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Research Article

A Reliability-Based Network Equilibrium Model with Adaptive Risk-Averse Travelers

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In this paper, route free-flow travel time is taken as the lower bound of route travel time to examine its impacts on budget time and reliability for degradable transportation networks. A truncated probability density distribution with respect to route travel time is proposed and the corresponding travel time budget (TTB) model is derived. The budget time and reliability are compared between TTB models with and without truncated travel time distribution. Under truncated travel time distribution, the risk-averse levels of travelers are adaptive, which are affected by the characteristics of the used routes besides the confidence level of travelers. Then, a TTB-based stochastic user equilibrium (SUE) is developed to model travelers' route choice behavior. Moreover, its equivalent variational inequality (VI) problem is formulated and a route-based algorithm is used to solve the proposed model. Numerical results indicate that route travel time boundary produces a great influence on decision cost and route choice behavior of travelers.

1. Introduction

Most studies indicate that travel time reliability (TTR) is an essential travel decision cost of travelers [1–3]. In particular, Abdel-Aty et al. (1977) pointed out that TTR was one of the major elements influencing commuters' route choice considerations [4]. However, the classical user equilibrium (UE) principle assumes that travelers are risk-neutral and their decision behavior only depends on the mean travel time in deterministic transportation networks. Obviously, it leaves the travel time uncertainty out of consideration and neglects the risk preferences of travelers (i.e., risk aversion and risk seeking). Meanwhile, empirical studies demonstrated that majority of travelers are risk-neutral and they would like to take an additional payment to avoid the congestion and travel risk [5, 6].

In transportation systems, uncertainty is unavoidable, which mainly derives from roadway capacity variation and travel demand fluctuation. Both supply and demand variations bring about travel time uncertainty. In order to model travelers' route choice behavior under uncertainty, a variety of traffic equilibrium models incorporating TTR are developed [7–11]. Typically, Lo et al. (2006) proposed the within budget time reliability (WBTR) model [12]. The notion of travel time budget (TTB) in WBTR model is given to represent the decision cost of travelers. The TTB refers to the sum of the mean travel time and a safety margin to arrive on time. The standard deviation of route travel time is used to measure the risk of route choice. However, the WBTR model disregards the effects of unreliable side of route travel time on route choice. By incorporating simultaneously the reliable and unreliable sides, the mean-excess traffic equilibrium (METE) model was developed by Chen and Zhou (2010) [13]. In the METE model, travelers are assumed to use such a route to make TTR ensured and unreliable impacts can be minimized. Subsequently, Chen et al. (2011) further introduced the perception errors of travelers to the METE model and proposed the stochastic mean-excess traffic equilibrium (SMETE) model [14]. In this case, the perceived travel time distribution is derived to model travelers' route choice behavior rather than the actual one. Different from the previous studies, Watling (2006) considered travelers' schedule delay and proposed a late arrive penalized user equilibrium (LAPUE) model [15]. Recently, the multiobjective equilibrium problems have been intensively investigated [16–18]. In this regard, Wang et al. (2014) provided a biobjective user equilibrium (BUE) condition, which is the state that no traveler can improve either his/her expected travel time or risk or both without worsening the other by unilaterally changing routes [17].

It can be found that most of aforementioned traffic equilibrium models with the TTR are based on the degradable transportation networks. As mentioned above, travel demand fluctuation would also lead to travel time uncertainty. Shao et al. (2006) developed the UE model with demand uncertainty [19]. Then, the model was further extended by considering travelers' perception errors and heterogeneity, and the reliability stochastic user equilibrium (RSUE) model was developed [20]. Lam et al. (2008) assumed the uncertainties mainly due to adverse weather condition and proposed a traffic equilibrium model with doubly uncertainties in supply and demand sides [21]. Zhou and Chen (2008) compared and analyzed the equilibrium results of three UE models with stochastic travel demand, using a simple network to illustrate the differences, and examined how these models address the travel risk [22]. Siu and Lo (2008) formulated a multiclass equilibrium model, in which the demand composes of infrequent travelers and commuters [23]. Sun and Gao (2012) assumed the probability distributions of travel demand and link capacities are unknown and developed the robust traffic equilibrium model [24]. In addition, considered uncertainties in demand and supply are also extended to the network design problems (NDPs) to develop the robust optimization models [25, 26].

In the above models, route travel time are commonly assumed to follow continuous normal distribution or lognormal distribution. However, these distributions imply that the lower boundary of route travel time approaches negative infinity or zero. To the best of our knowledge, route travel time is bounded and there is at least a lower boundary, namely, free-flow travel time. Yan et al. (2015) studied the effects of link speed limits on travelers' route choice and the network performance [27]. This study indicated that the mean and variance of total travel time can be reduced by imposing speed limits, but the total TTB of a network increases. Xu et al. (2017) showed clearly that the minimum travel time equals the link length divided by the speed limit value after imposing a speed limit scheme [28]. They indicated that a speed limit scheme on uncertain road networks would affect the network flow reallocation. In the study, the minimum travel time is derived from speed limits. Nevertheless, the route travel time without a speed limit should also have a lower bound, and the travel time uncertain profile would also change.

This paper aims to examine the impacts of route travel time boundary on travel cost and route choice of travelers. To this end, we firstly propose the probability density function (PDF) with the route free-flow travel time as a truncation of original distribution. Subsequently, the TTB model with truncated travel time distribution is derived and compared with the original TTB model in terms of budget time and TTR. Then, the TTB-based SUE models are developed to model the impacts of route travel time boundary on travelers' route choice.

The remainder of this paper is organized as follows. Section 2 presents the effect of route travel time boundary on PDF. Section 3 develops the TTB model with truncated travel time distribution under uncertainty. Section 4 gives the TTB-based SUE model. Then, we use numerical examples to illustrate the effect of route travel time boundary on network equilibrium in Section 5. Finally, Section 6 summarizes some concluding remarks.

2. Effect of Route Travel Time Boundary on Uncertainty

In this section, the route free-flow time is considered as the lower bound of route travel time, and the effects of route travel time boundary on uncertainty are examined by proposing a truncated PDF of route travel time. The relationships and differences between TTB models with and without route travel time boundary are derived.

Let us consider a stochastic network $G(N, A)$, which is composed of N nodes and A directed links with capacity variations. Let t_a^0 be the free-flow time, v_a the traffic flow, and C_a the capacity of link a, respectively. Then, link travel time $t_a(v_a)$ can be determined by the bureau of public roads (BPR) link performance function, i.e.,

$$t_a(v_a) = t_a^0\left(1+\beta\left(\frac{v_a}{C_a}\right)^n\right) \quad (1)$$

where β and n are, respectively, the deterministic parameters of the BPR link performance function. Assume that link capacity follows an uniform distribution, $C_a \sim U(\phi_a\overline{c}_a, \overline{c}_a)$, where $\overline{c}_a$ is the design capacity associated with link a and ϕ_a denotes the capacity degradable coefficient, $\phi_a \in [0,1)$. Further, we suppose that link capacity distributions are independent, and thus route travel time T_r is normally distributed $T_r \sim N(E(T_r), \sigma(T_r))$ based on the central limit theorem. The mean $E(T_r)$ and standard deviation $\sigma(T_r)$ of route travel time are, respectively,

$$E(T_r) = \sum_{a\in A}\delta_{ar}^{w}E(t_a), \quad \forall r \in R_w,\ w \in W \quad (2)$$

$$\sigma(T_r) = \sum_{a\in A}\delta_{ar}^{w}\sigma(t_a), \quad \forall r \in R_w,\ w \in W \quad (3)$$

where $E(t_a)$ and $\sigma(t_a)$ are, respectively, the mean and standard deviation of link travel time. δ_{ar}^{w} is a binary variable, which equals 1 when link a is on route r, and 0 otherwise. R_w is a set of all routes between OD pair w, and W is a set of all OD pairs in the transportation network.

Combining the derivation of the mean and standard deviation of link travel time in [12], (2) and (3) can be rewritten as follows.

$$E(T_r) = \sum_{a\in A}\left\{\delta_{ar}^{w}\cdot\left[t_a^0 + \beta t_a^0 v_\alpha^n \frac{1-\varphi_a^{1-n}}{\overline{c}_a^n (1-\varphi_a)(1-n)}\right]\right\} \quad (4)$$

$$\sigma(T_r) = \sqrt{\sum_{a\in A}\left[\delta_{ar}^{w}\cdot\beta^2 (t_a^0)^2 v_a^{2n}\left\{\frac{1-\varphi_a^{1-2n}}{\overline{c}_a^{2n}(1-\varphi_a)(1-2n)} - \left[\frac{1-\varphi_a^{1-n}}{\overline{c}_a^n (1-\varphi_a)(1-n)}\right]^2\right\}\right]} \quad (5)$$

2.1. Truncated Route Travel Time Distribution. This paper assumes that the route free-flow time is a determinate constant associated with specific route in the transportation network. Travel time uncertainty is only caused by the link capacity degradation. As derived above, route travel time is normally distributed under such case. A normal distribution implies that the lower bound or minimum route travel time is close to negative infinity. However, the minimum route travel time must be a positive constant, namely, the route free-flow travel time. In other words, the free-flow travel time is the lower bound of route travel time, and route travel time cannot be less than the free-flow travel time. More specifically, the probability of route travel time being less than the free-flow travel time should be zero. Hence, we develop the PDF with the free-flow travel time as a truncation of original distribution without the lower bound, i.e.,

$$f(T \mid \mu,\sigma,\overline{t}) = \begin{cases}\dfrac{f(T)}{1-F(\overline{t})}, & T > \overline{t}\\ 0, & others\end{cases} \quad (6)$$

where $f(\cdot)$ and $F(\cdot)$ are the PDF and CDF of the original normal distribution associated with route travel time T. $\overline{t}$ is the route free-flow travel time and $F(\overline{t}) = \Pr(T \le \overline{t})$. μ and σ are the mean and standard deviation of route travel time, respectively.

For the ease of elaboration, a single-route network with the mean of 20 and the standard deviation of 5 is used. Assume the free-flow travel time is 15. The PDFs of route travel time with and without lower bound are shown in Figure 1. The x-axis shows the change in route travel time. The blue solid line is the PDF curve without the lower bound. The green dotted line represents the PDF curve with the free-flow travel time as a truncation of the original distribution. From this figure, one can see that the original distribution is truncated and becomes taller due to the appearance of fixed denominator. Moreover, the probability equals zero when route travel time with the lower bound is less than free-flow travel time. Obviously, the new PDF still fulfills the conservation property of a valid PDF according to (6). In addition, rather than truncated continuous distributions, other modeling ways can also be used to model the impacts of the lower bound of route travel time. Xu et al. (2018) elaborated the reality and feasibility of truncated travel time distribution and its mathematical tractability [28].

2.2. Mean and Standard Deviation Derivations. Below, we derive the moments of route travel time T with the free-flow travel time as the lower bound. The truncated PDF in (6) can be rewritten as

$$f(T \mid \mu,\sigma,\overline{t}) = \frac{\left(1/\sqrt{2\pi}\sigma\right)\exp\left(-(T-\mu)^2/2\sigma^2\right)}{1-\Phi\left((\overline{t}-\mu)/\sigma\right)}, \quad \forall T > \overline{t} \quad (7)$$

where $\Phi(\cdot)$ is the CDF of the standard normal distribution. The mean and standard deviation of truncated travel time distribution are derived using the method of moment estimation as follows.

The mean of truncated travel time distribution $E(T \mid T > \overline{t})$ can be derived, i.e.,

$$E(T \mid T > \overline{t}) = \frac{\int_{\overline{t}}^{+\infty} T\left(1/\sqrt{2\pi}\sigma\right)\exp\left(-(T-\mu)^2/2\sigma^2\right)dT}{\Phi\left(-(\overline{t}-u)/\sigma\right)} \quad (8)$$

Let $x = (T-\mu)/\sigma$, we have $T = \mu + x\sigma$ and

$$\begin{aligned}\int_{\overline{t}}^{+\infty} T\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(T-\mu)^2}{2\sigma^2}\right)dT &= \frac{1}{\sqrt{2\pi}}\int_{(\overline{t}-\mu)/\sigma}^{+\infty}(\mu + x\sigma)\exp\left(-\frac{x^2}{2}\right)dT\\ &= \mu\Phi\left(\frac{\mu-\overline{t}}{\sigma}\right) + \frac{\sigma}{\sqrt{2\pi}}\exp\left(-\frac{(\overline{t}-\mu)^2}{2\sigma^2}\right)\end{aligned} \quad (9)$$

Then, substituting (4) into (3), we have the following.

$$E(T \mid T > \overline{t}) = \mu + \frac{\left(\sigma/\sqrt{2\pi}\right)\exp\left(-(\overline{t}-\mu)^2/2\sigma^2\right)}{\Phi\left((\mu-\overline{t})/\sigma\right)} \quad (10)$$

Obviously, it can be observed that $E(T \mid T > \overline{t}) > \mu$ from (5). It implies that the mean of truncated route travel time moves right and becomes larger compared to the original one.

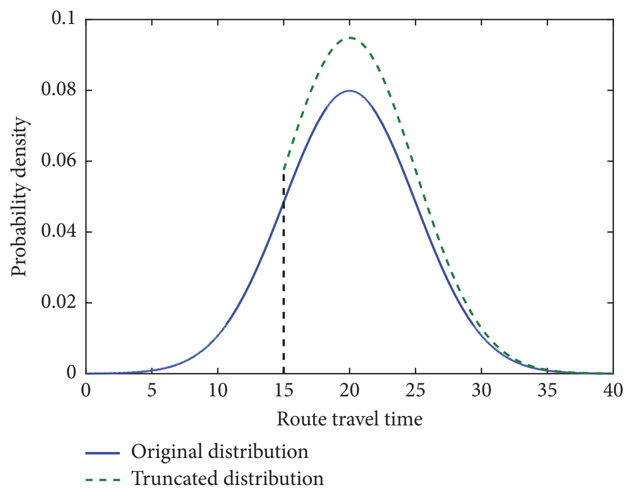

Figure 1: PDFs of route travel time with and without lower bound.

Similarly, the secondary moment of truncated travel time distribution can be derived, i.e.,

$$\begin{aligned} & E\left(T^{2}T > \bar{t}\right) \\ & = \frac{\int_{\bar{t}}^{+\infty} T^{2}\left(1/\sqrt{2\pi}\sigma\right)\exp\left(-\left(T-\mu\right)^{2}/2\sigma^{2}\right)dT}{\Phi\left(-\left(\bar{t}-u\right)/\sigma\right)} \\ & = \Phi^{-1}\left(-\frac{\bar{t}-u}{\sigma}\right)\int_{(\bar{t}-\mu)/\sigma}^{+\infty}\frac{\left(\mu+x\sigma\right)^{2}}{\sqrt{2\pi}}\exp\left(-\frac{x^{2}}{2}\right)dx \\ & = \Phi^{-1}\left(-\frac{\bar{t}-u}{\sigma}\right)\left(\int_{(\bar{t}-\mu)/\sigma}^{+\infty}\frac{\mu^{2}}{\sqrt{2\pi}}\exp\left(-\frac{x^{2}}{2}\right)dx\right. \\ & + \int_{(\bar{t}-\mu)/\sigma}^{+\infty}\frac{2x\mu\sigma}{\sqrt{2\pi}}\exp\left(-\frac{x^{2}}{2}\right)dx \\ & \left. + \int_{(\bar{t}-\mu)/\sigma}^{+\infty}\frac{x^{2}\sigma^{2}}{\sqrt{2\pi}}\exp\left(-\frac{x^{2}}{2}\right)dx\right) = \mu^{2}+\sigma^{2} \\ & + \frac{\sigma\left(\bar{t}+\mu\right)\exp\left(-\left(\bar{t}-\mu\right)^{2}/2\sigma^{2}\right)}{\sqrt{2\pi}\Phi\left(\left(\mu-\bar{t}\right)/\sigma\right)}. \end{aligned} \tag{11}$$

Then, the variance $D(T \mid T > \bar{t})$ and standard deviation $d(T \mid T > \bar{t})$ of truncated travel time distribution are, respectively,

$$D\left(T \mid T > \bar{t}\right) = \mathrm{E}\left(T^{2} \mid T > \bar{t}\right) - \left[\mathrm{E}\left(T \mid T > \bar{t}\right)\right]^{2} \tag{12}$$

$$d\left(T \mid T > \bar{t}\right) = \sqrt{D\left(T \mid T > \bar{t}\right)}. \tag{13}$$

3. TTB-Based Route Choice Model with Truncated Travel Time Distribution

In order to model travelers' route choice behavior under risk, different decision criterions have been proposed, such as TTB, METE, and PMETE. Xu et al. (2014) provided a detailed review on existing traffic equilibrium models under uncertainty [29]. This section restates the concepts of TTB and TTR and illustrates the effects of the route travel time boundary on them.

3.1. TTB without Route Travel Time Boundary

Definition 1. The TTB b_r^w on route r between OD pair w associated with a predefined confidence level ρ^w is defined as the sum of the expected travel time of travelers μ_r^w and a buffer time, the latter of which is the product of the standard deviation σ_r^w and travelers' risk-aversion level λ^w, i.e.,

$$b_r^w = \mu_r^w + \lambda^w \sigma_r^w, \quad r \in R_w,\ w \in W. \tag{14}$$

Definition 2. The TTR refers to the probability that travelers can arrive on time within the TTB b_r^w, i.e.,

$$P\left\{T_r^w \le b_r^w = \mu_r^w + \lambda^w \sigma_r^w\right\} = \rho^w, \quad r \in R_w,\ w \in W. \tag{15}$$

Thus, (15) can also be alternatively written as follows.

$$P\left\{\frac{T_r^w - \mu_r^w}{\sigma_r^w} \le \lambda^w\right\} = \rho^w \tag{16}$$

Mathematically, the risk-aversion level λ^w can be expressed as

$$\lambda^w = \Phi^{-1}\left(\rho^w\right), \quad \forall w \in W \tag{17}$$

where $\Phi^{-1}(\cdot)$ is the inverse function of the CDF of standard normal distribution. Obviously, travelers' risk-aversion level λ^w is only related to their expected on-time arrival probability ρ^w. In other words, the scale of λ^w for users with the same confidence level within one OD pair is identical.

3.2. TTB with Route Travel Time Boundary. The route free-flow time is viewed as a truncation of the original normal distribution, the resultant PDF is significantly different. In this case, the TTB and TTR with travel time boundary need to be derived and discussed.

For ease of the mathematical derivation, we assume that the expected travel time and risk have no changes under the truncated travel time distribution, and the TTB with route travel time boundary is as

$$\widehat{b}_r^w = \mu_r^w + \widehat{\lambda}_r^w \sigma_r^w, \quad \forall r \in R^w,\ w \in W \tag{18}$$

where $\widehat{b}_r^w$ and $\widehat{\lambda}_r^w$ are the TTB and risk-aversion level under truncated travel time distribution, respectively. $\widehat{\lambda}_r^w$ is no longer the constant which is only related to travelers' confidence level and it is also affected by the characteristics used routes (i.e., route travel time distribution and route free-flow travel time). More specifically, travelers' risk-aversion level $\widehat{\lambda}_r^w$ is context-based and adaptive, which would be proved and derived in the following.

According to Definition 2, the TTR under truncated distribution can be expressed as

$$P\left\{\widehat{T}_r^w \le \widehat{b}_r^w\right\} = \rho^w, \quad \forall r \in R^w,\ w \in W \tag{19}$$

Obviously, (19) can also be written as the conditional probability, i.e.,

$$P\left\{T_r^w \le \hat{b}_r^w \mid T_r^w > T_r^{w,0}\right\} = \rho^w, \quad \forall r \in R^w,\ w \in W \tag{20}$$

where $T_r^{w,0}$ is the free-flow travel time on route r between OD pair w. Mathematically, (20) can be alternatively written as

$$\frac{P\left\{T_r^{w,0} < T_r^w \le \hat{b}_r^w\right\}}{P\left\{T_r^w > T_r^{w,0}\right\}} = \rho^w, \quad \forall r \in R^w,\ w \in W \tag{21}$$

According to (21), we have the following.

$$P\left\{T_r^w \le \hat{b}_r^w\right\} = \rho^w\left(1 - F\left(T_r^{w,0}\right)\right) + F\left(T_r^{w,0}\right) \tag{22}$$

Then, we can derive

$$F\left(b_r^w\right) = \rho^w\left(1 - F\left(T_r^{w,0}\right)\right) + F\left(T_r^{w,0}\right) \tag{23}$$

Let $\rho^w(1 - F(T_r^{w,0})) + F(T_r^{w,0}) = \xi_r^w$, we have

$$\hat{b}_r^w = F^{-1}\left(\xi_r^w\right) \tag{24}$$

where $F^{-1}(\cdot)$ denotes the inverse function of the CDF of normal distribution.

3.3. Effect of Truncated Travel Time Distribution on TTB and TTR. In this subsection, some theoretical analysis on the TTB and TTR changes is given when considering route travel time boundary, and we illustrate the effects of truncated travel time distribution on TTB and TTR.

Proposition 3. *For route r between OD pair w, the truncated travel time distribution makes the corresponding TTR reduce under the same budget time b compared to the original distribution.*

Proof. Assume TTR_{with} and $TTR_{without}$ denote the within budget time reliability with and without travel time boundary, respectively, and the relationship between TTR_{with} and $TTR_{without}$ can be expressed as

$$\begin{aligned} &TTR_{with}(b) - TTR_{without}(b) \\ &\quad = \int_{T_r^{w,0}}^{b} \frac{f(T)}{1 - F\left(T_r^{w,0}\right)} dT - \int_{-\infty}^{b} f(T)\, dT \\ &\quad = \frac{F(b) - F\left(T_r^{w,0}\right)}{1 - F\left(T_r^{w,0}\right)} - F(b) = \frac{F\left(T_r^{w,0}\right)(F(b) - 1)}{1 - F\left(T_r^{w,0}\right)} \\ &\quad < 0 \end{aligned} \tag{25}$$

where $F(b) < 1$ is used in the above inequality. From (25), TTR_{with} is smaller than $TTR_{without}$, and the proof of Proposition 3 is completed. In addition, Proposition 3 also implies that ignoring route travel time boundary would result in overestimating the TTR. □

Proposition 4. *For route r between OD pair w, the truncated travel time distribution provides a longer TTB under the same confidence level ρ^w compared to the original distribution.*

Proof. Assume b_{with} and $b_{without}$ are the TTBs with and without travel time boundary, the WBTR under truncated travel time distribution can be expressed as follows.

$$\begin{aligned} &\int_{T_r^{w,0}}^{b_{with}} \frac{\left(1/\sqrt{2\pi}\sigma_r^w\right)\exp\left(-\left(T - \mu_r^w\right)^2/2\left(\sigma_r^w\right)^2\right)}{1 - \Phi\left(\left(T - \mu_r^w\right)/\sigma_r^w\right)} dT \\ &\quad = \rho^w \end{aligned} \tag{26}$$

Then, we transform (26) and have the following.

$$\begin{aligned} F\left(b_{with}\right) &= \rho^w \Phi\left(\frac{\mu_r^w - T_r^{w,0}}{\sigma_r^w}\right) + F\left(T_r^{w,0}\right) \\ &= \rho^w\left(1 - F\left(T_r^{w,0}\right)\right) + F\left(T_r^{w,0}\right) \end{aligned} \tag{27}$$

Similarly, the TTR under nontruncated travel time distribution can be derived, i.e.,

$$\int_{-\infty}^{b_{without}} \frac{1}{\sqrt{2\pi}\sigma_r^w} \exp\left(-\frac{\left(T - \mu_r^w\right)^2}{2\left(\sigma_r^w\right)^2}\right) dT = \rho^w \tag{28}$$

Meanwhile, (28) can be written as

$$F\left(b_{without}\right) = \rho^w \tag{29}$$

According to (27) and (29), we can obtain the difference of budget time between two types of distribution, i.e.,

$$\begin{aligned} &F\left(b_{with}\right) - F\left(b_{without}\right) \\ &\quad = \rho^w\left(1 - F\left(T_r^{w,0}\right)\right) + F\left(T_r^{w,0}\right) - \rho^w \\ &\quad = F\left(T_r^{w,0}\right)\left(1 - \rho^w\right) > 0. \end{aligned} \tag{30}$$

Because the CDF of normal distribution is monotonically increasing, $b_{with} > b_{without}$. Proposition 4 is proved. □

Proposition 5. *Travelers adopt a self-adaptive risk-aversion level $\hat{\lambda}_r^w$ under truncated travel time distribution, which is not only related to travelers' confidence level, but also affected by the used route.*

Proof. Combining (18) and (22), the TTR under truncated travel time distribution can be derieved, i.e.,

$$P\left\{T_r^w \le \mu_r^w + \hat{\lambda}_r^w \sigma_r^w\right\} = \rho^w\left(1 - F\left(T_r^{w,0}\right)\right) + F\left(T_r^{w,0}\right) \tag{31}$$

By rearranging terms of LHS in (31), and

$$P\left\{\frac{T_r^w - \mu_r^w}{\sigma_r^w} \le \hat{\lambda}_r^w\right\} = \rho^w\left(1 - F\left(T_r^{w,0}\right)\right) + F\left(T_r^{w,0}\right) \tag{32}$$

Then, $\hat{\lambda}_r^w$ can be derived as follows.

$$\hat{\lambda}_r^w = \Phi^{-1}\left(\xi_r^w\right) \tag{33}$$

Because ξ_r^w is associated with $T_r^{w,0}$ and ρ^w, the risk-aversion level $\hat{\lambda}_r^w$ is not only related to travelers' confidence level, but also affected by the used route. In addition, the difference of risk-aversion levels between different distributions can be expressed as follows.

$$\widehat{\lambda}_r^w - \lambda^w = \Phi^{-1}(\xi_r^w) - \Phi^{-1}(\rho^w) \tag{34}$$

Obviously, $\xi_r^w > \rho^w$, and $\Phi^{-1}(\cdot)$ is monotonically increasing function. So we have $\widehat{\lambda}_r^w > \lambda^w$. It implies that route travel time boundary has a great effect on travelers' risk attitudes. □

3.4. An Illustrative Example. An example network with two parallel routes connecting one origin and one destination is used to present the difference between the TTB model with and without truncated travel time distribution. In this example, the route travel time is assumed to follow a normal distribution $N(\mu, \sigma)$. The means and standard deviations of two route travel time are assumed to be $\mu_1 = 20$, $\mu_2 = 15$, $\sigma_1 = 5$, $\sigma_2 = 10$. The free-flow travel times are 15 min and 10 min, respectively. All users are assumed to be homogeneous with the same confidence $\rho = 0.90$.

The TTBs of route 1 with and without truncated travel time distribution are 26.89 and 26.41. And those of route 2 are 29.83 and 27.82, respectively. These imply that ignoring the effects of route travel time boundary would result in an exiguity of budget time. Travelers' risk-aversion level is only dependent on their confidence level when route travel time follows a general normal distribution. In this case, travelers' risk-aversion level can be calculated as 1.28. However, travelers' risk-aversion level not only depends on the expected on-time arrival condition, but also is affected by the characteristics of the used routes (i.e., travel time distribution and free-flow time) when considering route travel time boundary, which is a context-based and adaptive constant. The risk-aversion level of travelers using route 1 is 1.38, and that of those using route 2 is 1.48. Then, to reveal the difference of TTR incurred by different travel time distributions, we assume travelers' TTB is a fixed constant and its quantity is 28. The TTRs of route 1 with and without truncated travel time distribution are, respectively, 0.93 and 0.95. The results indicate that neglecting the travel time boundary would overestimate the travel time reliability.

Table 1 shows the effects of scale variations of route free-flow travel time on TTB and travelers' risk-aversion level. From this table, we can see that travelers adopt a longer budget time to ensure on-time arrival when they are faced with higher risk. As the free-flow travel time increases, or as longer minimum travel time is imposed on routes, the values of TTB and risk-aversion level increase. Intuitively, it is clear that the growth of the minimum travel time results in a longer budget time. Thus, travelers become more aversive to risk when a longer budget time is suggested. It can be also found that travelers' risk-aversion levels are different for using different routes. These results indicate that route travel time boundary has a great effect on travelers' TTBs and risk attitudes.

Table 1: Effect of route travel time boundary on TTB and traveler' risk attitudes.

Route #	Free-flow travel time	TTB	Risk aversion level
1	6	26.42	1.28
	9	26.45	1.29
	12	26.57	1.31
	15	26.89	1.38
	18	27.55	1.51
2	10	29.83	1.48
	11	30.10	1.51
	12	30.40	1.54
	13	30.73	1.57
	14	31.07	1.61

4. TTB-Based SUE Model

This section models the effects of route travel time boundary on the performance of network equilibrium. We also present the differences between both SUE models with and without route travel time boundary. In the SUE state, no traveler can improve his or her perceived TTB by unilaterally changing route. The perceived TTBs of all used routes are the same and less than those of unused routes.

4.1. TTB-SUE without Route Travel Time Boundary. In the TTB-SUE model, travelers are always inclined to choose the routes with the least perceived TTB. Travelers' route choice behavior follows the random utility maximization (RUM) principle; the perceived TTB B_r^w on route $r \in R_w$ can be expressed as

$$B_r^w = b_r^w + \varepsilon_r^w, \quad r \in R_w,\ w \in W \tag{35}$$

where b_r^w is given by (14) and the random terms ε_r^w is assumed to be identically and independently distributed (i.i.d.) Gumbel variate, $r \in R_w$, $w \in W$. According to the RUM principle, the route choice probability p_r^w on route $r \in R_w$ at equilibrium can be written as

$$p_r^w = \frac{\exp(-\theta b_r^w)}{\sum_{k \in R_w} \exp(-\theta b_k^w)}, \quad r \in R_w,\ w \in W \tag{36}$$

where θ is a dispersion parameter measuring the variability in perceived TTB. A small θ value implies a larger perceived error compared to the shortest route. On the contrary, a large θ value indicates a smaller perceived error associated with the shortest route. That is, travelers would be more inclined to the shortest route when θ value is large enough. The equilibrium flow pattern can be obtained as follows

$$f_r^w = d_w p_r^w = d_w \frac{\exp(-\theta b_r^w)}{\sum_{k \in R_w} \exp(-\theta b_k^w)}, \quad r \in R_w,\ w \in W \tag{37}$$

where d_w is travel demand on OD pair w. Apparently, the equilibrium flow pattern determined by the TTB-SUE model depends upon not only the considered route' TTB but also other alternatives' TTBs.

Mathematically, the perceived TTB B_r^w under the general normal distribution can be expressed as follows.

$$B_r^w = \frac{1}{\theta}\left(\ln f_r^w + 1\right) + b_r^w, \quad r \in R_w,\ w \in W \tag{38}$$

4.2. TTB-SUE with Route Travel Time Boundary. Compared to the TTB-SUE without route travel time boundary, the TTB-SUE with route travel time boundary uses a truncated travel time distribution to reflect the effects of route free-flow time on travelers' route choice behavior. In this case, travelers aim to minimize the perceived travel time budget with truncate travel time distribution. The new perceived travel time budget $\widehat{B}_r^w$ can be expressed as

$$\widehat{B}_r^w = \widehat{b}_r^w + \xi_r^w, \quad r \in R_w,\ w \in W \tag{39}$$

where $\widehat{B}_r^w$ is the perceived travel time budget under truncated travel time distribution. $\widehat{b}_r^w$ is given by (18) or (24). The random term ξ_r^w is assumed to be i.i.d. Gumbel variate, $r \in R_w$, $w \in W$. The route choice probability p_r^w on route $r \in R_w$ can be written as follows.

$$p_r^w = \frac{\exp\left(-\theta \widehat{b}_r^w\right)}{\sum_{k\in R_w}\exp\left(-\theta \widehat{b}_k^w\right)}, \quad r \in R_w,\ w \in W \tag{40}$$

Accordingly, we can obtain the equilibrium route flow pattern.

$$f_r^w = d_w p_r^w = \frac{\exp\left(-\theta \widehat{b}_r^w\right)}{\sum_{k\in R_w}\exp\left(-\theta \widehat{b}_k^w\right)}, \quad r \in R_w,\ w \in W \tag{41}$$

The perceived travel time budget B_r^w under truncated travel time distribution can be expressed as follows.

$$\widehat{B}_r^w = \frac{1}{\theta}\left(\ln f_r^w + 1\right) + \widehat{b}_r^w, \quad r \in R_w,\ w \in W \tag{42}$$

4.3. The Equivalent VI Formulation. Both TTB-SUE conditions can be written as follows.

$$f_r^w\left(f_r^w - d_w p_r^w\right) = 0, \quad f_r^w \geq 0 \tag{43}$$

So, logit-based TTB-SUE problem can be formulated equivalently as the following VI problem.

$$\sum_{w\in W}\sum_{r\in R_w}\left(f_r^{w*} - p_r^w\left(b\left(f_r^{w*}\right)\right)\cdot d_w\right)\left(f_r^w - f_r^{w*}\right) \geq 0, \quad \forall f_r^w \in \Omega \tag{44}$$

Obviously, (44) can be alternatively written as

$$\left(\mathbf{f}^* - \mathbf{p}\left(\mathbf{b}\left(\mathbf{f}^*\right)\right)\cdot\mathbf{d}\right)^{\mathrm{T}}\left(\mathbf{f} - \mathbf{f}^*\right) \geq 0, \quad \mathbf{f} \in \Omega \tag{45}$$

where Ω is the feasible flow set defined by the following.

$$\Omega = \left\{ f_r^w \geq 0 : \sum_{r\in R_w} f_r^w = d_w,\ w \in W \right\} \tag{46}$$

Theorem 6. *The VI problem (45) is equivalent to the equilibrium conditions of both TTB-SUE models.*

Proof. The proof of the theorem follows the works of Zhou et al. (2012) [30] and Bekhor et al. (2012) [31]. For the proof of the necessity, we suppose $\mathbf{f}^*$ is a solution of any TTB-SUE model; from the SUE condition in (37) or (41), the VI problem is satisfied naturally. Thus, any solution of both TTB-SUE models is a solution of the proposed VI problem. For the proof of sufficiency, let $\mathbf{f}^*$ be a solution of the VI problem. Without loss of generality, we fix a route $h \in R_v$ within OD pair v and construct a feasible route flow $\mathbf{f}$ such that $f_r^w = f_r^{w*}$, $(r,w) \neq (h,v)$, but $f_h^v \neq f_h^{v*}$. Then substituting them into (40), we have the following.

$$\left(f_h^{v*} - p_h^v\left(b\left(f_h^{v*}\right)\right)\cdot d_v\right)^{\mathrm{T}}\left(f_h^v - f_h^{v*}\right) \geq 0 \tag{47}$$

For each effective route $h \in R_v$, $v \in W$, it can be deduced that $f_h^v > 0$. So one can obtain the following.

$$f_h^{v*} - p_h^v\left(b\left(f_h^{v*}\right)\right)d_v = 0 \tag{48}$$

Thus, the SUE condition in (37) or (41) is satisfied, and the solution of the proposed VI problem is also the solution of TTB-SUE model. □

Theorem 7. *There exists at least one solution for the proposed VI problem in (45).*

Proof. According to the mathematical formulation of $\mathbf{b}(\mathbf{f})$, we know that it is a continuous function associated with $\mathbf{f}$, and $\mathbf{f}^* - \mathbf{p}(\mathbf{b}(\mathbf{f}^*))\cdot\mathbf{d}$ is also a continuous function of $\mathbf{f}$. In addition, since Ω is a nonempty, convex, and compact set, the proposed VI problem (45) has at least one solution.

However, the uniqueness of the solution of the equivalent VI problem cannot be guaranteed. The reason is that the solution to logit-based TTB-SUE is not unique in general and the strict monotonicity of the mapping $\mathbf{f}^* - \mathbf{p}(\mathbf{b}(\mathbf{f}^*))\cdot\mathbf{d}$ cannot be ensured. □

5. Solution Algorithm

The MSA algorithm is employed to resolve the proposed equilibrium problem in this paper. And the algorithm steps are expressed as follows.

Step 1 (initialization). For given OD demand d_w of OD pair w, randomly load a feasible route flow vector $\mathbf{f}^{(1)} \in \Omega$. Set the maximum iteration number N and the convergence tolerance $\varepsilon > 0$. Let the iteration counter $n := 1$.

Step 2 (cost calculation). Determine link flow vector $\mathbf{x}^{(n)}$ according to the relationship between link flows and route flows, and then calculate the mean $E^{(n)}(t_a)$ and standard deviation $\sigma^{(n)}(t_a)$ of link travel time. Accordingly, we can obtain the mean $E^{(n)}(T_r^w)$ and standard deviation $\sigma^{(n)}(T_r^w)$ of route travel time by (4) and (5). Thus, the travel time budget with truncated travel time distribution $\widehat{b}_r^{w(n)}$ can be derived by (24).

Table 2: Route characteristics of the three-route transportation network.

Route #	Free-flow travel time (min)	Capacity (veh/h)	Reliability (φ_a)
1	12	1000	0.5
2	30	2000	0.7
3	40	3000	0.9

Table 3: The equilibrium results of TTB-SUE without travel time boundary.

Route #	f_r^w	μ_r^w	σ_r^w	λ	b_r^w	B_r^w
1	1356.63	12.36	23.22		42.12	50.33
2	2115.83	35.31	4.97	1.28	41.68	50.33
3	1527.54	41.92	0.60		42.00	50.33

Table 4: The equilibrium results of TTB-SUE with travel time boundary.

Route #	f_r^w	μ_r^w	σ_r^w	$\widehat{\lambda}_r^w$	$\widehat{b}_r^w$	$\widehat{B}_r^w$
1	1281.98	12.29	18.52	1.64	42.64	50.79
2	2117.24	35.32	4.98	1.36	42.13	50.79
3	1600.78	42.32	0.07	1.28	42.41	50.79

Step 3 (flow assignment). Set the auxiliary route vector $\overline{\mathbf{f}}^{(n)}$, which can be yielded by (40) and (41).

$$\overline{f}_r^w = d_w p_r^w = \frac{\exp\left(-\theta \widehat{b}_r^w\right)}{\sum_{k\in R_w}\exp\left(-\theta \widehat{b}_k^w\right)}, \quad r \in R_w,\ w \in W \quad (49)$$

Step 4 (update). The route flow vector $\mathbf{f}^{(n)}$ can be updated by the following.

$$\mathbf{f}^{(n+1)} = \mathbf{f}^{(n)} + \frac{1}{n}\left(\overline{\mathbf{f}}^{(n)} - \mathbf{f}^{(n)}\right) \quad (50)$$

Step 5 (checking the convergence). If the iteration counter n exceeds the maximum iteration number N or (51) is satisfied, the algorithm stops and the present $\mathbf{f}^{(n+1)}$ is the equilibrium results.

$$\frac{\sqrt{\sum_{r\in R_w}\left(f_{rw}^{(n+1)} - f_{rw}^{(n)}\right)^2}}{\sum_{r\in R_w} f_{rw}^{(n)}} \le \varepsilon \quad (51)$$

Otherwise, make $n := n + 1$ and return to Step 2.

6. Numerical Example

6.1. A Three-Route Network. This section uses two transportation networks to demonstrate the proposed model and algorithm. The first network consists of three parallel routes shown in Figure 2. The characteristic values of the link parameters are given in Table 2. The parameters of BPR function in (1) are $\beta = 0.15$ and $n = 4$. The total traffic demand is assumed to be fixed and it is 15,000 vehicles per hour. Without loss of generality, travelers are assumed to be homogeneous with the identical expected on-time arrival probability $\rho = 0.90$.

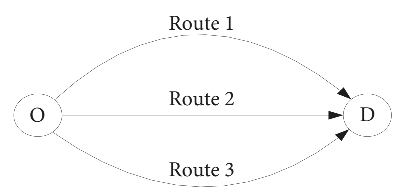

Figure 2: A three-route transportation network.

The dispersion parameter θ is assumed to be 1 in both TTB-SUE models. Tables 3 and 4 provide the equilibrium results of both SUE models. From the tables, one can see that two SUE models give quite different equilibrium results. It implies that route travel time boundary has a great impact on travelers' route choice behavior under uncertainty. In the TTB-SUE model without route travel time boundary, the risk-averse level is 1.28 no matter which route is used, which is only related to travelers' expected on-time arrival probability. In contrast, it is heterogeneous which is also affected by the used routes when considering route travel time boundary. It is plausible that travelers adopt different risk-aversion attitudes for diverse routes because of the physical performance difference between different routes. From the tables, we can see that the risk-aversion level is still 1.28 when using route 3. However, it is distinct when using other two routes, such as 1.64 for route 1 and 1.36 for route 2. As expected, longer travel time budget is suggested by the TTB-SUE model with travel time boundary than that without travel time boundary in terms of each route. In addition, the perceived travel time budgets using different routes are identical at equilibrium, and more perceived travel time budget is caused under truncated travel time distribution.

Figure 3 depicts the effects of the expected on-time arrival probability ρ on travelers' risk-aversion levels using different routes. As ρ increases, travelers' risk-aversion levels increase

Table 5: The equilibrium results of TTB-SUE with truncated travel time distribution.

O-D	Route #	Sequence of nodes	Route flow	Travel time budget	Perceived travel time budget	Expected travel time	Risk	Risk-aversion level
1-3	1	1-5-6-7-11-3	145.37	61.39	67.37	42.23	17.46	1.10
	2	1-5-6-10-11-3	57.90	62.31		49.52	12.39	1.03
	3	1-5-9-10-11-3	44.81	62.57		49.07	12.62	1.07
	4	1-12-6-7-11-3	231.02	60.93		42.91	16.20	1.11
	5	1-12-6-10-11-3	86.63	61.94		42.77	16.80	1.14
	6	1-5-9-13-3	37.27	62.75		50.06	11.72	1.08
4-2	7	4-5-6-7-8-2	260.01	73.37	79.93	47.73	24.05	1.07
	8	4-5-6-7-11-2	130.46	74.06		47.89	23.76	1.10
	9	4-5-6-10-11-2	53.96	74.94		55.18	18.68	1.06
	10	4-9-10-11-2	40.58	75.23		54.73	18.92	1.08
	11	4-5-9-10-11-2	314.99	73.18		55.20	17.88	1.01

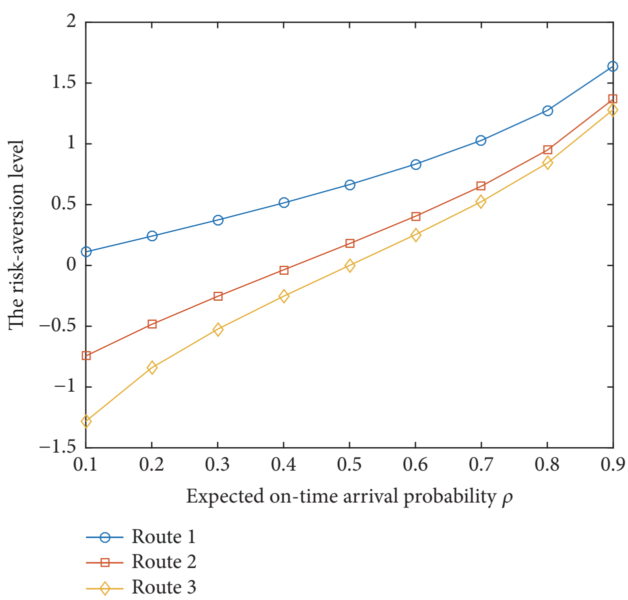

Figure 3: The risk-aversion level against ρ.

for each route. In comparison, travelers using route 1 of the worst reliability adopt largest risk-aversion levels to address the risk, and travelers on route 3 of the best reliability have the smallest risk-aversion attitudes.

Figures 4(a) and 4(b) show the impacts of different boundary scales of route travel time on the expected travel time and risk. Specifically, the expected travel time on three routes is always increasing as the boundary scale becomes larger. Meanwhile, the most reliable route (i.e., route 3) has the longest expected travel time, whereas using the most degradable route (i.e., route 1) spends the shortest expected travel time. As the boundary scale of route travel time increases, travel time risk on three routes is also always increasing. In addition, one can see that travelers using the most reliable route are confronted with the lowest travel time risk, and those using the shortest expected travel time encounter the highest travel time risk. These imply that travelers have to make a tradeoff between expected travel time and risk. These also demonstrate that travelers' risk attitude has to be incorporated into route choice modeling. Figure 4(c) plots the variation of perceived TTB with route travel time boundary in the equilibrium state. From this figure, we can see that the perceived TTB increases constantly when boundary scale of route travel time becomes larger. All of these indicate that route travel time boundary has a great influence on travelers' travel cost (i.e., expected travel time, risk, and perceived travel time budget).

6.2. Nguyen-Dupuis Network. In this section, the Nguyen-Dupuis network is used to demonstrate the solvability of the proposed model with route travel time boundary under uncertainty. This network has 13 nodes, 19 links, and 4 OD pairs. The BPR performance function ($\beta = 0.15, n = 4$) in (1) is adopted to compute link travel cost. Link specifications are provided in Figure 5. The potential demands are $d_{12} = 400$, $d_{13} = 800$, $d_{42} = 600$, and $d_{43} = 400$, respectively. The link capacity parameter and discrete parameter are set as $\varphi_a = 0.4$ and $\theta = 1.0$. All travelers in the test network are assumed to be homogeneous with $\rho = 0.8$.

First, the equilibrium results of TTB-SUE with truncated travel time distribution are shown in Table 5. Without loss of generality, we only present the equilibrium results of OD (1-3) and OD (4-2). As expected, the perceived TTBs at the equilibrium on all used routes for each OD pair are equal, which is satisfied with the TTB-based SUE condition. Travelers prefer to choose the routes with shorter TTBs. It can be also observed that the travel time risk decreases with the increase in expected travel time. The risk-aversion level is heterogeneous for different used routes within one OD pair, which is jointly affected by expected on-time arrival probability and route characteristics (i.e., route free-flow travel time and distribution). It is relatively larger for routes with higher risk. However, it does not mean that the larger the travel time risk, the higher the risk-aversion level. As mentioned above, route travel time boundary is a major influence factor of travelers' risk-aversion level.

Second, we further examine the joint impacts of expected on-time arrival probability ρ and discrete parameter θ on perceived TTB in Figures 6 and 7. From these figures, we can

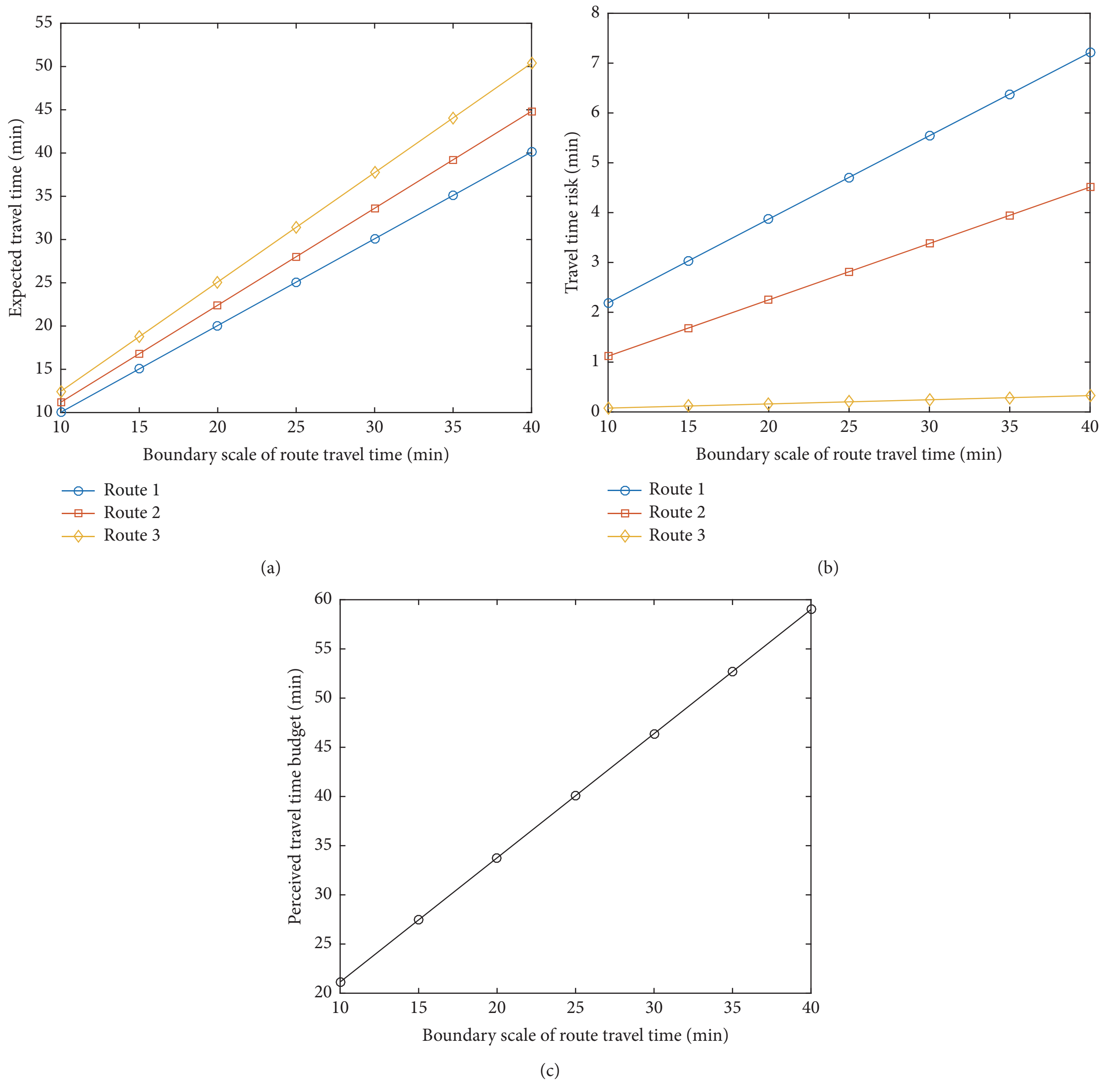

Figure 4: Expected travel time, risk, and perceived travel time budget on three routes under different boundary scale of route travel time. (a) Expected travel time, (b) travel time risk, (c) perceived travel time budget.

see that the perceived TTB decreases as θ increases. And the greater the expected on-time arrival probability ρ, the longer the perceived travel time budget. It implies that travelers with high requirement for arriving on time would make a longer budget time to avoid risk.

Then, the self-regulated averaging method (SRAM) proposed by Liu et al. (2009) is adopted to solve the model [32]. In the SRAM, the step size is $1/\tau^{(n)}$, where

$$\tau^{(n)} = \begin{cases} \tau^{(n-1)} + \Gamma, \ \Gamma > 1, & \text{if } \left\| \overline{\mathbf{f}}^{(n)} - \mathbf{f}^{(n)} \right\| \geq \left\| \overline{\mathbf{f}}^{(n-1)} - \mathbf{f}^{(n-1)} \right\| \\ \tau^{(n-1)} + \gamma, \ 0 < \gamma < 1, & \text{if } \left\| \overline{\mathbf{f}}^{(n)} - \mathbf{f}^{(n)} \right\| < \left\| \overline{\mathbf{f}}^{(n-1)} - \mathbf{f}^{(n-1)} \right\|. \end{cases} \quad (52)$$

In the MSA, $\tau^{(n)}$ is enlarged by an increment of one at each iteration. It can result in the next solution keeping farther away from the optimal solution than previous one and extremely slow convergence speed. However, $\tau^{(n)}$ depends on the information between iterations to adjust the choice of step size (i.e., speeding up or slowing down) in the SRAM. Meanwhile, the choice of the step size increment parameters Γ and γ is flexible, e.g., $\Gamma \in [1.5, 2.5]$ and $\gamma \in [0.01, 0.5]$.

Thus, we replace the step size $1/n$ with $1/\tau^{(n)}$ in (45). In addition, we assume the parameters $\theta = 1$, $\rho = 0.9$, $\Gamma = 1.5$, $\gamma = 0.5$, the maximum iteration number $N = 6000$, and the convergence tolerance $\varepsilon = 0.01$. Figure 8 compares the convergence performances of SRAM and MSA. The results

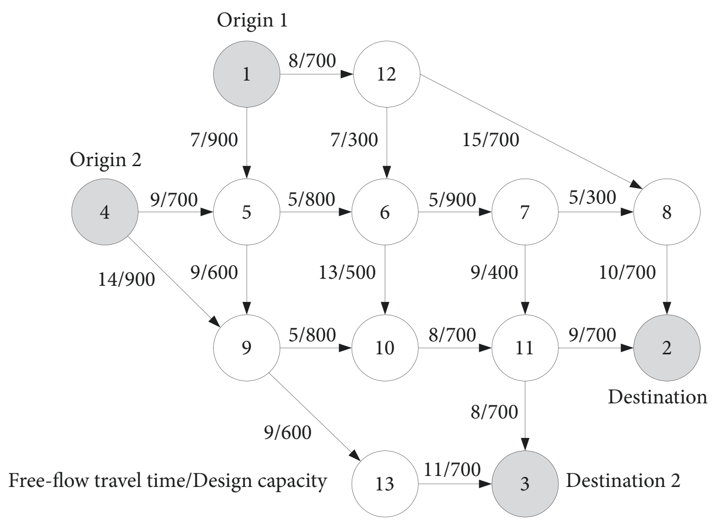

Figure 5: The Nguyen-Dupuis network and link specifications.

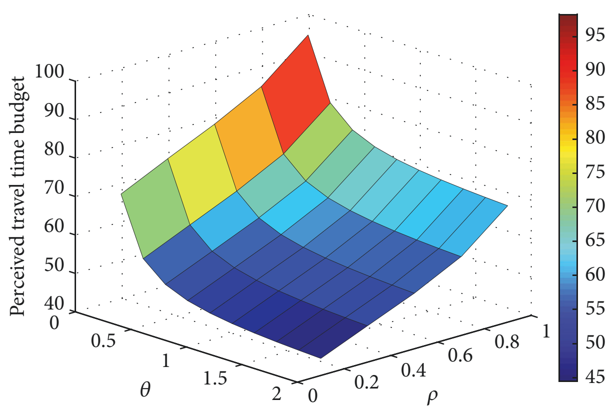

Figure 6: Joint impacts of parameters ρ and θ on perceived travel time budget (OD pair 1-3).

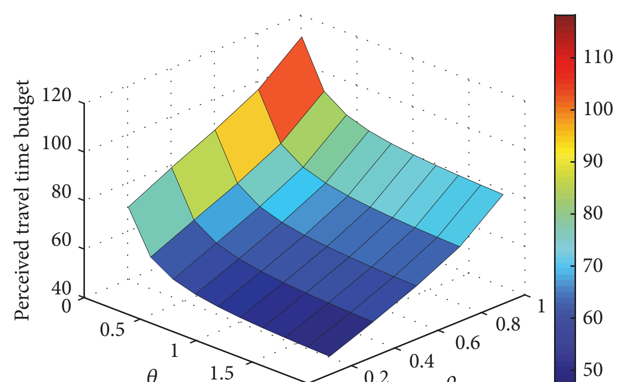

Figure 7: Joint impacts of parameters ρ and θ on perceived travel time budget (OD pair 4-2).

show that both SRMA and MSA can solve the model; the model is solvable. It is also clear that SRAM has a faster convergence speed than MSA.

7. Conclusions

This paper considers route free-flow travel time as the lower boundary of route travel time under uncertain networks. The travel time budget model with route travel time boundary is developed and derived. In such case, the budget time required by travelers is no less than that without route travel time boundary. Moreover, it has a lower travel time reliability under the same budget time. Travelers' risk-aversion attitude is no longer only associated with the expected on-time arrival condition, which is self-adaptive and depends on the chosen route.

To model traveler' route choice behavior when considering route travel time boundary, logit-based stochastic user equilibrium model based on travel time budget (TTB-SUE) is developed. A route-based solution algorithm is designed

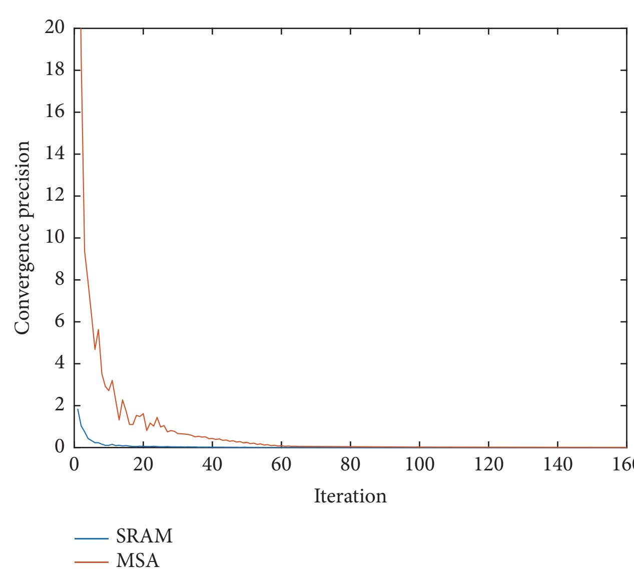

Figure 8: Convergence of the algorithms.

to solve the equivalent VI problem of the proposed model. The results indicate that route travel time boundary has a great impact on travelers' expected travel time, travel risk, and perceived travel time budget.

It should be noted that the model can be further extended to model its effects on mean-excess travel time (METT) proposed by Chen and Zhou (2010) [13] and bounded rational confidence level (BRCL) proposed by Sun et al. (2017) [33]. It can be also introduced and incorporated into dynamic travel assignment model. It would also be interesting to model travelers' risk-averse route choice decisions with multiclass users. These issues will be settled in the further research.

Data Availability

The experiment data in the submitted manuscript can be acheieved by running the program source of numerical example. The experiment program is written by Matlab 2015.

Conflicts of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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