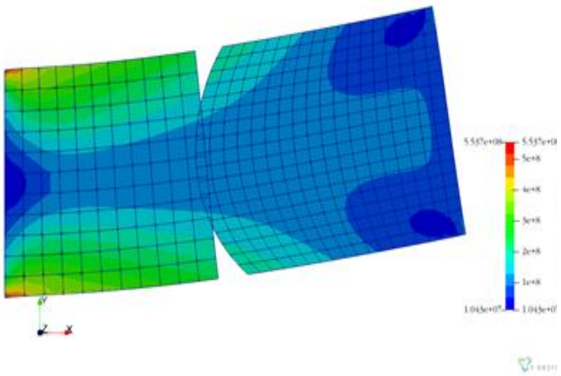

Figure 5: Continuous stress (Mises) distribution.

4.2 Tied contact: Kirsch problem

The Kirsch problem was solved to demonstrate stress continuity if proposed contact solution method used to refine the mesh close to a stress concentrator. Stress on the boundary is 1e6 Pa. Analytical solution gives maximal stresses at the boundary of the stress concentrator of 3e6 Pa. The stress distribution is presented on Figure 6 for non-conformal spectral element mesh of different orders refined nearby the stress concentrator. Results for the conformal mesh are presented on Figure 7.

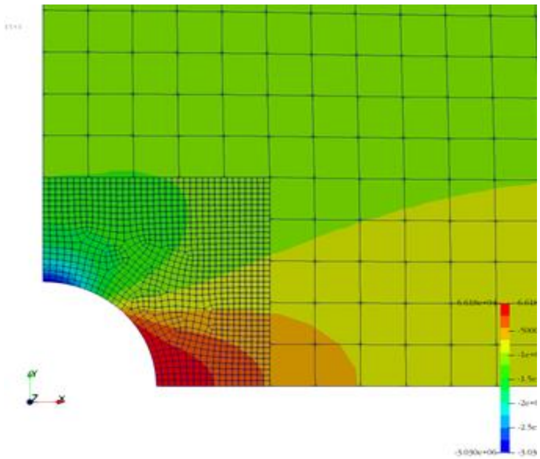

Figure 6: Non-conformal spectral element mesh of different orders refined nearby the stress concentrator.

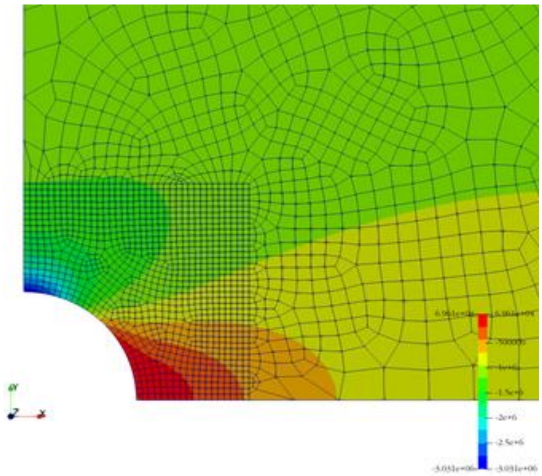

Figure 7: Conformal finite element mesh refined nearby the stress concentrator.

4.3 Tied contact with gap: Full waveform dynamic modeling

Lamb's problem was solved in order to demonstrate a continuous wave propagation thru the gap between two layers of a body produced by an intentional dissection of the isotropic semi-infinite media. This gap plays a role of inaccuracies happening in realistic geological layered models with geometrical artefacts and issues at the boundaries between layers. Amplitude of applied point load was given by Berlage source:

$$f(t) = A \cdot \frac{\omega_1^2 e^{-\omega_1 t}}{4}(sin(\omega_0 t)(-\frac{t^2}{\omega_1} + \frac{t}{\omega_1^2} + \frac{1}{\omega_1^3}) - cos(\omega_0 t)\sqrt{3}(\frac{t^2}{\omega_1} + \frac{t}{\omega_1^2})),$$

$$\omega_1 = \frac{\omega_0}{\sqrt{3}}, \omega_0 = 2\pi\omega \quad (18)$$

here $A = 1e8$ - magnitude, $\omega = 10Hz$ - frequency, t – time.

Source term of Berlage type shown on Figure 8. The SEM elements of third (top) and fourth (bottom) order where used. The gap between layers is about 0.2 of an element height (top layer). The results shown on figure 9 demonstrates continuous wave propagation through the gap. Figure 10 shows displacements and stress wave fields continuity despite the presence of a gap and non-conformal meshes of different orders used for the discretization of layers. A fully explicit Newmark scheme (2^{nd} order in time) was used for time integration. A time step computed accordingly to Courant's condition is turned out to be independent of the gap size and non-conformal interface between two layers i.e. it's exactly the same as for the case of solving the problem at the conformal SEM mesh of the same size (top layer).

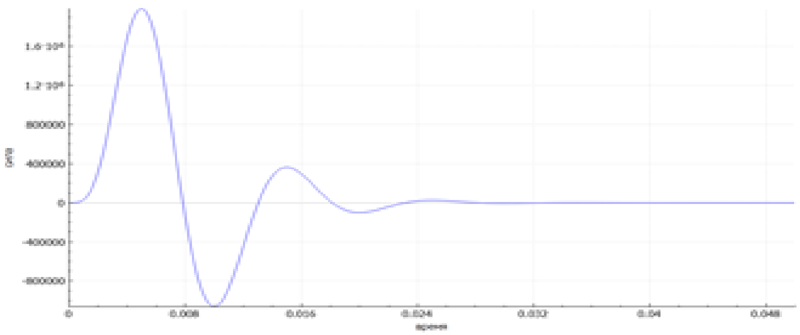

Figure 8: Wavelet for the source term

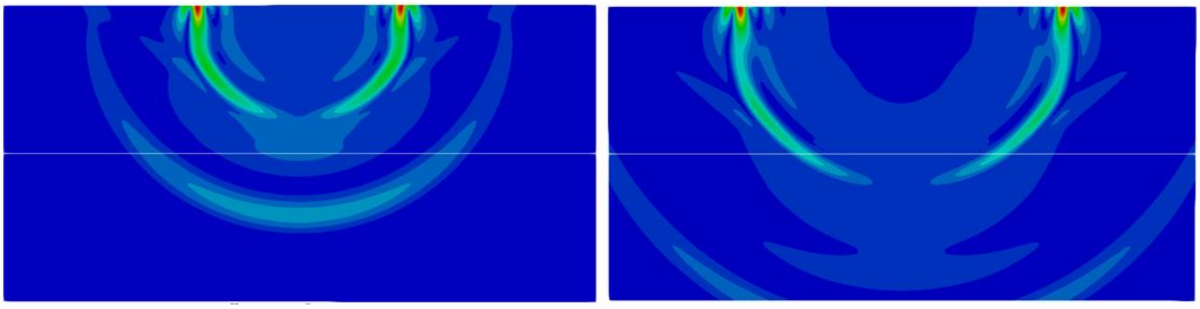

Figure 9: Displacement distribution (magnitude) at different time moments clearly demonstrates a continuity of wavefields through the gap between two layers

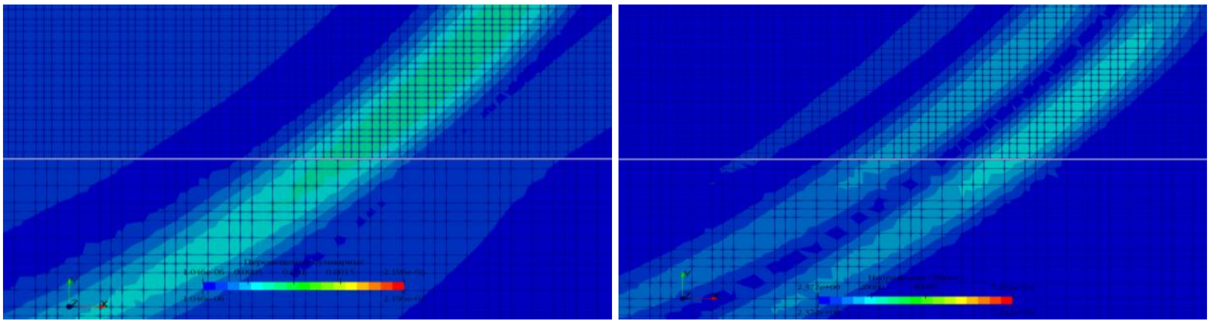

Figure 10: Displacement (left) and von Mises stress (right) wave fields near gap at non-conformal SEM meshes of different orders: top layer (third order), bottom layer (fourth order). Mark their continuity through the gap.

4.4 Seismic monitoring of fractured media

An example with a fractured zone seismic monitoring was considered. Quadrilateral and triangle spectral elements were used for the discretization of layers and fractures' boundaries inside them (Figure 12). A surface point source (Figure 11) excites a set of body waves reflected and refracted from the fractures' edges (Figure 13).

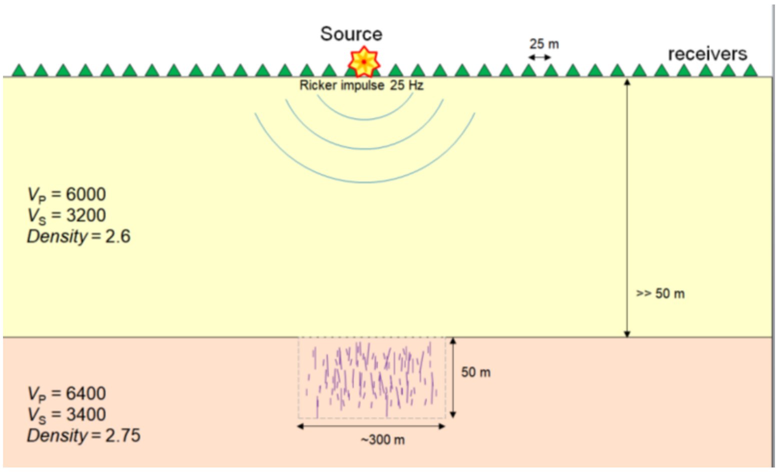

Figure 11: Seismic monitoring of fractured media – excitation scheme

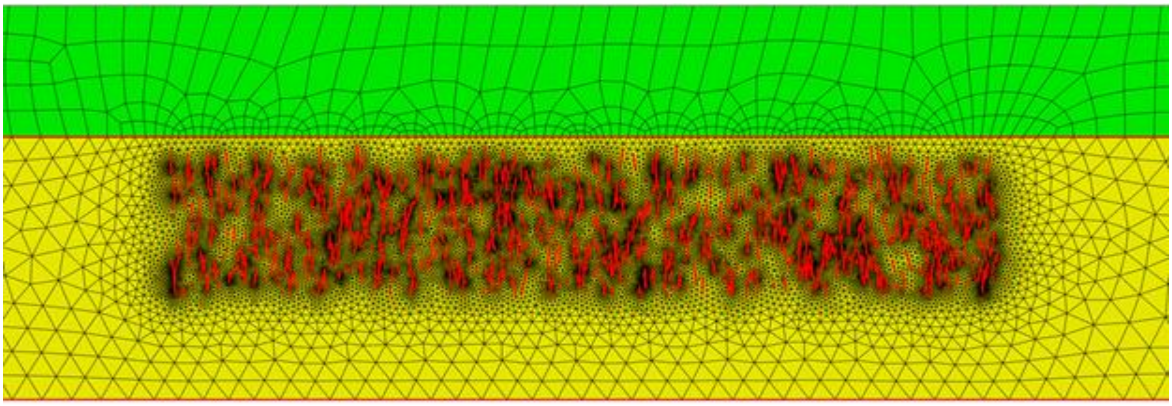

Figure 12: Fractured media unstructured hybrid SEM mesh

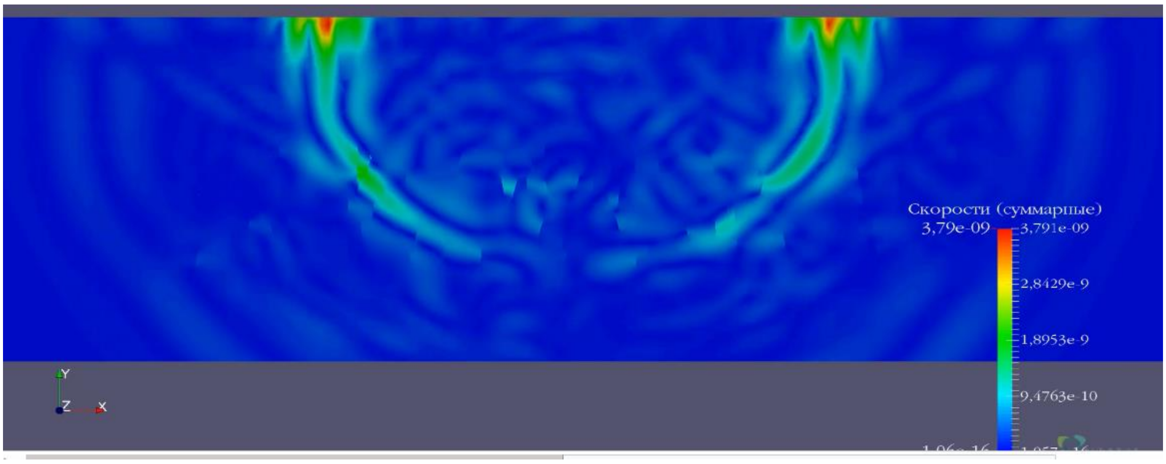

Figure 13: Fractured media survey simulation – velocity wave fields refraction at fractures

4.5 Localization of elastoplastic strains.

Additionally to mentioned above advantages, SEM also provides:

-Best numerical convergence properties for highly nonlinear problems with large plastic strains and their localization (no locking phenomena and other issues typical for FEM);

-Simplified numerical convergence analysis without a necessity to regenerate a mesh i.e. a convergence is achieved by simply increasing an approximation order of SEM.

As an example demonstrating these statements, a localization of the plastic shear bands nearby the wellbore due to the depression in the borehole was simulated using SEM. As it's seen at Figure 14 SEM accurately models a complicate localized structure of Luder's shear bands.

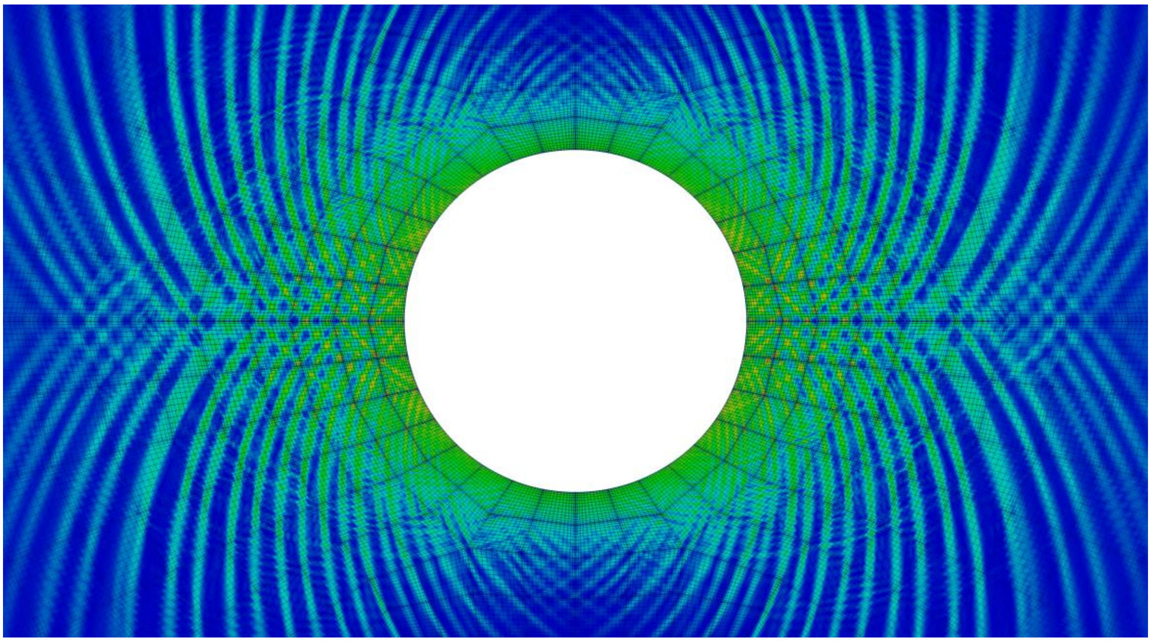

Figure 14: Localization of plastic shear bands due to the depression in the borehole at SEM mesh

5 CONCLUSION

An algorithm based on spectral element method was proposed for solving geomechanical problems (both stationary and non-stationary) at nonconformal unstructured hybrid SEM meshes of different orders which could be applied for the accurate discretization of models with geometrical issues like overlaps and gaps between different parts (layers) of the models. The developed algorithm provides a continuous solution (both in displacements and stresses) all over the domain even in case of presence of these geometrical artefacts and non-conformal SEM discretization. The latter greatly simplifies the process of mesh generation for the complicate models, and at the same time allows obtaining a mathematically correct result.

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