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*Research Article*

# Containerships Sailing Speed and Fleet Deployment Optimization under a Time-Based Differentiated Freight Rate Strategy

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This paper investigates the problem of containership sailing speed and fleet deployment optimization in an intercontinental liner shipping network. Under the consideration of the time value of container cargo, three kinds of impact of sailing speed changes on long legs of each liner route are analysed, and a time-based freight rate strategy is proposed. Then, the optimization problem is formulated as a mixed-integer nonlinear programming. Its goal is to maximize the total profits of a container liner shipping. To find the optimal solution to the model and improve the efficiency of model solution, a discretization algorithm is proposed. Numerical results verify the applicability of the proposed model and the efficiency of the algorithm. In addition, the time-based freight rate strategy is able to achieve more profit compared to a fixed freight rate strategy.

## 1. Introduction

Container liner shipping services play an important role in maritime freight transportation [1]. Its main task is the transportation of containerized cargo (containers) such as manufactured products, food, and garment [2]. The unit value of the containers is generally much higher than bulk cargo [3]. Hence, the sailing speed of containerships should be high (e.g., 20–25 knots) to deliver containers to their destination in short transit time (Wang and Meng [3]). In fact, short transit time is preferred by customers because it implies less inventory cost associated with cargoes in the containers [4] and more revenues gained from the increase of product sales for customers [5]. Offering short transit time is a competitive factor in liner shipping [6]. Therefore, the freight rate may be agreed between container shipping lines and customers based on the containers' transit time.

Container shipping lines usually face two types of customer demands: long-term contractual demand and spot market demand [7]. The freight rate for long-term demand is often fixed and contracted once a year, whereas the freight rate for spot market demand may be agreed between container shipping companies and shippers (customers) dynamically on daily/weekly basis [7]. As short transit time is preferred, spot market customers are willing to pay for saved time [8]. In other words, the freight rate increases as the sailing speed of transported cargo increases. It is beneficial for container shipping lines to employ a time-based differentiated freight rate strategy (hereafter TDFRS) for spot market customers when they decide containership sailing speed.

Fleet deployment is one of the most important problems which container shipping lines have to face. Fleet deployment is to determine the number and type of ships to be assigned to the shipping routes [9] over a planning period of, e.g., 6 months [10]. The sailing speed of containership is an important factor in the fleet deployment problem [11]. Reducing (or increasing) the sailing speed of containerships requires an increase (or decrease) in the number of deployed containerships in order to maintain a regular service frequency [12]. Besides, sailing speed has a significant impact on the total operating cost because it is closely related to the bunker consumption [13], which occupies a large proportion of the total operating cost [3]. It is thus meaningful for container shipping lines to investigate the optimization problem of sailing speed and fleet deployment (hereafter SSFD) under TDFRS.

*1.1. Literature Review.* Previous studies on the problem of SSFD assumed that containerships sail at fixed sailing speeds. Then, the problem of SSFD was simplified as the fleet deployment problem. The fleet deployment problem was first addressed in the literature by Perakis and Jaramillo [14]. It was formulated as a linear programming. This linear programming is improved as an integer linear programming by the two authors because the number of deployed containerships should be treated as an integer rather than a continuous variable. Powell and Perkins [15] extended the model developed by Jaramillo and Perakis [16] by adding ship lay-up costs to the objective function. The extended model is an integer programming. It optimizes fleet deployment for a container shipping liner. Christiansen et al. [17] proposed a mixed-integer linear programming formulation for the fleet deployment problem to determine voyages and lay-up times for each ship route during a given planning period. In those studies, cargo (container) shipment demand between two ports is assumed to be deterministic. Taking cargo shipment demand uncertainty into consideration, some researchers relaxed that assumption and further investigated the fleet deployment problem. Assuming that cargo shipment demand followed a normal distribution, Meng and Wang [18] developed a chance constrained programming for the fleet deployment problem formulation. However, the distribution which cargo shipment demand followed may be complicated and far away from a normal distribution. To relax that oftentimes restrictive assumption, Ng [19] proposed a distribution-free model for the fleet deployment problem. The distribution-free model only required the mean, standard deviation, and an upper bound of the cargo shipment demand. In all these existing models, the optimal sailing speeds decision were made exogenously and thus independent to the optimal fleet deployment decisions.

Due to high bunker prices caused by high sailing speed, container shipping lines began to adjust the sailing speed to reduce the total operating cost [20]. As indicated by Ronen [13], when bunker fuel prices hover at approximately 500 USD per ton, the bunker fuel cost accounts for about three quarters of the operating cost of a large containership. Engine theory and empirical data demonstrated that the daily bunker consumption of a containership is approximately proportional to the third power of its sailing speed [21]. Considering that reducing (or increasing) the sailing speed of containerships requires an increase (or decrease) in the number of deployed containerships in order to maintain a regular service frequency [12], some researchers are increasingly interested in the relationship between sailing speed optimization and fleet deployment optimization. Ronen [13] constructed a cost model of SSFD to analyse the trade-off between speed reduction and adding containerships to a container liner route and designed a simple procedure to identify the optimal sailing speed and number of containerships. Wang et al. [22] optimized sailing speed on each leg and number of containerships for a single liner route with the goal of minimizing the total cost of SSFD. Considering that a container shipping liner usually operates a group of liner routes, Gelareh and Meng [23] presented a mixed-integer nonlinear programming for the SSFD problem, in which the optimal sailing speeds for different ship types on different routes are interpreted as their realistic optimal travel times. Wang and Meng [11] also formulated the SSFD problem in a liner shipping network as a mixed-integer nonlinear programming. They employed an efficient outer-approximation method to transform the nonlinear programming model into an integer linear programming model which was solved by CPLEX. Wang et al. [24] proposed a practical tactical-level liner container assignment model for liner shipping companies, in which the container shipment demand is a nonincreasing function of the shipping time. However, considering the impact of shipping time on demand is difficult to measure in our manuscript, we assumed that demands are insensitive to shipping time. If cargo shipment demand and freight rate is assumed to be steady within a certain period, minimizing the total cost is tantamount to maximizing the total profit. Hence, the above studies chose the minimization of the total cost of transporting containers in a planning horizon as the objective function in the problem formulation. When the allocation of cargoes is considered, the minimization of the total cost is not tantamount to the maximization of the total profit anymore because the allocation has impact on the total freight revenue. Taking cargo allocation into consideration, Xia et al. [25] chose the maximization of the total profits as the objective function in the SSFD problem formulation.

These studies have significantly contributed to the development of mathematical programming for SSFD optimizations. However, to the best of the authors' knowledge, the problem of SSFD optimization under TDFRS is still open.

*1.2. Objective and Contributions.* The objective of this paper is to develop a model to achieve the optimal sailing speed of containership and the optimal number of deployed containerships under TDFRS with the goal of maximizing the total profit of a container shipping liner.

The contributions of this paper are threefold. First, it takes the initiative to address the SSFD problem under TDFRS while considering the time value of container cargo for an intercontinental liner network. This provides a reference for container shipping lines to design an optimal TDFRS and a freight rate for spot market customers. Consequently, the container cargo's time value is converted into revenue, which increases the total profit of the container shipping liner. Second, the SSFD problem under TDFRS is an extension to the SSFD problem in the literature [11] and it is formulated as a mixed-integer nonlinear programming. This modelling approach not only nests the model proposed by Wang and Meng [11] as a special case but also is more practical since it provides a reoptimal sailing speed decision-making approach on the two long legs of each liner route for the container shipping liner. Finally, managerial insights from the numerical experiment are obtained, providing significant references for container shipping lines.

The remainder of this paper is organized as follows. Section 2 describes the SSFD problem under TDFRS considering the time value of container cargo. In Section 3, the problem is formulated as a mixed-integer nonlinear programming. Section 4 proposes a discretization algorithm to solve the mixed-integer nonlinear programming. Section 5 presents the numerical example to demonstrate the performance of the proposed model and analyses its sensitivity to different parameters. Conclusions are presented in Section 6.

## 2. Problem Statement

Consider a container shipping liner that operates a set of intercontinental routes which regularly serves a group of calling ports. In practice, the calling ports on each liner route form a loop and the sequences of these calling ports are determined in advance. A string of homogeneous containerships is deployed on each liner route to provide service once a week. The SSFD problem faced by the container shipping liner is how to decide the sailing speed of the containership and fleet deployment with the consideration of time value of container cargo within a planning horizon. The length of the planning horizon is assumed to be 3–6 months, as it is the maximum period of time over which the cost parameters can be regarded as unchanged [26].

Table 1 shows an intercontinental liner network consisting of four Asia-Southwest America liner routes. Each of these four liner routes is deployed with either type 1 containership (10 060 TEUs) or type 2 containership (8 400 TEUs).

It can be seen from Table 1 that calling ports of each liner route can be divided into two types according to their geographical location (i.e., China and America). The voyage between two consecutive calling ports is defined as a leg. There are two long legs on each route and the rest are short legs. Take route 1 as an example. The ports of Lianyungang, Shanghai, and Ningbo are located in Asia and the other two ports are located in Southwest America. In route 1, Ningbo to Long Beach is one long leg on the head-haul direction, while Seattle to Lianyungang is the other long leg on the back-haul direction.

Under the consideration of the time value of container cargo, it is necessary to reoptimize sailing speed of containerships on long legs because it has three kinds of impact on SSFD optimization. First, it affects containerships' sailing time on long legs, which is an important part of the transit time of container cargoes between corresponding port pairs. Meanwhile, inventory cost associated with container cargoes changes, especially for shippers with high container cargo values. If shippers pay part of the saved inventory cost to the container shipping liner, the container shipping liner achieves an increased revenue generated by the transit time savings. Second, it influences the number of deployed containerships on each liner route because the number of deployed containerships needs to be changed according to containerships' sailing speed to maintain the weekly service frequency. Thus, a container shipping liner has to choose "fast steaming with less containerships" or "slow steaming with more containerships." Different choices result in different containership cost. Third, it significantly affects the bunker fuel cost on long legs due to the relationship between bunker fuel consumption and sailing speed. Meanwhile, for short legs on each liner route, the impact of the reoptimizing sailing speed is negligible because of short oceanic distance [4]. The increase in sailing speed on the short legs has little impact on the cargo transit time. For example, the short leg from Los Angeles to Oakland is 369 nautical miles, which can only save 2.65 hours when containership sailing speed increased from 19 to 22 knots. However, the saved shipping time is usually buffered because of possible delays at ports. On the contrary, the increase in sailing speed on long legs can greatly reduce time and thus have a significant impact on the container cargo time value. For example, the long leg from Ningbo to Long Beach is 5 761 nautical miles, which can save 41.35 hours when containership sailing speed increased from 19 to 22 knots. Therefore, the sailing speed of a containership on short legs does not need to be reoptimized and the containership is still sailing at the optimized speed without the consideration of the time value of container cargo.

The objective of the proposed optimization model is to reoptimize the sailing speed on each long leg on each liner route and the number of deployed containerships as well as the freight rate for the spot market customers. Before this, the optimal sailing speeds on each leg of each liner route need to be determined first without the consideration of the time value of the container cargo. The proposed models aim at maximizing the total profit. The total profit is calculated by the difference between the freight revenue and the total operating cost consisting of the bunker fuel cost and containership cost.

## 3. Problem Formulation

*3.1. Notion.* The notions used in this paper are introduced as follows:

Sets

$\Phi = \{1, 2, \ldots, R\}$: set of liner routes; $r \in \Phi$ is the route index

$\Omega = \{1, 2, \ldots, V\}$: set of containership types; $v \in \Omega$ is the containership type index

$A^r = \{1, 2, \ldots, Nr\}$: set of ports in one region on the intercontinental route $r$

$B^r = \{Nr + 1, Nr + 2, \ldots, Nr + Mr\}$: set of ports in the other region on the intercontinental route $r$

Parameters

$L_i^r$: oceanic distance on leg $i$ on route $r (i \in \{1, 2, \ldots, Nr + Mr\})$ (nautical miles)

$L_{Nr}^r$: oceanic distance on leg $Nr$ on route $r$, which is the distance on long leg $Nr$ on route $r$ in the head-haul direction (nautical miles)

Table 1: East Asia-U.S. West coast liner shipping network.

| Route | Containership type | Calling ports |
|---|---|---|
| 1 | 1 | Lianyungang ⟶ Shanghai ⟶ Ningbo ⟶ Long Beach ⟶ Seattle ⟶ Lianyungang |
| 2 | 2 | Tokyo ⟶ Qingdao ⟶ Shanghai ⟶ Ningbo ⟶ Los Angeles ⟶ Oakland ⟶ Tokyo |
| 3 | 2 | Qingdao ⟶ Shanghai ⟶ Ningbo ⟶ Los Angeles ⟶ Oakland ⟶ Qingdao |
| 4 | 1 | Taipei ⟶ Xiamen ⟶ Shekou ⟶ Yantian ⟶ Los Angeles ⟶ Oakland ⟶ Taipei |

$L^r_{Nr+Mr}$: oceanic distance on leg $Nr + Mr$ on route $r$, which is the distance on long leg $Nr + Mr$ on route $r$ in the back-haul direction (nautical miles)
$S^r_D$: designed sailing speed of a containership on route $r$ (knots)
$F^r_D$: fuel consumption at the designed sailing speed of the containership on route $r$ (tons)
$T^r_i$: port calling time at port $i$ on route $r$ of every voyage (hr)
$T^r_{ij}$: maximum acceptable transit time for containerships from port $i$ to port $j$ on route $r$ (hr)
$P$: bunker fuel price (USD/ton)
$C^r$: unit containership cost on route $r$ (USD/week)
$Y^r_{ij}$: initial freight rate for spot market customers from port $i$ to port $j$ on route $r$ (USD/TEU)
$D^r_{ij}$: average container demand for all customers from port $i$ to port $j$ on route $r$ (TEU/week)
$\alpha^r_{ij}$: freight rate discount for long-term contractual customers on the basis of initial freight rate from port $i$ to port $j$ on route $r$
$U^r$: unit container cargo value on route $r$ (USD/TEU)
$S^{r^*}_i$: optimal sailing speed on leg $i$ on route $r$ without considering the time value of container cargo (knots)
$S^r_{\min}$: minimum sailing speed of the containership on route $r$ (knots)
$S^r_{\max}$: maximum sailing speed of the containership on route $r$ (knots)
$\phi$: revenue sharing rate obtained by a container shipping liner, which is the proportion of revenue generated by the transit time savings $(0 \le \phi \le 1)$
$\rho$: coefficient of the time value of container cargo
$\gamma^r_{ij}$: container demand ratio of the spot market customers from port $i$ to port $j$ on route $r$
$N_v$: total number of type $v$ containerships owned by the container shipping liner

Variables

$S^r_i$: sailing speed on leg $i$ on route $r$ without considering the time value of container cargo (knots)
$s^r_{Nr}$: sailing speed on leg $Nr$ on route $r$ considering the time value of container cargo (knots)
$s^r_{Nr+Mr}$: sailing speed on leg $Nr + Mr$ of route $r$ considering the time value of container cargo (knots)
$m^r_v$: the number of deployed containerships $v$ on route $r$
$y^r_{ij}$: freight rate for the spot market customers from port $i$ to port $j$ on route $r$ considering the time value of container cargo (USD/TEU)

*3.2. Impact Analysis of Sailing Speed Changes.* According to the problem statement, the change of sailing speed has three kinds of impact on fleet deployment decisions considering the time value of the container cargo. The first impact is that it affects the transit time of containerships on the long legs on each liner route. If a container shipping liner increases sailing speeds of containerships, sailing time between port pairs can be shorten and the inventory cost of customers can be saved [8]. Since the container shipping liner achieves an increased revenue generated by the inventory cost saving, the increased revenue can be formulated as the saved inventory cost as shown in the following equation:

$$E^r_{Nr} = \frac{U^r \rho\left(\left(L^r_{Nr}/S^{r^*}_{Nr}\right) - \left(L^r_{Nr}/s^r_{Nr}\right)\right)}{24 \times 365}, \quad \forall r \in \Phi,$$
$$E^r_{Nr+Mr} = \frac{U^r \rho\left(\left(L^r_{Nr+Mr}/S^{r^*}_{Nr+Mr}\right) - \left(L^r_{Nr+Mr}/s^r_{Nr+Mr}\right)\right)}{24 \times 365}, \quad \forall r \in \Phi, \tag{1}$$

where $E^r_{Nr}$ represents the increasing revenue on the long leg $Nr$ and $E^r_{Nr+Mr}$ represents the increasing revenue on the long leg $Nr + Mr$.

The inventory (holding) cost per unit is a linear function of time in storage [27]. Accordingly, the inventory cost per unit of container cargoes in liner shipping is a linear function of shipping time. A shorter shipping time for container cargoes can save inventory costs for shippers. Thus, these shippers will be willing to pay higher freight rates to container shipping lines if their cargoes can be transported quicker. According to the study of Wang et al. [8], the time-sensitive freight rate for spot market customers can be modeled as a linear function on the inventory cost savings (increased revenue). Time-sensitive freight rate is expressed linearly by the inventory cost savings because inventory cost is a linear function of time. As a result, the time-sensitive freight rate can be obtained by adding a certain proportion of increased revenue into the initial freight rate. Then, the TDFRS can be formulated on the basis of the freight rate for spot market customers as follows:

$$y^r_{ij} = Y^r_{ij} + \phi E^r_{Nr}, \quad \forall r \in \Phi, i \in A^r, j \in B^r,$$
$$y^r_{ij} = Y^r_{ij} + \phi E^r_{Nr+Mr}, \quad \forall r \in \Phi, i \in B^r, j \in A^r. \tag{2}$$

It should be pointed out that when $\phi$ equals to 1, it is also reasonable. It means that the container shipping liner gains all the revenue generated by the transit time savings. Even so, cargo owners are still likely to seize the market owing to shorter transit time and, as a result, improve the market competitiveness.

The second impact is that it influences the number of deployed containerships, which is directly related to the voyage time on each liner route. The voyage time consists of the port calling time and the sailing time. To maintain a weekly service frequency, the following constraint should be satisfied:

$$\sum_{i=1}^{Nr-1}\frac{L_i^r}{S_i^{r^*}}+\frac{L_{Nr}^r}{s_{Nr}^r}+\sum_{i=Nr+1}^{Nr+Mr-1}\frac{L_i^r}{S_i^{r^*}}+\frac{L_{Nr+Mr}^r}{s_{Nr+Mr}^r}+\sum_{i=1}^{Nr+Mr}T_i^r\le 168m_v^r,\quad \forall r\in\Phi. \tag{3}$$

The third impact is that it affects bunker fuel cost according to the relationship between bunker fuel consumption and sailing speed [28]. Let $f(S_i^{r^*})$ be the daily bunker fuel consumption at the optimal sailing speed $S_i^{r^*}$ on the short leg $i$ on route $r$. Then, the daily bunker fuel consumption with respect to the optimal sailing speed can be calculated by

$$f\left(S_i^{r^*}\right)=\left(\frac{S_i^{r^*}}{S_D^r}\right)^3F_D^r,\quad \forall r\in\Phi, i\in\{1,2,\ldots,Nr-1\}\cup\{Nr+1,Nr+2,\ldots,Nr+Mr-1\}. \tag{4}$$

Similarly, the daily bunker fuel consumption on the long leg $Nr$ on route $r$ in the head-haul direction can be calculated by

$$f\left(s_{Nr}^r\right)=\left(\frac{s_{Nr}^r}{S_D^r}\right)^3F_D^r,\quad \forall r\in\Phi. \tag{5}$$

The daily bunker fuel consumption on the long leg $Nr+Mr$ of route $r$ in the back-haul direction can be calculated by

$$f\left(s_{Nr+Mr}^r\right)=\left(\frac{s_{Nr+Mr}^r}{S_D^r}\right)^3F_D^r,\quad \forall r\in\Phi. \tag{6}$$

Thus, the bunker fuel cost on each voyage $F^r$ on route $r$ can be calculated by

$$F^r=PF_D^r\left(S_D^r\right)^{-3}\frac{\left[\sum_{i=1}^{Nr-1}L_i^r\left(S_i^{r^*}\right)^2+L_{Nr}^r\left(s_{Nr}^r\right)^2+\sum_{i=Nr+1}^{Nr+Mr}L_i^r\left(S_i^{r^*}\right)^2+L_{Nr+Mr}^r\left(s_{Nr+Mr}^r\right)^2\right]}{24},\quad \forall r\in\Phi. \tag{7}$$

*3.3. Model Formulation.* Based on the consideration of the time value of container cargo, the SSFD problem under the TDFRS can be formulated as the following mixed-integer nonlinear programming model [M1]:

$$\begin{aligned}[M1]\max Z=&\sum_{r=1}^{R}\sum_{i=1}^{Nr}\sum_{j=Nr+1}^{Nr+Mr}\left[\gamma_{ij}^rD_{ij}^ry_{ij}^r+\alpha_{ij}^r\left(1-\gamma_{ij}^r\right)D_{ij}^rY_{ij}^r\right]+\sum_{r=1}^{R}\sum_{i=Nr+1}^{Nr+Mr}\sum_{j=1}^{Nr}\left[\gamma_{ij}^rD_{ij}^ry_{ij}^r+\alpha_{ij}^r\left(1-\gamma_{ij}^r\right)D_{ij}^rY_{ij}^r\right]\\&-\sum_{r=1}^{R}PF_D^r\left(S_D^r\right)^{-3}\frac{\left[\sum_{i=1}^{Nr-1}L_i^r\left(S_i^{r^*}\right)^2+L_{Nr}^r\left(s_{Nr}^r\right)^2+\sum_{i=Nr+1}^{Nr+Mr}L_i^r\left(S_i^{r^*}\right)^2+L_{Nr+Mr}^r\left(s_{Nr+Mr}^r\right)^2\right]}{24}-\sum_{r=1}^{R}C^rm_v^r,\end{aligned} \tag{8}$$

s.t.

$$E_{Nr}^r=\frac{U^r\rho\left(\left(L_{Nr}^r/S_{Nr}^{r^*}\right)-\left(L_{Nr}^r/s_{Nr}^r\right)\right)}{24\times 365},\quad \forall r\in\Phi,$$
$$E_{Nr+Mr}^r=\frac{U^r\rho\left(\left(L_{Nr+Mr}^r/S_{Nr+Mr}^{r^*}\right)-\left(L_{Nr+Mr}^r/s_{Nr+Mr}^r\right)\right)}{24\times 365},\quad \forall r\in\Phi, \tag{9}$$

$$\begin{aligned}y_{ij}^r&=Y_{ij}^r+\phi E_{Nr}^r,\quad \forall r\in\Phi, i\in A^r, j\in B^r,\\ y_{ij}^r&=Y_{ij}^r+\phi E_{Nr+Mr}^r,\quad \forall r\in\Phi, i\in B^r, j\in A^r,\end{aligned} \tag{10}$$

$$\sum_{i=1}^{Nr-1}\frac{L_i^r}{S_i^{r^*}}+\frac{L_{Nr}^r}{s_{Nr}^r}+\sum_{i=Nr+1}^{Nr+Mr-1}\frac{L_i^r}{S_i^{r^*}}+\frac{L_{Nr+Mr}^r}{s_{Nr+Mr}^r}+\sum_{i=1}^{Nr+Mr}T_i^r\le 168m_v^r,\quad \forall r\in\Phi, v\in\Omega, \tag{11}$$

$$\sum_{k=i}^{Nr-1}\frac{L_k^r}{S_k^{r^*}}+\frac{L_{Nr}^r}{s_{Nr}^r}+\sum_{k=Nr+1}^{j-1}\frac{L_k^r}{S_k^{r^*}}+\sum_{k=i}^{j}T_k^r\le T_{ij}^r,\quad \forall r\in\Phi, i\in A^r, j\in B^r, \tag{12}$$

$$\sum_{k=i}^{Nr+Mr-1}\frac{L_k^r}{S_k^{r^*}}+\frac{L_{Nr+Mr}^r}{s_{Nr+Mr}^r}+\sum_{k=1}^{j-1}\frac{L_k^r}{S_k^{r^*}}+\sum_{k=i}^{Nr+Mr}T_k^r+\sum_{k=1}^{j}T_k^r\le T_{ij}^r,\quad \forall r\in\Phi, i\in B^r, j\in A^r, \tag{13}$$

$$\sum_{r=1}^{R} m_v^r \le N_v, \quad \forall v \in \Omega, \tag{14}$$

$$S_{\min}^r \le s_{Nr}^r \le S_{\max}^r, \quad \forall r \in \Phi, \tag{15}$$

$$S_{\min}^r \le s_{Nr+Mr}^r \le S_{\max}^r, \quad \forall r \in \Phi, \tag{16}$$

$$\begin{aligned} & m_v^r \in Z^+, \\ & \forall r \in \Phi, \\ & \forall v \in \Omega. \end{aligned} \tag{17}$$

Objective function (8) expresses the maximization of the total profit. The sum of the first term and the second term expresses the total freight revenue of every voyage. The third term expresses the bunker fuel cost of every voyage. The fourth term expresses the containership cost of every voyage. The sum of the third term and the fourth term are the total operating cost paid by the container shipping liner. Constraint (9) represents the relationship between the inventory cost saving and the reoptimized sailing speed on long legs. Constraint (10) represents the relationship between time-sensitive freight rate and the initial freight rate for spot market customers. Constraint (11) indicates that the weekly service frequency should be maintained. Constraints (12) and (13) enforce the transit time for containerships from port $i$ to port $j$ on route $r$ to be no larger than a predetermined acceptable value. Constraint (14) ensures that the total number of containerships of each type not to exceed the limitation. Constraints (15) and (16) enforce the lower bound and the upper bound to the containership sailing speed, respectively. Constraint (17) requires the number of deployed containerships to be a positive integer variable.

## 4. Solution Algorithm

Model [M1] is a mixed-integer nonlinear programming model with nonlinear terms in its objective function (8) and constraints (11)–(13). Moreover, constraints (11)–(13) are nonconvex. The optimization problem of SSFD under TDFRS in our manuscript is an extension of study [22], which is NP-hard. Therefore, it is very difficult to solve model [M1] directly. An efficient and exact algorithm needs to be designed to find the solution to this problem.

It should be pointed out that one of the most important pieces of model [M1] is the optimal sailing speeds $S_i^{r^*}$ on leg $i$ on liner route $r$ without considering the time value of container cargo. The optimal sailing speed $S_i^{r^*}$ on each leg of the liner route can be obtained through model [M1] when $\phi$ equals to 0. This means the optimal sailing speed $S_i^{r^*}$ can be determined before optimizing the decision variable $S_i^r$(see [10]). Then, the sailing speed is reoptimized on the two long legs of each liner route under TDFRS.

In order to get the optimal solution, a discretization algorithm is designed to transform model [M1] into a mixed-integer one. Then, it can be solved by the linear optimization solvers. The steps of the discretization algorithm are described as follows:

Step 1: without the consideration of the time value of the container cargo, a new mathematical programming is formulated based on a fixed freight rate strategy (FFRS). That is, freight rates for spot market customers between port pairs do not change as sailing speed changes. Therefore, the sailing speeds $S_i^r$ on leg $i$ on route $r$ are treated as decision variables, and model [M1] can be simplified as model [M2]:

$$\begin{aligned} \text{[M2]}\max Z = & \sum_{r=1}^{R}\sum_{i=1}^{Nr}\sum_{j=Nr+1}^{Nr+Mr}\left[\gamma_{ij}^r D_{ij}^r Y_{ij}^r + \alpha_{ij}^r\left(1-\gamma_{ij}^r\right)D_{ij}^r Y_{ij}^r\right] \\ & + \sum_{r=1}^{R}\sum_{i=Nr+1}^{Nr+Mr}\sum_{j=1}^{Nr}\left[\gamma_{ij}^r D_{ij}^r Y_{ij}^r + \alpha_{ij}^r\left(1-\gamma_{ij}^r\right)D_{ij}^r Y_{ij}^r\right] \\ & - \frac{\sum_{r=1}^{R}\sum_{i=1}^{Nr+Mr}\left[PF_D^r\left(S_D^r\right)^{-3}L_i^r\left(S_i^r\right)^2\right]}{24} \\ & - \sum_{r=1}^{R} C^r m_v^r, \end{aligned} \tag{18}$$

s.t.

$$\sum_{i=1}^{Nr+Mr}\frac{L_i^r}{S_i^r} + \sum_{i=1}^{Nr+Mr} T_i^r \le 168 m_v^r, \quad \forall r \in \Phi,\ v \in \Omega, \tag{19}$$

$$\sum_{k=i}^{j-1}\frac{L_k^r}{S_k^r} + \sum_{k=i}^{j} T_k^r \le T_{ij}^r, \quad \forall r \in \Phi,\ i \in A^r,\ j \in B^r, \tag{20}$$

$$\begin{aligned} & \sum_{k=i}^{Nr+Mr}\frac{L_k^r}{S_k^r} + \sum_{k=1}^{j-1}\frac{L_k^r}{S_k^r} + \sum_{k=i}^{Nr+Mr} T_k^r + \sum_{k=1}^{j} T_k^r \le T_{ij}^r, \\ & \qquad \forall r \in \Phi,\ i \in B^r,\ j \in A^r, \end{aligned} \tag{21}$$

$$\begin{aligned} & m_v^r \in Z^+, \\ & \forall r \in \Phi, \\ & \forall v \in \Omega, \end{aligned} \tag{22}$$

$$S_{\min}^r \le S_i^r \le S_{\max}^r, \quad \forall r \in \Phi,\ i \in \{1, 2, \ldots, Nr + Mr\}, \tag{23}$$

$$\sum_{r=1}^{R} m_v^r \le N_v, \quad \forall v \in \Omega. \tag{24}$$

Step 2: linearize constraints (19)–(21) of model [M2] by defining the reciprocal of sailing speeds as new decision variables. Constraints (19)–(21) contain the reciprocal of the sailing speeds $S_i^r$, which is one of the factors causing the nonlinearity of model [M2]. To linearize these constraints, new decision variables are defined: $W_i^r = 1/S_i^r$, $W_{\min}^r = 1/S_{\max}^r$, and $W_{\max}^r = 1/S_{\min}^r$, $\forall r \in R$ and $i \in \{1, 2, \ldots, Nr + Mr\}$. Therefore, model [M2] can be reformulated as model [M3] as follows:

$$\begin{aligned}[\mathrm{M3}]\max Z = &\sum_{r=1}^{R}\sum_{i=1}^{Nr}\sum_{j=Nr+1}^{Nr+Mr}\left[\gamma_{ij}^{r}D_{ij}^{r}Y_{ij}^{r}+\alpha_{ij}^{r}\left(1-\gamma_{ij}^{r}\right)D_{ij}^{r}Y_{ij}^{r}\right]\\ &+\sum_{r=1}^{R}\sum_{i=Nr+1}^{Nr+Mr}\sum_{j=1}^{Nr}\left[\gamma_{ij}^{r}D_{ij}^{r}Y_{ij}^{r}+\alpha_{ij}^{r}\left(1-\gamma_{ij}^{r}\right)D_{ij}^{r}Y_{ij}^{r}\right]\\ &-\frac{\sum_{r=1}^{R}\sum_{i=1}^{Nr+Mr}PF_{D}^{r}\left(S_{D}^{r}\right)^{-3}L_{i}^{r}\left(W_{i}^{r}\right)^{-2}}{24}\\ &-\sum_{r=1}^{R}C^{r}m_{v}^{r},\end{aligned} \tag{25}$$

s.t.

$$\sum_{i=1}^{Nr+Mr}L_{i}^{r}W_{i}^{r}+\sum_{i=1}^{Nr+Mr}T_{i}^{r}\le 168m_{v}^{r},\quad \forall r\in\Phi,\ v\in\Omega, \tag{26}$$

$$\sum_{k=i}^{j-1}L_{k}^{r}W_{k}^{r}+\sum_{k=i}^{j}T_{k}^{r}\le T_{ij}^{r},\quad \forall r\in\Phi,\ i\in A^{r},\ j\in B^{r}, \tag{27}$$

$$\begin{aligned}&\sum_{k=i}^{Nr+Mr}L_{k}^{r}W_{k}^{r}+\sum_{k=1}^{j-1}L_{k}^{r}W_{k}^{r}+\sum_{k=i}^{Nr+Mr}T_{k}^{r}+\sum_{k=1}^{j}T_{k}^{r}\le T_{ij}^{r},\\ &\qquad \forall r\in\Phi,\ i\in B^{r},\ j\in A^{r},\end{aligned} \tag{28}$$

$$\begin{aligned}W_{i}^{r}&=\frac{1}{S_{i}^{r}},\\ W_{\min}^{r}&=\frac{1}{S_{\max}^{r}},\\ W_{\max}^{r}&=\frac{1}{S_{\min}^{r}},\\ &\forall r\in R,\ i\in\{1,2,\ldots,Nr+Mr\},\end{aligned} \tag{29}$$

$$\begin{aligned}&m_{v}^{r}\in Z^{+},\\ &\forall r\in\Phi,\\ &\forall v\in\Omega,\end{aligned} \tag{30}$$

$$W_{\min}^{r}\le W_{i}^{r}\le W_{\max}^{r},\quad \forall i\in\{1,2,\ldots,Nr+Mr\}, \tag{31}$$

$$\sum_{r=1}^{R}m_{v}^{r}\le N_{v},\quad \forall v\in\Omega. \tag{32}$$

All the constraints of model [M3] become linear constraints. Since the objective function (25) contains nonlinear terms, model [M3] is still a mixed-integer nonlinear programming. In the following, the solution efficiency of model [M3] is improved by taking the advantage of the special structure of model [M3] and the convexity of objective function (25).

**Theorem 1.** *The objective function (25) of model [M3] is convex in* $W_i^r$.

*Proof.* Let

$$h_{i}^{r}\left(W_{i}^{r}\right)=L_{i}^{r}W_{i}^{r-2}. \tag{33}$$

Since $h_i^r(W_i^r)$ is a differentiable function with respect to $W_i^r$ within the definition domain $[W_{\min}^r, W_{\max}^r]$, equations (34) and (35), respectively, express the first-order and the second-order derivative of $W_i^r$ with respect to $W_i^r$:

$$\frac{\mathrm{d}h_{i}^{r}\left(W_{i}^{r}\right)}{\mathrm{d}W_{i}^{r}}=-2L_{i}^{r}W_{i}^{r-3}, \tag{34}$$

$$\frac{\mathrm{d}^{2}h_{i}^{r}\left(W_{i}^{r}\right)}{\mathrm{d}W_{i}^{r2}}=6L_{i}^{r}W_{i}^{r-4}. \tag{35}$$

According to equation (35), the second derivative of function $h_i^r(W_i^r)$ is always greater than zero within the definition domain $[W_{\min}^r, W_{\max}^r]$. That is,

$$\frac{\mathrm{d}^{2}h_{i}^{r}\left(W_{i}^{r}\right)}{\mathrm{d}W_{i}^{r2}}>0. \tag{36}$$

Therefore, $h_i^r(W_i^r)$ is a convex function with respect to the new decision variables $W_i^r$ on each leg on each route. *According to the superposition of the convex function, it can be concluded that the objection function* (25) *of model [M3] is convex in* $W_i^r$. According to Theorem 1, the optimal value of $W_i^r$ can be efficiently obtained by the state-of-the-art mixed-integer linear programming solvers such as Gurobi.

Step 3: discretize the definition domain $[W_{\min}^r, W_{\max}^r]$ of decision variables $W_i^r$ in model [M3].

Considering the fact that the sailing speed is usually taken to the decimal point after the unit of knots, we define

$$Qr=\frac{\left(S_{\max}^{r}-S_{\min}^{r}\right)}{0.1}. \tag{37}$$

Therefore, the feasible domain of sailing speed can be discretely divided into $Qr+1\,(r\in\Phi)$ cells with equal intervals as follows:

$$S_{qr}^{r}=S_{\min}^{r}+0.1qr,\quad \forall r\in\Phi,\ qr\in\{0,1,\ldots,Qr\}. \tag{38}$$

Correspondingly, the definition domain of $W_{qr}^r$ are also divided into $Qr+1$ cells. That is,

$$W_{qr}^{r}=\frac{1}{S_{qr}^{r}},\quad \forall r\in R,\ qr\in\{0,1,\ldots,Qr\}. \tag{39}$$

Note that $W_{\min}^r = 1/S_{\max}^r, W_{\max}^r = 1/S_{\min}^r$, and the division of $W_i^r$ is shown in Figure 1.

The optimal sailing speed $S_i^{r^*}$ on leg $i$ on route $r$ can only be selected from the $Qr+1$ interval discrete values of $S_{qr}^r\,(r\in\Phi, qr\in\{0,1,\ldots,Qr\})$. Correspondingly, the optimal value of each new decision variable $W_i^r$ can only be selected in the set $\left\{W_{qr}^r, r\in\Phi, qr\in\{0,1,\ldots,Qr\}\right\}$. To indicate which value to adopt, a new binary variable on each leg on each liner route is defined as follows:

$$\beta_{i,qr}^{r}=\begin{cases}1, & W_{i}^{r}=W_{qr}^{r}\\ 0, & W_{i}^{r}\ne W_{qr}^{r},\end{cases}\quad \forall qr\in\{0,1,\ldots,Qr\}. \tag{40}$$

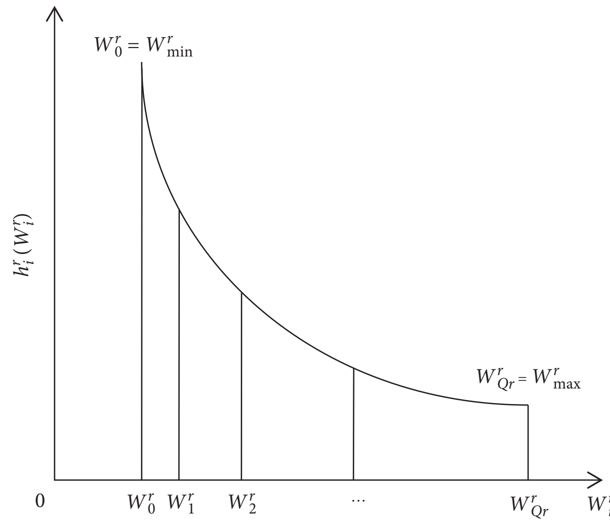


Figure 1: Discretization of the new decision variables.

Thus,

$$W_i^r = \sum_{qr=0}^{Qr} \beta_{i,qr}^r W_{qr}^r. \tag{41}$$

Then, model [M3] is equivalent to the following model [M4]:

$$\begin{aligned}
\text{[M4]}\max Z = {} & \sum_{r=1}^{R}\sum_{i=1}^{Nr}\sum_{j=Nr+1}^{Nr+Mr}\left[\gamma_{ij}^r D_{ij}^r y_{ij}^r + \alpha_{ij}^r\left(1-\gamma_{ij}^r\right) D_{ij}^r Y_{ij}^r\right] + \sum_{r=1}^{R}\sum_{i=Nr+1}^{Nr+Mr}\sum_{j=1}^{Nr}\left[\gamma_{ij}^r D_{ij}^r y_{ij}^r + \alpha_{ij}^r\left(1-\gamma_{ij}^r\right) D_{ij}^r Y_{ij}^r\right] \\
& - \frac{\sum_{r=1}^{R}\sum_{i=1}^{Nr+Mr}\left[PF_D^r\left(S_D^r\right)^{-3} L_i^r \sum_{qr=0}^{Qr}\beta_{i,qr}^r\left(W_{qr}^r\right)^{-2}\right]}{24} - \sum_{r=1}^{R} C^r m_v^r,
\end{aligned} \tag{42}$$

s.t.

$$\begin{aligned}
& \sum_{i=1}^{Nr+Mr} L_i^r \sum_{qr=0}^{Qr}\beta_{i,qr}^r W_{qr}^r + \sum_{i=1}^{Nr+Mr} T_i^r \le 168 m_v^r, \quad \forall r \in \Phi,\ v \in \Omega, \\
& \sum_{k=i}^{j-1} L_k^r \sum_{qr=0}^{Qr}\beta_{k,qr}^r W_{qr}^r + \sum_{k=i}^{j} T_k^r \le T_{ij}^r, \quad \forall r \in \Phi,\ i \in A^r,\ j \in B^r, \\
& \sum_{k=i}^{Nr+Mr} L_k^r \sum_{qr=0}^{Qr}\beta_{k,qr}^r W_{qr}^r + \sum_{k=1}^{j-1} L_k^r \sum_{qr=0}^{Qr}\beta_{k,qr}^r W_{qr}^r + \sum_{k=i}^{Nr+Mr} T_k^r + \sum_{k=1}^{j} T_k^r \le T_{ij}^r, \quad \forall r \in \Phi,\ i \in B^r,\ j \in A^r, \\
& W_{\min}^r \le \sum_{qr=0}^{Qr}\beta_{i,qr}^r W_{qr}^r \le W_{\max}^r, \quad \forall r \in \Phi,\ i \in \{1,2,\ldots,Nr+Mr\}, \\
& \sum_{r=1}^{R} m_v^r \le N_v, \quad \forall v \in \Omega, \\
& \sum_{qr=0}^{Qr}\beta_{i,qr}^r = 1, \quad \forall r \in \Phi,\ i \in \{1,2,\ldots,Nr+Mr\}, \\
& \beta_{i,qr}^r \in \{0,1\}, \quad \forall r \in \Phi,\ i \in \{1,2,\ldots,Nr+Mr\},\ qr \in \{0,1,\ldots,Qr\}, \\
& m_v^r \in Z^+, \\
& \forall r \in \Phi, \\
& \forall v \in \Omega.
\end{aligned} \tag{43}$$

Step 4: obtain the optimal sailing speed $S_i^{r^*}$ on leg $i$ on route $r$ without considering the time value of the container cargo. Model [M4] is an integer linear programming containing only binary variables $\beta_{i,qr}^r (r \in \Phi, \quad i \in \{1, 2, \ldots, Nr + Mr\}$ and $qr \in \{0, 1, \ldots, Qr\})$, and integer variables $m_v^r$. Model [M4] is a mixed-integer linear programming model which is standard and complete. Besides, the scale of decision variables has been reduced a lot because the optimal sailing speed $S_i^{r^*}$ on leg $i$ on route $r$ of is discretized into a finite number of values. Since Gurobi is the fastest and most powerful mathematical programming solver available for MIP problems [29]. The optimal value $W_i^{r^*}$ of model [M4] can be efficiently obtained by optimization solvers, such as Gurobi. Then, the optimal value of $S_i^{r^*}$ can be obtained.

Step 5: considering the time value of the container cargo, the mathematical programming is formulated under the TDFRS. Then, this formulated programming can be transformed into an integer linear programming following the methods of variable substitution and piecewise linearization in Steps 2 and 3. The transformation process is shown in the appendix:

$$
\begin{aligned}
[M5]\max Z = {} & \sum_{r=1}^{R}\sum_{i=1}^{Nr}\sum_{j=Nr+1}^{Nr+Mr}\left\{\gamma_{ij}^r D_{ij}^r\left[Y_{ij}^r + \frac{\phi U^r \rho\left(\left(L_{Nr}^r/S_{Nr}^{r^*}\right) - L_{Nr}^r\sum_{qr=0}^{Qr}\beta_{Nr,qr}^r W_{qr}^r\right)}{24\times 365}\right] + \alpha_{ij}^r\left(1-\gamma_{ij}^r\right)D_{ij}^r Y_{ij}^r\right\} \\
& + \sum_{r=1}^{R}\sum_{i=Nr+1}^{Nr+Mr}\sum_{j=1}^{Nr}\left\{\gamma_{ij}^r D_{ij}^r\left[Y_{ij}^r + \frac{\phi U^r \rho\left(\left(L_{Nr+Mr}^r/S_{Nr+Mr}^{r^*}\right) - L_{Nr+Mr}^r\sum_{qr=0}^{Qr}\beta_{Nr+Mr,qr}^r W_{qr}^r\right)}{24\times 365}\right] + \alpha_{ij}^r\left(1-\gamma_{ij}^r\right)D_{ij}^r Y_{ij}^r\right\} \\
& - \sum_{r=1}^{R} PF_D^r\left(S_D^r\right)^{-3}\frac{\left[\sum_{i=1}^{Nr-1}L_i^r S_i^{r^*2} + \sum_{i=Nr+1}^{Nr+Mr-1}L_i^r S_i^{r^*2} + L_{Nr}^r\sum_{qr=0}^{Qr}\beta_{Nr,qr}^r\left(W_{qr}^r\right)^{-2} + L_{Nr+Mr}^r\sum_{qr=0}^{Qr}\beta_{Nr+Mr,qr}^r\left(W_{qr}^r\right)^{-2}\right]}{24} \\
& - \sum_{r=1}^{R} C^r m_v^r,
\end{aligned}
\tag{44}
$$

s.t.

$$\sum_{i=1}^{Nr-1}\frac{L_i^r}{S_{ri}^*} + \sum_{i=Nr+1}^{Nr+Mr-1}\frac{L_i^r}{S_{ri}^*} + L_{Nr}^r\sum_{qr=0}^{Qr}\beta_{Nr,qr}^r W_{qr}^r + L_{Nr+Mr}^r\sum_{qr=0}^{Qr}\beta_{Nr+Mr,qr}^r W_{qr}^r + \sum_{i=1}^{Nr+Mr}T_i^r \le 168 m_v^r, \quad \forall r\in\Phi,\ v\in\Omega, \tag{45}$$

$$\sum_{k=i}^{Nr-1}\frac{L_k^r}{S_k^{r^*}} + \sum_{k=Nr+1}^{j-1}\frac{L_k^r}{S_k^{r^*}} + L_{Nr}^r\sum_{qr=0}^{Qr}\beta_{Nr,qr}^r W_{qr}^r + \sum_{k=i}^{j}T_k^r \le T_{ij}^r, \quad \forall r\in\Phi,\ i\in A^r,\ j\in B^r, \tag{46}$$

$$\sum_{k=i}^{Nr+Mr-1}\frac{L_k^r}{S_k^{r^*}} + L_{Nr+Mr}^r\sum_{qr=0}^{Qr}\beta_{Nr+Mr,qr}^r W_{qr}^r + \sum_{k=1}^{j-1}\frac{L_k^r}{S_k^{r^*}} + \sum_{k=i}^{Nr+Mr}T_k^r + \sum_{k=1}^{j}T_k^r \le T_{ij}^r, \quad \forall r\in\Phi,\ i\in B^r,\ j\in A^r, \tag{47}$$

$$W_{\min}^r \le \sum_{qr=0}^{Qr}\beta_{Nr,qr}^r W_{qr}^r \le W_{\max}^r, \quad \forall r\in\Phi, \tag{48}$$

$$W_{\min}^r \le \sum_{qr=0}^{Qr}\beta_{Nr+Mr,qr}^r W_{qr}^r \le W_{\max}^r, \quad \forall r\in\Phi, \tag{49}$$

$$\sum_{r=1}^{R} m_v^r \le N_v, \quad \forall v\in\Omega, \tag{50}$$

$$\sum_{qr=0}^{Qr}\beta_{Nr,qr}^r = 1, \quad \forall r\in\Phi, \tag{51}$$

$$\sum_{qr=0}^{Qr} \beta^r_{Nr+Mr,qr} = 1, \quad \forall r \in \Phi, \tag{52}$$

$$\beta^r_{Nr,qr} \in \{0,1\}, \quad \forall r \in \Phi,\ qr \in \{0,1,\ldots,Qr\}, \tag{53}$$

$$\beta^r_{Nr+Mr,qr} \in \{0,1\}, \quad \forall r \in \Phi,\ qr \in \{0,1,\ldots,Qr\}, \tag{54}$$

$$\begin{aligned} &m^r_v \in Z^+, \\ &\forall r \in \Phi, \\ &\forall v \in \Omega. \end{aligned} \tag{55}$$

Constraints (51) and (52) ensure that only a single sailing speed value is adopted on each of the long legs on each liner route, respectively.

Step 6: reoptimize the optimal sailing speed on long legas ($s^r_{Nr}$ and $s^r_{Nr+Mr}$) on liner route $r$ considering the time value of the container cargo. As mentioned in Section 2, the sailing speed of a containership on short legs does not need to be reoptimized. Model [M5] is an integer linear programming containing only binary variables $\beta^r_{Nr,qr}$ and $\beta^r_{Nr+Mr,qr}$ $(r \in \Phi, qr \in \{0,1,\ldots,Qr\})$ and integer variables $m^r_v$. The optimal value of $w^{r^*}_{Nr}$ and $w^{r^*}_{Nr+Mr}$ as well as the number of deployed containerships $m^{r^*}_v$ can be obtained by solving model [M5] using optimization solvers such as Gurobi. Then, the optimal sailing speed $s^{r^*}_{Nr} = 1/w^{r^*}_{Nr}$ in the head-haul direction and the optimal sailing speed $s^{r^*}_{Nr+Mr} = 1/w^{r^*}_{Nr+Mr}$ in the back-haul direction are obtained.

To summarize, the optimization results of the sailing speeds on each leg on each liner route and the number of deployed containerships under the FFRS can be obtained through Steps 1 to 4, and the optimization results under the TDFRS can be obtained through Steps 1 to 6.

## 5. Numerical Examples

*5.1. Parameter Settings.* To evaluate the applicability of the proposed model and the efficiency of the designed algorithm, a real-case example provided by COSCO Shipping Liner Co., Ltd is used in this experiment. In this example, the interested intercontinental liner network consists of four Asia-Southwest America liner routes, as shown in Table 1. For clear illustration, the calling ports on each liner route are numbered according to their sequences in Table 1. Take route 1 as an example. The calling port sequence is Lianyungang ⟶ Shanghai ⟶ Ningbo ⟶ Long Beach ⟶ Seattle ⟶ Lianyungang and is expressed as 1-2-3-4-5-1. The first three ports are located in Asia (denoted by set $A^1$) and the rest two ports are located in Southwest America (denoted by set $B^1$). The port calling time of each port on each liner route is shown in Table 2. Table 3 demonstrates the oceanic distance of each leg on each liner route.

There are two types of containerships: 10060 TEUs and 8 400 TEUs, as shown in Table 4. In addition, the maximum allowable port-to-port transit time for containerships is calculated according to the realistic operating data provided by the global container shipping liner. However, some data is unavailable due to commercial confidentiality, including the data on freight rates for spot market customers and the data on average container demand between port pairs of each liner route. Thus, the two parameters are determined in a reasonable manner. Since the data of the freight rates for spot market customers and port-to-port average container demand is too much, they are not listed in this paper.

The other parameters required by the proposed model are set as follows. The bunker fuel price is 500 USD per ton according to the bunker market situation. The coefficient of the time value of container cargo is 8%, and the unit container cargo value on each liner route is set to be 225 000 USD [8]. The container demand ratios of spot market customers between port pairs on each liner route are all set to be 60% and freight rate discounts for long contracted customers are all set to be 90%.

After the above required parameters are determined, the mixed-integer nonlinear programming proposed in Section 3 is transformed into an integer linear programming using the piecewise linearization algorithm presented in Section 4. The integer linear programming is solved by Gurobi 6.5. This algorithm is implemented by coding in AMPL.

*5.2. Result Analysis.* The optimization results under different freight rate strategies (FFRS and TDFRS) and different revenue sharing rates ($\phi = 0.5$ and $\phi = 1$) are shown in Table 5.

It can be seen from Table 5 that different freight rate strategies lead to different results obtained by the proposed optimization model. When the TDFRS is adopted, different revenue sharing rates lead to different optimization results.

Table 2: Port calling time of each port on each liner route.

| Routes | Port calling time (hr) | | | | | |
|---|---|---|---|---|---|---|
| | Port 1 | Port 2 | Port 3 | Port 4 | Port 5 | Port 6 |
| 1 | 20 | 48 | 20 | 83.5 | 35 | — |
| 2 | 8 | 28 | 36 | 36 | 72 | 35.5 |
| 3 | 22 | 22 | 20 | 61 | 24 | — |
| 4 | 16 | 18 | 12 | 29 | 46.5 | 22.5 |

Table 3: Oceanic distance of each leg on each liner route.

| Routes | Distance (nautical miles) | | | | | |
|---|---|---|---|---|---|---|
| | Leg 1 | Leg 2 | Leg 3 | Leg 4 | Leg 5 | Leg 6 |
| 1 | 356 | 235 | 5 761 | 1 148 | 5 122 | — |
| 2 | 1 110 | 375 | 235 | 5 758 | 369 | 4 560 |
| 3 | 375 | 235 | 5 758 | 369 | 5 413 | — |
| 4 | 197 | 327 | 85 | 6 356 | 369 | 5 635 |

Table 4: Containerships.

| Containership type | Capacity (TEUs) | Total owned containerships | Containership cost (USD/week) | Minimum speed (knots) | Maximum speed (knots) | Designed speed (knots) | Fuel consumption at designed speed (tons/day) |
|---|---|---|---|---|---|---|---|
| 1 | 10 060 | 12 | 269 500 | 18.0 | 28.0 | 23.0 | 222.9 |
| 2 | 8 400 | 13 | 245 000 | 18.0 | 28.0 | 22.5 | 208.4 |

Table 5: Results obtained by the proposed optimization model under FFRS and TDFRS.

| Freight rate strategy | | Routes | Sailing speed (knots) | | | | | | Number of containerships | Voyage time (hr) | Profit of each route (USD) | Total profit (USD) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Leg 1 | Leg 2 | Leg 3 | Leg 4 | Leg 5 | Leg 6 | | | | |
| FFRS | | 1 | 19.8 | 19.9 | *20.0** | 19.7 | *19.9** | — | 5 | 840 | 10 949 979 | 49 934 590 |
| | | 2 | 19.7 | 19.5 | 19.6 | *20.0** | 19.7 | *19.8** | 5 | 840 | 10 161 789 | |
| | | 3 | 18.0 | 19.5 | *19.2** | 18.0 | *18.0** | — | 5 | 803 | 11 209 699 | |
| | | 4 | 18.2 | 18.2 | 18.5 | *19.0** | 18.3 | *18.3** | 5 | 840 | 17 613 123 | |
| TDFRS | $\phi = 0.5$ | 1 | 19.8 | 19.9 | *20.2** | 19.7 | *19.7** | — | 5 | 840 | 10 950 651 | 49 948 395 |
| | | 2 | 19.7 | 19.5 | 19.6 | *20.7** | 19.7 | *19.0** | 5 | 840 | 10 162 840 | |
| | | 3 | 18.0 | 19.5 | *19.2** | 18.0 | *18.0** | — | 5 | 803 | 11 209 699 | |
| | | 4 | 18.2 | 18.2 | 18.5 | *19.8** | 18.3 | *19.3** | 5 | 811 | 17 625 205 | |
| | $\phi = 1.0$ | 1 | 19.8 | 19.9 | *24.6** | 19.7 | *23.7** | — | 5 | 745 | 11 202 298 | 51 697 312 |
| | | 2 | 19.7 | 19.5 | 19.6 | *23.4** | 19.7 | *21.8** | 5 | 777 | 10 271 035 | |
| | | 3 | 18.0 | 19.5 | *24.2** | 18.0 | *23.4** | — | 4 | 672 | 11 745 238 | |
| | | 4 | 18.2 | 18.2 | 18.5 | *25.6** | 18.3 | *25.0** | 4 | 671 | 18 478 741 | |

*Note.* * represents the optimal results.

When the revenue sharing rate is 0.5, the number of containerships on the four routes remains constant, which is different from the optimization results under FFRS. The variation of the sailing speed on the two long legs on each route is different. Specifically, the long leg sailing speed in the head-haul direction of routes 1 and 2 increase while those in the back-haul direction decrease. The sailing speed on the two long legs of route 3 remains unchanged, and the sailing speed on the two long legs of route 4 increases. It can be explained that the variations in the sailing speed on long legs affect the sailing time and consequently affect the total freight revenue. Meanwhile, the sailing speed affects the bunker fuel cost. Hence, to determine the optimal sailing speed on the long legs on each liner route, the container shipping liner has to make the trade-off between the total freight revenue and bunker fuel cost. If the average container demand in the head-haul direction (or the back-haul direction) is high, then the added value of the freight revenue is higher than the added value of the bunker fuel cost, and the container shipping liner increases its sailing speed on long legs (such as long legs in the head-haul direction of routes 1 and 2, and the long legs of route 4). If the average container demand in the head-haul direction (or the back-haul direction) is low, then the reduction of the total freight revenue is lower than the added value of bunker fuel cost, and the container shipping liner increases its sailing speed on long legs (such as long legs in the back-haul direction of routes 1 and 2). If the average container demand in the head-haul direction (or the back-haul direction) changes in a certain range, then the total profit is constant when the variations of the sailing speed is ignored, and the containerships remain the original sailing speed on long legs (such as long legs on route 3).

When the revenue sharing rate increases from 0.5 to 1, all the sailing speeds on the long legs of the four routes increase. With regard to the fleet deployment, the number of containerships on routes 1 and 2 remain constant, and the number of containerships on routes 3 and 4 reduce from 5 to 4. The reason is that, although increasing sailing speed on the long legs on each liner route results in the increase of bunker fuel cost, the total freight revenue obtained by the container shipping liner has more increase than the bunker fuel does. Moreover, the increase of sailing speed may lead to the reduction of the number of containerships and consequently reduce the containership cost. As a result, the increase in total freight revenue caused by increased sailing speed is higher than that of total operating cost caused by increased sailing speed. Therefore, the container shipping liner is expected to increase the sailing speed on long legs, and when the sailing speed of some routes increases to a certain value, the number of containerships decreases.

In addition, it can be seen from Table 5 that, compared with the voyage time under FFRS, the voyage time under TDFRS is shorter on the whole. The larger the revenue-sharing rate, the more obvious the shortened amount of the voyage time. This indicates that the TDFRS is beneficial for reducing the transit time of container cargoes and thus improve customer satisfaction.

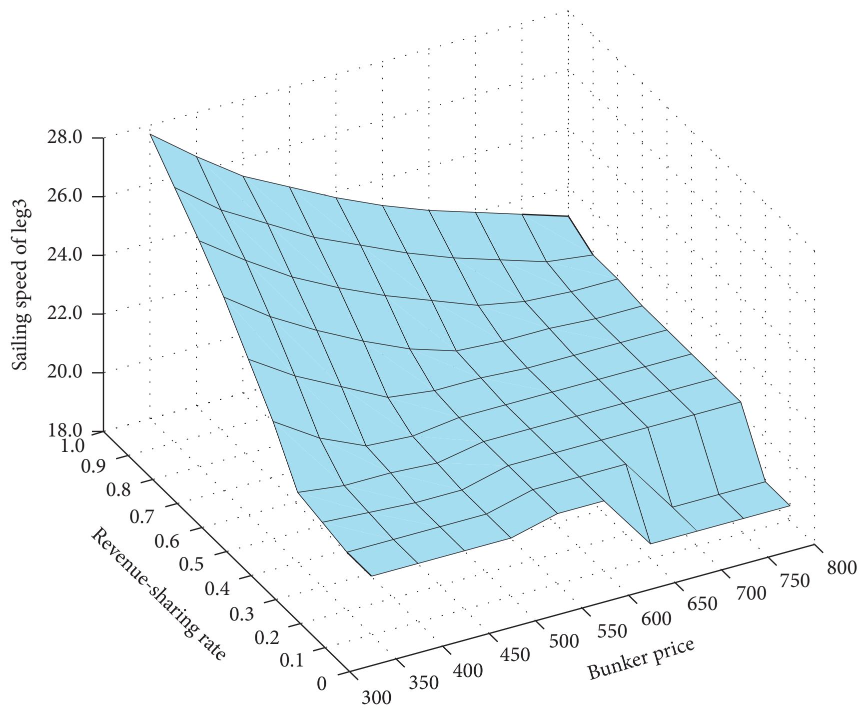
*5.3. Sensitivity Analysis.* Both the revenue-sharing rate and bunker fuel price have significant impact on the total profit achieved by the container shipper liner. To explore the relationship among the two factors and the sailing speed of long legs, the impact of the revenue sharing rate and bunker fuel price on the results obtained by the proposed optimization model under TDFRS is investigated. The bunker fuel price is ranged from 300 $/ton to 800 $/ton at 50 $/ton interval, and the revenue-sharing rate is ranged from 0.1 to 1.0 at a 0.1 interval. Sensitivity analysis results of the long legs' sailing speeds on the four routes are obtained. Considering the space limitations, only the optimal sailing speed on the long legs in the head-haul direction (leg 3) and in the back-haul direction (leg 5) of route 1 are demonstrated in this paper, as shown in Figure 2.

It can be seen from Figure 2 that the sailing speed changes on leg 3 and leg 5 of route 1 are not completely consistent. Specifically, when the revenue-sharing rate is between 0.1 and 0.2, the sailing speed on leg 3 first increases and then declines with the increase of the bunker fuel price. When the revenue-sharing rate is between 0.3 and 0.5, the sailing speed on leg 3 first decreases and then increase to a constant with the increase of the bunker fuel price. When the revenue-sharing rate is between 0.6 and 1.0, the sailing speed on leg 3 decreases with the growth of the bunker fuel price. With regard to the leg 5, when the revenue-sharing rate is between 0.1 and 0.2, the sailing speed first increases and then decreases with the increase of the bunker fuel price. When the revenue-sharing rate is between 0.3 and 0.4, the sailing speed first increases and then remains constant with the increase of the bunker fuel price. When the revenue-sharing rate is between 0.5 and 0.8, the sailing speed first decreases and then increases with the growth of the bunker fuel price. When the revenue sharing rate is between of 0.9–1.0, the sailing speed decreases with the growth of the bunker fuel price. When the bunker fuel price is fixed, the sailing speed of leg 3 increases with the growth of the revenue sharing rate. When the bunker fuel price is between 350 USD/ton and 600 USD/ton, the sailing speed of leg 5 first decreases and then increases with the growth of the revenue sharing rate. When the bunker fuel price is between 650 USD/ton and 800 USD/ton, the sailing speed of leg 5 first increases and then decreases and finally increases with the growth of the revenue sharing rate.

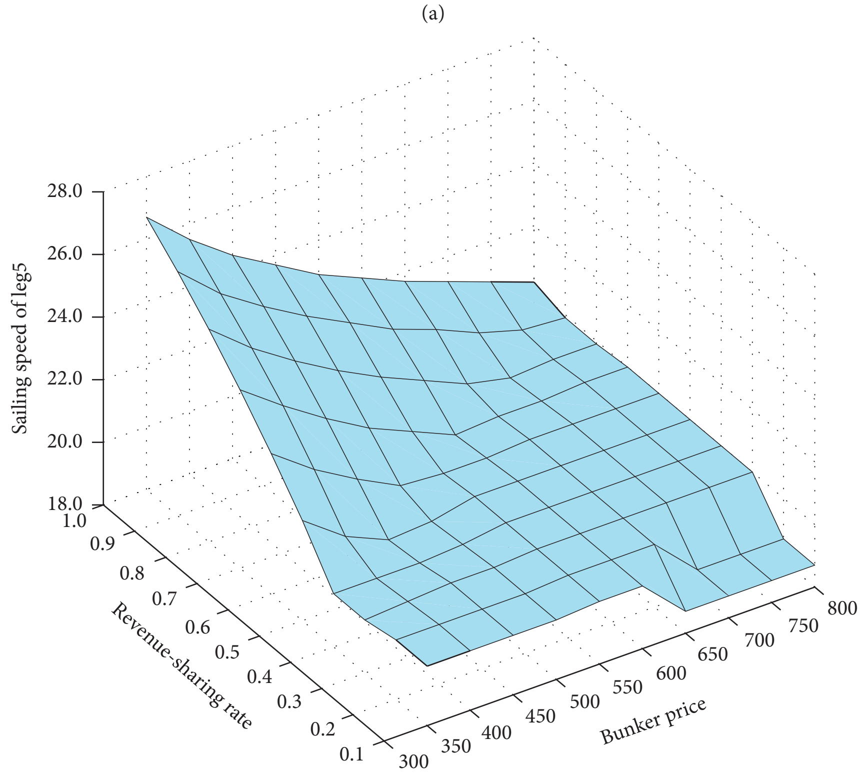
Generally, the increase of the bunker fuel price will result in the decrease of the sailing speed, while the increase in the revenue sharing rate will result in the growth of the sailing speed. Nevertheless, the above sensitivity analysis results of the long leg sailing speed are not completely consistent with the existing knowledge. The reason is that the SSFD problem under the TDFRS obtains optimization results under the full consideration of the total freight revenue, bunker fuel cost, and containership cost. The sailing speed changes in the head-haul direction and those in the back-haul direction have impact on each other.

Figure 3 depicts the changes of the total profit changes under different freight rate strategies (FFRS and TDFRS) and different parameters (revenue sharing rates and bunker fuel prices).

It can be seen from Figure 3 that there are some parameters (the overlapping part of the curved surfaces under the two freight rate strategies is demonstrated in shadow in Figure 3) under which the total profit under the TDFRS is equal to that under the FFRS. In most cases, the total profit under the TDFRS is greater than that under the FFRS. The greater the revenue-sharing rate is, the more profit the container shipping liner can achieve under TDFRS. In addition, as the revenue-sharing rate increases and the bunker fuel price decreases, the advantage of the TDFRS on the profit increment increases (that is, the gap between the two curved surfaces is getting bigger). This is because the higher the revenue sharing rate is, the higher the freight rate for spot market customers under the TDFRS is and more freight revenue the container shipping liner gets. In addition, the




(a)




(b)

Figure 2: Sensitivity of the optimal sailing speed of route 1: (a) leg 3 and (b) leg 5.

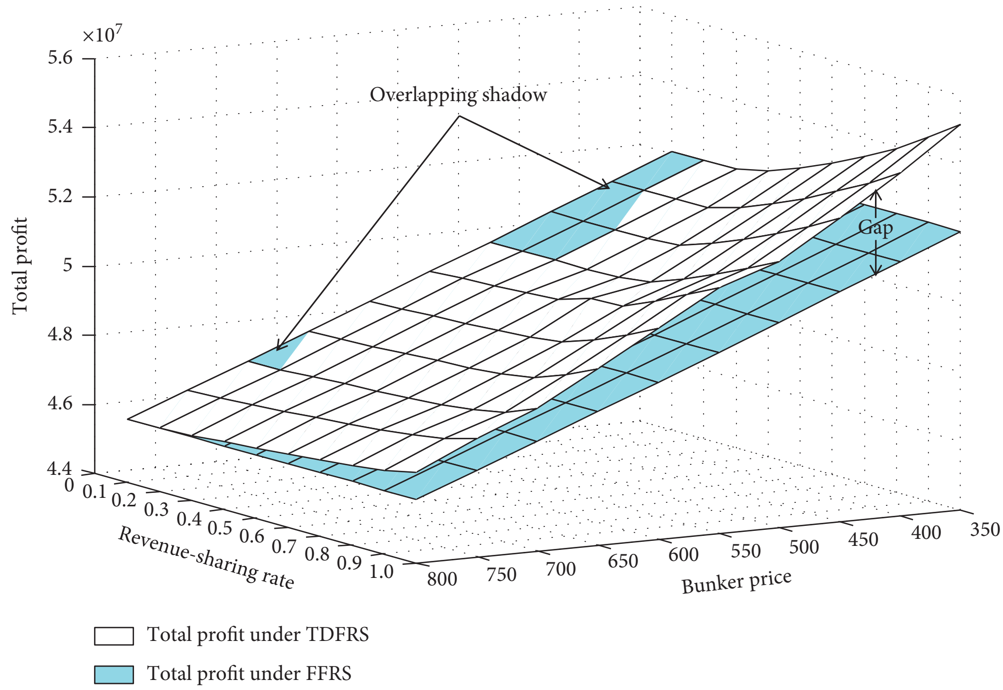



Figure 3: Changes of the total profit under the two freight rate strategies.

bunker fuel price reduction also has a positive effect on the overall sailing speed growth. This enables the container shipping liner to increase the freight rate for spot market customers and the total profit while the total bunker fuel cost reduces. Therefore, the total profit of the container shipping liner under TDFRS grows with the increase of the revenue-sharing rate and the decline of the bunker fuel price.

## 6. Conclusion

In this study, the SSFD problem in an intercontinental liner network with the consideration of the time value of the container cargo is investigated. The problem is first formulated as a mixed-integer nonlinear programming under the TDFRS. In consideration of the nonlinearity of the model, the piecewise linearization algorithm is designed to transform the model into an integer linear programming. The proposed model and the algorithm are evaluated by numerical examples. The results show that, considering the time value of the container cargo in the SSFD problem affects containerships sailing speed on long legs and the number of deployed containerships. Moreover, when the TDFRS is adopted for spot market customers, the optimization results obtained by the proposed model are able to not only increase the total profit for the container shipping liner but also provide a satisfactory level of service for customers.

It should be noted that the container demand between port pairs may fluctuate as freight rate changes. Therefore, the SSFD problem considering both the time value of the container cargo and the dynamic container demand should be explored in future research.

## Appendix

First, model [M1] linearization: the objective function (A.8) and constraints (A.9) and (A.10) in Section 3.3 all contain the reciprocal of the variables $s^r_{Nr}$ or $s^r_{Nr+Mr}$. Similarly, three sets of new decision variables are defined: $w^r_{Nr} = 1/s^r_{Nr}$ and $w^r_{Nr+Mr} = 1/s^r_{Nr+Mr}, \forall r \in \Phi$. Considering the time value of the container cargo, the freight rate for spot market customers $y^r_{ij}$ from port $i$ to port $j$ on route $r$ can be rewritten as

$$y^r_{ij} = Y^r_{ij} + \frac{\phi U^r \rho\left(\left(L^r_{Nr}/S^{r^*}_{Nr}\right) - L^r_{Nr} w^r_{Nr}\right)}{24 \times 365}, \quad \forall r \in \Phi,\ i \in A^r,\ j \in B^r,$$
$$y^r_{ij} = Y^r_{ij} + \frac{\phi U^r \rho\left(\left(L^r_{Nr+Mr}/S^{r^*}_{Nr+Mr}\right) - L^r_{Nr+Mr} w^r_{Nr+Mr}\right)}{24 \times 365}, \quad \forall r \in \Phi,\ i \in B^r,\ j \in A^r. \tag{A.1}$$

Define $W^r_{\min} = 1/S^r_{\max}$ and $W^r_{\max} = 1/S^r_{\min}$, $\forall r \in R$. Therefore, model [M1] can be reformulated as model [M3′]:

$$[\mathrm{M3}']\max Z=\sum_{r=1}^{R}\sum_{i=1}^{Nr}\sum_{j=Nr+1}^{Nr+Mr}\left\{\gamma_{ij}^{r}D_{ij}^{r}\left[Y_{ij}^{r}+\frac{\phi U^{r}\rho\left(\left(L_{Nr}^{r}/S_{Nr}^{r^*}\right)-L_{Nr}^{r}w_{Nr}^{r}\right)}{24\times 365}\right]+\alpha_{ij}^{r}\left(1-\gamma_{ij}^{r}\right)D_{ij}^{r}Y_{ij}^{r}\right\}$$
$$+\sum_{r=1}^{R}\sum_{i=Nr+1}^{Nr+Mr}\sum_{j=1}^{Nr}\left\{\gamma_{ij}^{r}D_{ij}^{r}\left[Y_{ij}^{r}+\frac{\phi U^{r}\rho\left(\left(L_{Nr+Mr}^{r}/S_{Nr+Mr}^{r^*}\right)-L_{Nr+Mr}^{r}w_{Nr+Mr}^{r}\right)}{24\times 365}\right]+\alpha_{ij}^{r}\left(1-\gamma_{ij}^{r}\right)D_{ij}^{r}Y_{ij}^{r}\right\}$$
$$-\sum_{r=1}^{R}PF_{D}^{r}\left(S_{D}^{r}\right)^{-3}\frac{\left[\sum_{i=1}^{Nr-1}L_{i}^{r}\left(S_{i}^{r^*}\right)^{2}+\sum_{i=Nr+1}^{Nr+Mr-1}L_{i}^{r}\left(S_{i}^{r^*}\right)^{2}+L_{Nr}^{r}\left(w_{Nr}^{r}\right)^{-2}+L_{Nr+Mr}^{r}\left(w_{Nr+Mr}^{r}\right)^{-2}\right]}{24}-\sum_{r=1}^{R}C^{r}m_{v}^{r}, \tag{A.2}$$

s.t.

$$\sum_{i=1}^{Nr-1}\frac{L_i^r}{S_i^{r^*}}+\sum_{i=Nr+1}^{Nr+Mr-1}\frac{L_i^r}{S_i^{r^*}}+L_{Nr}^{r}w_{Nr}^{r}+L_{Nr+Mr}^{r}w_{Nr+Mr}^{r}+\sum_{r=1}^{Nr+Mr}T_i^r\le 168m_v^r,\quad \forall r\in\Phi,\ v\in\Omega, \tag{A.3}$$

$$\sum_{k=i}^{Nr-1}\frac{L_k^r}{S_k^{r^*}}+\sum_{k=Nr+1}^{j}\frac{L_k^r}{S_k^{r^*}}+L_{Nr}^{r}w_{Nr}^{r}+\sum_{k=i}^{j}T_k^r\le T_{ij}^r,\quad \forall r\in\Phi,\ i\in A^r,\ j\in B^r, \tag{A.4}$$

$$\sum_{k=i}^{Nr+Mr-1}\frac{L_k^r}{S_k^{r^*}}+L_{Nr+Mr}^{r}w_{Nr+Mr}^{r}+\sum_{k=1}^{j-1}\frac{L_k^r}{S_k^{r^*}}+\sum_{k=i}^{Nr+Mr}T_k^r+\sum_{k=1}^{j}T_k^r\le T_{ij}^r,\quad \forall r\in\Phi,\ i\in B^r,\ j\in A^r, \tag{A.5}$$

$$\sum_{r=1}^{R}m_v^r\le N_v,\quad \forall v\in\Omega, \tag{A.6}$$

$$W_{\min}^r\le w_{Nr}^r\le W_{\max}^r,\quad \forall r\in\Phi, \tag{A.7}$$

$$W_{\min}^r\le w_{Nr+Mr}^r\le W_{\max}^r,\quad \forall r\in\Phi, \tag{A.8}$$

$$m_v^r\in Z^+,\forall r\in\Phi,\forall v\in\Omega. \tag{A.9}$$

All the constraints in model [M3′] are linear, but the third term of the objective function contains nonlinear functions of $w_{Nr}^r$ and $w_{Nr+Mr}^r$. Therefore, model [M3′] is still a mixed-integer nonlinear programming.

According to Theorem 1, it can be proved that the objective function of model [M3′] is also convex, and the value of $w_{Nr}^r$ and $w_{Nr+Mr}^r$ are uniform on each liner route.

Second, discretize the definition domain $[W_{\min}^r, W_{\max}^r]$ of the new decision variables $w_{Nr}^r$ and $w_{Nr+Mr}^r$. For the sailing speed on each long leg on each liner route, a new binary variable is defined as follows:

$$\beta_{Nr,qr}^r=\begin{cases}1, & w_{Nr}^r=W_{qr}^r\\ 0, & w_{Nr}^r\ne W_{qr}^r,\end{cases}\quad \forall r\in\Phi,\ qr\in\{0,1,2,\ldots,Qr\}, \tag{A.10}$$

$$\beta_{Nr+Mr,qr}^r=\begin{cases}1, & w_{Nr+Mr}^r=W_{qr}^r\\ 0, & w_{Nr+Mr}^r\ne W_{qr}^r,\end{cases}\quad \forall r\in\Phi,\ qr\in\{0,1,2,\ldots,Qr\}. \tag{A.11}$$

Then,

$$\begin{aligned} w_{Nr}^r&=\sum_{qr=0}^{Qr}\beta_{Nr,qr}^rW_{qr}^r,\\ w_{Nr+Mr}^r&=\sum_{qr=0}^{Qr}\beta_{Nr+Mr,qr}^rW_{qr}^r.\end{aligned} \tag{A.12}$$

Finally, the mixed-integer nonlinear programming model [M3′] can be transformed into an equivalent integer linear programming model [M5].

## Data Availability

All the data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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