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VARIATIONAL PROBLEM FOR HYDROGENERATOR THRUST BEARING

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Abstract. Reversible and irreversible heavy loaded thrust bearings of a hydrogenerator are investigated. The problem of oil wedge microgeometry profiling for load capacity optimization is considered. The analysis is based on optimization methods using variational calculus. The results of oil wedge microgeometry optimization for reversible and irreversible thrust bearings are presented.
Keywords: hydrogenerator, load capacity, optimization, thrust bearing.

1. Introduction

In conditions of dynamic development of the world hydropower and construction of a large number of hydropower plants in developing economics: Brazil, China, India, etc. it is important to ensure reliable trouble-free operation of the key structural elements of the plant. One of the most important structural elements is a hydrogenerator thrust bearing which perceives a major part of a load. Generally, the load on the thrust bearing is produced by a rotor, an impeller, and turbine shaft weight and water pressure on the impeller [1, 2]. The important operational parameter that characterizes the efficiency of the bearing is the load capacity of an oil wedge that is a nonlinear function of the gap magnitude. The main role performs the minimal oil film thickness; the thinner the oil layer, the higher the bearing load capacity. However, reduction of oil film thickness leads to decreasing the bearing stability under dynamic loads.

There are various classifications of the bearings: by geometrical characteristics, by perceived load, by number and type of supports and by kind of mounting. By the surface type realization bearings are divided into one-piece and segmented. One-piece bearing carrier surface is a surface coated with a relief profile. Such bearings are called profiled. Segmented or self-aligning acting bearings are the bearings which fixed part consists of separate segments, mounted on special supports that allow each segment to turn in the flow of liquid lubricant, forming an angle with the rotating disk surface. In order to provide greater load capacity, the support is moved relative to the segment axis by a certain value in the direction of rotation, creating eccentricity. Typically, the value of the eccentricity is 5-8 % of the segment length. Such bearings are called irreversible because they demonstrate the load capacity for only one rotation direction. In hydropower they are used for a wide range of devices on the majority of the existing plants.

However, in some cases, for example for devices with variable rotation direction of the turbine generators, it is necessary to set zero segments eccentricity to ensure the durability of the device. Such bearings are called reversible. Currently, the thrust bearing of this type is installed on Zagorskaya GAES , unique and the Russian only pumped storage power station.

In this work we consider the lubricant layer microgeometry profiling with the aim of optimal design of the hydrodynamic bearing for ensuring the maximum load capacity.

2. Optimization problem statement

Note that historically the first formulation of the considered problem in one-dimensional case goes back to the work by J.W. Rayleigh published in 1918 [5]. The Rayleigh results were much ahead of his time, were repeated later by S.Y. Maday only in 1967 [6]. In 1975 one of the authors together with V.A. Troitsky considered the spatial variational problem put by Rayleigh to gain the optimal shape profile for the rectangular gap region [3]. For the sector bearings considered here, one of the authors, jointly with Yu.V. Borisov obtained the first results in Ref. [7]. Here we enlarge the results of the previous works [3, 7] in relation to the hydrogenerator sector thrust bearings based on advanced computing technologies. It is worth noting that over the last years unique technologies of desired shape surface microgeometry manufacturing were developed, the optimization results can being implemented.

We consider the optimization problem of the thrust sector bearing microgeometry [3]. An example of such bearing is shown in Figure 1. One bearing segment with installation angles indication is presented in Figure 2. All the sectors are assumed to be identical and the sector angle $\Delta\varphi = 2\pi / N$, where N is the number of sectors. We assume that region Ω with boundary $\partial\Omega$ (Fig. 3), corresponding to one thrust bearing sector, is located in (r,φ) plane of cylindrical coordinates (r,φ). Plane (r,φ) moves in the φ direction with constant angular velocity ω.

Fig. 1. Michell thrust bearing [4].

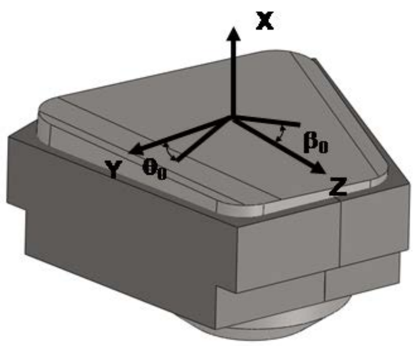

Fig. 2. Thrust bearing segment.

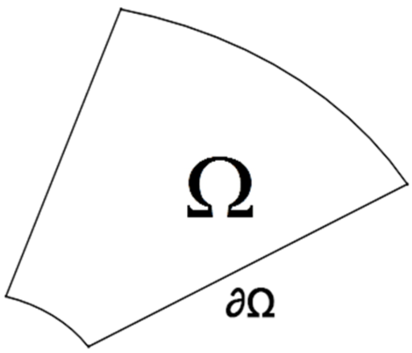

Fig. 3. Region Ω.

We will describe the profile shape of the lubricating layer by piecewise-smooth function $h(r,\varphi)$ and assume that h_{min} is its minimum value. We suppose that pressure field $p(r,\varphi)$ in the lubricant layer is described by the linear Reynolds equation written in the following dimensionless form:

$$div(h^3\nabla p - h\boldsymbol{V}) = 0 \text{ in } \Omega \tag{1}$$

Here the dimensionless pressure p and the coordinates r and φ are normalized correspondingly by ambient pressure p_a and dimensional values L_r and L_φ that characterize segment dimensions. Note that the critical parameter $\gamma=L_r/L_\varphi$ characterizing the elongation of region Ω in equation (1), here and throughout is put equal to 1 for simplification of calculations. The profile function h is normalized by h_{min}. The velocity vector $V_\varphi=(1,0)$ is normalized by the magnitude of $|V_\varphi|$. The boundary conditions for equation (1) correspond to zero pressure on the boundary $\partial\Omega$ of region Ω

$$p\big|_{\partial\Omega} = 0 \tag{2}$$

Note that equation (1) is an equation for the excess pressure $p(r,\varphi)$ in the lubricant layer in region Ω. According to shown above normalization character, the lubricant layer profile function $h(r,\varphi)$ should satisfy the restriction

$$h \geq 1 \tag{3}$$

In line with the last inequality, h_{min} is selected. Its value, corresponding to equilibrium stationary mode of thrust bearing operation, is specified usually from technological considerations.

The aim is construction of the lubricant layer profile $h(r,\varphi)$ that provides maximum sector load capacity. Thereby the negative value of the lifting force of the lubricant layer, normalized by $L_rL_\varphi p_a$, can by used as a variational problem functional. The negative sign is chosen according to traditional rules of the variational calculus for searching a functional minimum

$$W = -\int_\Omega p d\Omega \tag{4}$$

Thus we can formulate the optimization problem in such a way: find among continuous in Ω functions p that satisfy the boundary value problem for Reynolds equation (1, 2) and among piecewise continuous functions h satisfying the condition (3) those that provide minimum to functional (4). Further, we follow the approaches developed in Refs. [3, 8].

Introduce an auxiliary function $v(r,\varphi)$ and switch from the constraints of inequality (3) to constraints of equality

$$\psi = h - 1 - v^2 = 0 \tag{5}$$

Write down equation (1) in the form of following system of equations

$$div\boldsymbol{Q} = 0, \boldsymbol{Q} = h^3\nabla p - h\boldsymbol{V} \text{ in } \Omega, \tag{6}$$

where $\boldsymbol{Q}$ is the dimensionless volumetric flow vector, normalized by $h_{min}|\boldsymbol{V}_\phi|$. We satisfy the first of equations (6) by introducing continuous function $M(r,\varphi)$ in the following way:

$$\boldsymbol{Q} = rot(M\boldsymbol{k}) \ , \tag{7}$$

where $\boldsymbol{k}$ is a unit vector of Z axis. We form auxiliary functional

$$J = \int_\Omega f(p, \nabla p, \boldsymbol{Q}, M, h, v) d\Omega \tag{8}$$

with augmented function f which includes the system of problem restrictions[1] (5-7)

$$f = -p + \left(\lambda_0, \boldsymbol{Q} - rot\left(M\boldsymbol{k}\right)\right) + \left(\lambda_1, \boldsymbol{Q} - h^3\nabla p + h\boldsymbol{V}\right) + \lambda_2\psi \ , \tag{9}$$

Here λ_0, λ_1 and λ_2 are the functional Lagrange multipliers, the first two being vectors.

In the articles mentioned above [3, 8] it is shown that in the case of a rectangular region Ω, the optimal function h has one line of discontinuity that starts at the extreme front points of the region and separates the front part of the region with $h > 1$ and back part with $h = 1$. In this case in the front part of the region ($h > 1$), pressure increases everywhere but at $h = 1$ it falls everywhere to an ambient value.

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[1] Note, that the symbol $(\boldsymbol{a},\boldsymbol{b})$ we define the scalar product of vectors.

Let us briefly discuss the numerical procedure for constructing profile *h* used in Refs. [3, 8]. First of all, note that at each step of the iterative procedure, in addition to the boundary value problem for Reynolds equation (1, 2), the boundary value problem for determining Lagrange multiplier λ, which is related to Lagrange multiplier λ_0 by λ_0 = grad λ, is also solved. Besides at each step the point value of the profile function is determined; at the points where h> 1 we have Euler-Lagrange equation:

$$3h^2 = (\nabla\boldsymbol{\lambda}, \boldsymbol{V}) / (\nabla\boldsymbol{\lambda}, \nabla p) \tag{10}$$

The discontinuity line position of function h is determined on the basis of Erdmann-Weierstrass conditions [2] from which, in particular, we find the equation for the line of discontinuity:

$$\left[-\frac{\partial\lambda}{\partial\tau}\frac{\partial M}{\partial n} + \frac{\partial\lambda}{\partial n}h^3\frac{\partial p}{\partial n}\right]_-^+ = 0\,, \tag{11}$$

where, $[\ ]_-^+$ is value of difference to the right and left of the discontinuity line, $\boldsymbol{n}$ and $\boldsymbol{\tau}$ normal and tangent vectors to function *h*. The equation (11) can be simplified – according to equation (6) one obtains $-\partial M/\partial n = Q_\tau = h^3\,\partial p/\partial\tau - hV_\tau$ and final discontinuity line equation can be written in the following form:

$$\left[h^3(\nabla\lambda, \nabla p) - h\frac{\partial\lambda}{\partial\tau}V_\tau\right]_-^+ = 0 \tag{12}$$

Let us briefly consider the analysis of the system of necessary conditions. First of all, we point out that a correct analysis of this system presupposes knowledge of the solutions properties of partial differential equations of elliptic type [9], which include equations for Lagrange multiplier λ and Reynolds equation. Restricting ourselves to the final results, note that an analysis of the solutions properties of boundary value problems for pressure *p* and Lagrange multiplier λ allows do conclusions about the behavior of gradients ∇p and $\nabla\lambda$ on the domain boundary, under the additional assumption of the smoothness of the optimal solution everywhere except for the corner points of domain Ω. Taking into account the constancy of vector $\boldsymbol{V}$, we can conclude about the sign of scalar product $(\nabla\lambda, \boldsymbol{V})$. Together with Erdmann-Weierstrass conditions, it is possible to make an important conclusion about the existence of single discontinuity line γ for the profile function *h* in the region Ω that separates region *h* = 1 from region *h* > 1.

3. Optimization results for rectangular region statement

In this work the special case for rectangular region Ω with profile consisted of two parts: straight line and parabola (Fig. 4) was considered. Here, *a* and b are the geometrical parameters which define parabola curvature. The parameters are used as optimization variables during optimization procedure. Using special code IOSO, the optimization problem was solved. There are only one objective function and two variable parameters. As an objective function, the maximum of pressure integral over the lubricant layer surface was used. Parameters *a* and *b* are varied in the following range $a \in [0.03; 0.35]$, $b \in [0.5; 0.9]$. Global size of the rectangle is 1x1.

To solve the optimization problem, the CFD mesh for investigated domain was generated and the hydrodynamics problem, using Navier–Stokes equations, was solved on the basis of numerical approach and commercial CFD code ANSYS/CFX. The numerical simulation of the problem was carried out using St. Petersburg Polytechnic Supercomputer Center. Totally about 70 iterations were done before the maximum of pressure integral was achieved. As a result, the optimal parameters *a* and *b* were found; they are 20.2 and 77.5 correspondingly. In Figure 5 the pressure distribution for the optimum profile is shown; in Figure 6 the dependence of the maximum pressure on a coordinate for section y=0 is demonstrated.

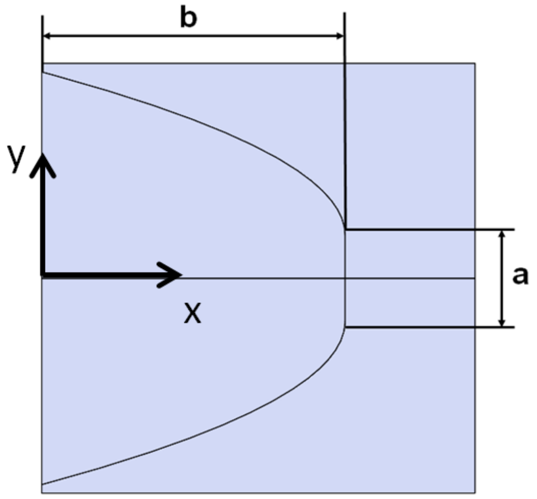

Fig. 4. Profile parameters for optimization procedure.

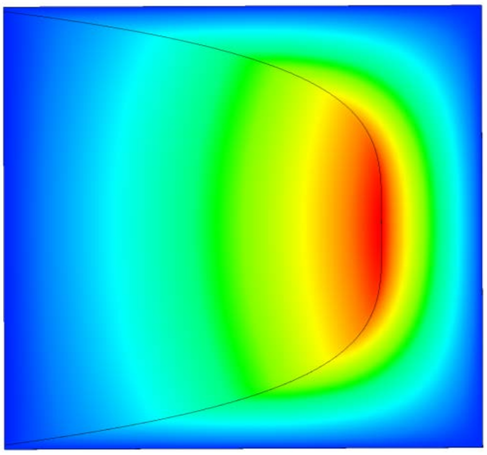

Fig. 5. Pressure distribution for the optimum profile.

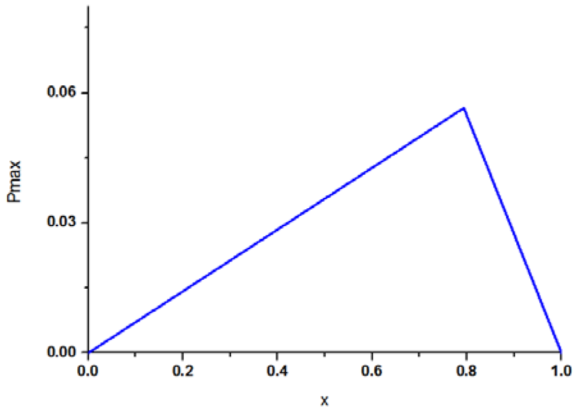

Fig. 6. Dependence of maximum pressure on a coordinate.

4. Conclusions

In this work the variational problem for hydrogenerator thrust bearing is considered. From problem statement above the important conclusion about the existence of a single discontinuity line of the profile function h in the region was made. This line separates the region h = 1 from the region h > 1. The optimization problem using commercial codes is solved for a rectangular region. Current results could be used in future investigations for a wide range of thrust bearing with different profile forms.

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